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Forecasting the Risks of Stability Loss for Nonlinear Supply Energy Systems [1]

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Abstract: The paper presents methods for studying the dynamics of nonlinear processes in relation to assessing the risks of stability loss on the basis of accumulated knowledge about the operation of power supply systems and methods for analyzing power supply modes. The methods are a complex of linear point discrete predictive identification models for a wide class of nonlinear objects, as well as spectral decompositions of Gramians for linear models.

Keywords: process identification; knowledge base; associative search models; Gramian method

1. INTRODUCTION

Reliable power supply with high power quality of industrial enterprises determines their high productivity. Therefore, the reliability of power supply systems plays just as important a role for the efficiency of an enterprise as: modernization of equipment, the use of modern management technologies and the introduction of modern information technologies. Reliable power supply with high power quality of industrial enterprises determines their high productivity.

Therefore, the reliability of power supply system plays no less important role for the efficiency of an enterprise than: modernization of equipment, the use of modern management technologies and the introduction of modern IT. For areas where AC motors are the main consumers, it is especially important to predict the approach of the state of processes to the boundaries of stability.

Spectral methods have been widely used over the past decades for studying the stability of linear and, to a lesser extent, nonlinear discrete and continuous systems (Bunse-Gerstner, et al., 2010; Shaker and Tahavori, 2014). This paper describes the possibility of studying the stability of nonlinear systems using point identification models.

The method of spectral expansions of controllability and observability Gramians has been successfully applied to study the stability of continuous dynamical systems. The Gramian method (Yadykin, 2010) provides an effective tool for analyzing the stability degree of power systems. It enables the investigation of system dynamics on the basis of a new mathematical technique for solving Lyapunov and Sylvester equations (Antoulas, 2006).

The method is based on the decomposition of the Gramian matrix, which is the solution of Lyapunov or Sylvester equations, into the spectrum of the matrices of these equations.

The development of the Gramian method for discrete dynamical systems was presented in (Yadikin, et al., 2016; Bakhtadze and Yadykin, 2019). A systematic approach was developed to evaluating the risk of stability loss for dynamic systems based on the use of accumulated knowledge about the functioning of energy supply systems and new methods for the analysis of energy planning modes.

The study consists of two stages: for a specific point in time, a linear discrete predictive model of the process under study is constructed. Further, the Gramian method is used to predict the approach of the nonlinear system state to the boundary of the stability area.

To develop the model, identification methods and algorithms based on the formation and analysis of knowledge processing – associative search algorithms (Bakhtadze, et al., 2011, 2016) were used. The methods use: linear point discrete predictive smart models of a wide class of nonlinear non-stationary objects, wavelet analysis and spectral decompositions of Gramians for these linear models.

This paper is devoted to the development of these methods. First of all, a modified associative search method is presented, which makes it possible to build a point identification model of the process under study. The method focuses on the study of transfer functions. The development

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of the Gramian method is presented below. It is shown that the use of the eigenvectors of the dynamics matrix significantly simplifies the calculation of the controllability and observability sub-grammians.

2. METHOD OF DEVELOPMENT OF THE PREDICTIVE MODEL

2.1 Associative search method

At a time when the available a priori information on the dynamic properties of the processes under study is insufficient to achieve the required quality of control by traditional methods, for nonlinear, as well as poorly structured and poorly formalized systems, it is effective to use modeling methods based on machine learning and knowledge bases. To build a predictive model of power supply systems, the associative search method proved to be effective.

Knowledge can be interpreted as patterns extracted from process data through data mining. In the associative search algorithm, knowledge is formalized in the form of templates of input and output variables that describe the functioning of the process. On the basis of data mining, the values of the input variables (in the general case, vector) are selected from the archive system, which are close to the current value of the input variable in the sense of a certain criterion.

The selected values of the historical data correspond to the actual values of the outputs. (Bakhtadze, et al., 2011). Next, a joint system of linear equations is formed. The only solution to this system by the method of least squares gives the values of the coefficients of the linear model (which is built for a specific point in time), as well as the output forecast.

2.2 Modeling using transfer functions

In a number of cases, to construct point linear models of nonlinear systems, it is advisable to build a model of the discrete transfer function of the system.

For example, for the stable operation of the power supply system, it is necessary to be able to assess the performance of the equipment. Evaluation is carried out mainly during commissioning of equipment or after major repairs by means of proof tests. It is practically impossible to carry out such tests in the course of real operation, since this requires shutting down the facility for a sufficiently long time. Moreover, the testing process is quite expensive.

The development of a methodology for on-line assessment of the current state of equipment and its prediction for some time ahead on the basis of intelligent analysis of normal operation data effectively supports the control of the power supply system. Let the model of the discrete transfer function ($u \in R^1, y \in R^1$), obtained using the associative search algorithm for a specific moment in time, is as follows:

$$W(z) = \frac{Y(z)}{U(z)} = \frac{b_0 z^m + b_1 z^{m-1} + b_2 z^{m-2} + \cdots b_m}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n}, \quad (1)$$

where $U(z)\ and\ Y(z)$ are z-transformations of discrete input and output vectors, respectively. For the case when $b_m \neq 1, b_0 = b_1 = \cdots = b_{m-1} \neq 0, m \leq n$, we have:

$$\begin{aligned} &x_1[i] = y[i]; \\ &x_2[i] = y[i+1] + x_1[i+1]; \\ &\vdots \\ &x_n[i] = y[i+n-1] + x_{n-1}[i+1]; \\ &y[i+n] = x_n[i+1]. \end{aligned} \quad (2)$$

Taking into account the introduced variables it is possible to obtain an expression for the matrix-vector form of the system in the state space. In the case $u \in R^n, y \in R^m$ we have an $m \times n$ transfer matrix, whose elements are fractional rational functions.

The algorithm uses clustering methods in relation to the values of the elements of the transfer matrix, calculated in the course of experiments on the measured values of inputs and outputs.

3. DETERMINING STATIC STABILITY DEGREE BY GRAMIAN METHOD

From the methods used for solving the discrete Lyapunov equation (Antoulas, 2006), we have chosen the one offered in (Godunov, 1998). It applies the Fourier transform and *z*-transform to the discrete Lyapunov equation. The solution of the Lyapunov equation is an integral in the complex area of the product of resolvents of two matrices: the dynamics matrix and its transposed and adjoint one.

So, we investigate the stability of the linear model described above. Let the linear stationary discrete time-invariant system be as follows:

$$\begin{aligned} &x(k+1) = Ax(k) + Bu(k), x(0) = 0, \\ &y(k) = Cx(k), \end{aligned} \quad (3)$$

where $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^m, y(k) \in \mathbb{R}^m$.

Suppose the matrices $A_{[n\times n]}, C_{[m\times n]}, B_{[n\times m]}$ are the real ones, where *m, n* integer positive numbers, $m \leq n$. Suppose that the system (3) is stable, fully controllable and observable, all matrix A eigenvalues are distinct ones.

Suppose that transfer function of the system (3) is strictly proper. Consider the following algebraic discrete Lyapunov (Stein) equation of the form (Antoulas, 2006):

$$AP^cA^* + BB^* = P^c, A^*P^oA + C^*C = P^o. \quad (4)$$

If all eigenvalues of the matrix A are distinct then the matrix can be transformed to diagonal form by means of the similarity transform:

$$x_d = \mathrm{T}x,\ x_d(k+1) = \Lambda x_d(k) + B_d u(k), y_d(k) = C_d x_d(k),$$

$$\Lambda = TAT^{-1}, B_d = TB, C_d = CT^{-1}, \quad (5)$$

or

$$A = [u_1 \quad u_2 \cdots \quad u_n]\begin{bmatrix} z_1 & 0 & 0 & 0 \\ 0 & z_2 & & 0 \\ 0 & & \ddots & \\ 0 & 0 & & z_n \end{bmatrix}\begin{bmatrix} v_1^* \\ v_2^* \\ \vdots \\ v_n^* \end{bmatrix} = T^{-1}\Lambda T,$$

$$TV = VT = \mathrm{I}, \quad (6)$$

where the matrix T^{-1} consists of the right eigenvectors u_i, and matrix T consists of the left eigenvectors V_i^* corresponding to the eigenvalues z_i. The last equality is a condition for eigenvectors normalization.

Theorem. Consider LTI MIMO – the real discrete system with (A, B, C) presentation in the form, where the matrices A, B, are the real ones. Suppose the system is stable, fully controllable and observable, it's transfer function is strictly proper and the matrix A eigenvalues are distinct ones.

Then the following matrix equalities hold:

$$P^c = \frac{1}{2\pi i}\int_\gamma \sum_{k=1}^n u_k v_k^* \frac{1}{z-z_k} BB^* z^{-1}(z^{-1}I - A^*)^{-1}dz \quad \forall z: |z| < 1, \quad (7)$$

$$P^o = \frac{1}{2\pi i}\int_\gamma \sum_{k=1}^n z^{-1}(z^{-1}I - A^*)^{-1}C^*Cu_k v_k^* \frac{1}{z-z_k}dz \quad \forall z: |z| < 1, \quad (8)$$

$$P_k^c = \frac{1}{2\pi i}\int_\gamma R_k \frac{1}{z-z_k} BB^* z^{-1}(z^{-1}I - A^*)^{-1}dz \ \forall z: |z| < 1, \quad (9)$$

$$P_k^o = \frac{1}{2\pi i}\int_\gamma z^{-1}(z^{-1}I - A^*)^{-1}C^*CR_k \frac{1}{z-z_k}dz \ \forall z: |z| < 1, \quad (10)$$

where: R_k is the residue of the matrix A resolvent in the point equal to the matrix eigenvalue, u_k is matrix A right eigen vector, v_k^* is matrix A left eigen vector.

The equalities (7)-(10) define the semi-decomposition of the infinite controllability and observability Gramians on the eigenvalues set to belong of unit circle:

$$P^c = \sum_{k=1}^n \sum_{\rho=1}^n P_{k,\rho}^c, P_{k,\rho}^c = \frac{1}{1-z_\rho z_k} u_k v_k^* BB^* u_\rho v_\rho^*, \forall z: |z| < 1, \quad (11)$$

$$P^o = \sum_{k=1}^n \sum_{\rho=1}^n P_{k,\rho}^o,$$

$$P_{k,\rho}^o = \frac{1}{1-z_\rho z_k} u_k v_k^* C^*Cu_\rho v_\rho^* \ \forall z: |z| < 1, \quad (12)$$

and the equalities (3)-(6) define full decomposition of the infinite Gramians on the eigenvalues set to belong interior of the unite circle on complex plane. Expressions for Gramians spectral decomposition one can simplify.

Proof. The equation (4) solution in frequency domain looks as follows:

$$P^c = \frac{1}{2\pi}\int_0^{2\pi}(e^{-i\theta} - A)^{-1}BB^*(e^{-i\theta} - A^*)^{-1}d\theta, \quad (13)$$

$$P^o = \frac{1}{2\pi}\int_0^{2\pi}(e^{-i\theta} - A^*)^{-1}C^*C(e^{-i\theta} - A)^{-1}d\theta. \quad (14)$$

The following variable change $e^{i\theta} = z$ is made in the integrals:

$$P^c = \frac{1}{2\pi i}\int_\gamma (z^{-1}I - A)^{-1}BB^*z^{-1}(zI - A^*)^{-1}dz, \quad (15)$$

$$P^o = \frac{1}{2\pi i}\int_\gamma (z^{-1}I - A^*)^{-1}C^*Cz^{-1}(zI - A)^{-1}dz, \quad (16)$$

where γ is a unit circle, moving counter clockwise. So, all eigenvalues are distinct ones, and we have the following resolvent decomposition:

$$(Iz - A)^{-1} = \sum_{k=1}^n \frac{u_k v_k^*}{z-z_k},$$

$$(Iz^{-1} - A)^{-1} = \sum_{k=1}^n \frac{u_k v_k^*}{z^{-1}-z_k}.$$

We introduce the following designations:

$$[Res(zI - A)^{-1}, z = z_k] = u_k v_k^* = R_k,$$

$$BB^* = Q_d = [q_{d\rho,k}], C^*C = W_d = [w_{d\rho,k}].$$

After substituting this formula in equations (15), (16) we obtain:

$$P^c = \frac{1}{2\pi i}\int_\gamma \sum_{k=1}^n u_k v_k^* \frac{1}{z-z_k} BB^*z^{-1}(z^{-1}I - A^*)^{-1}dz, \quad (17)$$

$$P^o = \frac{1}{2\pi i}\int_\gamma \sum_{k=1}^n z^{-1}(z^{-1}I - A^*)^{-1} C^*C R_k \frac{1}{z-z_k}dz. \quad (18)$$

Let us introduce the following designations for each sub-Gramian:

$$P_k^c = \frac{1}{2\pi i}\int_\gamma u_k v_k^* \frac{1}{z-z_k} BB^*z^{-1}(z^{-1}I - A^*)^{-1}dz, \quad (19)$$

$$P_k^o = \frac{1}{2\pi i}\int_\gamma z^{-1}(z^{-1}I - A^*)^{-1}C^*Cu_k v_k^* \frac{1}{z-z_k}dz. \quad (20)$$

The integrands in (19), (20) are the analytic functions over the whole complex plane with the exclusion of particular points $= z_k$.

By means of the Caushy residue theorem, we have the Gramians spectral semi-decomposition in following way:

$$P^c = \sum_{k=1}^n P_k^c, P_k^c = u_k v_k^* BB^*[z^{-1}(z^{-1}I - A^*)^{-1}]_{z-z_k}, \quad (21)$$

$$P^o = \sum_{k=1}^n P_k^o, P_k^o = [z^{-1}(z^{-1}I - A^*)^{-1}]_{z-z_k} C^*Cu_k v_k^*. \quad (22)$$

We transform the matrix $[z^{-1}(z^{-1}I - A^*)^{-1}]_{z-z_k}$ by using matrix resolvent decomposition to vulgar fractions

$$[z^{-1}(z^{-1}I - A^*)^{-1}]_{z=z_k} = \\ = \frac{1}{z_k}\lim_{z^{-1}\to z_k}(z^{-1} - z_k)(z^{-1}I - A^*)^{-1} = \frac{1}{z_k} u_k v_k^*.$$

By substituting expressions (21), (22) to the above formulae we obtain the full decomposition of the Gramians in the form:

$$P^c = \sum_{k=1}^{n}\sum_{\rho=1}^{n} P^c_{k,\rho}\,, P^c_{k,\rho} = \frac{1}{1-z_\rho z_k} u_k v_k^* B B^* \left(u_\rho v_\rho^*\right)^* \quad (23)$$

$$P^o = \sum_{k=1}^{n}\sum_{\rho=1}^{n} P^o_{k,\rho}\,, P^o_{k,\rho} = \frac{1}{1-z_\rho z_k} u_k v_k^* C^* C \left(u_\rho v_\rho^*\right)^* \quad (24)$$

Corollary. The Gramians of diagonalized system are the solution of Lyapunov equations:

$$\Lambda P^c_d \Lambda^* + B_d B^*_d = P^c_d, \quad (25)$$

$$\Lambda^* P^o_d \Lambda + C^*_d C_d = P^o_d, \quad (26)$$

which are defined from the formulae:

$$P^c_{dk,\rho} = \frac{1}{1-z_\rho z_k} u_k v_k^* B_d B_d^* \left(u_\rho v_\rho^*\right)^* \ \forall z{:}\, |z| < 1, \quad (27)$$

$$P^o_{dk,\rho} = \frac{1}{1-z_\rho z_k} u_k v_k^* C_d^* C_d \left(u_\rho v_\rho^*\right)^* \ \forall z{:}\, |z| < 1. \quad (28)$$

The controllability Gramian P^c_d is linked with the Gramian P^c by equation

$$P^c = T^{-1} P^c_d (T^{-1})^*.$$

The observability Gramian P^c_d is linked with Gramian P^c by similar equation

$$P^o = T^* P^o_d T.$$

We introduce the new designation $\mathbf{1}_{ij}$ for the matrix with all zeros except for the element "ij", which is equal to one (1):

$$\mathbf{1}_{ij} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & & 0 \\ \vdots & 0 & 1 & 0 & \vdots \\ 0 & & 0 & & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

For diagonalized matrix A, the following expressions are valid:

$$(Iz - \Lambda)^{-1} =$$
$$= \sum_{k=1}^{n} R_k\,(z - z_k)^{-1} = \sum_{k=1}^{n} 1_{kk}\,(z - z_k)^{-1}, R_k = 1_{kk}.$$

Consider the spectral decomposition of the controllability and observability Gramians by pairwise combinational spectrum of the dynamics matrix. In this case, the formulae (27), (28) have the form:

$$P^c_{d\rho,k} = \frac{1}{1-z_\rho z_k} \mathbf{1}_{\rho\rho} \boldsymbol{Q_d} \mathbf{1}_{kk} = \frac{1}{1-z_\rho z_k} \mathbf{1}_{\rho,k} \boldsymbol{q}_{d\rho,k},$$

$$P^o_{d\rho,k} = \frac{1}{1-z_\rho z_k} \mathbf{1}_{\rho\rho} \boldsymbol{W_d} \mathbf{1}_{kk} = \frac{1}{1-z_\rho z_k} \mathbf{1}_{\rho,k} \boldsymbol{w}_{d\rho,k}.$$

Note that the premultiplication of the matrix $\boldsymbol{Q_d}$ by the matrix $\mathbf{1}_{\rho\rho}$ and the post-multiplication by the matrix $\mathbf{1}_{kk}$ allows us to cut from the matrix its element located in the intersection of the column "k" and the row "p". For the diagonalized system, we have the following formulae:

$$P^c_d = \oplus \sum_{\rho,k} \mathbf{1}_{\rho,k} \frac{\boldsymbol{q}_{dk,\rho}}{1-z_\rho z_k}, P^o_d = \oplus \sum_{\rho,k} \mathbf{1}_{\rho,k} \frac{\boldsymbol{w}_{dk,\rho}}{1-z_\rho z_k}.$$

These simple and compact expressions allow computing sub-Gramians by computing their n^2 elements. They are simpler than the common formulae for Gramian spectral expansion. We have got spectral separable Gramians decomposition in the form of a direct sum of n^2 sub-Gramians corresponding to the decomposition of controllability and observability Gramians by pairwise combinational eigenvalues of the dynamics matrix's spectrum.

The Gramians method can be used simultaneously for state monitoring and control of large-scale power systems, in particular, for static stability analysis; for developing stability estimator; detecting dangerous free and forced oscillations, and assessing the resonant interaction of dangerous oscillations (Iskakov, Yadikin, 2019; Yadikin, et al., 2016).

As it is known (Antoulas, 2006), the necessary and sufficient condition for the energy stability of the system in terms of the square of the H_2 norm of the transfer function has the form:

$$\|\mathbf{G}(z)\|_2^2 < +\infty.$$

We define the stability loss risk functional as follows:

$$J(z_1, z_2, \ldots, z_n) = \|\mathbf{G}(z_1, z_2, \ldots, z_n)\|_2^2.$$

As the system approaches the stability threshold caused by the approaching of the characteristic equation roots to the imaginary axis, the risk functional approaches infinity.

Let us define the acceptable risk of stability loss in the form

$$J(z_1, z_2, \ldots, z_n) = N_{perm}.$$

We will consider any system as *conditionally unstable* if all its roots are in the left half-plane, but the functional of the stability loss risk exceeds the established acceptable risk value. Accordingly, we will consider the system *conditionally stable* if

$$J(z_1, z_2, \ldots, z_n) < N_{perm}.$$

4. CASE STUDIES: AUTOMATIC REMOTE DIAGNOSIS OF READINESS FOR GENERAL PRIMARY FREQUENCY CONTROL IN THE POWER SYSTEM

In the event of an emergency situation in the unified power system, leading to a voltage drop in the network, all power plants carry out "primary frequency control" by changing the power through automatic controllers of the rotational speed of turbine units, the performance of boilers, nuclear reactors, etc. Each power plant does this in accordance with a specific contract.

The analysis of the power systems dynamic properties is convenient to carry out on the basis of the transient mode monitoring technique, implemented on special equipment for recording and transmitting information in real time. Modern primary frequency control systems contain digital generator excitation models, rotation turbine speed controllers, dynamic load models, protection and automation models. In the event of an emergency situation that leads to a voltage drop in the network, all power plants carry out primary frequency control – by changing the power through automatic controllers of the rotational speed of turbine units, the productivity of boilers, nuclear reactors, etc.

Primary frequency control should be carried out by fixed power stations that have the prescribed primary control characteristics. The participation of a particular unit in the primary frequency control is determined by a set of parameters that can change over the life of the equipment. If the characteristics of the equipment do not meet the generally accepted standards, it can lead to the loss of stability of the entire power system in the event of high frequency fluctuations. Therefore, it is very important for the normal operation of the power system to diagnose the current state of the units and their readiness for primary frequency control (Bakhtadze and Yadykin, 2019; Bakhtadze et al., 2010; Iskakov and Yadykin, 2019; Yadikin et al., 2016).

The only diagnostic technique available today is control testing. However, for the entire testing period, it is required to disable the generation process, which is too expensive. We proposed a technique for remote diagnostics of the readiness of generating facilities to control the primary frequency by aggregate responses to sudden frequency changes in normal operation.

Proof tests include checking the speed governors for each turbine; joint testing of the power unit; section of the heat block with a common steam pipe. The main requirements to be met by generating equipment are as follows:

- the aggregate of the main and auxiliary equipment, automation devices for power units, power plants and their operating modes should allow, within the prescribed load limits the amplitude of the primary control up to 20% of the rated power;
- when the power of the turbine unit changes in the range of ± 10% of the nominal value, the power value is formed by the basic and auxiliary equipment, as well as equipment for the automation of technological processes of the power unit / station. In this case, the rotation speed controller must provide the specified transient time.

Automatic regulation of operating mode parameters (the position of the turbine regulator, the inlet pressure of the steam turbine, etc.) should ensure that the experimental transient response of the primary frequency control is close to the required one (which is verified during certification tests).

The results of certification tests (empirical transient characteristics and parameter estimates), as well as the type of turbine, frequency slope and deadband of turbine speed controllers, slopes and deadbands of frequency shifts of power controllers, constitute the content of the knowledge base of the automatic diagnostic system.

An aggregated dynamic model giving the parameters estimation, as well as the statistical and dynamic properties of certain parameters and their relationships, is created on the basis of the associative search technique.

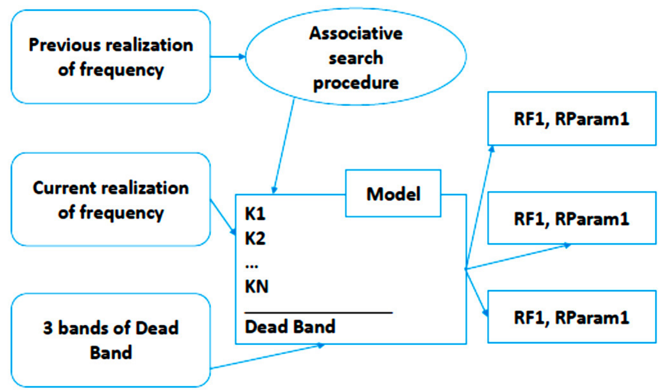

Fig. 1. Frequency analysis by intelligent model. RP1, RP2, RP3 – power realizations values.

Monitoring changes in the parameters of associative models for the identification of aggregates, which are developed on the basis of empirical data on the functioning of a real power system, makes it possible to identify and predict negative trends in the operation of a certain aggregate and its control system, which can degrade the quality of the aggregates in the primary frequency control system.

5. CONCLUSIONS

A method for constructing a predictive linear discrete model of the state of a nonlinear system is presented. To build a model, identification methods and algorithms are used based on the formation and analysis of knowledge processing – associative search algorithms.

Next, linear system stability by means of Gramian method is investigated. The use of the eigenvectors of the dynamics matrix greatly simplifies the calculation of controllability and observability sub-Grammians. The sub-Gramian matrix is the product of one scalar and three matrix factors.

The central matrix is the matrix of the right-hand sides of the Lyapunov equations, the left bordering matrix is the product of the right eigenvectors of the dynamics matrix by the conjugate left eigenvectors, the right bordering matrix is conjugate to the left one.

It is shown how the proposed method makes it possible on the base of data mining to perform dynamic predictive remote diagnosis for general primary frequency control in the power system problem.

Case studies have demonstrated high accuracy of the estimates obtained with help of associative search algorithm.

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