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DOI:

https://doi.org/10.3390/math10091404

Link to publication record in Manchester Research Explorer

Citation for published version (APA): Oyekola, O. (2022). How Resilient Is the U.S. Economy to Foreign Disturbances? *Mathematics*, 10(9). https://doi.org/10.3390/math10091404

#### Published in:

Mathematics

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Article

# How Resilient Is the U.S. Economy to Foreign Disturbances?

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**Abstract:** We assess the relative importance of domestic and foreign disturbances in explaining fluctuations in key macroeconomic variables and find that both types of shocks are equally important. We reach this conclusion within a constructed two-sector open economy DSGE model context, where we isolate the relative contributions of each group of disturbances to post-WWII U.S. business cycles. Our approach is to apply the indirect inference method to test the model's fit against a four-equation VAR(1) of output, real exchange rate, energy use, and consumption. Our main result is that foreign disturbances are pivotal to driving movements in these home variables; accounting for 38% of the variability in aggregate output, 73% of the variation in the real exchange rate, 45% of the variance of energy use, and 84% of the volatility of consumption. Further, foreign disturbances are also identified to be crucial for some other home macroeconomic variables, explaining larger fractions in changes to investment, labour hours, and real interest rate. However, the U.S. economy appears to be resilient to foreign disturbances with respect to certain macroeconomic variables; in particular, exports, imports, real wages, and domestic absorption.

**Keywords:** foreign disturbances; DSGE model; open economy macroeconomics; indirect inference; U.S. economy

MSC: 49M05; 91B51; 91B55; 91B66; 91B76

Citation: Oyekola, O. How Resilient Is the U.S. Economy to Foreign Disturbances? *Mathematics* **2022**, *10*, 1404. https://doi.org/10.3390/ math10091404

Academic Editor: David Barilla

Received: 21 February 2022 Accepted: 12 April 2022 Published: 22 April 2022

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## 1. Introduction

The conventional view is that U.S. economy shocks represent a major source of business cycles in most countries. Seen in both theoretical and empirical studies, this argument seeks to explain the external origins of aggregate macroeconomic fluctuations in other economies as being to a large extent due to the U.S. having been the world's foremost economic power for so long. Hence, a large strand of the literature has been committed to evaluating the international spill over effects and transmission mechanisms of U.S. economic activities ([1–5]). One shortcoming of these studies is that they are silent on the potential reverse effects, where disturbances originating in the rest of the world (the "foreign country") can play a non-negligible role in causing changes to U.S. economic outcomes. There is sufficient evidence to indicate that this has been the case on many occasions and that the U.S. itself is susceptible to foreign disturbances ([6–9]).

In this paper, we contribute to the debate on the relative importance of internal and external shocks as sources of U.S. macroeconomic fluctuations. To this end, we focus on twelve shocks; of which, nine originate within the U.S. economy (two sectoral productivity shocks, two sectoral energy efficiency shocks, preference shock, two sectoral investment-specific technology shocks, government spending shock, and labour supply shock) and the remaining three have their roots in the foreign country (world demand shock, oil price shock, and shock to the price of imported energy-intensive sector goods). We then ask the following questions: To what extent are U.S. macroeconomic fluctuations driven by foreign disturbances? Which indicators of economic activities do U.S. shocks, rather than foreign shocks, have larger effects on? In seeking answers to these questions, we

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hope to galvanise new interest, amongst academics and policymakers, in how models of the U.S. economy are designed and interpreted.

Our main results underscore the importance of both domestic and foreign disturbances for U.S. macroeconomic dynamics. Employing a four-equation VAR(1) of output, real exchange rate, energy use, and consumption, we find that the estimated model is able to reproduce the features of the U.S. data jointly, passing the Wald test with a transformed Mahalanobis distance of 1.46, thereby yielding a non-rejection of the model by the data at a 95% confidence level. The findings show that foreign disturbances are pivotal to driving movements in many U.S. macro variables, accounting for 38% of the variability in aggregate output, 73% of the variation in the real exchange rate, 45% of the variance of energy use, and 84% of the volatility of consumption. Foreign disturbances are also identified to be crucial for many other macro variables, explaining larger fractions in changes to investment (63%), labour hours (76%), and real interest rate (73%). These results are consistent with Christiano et al. [10] and Meenagh et al. [9] on the relevance of the open economy dimension (i.e., foreign shocks) in accounting for key macro variables. However, the U.S. economy tends to be more resilient to foreign disturbances when it comes to certain macroeconomic variables, such as exports, imports, real wages, and domestic absorption.

We make both theoretical and empirical contributions. On the theoretical side, the model used for our exercise is constructed in the spirit of the early international real business cycle (IRBC) literature ([11,12]). Our main departure from that literature is that we have adopted the production structure popularised by Kim and Loungani [13]—see also Backus and Crucini [14]—such that we can introduce oil price shock (Different factors have been credited as the source of oil shocks (e.g., disruptions in supply, changes in demand, and precautionary motives); see, for example, Stock and Watson [15] for a review of this literature. While we note that none of these explanations have gained universal acclaim in this ongoing debate, it is worth mentioning that the scope of our study and econometric strategy do not require us to distinguish between these different causes of oil price shocks; they enter implicitly in the model's error terms). Our model builds on these lines of research, where oil (like labour and capital) enters as a factor of production, with the U.S. characterised as an oil importer. A novel feature of our model is that the U.S. economy is disaggregated into energy-intensive and non-energy-intensive production sectors, as in Meenagh et al. [9]. We assume that goods from both sectors are traded internationally and that they form parts of consumption, investment, and government spending in both world economies. Following Bodenstein et al. [16], Schmitt-Grohé and Uribe [17], and Melek et al. [18], we utilise a model with no nominal rigidities as a benchmark to establish how well the model behaviour fits with the U.S. data dynamics.

On the empirical side, we apply the indirect inference method ([19–21]) on U.S. data over the period 1949–2013. The model is evaluated on a set of initial parameters put forward as true for our model, and thereafter *indirectly* estimated to achieve parameter values that move the model closest to the data dynamics that we seek. Hence, in estimation, we further apply the indirect inference techniques to search for alternative sets of coefficients that could do better in replicating the behaviour of U.S. data. This is the fundamental reason for using indirect inference: it provides a test of the model *itself* rather than that of a selected set of parameter values, whereas the more commonly used Bayesian method ([22,23]) does not guarantee this. Further, compared to frequentist methods, Bayesian estimation relies on priors, which in macroeconomics is still controversial because for some coefficients we may not have a strong understanding of what the priors should be—we find this to be the case for our model. To make progress, therefore, in building up support for modelling tactics, we require a method that tests the model against the data, without appeal to priors. The advantage of using indirect inference is that it provides a highly powerful test of our model against the data.

Our paper is closely related to the work of Meenagh et al. [9], who examined the effects of global shocks, relative to productivity, mark-up, and demand shocks, in account-

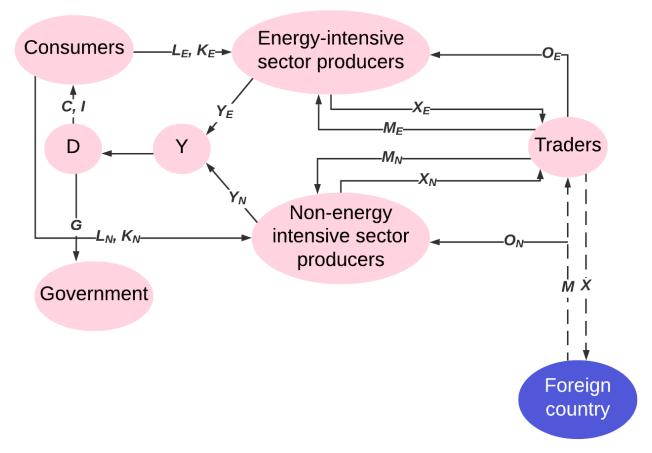
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ing for U.S. business cycle fluctuations. However, the current paper differs in three important ways. First, we assume that financial markets are complete following Backus and Crucini [14]. Second, we estimate our model using stationary data and VAR auxiliary model in four macro variables (output, real exchange rate, energy use, and consumption), while Meenagh et al. [9] employ non-stationary data and VECM auxiliary model in two macro variables (output and real exchange rate). Third, we follow Blankenau et al. ([24], p. 874) in using "the observable endogenous variables and the orthogonality conditions implied by the Euler equations to recover the exogenous shocks…" (Further details are provided in Appendix C). Whereas, Meenagh et al. [9] derived the parameters of the observed shock processes by making use of their corresponding actual data.

The remainder of this paper is organised as follows. Section 2 develops the one-commodity, two-sector, two-country open economy DSGE model. Section 3 discusses the method of indirect inference and the data used for model evaluation and estimation. Section 4 details the results. The final section concludes.

#### 2. The DSGE Model

In this section, we describe the general features of the DSGE model used for our analysis. The model was developed in Oyekola [25], which is a streamlined version of that considered by Meenagh et al. [9]. The model assumes that each economy is populated by four agents (consumers, producers, traders, and a government). It is also assumed that the finished goods of the two sectors are imperfect substitutes for similar products being produced abroad. We have assumed for our purpose that all markets are perfectly competitive and that there are no nominal rigidities ([16–18]), thereby taking seriously the critique of Chari et al. [26]. Figure 1 gives a visual representation of key macro variables and the interconnectedness between home and foreign countries.



**Figure 1.** The structure of the model.

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On the consumer side, households demand composite consumption good, C, make decisions on investment, I, and supply aggregate labour hours, L, to home producers. Households own firms in the two production sectors and provide them with the required physical capital stock, K, assumed to be subject to adjustment costs. All households have access to capital markets and can invest in a riskless one-period bond, B. On the supplier side, there are two production sectors consisting of firms producing two types of goods with different levels of energy intensities. The firms requiring a greater amount of energy for production make up the energy-intensive, E, sector producing energy-intensive goods,  $Y_{E}$ , and the remaining firms comprise the non-energy-intensive, N, sector producing nonenergy-intensive goods, Y<sub>N</sub>. Production in both sectors uses three factors of production, namely: labour ( $L_E$  and  $L_N$ ), capital ( $K_E$  and  $K_N$ ), and energy ( $O_E$  and  $O_N$ ). We assume that both labour and physical capital are mobile across sectors but are immobile internationally. Domestic firms, however, import their primary energy requirements. Just as aggregate consumption and investment by households are a composite of home and foreign goods (M), so is the government consumption spending, G. The foreign country also demands goods produced by home firms (X) (We note that both imports and exports are aggregates of two types of goods,  $M(M_E,M_N)$  and  $X(X_E,X_N)$ . Moreover, we have assumed symmetry of model structure between home and foreign economies).

For brevity, we mainly present the specific functional forms on preferences and technologies for the home country, as well as show the government's behaviour to be fully Ricardian. The listing of the model's equilibrium conditions, the full log-linearised representations, and the steady state values are documented in Appendix A. In what follows, G and  $\varepsilon_t^g$  are used interchangeably, and  $i \in \{H, F\}$  (H refers to the home economy and H the foreign economy) and H (H refers to energy-intensive sector and H the non-energy-intensive sector) will be used when we are distinguishing, respectively, between economies and sectors.

## 2.1. Consumers

The representative household maximises the expected utility function described by:

$$E_t \sum_{t=0}^{\infty} \beta^t \, \varepsilon_t^c \left[ \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \varepsilon_t^l \frac{(L_t)^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where  $E_t$  is an expectation's operator,  $C_t$  is a consumption bundle, and  $L_t$  is labour hours;  $\beta \in (0,1)$  is the intertemporal subjective discount rate,  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution in consumption,  $\varphi \geq 0$  is the inverse of the Frisch elasticity of labour supply, and h > 0 is the parameter of habit persistence.  $\varepsilon_t^c$  is the preference shock and  $\varepsilon_t^l$  is the labour supply shock.

Following Basu and Kimball [27], we assume that capital depreciation is a function of the intensity of its usage, and as in Dhawan and Jeske [28], we also assume that the accumulation of capital entails adjustment costs. Hence, the household's accumulation of the physical capital stocks,  $K_{j,t}$ , that are rented to each sector evolves according to the following law of motion:

$$K_{j,t} = \left(1 - \delta_{j,0} - \frac{\delta_{j,1}}{\delta_{j,2}} \left(U_{j,t}\right)^{\delta_{j,2}}\right) K_{j,t-1} + \varepsilon_t^{ij} I_{j,t} - \frac{\psi_j}{2} \left(\frac{K_{j,t} - K_{j,t-1}}{K_{j,t-1}}\right)^2 K_{j,t-1} \tag{2}$$

where  $I_{j,t}$  denotes sectoral investments,  $\delta_{j,0} \ge 0$  is the intercept of the depreciation function,  $\delta_{j,1} > 0$  is the slope of the depreciation function,  $\delta_{j,2} > 1$  is the elasticity of marginal depreciation with regards to the capital utilisation rate, and  $\psi_j \ge 0$  governs the costliness of capital installation.  $\varepsilon_t^{ij}$  is the sectoral investment-specific technological shock.

Given the above, we can express the household's budget constraint as:

$$C_t + I_t + E_t(F_{t+1}B_{t+1}) + T_t = W_tL_t + R_{E,t}U_{E,t}K_{E,t-1} + R_{N,t}U_{N,t}K_{N,t-1} + B_t + \Pi_t$$
 (3)

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where  $I_t$  is gross investment,  $F_{t+1}$  is the stochastic discount factor,  $B_{t+1}$  is the payoff in period t+1 of the portfolio held at the beginning of period t,  $T_t$  is lump-sum taxes (or transfers),  $W_t$  is wage rate,  $R_{j,t}$  is sector-specific return on capital services,  $U_{j,t}$  is sector-specific capital utilisation rate, and  $\Pi_t$  is the generated profits of domestic producers and traders.

#### 2.2. Producers

The outputs,  $Y_{j,t}$ , of both sectors are produced by competitive firms using the constant elasticity of substitution (CES) technology that employs three factors (labour hours,  $L_{j,t}$ , capital services,  $U_{j,t}K_{j,t-1}$ , and imported energy,  $O_{j,t}$ :

$$Y_{j,t} = \varepsilon_t^{\gamma j} \left( L_{j,t} \right)^{1-\alpha_j} \left[ \theta_j \left( U_{j,t} K_{j,t-1} \right)^{-\nu_j} + \left( 1 - \theta_j \right) \left( \varepsilon_t^{oj} O_{j,t} \right)^{-\nu_j} \right]^{-\frac{\alpha_j}{\nu_j}} \tag{4}$$

where  $\alpha_j$ ,  $\theta_j \in (0,1)$ , and  $\nu_j \in (0,\infty)$  measure output elasticity of labour hours  $(1-\alpha_j)$ , weight of capital services in production, and production elasticity of substitution, respectively.  $\varepsilon_t^{\nu j}$  is sectoral productivity shock and  $\varepsilon_t^{oj}$  is sectoral energy efficiency shock.

The period t profits of sectoral producers are given by:

$$\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t L_{j,t} - R_{j,t}U_{j,t}K_{j,t-1} - \varepsilon_t^{po}O_{j,t}$$
(5)

where  $P_{j,t}$  stands for the relative prices of sectoral goods and  $\varepsilon_t^{po}$  the exogenous world price of oil.

#### 2.3. Traders

As mentioned above, we assume, following Backus et al. [29], that goods produced in both sectors of the home economy are imperfect substitutes for similar goods produced in equivalent sectors in the foreign economy. The new feature of our modelling strategy is that we have two goods from each country rather than one. Given this, consumption, investment, and exogenous government spending in both countries are assumed to be bundles of four goods. In the home economy, the respective definitions are  $C_t = \Phi_C(C_{i,t}^j)$ ,  $I_t = \Phi_I(I_{i,t}^j)$ , and  $G_t = \Phi_G(G_{i,t}^j)$ , where the aggregator functions  $\Phi_C$ ,  $\Phi_I$ , and  $\Phi_G$  (and all the ones defined hereafter) are assumed to be increasing and homogeneous of degree one in their arguments.

Meanwhile, to maintain the focus of our analysis on the effects of foreign disturbances on aggregate economic activities, we use total spending by households and government in the home economy to bundle all good types. Formally, this is given as:

$$D_t = C_t + I_t + G_t \tag{6}$$

where  $D_t$  is domestic absorption (and can now be interpreted as a composite of the four final goods in this world economy). Hence, we can write that:

$$D_{t} = \left(\kappa^{\frac{1}{\phi}}(D_{H,t})^{\frac{\phi-1}{\phi}} + (1-\kappa)^{\frac{1}{\phi}}(M_{t})^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$
(7)

where  $D_{H,t}$  is domestic absorption of goods produced in the home economy,  $M_t$  is total imports,  $\phi > 0$  is the elasticity of substitution between home and foreign goods, and  $\kappa \in (0,1)$  is the home bias parameter.

As shown by Backus et al. [29], Equation (7) will suffice if, as they did, we are modelling two countries with two goods. However, we need another level of disaggregation because we have a model consisting of two countries and four goods. We do this by defining domestic absorption and aggregate imports of goods in the home economy as functions of energy-intensive and non-energy-intensive goods:

$$D_t = (\gamma^{\frac{1}{\varsigma}} (D_{E,t})^{\frac{\varsigma-1}{\varsigma}} + (1-\gamma)^{\frac{1}{\varsigma}} (D_{N,t})^{\frac{\varsigma-1}{\varsigma}})^{\frac{\varsigma}{\varsigma-1}}$$
(8)

and

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$$M_{t} = \left(\chi^{\frac{1}{\eta}} (M_{E,t})^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} (M_{N,t})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$
(9)

where  $D_{j,t}$  and  $M_{j,t}$  are, respectively, the total demand of the home economy's households and government of sector j goods produced in home and foreign economies.  $\varsigma, \eta > 0$  are the elasticity of substitution parameters across the sectoral goods, and  $\gamma, \chi \in (0,1)$  are the bias parameters for the energy-intensive goods.

The profits of traders can be written as:

$$\Pi_{T,t} = \Pi_{\Phi_{D,t}} + \Pi_{\Gamma_{D,t}} + \Pi_{\Gamma_{M,t}} \tag{10}$$

where  $\Pi_{\Phi_D,t} = P_t D_t - P_{H,t} D_{H,t} - M_t$ ,  $\Pi_{\Gamma_D,t} = P_t D_t - P_{E,t} D_{E,t} - P_{N,t} D_{N,t}$ , and  $\Pi_{\Gamma_M,t} = M_t - \varepsilon_t^{fe} M_{E,t} - \varepsilon_t^{fn} M_{N,t}$ . Consumer price index (CPI) is defined as  $P_t = (\gamma(P_{E,t})^{1-\varsigma} + (1-\gamma)(P_{N,t})^{1-\varsigma})^{1/(1-\varsigma)}$ ; this is also the real exchange rate. The only other price that enters the (solved) log-linearised model is the exogenous world price of imported energy-intensive goods,  $\varepsilon_t^{fe}$ .  $P_{H,t}$  and  $\varepsilon_t^{fn}$ , which are, respectively, the price index of composite goods produced in the home economy and the exogenous world price of non-energy-intensive goods, do not show up in the equations used for model simulation. In an empirical paper on the associations between world shocks, world prices, and business cycles in 138 countries covering the period 1960–2015, Fernández et al. [30] documented that it is important to specify multiple world prices rather than one to elicit the true effects of foreign shocks on the domestic output of a country. Our modelling strategy is thus consistent with their empirical proposition and finding.

#### 2.4. Government

The government is assumed to balance its budget in each fiscal period t. As such, the exogenous government spending level,  $\varepsilon_t^g$ , per period is covered by tax revenues and borrowing from the households:

$$\varepsilon_t^g + B_t = T_t + E_t(F_{t+1}B_{t+1}) \tag{11}$$

where the assumption is that the government can raise or reduce taxes or transfers through the Treasury and can increase or decrease the short-term nominal interest rate through the Federal Reserve to achieve its policy stance ([31]).

#### 2.5. Foreign Economy

All the agents—consumers, producers, traders, and the government—in the foreign economy are making identical decisions to those of the home economy. Here, we focus on the two equilibrium conditions needed to close the trade account between the two countries, namely: aggregate imports and imports of energy-intensive goods by traders in the foreign economy. To avoid redundancy, we do not describe variables and parameters of the foreign economy with similar meanings to their home economy counterparts. First, we assume that the exogenous total foreign absorption,  $D_{F,t}$ , can be written analogously as:  $D_{F,t} = C_{F,t} + I_{F,t} + G_{F,t}$ . Noting also that  $D_{F,t}$  is a composite of foreign and home goods, we write that:  $D_{F,t} = \Phi_{D_F}(D_{W,t}, M_{F,t}) = \Phi_{D_F}(D_{W,t}, X_t)$ , where  $D_{W,t}$  is the foreign absorption of goods produced in the ROW, and  $M_{F,t} = X_t$  is either total imports (from the viewpoint of foreign traders) or total exports (from the standpoint of home traders). Adopting the latter representation leads to:

$$\varepsilon_t^{df} = ((\kappa_F)^{\frac{1}{\phi_F}} (D_{Wt})^{\frac{\phi_F - 1}{\phi_F}} + (1 - \kappa_F)^{\frac{1}{\phi_F}} (X_t)^{\frac{\phi_F - 1}{\phi_F}})^{\frac{\phi_F}{\phi_F - 1}}$$
(12)

where  $\varepsilon_t^{df}$  has been used to replace  $D_{F,t}$  to keep with our representation of exogenous variables. Moreover, our discussion so far dictates that  $\varepsilon_t^{df}$  is likewise a composite of four types of goods. Hence, aggregate exports can be expressed as:

$$X_{t} = ((\chi_{F})^{\frac{1}{\eta_{F}}}(X_{E,t})^{\frac{\eta_{F}-1}{\eta_{F}}} + (1 - \chi_{F})^{\frac{1}{\eta_{F}}}(X_{N,t})^{\frac{\eta_{F}-1}{\eta_{F}}})^{\frac{\eta_{F}}{\eta_{F}-1}}$$
(13)

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### 2.6. Market Clearing

We close the model by imposing the following clearing conditions on the markets. Total investment of the home economy is given as the sum of the sectoral investments:

$$I_t = I_{E,t} + I_{N,t} (14)$$

Likewise, total output is given as the sum of outputs of both sectors of the home economy:

$$Y_t = Y_{E,t} + Y_{N,t} (15)$$

Further, the output of each sector is either absorbed into the home economy or sold to the foreign economy. Hence, the energy-intensive goods market clears in the home economy:

$$Y_{E,t} = D_{E,t} + X_{E,t} - M_{E,t} (16)$$

where, by Walras' law, the market also clears for non-energy-intensive goods.

In the factor markets, equilibrium requires that labour and energy markets clear:

$$L_t = L_{E,t} + L_{N,t} \tag{17}$$

$$O_t = O_{E,t} + O_{N,t} (18)$$

Further, current account constraint in the home economy is satisfied by:

$$P_t X_t = M_t + \varepsilon_t^{po} O_t \tag{19}$$

Finally, given that current account holds in equilibrium, total demand will also be equal to total supply:

$$D_t = Y_t \tag{20}$$

## 2.7. Exogenous Shocks

Business cycle fluctuations in our model are driven by stochastic disturbances to twelve shocks: nine home shocks and three foreign shocks. The nine home shocks include four productivity shocks (two sectoral total factor productivity shocks,  $\varepsilon_t^{yj}$ , and two sectoral energy efficiency shocks,  $\varepsilon_t^{oj}$ ), four demand shocks (preference shock,  $\varepsilon_t^c$ , two sectoral investment-specific technology shocks,  $\varepsilon_t^{ij}$ , and government spending shock,  $\varepsilon_t^g$ ), and a labour market shock (labour supply shock,  $\varepsilon_t^l$ ). The three foreign shocks are the world price of crude oil,  $\varepsilon_t^{po}$ , price of imported energy-intensive sector goods,  $\varepsilon_t^{fe}$ , and world demand,  $\varepsilon_t^{df}$ . All twelve shocks are assumed to follow first-order autoregressive (AR(1)) processes, with *i.i.d.* error terms.

### 2.8. Equilibrium Condition

Equilibrium consists of a set of endogenous stochastic processes  $\{C_t, I_t, I_{j,t}, L_t, L_{j,t}, K_{j,t}, U_{j,t}, O_t, O_{j,t}, Y_t, Y_{j,t}, D_t, D_{E,t}, M_t, M_{E,t}, X_t, X_{E,t}, P_t, P_{j,t}, W_t, r_t, R_{j,t}\}$  for  $t = 0, ..., \infty$ , satisfied by the first-order conditions of the consumers, producers, and traders, given the initial conditions  $C_{t-1}$ ,  $K_{j,t-1}$ , and  $B_t$ , the realisations of a set of exogenous stochastic processes  $\{\varepsilon_t^c, \varepsilon_t^l, \varepsilon_t^{ij}, \varepsilon_t^{yj}, \varepsilon_t^{oj}, \varepsilon_t^{po}, \varepsilon_t^g, \varepsilon_t^{fe}, \varepsilon_t^{df}\}$  for  $t = 0, ..., \infty$ , and the markets for labour, capital, energy, goods, and bonds all clear.

## 3. Indirect Inference Methodology and Data

## 3.1. Indirect Inference Methodology

We examine the capacity of the model in fitting the data by using the method of indirect inference initially proposed in Minford et al. [32]. The approach uses an auxiliary model that is completely independent of the theoretical model to produce a description of the data against which the theoretical model's performance is assessed indirectly. Such a Mathematics 2022, 10, 1404 8 of 33

description can be summarised either by the estimated parameters of the auxiliary model or by functions of these; we will call these the descriptors of the data. While these are treated as the "reality", the theoretical model being evaluated is simulated to find its implied values for them. Although model evaluation and estimation by indirect inference are related, with the common feature being the use of an auxiliary time-series model in addition to the structural macroeconomic model, they are not the same. Given, therefore, the not-so-familiar nature of the method of indirect inference, we discuss below how to use it for both model evaluation and estimation.

Indirect inference has been widely used in the estimation of structural models; see, for example, Canova [2], Gouriéroux et al. [19], and Smith [20]. Here, we make additional use of the indirect inference techniques as a framework for evaluating a calibrated or already, but perhaps partially, estimated structural model. In estimating the model, we choose the parameters of the structural model such that when we simulate this model it produces estimates of the auxiliary model akin to those that are derived from actual data. The parameters that are optimal for the structural model are those selected to minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. The standard functions selected for this purpose include the actual coefficients, the scores, or the impulse responses. In evaluating the model, we take the parameters of the structural model as given. The goal here is to determine how well the auxiliary model estimated on simulated data (that is constructed from the given estimates of a structural model-i.e., a true model, the null hypothesis) performs when judged against the auxiliary model estimated based on actual data. The idea is that the predictions of the structural model, in terms of the impulse responses, moments, and time-series properties of the simulated data, should statistically match the ones from actual data if the structural model is true. The distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates, are the bases for this appraisal.

We note that the testing procedure by indirect inference requires that we first calculate the errors implicit in the hitherto calibrated or estimated model and the data, which are known as the structural errors. We employ the model equations and the data to extract these errors directly. As certain structural equations may require the computation of expectations, the method we utilise is the robust instrumental variables approach ([33,34]). This method entails the calculation of the fitted values from a VAR(1) by setting the lagged endogenous data as instruments. Second, the errors are bootstrapped and used to generate for each bootstrap new data based on the structural model. The bootstraps in our tests are all drawn as time vectors so contemporaneous correlations between the innovations are preserved. An auxiliary time-series model is then fitted to each set of data and the sampling distribution of the coefficients of the auxiliary time-series model is obtained from these estimates of the auxiliary model. Lastly, a Wald statistic is computed to determine whether functions of the parameters of the time-series model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

We use as the auxiliary model a VAR(1) that includes output, real exchange rate, energy use, and consumption. In practice, the auxiliary model is represented by:  $Ax_t = B(L)x_{t-1} + \epsilon_t$ , where A and B(L) are, respectively, an n by n matrix of coefficients and polynomials in the lag operator, L, and  $\epsilon_t$  is an n by 1 vector of a mean zero, serially uncorrelated random structural disturbance, such that  $E(\epsilon_t, \epsilon_t') = \Sigma$  represents its finite diagonal variance-covariance matrix.  $A^{-1}B(L)$  is treated as the descriptors of the data for the VAR coefficients on the endogenous variables, and  $var[\epsilon_t]$  is treated as the VAR error variances (Our choice of the auxiliary model exploits the fact that the solution to a log-linearised DSGE model can be represented as a restricted VARMA model, which can be closely represented by a VAR (see, for example, Canova [2]). The implication is that the true VAR should be of infinite order, at least if any DSGE model is the true model. However, for the same reason that we have not raised the VAR order above one, we have also not added any MA elements. As DSGE models do better in meeting the challenge, this could be considered. Moreover, recent surveys based on Monte Carlo experiments found

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that a VAR(1) with two variables typically gives the right level of power for testing macro models ([35,36]). This is because a more rigorous pass criterion is set, the higher the number of variables and lags. We have therefore restricted the number of variable combinations and lag order to a maximum of four for the auxiliary VAR(1) model for which we present results below. The importance of this directed Wald test, according to Le et al. [21], is that it helps to narrow down the economic questions that can be addressed using the constructed structural economic model). Based on these descriptors, the Wald statistic is computed to serve as a test of whether the observed dynamics and volatility of the selected variables are reproduced by their DSGE-model simulation of joint distribution at a given confidence level. This Wald statistic is given by:

$$(\Theta - \overline{\Theta})' \sum_{(\Theta\Theta)}^{-1} (\Theta - \overline{\Theta}) \tag{21}$$

where  $\Theta$  is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with  $\overline{\Theta}$  and  $\Sigma_{(\Theta\Theta)}$  capturing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively. The joint distribution of the vector of VAR estimates,  $\Theta$ , is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is, therefore, an estimate of the small sample distribution. Such a distribution is generally more accurate for small samples than the asymptotic distribution. It is also shown to be consistent by Le et al. [21] given that the Wald statistic is "asymptotically pivotal". Further, these authors also showed that it had quite good accuracy in small sample Monte Carlo experiments. Specifically, they found on stationary data, as we use in this study, that the bias due to bootstrapping was just over 2% at the 95% confidence level and 0.6% at the 99% level.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones ( $H_0$ , the null hypothesis); they can be derived from calibration, estimation, or a combination of both. Regardless of how parameter values are derived, the test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? Of course, only one true model with one set of coefficients is possible. Nevertheless, we may have chosen coefficients that are not exactly right numerically, so that the same model with other coefficient values could be correct. Only when we have examined the model with all coefficient values that are feasible within the model theory will we have properly tested it. The inference is made by comparing the percentile of the Wald distribution in which the test statistic falls within the chosen size of the test. For instance, for 5% significance, a percentile above 95% marks rejection. We can likewise present this information using transformed Mahalanobis distance based on the same joint distribution, which is normalised as a t-statistic (The Mahalanobis distance is the square root of the Wald value, and as the square root of a chi-squared distribution, it can be converted into a t-statistic by adjusting the mean and the size. This is then normalised to ensure that the resulting t-statistic is 1.645 at the 95% point of the distribution. Written mathematically, this is:  $t = 1.645\{(\sqrt{2w^a} - \sqrt{2n}) \div (\sqrt{2w^{0.95}} - \sqrt{2n})\}$ , where t is the normalised t value,  $w^a$  is the Wald statistic calculated from actual data,  $w^{0.95}$  is the Wald statistic for the 95th percentile of the simulated data, and n is the degrees of freedom. The method described here is based on the method of transforming chi-squared distribution into the standard normal distribution of Wilson and Hilferty [37]). Non-rejection of the null hypothesis is taken to indicate that dynamic behaviour of the structural macroeconomic model is not materially different from that of the actual data. Rejection is taken to imply that the structural macroeconomic model is incorrectly specified. For this reason, we later extend our procedure by a further search algorithm, in which we seek alternative coefficient sets that could do better in the test.

Thus, we calculate the minimum-value full Wald statistic using an algorithm in which search takes place over a wide range around the initial values (We use the simulated annealing algorithm due to Ingber [38], which mimics the behaviour of the steel cooling process in which steel is cooled, with a degree of reheating at randomly chosen

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moments in the cooling process, thereby ensuring that the defects are minimised globally. Similarly, the algorithm searches in the chosen range, and as points that improve the objective are found, it also accepts points that do not improve the objective. We find that this algorithm improves substantially here on a standard optimisation algorithm; Chib et al. [39] report that in their experience the simulated annealing algorithm deals well with distributions that may be highly irregular in shape, and much better than the Newton-Raphson method. Our method is to apply our standard testing procedure: we take a set of model parameters (excluding error processes), extract the resulting residuals from the data using the limited information maximum likelihood (LIML) method, find their implied autoregressive coefficients (AR(1), here given stationary data), and then bootstrap the implied innovations with this full set of parameters to find the implied Wald value. The simulated annealing algorithm then minimises this). In effect, this is the estimation of the model by the method of indirect inference. However, here, this estimation is being performed to find whether the model can be rejected in itself and not for the sake of finding the most satisfactory estimates of the model parameters. Nevertheless, of course the method does this latter task as a byproduct so that we can use the resulting unrejected model to represent the best available estimated version. The merit of this extended procedure is that we are comparing the best possible versions of each model type when finally conducting our comparison of model compatibility with the data.

#### 3.2. Data

The raw data were gathered from a variety of sources, including the U.S. Bureau of Economic Analysis (BEA), Bureau of Labour Statistics (BLS), Federal Reserve Economic Data (FRED), and Energy Information Administration (EIA). The data used in the empirical exercises are yearly observations that cover the period 1949 to 2013. We note that all real quantities are in per capita terms and logged, with the home variables deflated by the U.S. CPI, while the rest of the world CPI is used to deflate foreign series. Moreover, all series were filtered using Hodrick-Prescott's procedure, where we have set the smoothing parameter to 400, as in Kim and Loungani [13]. To create empirical counterparts to the variables in our model, we follow Meenagh et al. [9] to construct aggregate measures and disaggregate the data into energy-intensive and non-energy-intensive sectors. In particular, we collect aggregate data on output, consumption, investment, exports, imports, labour hours, and primary energy (crude oil) demand by producers, domestic absorption, real exchange rate, and real interest rate. In addition to these, we also obtain sectoral observations on output, investment, labour hours, capital stock, capital utilisation rate, energy consumption, and the sectoral prices of goods. Finally, we gather, in relation to energy-intensive goods, observable time-series for domestic absorption, exports, and imports. For detailed data sources and definition of variables, see Appendix B; interested readers can also find line plots of the data in Oyekola [25].

#### 4. Quantitative Results

Using the method of indirect inference, the model is evaluated against U.S. data for the 1949–2013 period. As pointed out in Section 3, as the number of variables and the VAR order are raised, so is the power of the indirect inference test. Hence, the primary focus of this paper is on the ability of the model to replicate the joint distribution of the chosen macroeconomic variables. Towards this end, the main macroeconomic variables that we considered are: (1) aggregate output, which serves as a measure of the home economy's overall economic performance, (2) real exchange rate, which serves as a measure of the home economy's competitiveness against the rest of the world, (3) energy use since it is an input whose exogenously determined world price has historically had significant adverse effects on the home economy, and (4) consumption, which may serve as a measure of well-being of households in the home economy. Based on the calibrated parameters, the model is rejected with a Wald percentile of 100 and transformed Mahalanobis distance of 40.14 (The calibrated parameter values used to initially test the model and to kick-start

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the simulated annealing algorithm, along with the implied persistence and volatility values for the shock processes are reported in Appendix C. Additionally, Appendix C contains values of the long-run structural parameters that are kept fixed throughout the empirical work).

We are thus compelled to test the model itself more fully by searching over the full range of potential values permitted by the model's theory, to find the best set of values from the model's viewpoint: if rejected on that set, then the model itself is rejected. Such a search is the purpose of indirect inference estimation. Using these estimation procedures, we optimally locate values for a set of parameters within the model's theoretical structure, such that the model is not rejected according to the Wald test statistics, when we adopt conventional significance levels. In the rest of this section, we report the estimated parameters, discuss the assessment of model fit, and investigate how important foreign disturbances are to the U.S. economy.

#### 4.1. Parameter Estimates

Table 1 documents the estimated parameter values. The estimate for the inverse of the elasticity of substitution in consumption is 5.59, which is well within the range found in many DSGE models ([40,41]). The consumption elasticity of 0.18 found here implies that households are less willing to smooth consumption across time in response to a change in the real interest rate. The estimated value of the habit formation parameter (h = 0.38) suggests that a household's current consumption is less dependent on its level in the previous period. Our reading of this is that this may be due to the yearly data frequency, as it suggests that households are likely to adjust more to shocks over the course of a year relative to a quarter. The high value for Frisch elasticity of labour supply  $(1/\varphi = 0.12)$  signifies that labour hours react more to changes in real wages.

The estimated values for the share of capital services and energy demand in production in both sectors are low but not unreasonable ( $\nu_E = 0.07$ ). The values imply that the two production functions are closer to the Cobb–Douglas form than the assumed general CES form, particularly in the non-energy-intensive sector; see Kim and Loungani [13]. Both estimates for the adjustment cost parameters are close to zero, although they have different quantitative impacts on the model. As documented, the value of  $\psi_E = 0.003$  compared to  $\psi_N = 0.002$  means that the marginal user cost in the energy-intensive sector responds by 50% more to changes in interest rate than does the marginal user cost in the accumulation of non-energy-intensive sector capital goods.

Further, the estimate of the marginal costs of capital utilisation is 32% in the energy-intensive sector, while it is 44% in the non-energy-intensive sector. These different estimates simply indicate that return to investment in the latter sector is higher than in the former one. Our estimates for the elasticities of capital utilisation rates are 5.51 and 4.32, respectively, for the energy-intensive and non-energy-intensive sectors. All the elasticities of substitution parameters in the aggregator functions are sensibly in the ballpark of other estimates found in the existing trade literature.

Parameter	Value	Description
$\varphi$	8.64	Frisch elasticity of labour supply
σ	5.59	Elasticity of substitution in consumption
$ u_E$	0.12	Elasticity of substitution between $K_E$ and $O_E$
$ u_N$	0.07	Elasticity of substitution between $K_N$ and $O_N$
$lpha_E$	0.43	Elasticity of output to labour hours plus 1 in the energy-intensive sector
$\alpha_N$	0.39	Elasticity of output to labour hours plus 1 in the non-energy-intensive sector
h	0.38	Habit formation in consumption
$\delta_E$	0.32	Marginal cost of capital utilisation in the energy-intensive sector
$\delta_{\scriptscriptstyle N}$	0.44	Marginal cost of capital utilisation in the non-energy-intensive sector

**Table 1.** Estimates of structural parameters by indirect inference.

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$\delta_{E.2}$	5.51	Elasticity of capital utilisation rate in the energy-intensive sector
$\delta_{N,2}$	4.32	Elasticity of capital utilisation rate in the non-energy-intensive sector
$\psi_{\scriptscriptstyle E}$	0.003	Adjustment cost parameter for capital in the energy-intensive sector
$\psi_{\scriptscriptstyle N}$	0.002	Adjustment cost parameter for capital in the non-energy-intensive sector
$oldsymbol{\phi}$	0.16	Elasticity of substitution between $D_H$ and $M$
$oldsymbol{\phi}_F$	34.8	Elasticity of substitution between $D_W$ and $X$
$\eta$	0.51	Elasticity of substitution between $M_E$ and $M_N$
$\eta_F$	2.12	Elasticity of substitution between $X_E$ and $X_N$
ς	0.27	Elasticity of substitution between $D_E$ and $D_N$
γ	0.48	Bias parameter for energy-intensive goods
$ heta_E$	0.963	Weight of capital services in the energy-intensive sector
$ heta_N$	0.983	Weight of capital services in the non-energy-intensive sector

Using the estimated parameters and the indirect inference procedures discussed above, we extract the residuals and shocks of the model; these are depicted in Figures 2 and 3, respectively, with the implied persistence and volatility values for these shock processes reported in Table 2. We observe that all the shocks are mildly persistent, with the highest AR coefficient being that of the world demand shock and the lowest that of productivity shock in the non-energy-intensive sector. It is also observed that world demand, labour supply, energy efficiencies in both sectors, oil price, and preference shocks are the most volatile.

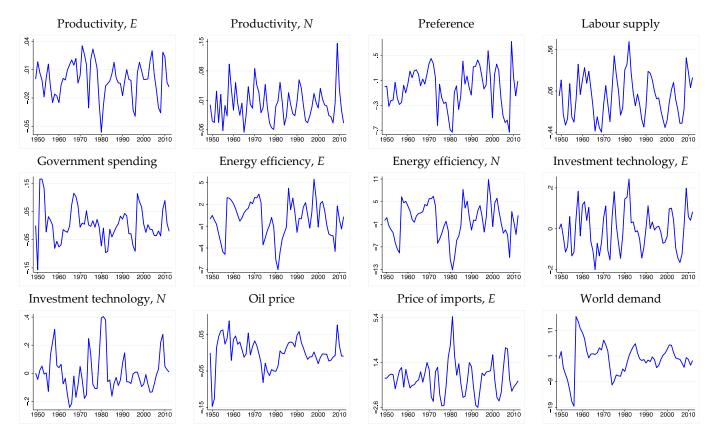


Figure 2. Residuals.

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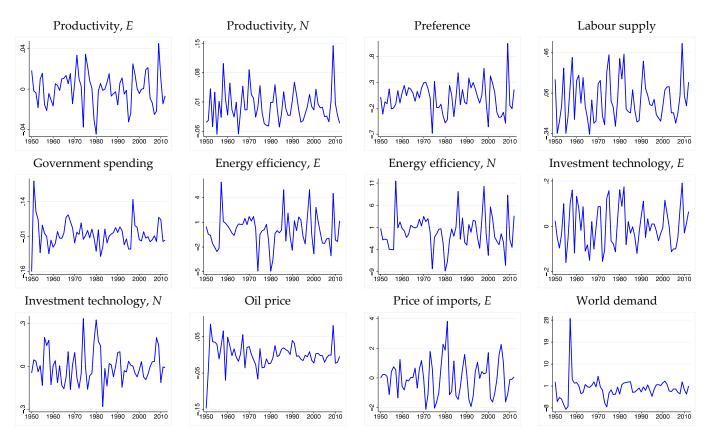


Figure 3. Shocks.

**Table 2.** Estimates of the shock processes by indirect inference.

Persistence	Value	Volatility	Value	Description
$ ho_c$	0.58	$\sigma_c$	1.21	Preference shock
$ ho_l$	0.58	$\sigma_l$	0.22	Labour supply shock
$ ho_{ie}$	0.46	$\sigma_{ie}$	0.09	Investment technology in the energy-intensive sector shock
$ ho_{in}$	0.57	$\sigma_{in}$	0.11	Investment technology in the non-energy-intensive sector shock
$ ho_{po}$	0.44	$\sigma_{po}$	0.30	Oil price shock
$ ho_{ye}$	0.46	$\sigma_{ye}$	0.02	Productivity in the energy-intensive sector shock
$ ho_{yn}$	0.22	$\sigma_{yn}$	0.04	Productivity in the non-energy-intensive sector shock
$ ho_{oe}$	0.60	$\sigma_{oe}$	2.08	Energy efficiency in the energy-intensive sector shock
$ ho_{on}$	0.60	$\sigma_{on}$	3.93	Energy efficiency in the non-energy-intensive sector shock
$ ho_g$	0.41	$\sigma_g$	0.06	Government spending shock
$ ho_{fe}$	0.56	$\sigma_{\!fe}$	0.04	Imported energy-intensive goods price shock
$ ho_{df}$	0.66	$\sigma_{df}$	4.62	World demand shock

#### 4.2. Assessing the Model Fit

Table 3 shows the Wald test results based on the above estimates. The Wald percentile reports the test statistic for where the percentile of the Wald distribution based on data VAR(1) lies, which in this case is the 93rd percentile. This is confirmed by the reported transformed Mahalanobis distance of 1.46, thus yielding a non-rejection of the model by the data at a 95% confidence level. Additionally, we document in Table 4 the estimates of the auxiliary VAR coefficients given the estimated structural errors alongside their 95% bounds from the model simulations. Regarding the individual distributions of the VAR coefficients, all but one of the sixteen dynamic VAR parameters lie within the 95% confidence bounds. The only cross-effect that lies outside of the 95% bounds is the response of energy use to consumption. Notwithstanding, the dynamic fit of the model to the data

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passes the Wald test at the 95% confidence level, with approximately a 92nd percentile of the Wald distribution and a transformed Mahalanobis distance of 1.3. Meanwhile, all four data variances are overpredicted by the model, with a Wald percentile of 98.1 and an associated transformed Mahalanobis distance of 2.61.

**Table 3.** Indirect inference test results.

Statistic	Value	Description
w	93.1	Wald percentile (Y, P, E, C)
t	1.46	Transformed Mahalanobis distance (Y, P, E, C)

Table 4. Auxiliary model (VAR) coefficients and bootstrap bounds.

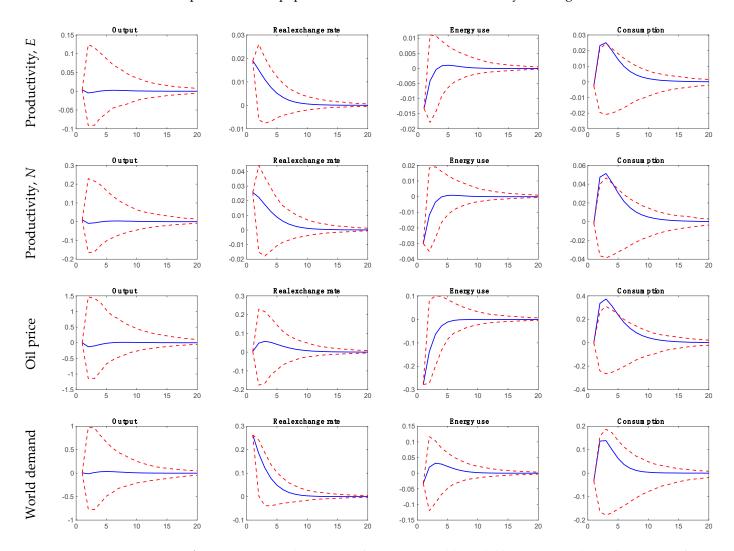
		95%	95%		
Parameter	Actual	Lower	Upper	In/Out	Description
	Value	Bound	Bound	III/Out	Description
Panel	! A: Directed V	Vald for data dyna	mics		
$Y_{Y}(-1)$	0.4913	0.0316	1.0204	In	Output on lagged output
$Y_P(-1)$	-0.0076	-0.0628	0.0982	In	Output on lagged real exchange rate
Y_O(-1)	-0.0354	-0.072	0.0646	In	Output on lagged energy use
$Y_C(-1)$	0.108	-0.0767	0.1086	In	Output on lagged consumption
$P_Y(-1)$	0.0047	-2.2335	3.3631	In	Real exchange rate on lagged output
P_P(-1)	0.6797	0.0987	0.8771	In	Real exchange rate on lagged real exchange rate
P_O(-1)	0.1341	-0.302	0.4652	In	Real exchange rate on lagged energy use
$P_C(-1)$	0.4435	-0.4135	0.599	In	Real exchange rate on lagged consumption
$O_Y(-1)$	0.4621	-5.1792	4.1954	In	Energy use on lagged output
O_P(-1)	-0.1641	-0.8366	0.6663	In	Energy use on lagged real exchange rate
O_O(-1)	0.4959	-0.2948	1.0473	In	Energy use on lagged energy use
O_C(-1)	-1.2027	-1.0638	0.7703	Out	Energy use on lagged consumption
$C_{Y}(-1)$	0.0697	-0.6549	1.8098	In	Consumption on lagged output
$C_P(-1)$	0.0041	-0.2242	0.1765	In	Consumption on lagged real exchange rate
$C_{O}(-1)$	-0.0198	-0.1247	0.2351	In	Consumption on lagged energy use
$C_{C}(-1)$	0.5978	0.4833	0.9146	In	Consumption on lagged consumption
w	91.7				Wald percentile
t	1.30				Transformed Mahalanobis distance
Panel	B: Directed W	'ald for data volati	lities		
Var(Y)	0.0009	0.0019	0.0068	Out	Variance of output
Var(P)	0.0314	0.0364	0.203	Out	Variance of real exchange rate
Var(O)	0.0413	0.1614	0.4604	Out	Variance of energy use
Var(C)	0.0004	0.0198	0.0633	Out	Variance of consumption
w	98.1				Wald percentile
t	2.61				Transformed Mahalanobis distance

An additional test that we subject our estimated model to is to compare its performance against an empirical benchmark. To do this, we turn next to the VAR impulse response functions of output, real exchange rate, energy use, and consumption to four of the twelve shocks in our model (see Figure 4) (The shocks reported are the two sectoral productivity shocks, oil price shock, and world demand shock. The VAR impulse response functions for the remaining shocks in the paper are shown in Appendix D). We note that the VAR shocks have been identified using the structural model. In the figure, there appears to be congruence in the responses of both the model and the data to all the shocks for output, real exchange rate, and energy use, with their responses placed inside the 95% bounds both in the short and the long term. This is an important result because it

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provides support for the usability of the model for policy-related work by adapting the structural impulse response functions to determine the influences of shocks and in creating appropriate policy responses ([40]). Nevertheless, we cannot say the same for consumption as there is a consistent short-run difference between the model and the data; this is especially pronounced in the cases of productivity shock in the non-energy-intensive sector and oil price shock (The same is found for the two sectoral energy efficiency shocks and the imported energy intensive goods price shock; see details in Appendix D).

Overall, the ability of the model to replicate both the joint and individual features of the U.S. data has been encouraging. On that note, and given the coexistence of both home and foreign disturbances in our model, we now proceed to seek an answer to the main question of our paper: How resilient is the U.S. economy to foreign disturbances?



**Figure 4.** VAR impulse response functions. *Note*: blue solid lines: estimated impulse response function; red dashed lines: 95% confidence intervals.

## 4.3. How Important Are Foreign Disturbances?

Our primary approach to identifying the contribution of foreign disturbances to business cycles in the U.S. relative to those arising from the home economy is to utilise the method of variance decomposition. We, therefore, report the variance decomposition for the aggregate macroeconomic variables in Table 5. Variance decompositions for the sectoral macro variables are contained in Appendix D. In our variance decomposition analy-

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sis, we consider as foreign disturbances the shocks to oil price, the price of imported energy-intensive goods, and world demand. The remaining nine shocks are treated as home disturbances.

Focusing first on the four macroeconomic variables used in the auxiliary VAR(1) model, what we see from Table 5 is that foreign disturbances explain 38% of the variability in aggregate output (Y), 73% of the variation in the real exchange rate (P), 45% of the variance of energy use (O), and 84% of the volatility of consumption (C). As shown, these contributions are due mainly to world demand, except for energy use, in which case, the main driving force is oil price shock. In each case, the rest of the fluctuations are accounted for by home disturbances. Concerning the other macroeconomic variables, we found that foreign disturbances also explain larger fractions in the variances of investment I (63%), labour hours L (76%), and real interest rate r (73%). These results are consistent with Christiano et al. [10] and Meenagh et al. [9] on the relevance of the open economy dimension (i.e., foreign shocks) in accounting for key macro variables.

				Н		Foreign disturbances								
	$oldsymbol{arepsilon_t^c}$	$oldsymbol{arepsilon}_t^l$	$oldsymbol{arepsilon}_t^{ie}$	$oldsymbol{arepsilon}_t^{in}$	$arepsilon_t^{ye}$	$\varepsilon_t^{yn}$	$oldsymbol{arepsilon_{t}^{oe}}$	$oldsymbol{arepsilon}^{on}$	$arepsilon_t^g$	$\Sigma_H$	$arepsilon_t^{po}$	$arepsilon_t^{fe}$	$arepsilon_t^{df}$	$\Sigma_F$
Υ	21.07	0.12	0.02	0.53	0.00	0.05	35.14	5.56	0.04	62.53	0.55	0.00	36.92	37.47
C	1.59	0.30	1.20	0.62	0.56	0.33	8.81	1.80	0.42	15.64	0.17	0.00	84.19	84.36
I	31.40	1.15	2.06	0.26	1.08	1.00	0.55	0.00	0.00	37.50	0.00	0.00	62.50	62.50
X	87.99	0.71	0.37	0.32	0.14	0.31	0.15	0.54	0.12	90.65	0.01	0.00	9.34	9.35
M	49.32	0.33	0.00	0.77	0.00	0.13	37.26	6.63	0.08	94.54	0.61	0.00	4.85	5.46
L	14.09	3.60	1.37	0.35	0.63	0.40	3.04	0.70	0.16	24.33	0.06	0.00	75.60	75.67
P	26.25	0.21	0.11	0.10	0.04	0.09	0.04	0.16	0.04	27.04	0.00	0.00	72.96	72.96
W	78.08	0.22	0.78	0.03	0.33	0.27	5.00	0.74	0.18	85.63	0.08	0.00	14.29	14.37
r	15.18	0.32	7.79	0.29	0.44	0.19	2.19	0.73	0.21	27.35	0.06	0.00	72.59	72.65
O	29.60	0.2	0.00	0.45	0.00	0.08	21.21	3.81	0.05	55.40	41.99	0.00	2.61	44.60

**Table 5.** Variance decomposition for macroeconomic aggregates.

Note: Home disturbances include shocks to preference  $\varepsilon_t^c$ , labour supply  $\varepsilon_t^l$ , investment technology in the energy-intensive sector  $\varepsilon_t^{ie}$ , investment technology in the non-energy-intensive sector  $\varepsilon_t^{in}$ , productivity in the energy-intensive sector  $\varepsilon_t^{v}$ , productivity in the non-energy-intensive sector  $\varepsilon_t^{v}$ , energy efficiency in the energy-intensive sector  $\varepsilon_t^{oe}$ , energy efficiency in the non-energy-intensive sector  $\varepsilon_t^{on}$ , and government spending  $\varepsilon_t^{g}$ . Foreign disturbances are shocks to the exogenous world variables: oil price  $\varepsilon_t^{po}$ , price of imported energy-intensive goods  $\varepsilon_t^{fe}$ , and world demand  $\varepsilon_t^{df}$ .  $\Sigma_H$  and  $\Sigma_F$  represent, respectively, the sum of home and foreign disturbances. The variables are output Y, consumption C, investment I, exports X, imports M, labour hours L, real exchange rate P, wages W, interest rates r, and energy demand O.

However, the U.S. economy tends to be more resilient to foreign disturbances with respect to certain macroeconomic variables. In particular, home shocks account for 91%, 95%, and 86%, respectively, of exports, imports, and real wages. The home shocks are also more important for domestic absorption (55%). Of the contributions by home disturbances, preference shocks, on average, dominate, thus explaining the bulk of the variations in investment (31%), exports (88%), imports (49%), labour hours (14%), real exchange rate (26%), real wages (78%), real interest rate (15%), and energy use (30%). The energy efficiency shock in the energy-intensive sector, which seems to be crowding out the other supply shocks (e.g., sectoral productivity shocks), also plays a prominent role.

## 4.4. What Is the Relative Contribution of Foreign Disturbances to the Recent U.S. Boom-Bust Cycles?

We next look at the time paths of the contributions of the different structural shocks to output, real exchange rate, energy use, and consumption in Figure 5. Although the depicted historical decompositions are for the year 2000 onwards, they are derived from estimates of the model over the full sample period of 1949 to 2013. The importance of this

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type of analysis is that it can help to shed further light on how the model interprets the developments of booms and busts in the U.S. since the year 2000; for an alternative story, see, for example, In't Veld et al. [8].

The energy efficiency shock in the energy-intensive sector played a dominant role in output movements during this period (panel A). This has been particularly evident in the surge of output in 2000, 2001–2002, and 2010. Similarly, this shock also accounted for the largest output drops during the Great Recession of 2007–2009. Although energy efficiency shock in the non-energy-intensive sector has been important to a lesser extent, its contribution from 2000–2013 has reinforced that of energy efficiency shock in the energy-intensive sector. We also observe that shocks to the price of imported energy-intensive goods and world demand are relevant to explaining increases and decreases in output during this period.

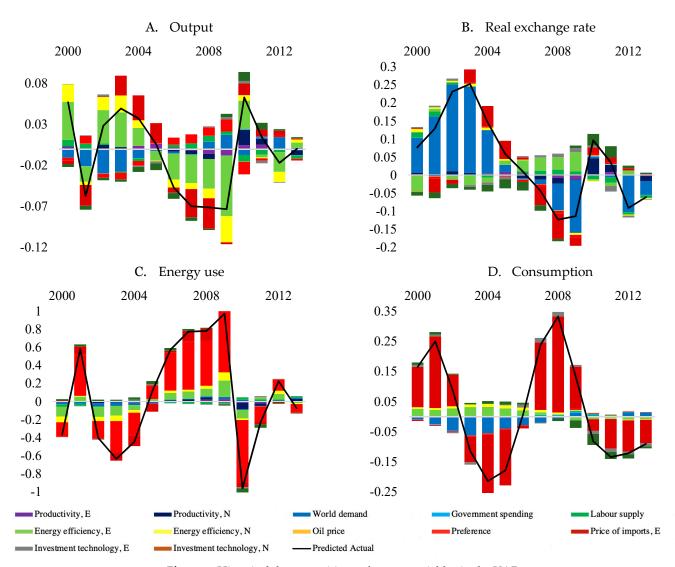


Figure 5. Historical decompositions of macro variables in the VAR.

With regard to the real exchange rate, panel B of Figure 5 shows that its fluctuations are mostly explained by shocks to world demand, followed by shocks to the price of imported energy-intensive goods and energy efficiency in the energy-intensive sector. As shown, the influence of world demand was more pronounced between 2000 and 2004, during the Great Recession, and in 2012–2013. Our findings also emphasise the significant roles played by preference shock in energy use (panel C) and shock to the price of imported energy-intensive goods in consumption (panel D).

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In summary, the historical decomposition provides support for the results from the variance decomposition confirming that both the home and foreign disturbances are effective in driving the U.S. business cycles. Further, it points out that different shocks were the most important to different macroeconomic variables and over different time horizons.

#### 5. Conclusions

The rise in economic influence of countries such as China, relative to the U.S., over recent decades suggests that the U.S. economy, more than ever before, will increasingly become exposed to international shocks. In this paper we test for, and find evidence of, the effects of foreign disturbances in the U.S. economy. The context for our analysis is a tractable one-commodity, two-sector, two-country open economy DSGE model. As with Meenagh et al. [9], we adopt the indirect inference method to match the model's behaviour with that of the U.S. data over the period 1949-2013. By using stationary data and engaging a VAR(1) in four variables (output, real exchange rate, energy use, and consumption), this paper establishes foreign disturbances as a force to reckon with when explaining the prime drivers of movements in U.S. business cycle fluctuations. Overall, we add to both the theoretical and empirical literature on the relative importance of foreign disturbances. While there abounds a large body of work in this area using U.S. economic policies and activities as sources of external shocks for other economies, studies investigating the reverse effects remain rather sparse. Given these results, we cannot but wonder: could it be that the U.S. economy now catches cold more frequently, when the rest of the world sneezes? We submit that more work will need to be done in this area to arrive at a consensus and we have put this work forward to provide a framework that academics and policymakers can build on.

Thus, we propose that our work can be further improved to advance this line of research in the following ways. As it is, our interest here has been to put forward first a stylised framework and preliminary inquiry into whether foreign disturbances matter for the U.S. economy. To that extent, we focus on showing the ability of a flex price DSGE model, with two sectors possessing different levels of energy intensities, to offer predictive power. We have also not explicitly discriminated between the sources of oil price shocks as advanced by Kilian [42]. Further, we have not explicitly incorporated heterogenous agents, allowed for a more involving role for the government regarding domestic and import-related taxes, or introduced financial frictions. These issues, which may be helpful to provide robustness and for monetary policy and welfare analyses, are left as interesting avenues for future work.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The data used for this study are publicly available at the U.S. Bureau of Economic Analysis (BEA), Bureau of Labour Statistics (BLS), Federal Reserve Economic Data (FRED), and Energy Information Administration (EIA).

Conflicts of Interest: The authors declare no conflict of interest.

## Appendix A. Technical Appendix

In what follows,  $i \in \{H, F\}$  (H refers to the home economy and F the foreign economy) and  $j \in \{E, N\}$  (E refers to energy-intensive sector and N the non-energy-intensive sector) will be used when we are distinguishing, respectively, between economies and sectors. Also note that we have used  $E_t F_{t+1} = 1/(1 + r_t)$ .

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This section lists the home economy's equilibrium conditions. Risk-sharing condition:

$$\frac{\varepsilon_t^c}{(C_t - hC_{t-1})^{\sigma}} = \beta(1 + r_t)E_t \left(\frac{\varepsilon_{t+1}^c}{(C_{t+1} - hC_t)^{\sigma}}\right)$$
(A1)

Intratemporal labour equation:

$$(C_t - hC_{t-1})^{\sigma} \varepsilon_t^l(L_t)^{\varphi} = W_t \tag{A2}$$

Capital utilisation rate:

$$R_{j,t}U_{j,t}\varepsilon_t^{ij} = \delta_{j,1}(U_{j,t})^{\delta_{j,2}} \tag{A3}$$

Investment Euler equation:

$$\left(1 + \psi_{j}\left(\frac{K_{j,t} - K_{j,t-1}}{K_{j,t-1}}\right)\right) = \beta E_{t} \frac{\varepsilon_{t+1}^{c}}{\varepsilon_{t}^{c}} \frac{\varepsilon_{t+1}^{ij}}{\varepsilon_{t+1}^{c}} \frac{\varepsilon_{t}^{ij}}{(C_{t} - hC_{t-1})^{\sigma}} \left(1 - \delta_{j,0} - \frac{\delta_{j,1}(U_{j,t+1})^{\delta_{j,2}}}{\delta_{j,2}} \left(1 - \delta_{j,2}\right) - \frac{\psi_{j}}{2} \left(\frac{K_{j,t+1} - K_{j,t}}{K_{j,t}}\right)^{2} + \psi_{j} \left(\frac{K_{j,t+1} - K_{j,t}}{K_{j,t}}\right) \frac{K_{j,t+1}}{K_{j,t}}\right) \tag{22}$$

Law of motion for capital accumulation:

$$K_{j,t} = \left(1 - \delta_{j,0} - \frac{\delta_{j,1} (U_{j,t})^{\delta_{j,2}}}{\delta_{j,2}}\right) K_{j,t-1} + \varepsilon_t^{ij} I_{j,t} - \frac{\psi_j}{2} \left(\frac{K_{j,t} - K_{j,t-1}}{K_{j,t-1}}\right)^2 K_{j,t-1}$$
(A5)

Aggregate investment:

$$I_t = I_{E,t} + I_{N,t} \tag{A6}$$

Production function:

$$Y_{j,t} = \varepsilon_t^{yj} (L_{j,t})^{1-\alpha_j} \left[ \theta_j (U_{j,t} K_{j,t-1})^{-\nu_j} + (1-\theta_j) (\varepsilon_t^{oj} O_{j,t})^{-\nu_j} \right]^{-\frac{\alpha_j}{\nu_j}}$$
(A72)

Aggregate output:

$$Y_t = Y_{E,t} + Y_{N,t} \tag{A8}$$

Labour demand:

$$(1 - \alpha_i) P_{i,t} Y_{i,t} = W_t L_{i,t} \tag{24}$$

Demand for capital services:

$$\alpha_{j}\theta_{j}P_{j,t}Y_{j,t} = R_{j,t} \left[\theta_{j} (U_{j,t}K_{j,t-1})^{-\nu_{j}} + (1-\theta_{j}) (\varepsilon_{t}^{oj}O_{j,t})^{-\nu_{j}}\right] (U_{j,t}K_{j,t-1})^{\nu_{j}+1} \quad (A10)$$

Energy demand:

$$\alpha_{j}(1-\theta_{j})P_{j,t}Y_{j,t}\varepsilon_{t}^{oj} = \varepsilon_{t}^{po}\left[\theta_{j}(U_{j,t}K_{j,t-1})^{-\nu_{j}} + (1-\theta_{j})(\varepsilon_{t}^{oj}O_{j,t})^{-\nu_{j}}\right](\varepsilon_{t}^{oj}O_{j,t})^{\nu_{j}+1}$$
(A11)

Aggregate labour hours:

$$L_t = L_{E,t} + L_{N,t} \tag{A12}$$

Aggregate energy usage:

$$O_t = O_{Et} + O_{Nt} \tag{A13}$$

Demand for aggregate imports:

$$M_t = (1 - \kappa) \left(\frac{1}{P_t}\right)^{-\phi} D_t \tag{A14}$$

Demand for energy-intensive sector imports:

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$$M_{E,t} = \chi \left(\varepsilon_t^{fe}\right)^{-\eta} M_t \tag{A15}$$

Domestic absorption of energy-intensive sector goods:

$$D_{E,t} = \gamma \left(\frac{P_{E,t}}{P_t}\right)^{-\varsigma} D_t \tag{A16}$$

Supply of aggregate exports:

$$X_t = (1 - \kappa_F)(P_t)^{-\phi_F} \varepsilon_t^{df} \tag{A17}$$

Supply of energy-intensive sector exports:

$$X_{E,t} = \chi_F \left(\frac{P_{E,t}}{P_t}\right)^{-\eta_F} X_t \tag{A18}$$

Real exchange rate:

$$P_t = (\gamma (P_{E,t})^{1-\varsigma} + (1-\gamma)(P_{N,t})^{1-\varsigma})^{1/(1-\varsigma)}$$
(A19)

Goods market clears for energy-intensive sector:

$$Y_{E,t} = D_{E,t} + X_{E,t} - M_{E,t} (A20)$$

Feasibility constraint for current account:

$$P_t X_t = \varepsilon_t^{po} O_t + M_t \tag{A21}$$

Aggregate domestic absorption:

$$D_t = C_t + I_t + \varepsilon_t^g \tag{25}$$

Resource constraint:

$$Y_t = D_t \tag{A23}$$

Appendix A.2. Log-Linearised Equilibrium Conditions

This section lists the log-linearised home economy's equilibrium conditions that we use for empirical analysis.

Risk-sharing condition:

$$c_{t} = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_{t}c_{t+1} + \frac{1-h}{\sigma(1+h)}(\varepsilon_{t}^{c} - E_{t}\varepsilon_{t+1}^{c} - r_{t})$$
(A24)

Intratemporal labour equation:

$$\varphi l_t = w_t - \frac{\sigma}{(1-h)} (c_t - hc_{t-1}) - \varepsilon_t^l$$
(A25)

Investment Euler equation:

$$\varepsilon_{t}^{c} - \varepsilon_{t}^{ij} - \frac{\sigma}{(1-h)} (c_{t} - hc_{t-1}) + \psi_{j} (k_{j,t} - k_{j,t-1}) 
= E_{t} \varepsilon_{t+1}^{c} - E_{t} \varepsilon_{t+1}^{ij} - \frac{\sigma}{(1-h)} (E_{t} c_{t+1} - hc_{t}) + \beta \delta_{j,1} (u_{j})^{\delta_{j,2}} (\delta_{j,2} - 1) E_{t} u_{j,t+1} 
+ \beta \psi_{j} (k_{j,t+1} - k_{j,t})$$
(A26)

Law of motion for capital accumulation:

$$k_{j,t} = (1 - \delta u_j) k_{j,t-1} - \delta_{j,1} (u_j)^{\delta_{j,2}} u_{j,t} + \frac{i_j}{k_j} (\varepsilon_t^{ij} + i_{j,t})$$
(A27)

Aggregate investment:

$$i_t = \frac{i_E}{i} i_{E,t} + \frac{i_N}{i} i_{N,t}$$
 (A28)

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Production function:

$$y_{j,t} = \varepsilon_t^{yj} + (1 - \alpha_j)l_{j,t} + \frac{\alpha_j}{1 + \frac{1 - \theta_j}{\theta_j} \left(\frac{o_j}{k_i}\right)^{-\nu_j}} (u_{j,t} + k_{j,t-1}) + \frac{\alpha_j}{1 + \frac{\theta_j}{1 - \theta_j} \left(\frac{o_j}{k_i}\right)^{\nu_j}} (\varepsilon_t^{oj} + o_{j,t})$$
(A29)

Aggregate output:

$$y_{t} = \frac{y_{E}}{y} y_{E,t} + \frac{y_{N}}{y} y_{N,t}$$
 (A30)

Labour demand:

$$p_{j,t} + y_{j,t} = w_t + l_{j,t} (A31)$$

Demand for capital services:

$$\left(\delta_{j,2} + \nu_{j} - \frac{\nu_{j}}{1 + \frac{1 - \theta_{j}}{\theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{-\nu_{j}}}\right) u_{j,t} - p_{j,t} - y_{j,t} - \varepsilon_{t}^{ij} 
= \left(\frac{\nu_{j}}{1 + \frac{1 - \theta_{j}}{\theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{-\nu_{j}}} - \nu_{j} - 1\right) k_{j,t-1} + \frac{\nu_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{\nu_{j}}} (\varepsilon_{t}^{oj} + o_{j,t})$$
(A32)

Energy demand:

$$\left(1 + \nu_{j} - \frac{\nu_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{\nu_{j}}}\right) o_{j,t} - p_{j,t} - y_{j,t} - \varepsilon_{t}^{po}$$

$$= \frac{\nu_{j}}{1 + \frac{1 - \theta_{j}}{\theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{-\nu_{j}}} \left(u_{j,t} + k_{j,t-1}\right) + \left(\frac{\nu_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{\nu_{j}}} - \nu_{j}\right) \varepsilon_{t}^{oj} \tag{A33}$$

Aggregate labour hours:

$$l_{t} = \frac{l_{E}}{l} l_{E,t} + \frac{l_{N}}{l} l_{N,t}$$
 (A34)

Aggregate energy usage:

$$o_t = \frac{o_E}{o} o_{E,t} + \frac{o_N}{o} o_{N,t} \tag{A35}$$

Demand for aggregate imports:

$$m_t = \phi p_t + d_t \tag{A36}$$

Demand for energy-intensive sector imports:

$$m_{E,t} = -\eta \varepsilon_t^{fe} + m_t \tag{A37}$$

Domestic absorption of energy-intensive sector goods:

$$d_{E,t} = -\varsigma(p_{E,t} - p_t) + d_t \tag{A38}$$

Supply of aggregate exports:

$$x_t = -\phi_F p_t + \varepsilon_t^{df} \tag{A39}$$

Supply of energy-intensive sector exports:

$$x_{E,t} = -\eta_F (p_{E,t} - p_t) + x_t \tag{A40}$$

Real exchange rate:

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$$p_t = \gamma \left(\frac{p_E}{p}\right)^{1-\varsigma} p_{E,t} + (1-\gamma) \left(\frac{p_N}{p}\right)^{1-\varsigma} p_{N,t}$$
 (A41)

Goods market clears for energy-intensive sector:

$$y_{E,t} = \frac{d_E}{y_E} d_{E,t} + \frac{x_E}{y_E} x_{E,t} - \frac{m_E}{y_E} m_{E,t}$$
 (A42)

Feasibility constraint for current account:

$$p_t + x_t = \frac{o}{px} \left( \varepsilon_t^{po} + o_t \right) + \frac{m}{px} m_t \tag{A43}$$

Aggregate domestic absorption:

$$d_t = \frac{c}{d}c_t + \frac{i}{d}i_t + \frac{g}{d}g_t \tag{A44}$$

Resource constraint:

$$y_t = d_t \tag{A45}$$

Appendix A.3. Steady State

This section lists the steady state of the home economy equilibrium conditions. From Equation (A1):

$$r = \frac{1}{\beta} - 1 \tag{A46}$$

Combining Equation (A3) with Equation (A4):

$$R_i = r + \delta u_i \tag{A47}$$

Combining Equations (A10) and (A11), using Equations (A46) and (A47):

$$\frac{o_j}{k_j} = \left(\frac{1 - \theta_j}{\theta_j} \frac{1 - \beta(1 - \delta u_j)}{\beta}\right)^{\frac{1}{1 + \nu_j}} \tag{A48}$$

Using Equation (A10), we show that:

$$\frac{k_{j}}{l_{j}} = \left(\frac{\alpha_{j}\beta\theta_{j}p_{j}}{1 - \beta(1 - \delta u_{j})}\right)^{\frac{1}{1 - \alpha_{j}}} \left(\theta_{j} + \frac{1 - \theta_{j}}{\left(\frac{1 - \theta_{j}}{\theta_{i}} \frac{1 - \beta(1 - \delta u_{j})}{\beta}\right)^{\frac{\nu_{j}}{1 + \nu_{j}}}}\right)^{-\frac{\alpha_{j} + \nu_{j}}{\nu_{j}(1 - \alpha_{j})}} \tag{A49}$$

From Equation (A5), we derive that:

$$\frac{i_j}{k_i} = \delta u_j \tag{A50}$$

Substituting Equations (A48) and (A49) into Equation (A7):

$$\frac{y_j}{l_j} = \left(\frac{\alpha_j \beta \theta_j p_j}{1 - \beta (1 - \delta u_j)}\right)^{\frac{1}{1 - \alpha_j}} \left(\theta_j + \frac{1 - \theta_j}{\left(\frac{1 - \theta_j}{\theta_j} \frac{1 - \beta (1 - \delta u_j)}{\beta}\right)^{\frac{\nu_j}{1 + \nu_j}}}\right)^{\frac{\alpha_j^2 - 2\alpha_j - \nu_j}{\nu_j (1 - \alpha_j)}} \tag{A51}$$

Then, the above sector-specific variables sum to give their aggregate counterparts:

$$o = o_E + o_N \tag{A52}$$

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$$i = i_E + i_N \tag{A53}$$

$$y = y_E + y_N \tag{A54}$$

Further, combining Equations (A14) and (A23):

$$p = \left(\frac{1}{1 - \kappa} \frac{m}{\nu}\right)^{\frac{1}{\phi}} \tag{A55}$$

From Equations (A16) and (A23):

$$p_E = \left(\frac{1}{\gamma} \frac{d_E}{v}\right)^{-\frac{1}{\varsigma}} p \tag{A56}$$

Rewriting Equation (A19):

$$p_N = \left(\frac{(p)^{1-\varsigma} - \gamma(p_E)^{1-\varsigma}}{1-\gamma}\right)^{\frac{1}{1-\varsigma}} \tag{A57}$$

Using Equations (A22) and (A23), we derive:

$$\frac{c}{y} = 1 - \frac{i}{y} - \frac{g}{y} \tag{A58}$$

Combining Equations (A14) and (A15), we obtain:

$$m_E = \chi (1 - \kappa) p^{\phi} y \tag{A59}$$

Finally, combining Equations (A17) and (A18) gives:

$$x_E = \chi_F (1 - \kappa_F) (p_E)^{-\eta_F} (p)^{\eta_F - \phi_F}$$
 (A60)

## Appendix B. Data Appendix

This section describes the U.S. annual data used for model evaluation and estimation. The period covered is 1949 to 2013. The raw data were taken from a variety of sources, which, unless stated otherwise, are the U.S. Bureau of Economic Analysis (BEA), Bureau of Labour Statistics (BLS), Federal Reserve Economic Data (FRED), and Energy Information Administration (EIA). We note that all data are seasonally adjusted, in constant prices, per capita terms, and logged, except when stated otherwise. The model and the estimating framework both necessitate compiling a dataset on aggregate measures for the single final output, consumption, investment, labour hours, oil use, exports, and imports. Further, we require observed time-series on the real exchange rate, wages, and interest rates. Besides, we need empirical counterparts for sectoral capital stocks and capital utilisation rates. We note that, as in the model, aggregates of output, investment, labour hours, and oil use are obtained as the sum of their respective sectoral values.

In general, we define data for the energy-intensive sector as containing the following industries: agriculture, mining, utilities, construction, manufacturing, and transportation. Wholesale and retail trade, information, finance, professional and business services, educational services, arts, and other non-government services make up the non-energy-intensive sector. Practical issues faced in constructing the variables on the above-defined grounds imply the need to comment on some of our data definitions. To begin with, in measuring sectoral output, we split the output of the public sector into two due to the lack of sufficient disaggregation of government output and added half each to the summed value-added of the relevant industries for each of energy- and non-energy-intensive sectors.

For the investment series, we combined investment in private fixed assets, equipment, structures, and intellectual property products with the series for consumer durables (both of which are classified by type of product). Then, starting with the consumption of durable goods, we assign into investments that are non-energy-intensive furnishings and

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durable household equipment, recreational goods and vehicles, and other durable goods. Investment in energy-intensive durable goods is given as the residual. Further, investment in energy-intensive goods is given by the sum of equipment and structures less residential equipment and improvements. We define investment in non-energy-intensive-type goods as the sum of residential equipment, improvements, and intellectual property products.

Hours worked is obtained by following the procedure of Herrendorf et al. [43], which involves combining GDP-by-Industry data reported using the North American Industry Classification System (NAICS) classification with the Income-and-Employment-by-Industry data reported with three different classifications over the sample period (the Standard Industrial Classification (SIC) from 1949 to 2000 (SIC72 for pre-1987 and SIC87 between 1987 and 2000) and NAICS since 2001). In particular, the former data representations follow the classification we would prefer, while the latter provides the industry-level information we require for assignment into energy- and non-energy-intensive sectors.

Formally, the sectoral labour hours are computed using:

$$L_{j} = NAICS_{ft}^{HE} + \frac{NAICS_{ft}^{HE}}{NAICS_{ft}^{NE}} \times NAICS_{se}$$
(A61)

$$NAICS_{ft}^{HE} = SIC_{ft}^{HE} \times \frac{NAICS_{ft}^{NE}}{SIC_{ft}^{NE}}$$
 (61)

$$NAICS_{ft}^{NE} = SIC_{ft}^{NE} \times \frac{NAICS_{ftpt}^{NE}}{SIC_{ftpt}^{NE}}$$
 (62)

$$NAICS_{se} = SIC_{se} \times \frac{NAICS_{ftpt}^{NE}}{SIC_{ftpt}^{NE}}$$
(63)

where HE = number of hours employed, NE = number of employees, ft = full-time, se = self-employed, and ftpt = full-time part-time.

We take the total energy consumption in the economy to be the aggregate consumption of primary energy. That is, the consumption of fossil fuels comprising petroleum, coal, and natural gas (measured in trillions of British thermal units (BTUs)) in the private sector, excluding the electric power sector. We collectively refer to these fossil fuels as oil. We do not, however, include the consumption of renewables (geothermal, solar/PV, and biomass) and electricity for both theory and data reasons. On the data, if one chooses to use, for instance, total primary energy consumption data, there are no data for biomass consumption until 1981. Moreover, we excluded the electric-power-generating sector, which would have been classed as a highly energy-intensive sector, given that close to 70% of all primary energy is used or lost as this sector provides electricity to the final consumers. We have, however, not included it because we have not modelled an energy-producing sector, which would have to be the case if we had incorporated electricity into our total for energy consumption.

Hence, aggregate energy consumption in the U.S. is formally given by the dollar value of total primary energy use:

$$VTOU = P_{0,t} \times \frac{O_t \times 1 \text{ trillion} \div \Re \times 1 \text{ million}}{1 \text{ billion}}$$
(A65)

where  $\aleph = 5.78$  represents the conversion factor for relating BTUs to barrels of oil.

Oil consumption is provided for four end-use sectors: namely, the industrial, transportation, residential, and commercial sectors. Given a lack of further disaggregation, we use the primary energy consumption in both the industrial and transportation sectors as a proxy for energy use in the energy-intensive sector, and primary energy consumption in both the residential and commercial sectors as a proxy for energy use in the non-energy-intensive sector. Prices of energy- and non-energy-intensive goods are derived based on

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chain-type price indexes for value-added by industry. For the price of energy-intensive goods, we use the weighted average from agriculture, mining, utilities, construction, manufacturing, and transportation. We utilise the weighted average from wholesale and retail trade, information, finance, professional and business services, educational services, arts, and other non-government services for the price of non-energy-intensive goods.

Following the constructions of energy-intensive and non-energy-intensive investment goods above, we construct the sectoral physical capital stocks. The energy-intensive sector capital stock is the sum of non-residential equipment and structures. The physical capital stock of the non-energy-intensive sector is obtained as the sum of residential equipment and structures, and intellectual property products. Further, non-energy-intensive-type capital stock is taken as the sum of furnishings and durable household equipment, recreational goods and vehicles, and other durable goods, such that capital stock in the energy-intensive-type consumption durable goods is given by motor vehicles and parts. For the energy-intensive sector capital utilisation rate, we use capacity utilisation rate for total manufacturing industry. Meanwhile, capacity utilisation rate for motor vehicles and parts is used to proxy capital utilisation rate in the non-energy-intensive sector.

For wages, we use the real index of hourly compensation. Interest rate is the three-month Treasury bill rate for 1949–1954 ([41]), where we have converted their quarterly data into annual data by averaging, and we use the federal funds rate for 1955–2013. The real exchange rate is U.S. CPI for all urban consumers relative to ROW CPI. Consumption is measured using personal consumption expenditures, less durable goods.

## Appendix C. Calibration

The assessment of the quantitative workings of the model can only begin when we have chosen values for the model parameters, such that we are able to simulate the model. Further, indirect inference estimation that we employ needs starting parameter values. In this section, we discuss how we obtain these values (see Table A1). We set the Frisch elasticity of labour supply  $\varphi$  equal to 5 (A value of zero for  $\varphi$  implies perfect labour mobility between sectors. This type of perfect factor mobility is prevalent in the RBC literature especially as it relates to the labour market activities. However, suffice it to say that this is as plausible as, for example, the degree of sector-specific skilled labour that is needed. Hence, as  $\varphi \to \infty$  so does the degree of sector specificity. We are thus inclined to begin the analysis from a more Walrasian context such that we set  $\varphi$  closer to 0, fix consumption elasticity  $\sigma$  at 2, and preserve the CES form of the production functions by setting the respective sector's elasticity of substitution between capital services and efficient energy use,  $v_E$  and  $v_N$ , equal to 0.7; Kim and Loungani ([13], Table 2, p. 180) provide a justification for using this value. They also considered a value of 0.001 suggesting a Cobb-Douglas form and high elasticity of substitution between capital services and energy use. We, however, stick to the parameter value that preserves the general form of specification and leave the optimal choice of parameter value to be decided at the estimation stage later on). Further, we suppose that there is some degree of habit formation h for agents in this model, setting the initial parameter value to 0.7, which is in line with previous estimates in the literature for a developed country such as the U.S. A very small value of 0.001 is chosen for the parameters that relate to the adjustment costs of capital,  $\psi_E$  and  $\psi_N$ , following a popular practice in the literature.

Moving on to the component parameters of the two depreciation functions, the steady state implies that  $\delta_{j,0} + \delta_{j,1} (u_j)^{\delta_{j,2}}/\delta_{j,2}$  for which we note that only four of their six parameters needed identifying; these are  $\delta_{j,0}$  and  $\delta_{j,2}$ . So, conditional on the values of the discount factor and the real rental rates for capital services in the two sectors, we calibrate the parameters governing the elasticities of marginal depreciations with respect to capital utilisation rates as  $\delta_{j,2} = \beta \delta_{j,1} u^{\delta_{j,2}}/(\beta(1+\delta_{j,1} u^{\delta_{j,2}})-1)$ . This expression yields 1.463 and 1.694, respectively, for the energy-intensive and non-energy-intensive parameters, which

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are reasonably located in the range found in the literature (Basu and Kimball [27] suggested the upper bound of 2 based on a 95% confidence band. Further, to calibrate this parameter, we have gone for the more restricted form of the depreciation function by setting  $\delta_{i,0} = 0$ . Basu and Kimball [27], though, used the more general form in their empirical work and concluded that there is no statistical evidence in support of the non-zero value for the fixed component of the depreciation function as assumed by many other authors in the literature. See, for example, Greenwood et al. [44] and Burnside and Eichenbaum [45]. We observe that our values are not far from those usually employed in the literature. For example, Greenwood et al. used a value of 1.42, while Burnside and Eichenbaum, using their factor-hoarding model and data on output and capital, calibrated  $\mu$  to be 1.56 ( $\Delta$ = 0.56). We must note that the specification for time-varying depreciation is less general in these other studies. Statistically, however, both values are not rejected by the data. This is performed noting one of the concerns of Basu and Kimball [27] that "...our method makes clear that  $\Delta$  is a parameter that needs to be estimated, and in fact is not pinned down very precisely by the data because it has to be estimated as the reciprocal of a fairly small number. Thus, even the small standard error of the reduced-form parameter necessarily implies that there is large uncertainty about the structural parameter  $\Delta$ . Consequently, economic modellers should conduct sensitivity analysis of their results using a wide range of values for this parameter." In addition, "variable depreciation does not seem a significant source of error in the capital stock figures reported by the BEA." Specifically, they concluded that this issue "strikes us as second-order." This thus supports our empirical approach because it is essentially aimed at achieving optimal calibration and is also efficient in dealing with errors in the model, irrespective of how they are introduced. So, once we have assessed the performance of the model under this calibration approach, the above arguments make more important our next empirical exercise, which is to estimate the model's underlying structural parameters). Moreover, with no loss of generality, we fix the values for  $\delta_{i,1}$  at unity, as in Burnside and Eichenbaum [45], Boileau and Normandin [46], and Leduc and Sill [47]. The idea is that  $\delta_{i,1}$  and  $u_i$  are admitted into the model only jointly as  $\delta_{j,1}u^{\delta_{j,2}}$ , such that  $\delta_{j,1}=1$  has a trivial implication that  $\delta_{j,1}u^{\delta_{j,2}}=u^{\delta_{j,2}}$ . Additionally, using household's optimality conditions with regards to capital utilisation rates conditioned on the values for the respective sector's rental rate of capital in the steady state, we can show that  $\delta_{i,1}u^{\delta_{j,2}}=R_i=1/\beta-(1-\delta u_i)$ . This simplifies to give 0.132 and 0.102 that are reported in Table A1.

**Table A1.** Initial parameter values used to start the simulated annealing algorithm.

Parameter	Value	Description
$\varphi$	5	Frisch elasticity of labour supply
σ	2	Elasticity of substitution in consumption
$ u_E$	0.7	Elasticity of substitution between $K_E$ and $O_E$
$ u_N$	0.7	Elasticity of substitution between $K_N$ and $O_N$
$lpha_E$	0.43	Elasticity of output to labour hours plus 1 in the energy-intensive sector
$\alpha_N$	0.28	Elasticity of output to labour hours plus 1 in the non-energy-intensive sector
h	0.7	Habit formation in consumption
$\delta_E$	0.132	Marginal cost of capital utilisation in the energy-intensive sector
$\delta_N$	0.102	Marginal cost of capital utilisation in the non-energy-intensive sector
$\delta_{E,2}$	1.463	Elasticity of capital utilisation rate in the energy-intensive sector
$\delta_{N,2}$	1.694	Elasticity of capital utilisation rate in the non-energy-intensive sector
$\psi_{\scriptscriptstyle E}$	0.001	Adjustment cost parameter for capital in the energy-intensive sector
$\psi_N$	0.001	Adjustment cost parameter for capital in the non-energy-intensive sector
$\phi$	1.5	Elasticity of substitution between $D_H$ and $M$
$\phi_F$	1.5	Elasticity of substitution between $D_F$ and $X$
$\eta$	0.44	Elasticity of substitution between $M_E$ and $M_N$

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$\eta_F$	0.44	Elasticity of substitution between $X_E$ and $X_N$
ς	0.9	Elasticity of substitution between $D_E$ and $D_N$
γ	0.55	Bias parameter for energy-intensive goods
$ heta_E$	0.990	Weight of capital services in the energy-intensive sector
$ heta_N$	0.996	Weight of capital services in the non-energy-intensive sector
w	100	Wald percentile (Y, P, E, C)
t	40.14	Transformed Mahalanobis distance (Y, P, E, C)

Parameters governing the elasticities of labour hours in the energy-intensive and non-energy-intensive sectors,  $\alpha_E$  and  $\alpha_N$ , are 0.43 and 0.28, respectively, being calibrated to match the respective sector's capital-output ratios. The values chosen for the elasticities of substitution parameters in the aggregator functions are all standard in the trade literature ([48–50]) and U.S. data:  $\phi = \phi_F = 1.5$ ,  $\eta = \eta_F = 0.44$ ,  $\varsigma = 0.9$ , and  $\gamma = 0.55$ . Then, the parameters governing the weight of capital services in both sectors  $\theta_j$  are implicitly estimated throughout. First, based on calibrated values and later using estimated values, given the fixed parameters obtained from target steady state ratios of the model. In practice, we use the expression:

$$\theta_{j} = \frac{1}{1 + \frac{1}{1/\beta - (1 - \delta u_{j})} \left(\frac{o_{j}}{k_{j}}\right)^{1 + \nu_{j}}}$$
(A66)

where the values change mainly with the parameter  $v_i$ .

The values of all the remaining structural model parameters are fixed throughout the empirical investigation (see Table A2). As an example, these include the discount factor,  $\beta$ , which we set at 0.96, indicating that the annual real rate of interest is 4% (a value found to be consistent with the average post-WWII interest rate in the U.S.). All the other parameters capture the long-run average data values over the sample period.

**Table A2.** Fixed structural parameter and steady state ratios.

Parameter	Value	Description
β	0.96	Discount factor
$\delta u_{\scriptscriptstyle E}$	0.09	Marginal cost of capital utilisation in the energy-intensive sector
$\delta u_N$	0.06	Marginal cost of capital utilisation in the non-energy-intensive sector
$p_O$	1	Price of crude oil
$o_E/k_E$	0.011	Energy-capital ratio in the energy-intensive sector
$o_N/k_N$	0.014	Energy-capital ratio in the non-energy-intensive sector
$i_E/k_E$	0.08	Investment-capital ratio in the energy-intensive sector
$i_N/k_N$	0.17	Investment-capital ratio in the non-energy-intensive sector
$i_E/i$	0.7	Share of investment in the energy-intensive sector to aggregate investment
$i_N/i$	0.3	Share of investment in the non-energy-intensive sector to aggregate investment
$h_E/h$	0.4	Share of labour hours in the energy-intensive sector to aggregate labour hours
$h_N/h$	0.6	Share of labour hours in the non-energy-intensive sector to aggregate labour hours
$o_E/o$	0.78	Share of oil use in the energy-intensive sector to aggregate oil use
$o_N/o$	0.22	Share of oil use in the non-energy-intensive sector to aggregate oil use
$y_E/y$	0.41	Ratio of energy-intensive output to total output
$y_N/y$	0.59	Ratio of non-energy-intensive output to total output
g/d	0.21	Share of government consumption spending in domestic absorption
i/d	0.3	Share of investment in domestic absorption
c/d	0.49	Share of consumption in domestic absorption
$d_E/y_E$	1.385	Ratio of domestic absorption to output in the energy-intensive sector
$x_E/y_E$	0.1573	Ratio of exports to output in energy-intensive sector
$m_E/y_E$	0.205	Ratio of imports to output in energy-intensive sector

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$d_E/d$	0.37	Ratio of absorption of energy-intensive goods to total domestic absorption
i/y	0.308	Share of investment to total output
g/y	0.215	Share of government consumption spending in total output
o/y	0.037	Share of energy use in total output
x/y	0.08	Share of exports in total output
m/y	0.092	Share of imports in total output
c/y	0.268	Share of private consumption in total output
$p_{M}$	1	Price of imported goods
r	0.04	Interest rate
w	1	Wages

In addition, there are 12 autocorrelation parameters and 12 standard deviations of innovations that make up the model's structural shock processes. To calibrate these parameters, we assume that the twelve exogenous processes follow AR(1) stationary processes in logarithm. Further, we assume that the innovations are serially uncorrelated, such that the 24 parameters are calculated based on twelve derived series. More specifically, the eight behavioural errors (preference shock, labour supply shock, two sectoral productivity shocks, two investment technology shocks, and two sectoral energy efficiency shocks) and the four exogenous processes (government spending shock, oil price shock, world demand shock, and imported energy-intensive goods price shock) are calculated part-sequentially as using model equations and actual data:

$$\varepsilon_t^g = \left(\frac{g}{y}\right)^{-1} \left(y_t - \frac{c}{y}c_t - \frac{i}{y}i_t\right) \tag{A67}$$

$$\varepsilon_t^l = w_t - \varphi l_t - \frac{\sigma}{(1-h)} (c_t - hc_{t-1}) \tag{A686}$$

$$\varepsilon_t^{fe} = \frac{m_t - m_{E,t}}{\eta} \tag{A696}$$

$$\varepsilon_t^{df} = x_t + \phi_F p_t \tag{A706}$$

$$\varepsilon_t^{po} = \left(\frac{o}{px}\right)^{-1} \left(p_t + x_t - \frac{o}{px}o_t - \frac{m}{px}m_t\right) \tag{A71}$$

$$\varepsilon_{t}^{oj} = \left(\frac{\nu_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{\nu_{j}}} - \nu_{j}\right)^{-1} \left(\left(1 + \nu_{j} - \frac{\nu_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{o_{j}}{k_{j}}\right)^{\nu_{j}}}\right) o_{j,t} - p_{j,t} - \nu_{j,t} - \nu$$

$$\varepsilon_{t}^{yj} = y_{j,t} - \left(1 - \alpha_{j}\right) l_{j,t} - \frac{\alpha_{j}}{1 + \frac{1 - \theta_{j}}{\theta_{j}} \left(\frac{\theta_{j}}{k_{j}}\right)^{-\nu_{j}}} \left(u_{j,t} + k_{j,t-1}\right) - \frac{\alpha_{j}}{1 + \frac{\theta_{j}}{1 - \theta_{j}} \left(\frac{\theta_{j}}{k_{j}}\right)^{\nu_{j}}} \left(\varepsilon_{t}^{oj} + o_{j,t}\right)$$
(A73)

$$\varepsilon_t^c = \frac{\sigma(1+h)}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1} - \frac{\sigma}{1-h}E_t c_{t+1} + r_t \tag{A74}$$

$$\varepsilon_t^{ij} = \varepsilon_t^c - \frac{\sigma}{(1-h)} (c_t - hc_{t-1}) + \psi_j (k_{j,t} - k_{j,t-1}) + \frac{\sigma}{(1-h)} (E_t c_{t+1} - hc_t)$$

$$-\beta \delta_{i,1} (u_i)^{\delta_{j,2}} (\delta_{i,2} - 1) E_t u_{i,t+1} - \beta \psi_i (k_{i,t+1} - k_{i,t})$$

$$(A756$$

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Nine of the above equations are without expectations such that the structural errors are backed out directly as residuals. For Equations (A74) and (A75) that are with expectations, the residuals are derived using the instrumental variable method recommended by McCallum [33] and Wickens [34], where the instruments are the lagged values of the endogenous variables. We then fit a univariate AR(1) model to each of the constructed series for the shocks. We have thus followed Blankenau et al. ([24], p. 874) in using "the observable endogenous variables and the orthogonality conditions implied by the Euler equations to recover the exogenous shocks..." This allows us to use the model equivalent of the four observed exogenous variables, such that we have maintained one of the early open economy model assumptions in the lineage of Fleming [51] and Mundell [52] that treat current account transactions mainly as residuals. In Meenagh et al. [9], we relax this manner of deriving the parameters of the observed shock processes, making use of their corresponding actual observations. This other approach follows the literature that interprets changes in the current account as emerging from planned behaviour of agents (see, for example, Sachs [53], Aizenman [54], Frenkel and Razin [55], Razin [56], and Dornbusch [57] for earlier accounts). Table A3 documents the results.

Table A3. Starting values of the shock processes.

Parameter	Value	Parameter	Value	Description
$ ho_c$	0.58	$\sigma_c$	1.20	Preference shock
$ ho_l$	0.49	$\sigma_l$	0.12	Labour supply shock
$ ho_{ie}$	0.30	$\sigma_{ie}$	0.07	Investment technology in the energy-intensive sector shock
$ ho_{in}$	0.30	$\sigma_{in}$	0.07	Investment technology in the non-energy-intensive sector shock
$ ho_{po}$	0.54	$\sigma_{po}$	0.31	Oil price shock
$ ho_{ye}$	0.49	$\sigma_{ye}$	0.02	Total factor productivity in the energy-intensive sector shock
$ ho_{yn}$	0.26	$\sigma_{yn}$	0.03	Total factor productivity in the non-energy-intensive sector shock
$ ho_{oe}$	0.59	$\sigma_{oe}$	0.52	Energy efficiency in the energy-intensive sector shock
$ ho_{on}$	0.59	$\sigma_{on}$	0.55	Energy efficiency in the non-energy-intensive sector shock
$ ho_g$	0.41	$\sigma_g$	0.06	Government spending shock
$ ho_{fe}$	0.56	$\sigma_{\!fe}$	0.04	Imported energy-intensive goods price shock
$ ho_{df}$	0.61	$\sigma_{df}$	0.20	World demand shock

## Appendix D. Additional Table and Figure

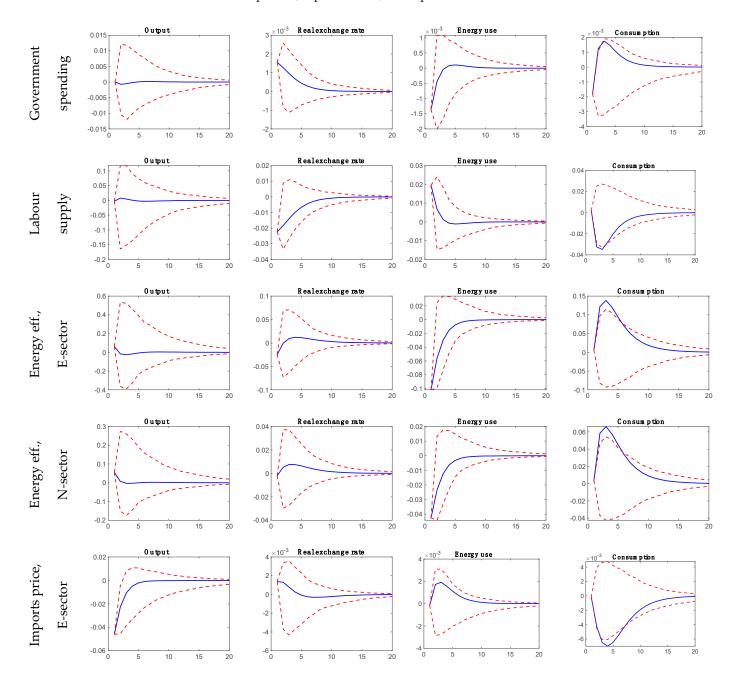
**Table A4.** Variance decomposition for sectoral variables.

				Н		Foreign Disturbances								
	$oldsymbol{arepsilon}_t^c$	$oldsymbol{arepsilon}_t^l$	$oldsymbol{arepsilon_t^{ie}}$	$arepsilon_t^{in}$	$arepsilon_t^{ye}$	$arepsilon_t^{yn}$	$oldsymbol{arepsilon}_t^{oe}$	$arepsilon_t^{on}$	$oldsymbol{arepsilon}_t^g$	$\Sigma_H$	$arepsilon_t^{po}$	$oldsymbol{arepsilon_t^{fe}}$	$arepsilon_t^{df}$	$\Sigma_F$
Υ	85.14	0.68	0.36	0.31	0.13	0.30	0.00	0.36	0.12	87.40	0.00	0.00	12.60	12.60
I	84.65	1.04	1.02	0.60	0.26	0.30	0.28	0.18	0.08	88.41	0.00	0.00	11.59	11.59
X	88.20	0.70	0.35	0.34	0.13	0.31	0.15	0.56	0.12	90.85	0.01	0.00	9.13	9.15
M	47.36	0.31	0.00	0.74	0.00	0.13	35.78	6.37	0.08	90.78	0.58	3.97	4.66	9.22
L	90.47	0.51	0.63	0.14	0.25	0.33	0.97	0.51	0.15	93.95	0.02	0.00	6.02	6.05
P	24.33	0.15	0.03	0.20	0.01	0.07	0.03	0.26	0.04	25.13	0.00	0.00	74.87	74.87
O	74.79	0.68	0.32	0.27	0.15	0.27	11.60	0.23	0.09	88.41	9.26	0.00	2.33	11.59
D	25.14	0.18	0.00	0.42	0.00	0.07	35.70	5.33	0.04	66.89	0.55	0.00	32.55	33.11
K	18.21	1.12	0.04	1.60	0.38	0.05	0.93	0.02	0.00	22.35	0.01	0.00	77.64	77.65
U	73.41	0.08	0.13	0.09	0.00	0.30	0.10	0.63	0.20	74.95	0.01	0.00	25.04	25.05
Υ	91.04	0.71	0.29	0.44	0.11	0.31	1.27	1.05	0.13	95.34	0.04	0.00	4.62	4.66
I	54.86	1.21	1.75	0.00	0.75	0.73	0.06	0.05	0.01	59.42	0.00	0.00	40.58	40.58
L	74.11	1.37	0.99	0.01	0.42	0.42	0.04	0.08	0.03	77.48	0.00	0.00	22.52	22.52
P	40.00	0.93	2.23	0.52	0.98	0.37	0.17	0.18	0.00	45.39	0.00	0.00	54.61	54.61

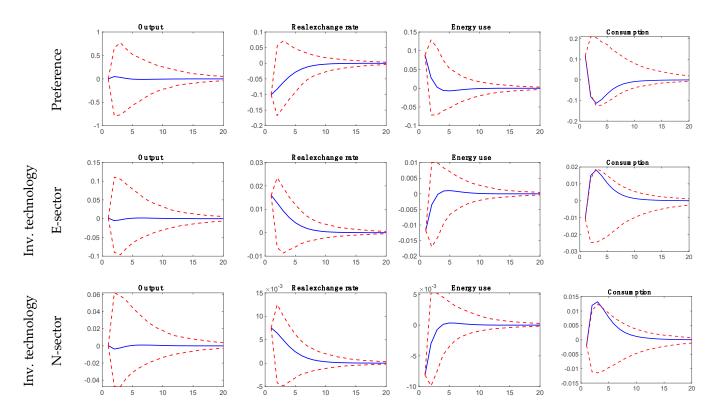
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0	75.84	0.81	0.82	0.03	0.35	0.33	0.90	1.37	0.07	80.52	2.37	0.00	17.11	19.48
K	68.15	0.01	0.49	2.04	0.25	0.26	6.61	1.44	0.38	79.63	0.15	0.00	20.22	20.37
U	9.81	1.44	2.98	0.06	1.34	1.32	1.34	0.18	0.05	18.51	0.03	0.00	81.46	81.49

Note: Home disturbances include shocks to preference  $\varepsilon^c_t$ , labour supply  $\varepsilon^l_t$ , investment technology in the energy-intensive sector  $\varepsilon^{ie}_t$ , investment technology in the non-energy-intensive sector  $\varepsilon^{in}_t$ , productivity in the energy-intensive sector  $\varepsilon^{ye}_t$ , productivity in the non-energy-intensive sector  $\varepsilon^{ye}_t$ , energy efficiency in the energy-intensive sector  $\varepsilon^{oe}_t$ , energy efficiency in the non-energy-intensive sector  $\varepsilon^{oe}_t$ , and government spending  $\varepsilon^g_t$ . Foreign disturbances are shocks to the exogenous world variables: oil price  $\varepsilon^{po}_t$ , price of imported energy-intensive goods  $\varepsilon^{fe}_t$ , and world demand  $\varepsilon^{df}_t$ .  $\Sigma_H$  and  $\Sigma_F$  represent, respectively, the sum of home and foreign disturbances. The variables are output Y, investment I, exports X, imports M, labour hours L, prices P, energy demand O, domestic absorption D, capital stock K, and capital utilisation rate U.



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**Figure A1.** Additional VAR impulse response functions. *Note*: blue solid lines: estimated impulse response function; red dashed lines: 95% confidence intervals.

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