A Trellis-based Passive Beamforming Design for an Intelligent Reflecting Surface-Aided MISO System

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Abstract—In this paper, the downlink transmission of an intelligent reflecting surface (IRS)-assisted multiple-input single-output (MISO) system is investigated where the IRS elements are selected from a predefined discrete set of phase shifts. We minimize the mean square error (MSE) of the received symbols in the system via optimizing the phase shifts at the IRS jointly with beamforming vectors at the base station (BS) and equalizers at the user terminals. In order to find the optimal IRS phase shifts, a trellis-based structure is used that smartly selects the discrete phases. Moreover, for the sake of comparison, a semi-definite programming (SDP)-based discrete phase optimization is also presented. The BS beamformer and the optimal equalizers are determined via closed-form solutions. Numerical results demonstrate that the trellis-based scheme has better performance compared to other discrete IRS phase shift designs, such as SDP and quantized majorization-minimization technique, while maintaining a very low computational complexity.

Index Terms—Multiple-input single-output, Intelligent Reflecting Surface, Reconfigurable Intelligent Surface, Trellis.

I. INTRODUCTION

INTELLIGENT reflecting surface (IRS) has been introduced as a promising technique for providing a low-cost spectrum and energy efficient wireless solution that achieves high performance by reconfiguring the wireless propagation environment [1]. An IRS is a meta-surface with passive radio elements that reflect the RF waves towards a specific direction via passive reflection beamforming. Hence, deploying such a structure requires a large number of passive elements without any power amplifiers, which makes it a low-cost and power efficient technology. Motivated by these beneficial features, the topic of IRS-aided multiple-input single-output (MISO) systems has seen a significant surge in popularity among researchers in the field of 5G communications and beyond.

Recently, various passive beamforming schemes have been studied in the literature for different IRS-aided systems [2]. In [3], a multiple-input single-output (MISO) system was considered, in which an IRS was applied to assist the communication from the multi-antenna base station (BS) to multiple single-antenna users. Specifically, two solutions for IRS phase optimization problem were proposed based on semi-definite relaxation (SDR) and alternate optimization tech-

niques. The authors in [4] studied the effects of channel estimation error in an IRS-aided MISO system, and they presented an alternating algorithm for joint optimization of the BS and IRS beamformers using the Lagrangian method and the majorization-minimization (MM) technique, respectively. In [5], the authors considered the joint active and passive discrete beamforming optimization problem in an IRS-aided MIMO system to minimize the total transmit power. They presented an optimal solution for discrete phase optimization where the complexity was exponential with the number of IRS elements. To decrease the complexity, they designed a sub-optimal solution by using an iterative algorithm, where in each step, only one phase shift was optimized while the other phase shifts remained constant. This process was repeated until convergence was achieved. In [6], the energy efficiency maximization problem was studied through the joint power allocation and discrete phase optimization. The authors relaxed the problem by solving it for a continuous phase shift vector, and they used the quantization method to obtain discrete phase shifts. It was shown that in terms of average energy efficiency, there is a large gap between the discrete and infinite-resolution phase shifters, but in terms of average sum-rate, the quantization method has a near-optimal performance when the number of IRS elements is large. In [7] a mathematical-based approach is presented to handle the optimization of discrete IRS elements by using some auxiliary continuous variables and inserting some penalty terms into the objective function for converting these continuous variables and replacing non-convex components with approximated convex ones through using the successive convex approximation method. However this method seems to be too sub-optimal as there would be no well-optimized policies for converting the continuous variables into discrete ones, other than weighted penalty terms following some linear approximations.

It is worth noting that most methods in the literature deal with designing continuous phase shifts, which is impractical in real scenarios. In addition, few works devoted to adopting discrete phase shifts, either impose very high computational complexity, or are sub-optimal. Hence, this paper aims to propose a smart discrete IRS design by employing a trellis-based realization which results in a significantly low-complexity solution for its corresponding optimization problem.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we consider the downlink transmission of a multi-user MISO system aided by an IRS. The system

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contains a BS with M active antennas serving K single-antenna users. The downlink transmission is assisted by an IRS with N passive elements. Due to path loss, we only consider the first pilot signal that is reflected by the IRS and we ignore the signals that are reflected two or more times. Hence, the received signal by the ith user is written as $y^{[i]} = \sum_{k=1}^{K} \mathbf{g}^{[i]}\mathbf{v}^{[k]}s^{[k]} + n^{[i]}$, where $\mathbf{g}^{[i]} = \mathbf{h}_r^{[i]H}\mathbf{\Psi G} + \mathbf{h}_d^{[i]H}$ is the effective channel between the ith user and the BS, $\mathbf{v}^{[k]} \in \mathbb{C}^{M \times 1}$ is the active beamforming vector for the ith user, $s^{[k]}$ is the desired symbol for the ith user with unit power, and $n^{[i]}$ is the additive white Gaussian noise at the ith user with zero mean and variance σ^2. The IRS-user, BS-IRS and BS-user channels are denoted by $\mathbf{h}_r^{[i]} \in \mathbb{C}^{N \times 1}$, $\mathbf{G} \in \mathbb{C}^{N \times M}$ and $\mathbf{h}_d^{[i]} \in \mathbb{C}^{M \times 1}$, respectively. The IRS phase shift matrix is shown by $\mathbf{\Psi}$ which is a diagonal matrix with the discrete entries $\psi_k \in \mathcal{B}$, where $\mathcal{B}$ is the set of N_{IRS} possible discrete phases, shown by $\mathcal{B} = \{e^{\frac{j2\pi m}{N_{IRS}}}, m = 1, ..., N_{IRS}\}$. Each user applies a one-tap equalizer, denoted by $c^{[i]}$, to the received signal to detect the transmit symbol. Therefore, the estimated symbol for the ith user is $\hat{s}^{[i]} = c^{[i]}y^{[i]}$. In this paper, the criteria of minimizing the MSE is employed. To this end, we form the following optimization problem

$$\min_{\mathbf{V}, c^{[i]}, \mathbf{\Psi}} \quad \sum_{i=1}^{K} E|\hat{s}^{[i]} - s^{[i]}|^2$$
$$\text{s.t.} \quad \text{Tr}(\mathbf{VV}^H) \leq P, \quad (1)$$
$$\psi_k \in \mathcal{B}, k = 1, ..., N,$$

where $\mathbf{V} = [\mathbf{v}^{[1]}, ..., \mathbf{v}^{[K]}] \in \mathbb{C}^{M \times K}$, and P is the maximum transmit power at the BS. Note that the expectation is taken over the data symbols and the additive Gaussian noise received at the users. We assume that $E\{\mathbf{ss}^H\} = \mathbf{I}$ and $E\{\mathbf{s}s^{[i]*}\} = \mathbf{1}_i$, where $\mathbf{s} = [s^{[1]}, ..., s^{[K]}]^T$, and $\mathbf{1}_i$ is an all zero vector with a 1 at the ith element. With these assumptions at hand, the objective function in (1) can be rewritten as

$$MSE = \sum_{i=1}^{K} |c^{[i]}|^2 \text{Tr}(\mathbf{V}^H \mathbf{g}^{[i]H} \mathbf{g}^{[i]} \mathbf{V})$$
$$- 2\text{Re}\{c^{[i]} \mathbf{g}^{[i]} \mathbf{v}^{[i]}\} + |c^{[i]}|^2 \sigma^2 + 1. \quad (2)$$

It can be recognized that the optimization problem in (1) is non-convex. Therefore, we propose an alternating optimization technique to find solutions for each variable alternatively.

III. PROPOSED SOLUTION

In this section, an alternating optimization technique is presented to solve the problem in (1). To do so, in each step, the problem is solved with respect to only one variable, and this continues until some convergence criteria is met. In the following, the steps of this algorithm are presented.

A. Optimization of the equalizers

In the first step, given a fixed $\mathbf{V}$ and $\mathbf{\Psi}$, the optimal equalizer for each user is calculated. The optimization problem with respect to $c^{[i]}$ would be

$$\min_{c^{[i]}} \quad \sum_{i=1}^{K} |c^{[i]}|^2 \text{Tr}(\mathbf{V}^H \mathbf{g}^{[i]H} \mathbf{g}^{[i]} \mathbf{V})$$
$$- 2\text{Re}\{c^{[i]} \mathbf{g}^{[i]} \mathbf{v}^{[i]}\} + |c^{[i]}|^2 \sigma^2. \quad (3)$$

The solution to this problem is the Wiener filter [8],

$$c^{[i]} = \frac{\mathbf{v}^{[i]H} \mathbf{g}^{[i]H}}{\mathbf{g}^{[i]} \mathbf{VV}^H \mathbf{g}^{[i]H} + \sigma^2}. \quad (4)$$

B. Optimization of the BS beamformer

In the second step, given fixed $\mathbf{\Psi}$ and $c^{[i]}$, the optimization problem in (1) is solved with respect to $\mathbf{V}$. To this end, the optimization problem is rewritten as

$$\min_{\mathbf{V}} \quad \text{Tr}\left(\mathbf{V}^H \left(\sum_{i=1}^{K} |c^{[i]}|^2 \mathbf{g}^{[i]H} \mathbf{g}^{[i]}\right) \mathbf{V}\right)$$
$$- 2\text{Re}\left\{\sum_{i=1}^{K} c^{[i]} \mathbf{g}^{[i]} \mathbf{v}^{[i]}\right\} \quad (5)$$
$$\text{s.t.} \quad \text{Tr}(\mathbf{VV}^H) \leq P,$$

which can be reformulated by

$$\min_{\mathbf{V}} \quad ||\mathbf{VD} - \mathbf{I}||_F^2$$
$$\text{s.t.} \quad \text{Tr}(\mathbf{VV}^H) \leq P, \quad (6)$$

where $\mathbf{D} = \mathbf{CH}$, $\mathbf{C} = \text{diag}([c^{[1]}, ..., c^{[K]}])$ and $\mathbf{H} = [\mathbf{g}^{[1]T}, ..., \mathbf{g}^{[K]T}]^T$. This optimization problem in (6) is in the form of a least square problem. By assuming that the constraint does not exist, the solution to this optimization problem is

$$\mathbf{V} = \mathbf{D}^H(\mathbf{DD}^H)^{-1}. \quad (7)$$

In case $\text{Tr}(\mathbf{VV}^H) \leq P$, the solution stands, but if $\text{Tr}(\mathbf{VV}^H) > P$, the beamforming matrix $\mathbf{V}$ needs to be projected into a ball centered at the origin with radius P. Hence, $\mathbf{V}$ needs to be updated by

$$\mathbf{V} = \frac{\sqrt{P}\mathbf{V}}{\sqrt{P}||\mathbf{V}|| + \max\{0, \sqrt{P} - ||\mathbf{V}||\}}. \quad (8)$$

C. Optimization of the IRS discrete phase shifters

In the final step, the problem is solved with respect to $\mathbf{\Psi}$, for fixed values of $\mathbf{V}$ and $c^{[i]}$. To this end, the optimization problem becomes

$$\min_{\mathbf{\Psi}} \quad \sum_{i=1}^{K} |c^{[i]}|^2 \text{Tr}(\mathbf{V}^H \mathbf{g}^{[i]H} \mathbf{g}^{[i]} \mathbf{V})$$
$$- 2\text{Re}\{c^{[i]} \mathbf{g}^{[i]} \mathbf{v}^{[i]}\} \quad (9)$$
$$\text{s.t.} \quad \psi_k \in \mathcal{B}, k = 1, ..., N.$$

In the following, we propose a trellis-based method while introducing the semi-definite programming (SDP)-based alternative for the sake of comparison.

1) Trellis-based Solution

We assume discrete phase shifters for the IRS, i.e. the phases are selected from a set of discrete phases, referred to as $\mathcal{B}$. Therefore, assuming there are N_{IRS} possible phases in $\mathcal{B}$, there are $(N_{IRS})^N$ ways to allocate phases to the IRS elements. In the literature, this problem has either been solved by using the exhaustive search algorithm, which imposes a large computational complexity on the system, or it has been addressed by using sub-optimal techniques such as quantization. In spite of its low computational complexity, it can be shown that the quantization technique inflicts an inevitable loss to the system performance. To address these issues, here we employ a smart phase selection method with

low computational complexity, using trellis. Note that this technique can be easily employed in other system models.

To use the trellis method, the objective function must be a summation of real terms, where the jth term is a function of only the first j variables. In other words, using simple linear algebra, we reformulate the objective function in (9) as

$$
\begin{aligned}
f(\mathbf{\Psi}) = \sum_{k=1}^{N} \mathrm{Re} \Big\{ & \psi_k \big(\sum_{i=1}^{K} |c^{[i]}|^2 \mathbf{q}_k^{[i]} \mathbf{V}\mathbf{V}^H \mathbf{h}_d^{[i]} \big) \\
& + \psi_k \sum_{n=1}^{k-1} \psi_n^* \big(\sum_{i=1}^{K} |c^{[i]}|^2 \mathbf{q}_k^{[i]} \mathbf{V}\mathbf{V}^H \mathbf{q}_n^{[i]H} \big) \\
& - \psi_k \big(\sum_{i=1}^{K} c^{[i]} \mathbf{q}_k^{[i]} \mathbf{v}^{[i]} \big) \Big\},
\end{aligned}
\tag{10}
$$

where $\mathbf{q}_k^{[i]} \in \mathbb{C}^{1 \times M}$ is the kth row of the matrix $\mathbf{G}_c^{[i]} = \mathrm{diag}(\mathbf{h}_r^{[i]H})\mathbf{G}$, which represents the BS-IRS-user cascaded channel for the ith user. As shown in (10), to optimally select the phase of the kth element, we require only the phases of the first $k-1$ elements. Therefore, we can use trellis to solve the following optimization problem

$$
\begin{aligned}
\min_{\mathbf{\Psi}} \quad & f(\mathbf{\Psi}) \\
\mathrm{s.\,t.} \quad & \psi_k \in \mathcal{B}, k = 1, ..., N.
\end{aligned}
\tag{11}
$$

To solve the problem in (11), we first introduce a variable T which represents the memory window of the trellis, i.e. in each step, the trellis takes T of the variables ψ_k to form its states. Each of the variables ψ_k can have N_{IRS} possible phase choices. Hence, the trellis structure can be formed as in Fig. 1, with $(N_{IRS})^T$ states, each having N_{IRS} outgoing branches with labels selected from $\mathcal{B}$.

In the first stage, the first T variables are selected as initial memory, and their $(N_{IRS})^T$ permutations of phase selection are considered as the initial states. To move from current states to the next, in the nth stage where the memory includes nth to $(n+T-1)$th variables, the memory window moves to the $(n+1)$th to $(n+T)$th variables. Then, the next states are formed by considering all possible phase selections for the $(n+T)$th variable. There are multiple branches entering each states; therefore, to select one branch and remove the others, we use the MSE benchmark. In other words, at the nth stage, we calculate the value of (10) for the $(n+T)$th variable (ψ_{n+T}), and the benchmark for each branch is the $(n+T)$th term of (10). At each stage, the cumulative benchmark of branches are calculated by adding the branch benchmarks to the cumulative benchmark of their originating paths, and then among all branches entering the same state, all but one with the least cumulative benchmark are removed. The algorithm terminates after $N-T$ stages, when the window has covered all variables. Finally, the path with the least cumulative benchmark is selected as the final solution to the problem in (11). Algorithm 1 summarizes the trellis method.

2) SDP-based Solution

To facilitate a benchmark for the trellis-based solution, the SDP method could be employed for solving problem (9).

Algorithm 1 Trellis Algorithm to solve (11)

Input : $\mathcal{B}, \mathbf{\Psi}^{initial}$
Initialize all possible permutations of $\psi_1, \ldots, \psi_T$
  for $i = T+1, \ldots, N$ **do**
    for $j = 1, \ldots, N_{IRS}$ **do**
      $\psi_i = \mathcal{B}(j)$;
      Calculate (10) as the benchmark;
    end for
    Eliminate all paths except the one with the minimum benchmark value;
  end for
  Choose $\psi_i, \ldots, \psi_N$, such that they lead to the minimum cumulative benchmark value.
Output : $\mathbf{\Psi} = \mathrm{diag}([\psi_i, \ldots, \psi_N])$

To this end, by defining $\mathbf{f} = [\psi_1, ..., \psi_N]$ the optimization problem (9) would be reformulated as

$$
\begin{aligned}
\min_{\mathbf{f}} \quad & \mathrm{Tr}(\mathbf{\Gamma}\mathbf{f}^H\mathbf{f}) + 2\,\mathrm{Re}\{\mathrm{Tr}(\boldsymbol{\gamma}\mathbf{f})\} \\
\mathrm{s.\,t.} \quad & \psi_k \in \mathcal{B}, k = 1, ..., N,
\end{aligned}
\tag{12}
$$

in which $\mathbf{\Gamma} = \sum_{i=1}^{K} |c^{[i]}|^2 \mathbf{G}_c^{[i]} \mathbf{V}\mathbf{V}^H \mathbf{G}_c^{[i]H}$ and $\boldsymbol{\gamma} = \sum_{i=1}^{K} \left(|c^{[i]}|^2 \mathbf{G}_c^{[i]} \mathbf{V}\mathbf{V}^H \mathbf{h}_d^{[i]} - c^{[i]} \mathbf{G}_c^{[i]} \mathbf{v}^{[i]} \right)$. Now, by assuming $\mathbf{f}' = [\mathbf{f}, 1]$ and $\mathbf{\Delta} = \begin{bmatrix} \mathbf{\Gamma} & \boldsymbol{\gamma}^H \\ \boldsymbol{\gamma} & 0 \end{bmatrix}$, the equivalent optimization problem can be written by

$$
\begin{aligned}
\min_{\mathbf{F}} \quad & \mathrm{Tr}(\mathbf{\Delta}\mathbf{F}) \\
\mathrm{s.\,t.} \quad & \mathrm{diag}(\mathbf{F}) = \mathbf{1}_{(N+1)\times 1}, \quad \mathrm{Rank}(\mathbf{F}) = 1, \\
& \mathbf{F} \succeq 0, \quad \psi_k \in \mathcal{B}, k = 1, ..., N,
\end{aligned}
\tag{13}
$$

where $\mathbf{F} = \mathbf{f}'^H\mathbf{f}'$. By relaxing the optimization problem in (13) and removing the rank one constraint, we have

$$
\begin{aligned}
\min_{\mathbf{F}} \quad & \mathrm{Tr}(\mathbf{\Delta}\mathbf{F}) \\
\mathrm{s.\,t.} \quad & \mathrm{diag}(\mathbf{F}) = \mathbf{1}_{(N+1)\times 1} \\
& \mathbf{F} \succeq 0, \quad \psi_k \in \mathcal{B}, k = 1, ..., N,
\end{aligned}
\tag{14}
$$

For a rank-one matrix $\mathbf{F}$, vectors $\mathbf{f}'$ and $\mathbf{f}$ would be straightforwardly recovered from $\mathbf{F}$ and mapped into the discrete phases, otherwise, in case the rank of $\mathbf{F}$ is $r > 1$, the following procedure could be adopted

1) Solve SDP-driven problem (14) to reach an optimal solution $\mathbf{F}^*$. Owing to the fact that $\mathbf{F}^*$ is a positive semi-definite matrix, the Cholesky decomposition could be used as $\mathbf{F}^* = \mathbf{L}\mathbf{L}^H$.
2) Consider a random vector $\mathbf{w}$ following the distribution $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N+1})$.
3) For $s = 1, 2, \ldots, N+1$, let $\hat{\psi}_s = h(\mathbf{l}_s\mathbf{w})$, where $\mathbf{l}_s$ denotes the s-th row of $\mathbf{L}$ and the function $h()$ is defined as follows

$$
h(x) = \begin{cases} 1, & \arg(x) \in [\frac{-\pi}{N_{IRS}}, \frac{\pi}{N_{IRS}}) \\ e^{\frac{j2\pi}{N_{IRS}}}, & \arg(x) \in [\frac{\pi}{N_{IRS}}, \frac{3\pi}{N_{IRS}}) \\ \vdots \\ e^{\frac{j2\pi(N_{IRS}-1)}{N_{IRS}}}, & \arg(x) \in [\frac{(2N_{IRS}-3)\pi}{N_{IRS}}, \frac{(2N_{IRS}-1)\pi}{N_{IRS}}) \end{cases}
\tag{15}
$$
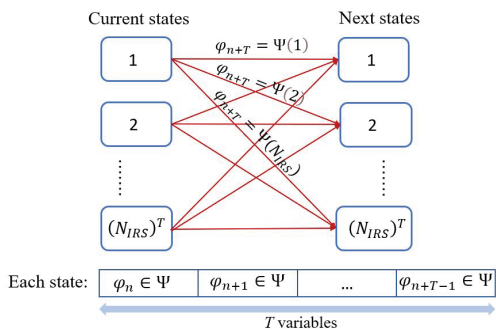
Figure 1: Trellis-based design for determination of $\boldsymbol{\Psi}$.

Algorithm 2 Overall Algorithm to solve (1)

Initialize : $\boldsymbol{\Psi}, \mathbf{V}, c^{[i]}$ such that all constraints are met.
RepeatUntil : convergence criteria are met.
  Phase1 : For fixed $\boldsymbol{\Psi}$ and $\mathbf{V}$, find $c^{[i]}$ using (4).
  Phase2 : For fixed $c^{[i]}$ and $\boldsymbol{\Psi}$ find $\mathbf{V}$ using (7).
      if $\text{Tr}(\mathbf{V}\mathbf{V}^H) > P$:
        Update $\mathbf{V}$ using (8)
  Phase3 : For fixed $c^{[i]}$ and $\mathbf{V}$ find $\boldsymbol{\Psi}$ via trellis/SDP.
Output : $\boldsymbol{\Psi}, \mathbf{V}, c^{[i]}$

Note that $\hat{\psi}_s \in \mathcal{B}$, $s = 1, \ldots, N$, satisfies the constraint of the optimization problem in (9) and $\hat{\boldsymbol{\Psi}} = \text{diag}\left([\hat{\psi}_1, \ldots, \hat{\psi}_N]\right)$ implies the final solution. It can be shown, this method leads to an $\frac{(N_{IRS} \sin(\frac{\pi}{N_{IRS}}))^2}{2N_{IRS}}$-approximation.

D. Overall Algorithm

In this section, the algorithm to solve the optimization problem in (1) is presented. We use a three phase alternating optimization technique. In the first phase the equalizers are optimized, in the second phase the optimal BS beamformer is determined and in the final phase the trellis method is employed for the IRS phase selection. The steps of the overall algorithm are explained in Algorithm 2.

IV. COMPLEXITY ANALYSIS

The proposed scheme in Algorithm 2 is iterative. In each iteration, the optimal values for $c^{[i]}$ and $\mathbf{V}$ are calculated by (4) and (7), respectively, which has the combined computational complexity in the order of $\mathcal{O}(K^3 + 2K^2M + MK)$. Also, the discrete IRS phase shifts are determined in each iteration via the trellis method in Algorithm 1. The trellis-based algorithm consist of $N - T$ stages. The optimization problem in (11) consists of $(N_{IRS})^T$ states with N_{IRS} branches entering each state. Hence, a total of $(N - T)(N_{IRS})^{T+1}$ comparisons are needed. This number is negligible compared to an exhaustive search approach with $(N_{IRS})^N$ comparisons, especially for large values of N. To have a comparison benchmark for the proposed method, we compare the trellis-based phase selection scheme with the SDP method, the MM-based method in [4] and the presented successive refinement algorithm in [5]. The

computational complexity of the SDP-based method using the interior point algorithm with an accuracy $\epsilon > 0$, would be $\mathcal{O}((N + 1)^{3.5})N_{IRS} \log(1/\epsilon) + 2N$ [9]. On the other hand, the complexity of the MM-based scheme is on the order of $\mathcal{O}(n(NM + N^2))$, where n represents the number of MM iterations, and the complexity of the method in [5] is on the order of $\mathcal{O}\left(\hat{I}_{itr}N_{IRS}(K^3 + K^2M + KMN)\right)$, where $\hat{I}_{itr} > N$ is the number of iterations required for achieving the convergence of the successive refinement algorithm. To have a better understanding, consider a system with $M = K = 4$ and $N = 100$. By using a trellis structure with $T = 2$ and $N_{IRS} = 2$, the complexity of the trellis-based method is 10^{-3} times that of the MM method in [4] and the successive refinement method in [5], and 10^{-5} times that of the SDP scheme. We show in Section VI that a trellis with $T = N_{IRS} = 2$ is enough for having a near-optimal performance.

V. INTERFERENCE ANALYSIS

Here, we aim to investigate the impact of presence of the IRS from the interference point of view. To this end, using the definition of $\mathbf{g}^{[i]} = \mathbf{f}\mathbf{G}_c^{[i]} + \mathbf{h}_d^{[i]H}$, the MSE in (2) is reformulated as $MSE = MSE_{\text{IRS-free}} + J_{\text{IRS}}$, where

$$MSE_{\text{IRS-free}} = \sum_{i=1}^{K} |c^{[i]}|^2 \, \text{Tr}(\mathbf{h}_d^{[i]H} \mathbf{V}\mathbf{V}^H \mathbf{h}_d^{[i]})$$
$$- 2\,\text{Re}\{c^{[i]}\mathbf{h}_d^{[i]H}\mathbf{v}^{[i]}\} + |c^{[i]}|^2\sigma^2 + 1, \quad (16)$$

represents the MSE in case the system is not equipped with an IRS and

$$J_{\text{IRS}} = \sum_{i=1}^{K} |c^{[i]}|^2 \, \text{Tr}(\mathbf{f}\mathbf{G}_c^{[i]}\mathbf{V}\mathbf{V}^H\mathbf{G}_c^{[i]H}\mathbf{f}^H)$$
$$+ 2|c^{[i]}|^2 \, \text{Re}\{\text{Tr}(\mathbf{V}^H\mathbf{h}_d^{[i]}\mathbf{f}\mathbf{G}_c^{[i]}\mathbf{V})\} - 2\,\text{Re}\{c^{[i]}\mathbf{f}\mathbf{G}_c^{[i]}\mathbf{v}^{[i]}\}, \quad (17)$$

stands for the IRS impact on the system. Note that, equipping the propagation environment with the IRS would lead to either a constructive impact or a destructive impact on the system performance based on the value of (17). In particular, since $MSE_{\text{IRS-free}}$ has a positive value, in order to provide a constructive impact, J_{IRS} in (17) should be negative. On the other hand, owing to the positive value of MSE, we have $|J_{\text{IRS}}| < MSE_{\text{IRS-free}}$. In section VI, the proposed method is evaluated in terms of the interference mitigation.

VI. NUMERICAL RESULTS

In this section, the performance of the smart trellis-based IRS phase selection algorithm is evaluated in terms of MSE per user. It is assumed that the channel vectors $\mathbf{h}_r^{[i]}$, $i = 1, \ldots, N$, and $\mathbf{G}$ are modeled by the Ricean fading with a Ricean factor of 10, while $\mathbf{h}_d^{[i]}$, $i = 1, \ldots, N$, is a Rayleigh flat fading. The large-scale path loss is modeled by $\beta(d) = \beta_0 d^{-\eta}$, where $\beta_0 = -30$ dB is the path loss at the distance of 1 m, d is the distance between the two nodes of the link in meters and η is the path loss exponent which is set to $\eta = 2$ for
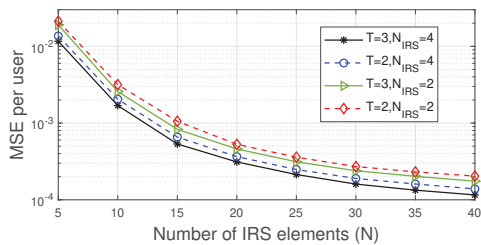
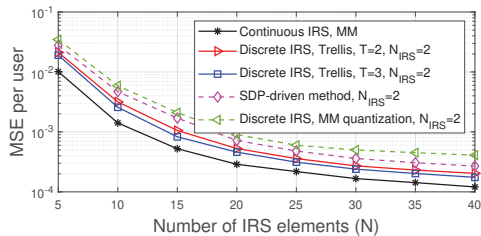Figure 2: MSE vs. N, for different values of T and N_{IRS}.

Figure 3: MSE vs. N, for different IRS beamforming methods.

channels with line of sight components and $\eta = 3$ for channels without line of sight components. The system is considered to have a BS with $M = 4$ antennas serving $K = 4$ single-antenna users randomly located within a 100 m vicinity of the BS. An IRS, located at the distance of 100 m from the BS, aids this communication by re-configuring the propagation environment. The IRS is equipped with N discrete elements with the resolution $N_{IRS} = 4$. The maximum power of the BS is $P = 10$ and the power of noise at each receiver is $\sigma^2 = 0$ dB. In the trellis structure, the memory of the trellis is $T = 2$, and we use a 4-PSK modulation to drive our results.

Fig. 2 demonstrates the performance of the trellis-based method versus N, for different values of T and N_{IRS}. As shown here, with larger trellis memory, the performance becomes better, since the trellis can take more IRS elements into consideration. Also, the higher the resolution of the IRS elements is, the lower the MSE gets. However, take into consideration that the better performance gained by higher values of T and N_{IRS} comes at the cost of higher computational complexity, as discussed in section IV. So, there is a trade-off between performance and complexity. Fig. 3 compares the performance of the trellis-based scheme to the MM method presented in [4]. The black graph is the solution of the MM method in case of having continuous IRS and the purple one is the case where in each iteration of the MM algorithm, the phase shifts are quantized with the resolution of $N_{IRS} = 2$. As shown in this figure, the trellis algorithm has a better performance compared to not only the quantized MM method with the same resolution but also the SDP-driven method. Moreover, by comparing the figures 3 and section IV, it can be concluded that a trellis structure with a resolution of $N_{IRS} = 2$ and the memory $T = 2$ is very efficient in terms of complexity and it has almost the same performance as the continuous IRS case solved with the MM method. In Fig. 4, the effect of an IRS employing the trellis-based scheme with $T = 2$

and $N_{IRS} = 2$ is investigated in two different systems with parameters $K = M = 4$ and $K = M = 6$. It is shown that the value of J_{IRS} is negative for different realizations of the system model, and the MSE which is given in (2) is reduced by increasing N. Thanks to such a negative value for J_{IRS}, it can be claimed that employing the IRS with a large number of reflectors results in an obvious constructive impact on the system performance from MSE standpoint in comparison with the case in which the system is not equipped with the IRS.
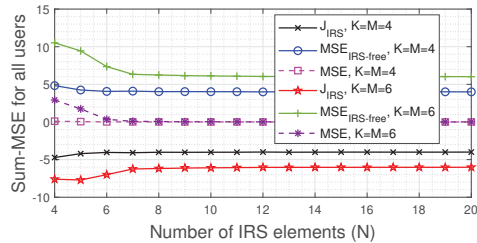

Figure 4: Interference versus number of IRS elements (N).

VII. CONCLUSION

In this paper, a smart phase selection scheme based on the trellis algorithm is presented for discrete phase optimization of an IRS-aided MISO system. This algorithm is compared to the MM solution in the literature, which was aimed for continuous IRS. The numerical results demonstrate that while the trellis-based scheme has much lower computational complexity than the MM method, the performance loss caused by the discrete phase shifts is negligible.

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