Journal of Humanistic Mathematics

Volume 12 | Issue 1 January 2022

The Problem of Words: Learning to Teach Mathematics When Numbers and Languages Mix

Gladys Krause
William & Mary

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Gladys Krause

School of Education, William & Mary, Williamsburg, Virginia, USA
ghkrause@wm.edu

Synopsis

In this piece I propose a perspective shift, from a simplified view of mathematics story problems to a more academically rigorous perspective that integrates mathematical proficiency and language practices crucial for educating bilingual students. The data presented in this article provide a window into what preparing bilingual pre-service teachers to teach mathematics might involve. I discuss issues that arise in the context of preparing Spanish-English bilingual pre-service teachers in a way that can inform their practice in linguistically and culturally diverse classrooms.

Keywords: *bilingual education, pre-service teachers, elementary education, mathematics education*

Sandra was a bilingual pre-service teacher (BPST) enrolled in my bilingual elementary mathematics methods course at a university in the southern United States. She was part of a cohort of 22 bilingual pre-service teachers seeking certification in elementary bilingual education. Sandra and most of her peers in the program were bilingual in Spanish and English and received all of their formal education in English, their second language. Spanish was a language they spoke only at home and used less and less often as they advanced in their education. By the time they arrived in my class, the majority of the BPSTs favored English over Spanish, at least in the classroom context, and it seemed evident that Spanish had become their second language.

As part of my math methods course, the BPSTs were required to create a series of word problems in both Spanish and English for the elementary students they would work with during the semester. In composing the problems, they were instructed to include three number choices, allowing for differentiation in the level of difficulty of the problems. In addition, they needed to provide a justification of why each number choice was a good fit for both the problem and the particular student with whom they were working.

The following was one of the problems written by Sandra:

> Selena has her own slime business. Last week, she sold _____ batches of slime, and this week she sold _____. How many batches of slime did Selena sell all together?
>
> *Selena tiene su propio negocio de slime. La semana pasada, vendió _____ contenedores de slime, y esta semana vendió _____. En total, ¿cuántos contenedores de slime vendió Selena?*

The number choices Sandra chose for the problem were $(12, 7)$, $(31, 19)$, and $(200, 130)$. She reasoned as follows for the appropriateness of these choices:

> *Para la primera opción, escogí 12 y 7 porque son fáciles de sumar, y quiero que se sienta cómoda en empezar a trabajar conmigo. Siento que para esta opción va a usar conteo de dedos, ya que son cantidades chicas. Para la segunda opción escogí 31 y 19, que son un poco más difícil. Para esta opción yo creo que lo va a resolver como:*
>
> $$\begin{array}{r} 31 \\ +\,19 \\ \hline \end{array}$$
>
> *Estos números los escogí porque requieren que reagrupe. Para la última opción escogí 200 y 130. Aunque parece ser más fácil que la segunda opción, en su entrevista la estudiante me dijo que sumar números con valor tan grande es algo que se le dificulta mucho.*

> [For the first option, I chose 12 and 7 because they are easy to add, and I want [the student] to feel comfortable in starting to work with me. I feel that for this option [the student] is going to use her fingers to count, since they are small quantities. For the second option I chose 31 and 19, which are a bit more difficult. For this option I think [the student] will solve it like:
>
> $$\begin{array}{r} 31 \\ +\,19 \\ \hline \end{array}$$
>
> These numbers I chose because they require regrouping. For the last option I chose 200 and 130. Although it seems to be easier than the second option, in her interview the student told me that adding numbers with such a large value is something that is very difficult for her.]

A number of questions came to my mind as I read the problem and her reasoning.

What is *slime*? Is there a word in Spanish for *slime*? What is a *contenedor*? As a native Spanish speaker myself, I noticed these two words the first time I saw the problem.

Regarding Sandra's number choices, why are 12 and 7 easier to add? If Sandra anticipated that the student would use her fingers to add the first number set, why didn't she also expect the student to use her fingers to add the second? If it is difficult for the student to add three-digit numbers, why did Sandra give her an addition problem with three-digit numbers? How did Sandra know these numbers were a good fit for the student with whom she was working?

The purpose of this article is not to answer the questions above. Rather, in this article I will explore two themes that arise when searching for answers to the questions above: (1) word choice, and (2) selection of numbers when writing word problems for elementary school students.

My intention is not to present a generalization or recipe for teaching; as Barwell [3] observed, "[l]anguage, learning and society are too complex for that" (page 2). Instead, I propose shifting from a simplified view of mathematics

story problems to a more academically rigorous perspective that integrates mathematical proficiency and language practices crucial for educating bilingual students. Sandra's problem provides an exemplary window into what preparing BPSTs might involve. In this paper I discuss issues that arise in the context of preparing Spanish-English bilingual pre-service teachers in a way that can inform their practice in linguistically and culturally diverse classrooms.

Research and policy have repeatedly called for mathematics instruction for bilingual students that maintains high standards and high cognitive demand [21, 23]. However, the presumption that instruction will take place in English only still underlies this call. Moreover, while researchers agree that the goals of high-cognitive-demand instruction and high standards for teaching and learning mathematics should apply to bilingual students and the preparation of future bilingual teachers, few studies have addressed certain common issues arising in practice in a way that offers practical and executable solutions. This paper intends to make a small contribution in this direction.

1. A Word on Terminology

Throughout this article I use the term "bilingual" to refer to students, classrooms, pre-service teachers, and teachers who speak both English and Spanish, and "Spanish-English bilingual classrooms" to describe classrooms that intentionally serve the needs of Spanish-speaking bilingual students [12, 18]. I use the phrase "second language" to refer to students, teachers, or pre-service teachers actively engaged in learning English (if Spanish is their first language) or Spanish (if English is their first language). Finally, I use the term "Latinx" instead of "Latino" to be more inclusive of diverse genders and identities in referring to any person of Latin American descent residing in the United States.

2. Background

The research literature identifies word problems as the task most investigated by researchers interested in the teaching and learning of mathematics with bilingual learners [1, 4, 9, 22]. Barwell [4] notes that this interest positions word problems at the intersection of mathematics and language use.

He describes this intersection as demanding a higher level of thinking from bilingual students, who need to balance specific linguistic forms (e.g. vocabulary such as *slime* and *batches*, but also more complicated grammatical structures) with an understanding of different scenarios (e.g., is "selling batches of slime" a culturally salient situation for the student?) in a mathematics context.

Teaching mathematics through word problems has become increasingly valued as an approach to mathematics instruction [5, 16, 24], mainly because using a problem-based approach suggests that students can make connections between mathematical ideas that are familiar to them by solving these problems in a variety of ways. However, the complexity involved in parsing the linguistic form used in the statement of a particular problem while making connections with the relevant mathematics is not addressed in the theory supporting problem-based instruction. Moreover, the complexity involved in composing problems that balance these considerations has not been discussed as broadly in the research.

To chip away at this complexity, educators have recognized the need to identify the knowledge and skill base required to prepare early educators to work effectively with bilingual children [10, 26]. Zavala [25] states, "as teacher educators, we need to strive towards helping our pre-service teachers develop the ability to focus on the math content and equitable participation of students from different racial and linguistic backgrounds simultaneously, and give them tools to do so" (page 63). However, research discussions of practical tools for this skill base are few and far between.

My work here, focusing on bilingual issues surrounding number choice in word problems, responds to this need. Mathematics educators must provide BPSTs with the tools to develop mathematics word problems that contribute effectively to preparing the bilingual students they serve. Because of the bilingual nature of this work, language is naturally foregrounded. However, language is not the only factor we need to consider when learning how to write mathematics word problems. The numbers in the problem also play an important role in determining the content and level of difficulty that these word problems are meant to address [6, 17].

Land [17] identified four components of creating story problems; these are: (a) deciding what mathematics the problem should address, (b) selecting a type of problem, (c) choosing a story context, and (d) choosing numbers.

Each component can influence how accessible a problem is for a student as well as the mathematics the problem addresses. However, when teachers plan problems for instruction, they may or may not attend to these components; if they do, their decisions may be informed by a variety of considerations.

In this paper, I analyze the considerations two cohorts of BPSTs identified as influencing how they wrote story problems and the numbers they selected for such problems. The problems I discuss are drawn from the BPSTs' classroom work over the course of a semester, their reflections after writing the problems and interviewing students, and the justifications they provided for their own instructional decisions in selecting the problem type and numbers.

3. Classroom Setting and Writing Problems

The problems discussed here come from a course I taught during Spring 2018 for two cohorts of BPSTs. One class had 22 students, 20 of whom identified as Latinx or Hispanic and two of whom identified as White. The other class had nine students, seven of whom identified as Latinx and two of whom identified as White. All 31 students were Spanish-English bilingual, a requirement to enter the program. I taught this class completely in Spanish.

In my course we explored what it means to learn and teach mathematics with understanding and how we can help students from diverse cultural, racial, social, and linguistic backgrounds appreciate its beauty and power. I placed particular emphasis on how children think about mathematics and on learning to use what we know about children's thinking to design and adapt instructional tasks. We also discussed the roles of students and teachers in the classroom and how to foster an equitable classroom environment that encourages rich discussion of mathematics.

The required textbook was *Children's Mathematics: Cognitively Guided Instruction* [7], I also used excerpts from *The Impact of Identity in K-8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices* [2]; *Elementary and Middle School Mathematics: Teaching Developmentally* [24]; and *Beyond Good Teaching: Advancing Mathematics Education for ELLs* [8], as well as articles from various journals. In preparing to interview students, BPSTs viewed video examples of clinical interviews; read a chapter on conducting clinical interviews [13]; and had substantial conversations about problem types, students' mathematical strategies, and how teachers can elicit mathematical thinking through questioning.

The set of problems discussed here comes from an assignment in this course that required the BPSTs to write a set of problems for a target student at their placement. They were asked to explain their reasoning for the problem type, the context, and the numbers they chose for that particular student. They were instructed to write the problem in both English and Spanish.

In the process of creating word problems, language, culture, and content can quickly become tightly intertwined. I chose to present this particular set of problems to illustrate how this intertwining occurred commonly in the context of my class. Although my students represent only a small fraction of the broader corpus of general BPSTs, they nevertheless provide exemplars of certain types of difficulties and their possible resolutions.

4. Analysis of Language and Mathematics Content

4.1. Use of Language to Develop Mathematics Tasks

Hiebert *et al.* [15] identified three components of word problems that generally make problem-based teaching productive: (1) the problem is accessible to a range of students, (2) the problem can be solved in different ways, and (3) the problem addresses worthwhile mathematics. Sandra's problem addresses worthwhile mathematics. It is a Join Result Unknown [7] problem appropriate for the grade level and student with whom she was working. The problem can be solved in different ways, inasmuch as the student can use multiple strategies to solve the problem.

The accessibility of the problem, however, could be affected by its contexts. Slime, that sticky substance you can mold and play with that seems to be popular among elementary school children, is not called *slime* in Spanish. A search of a popular Spanish-English dictionary [19] representing regional Spanish variations from countries in Central and South America brings up the words *limo*, *baba*, *lodo*, *cieno*, or *légamo* as possible translations for *slime*. In other words, generally slime is not the word used in Spanish to describe slime.

Moreover, an informal survey of people from Perú, Colombia, Panamá, El Salvador, México, and Chile regarding what they understand from the words *limo*, *baba*, *lodo*, *cieno*, or *légamo* yields a diverse range of referents, such as dirt, saliva, bog, etc. In the context of the word problem, then, *slime* serves as a coinage: it has no meaning beyond Sandra's intended referent, *slime*.

Moreover, an attempt to translate the intended concept with a more current and widely used Spanish term yields a multitude of interpretations or meanings depending on the particular variety of Spanish a speaker employs, such that the resulting impression given to the speaker might not fit with the intended content of the problem. That is, translation even into a more "standard" Spanish can itself impose an obstacle to understanding.

In his study on using word problems for mathematics instruction in the bilingual classroom, Barwell [4] found that students, when assigned the task of creating word problems, drew on their personal experiences in writing them. In my experiences working with bilingual pre-service teachers, I see them using their personal experiences as well. In the example above, the word problem Sandra created uses a mixture of two languages, but it seems that *slime* is being interpreted as a word Sandra uses in Spanish as well.

For this particular group of BPSTs, despite being Latinx, their schooling has been influenced heavily by the English-language-dominant culture and many may no longer be as familiar with the culture many of their students bring into the classroom. That is, as BPSTs in the southern United States, they will likely work with Latinx students whose cultural background has become less prominent in their own thinking. This is a particularly important observation for educators of bilingual teachers in the U.S. The structure we currently have for preparing our bilingual teachers is founded on what a monolingual and monocultural group of students need.

As a field, we currently have few resources that speak to the needs of bilingual multicultural students in mathematics instruction. The components of word problems identified by Hiebert *et al.* [15] are important; however, because they are also broad they mask the complexity of the language when designing word problems. For example, consider the following problem written by Lucía:

> *Rubí ayudó a lavar 24 platos a su mamá, para celebrar el cumpleanos de su hermanito Axel. Ella colocó 6 platos en cada mesa. ¿Cuántas mesas pudo llenar con los platos?*

Lucía translated the problem as follows:

> Rubí helped to wash 24 dishes for her mother to celebrate her little brother Axel's birthday party. How many tables could she set with plates, if she used 6 plates for each table?

The word *llenar* is not the equivalent of set, at least in that particular context: it is more suggestive of "filling in." To a native Spanish speaker that can make a big difference, particularly if students are following the action stated in the problem to directly model the action it describes. That is, the act of "filling in" does not seem to match the act of "setting" a table.[1]

4.2. Selecting Numbers to Develop Mathematics Tasks

In a study of number choice for equal sharing fraction problems, Empson, *et al.* [11] describe the specific numbers in Equal Sharing problems as a factor that can influence (a) the accessibility of a problem for a child, (b) the knowledge a child might use to solve the problem, and (c) the mathematics that can be addressed. In an earlier study, Greer [14] described how the numbers in elementary story problems have a significant influence on which arithmetic operation is selected in solving the problem.

Let us once again look at Sandra's number choices for the problem she wrote. She stated that for 12 and 7, she would expect the student to use her fingers to add the numbers. This information is supported by her observations in the classroom. In her explanation for 31 and 19, however, she anticipated that her student would use the standard algorithm of addition, also noting that the student would be required to re-group here.

Sandra's second number choice is quite appropriate. The numbers are well chosen because the student can "change" the the first number from 31 to 30 by removing a 1, add 19, and add the 1 back, or change the second number 19 to 20 by adding a 1, then add 31 and subtract a 1 to balance things out. In either case, the operations the student needs to perform are very similar, but the student has an opportunity to solve the problem without using any standard procedures if he or she does not remember one or the other of them. Also, the numbers provide space for the student to compose and decompose them in a mathematical way that can be used to support her mathematics understanding.

Sandra stated that her last number choice (200, 130) was based on her student's statement that adding larger numbers was difficult for her.

[1] Perhaps a pairing that a student might more easily model would be *set* ∼ *arreglar* ("arrange, order"), or even *set* ∼ *alistar* ("ready, prepare").

She provided no additional detail for this justification. In my experience working with pre-service teachers, this justification is common. It is based on a simple axiom: Because this concept is difficult, I will give students more exercises on this concept.

However, *why* the concept is difficult for the student is not something pre-service teachers typically pay attention to. Generally, they make instructional decisions based on what students do not understand or what they find difficult. However, this focus will eventually make the process of instruction more challenging because the root of the difficulty has not been identified. In that sense, focusing on the knowledge the student might need to solve the problem will provide a more accurate justification and result in a more productive interaction.

I want to highlight here the importance of ensuring that pre-service teachers consider this full line of reasoning and remain aware of the broad range of possible strategies. They should also plan for how they might proceed in light of strategies they have never considered as they engage in the process of learning about writing word problems. This challenge is not specific to BPSTs. However, it is critical for teacher preparation programs that focus on preparing BPSTs to recognize that language preparation needs to go hand in hand with pedagogical practices for teaching mathematics. The two must be incorporated into one unit.

5. Conclusions

The exercise of engaging BPSTs in writing their own word problems provided two crucial lessons. First, it enabled them to identify the role that context and culture play in language use when writing mathematics word problems. For example, consider how the mere change from *slime* to *baba* (saliva) in the Spanish version of Sandra's word problem could have affected the context of the problem. The BPST would certainly have needed to change the problem in the Spanish version. The task of writing the problems in both languages promoted language learning and cultural awareness in the particular context of my classroom. These are in fact principal objectives of my work in preparing BPSTs. We had an opportunity to talk about the multiple meanings for the translation of *slime*, and we understood the impact a single word could have on the course of a well-planned mathematical task.

Second, this exercise allowed BPSTs to recognize the importance of selecting the numbers in the problem. Number selection can influence the operations children use to solve a problem, demonstrating the knowledge they possess of a particular concept. Going back to Sandra's number choices and her reasons for selecting them, if larger numbers are more difficult for a student to add, the student could potentially decompose the numbers into a set of smaller numbers in order to solve the problem. In this way, the knowledge and understanding students possess is evidenced through the approach they select, which is influenced by the BPST's strategic selection of the numbers.

In 2018, the National Academies of Sciences, Engineering, and Medicine identified practices that have long been recognized as ineffective in, and in fact detrimental to, fostering student engagement in STEM-related content, yet which teachers are still utilizing in working with students whose first language is not English [20]. The same report noted that this finding was unsurprising given that teachers continue to be ill-equipped to support the learning experiences of this student population. If we already know this, what is delaying a change?

The ideas shared here suggest the need for a conversation in which we study more rigorously what must be required in teacher preparation programs, in particular those programs that prepare BPSTs. Becoming a bilingual teacher is a knowledge-intensive task that requires integrating both the mathematical proficiency and the language practices that are fundamental to educating bilingual students. I encourage the bilingual mathematics education community to build on the ideas presented here to develop instructional practices that support the teaching practices of BPSTs in ways that equip them to work effectively with Spanish-English bilingual students.

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