Information length in dynamical systems

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Abstract: Information geometry or the application of differential geometry to the information sciences has become a promising tool for the analysis of complex dynamical systems. From the information geometry tools, we have the concept of information length defined as the integral of the information rate and describing the total amount of statistical changes that a time-varying probability distribution takes through time.
Throughout this poster, we introduce the definition of information length and its application to stochastic thermodynamics, abrupt events detection, time series analysis and control engineering. All this is in the light of linear non-autonomous stochastic systems.

Keywords: Information geometry, Information length, Stochastic dynamics, Abrupt evets, Time series, Control theory

Information Length (IL)

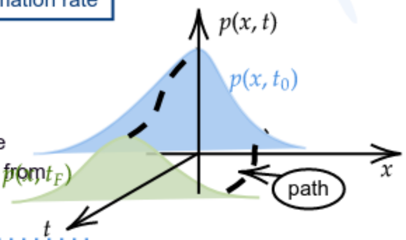
Definition

$$\mathcal{L}(t) = \int_{t_0}^{t_F} \Gamma(\tau)d\tau = \int_{t_0}^{t_F} \sqrt{\int_{\mathbb{R}^n} dx \frac{[\partial_\tau p(x;\tau)]^2}{p(x;\tau)}} d\tau$$

information rate

Interpretation

Information length is a metric of the total of statistical states that the system passes through from $p(x,t_0)$ to $p(x,t_F)$.



We focus on its application to

Linear non-autonomous stochastic systems

Langevin equation

$$\dot{\zeta}(t) = A\zeta(t) + Bu(t) + \xi(t)$$

Assuming

$$\begin{cases} \zeta = [\zeta_1, \zeta_2, \cdots, \zeta n]^\top \\ A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times 1}, u \in \mathbb{R} \\ \xi = [\xi_1, \xi_2, \ldots, \xi_n]^\top \\ \langle \xi_i(t)\rangle = 0 \\ \langle \xi_i(t)\xi_j(t')\rangle = 2D_{ij}\delta(t-t') \\ t, t' \in \mathbb{R},\ i,j = 1,2,\ldots \end{cases}$$

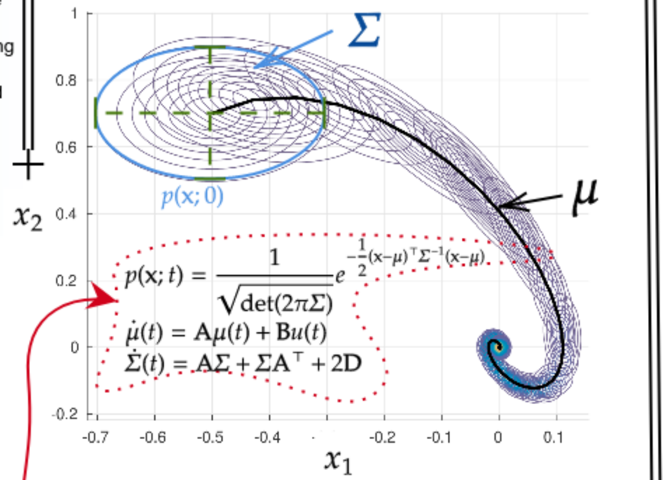
Corresponding Fokker-Planck equation

$$\frac{\partial p(x;t)}{\partial t} = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}[(A_{ij}x_j + B_i u(t))p(x;t)] + \sum_{i,j=1}^{n} D_{ij}\frac{\partial^2}{\partial x_i \partial x_j}p(x;t)$$

Solution

Example:

2nd order stochastic dynamical system [1]



$$p(x;t) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} e^{-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)}$$

$$\dot{\mu}(t) = A\mu(t) + Bu(t)$$

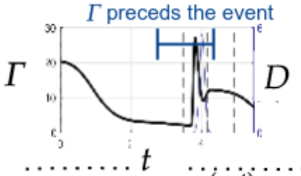
$$\dot{\Sigma}(t) = A\Sigma + \Sigma A^\top + 2D$$

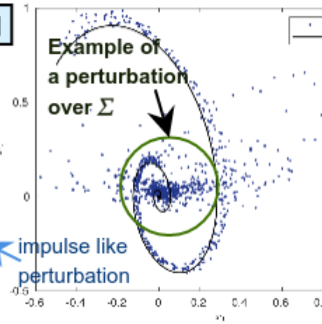
IL in Linear non-autonomous stochastic systems [1]

Compute using $\Gamma(t) = \dot{\mu}^\top \Sigma^{-1}\dot{\mu} + \frac{1}{2}\mathrm{Tr}\left(\Sigma^{-1}\dot{\Sigma}\right)^2$

IL in abrupt events prediction [2]

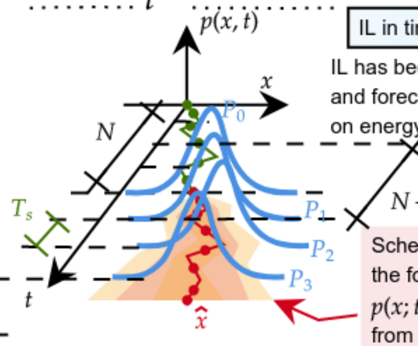
The value of information rate Γ can be used to detect perturbations over μ and Σ [2].





IL in time series analysis [3]

IL has been introduced to study and forecast the fluctuations on energy consumption [5].



Schematic showing the forecasting of $p(x;t)$ to compute IL from a time series (for further details, see [5]).

Applications

IL in Stochastic Thermodynamics [5]

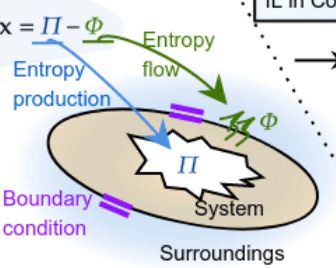
IL can be related to the entropy rate $\dot{S}$

$$\dot{S} = \int_{\mathbb{R}^n} \dot{p}(x;t)\ln(p(x;t))dx = \Pi - \Phi$$

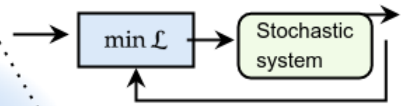
using $\Gamma^2 \leq \mathcal{E}_u := \mathrm{Tr}(\Sigma^{-1})\Pi\mathrm{Tr}(D) + \dot{S}^2 - 2g(H)$

where $g(H) := \sum_{i<j}^{n} \lambda_i\lambda_j$

s.t. λ_i are the eigenvalues of the matrx $H = \Sigma^{-1}D + A$



IL in Control Engineering [6]



Design and implementation of minimun variability control through the minimisation of IL in real time [6].

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