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Makers Do Math!
Legitimizing Informal Mathematical Practices Within Making Contexts

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Abstract

In this paper, we argue that making activities within non-formal learning environments (e.g., museums, libraries) provide opportunities to engage youth in what we define as mathematical practices for making, everyday mathematical practices within the context of making activities. The mathematical practices identified from two non-formal school-based contexts highlighted three mathematical practices for making: informal measurement, spatial reasoning, and curiosity. These practices are identified in prior scholarship as being beneficial and foundational for the understanding of mathematical concepts. As educators and researchers turn to non-formal and informal contexts, with an eye toward understanding ways youth engage in the activity of making, descriptions of mathematical practices for making build upon prior everyday mathematical practices and open up a new landscape of inquiry.

Keywords: *mathematics, making activities, informal measurement, spatial reasoning, curiosity.*

1. Introduction

> . . . because our parents and our grandparents created the world's largest economy and strongest middle class not by buying stuff, but by building stuff — by making stuff, by tinkering and inventing and building; by making and selling things first in a growing national market and then in an international market . . . [75]

As exemplified in this quote, making is not a new phenomenon; yet, in the 21st century, researchers have explored its potential to engage youth in the process of "designing, building, modifying, and/or repurposing material objects, for play or useful ends, oriented toward making a 'product' of some sort that can be used, interacted with, or demonstrated" [58, page 3]. Scholarship has documented the practices gained by youth through the processes of creating a prototype or model of a product (i.e., making) such as (a) developing creative thinking and problem-solving skills [6, 59]; (b) persisting through constraints and challenges [85, 112]; and (c) collaborating with peers toward a shared goal [54]. While making integrates and encompasses a range of disciplines such as art, engineering, science and technology [5, 88], less is known about youths' engagement in mathematics in making-related environments or making spaces. The majority of mathematics education and scholarship is grounded in formal learning environments [79] or implemented less frequently in out-of-classroom contexts than other disciplines such as technology and science [18].

School and academic mathematics can be defined as practices of students and teachers and practices of academic mathematicians, respectively [68]. More specifically, school mathematical practices are described as those practices used by mathematically proficient students [82]. For example, mathematically proficient students should be able to apply their knowledge and understanding of mathematics to solve problems situated in everyday contexts [20]. Yet, students often have narrow views of how they use mathematics outside of school, as their perceptions of mathematics are grounded in school activities such as calculating sums and estimation, as opposed to mathematical practices such as ways of thinking and sense-making [36, 63]. Additionally, Nicol [74], Stevens [94], and Nasir and Hand [71] argue that focusing on school and academic mathematical practices rather than everyday mathematical practices limits students' learning of mathematics and may lead to deficit labeling of students.

More broadly, school mathematics too often defines what counts as mathematics [72], which, for many then, is narrowly focused on a set of standards that must be met through successful performance on tests or sanctioned mathematical practices. However, we do know that the knowing and doing of mathematics outside the "norm" is consequential for mathematics participation, identity, and sense of belonging [31, 44, 76].

Given the above context, Arcavi [2] concluded that more research is needed to discover and uncover mathematical practices in different situations and environments; particularly, situations that may seem non-mathematical to students and educators. We contend that broadening our views as to what constitutes a mathematical practice will push us as mathematicians and educators to think about where and how learning happens; where and how students are afforded opportunities to explore mathematics and engage in mathematical practices within cultural and authentic contexts [24, 26, 39].

In this paper, we consider environments characterized as non-formal [30], specific contexts providing structured, educator-directed making activities that offer opportunities for youth exploration.[1] These making activities are typically not designed to engage youth in mathematical practices or knowledge construction and application; instead they are designed with principles of making practices (e.g., collaboration) and integration of new tools and technologies (e.g., conductive tape) at the forefront.

In what follows, we argue that making activities within non-formal learning environments (e.g., museums, libraries) provide opportunities to engage youth in what we define as mathematical practices for making: everyday mathematical practices within the context of making activities. We ground this argument in Civil's [16] everyday mathematical practices and illustrate three specific mathematical practices — informal measurement, spatial reasoning, and curiosity. These practices are identified in the literature as supporting knowledge construction of mathematics through experiences and in-

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[1] Here, making refers to a specific type of activity people engage in at specific locations, sometimes called "makerspaces'. The latter are usually "collaborative work space(s) inside a school, library or separate public/private facility for making, learning, exploring and sharing that uses high tech to no tech tools. These spaces are open to kids, adults, and entrepreneurs and have a variety of maker equipment including 3D printers, laser cutters, cnc machines, soldering irons and even sewing machines" (from `https://www.makerspaces.com/what-is-a-makerspace/`, last accessed on January 15, 2022.

formal meaning-making explorations in out-of-school contexts [26, 43]. The significance of this paper is the identification and illustration of mathematical practices for making that extend Civil's everyday mathematical practices to the domain of non-formal making learning environments.

We begin with defining and characterizing everyday mathematical practices and mathematical practices for making (Section 2). Next, we ground informal measurement, spatial reasoning, and curiosity in the literature (Section 3). After describing the study contexts from which we draw our examples (Section 4), we highlight the ways youths were engaged in the three identified mathematical practices while making (Section 5). We conclude by arguing for the recognition of and legitimization for youth's informal ways of thinking about and doing mathematics in a variety of learning environments through making activities (Section 6). A brief final section (Section 7) summarizes this paper, contextualizing it in the broader projects of clarifying and expanding the construct of mathematical practice and improving the general societal perceptions of mathematics.

2. Theoretical Grounding

In this study, everyday mathematical practices are defined as mathematical practices that youths and adults engage in outside of school and in contrast to mathematical practices in formal school environments [68].[2] Civil [16] characterized mathematics outside of school as (a) occurring through an apprenticeship model, (b) working on contextualized problems, (c) providing agency to the person working on the task, and (d) involving mathematics that is not apparent but hidden in the process. Additionally, everyday mathematical practices are socially valued and grounded in applied problems in everyday contexts [36]. For example, home and communal activities such as crocheting [22], basket weaving [1], sewing [40], laying carpet [61], cornrow designs in hair [27], money transactions and cooking [84], and Latinx jazz, mambo, and salsa compositions [100] have been found to engage youth in everyday mathematical practices and inform their mathematical thinking.

We consider mathematical practices for making as a specialized form of everyday mathematical practices as characterized by Civil [16]. The mathematical

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[2] We agree with Carraher [12] that framing everyday and school mathematics in opposition are problematic, but that "any pair of terms is going to raise issues" (page 27).

practices for making occur within a making space or environment and afford youth opportunities to engage in making activities that promote mathematical practices through informal measurement, spatial reasoning, and curiosity, as evidenced in our video data. Similar to Civil [16] and grounded in maker education research [7, 53], we characterize mathematical practices for making as

- occurring through social interactions in which youths and adults are teaching and learning from one another [7, 81];
- working through making activities that are contextualized and promote youth exploration, imaginative play, and/or inquiry of mathematics [33, 81];
- providing opportunities for taking an active role in personalization of the making process and/or object through mathematical strategies, tools (e.g., dynamic software, measuring tape, fingers), and/or risk taking (e.g., trying a non-traditional approach to the problem through the use of mathematics) [55]; and
- involving mathematics that may not be apparent to youths and/or educators, but hidden within the process of making as it does not resemble mathematical work in school settings [94].

Although we use this characterization of mathematical practices for making to identify opportunities to engage youths in everyday mathematical activities in non-formal making learning environments (i.e., afterschool program and free period in school), we contend this characterization is applicable to a range of making spaces.

3. Literature Review

In this paper, we describe three mathematical practices for making — informal measurement, spatial reasoning, and curiosity. In this section, we define each practice before situating it within relevant scholarship that highlights students' use and understanding of these practices as constructed in their everyday activities.

3.1. Informal measurement

We define informal measurement as intuitive and cultural approaches for determining the size of an attribute through the use of non-geometric tools and nonstandard units such as straws, feet, rope, and estimation [104]. Educators in a museum setting referred to informal measurement as intuitive precision or measurement in relation to existing pieces and objects. These educators argued that a craftmanship version of precision should be accounted for within mathematical practices.[3] As argued by researchers, the use of informal forms of measurement is beneficial and foundational for all grade levels and a practical skill we use every day (e.g., parking a car, carrying a large object through a door, distance) [37, 104]. As confirmed by Smith, van den Heuvel-Panhizen, and Teppo [93], "measurement is among the most sensible, contextually situated and practical domains of mathematics for students" (page 618).

In a measurement activity, the tool used is often determined by the social and cultural nature of the activity itself; it is linked to a purpose [62, 78]. Owens and Kaleva [78], for instance, described how a rope was employed as a tool to determine and compare the height and girth of a pig during bride-price and other recognition ceremonies of cultural groups in Papua New Guinea. A wide range of activities involved non-standard units of length — use of paces to measure the width of a garden plot, use of string to measure the circumference of shells, and use of sticks with a mark to represent some unit for the construction of a canoe. In addition, the practice of measurement is social and cultural in nature as exemplified by the unit of measurement [25, 37, 78, 95]. For example, 56 Indigenous students in Years 3 to 6 in Aboriginal schools in north-western Australia were asked, "how far is it to the [Fitzroy] river" [37]. Of the 56 students, none used a standard length of measurement such as meters, but instead used the time it would take to get (e.g., walk) to the river. This method was noted by Grootenboer and Sullivan [37] as reasonable and relevant within the context of the question and within the students' everyday experiences.

As identified in these examples, and as argued by other scholars [26, 56, 64], mathematical knowledge of measurement concepts is built through experiences and exploration in cultural and meaningful contexts, most often in

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[3] J. Barnes, personal communication, March 8, 2018.

out-of-school contexts. However, students' understanding of measurement as constructed in their everyday activities is often suppressed and de-valued in school contexts, as home and school mathematics are characterized by different social and cultural systems [25, 68]. Scholars in mathematics education have argued for the transition to formal measurement to build upon students' use of non-standard units and tools [19, 43]. Further, scholars have argued that this process be cyclical, where the stages of development between formal and informal approaches to measurement inform one another, as opposed to the application of measurement concepts showing up at the final stages [56].

In supporting and making connections between in- and out of school learning opportunities, youth "become more mathematically powerful" [56, page 30]. As such, measurement concepts should be developed through contextual open-ended questions and problems; questions and problems that are not foreign but cultural and situational, have more than one solution, and afford educators' insights into students' learning [37, 56]. Making and tinkering activity may support youths' development of measurement in that the activities are grounded in youths' interests, contextualized and situational, and afford exploration and choice [5, 81].

3.2. Spatial reasoning

Spatial reasoning is defined in this study as the ability to perform mental manipulations of visual representations, the ability to transform objects and spatial configurations into other visual arrangements, an awareness of spatial components, and the ability to discover relationships between components and new visual arrangements [3]. Examples of spatial reasoning activities include orienting, decomposing, diagramming, symmetrizing, transforming, scaling, and visualizing [111]. An extensive body of scholarship has consistently shown a strong relationship between spatial ability and success in mathematics [35, 42, 73, 99]. Individuals with high performance on spatial reasoning measures are more likely to pursue and obtain a career in a STEM field [107]. As Mix and Chen assert [66], "the relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are connected" (page 206). Additionally, more recent research has concluded that the development of spatial reasoning of young children serves as a strong predictor of later mathematics achievement [38, 106].

Spatial reasoning is often internal to individual learners and expressed through actions as opposed to verbal acts of communication [111]. Such reasoning cannot be disassociated with other mathematical concepts but instead should be acknowledged for its complex interplay with many aspects of doing mathematics [111]. Scholars have identified positive association and application of spatial reasoning with other mathematical representations, concepts, and reasoning [35, 38, 70], such as fluency in shifting between two-dimensional and three-dimensional spaces [34], geometrical foundations such as characteristics of triangles [13] and parallel lines [92], and the relationship between quantitative concepts and spatial situations such as calculating the number of squares in an array [4]. Further, research has supported the development of spatial reasoning within activities that occur across contexts (e.g., preschool centers, home) — block play [13], robotics [34], puzzles [53], perspective drawing [102], and hand tools (e.g., screwdriver) [45]. In general, these activities afford learners opportunities to experiment with spatial orientations and transformations, decomposing and recomposing figures, symmetry, and mental imagery.

Regardless of the cumulative evidence that highlights the importance of spatial reasoning and its development in young children prior to entering formal schooling, little time is dedicated to spatial reasoning in young children's early formal schooling experiences [103] and receives little attention in the mathematics standards adopted in countries such as Canada and the United States [10]. Together, this lack of attention may lead to a deterioration of children's spatial reasoning skills developed prior to formal schooling [69]. Francis, Khan, and Davis [34] even explored how requiring students to sit idle may limit the development of spatial reasoning; their research highlighted movements often characterized as fidgeting or distracting as co-occurring with formal acts and behaviors of spatial reasoning.

Similar to informal measurement, spatial reasoning skills and concepts are built through experiences and exploration in cultural and meaningful contexts [62, 111]. Therefore, school mathematics curricula should be reconceptualized to incorporate opportunities to build spatial reasoning as a means for problem solving and making decisions within cultural and meaningful daily events [24].

3.3. Curiosity

In this paper, we align with Mehta, Keenan, Henriksen, and Mishra's [65] description of curiosity as foundational to participation in STEM and building a sense of wonder.

> A cognitive-emotional desire to seek, to anticipate, and to understand and/or solve problems or phenomena. This is the intellectual equivalent of an itch that must be scratched. The desire to learn about the unknown is, arguably, a fundamental human trait. We are capable of reacting to feelings of awe, admiration, and respect with sense of curiosity that kindles a desire to seek, anticipate, and solve problems and answer questions, in essence, to understand . . . Reacting to nature, one may feel like a detective who wants to solve new mysteries. [65, page 131]

These problems or phenomena are encountered within our daily lives and experiences [77].

Curiosity can also be discipline specific [80]. For example, Weible and Zimmerman [110] described curiosity in science as three interrelated components: stretching or seeking out novel experiences and information, embracing or testing out experiences that are unpredictable, and science practices or participating in science practices for the attainment of a scientific understanding. Further, research has identified curiosity as a foundation for individuals who pursue and enter a STEM field [96, 105] and as a basis for a parent's ability to build upon and enhance their children's curiosity as children traverse schooling [38].

Human environment and social interactions support and/or hinder curiosities with significant developmental and educational outcomes [29]. As young children, and even as adults, we are naturally curious about our experiences within and observations of our world — why do leaves change their color? why do birds build nests? why might every even number greater than two be the sum of two primes? In fact, there is evidence that curiosity begins in infancy [29, 57]. However, as argued by researchers, providing opportunities for learners to be curious in formal schooling is limited at best and is a necessity that should be cultivated as opposed to being minimized [28, 29]. For example, Engel [28] found that on average curiosity was explicitly expressed by 21 elementary students less than one time every two-hour period. Engel explained this finding as situated within our current schooling model of addressing standards and evaluating student proficiency as defined through standardized testing. Leas *et al.* [52] further noted how curiosity was constrained by cookbook or procedural activities that limited authentic learning and reasoning.

Given these constraints of school -based learning, learning environments outside the formal school context may serve as an avenue to support students' interests and curiosities, opportunities that are so often eliminated in classroom settings [6, 98]. In particular, making activities are developed in such a way to foster curiosity and wonderment [48].

4. Method

The mathematical practices for making examples provided in this paper are from two prior research studies conducted in two non-formal learning environments that provided making and tinkering activities for youth in grades 3-6 [85, 87]. In particular, these making spaces were physically located in a school setting, and making activities were designed to connect to academically valued learning [97].

The first was an afterschool program developed and implemented by a local science museum located in the midwestern region of the United States. The afterschool program, entitled After School EdVentures (ASE), occurred in a local elementary school. The purpose of the program was to engage youth with STEM-enriched activities, as well as support the state's academic content and process standards in science. The program occurred twice a week for 45 minutes each day.

ASE staff developed and implemented two units, one focused on electricity/circuitry concepts and the second focused on engineering design processes. The first unit was taught in a six-week period spanning from November 2015 to December 2015 and created around utilizing innovative low-tech, yet easy to use, material and tools (e.g., conductive tape, LED lights, screwdrivers) grounded in the tenets of maker education [60]. The second unit was facilitated in a six-week period spanning from April 2016 to May 2016. The goal of this second unit was to extend youths' engagement in the iterative process and to promote collaboration among youth as the majority of activities involved a team challenge. Although these two units were overtly activities situated in the domains of science and engineering, we expected that mathematical practices for making would be used, based on research highlighting that mathematics is used in concert with other disciplines in making spaces [83], as well as Civil's [16] characteristic of mathematics being a hidden process within everyday activities.

The second context was from three classes, a third grade, a fourth grade, and a fifth grade class, in an intermediate public school (i.e., Grades 3-5) located in the northeast region of the United States. Youth from the three identified classes were invited to participate in three days of STEM-related making activities in the "TinkerLab" during a 30-minute free period at the beginning of June 2019. These activities were created and initiated by the TinkerLab teacher with the intent of promoting the application of mathematical concepts and skills.

4.1. Data Sources

Prior to collecting data, we obtained approval from the relevant Institutional Review Boards. We informed caregivers of the study and asked them to return an informed consent form if they wished to opt their child out of the study. We also explained the study to the children involved and asked for daily assent before they were allowed to volunteer to wear a chest-mounted GoPro camera. Our use of GoPro cameras was intentional; these cameras are equipped with a wide-angle lens, providing an expansive field of view of each individual's engagement with tools and material, as well as interactions with facilitators and peers. This also allowed the research team to view the data from the perspective of the children themselves as opposed to relying on a stationary or researcher-held camera [11].

Within the first study, we accumulated 54 videos from 11 youth participants in the electricity/circuitry unit and 47 videos from 8 youth participants in the engineering design unit. Each video was 20 to 45 minutes in length. In the second study, we collected 14 videos of the first activity, 14 videos of the second activity, and 6 videos of the third activity. Each video was approximately 15 to 22 minutes in length.

4.2. Description of selected mathematical practices for making

The mathematical practices for making came out of the results of a study that aimed to use the video data to look for and examine incidences of school mathematics practices as defined by the eight Standards for Mathematical Practice in the Common Core State Standards [20]. Throughout our coding process, we documented other instances in which we each perceived youth to be engaged in mathematics practices not necessarily grounded within the eight standards for mathematical practice. We did not establish criteria for these "other" instances beforehand as they were not the focus of our initial analysis and instead were grounded in our own understanding of mathematics. At the conclusion of the initial analysis, we met to discuss our codes, including instances in which we coded for practices not aligned with the eight mathematical practices defined in the Common Core [20].

It was apparent that what we observed as practices were not aligned with the standards for mathematical practice. For example, one of the making activities was creating and using tightly rolled sheets of paper (i.e., beams) to create a structure that would hold the most composition notebooks. One youth, Kaylee, decided to build a cube. We observed Kaylee place one arm horizontally across the tops of two vertical beams perceived to be straight. She "marked" this length on her arm before placing a beam across her arm and cutting the same length as her marked arm. Our memo on this moment noted "Kaylee measured the length of the distance between the two pieces of paper using her arm, a tool at her disposal, but not one necessarily recognized for its precision and accuracy; does not produce a standard unit of measurement. What would Kaylee say? It is the length from this vein to this mole on my arm."

Such memos motivated the team to consider what we noticed in the data that resonated with prior research. Specifically we wanted to connect what we were seeing with the mathematical ways of understanding concepts as experienced in daily and cultural contexts. We revisited the video data with the following question in mind: In what ways do youth engage in mathematical practices for making in non-formal learning environments? The mathematical practices for making that we developed were derived from our alignment of the practices in the literature and practices we identified youth using through making activities.

5. Insights

Through our analysis, we observed youth engaged in the following everyday mathematical practices for making within the non-formal making programs: informal measurement, spatial reasoning, and curiosities. These mathematical practices for making were not observed within isolated activities, but spanned the video data set. In this section, we describe and share evidence or examples of each mathematical practice for making [12]. The intent of the examples is to illustrate and characterize the ways that youth engaged in the doing of and thinking about mathematics within different making activities.

In addition, the examples were chosen based on quality of the videos and capturing images. Pseudonyms are used throughout the manuscript.

5.1. Informal measurement

Children were observed using their bodies as a form of measurement in the making activities. One example is from the first engineering design making activity developed as part of the ASE afterschool program. Youth were tasked with constructing 3D structures/models from nets. Daniel first created a house by folding the nets for a cube and a triangular pyramid (see in Figure 1A). He decided to add a garage; yet, there was no more available nets for a cube. Daniel placed his thumb on one vertex of the house and his index finger on an adjacent vertex to measure the length of one edge of a cube. As observed in Figure 1B, he then used this length measurement, represented by the space between his thumb and index finger, to draw lines segments of a net. The utilization of a non-standard unit of length was reasonable to Daniel within the context of the making activity and was a tool that was readily available as youth were not provided with a ruler or another standard measuring tool.

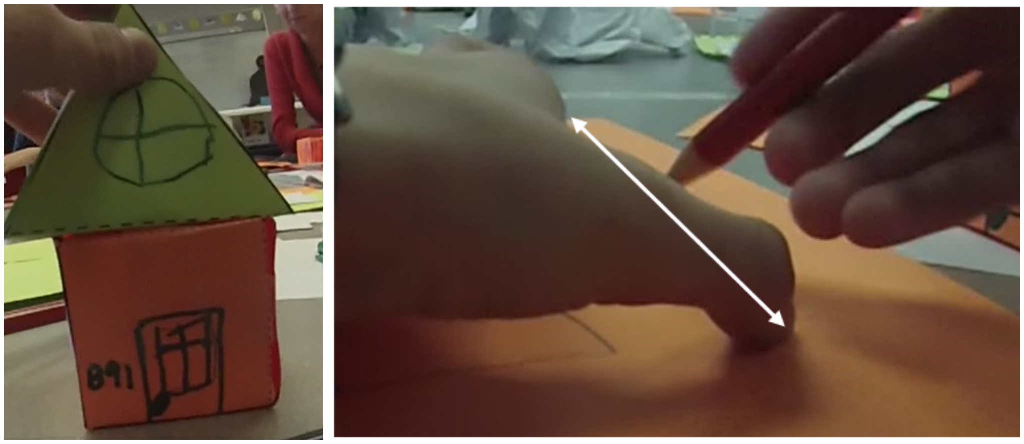

Figure 1: A) Daniel's house using two nets. B) Daniel is using the length between his thumb and index finger to draw a line (indicated by white arrow) the approximate length of one edge of the cube.

The use of body parts was not the only non-standard mathematical tool used within the making process [78]. Youth were observed using object-to-object comparison as a form of measurement, which has been found to be a common practice of tradespeople (e.g., carpet laying [62]) and referred to by museum educators as an intuitive form of measurement.[4] The majority of these instances were observed within the engineering design processes unit. For instance, youth were challenged with building the tallest tower using ten pieces of spaghetti and ten small marshmallows. Janelle was observed using spaghetti pieces as a "ruler." As illustrated in Figure 2A, Janelle aligned one spaghetti noodle to a piece in her prototype, a piece situated between two marshmallows. She broke the spaghetti noodle into two pieces, one piece being approximately the same length of that measured, and included the measured piece in between two marshmallows (see Figure 2B and Figure 2C) before incorporating them into her prototype.

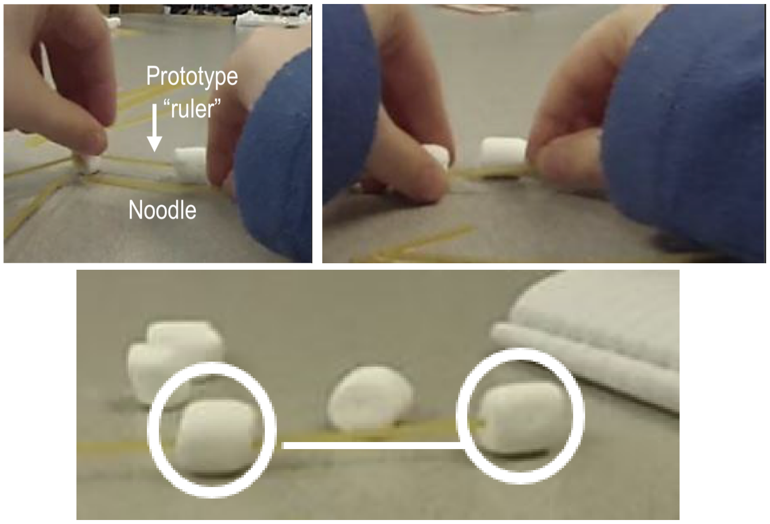

Figure 2: These figures show Janelle's use of a spaghetti noodle for object-to-object comparison in the construction of a tower.

Some children also used an object-to-object comparison method in building a paper rollercoaster or a marble run. To suspend the rollercoaster, they constructed supports (i.e., triangular prisms) by folding 8.5-inch by 11-inch sheets of paper into fourths. They were often observed aligning rollercoaster supports and cutting the supports at the same height. As observed in Figure 3A, Allie aligned a 11-inch tall support to a shorter support and used scissors to create supports of the same height (see Figure 3B). Some youth used this

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[4] J. Barnes, personal communication, March 8, 2018.

technique of comparing the support heights to determine an appropriate height of additional supports so that the rollercoaster sloped downward (see Figure 3B and Figure 3C).

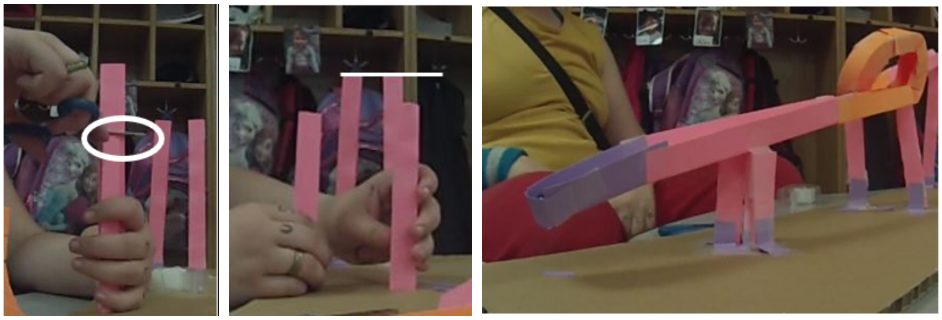

Figure 3: Object-to-object comparison of roller coaster supports.

In a last example of informal measurement as an everyday mathematical practice for making, we observe what Bright [9] referred to as measurement estimation, which is a physical measurement in the absence of a standard instrument such as a ruler. During a making activity in which youth were asked to construct an object from folding $1'' \times 11''$ strips of paper into shapes (e.g., circles, rectangles), we first observed Colin estimating one-third the length of the strip of paper as marked by his thumb in Figure 4A. Next, he folded the strip of paper in half (see Figure 4B), then slid one end of the strip of paper until about one-third of the length from the top or other end of the strip of paper (see Figure 4C). We believe this length is about the same place in which Colin marked this estimation with his thumb.

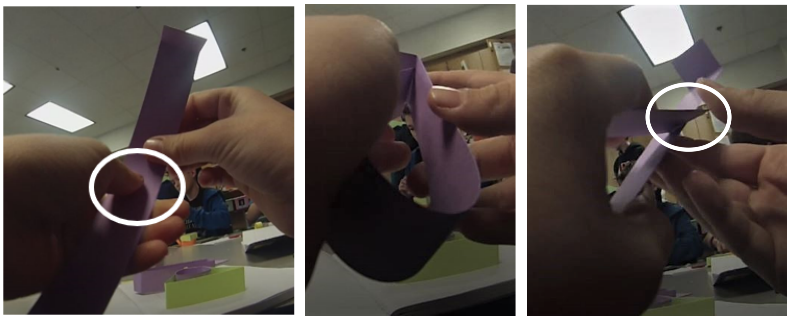

Figure 4: Colin's use of measurement estimation to fold a strip of paper about one-third from one end.

Spatial Reasoning

Making activities in the informal learning settings afforded youth opportunities to manipulate or transform an object or tool, actions that are representative of spatial reasoning [111]. For example, the first day in the TinkerLab was free-making time. Youth had access to a range of material, resources, and tools (e.g., plastic cups, tape, Legos) to make a physical and/or digital object. Marvin chose to create an object from pattern blocks. The resulting object was symmetrical. The concept of symmetry is foundational in spatial reasoning [91]. In this example, after laying the triangle pattern block on the table (Figure 5A), Marvin used his thumb and index finger to rotate the pattern block in Figure 5B and Figure 5C. He then fit the triangle in the space between a square and trapezoid (Figure 5D).

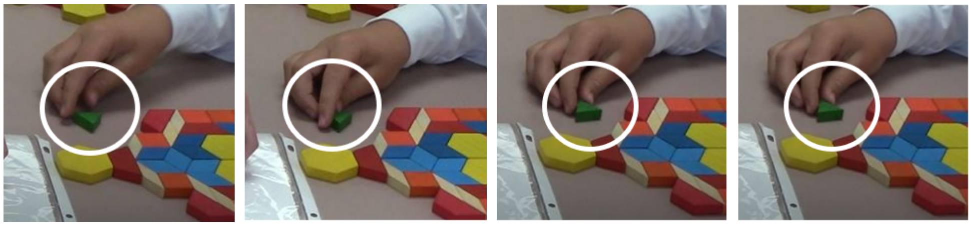

Figure 5: Marvin's transformation of a pattern block.

This pattern block is outlined with a black triangle in the final design (see Figure 6). Martin transformed the block by rotating it to align the angle of the triangle with the angle created by the square and trapezoid. To accomplish the symmetric pattern, Marvin likely created a mental construct — a vertical line of symmetry — about which his pattern block was reflected.

Figure 6: Marvin's pattern blocks creation.

Another example of the use of rotation was observed when children were asked to take apart electronic devices (e.g., cd player, keyboard, toy flashlight) to discuss circuitry and repurpose components into a piece of art. Screwdrivers were the most common tool utilized. In operating the screwdriver, children had to "take into account the spatial relation between the handle and the artifact's functional end," as well as "whether these two components lie in the same spatial plane (e.g., aligning a flat-head screwdriver into the groove of a screw)" [45, page 228]. As Jung and colleagues [45] point out, using such handheld tools requires translating the screwdriver to an object (i.e., electronic device) while also orienting or rotating the screwdriver in relation to the screw.

In Figure 7, Billie is observed taking apart a cd player. In Figure 7A, she is orienting the head of the screwdriver into the head of the screw. Similarly, in Figure 7B, Billie attempted to orient a wire cutter under a set of wires. This took several tries as the wires were securely attached to the blue plastic cover. As this example illustrates, Billie demonstrated spatial reasoning through anticipating where to position and orient the tool (e.g., screwdriver) to then produce physical activity that resulted from the anticipated orientation and position of the tool.

Figure 7: Billie's orientation of two tools — screwdriver and wire cutter — to take the electronic device apart.

A last example is from the circuitry unit developed as part of the ASE afterschool program and exemplifies how visualization of space is utilized by youth in making activities in anticipation of where to place an object [3]. Youth were asked to build something using LEGOs and magnetic bits (i.e., littleBits, [51]) that snap together to form a circuit. Travis was building a rectangular structure to hold a fan Bit (see Figure 8A). Throughout the construction of this structure, Travis did not measure the length and width of the fan Bit nor hold the fan Bit in place and build around it. Instead, Travis visualized the amount of physical space needed in his LEGO construction for the fan Bit to fit and be supported upright. For example, Travis attempted to place the fan Bit inside the rectangular structure, but the fan Bit sat on top. He stated, "I forgot I've got to make it wider … Every time I have to make it wider and wider." Travis then proceeded to remove the purple LEGO piece to widen the structure (see Figure 8B). In the next instance, Travis stated, "Is this wide enough? Yes, I made it too wide." Notice the gap between the fan Bit and the blue and green LEGOs in Figure 8C, illustrating a failed attempt of visualizing an appropriate amount of space.

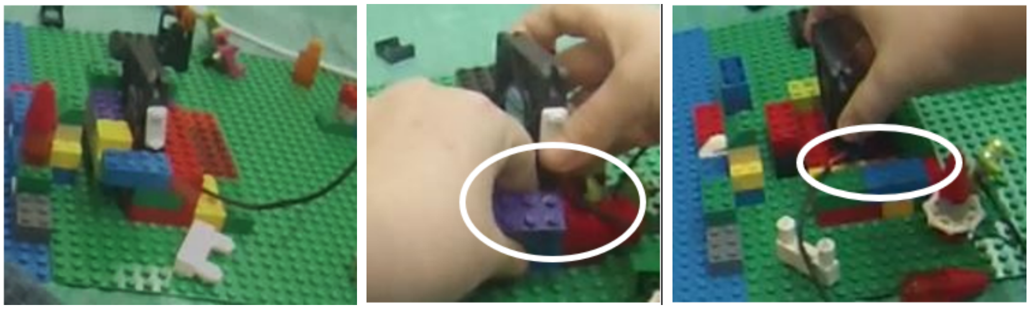

Figure 8: Travis's attempts to create a "perfect" Lego structure to house a fan Bit.

Curiosity

In the contexts we observed, children engaged in making tasks that became a springboard for posing questions of curiosity [65]. These questions were, on the surface, not grounded in school mathematics or in formal ways of thinking about mathematics; the mathematics was hidden within the process of exploring their questions through making [94]. This is in contrast with viewing mathematical curiosity as posing questions upon completion of an "interesting" problem [47].

Many examples of curiosities we observed in this study can be characterized by the asking of a "what if" question. This is a practice associated with in-the-moment puzzlement and wonder, deeper thinking, exploration, and motivation and interest in a particular concept [14, 15, 80]. For instance, in the ASE program, youth were provided with plastic cups and challenged with building the tallest tower in the shortest amount of time. At the conclusion of the challenge, Kellie decided to "build around herself." She began enclosing herself in a circular tower of plastic cups, yet the tower was extending vertically as opposed to completely enclosing Kellie. After some time, Kellie posed a serendipitous question, "What if we bring the cups in a little bit on every level and make a little roof?" In other words, Kellie became interested in and curious about constructing a hemispherical dome as a roof.

At another instance, near the end of the engineering unit in the ASE program, youth were tasked with building a ramp out of a limited number of wooden blocks on one end of a gym with the goal of releasing a ball to roll between two cones at the other end of the gym (see Figure 9).

Figure 9: One example of a ramp created by Kellie and Chelsie.

We observed Kellie and Chelsie posing what-if questions to one another as part of the engineering design process. The questions were embedded in mathematical thinking around ways to manipulate the ramp to ensure the ball would roll between the two cones on an uneven gym floor. For instance, after several attempts, Chelsie stated "I noticed that the ball always wants to move this way. What would happen if we angle the ramp more this way because even if [the ball] goes all the way over there, it could still go in." After shifting the angle of the ramp was not as successful as hoped, Kellie and Chelsie changed their course of action. Kellie asked, "What do you think would happen if we extended the length of the tracks?" Chelsie immediately countered with "Yeah, but what if we build the structure taller so that it [the ball] goes faster?"

One last example of a curiosity may be characterized as searching for an unknown. Shin and Kim [90] refer to this as forward curiosity. A group of youth were collectively interested in stacking the cups inside one another to make an arch or a parabola (see Figure 10). They began by constructing two tall vertical towers and attempted to bring them together to connect in the middle. Questions from this construction included: How tall should the towers be to stand on its own? How far apart should the leg of the towers be to be so that the tops of the towers join in the middle of the arch? What additional support might be needed to hold the base of the tower? The youth continued to explore and test these ideas until it was time to clean up.

Figure 10: One group's attempt to create an arc with two towers of stacked plastic cups.

6. Discussion

In this paper, we argue that, and illustrate how, making activities provide opportunities for youth to engage in a specialized form of everyday mathematical practices as characterized by Civil [16], which we call mathematical practices for making. Mathematical practices for making occur through social interactions in context. Social interactions offer moments for youth to teach and learn from one another. Making activities are contextualized and promote exploration, play, and inquiry of mathematics. In addition, mathematical practices for making allow for personalization and the use of mathematics that is embedded in the activity. Youth are afforded opportunities to personalize the making process through use of mathematical strategies and tools, as well as non-traditional mathematical strategies and tools. Mathematics within the process of making is not explicit, but lies hidden within the process.

The mathematical practices identified from two non-formal school-based contexts highlighted three mathematical practices for making: informal measurement, spatial reasoning, and curiosity. These practices are identified in prior scholarship as being beneficial and foundational for the understanding of mathematical concepts such as numerical and arithmetic representations [35, 38, 70] and standard units of measurement [104]. These practices are also utilized in workplace environments [62] and associated with pursuing and obtaining a career in a STEM field [105, 107].

Aligned with the argument of Nemirovsky, Kelton, and Civil [72] regarding informal mathematics education, we view making spaces as new social and learning spaces in which youth are able to experience, express, and build upon mathematical practices in a context that is not bounded by standardized tests and textbooks. Formal or school-based tasks, while powerful forms of learning, often strip the complexity and authenticity of engaging in "worldly" problems [17, 68, 72]; making and tinkering tasks give rise to mathematical practices of making when the problem and process demand it. For example, we observed instances of youth using their body as a form of measurement when needed to accomplish the goal of the making task.

The significance of these mathematical practices for making within non-formal learning contexts lies in the potential to build upon youths' everyday experiences and explorations in cultural and meaningful contexts [24, 26, 28]. Furthermore, these practices can help youth make connections that support and reach "across the divide between formal and informal learning, pushing us to think more expansively about where and how learning happens" [39, page 498].

Too often, such opportunities to engage in everyday mathematics, and everyday mathematics for making, are viewed as illegitimate ways of thinking in formal schooling environments [25, 28, 34], and devalued by parents as inappropriate ways to engage in mathematics [21, 109]. Together with other researchers [24, 37, 56], we contend there is value in maker spaces and activities. Documenting children's actions in such spaces provides insights into intuitive thinking that can serve as the basis for curriculum development and instructional practices that build from children's own intuitive mathematical understanding in school mathematics curriculum. Additionally, we contend that recognizing and leveraging children's mathematical practices of making will support their sense of belonging within a STEM community or discipline [31] and their identity as a mathematics learner [44, 76].

Beyond the school curriculum, legitimizing children's ways of doing mathematics in out-of-school contexts and capitalizing on their informal ways of thinking mathematically can encourage them to view mathematics as a human endeavor that supports them in reaching their potential as individuals and citizens in creating a better world for self and others [23]. Similar to everyday mathematical practices, mathematical practices for making are grounded in everyday contexts that also have applications to home, communal, and artistic activities such as basket weaving [1], cornrow designs in hair [27], and carpentry [67]. Children, as newcomers or peripheral participants engaged in making activities, are afforded the opportunity to begin developing the mathematical skills and practices to move toward full participation in daily life and grow into life-long learners, engaged citizens, community members, and "influential other[s]" [8, 49, 50].

We urge scholars to collect additional video data in a variety of making spaces (e.g., community centers, homes, libraries, and playgrounds) to continue to build upon our preliminary exploration here of ways youth are engaged in mathematical practices for making. What additional mathematical practices for making do youth utilize in such activities? For example, what heuristic strategies and practices (e.g., systematic experimentation) might youth employ through the making process? It can be argued that the value of heuristics in making undergirds the formal decision-making in solving mathematical problems [89, 108]. How are these practices shaped by peers, siblings, educators, and parents? How are these practices influenced by the available tools and materials [46]? In what ways are youth resourceful in their thinking about mathematics when tools such as rulers are not available? How might these practices develop over time and transfer to other learning contexts?

There are also opportunities to expand this investigation to a range of age groups and communities as making is an activity that spans young children to adults [41] and occurs on a global scale [84]. How do mathematical practices for making look similar and/or different across age bands and culture? Legitimizing ways of doing mathematics through using these practices as a human endeavor may serve as a foundation for engaging learners in complex and authentic problems grounded in everyday experiences.

We acknowledge that the mathematical practices for making identified in this paper are limited by our own understanding of and perspective on mathematics as teacher educators, researchers, and learners of mathematics, as well as our knowledge of and perspective on makerspaces as learning environments.

As Civil asks, “how can we “uncover” the mathematics in contexts in which we may have no experience with or may look very different from our background in academic mathematics?” [17, page 53] Formal education is like a rubber band that keeps pulling us back into our “truths” of what constitutes learning. For example, when implementing making activities in a non-formal school-based context, we noted in ourselves a reliance on step-by-step instructions as “experts” of the content as opposed to allowing for exploration and experiences with failure [87]. This pull is disproportionate to the time we spend in classrooms, considering we only spend an average of 5% of our lives in classrooms [32].

7. Conclusion

Civil’s description of everyday mathematical practices [16] laid the foundation for our thinking around ways mathematics is practiced outside of formal schooling. The significance of this study is the application of Civil’s definition to the domain of non-formal making learning environments. The identification and illustration of mathematical practices for making extends the robust descriptions of mathematical practices in the context of everyday practices toward environments designed to support the development and creativity of youth as they seek to contribute to social situations beyond the classroom.

In addition, this paper begins to address Arcavi’s [2] call for the discovery of mathematical practices that are often viewed as non-mathematical to youth and adults and build an understanding of ways youth utilize and apply mathematics in out-of-class contexts, contexts that have been developed from need, but have not been researched as much as formal mathematics classrooms [78]. As researchers turn to non-formal and informal contexts, with an eye toward understanding ways youth engage in the activity of making, descriptions of mathematical practices for making open up a new landscape of inquiry. In such a landscape, informal measurement, spatial reasoning, and curiosity are used as tools within the texture of activity. We also hope that further exploration of whether (and if so, how) mathematical practices for making may develop and serve the evolving goals of a maker will contribute to societal understanding of mathematics as lived rather than done.

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