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The informativeness/complexity trade-off in the domain of Boolean connectives

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Abstract

I apply the model of semantic universals in terms of informativeness/complexity trade-off (Kemp, Xu, and Regier, 2018) to Boolean connectives. The model explains the cross-linguistic absence of the connective NAND, once we incorporate theoretical insights from Horn (1972) and Katzir and Singh (2013). The lack of NAND follows if languages optimise the trade-off between (a) simplicity of the lexicon measured in terms of primitive symbols and (b) informativeness of the lexicon measured in terms of accurate transfer of information, given scalar implicature. The analysis demonstrates that the model provides a promising insight into the nature of lexicalisation in logical vocabularies.

Keywords: Boolean connectives, NAND, Complexity/informativeness tradeoff, lexicalisation, semantic universals.

1 Introduction

Investigation of semantic universals has been highly successful in the the lexical semantics of logical vocabularies (Horn 1972; Barwise and Cooper 1981; Keenan and Stavi 1986; see von Fintel and Matthewson 2008; Steinert-Threlkeld and Szymanik 2019 for reviews). However, there are still open questions concerning *explanations* of observed semantic universals. The most fundamental ones among such open questions include why attested universals exist and whether it is possible to have a *unified* explanation for different universals observed in different domains of vocabulary.

For domains of content words, such as colour terms and kinship terms, research has shown that universal semantic patterns can be explained by a principle favouring a linguistic system that supports an optimal trade-off between INFORMATIVENESS and COMPLEXITY of linguistic expressions (Kemp, Xu, and Regier, 2018). Importantly, this principle is highly general, and suggests that seemingly disparate semantic phenomena may in fact reflect a unified pressure. Thus, it is of theoretical interest to investigate whether this model also explains semantic universals in logical vocabularies.[1] In a recent study, Steinert-Threlkeld (2019) has shown that properties of natural language quantifiers described in Keenan and Paperno (2012); Paperno and Keenan (2017) follow the pattern predicted by the trade-off model. Similarly, Denić, Steinert-Threlkeld, and Szymanik (2020) show that the model predicts known typological generalisations in the domain of indefinite pronouns due to Haspelmath (1997). In addition, Carcassi (2020) reports that the monotonicity universal of gradable adjectives is correctly predicted by a computational model of language evolution that builds in the pressures from informativeness and simplicity. Singh (2019) is also relevant in this context, as he considers the trade-off between informativeness and cost in the computation of processing cost associated with implicature/exhaustification.

This paper is an attempt to show that the model in terms of the informativeness/complexity trade-off (henceforth the TRADE-OFF MODEL for short) is also applicable to the domain of BOOLEAN CONNECTIVES, and that it offers an explanation of a well-known semantic universal in the domain, i.e., the lack of NAND—the connective such that $\ulcorner p$ NAND $q \urcorner$ means 'not (p and q)'—once we incorporate the theoretical insights from Horn (1972) and Katzir and Singh (2013). In a nutshell, the lack of NAND follows if languages optimise the trade-off between (a) simplicity of the lexicon measured by the number of metalanguage symbols necessary to express the meaning of the connectives and (b) informativeness of the lexicon measured by the fine-grainedness of the semantic space partitioned by the lexicon, *given* scalar implicature. The analysis takes its inspirations from an existing analysis by Katzir and Singh (2013) (K&S) (who in turn improves on Horn 1972), but it also diverges from it in important respects. First, it explains the absence of several unattested inventories that are not ruled out by K&S. Second, it replaces K&S's hypotheses about lexicalisable combinations of primitive semantic elements to a separate set of hypotheses, i.e., one in terms of the optimal trade-off between complexity and informativeness and one in terms of commutativity of lexicalisable connectives.[2]

The rest of the paper is structured as follows. In Sect. 2, I will introduce the theory in terms of the informativeness/complexity trade-off as a general model for semantic universals. In Sect. 3, I will review existing explanations of the cross-linguistic absence of NAND due to Horn (1972) and Katzir and Singh (2013), and point out their limitations. At the same time, I will draw attention to conceptual similarities between the Horn/K&S account and the trade-off model. Building on this conceptual similarity, the Horn/K&S account will be reformulated within the trade-off model in Sect. 4. Within this section, I will also discuss empirical validity of the account's predictions, first with respect to the restricted class of inventories that only contain connectives in the four 'corners' of the square of opposition, and later with respect to all non-empty combinations of Boolean connectives. I will then discuss issues pertaining to the choice of parameters in the model in Sect. 5, before concluding in Sect. 6.

2 Informativeness/complexity Trade-off as a Mdel for Semantic Universals

The model of semantic universal and variation proposed by Kemp, Xu, and Regier (2018) is stated in terms of two pressures that can be taken to shape semantic systems in general: SIMPLICITY and INFORMATIVENESS. The former is a preference for a simpler system that requires less cognitive cost while the latter is a preference for a more informative system that ensures that the meaning intended by the sender is accurately recovered by the addressee as much as possible. These two pressures compete with each other. The simpler a system is, less guarantee there is that the intended meaning can be accurately recovered. The more informative and fine-grained a system is, more complex it is. This situation is schematically represented in Fig. 1. The dots in the figure represent semantic systems ranked by their simplicity and informativeness. The region without dots are not reachable because it would involve a degree of informativeness without its necessary cost for simplicity in a human language. The model states that natural semantic systems reside in the region represented by the gray dots, i.e. ones that make an optimal trade-off between simplicity and informativeness. The languages may differ depending on *how* they make the trade-off (how much they sacrifice simplicity for informativeness), but the variation is constrained in that all languages lie in the optimal frontier represented by the blue dots.

The model has provided explanations for systematic cross-linguistic patterns in lexical semantics across several domains of content words, including kinship terms (Kemp and Regier, 2012), colour terms (Regier, Kemp, and Kay, 2015), and artefact terms (Xu,

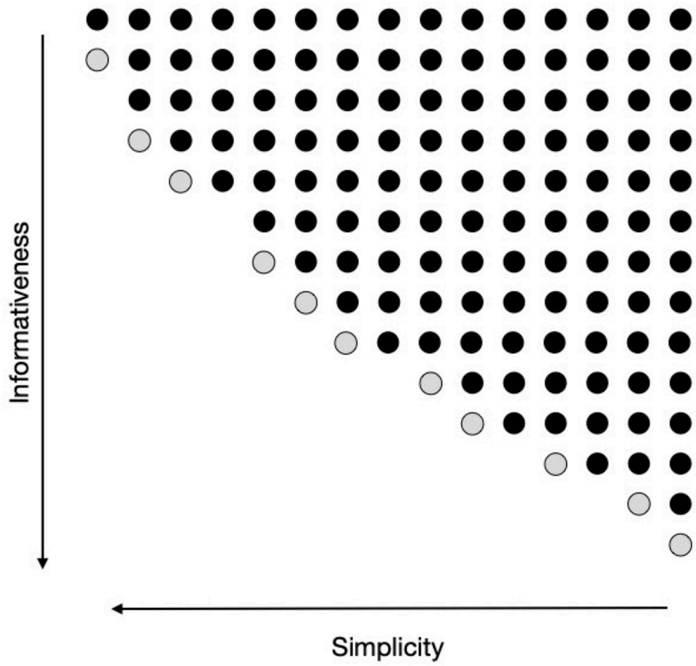

Figure 1: Trade-off between simplicity and informativeness (after Kemp, Xu, and Regier 2018)

Regier, and Malt, 2016). For example, Regier, Kemp, and Kay (2015) show that the inventory of colour terms predicted by the model given the size of the inventory closely tracks the actual cross-linguistic data of colour inventories (Cook, Kay, and Regier, 2005), providing explanations for typological implicational generalisations suggested in earlier literature (Berlin and Kay, 1969).

3 Existing Accounts

There is a distinct body of literature within formal semantics that concerns the explanation of semantic universals in logical vocabularies. One of the earliest and most well-known such explanations is the one put forth by Horn (1972) for the generalisation that natural languages lack the connective NAND.[3] Horn's explanation is stated in terms of two conditions, which can be roughly stated as follows, following the formulation given in

Katzir and Singh (2013):

(1) a. **Gricean Condition**: Let X and Y be two inventories of logical operators in the Aristotelian square of opposition (i.e., AND, OR, NOR, NAND) that cover the same semantic space. If $Y \subset X$, X cannot be lexicalised.

b. **Negation Condition**: Let X and Y be two inventories of logical operators in the Aristotelian square of opposition (i.e., AND, OR, NOR, NAND) that cover the same semantic space. If X contains more instances of negation than Y, X cannot be lexicalised.

The notion of semantic coverage is crucially defined by taking into account the possibility of scalar implicature:

(2) **Semantic Coverage**: X COVERS z if z is either a member of X or the scalar implicature of some member of X

To see how the above conditions explain the lack of NAND, let us first consider the full inventory of Aristotelian four corners:

(3) $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$

From the perspective of the Gricean Condition, this inventory is sub-optimal. This is so because, once we take scalar implicature into account, the same semantic coverage can be achieved by an inventory with fewer operators, e.g., the one in (4) below:

(4) $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$

This is so because OR can implicate NAND given the presence of the stronger alternative AND. The Gricean Condition thus states that (3) cannot be lexicalised given the presence of (4).

Now, Horn argues that the Gricean Condition is necessary but not sufficient as an explanation for the absence of NAND. To see this, consider another three-membered inventory as follows:

(5) $\{$AND, NOR, NAND$\}$

This inventory can achieve the same Semantic Coverage as (4) (and hence as the four-membered inventory in (3)). This is so since NAND in (5) can implicate OR, given the presence of the stronger alternative NOR. The Gricean Condition alone does not distinguish between (4) and (5), which have the same number of operators and the same Semantic Coverage. This is where the Negation Condition in (1b) comes in. Assuming that NAND and NOR involve negation in its semantic representation, but OR and AND don't, the inventory in (4) involves one occurrence of negation but (5) involves two. Given the Negation Condition, this makes (5) less optimal in comparison with (4).

The other two possible three-membered inventories do not achieve the same Semantic Coverage as the four-membered inventory, even with scalar implicature:

(6) a. $\{$AND, OR, NAND$\}$ does not cover NOR

b. $\{$OR, NAND, NOR$\}$ does not cover AND

Similarly, inventories with fewer operators don't achieve the Semantic Coverage of (3-5). Thus, the combination of the Gricean Condition and the Negation Condition dictates that the NAND-less inventory in (4) is the only lexicalisable inventory with the full semantic coverage of Aristotelian four corners.

Katzir and Singh (2013) point out that Horn's (1972) account does not extend to broader typological patterns involving Boolean connectives outside of the four corners, e.g. the lack of XOR. To address this issue, Katzir and Singh (2013) propose that connectives involve primitive semantic building blocks and a restricted set of their combinations, following Keenan and Stavi's (1986) account of determiner meanings. Specifically, they stipulate that a connective meaning may consist of a disjunction or a conjunction and a negation optionally applied to it. This guarantees that no Boolean connective outside of the square can be lexicalised, including XOR. On top of this, the two conditions in (1) operate on inventories consisting of the four corners that are in principle allowed by the building blocks. This rules out e.g. the four-member inventory $\{\text{AND}, \text{OR}, \text{NOR}, \text{NAND}\}$. K&S show that the relevant stipulation applies beyond connectives and to any syntactic category and semantic type: logical operators across categories and types are restricted to generalised disjunction and conjunction. This is formalised in terms of an ordering-based perspective (Keenan and Faltz, 1985), i.e., by positing infimum and supremum as semantic primitives and allowing negation over these as an allowable operation.

Thus, Katzir and Singh's (2013) account improves on Horn (1972) by enabling an explanation of typological generalisations concerning connectives outside of the four corners, while also showing that the explanation can be generalised to account for wider lexicalisation patterns in logical vocabularies. However, an empirical issue still remains. The issue concerns inventories involving connectives in the four corners theoretically allowed by the account. Nothing in Horn's and K&S's account outlined above precludes an inventory from having a Semantic Coverage that is smaller than the Aristotelian four corners. In fact, the account predicts that an inventory is lexicalisable if it is the most optimal one from the point of view of the Gricean Condition and the Negation Condition, i.e., it does not have a competing inventory that achieves the same amount of Semantic Coverage either with fewer connectives or with the same number of connectives but with fewer overall instances of negation. In particular, the following inventories are predicted to be possible by Horn/K&S:

(7) a. $\{\text{AND}, \text{OR}\}$ b. $\{\text{AND}\}$ c. $\{\text{OR}\}$

Each of these inventories is the most optimal one in view of Horn's two conditions among the ones with the same semantic coverage. To the extent that the inventories in (7) are concerned, this is a welcome prediction, since all the inventories in (7) have been reported to exist in the typological literature. For example, Mous (2004) reports that Iraqw (Southern Cushitic, Tanzania) has the inventory of connectives in (7a). Wari' (Chapacura-Wanam, Brazil) is argued to have only a conjunctive connective, thus having the inventory in (7b) (Mauri, 2008). Finally, (7c) is reported to be the inventory of connectives in Warlpiri (Pama-Nyungan, Australia) (Bowler, 2015).[4]

However, for the same reasons why the account predicts (7) to be possible, it also predicts the following inventories to be possible:

(8)
a. $\{\mathsf{AND}, \mathsf{NAND}\}$
b. $\{\mathsf{AND}, \mathsf{NOR}\}$
c. $\{\mathsf{OR}, \mathsf{NOR}\}$
d. $\{\mathsf{OR}, \mathsf{NAND}\}$
e. $\{\mathsf{NOR}, \mathsf{NAND}\}$
f. $\{\mathsf{NAND}\}$
g. $\{\mathsf{NOR}\}$

Each of these inventories is the most optimal combination of connectives from the point of view of the two conditions in (1). In fact, aside from $\{\mathsf{NOR}, \mathsf{NAND}\}$, each inventory in (8) is the only one that has the specific Semantic Coverage that it has (while $\{\mathsf{NOR}, \mathsf{NAND}\}$ has the same Semantic Coverage as $\{\mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$ due to the fact that NAND scalar-implicates OR). To my knowledge, none of the inventories in (8) is reported in a natural language. This can be taken to be a problem for Horn as well as for K&S.[5]

At this point, it may be clear to the reader that the accounts based on the two conditions in (1) are similar to the informativeness/complexity trade-off model at its conceptual level. The two conditions in (1) together amount to pressure from simplicity, given a certain level of informativeness. This is so because both the number of connectives—constrained by the Gricean Condition—and the number of negation—constrained by the Negation Condition—can be taken to contribute to the complexity of an inventory. Also, Horn/K&S's notion of Semantic Coverage is intuitively similar to the notion of informativeness in the trade-off model. When an inventory has a wider semantic coverage than another, then the former inventory can be said to be more informative than the latter, since it allows speakers to convey more meanings than would be possible in the latter inventory. Although Horn/K&S don't explicitly discuss pressure from informativeness, it is reasonable to assume that there is a general preference for an inventory with more semantic coverage, given a certain level of complexity of the inventory. In the next section, I will give a concrete reformulation of Horn/K&S's account within the trade-off model. The reformulation will directly incorporate the insights of Horn/K&S's two conditions as well as the assumption about the primitiveness of conjunction, disjunction, and negation. However, it will also overcome the empirical issue discussed above. This is due to the fact that the reformulation replaces K&S's hypotheses regarding constraints on lexicalisation with a new set of hypotheses that allow more general comparisons between inventories with respect to their fitness for lexicalisation.

4 Informativeness/complexity Trade-off for Boolean Connectives

Following the trade-off model, I hypothesise that inventories of Boolean connectives in natural language optimise the trade-off between informativeness and simplicity, where these two measures are defined as follows:

- **Complexity** of an inventory is measured by the sum of the number of symbols in Propositional Logic containing $\neg$, $\wedge$, and $\vee$ necessary to represent all connectives in the inventory.
- **Informativeness** of an inventory is measured by the likelihood that the meaning intended by the sender is accurately recovered by the addressee, *given* scalar implicature.

More precisely, the hypothesis is that natural inventories are PARETO-OPTIMAL with respect to informativeness and complexity. An inventory is Pareto-optimal with respect to two measures iff, given its value in one measure, there is no other inventory that improves on it with respect to the other measure. Thus, an inventory L is Pareto-optimal with respect to informativeness and complexity iff we cannot find another inventory L' that is as simple as L but is more informative than L nor another inventory L'' that is as informative as L but is simpler than L.

As suggested in the previous section, insights from Horn and K&S are incorporated in these measures. In particular, in line with K&S, complexity is measured under the assumption that negation ($\neg$), disjunction ($\vee$), and conjunction ($\wedge$) are the only primitive semantic building blocks. Also, scalar implicature is taken into account in the measurement of informativeness, following Horn/K&S's idea that scalar implicature feeds Semantic Coverage. However, unlike K&S, the model does not preclude any logically possible Boolean connective from the set of in-principle lexicalisable connectives (though I will consider restricting lexicalisable connectives to commutative ones in Sect. 4.3.2.). For example, the connective XOR is included as an *in-principle* lexicalisable connective that the model considers when it calculates the trade-off between informativeness and complexity of various inventories.

In this section, I will introduce a formal model of Boolean connective inventories and discuss in detail how their complexity and informativeness are measured. I take the exact measurements adopted in this paper to be mere approximations guided by the insights from Horn/K&S, as my goal is to show that the Horn/K&S account can be reformulated within the trade-off model, while preserving their virtues and overcoming their limitations. This said, it is worth mentioning that informativeness-simplicity trade-off based on the two measures along the lines of what I will introduce below has been independently shown to track properties of quantifiers in natural language by Steinert-Threlkeld (2019).[6]

Needless to say, the exact choice of the parameters should ideally be tested based on further independent empirical evidence. In Sect. 5, I will discuss several specific issues pertaining to the choice of parameters.

4.1 The Model of Boolean Connective Inventories

An inventory of Boolean connectives is any nonempty subset of the set CON of 16 Boolean connectives. These connectives are labeled as follows, with their semantics defined as in the truth table in Table 1:

(9) $$CON = \left\{ \begin{array}{c} \mathsf{P}, \mathsf{Q}, \mathsf{TAU}, \leftarrow, \rightarrow, \leftrightarrow, \mathsf{AND}, \mathsf{ONLYP}, \mathsf{ONLYQ}, \\ \mathsf{NAND}, \mathsf{XOR}, \mathsf{NOTP}, \mathsf{NOTQ}, \mathsf{NOR}, \mathsf{CONT} \end{array} \right\}$$

The set $\mathcal{L}$ of all possible inventories thus has $2^{16} - 1 = 65,535$ members:

(10) $$\mathcal{L} := Pow(CON) - \varnothing$$

I will formally represent the semantics of each connective as a subset of the four-world universe $W := \{w_1, w_2, w_3, w_4\}$. That is, for each connective $c \in CON$, $[\![c]\!] = \{\, w \in W \mid \text{column } c \text{ has value 1 in row } w \text{ in Table 1} \,\}$.

subsectionMeasuring Simplicity/Complexity

The complexity of an inventory is the sum of the (smallest) number of symbols in Propositional Logic (PL) necessary to represent the connectives in the inventory. Here, I take PL to be one that has three connectives: $\neg$, $\wedge$ and $\vee$, and propositional letters p and q for the left-hand side and right-hand side propositional argument. Although the account should ideally be grounded in a theory of the cognitive complexity of Boolean concepts (e.g., Feldman, 2000; Piantadosi, Tenenbaum, and Goodman, 2016), I will adopt a simplistic measure of complexity here, inheriting K&S's assumption that the the primitive semantic building blocks consist of conjunction, disjunction, and negation. Table 2 lists the PL translations of the 16 Boolean connective meanings.[7]

	P	Q	TAU	OR	$\leftarrow$	$\rightarrow$	$\leftrightarrow$	AND	ONLYP	ONLYQ	NAND	XOR	NOTQ	NOTP	NOR	CONT
w_1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
w_2	1	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0
w_3	0	1	1	1	0	1	0	0	0	1	1	1	0	1	0	0
w_4	0	0	1	0	1	1	1	0	0	0	1	0	1	1	1	0

Table 1: Truth table for all possible Boolean connective meanings

label	**formula**	**length**
P	p	1
Q	q	1
TAU	$p \vee \neg p$	4
OR	$p \vee q$	3
$\leftarrow$	$\neg p \vee q$	4
$\rightarrow$	$p \vee \neg q$	4
$\leftrightarrow$	$(p \wedge q) \vee \neg(p \vee q)$	8
AND	$p \wedge q$	3
ONLYP	$p \wedge \neg q$	4
ONLYQ	$\neg p \wedge q$	4
NAND	$\neg(p \wedge q)$	4
XOR	$(p \vee q) \wedge \neg(p \wedge q)$	8
NOTQ	$\neg q$	2
NOTP	$\neg p$	2
NOR	$\neg(p \vee q)$	4
CONT	$p \wedge \neg p$	4

Table 2: Propositional Logic representations of connectives and their length

Based on the length of the formulas, the complexity of an inventory is calculated by simply summing up the length of the formulas in the inventory. Below are some examples:

$$
\begin{aligned}
C(\{\text{AND, OR, NAND, NOR}\}) &= 3+3+4+4 = 14 \\
C(\{\text{AND, OR, NOR}\}) &= 3+3+4 \qquad = 10 \\
C(\{\text{AND, NOR, NAND}\}) &= 3+4+4 \qquad = 11
\end{aligned}
$$

Note that the comparison between these inventories based on our measure of complexity follows the one based on the Gricean Condition and the Negation Condition, discussed in the previous section. These three inventories have the same 'Semantic Coverage' in the sense of Horn/K&S, but {AND, OR, NOR} is simpler than {AND, OR, NAND, NOR } by virtue of the fact that the former has fewer connectives in it, and {AND, OR, NOR} is simpler than {AND, NOR, NAND} by virtue of the fact that the PL representation of OR is shorter than that of NAND, due to the presence of $\neg$ in the latter.

4.2 Measuring Informativeness

Following Horn/K&S, our measure of informativeness takes into account scalar implicature. Following K&S, I adopt Fox's (2007) formulation of scalar implicature, where the result of applying scalar implicature to p given a set A of alternatives is the intersection of p with the the negation of Innocently Excludable (IE) alternatives for p in A. This is formally stated as follows:[8]

(11) a. $\mathsf{ScalarImp}(p, A) := p \cap \bigcap \{ \overline{p'} \mid p' \in \mathsf{IE}(p, A) \}$

b. $\mathsf{IE}(p, A) :=$

$\bigcap \{ A' \subseteq A \mid A'$ is a maximal subset of A s.t. $p \cap \bigcap \{ \overline{p'} \mid p' \in A' \} \neq \varnothing \}$

Given this formulation of scalar implicature, we derive the strengthened meaning of connective c in inventory L, $[\![c]\!]^+_L$, as the result of applying scalar implicature to the meaning of c, $[\![c]\!]$, with the whole inventory L as its alternatives:

(12) $[\![c]\!]^+_L := \mathsf{ScalarImp}([\![c]\!], [\![L]\!])$ where $[\![L]\!] := \{ [\![c]\!] \mid c \in L \}$

The informativeness of an inventory L is then measured using notions of communicative success, based on the speaker's intended message (represented as a particular possible world) w, the addressee's interpretation w', and the utility (u) of interpretation w' given w, as follows (Skyrms, 2010; Steinert-Threlkeld, 2019):

(13) $I(L) =$

$$\sum_{w \in W} \overbrace{P(w)}^{\text{prior prob. of } w} \sum_{c \in L} \overbrace{P(c|w)}^{\text{prob. of uttering } c \text{ given } w} \sum_{w' \in W} \overbrace{P(w'|c)}^{\text{prob. of interpreting } c \text{ as } w'} \cdot \overbrace{u(w, w')}^{\text{utility of } w' \text{ given } w}$$

I assume this particular measure of informativeness since it has been independently shown to lead to empirically accurate predictions of the trade-off model in other domains of the logical vocabulary, i.e., quantifiers (Steinert-Threlkeld, 2019) and indefinite pronouns (Denić, Steinert-Threlkeld, and Szymanik, 2020). Furthermore, as I will illustrate shortly, the measure allows an intuitive characterisation of informativeness that penalises both non-specificy in meanings and semantic overlaps between different expressions.

I will assume flat priors and flat conditional probabilities based on the strengthened meanings of the connectives. Thus, $P(w) = \frac{1}{4}$ for every $w \in W$. $P(c|w)$ is the conditional probability of the speaker truthfully uttering c given that they know they live in world w. With flat conditional probability, if $w \in [\![c]\!]^+_L$, $P(c|w) = \frac{1}{n}$ where $n = |\{ c' \in L \mid w \in [\![c']\!]^+_L \}|$ (i.e., the number of connectives that are true in w given their strengthened meanings); otherwise, $P(c|w) = 0$. Similarly, if $w' \in [\![c]\!]^+_L$, $P(w'|c) = \frac{1}{n}$ where $n = |[\![c]\!]^+_L|$ (i.e. the number of worlds that make c true given its strengthened meaning); otherwise, $P(w'|c) = 0$. As for the utility function u, I will assume a binary function that assigns 1 if the intended message is correctly recovered and 0 otherwise. That is:

(14) $u(w, w') := \begin{cases} 1 & \text{if } w' = w \\ 0 & \text{otherwise} \end{cases}$

I will consider a non-binary utility function in Sect. 5.

Given these assumptions, the informativeness value defined in (13) can be simplified as follows, where $L(w) := \{ c' \in L \mid w \in [\![c']\!]^+_L \}$:

(15)

$$I(L) = \frac{1}{4} \sum_{w \in \{w' | L(w') \neq \varnothing\}} \sum_{c \in L(w)} \frac{1}{|L(w)|} \frac{1}{|[\![c]\!]_L^+|}$$

Intuitively, $\frac{1}{|L(w)|}$ can be understood as a factor that penalises overlap in meanings between different connectives. Given a certain connective c and a world w, where the strengthened meaning of c is true in w, the more connectives the speaker could use in w in addition to c, there is less chance that c is used. This can lead to overall informativity loss, as will be exemplified shortly. On the other hand, $\frac{1}{|[\![c]\!]_L^+|}$ can be understood as a factor that penalises non-specificity in meanings. The more worlds there are in which a connective can be true, less likely it is that using the connective results in accurate communication.

Here are some examples that illustrate how informativeness is calculated according to the parameters set out above. Suppose we have $L = \{\mathsf{AND}, \mathsf{OR}\}$ as our inventory. We have $[\![\mathsf{AND}]\!]_L^+ = \{w_1\}$ and $[\![\mathsf{OR}]\!]_L^+ = \{w_2, w_3\}$ (see Figure 2(a) for a depiction). In this case, the informativeness value of L, $I(L)$, can be calculated as follows:

i) If the speaker intends w_1, then they can only choose AND to convey this $[P(\mathsf{AND}|w_1) = 1]$. AND can only be interpreted to convey w_1 $[P(w_1|\mathsf{AND}) = 1]$. Thus, the probability of communicative success given that the speaker intends w_1 is 1.

ii) If the speaker intends w_2, then they can only choose OR to convey this $[P(\mathsf{OR}|w_2) = 1]$. OR can be interpreted to convey w_2 or w_3. The communication will be successful only if the addressee interprets the message as w_2 $[P(w_2|\mathsf{OR}) = \frac{1}{2}]$. Thus, the probability of communicative success given that the speaker intends w_2 is $1 \cdot \frac{1}{2} = \frac{1}{2}$.

iii) If the speaker intends w_3, then they can only choose OR to convey this $[P(\mathsf{OR}|w_3) = 1]$. OR can be interpreted to convey w_2 or w_3. The communication will be successful only if the addressee interprets the message as w_3 [$P(w_3|\mathsf{OR}) = \frac{1}{2}$]. Thus, the probability of communicative success given that the speaker intends w_3 is $1 \cdot \frac{1}{2} = \frac{1}{2}$.

iv) If the speaker intends w_4, then there is no connective they can use to achieve successful communication.

We thus have $I(\{\mathsf{AND}, \mathsf{OR}\}) = \frac{1}{4}(1 + \frac{1}{2} + \frac{1}{2} + 0) = \frac{1}{2}$.

Next, let us consider $L' = \{\mathsf{AND}, \mathsf{P}, \mathsf{Q}\}$. Here, we have $[\![\mathsf{AND}]\!]^+_{L'} = \{w_1\}$; $[\![\mathsf{P}]\!]^+_{L'} = \{w_2\}$; $[\![\mathsf{Q}]\!]^+_{L'} = \{w_3\}$ (see Figure 2(b)). In this case, the situation is exactly the same as above for (i) and (iv). Only, in (ii) and (iii), the speaker can use designated connectives—P and Q respectively—and their messages will be fully recovered by the addressee. Thus, we have $I(L') = \frac{1}{4}(1 + 1 + 1 + 0) = \frac{3}{4}$. This exemplifies the fact that the more fine-grained the set of strengthened meanings, the greater the informativeness, since fine-grainedness of meanings contributes to higher likelihood that the speaker's intended message is correctly recovered by the addressee.

Finally, let us consider $L'' = \{\mathsf{AND}, \mathsf{OR}, \mathsf{XOR}\}$. In this case, scalar implicature does not strengthen any of the connectives.[9] That is, we have $[\![\mathsf{AND}]\!]^+_{L''} = \{w_1\}$; $[\![\mathsf{OR}]\!]^+_{L''} = \{w_1, w_2, w_3\}$; $[\![\mathsf{XOR}]\!]^+_{L''} = \{w_2, w_3\}$ (see Figure 2(c)). In this case, due to the overlap in meanings, L'' will have less informativeness than L. Concretely, if the speaker intends w_1, they can use AND and OR with $\frac{1}{2}$ probability each. In the AND case, the intended meaning is always correctly recovered. In the OR case, it is correctly recovered with $\frac{1}{3}$ probability. Thus, the probability of successful communication given that the speaker intends w_1 is $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{3}$. Comparing this with case (i) in the calculation of the informativeness for L above makes it clear that communicative success is less likely under L'' than under L when the speaker intends w_1. Similar considerations apply in cases where the speaker intends other meanings. The overall informativeness for L'' turns out to be $I(L'') = 0.375$. Thus, we see that adding an additional connective to L results in lower informativeness value. This is an example of a case where the overlap in meanings results in the overall informativity loss.[10]

Note further that this informativeness measure replicates the prediction of the Fox/K&S model regarding the equivalence in Semantic Coverage between $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$ and $\{\mathsf{AND}, \mathsf{NAND}, \mathsf{NOR}\}$ due to the fact that the sets of strengthened meanings of these two inventories turn out to be the same (see Figure 2(d)).

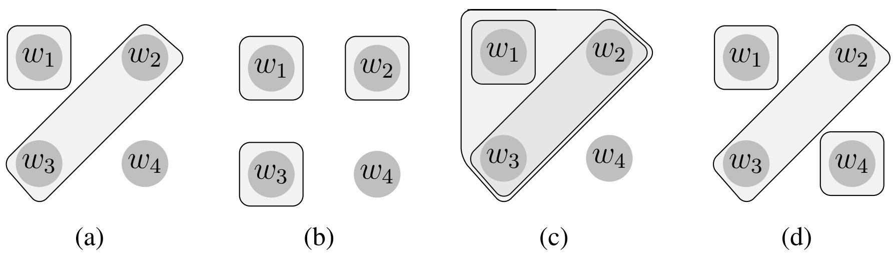

Figure 2: Strengthened meanings for (a) $\{\mathsf{AND}, \mathsf{OR}\}$, (b) $\{\mathsf{AND}, \mathsf{P}, \mathsf{Q}\}$, (c) $\{\mathsf{AND}, \mathsf{OR}, \mathsf{XOR}\}$ and (d) $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$ and $\{\mathsf{AND}, \mathsf{NAND}, \mathsf{NOR}\}$

4.3 Model Predictions

A Python script was written to generate the complexity and the informativeness values of all 65,535 inventories of Boolean connectives, according to the measures given above. The script and its output are available at `https://github.com/wuegaki/connectives`. In the following two sections, I will discuss predictions of the model, first focusing on those consisting of connectives in the Aristotelian four corners (i.e., AND, OR, NAND, and NOR) (henceforth FOUR-CORNER INVENTORIES) and then extending the discussion to all possible inventories consisting of 16 Boolean connectives.

4.3.1 Four-corner Inventories

The predictions for four-corner inventories are summarised in Table 3. A plot of the inventories with respect to informativeness and complexity are given in Figure 3. As can

inventory	complexity	informativeness
{AND, OR, NAND, NOR}	14	0.5
{AND, NAND, NOR}	11	0.75
{OR, NAND, NOR}	11	0.5833
{OR, AND, NOR}	**10**	**0.75**
{OR, AND, NAND}	10	0.5833
{NAND, NOR}	8	0.5
{AND, NOR}	7	0.5
{OR, NOR}	7	0.5
{AND, NAND}	7	0.5
{OR, NAND}	7	0.5
{OR, AND}	**6**	**0.5**
{NOR}	4	0.25
{NAND}	4	0.25
{AND}	**3**	**0.25**
{OR}	**3**	**0.25**

Table 3: Complexity and informativeness for four-corner inventories. Inventories that are Pareto-optimal with respect to simplicity and informativeness are boldfaced.

be seen from the table and the plot, the model predicts four inventories—{AND, OR, NOR}, {AND, OR}, {AND}, and {OR}—to be the Pareto-optimal ones. As discussed in Sect. 3, all of these inventories are reported to exist in a natural language. Furthermore, the current model overcomes one of the empirical problems with the Horn/K&S account, as it predicts that no inventories other than the above four are Pareto-optimal. For example, although the Horn/K&S account predicts {NAND} to be an inventory that can in principle be lexicalised (see Sect. 3), the current model rules out {NAND} since it is more complex than {AND} and {OR} which are equally informative. In other words, our measure of informativeness is general enough to compare the whole range of inventories (allowing us, for example, to equate the

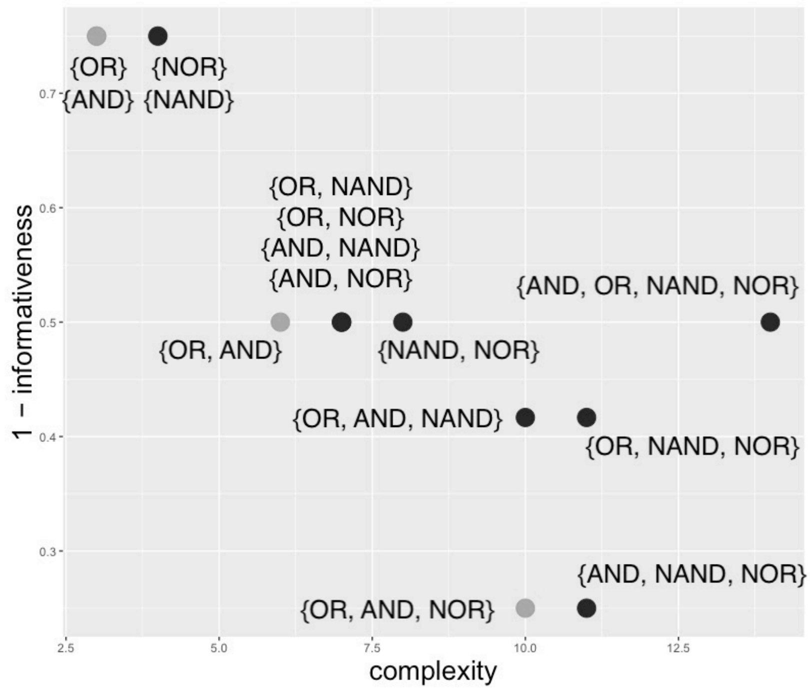

Figure 3: Four-corner inventories plotted with respect to informativeness and simplicity. Pareto-optimal inventories are in gray.

informativeness of {NAND} with that of {AND} and {OR}) whereas Horn/K&S's notion of Semantic Coverage is not general enough to allow similar comparisons across inventories.[11]

4.3.2 All Possible Boolean Connective Inventories

The generality of our model allows us to investigate its predictions with respect to all 65,535 possible inventories of Boolean connectives. The full output of the model is plotted in Figures 4 and 5. As can be seen in Table 4(a), Pareto-optimal inventories in the full model output do not track attested cross-linguistic patterns as well as those in the partial output for four-corner inventories. On the one hand, the lack of NAND is still correctly predicted since no inventory in Table 4(a) contains NAND. On the other hand, it makes a number of problematic predictions, including the lack of OR in any

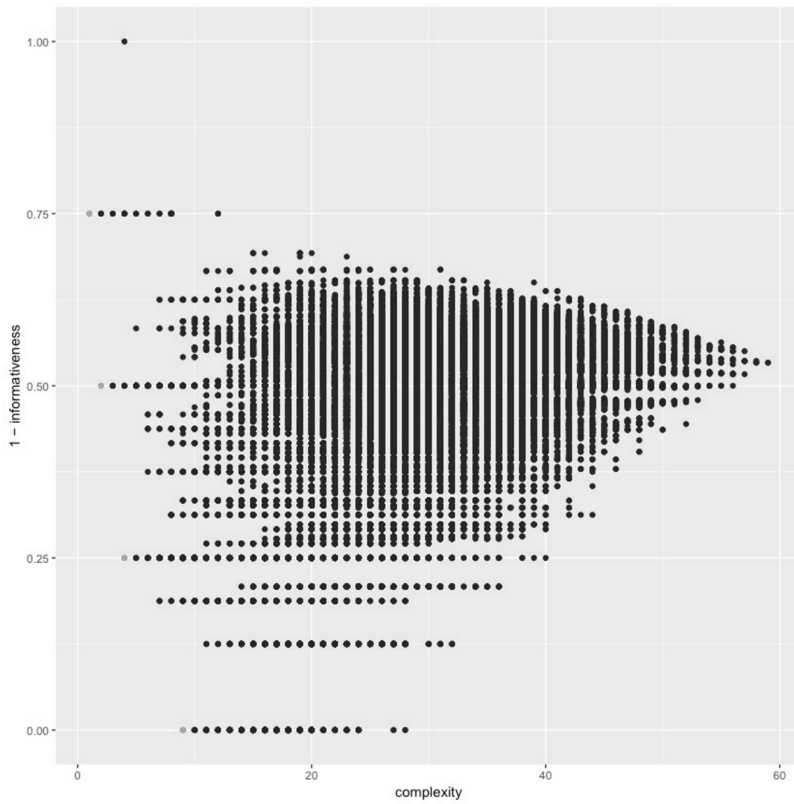

Figure 4: Plot of all 65,535 possible inventories of Boolean connectives with respect to informativeness and complexity. Inventories in the Pareto-optimal frontier listed in Table 4(a) are in gray.

Pareto-optimal inventory.

In this context, it is worth noting another property of the inventories in Table 4(a): every one of them involves a *non-commutative* connective (i.e., those connectives con such that $[\![p \text{ con } q]\!] = [\![q \text{ con } p]\!]$) whereas connectives in the four corners are all commutative.[12] With respect to the property of commutativity, the 16 Boolean connectives can be divided into the following two classes:

(16) a. **Non-commutative connectives**

P, Q, NOTP, NOTQ, $\rightarrow$, $\leftarrow$, ONLYP, ONLYQ

b. **Commutative connectives**

(a)

inventory	**complexity**	**informativeness**
$\{\mathsf{P}, \mathsf{Q}, \mathsf{AND}, \mathsf{NOR}\}$	9	1
$\{\mathsf{P}, \mathsf{Q}, \rightarrow, \mathsf{AND}\}$	9	1
$\{\mathsf{P}, \mathsf{Q}, \leftarrow, \mathsf{AND}\}$	9	1
$\{\mathsf{TAU}, \mathsf{P}, \mathsf{Q}, \mathsf{AND}\}$	9	1
$\{\mathsf{P}, \mathsf{Q}, \mathsf{AND}, \mathsf{NOTP}\}$	7	0.8125
$\{\mathsf{P}, \mathsf{Q}, \mathsf{AND}, \mathsf{NOTQ}\}$	7	0.8125
$\{\mathsf{P}, \mathsf{Q}, \mathsf{NOTP}\}$	4	0.75
$\{\mathsf{P}, \mathsf{Q}, \mathsf{NOTQ}\}$	4	0.75
$\{\mathsf{P}, \mathsf{Q}\}$	2	0.5
$\{\mathsf{P}\}$	1	0.25
$\{\mathsf{Q}\}$	1	0.25

(b)

inventory	**complexity**	**informativeness**
$\{\mathsf{OR}, \mathsf{AND}, \mathsf{NOR}\}$	10	0.75
$\{\mathsf{TAU}, \mathsf{OR}, \mathsf{AND}\}$	10	0.75
$\{\mathsf{OR}, \mathsf{AND}\}$	6	0.5
$\{\mathsf{OR}\}$	3	0.25
$\{\mathsf{AND}\}$	3	0.25

Table 4: (a): Pareto-optimal inventories among all 65,535 possible combinations of Boolean connectives; (b): Pareto-optimal inventories among combinations of *commutative* Boolean connectives ($2^8 - 1 = 255$ in total).

TAU, CONT, OR, $\leftrightarrow$, AND, NAND, XOR, NOR

In the earlier literature, Gazdar and Pullum (1976) and Gazdar (1979: 74-78) have hypothesised that connectives that are in principle lexicalisable in natural language are restricted to the commutative class in (16b). Gazdar (1979) in particular has shown that this constraint is derived once we take connectives as a function that takes a *set* (rather than a tuple) of truth values as its argument, following an earlier suggestion by McCawley (1972). Following these authors, I take it as a viable hypothesis that natural language connectives are always commutative although I have to leave open the exact explanation for the existence of this constraint.[13]

As such, the prediction of the model only for inventories containing the commutative connectives in (16b) is also considered. The Pareto-optimal inventories among the combinations of the commutative connectives are listed in Table 4(b). These inventories seem to track the attested pattern more accurately than those in Table 4(a). The only unattested inventory in Table 4(b) is $\{\mathsf{TAU}, \mathsf{OR}, \mathsf{AND}\}$, which is arguably ruled out for independent grounds: TAU is non-lexicalisable in natural language because it systematically derives a trivial meaning, i.e., tautology. There is growing evidence in formal semantics that systematically trivial meanings resulting from logical vocabularies leads to ungrammaticality (e.g., Barwise and Cooper, 1981; von Fintel, 1993; Gajewski, 2002; Fox and Hackl, 2007; Chierchia, 2013; Del Pinal, 2019). Given this link between semantic triviality and ungrammaticality, a sentence containing TAU would always be ungrammatical, regardless of the clauses it coordinates. This would explain the lack of TAU in the grammar of any natural language.[14]

4.4 Interim Summary

The Horn/K&S account of the lack of NAND can be reformulated within the informativeness/complexity trade-off model due to Kemp and Regier (2012); Regier, Kemp, and Kay (2015); Kemp, Xu, and Regier (2018). According to this model, inventories of connectives in natural languages reside in the Pareto-optimal frontier of their complexity (measured as the number of Propositional Logic operators necessary to express the connective meanings) and informativeness (measured in terms of the notion of communicative success, given scalar implicature).

In this section, we first considered the prediction of the model with respect to those inventories only containing connectives in the Aristotelian four corners (Sect. 4.3.1). Not only does the model predict the four empirically attested inventories—{AND, OR, NOR}, {AND, OR}, {AND} and {OR}—to be Pareto-optimal among the restricted class of inventories, but it also overcomes the problem with Horn/K&S in ruling out unattested inventories, e.g., {NOR}, as sub-optimal. We then moved on to consider the broader prediction of the model (Sect. 4.3.2) with respect to possible combinations of Boolean connectives in general, not just those in the Aristotelian corners. Although the Pareto-optimal inventories among the full set of inventories (65,535 in total) do not seem to track the empirical patterns accurately, restricting the overall domain of inventories to those containing *commutative* connectives allows us to make empirically plausible predictions.

5 Alternative Parameter Settings

The model proposed in the previous section relies on a number of parameter choices and assumptions, e.g., a precise definition of complexity and the utility function involved in the calculation of informativeness. Since the goal of this paper is to reformulate the Horn/K&S account within the trade-off model and discuss its advantages, these parameters have been generally chosen on the basis of theoretical simplicity and/or similarity to the assumptions made in Horn/K&S. Nevertheless, ideally, different choices of parameters should be compared based on the empirical validity of their predictions and tested based on independent evidence. In this section, I will briefly discuss three notable issues pertaining to the choice of parameters, although detailed empirical investigations have to be left for another occasion.

Scalar implicature Following Horn/K&S, our measure of informativeness takes into account scalar implicature as a crucial ingredient. Scalar implicature is indeed necessary to derive empirically correct predictions. This can be illustrated by considering the informativeness of the following inventories in models with and without scalar implicature.

(17) $\{\mathsf{OR}, \mathsf{AND}\}$

(18) a. $\{\mathsf{OR}, \mathsf{NOR}\}$ b. $\{\mathsf{AND}, \mathsf{NOR}\}$ c. $\{\mathsf{AND}, \mathsf{NAND}\}$

As shown in Table 3, these inventories have the same informativeness value (= 0.5) if they are calculated on the basis of the strengthened meanings resulting from scalar implicature. This makes (17) more optimal than those in (18), given its relative simplicity. On the other hand, without scalar implicature, the informativeness value for (17) (= 0.325) will be lower than that for those in (18) (= 0.5), making the inventories in (18) Pareto-optimal among the four-corner inventories. To my knowledge, none of the inventories in (18) is attested in the typological literature. Hence, scalar implicature is an important ingredient of the current model in deriving the empirically correct predictions (see also fn. 8 for a note on the choice of the particular implementation of scalar implicature).

Utility function In Sect. 4.2, the utility function has been defined as a binary function, returning 1 if the speaker's intended message is accurately recovered by the addressee, and 0 otherwise. However, we could also conceive of a more nuanced version of utility function with a partial value assigned to partial communicative success. Such a weighted utility can be defined as follows:

(19) a. $u(w, w') = \frac{1}{1+d(w,w')}$ [$d(w, w')$: the 'distance' between w and w']

b. $$d(w, w') = \begin{cases} 0 & \text{if } w = w' \\ 2 & \text{if } (w = w_1 \text{ and } w' = w_4) \text{ or } (w = w_4 \text{ and } w' = w_1) \\ 1 & \text{otherwise} \end{cases}$$

Intuitively, the weighted utility function assigns 1 if the intended meaning is completely recovered by the addressee, 1/2 if only the truth value of one of the coordinates is correctly recovered, 1/3 if the truth value of neither coordinate is correctly recovered.

The prediction of a model with the weighted utility is similar to the one based on the binary utility function discussed in Sect. 4, but with several intriguing differences, as can be seen in the list of Pareto-optimal inventories among inventories containing only the binary connectives in Table 5. In particular, with the weighted utility, empirically unattested inventories $\{\mathsf{OR}, \mathsf{NOR}\}$ and $\{\mathsf{AND}, \mathsf{NAND}\}$ turn out to be Pareto-optimal among the class of commutative connectives, although they aren't if the binary utility is used.

Although the notion of utility is not relevant for the Horn/K&S's account, weighted

inventory	**complexity**	**informativeness**
$\{\mathsf{OR}, \mathsf{AND}, \mathsf{NOR}\}$	10	0.875
$\{\mathsf{TAU}, \mathsf{OR}, \mathsf{AND}\}$	10	0.875
$\{\mathsf{OR}, \mathsf{NOR}\}$	7	0.75
$\{\mathsf{AND}, \mathsf{NAND}\}$	7	0.75
$\{\mathsf{TAU}, \mathsf{AND}\}$	7	0.75
$\{\mathsf{TAU}, \mathsf{OR}\}$	7	0.75
$\{\mathsf{OR}, \mathsf{AND}\}$	6	0.625
$\{\mathsf{TAU}\}$	4	0.604
$\{\mathsf{OR}\}$	3	0.5

Table 5: Pareto-optimal inventories predicted on the basis of the weighted utility function in (19) among combinations of commutative Boolean connectives.

utility along the lines of (14) has been employed for independent purposes (e.g., Jäger, 2007; Steinert-Threlkeld, 2019). Further investigation is required to assess the empirical validity of a particular choice of utility functions and the distance metric.

Prior The flat prior assumed in the model is also important for it to predict the correct lexicalization patterns. To see this, consider the comparison between $\{\mathsf{AND}, \mathsf{OR}\}$ and $\{\mathsf{AND}, \mathsf{NAND}\}$ again. It can be shown that $I(\{\mathsf{AND}, \mathsf{NAND}\}) > I(\{\mathsf{AND}, \mathsf{OR}\})$ if $2 \cdot P(w_4) > P(w_2) + P(w_3)$. That is, with such a non-flat prior, the model makes an incorrect prediction that $\{\mathsf{AND}, \mathsf{NAND}\}$ is more informative than $\{\mathsf{AND}, \mathsf{OR}\}$. This fact is potentially crucial for the theoretical interpretation of the model, as it may suggest that its success is dependent on its *encapsulation* with a restricted set of priors (Fox and Katzir, 2021), as opposed to its integration within a domain-general probabilistic model of communication that in principle allows non-flat priors.

6 Conclusions

In this paper, I have reformulated an existing explanation of the cross-linguistic absence of NAND due to Horn (1972) and Katzir and Singh (2013), by employing the informativeness/complexity trade-off model (Kemp, Xu, and Regier, 2018), which has been proposed as a general model for semantic universals. According to the reformulated account, inventories of connectives in natural languages optimise the trade-off between informativeness (measured in terms of the notion of communicative success, *given* scalar implicature) and complexity (measured in terms of the number of symbols required to express the connective meanings in Propositional Logic).

Within the restricted class of inventories consisting of connectives in the Aristotelian four corners (AND, OR, NAND, and NOR), the model makes empirically accurate predictions: it predicts that none of the possible inventories that make the optimal trade-offs involve NAND. Moreover, it predicts all and only empirically attested inventories (i.e., $\{\mathsf{AND}\}$, $\{\mathsf{OR}\}$, $\{\mathsf{AND}, \mathsf{OR}\}$ and $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$) to be the optimal inventories in the class. This is an improvement over Horn (1972) and Katzir and Singh (2013), who predict some of the unattested inventories to be in principle realisable in a natural language.

The model is generalisable to all possible combinations of 16 Boolean connectives (66,535 in total). The empirical validity of the full prediction of the model with respect the 66,535 inventories seems to be mixed. On the one hand, none of the inventories that are optimal in the relevant sense includes NAND, giving further support to the idea that its cross-linguistic absence is explained in terms of the general notion of informativeness and complexity. On the other hand, the optimal inventories include those that contain connectives that are arguably non-existent in natural language, such as the material implication $\rightarrow$. However, if we restrict our attention to the class of inventories that contain *commutative* connectives (255 in total), the Pareto-optimal inventories turn out to be precisely the four attested inventories plus one unattested inventory $\{\mathsf{TAU}, \mathsf{OR}, \mathsf{AND}\}$, which is arguably ruled out given an independent restriction against logically trivial meanings.

Overall, the current paper provides a proof of concept for an account of semantic universals in the domain of connectives based on the informativeness/complexity trade-off model. Along with similar recent studies in other domains of logical/functional vocabulary (Steinert-Threlkeld, 2019; Carcassi, 2020; Denić, Steinert-Threlkeld, and Szymanik, 2020; Enguehard and Spector, 2021), the current paper offers a promising prospect for the explanation of cross-linguistic universals in logical vocabularies within the trade-off model, motivating further research in the domain. Particularly interesting issues include empirical underpinnings of parameters affecting the model and the extent of their domain-specificity, as well as the model's compatibility with grammar-internal explanations of semantic universals (e.g., Romoli, 2015).

Notes

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[1]In this context, it is also worth mentioning that Chemla, Buccola, and Dautriche (2018) and Enguehard and Chemla (2021) have successfully extended a semantic constraints on content vocabularies—connectedness—to logical vocabularies/operations, i.e., quantifiers and exhaustification.

[2]While writing up the current paper, I was made aware of a closely related study by Enguehard and Spector (2021), who independently propose an account of the lexicalisation gap in the 'O-corner' in the square of opposition in the quantificational domain, based on the notion of informativeness/cost trade-off. In contrast to Enguehard and Spector (2021), who fully integrate their analysis within (a version of) the Rational Speech Act model, my goal in the current paper is to show that a fairly conservative reformulation of existing accounts by Horn (1972) and Katzir and Singh (2013) employing the trade-off model provides a satisfactory account of the lexicalisation pattern in the domain of connectives. One major difference between my analysis and Enguehard and Spector (2021) is that my analysis crucially assumes that negation adds to the complexity of inventories while Enguehard and Spector do not rely on this assumption. At the same time, the scope of the analysis is also different. Whereas I consider all possible combinations of Boolean connectives, Enguehard and Spector only demonstrate a comparison of two inventories: $\{A, E, I\}$ and $\{A, E, O\}$. See also Carcassi and Sbardolini (2021) for a related analysis based on a version of update semantics and a tradeoff between *cognitive complexity* and *use complexity*. I would like to leave a more extensive comparison of the current analysis with these alternative accounts to another occasion.

[3]The same generalisation specifically in the domain of connectives is also noted by Zwicky (1971: 124) while the broader generalisation about the lack of a lexical operator corresponding to the O-corner in the square of opposition is discussed by Löbner (1983);

Hoeksema (1999); Jaspers (2005); Seuren (2006), and von Fintel and Matthewson (2008).

[4]It should be noted that connectives in Warlpiri under Bowler's (2015) analysis achieves the semantic coverage of $\{\mathsf{AND}, \mathsf{OR}\}$ based on recursive exhaustification of OR. See also fn. 8.

[5]One might attempt to defend Horn/K&S by saying that, e.g., $\{\mathsf{NAND}\}$ should be ruled out because of the presence of $\{\mathsf{OR}\}$. After all, NAND and OR are comparable in their 'semantic coverages' in that both of them are true in exactly three 'rows' in a truth table. Note, however, that Horn/K&S's definition of Semantic Coverage is stated in terms of the specific corners of the square of opposition, and not in terms of the number of rows in a truth table. Thus, according to this definition, $\{\mathsf{NAND}\}$ and $\{\mathsf{OR}\}$ have separate Semantic Coverages. One might also point to various kinds of 'unnaturalness' of the inventories in (8). E.g., some of them involve a negated connective without having its positive counterpart. However, constraints based on such 'naturalness' are not part of Horn/K&S's theory par se. In contrast, my proposal to be discussed below can provide a principled explanation of their absence based on the informativeness/complexity trade-off.

[6] However, crucially, the lack of O-corner quantifiers, e.g., 'NALL' meaning 'not all', is not explained by the properties of natural language quantifiers considered by Steinert-Threlkeld (2019).

[7]Parentheses are not taken into account in the length measure in Table 2. The rationale for this is that the relevant notion of complexity only concerns the number of primitive elements in the formula, and that properties of the hierarchical structure of the combination of primitive elements do not by themselves contribute to complexity. It turns out that including parentheses in the length measure changes the prediction of the model slightly. The list of Pareto-optimal inventories will not include the inventories containing NOR. That is, Pareto-optimal inventories among 65,535 combinations of connectives will be those 10 inventories in Table 4(a) without the top one, and Pareto-optimal inventories

among combinations of commutative connectives will be those 4 inventories in Table 4(b) without the top one. This result is interesting in view of the relatively questionable empirical status of NOR as a lexical item in many languages (Gazdar, 1979; von Fintel and Matthewson, 2008).

[8]As far as I can see, it is not crucial for our purposes to define scalar implicature in this particular fashion. One may, for example, derive scalar implicatures as a combination of the primary implicature and the secondary implicature as in Sauerland (2004). However, we would presumably derive different predictions if scalar implicature is allowed to apply recursively, strengthening e.g., $\{\mathsf{OR}, \mathsf{P}, \mathsf{Q}\}$ into $\{\mathsf{AND}, \mathsf{P}, \mathsf{Q}\}$ (Fox 2007; Bowler 2015; cf. Singh et al. 2016; Bar-Lev and Fox 2020). It may also be possible to incorporate scalar implicature directly into the model of informativeness by assuming communicative agents according to the Rational Speech Act (RSA) model (e.g., Frank and Goodman, 2012). I would like to leave detailed investigation of the consequences of adopting different models of scalar implicature to a different occasion.

[9]This is so since the presence of XOR makes AND not innocently excludable relative to the prejacent OR.

[10]Consequently, the maximal inventory containing all 16 connectives does not have the highest possible informativeness value due to the abundant overlaps in meanings. It in fact has the informativeness value 0.46875, which is lower than e.g., $I(\{\mathsf{AND}, \mathsf{OR}\}) = 0.5$.

[11]Another point worth mentioning concerns the prediction with respect to the inventory $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$. Although the Horn/K&S account predicts this inventory to have the same Semantic Coverage as $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$, our model predicts $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$ to be less informative than $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}\}$. This is because our model of scalar implicature predicts neither OR nor NOR to be strengthened in $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$, due to the fact that the set of innocently excludable alternatives to OR among $\{\mathsf{AND}, \mathsf{OR}, \mathsf{NOR}, \mathsf{NAND}\}$ is just $\{\mathsf{NOR}\}$, and similarly that to NAND is

{AND}.

[12]I am very much indebted to Moysh Bar-Lev for discussion on this point.

[13]It is also conceivable that non-commutative connectives are inherently more complex than commutative ones, given that the former are order-sensitive while the latter aren't. However, it is not straightforward how order-sensitivity can be incorporated as part of the complexity measure as currently defined.

[14]One might consider an alternative constraint that restricts the set of lexicalisable connectives to those where both sentential arguments are RELEVANT to the truth value of the whole sentence, as e.g., formalised in Fox (2008: 250, (11)). This would rule out P, Q, NOTP, NOTQ, TAU, and CONT. It turns out, however, that running the trade-off model as defined above on inventories consisting of combinations of 10 remaining connectives does not eliminate unattested inventories. The Pareto-optimal inventories given the 10 connectives are { OR, AND, ONLYP, NOR }, { OR, AND, ONLYQ, NOR }, { OR, AND, NOR }, { OR, →, AND, ONLYQ }, { OR, ←, AND, ONLYP }, { OR, AND, ONLYQ }, { OR, AND, ONLYP }, { OR, →, AND }, { OR, ←, AND }, { OR, AND }, { OR }, and { AND }. This shows that the constraint in terms of relevance in the sense above, taken together with the trade-off model, is not sufficient in predicting the typological generalisations. I thank an anonymous reviewer for suggesting this possibility.

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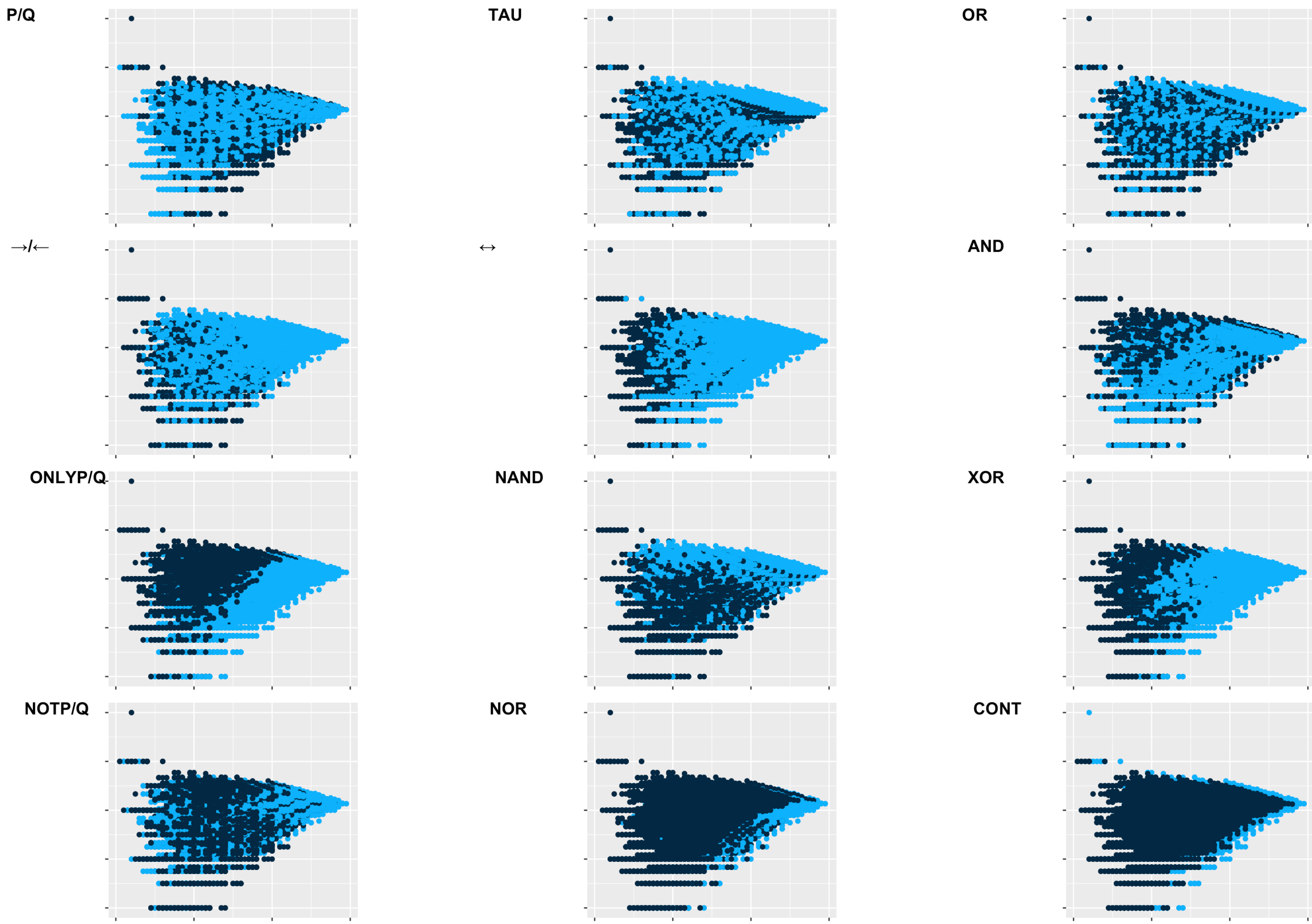

Figure 5: Plots of all possible inventories of Boolean connectives with blue dots indicating inventories containing individual connectives.