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Essays in Monetary Economics and Finance

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 30,166 words.

Statement of conjoint work

I confirm that Chapter 2 was jointly co-authored with Alkiviadis Georgiadis-Harris (LSE), and I contributed 50% of this work.

I confirm that Chapter 3 was jointly co-authored with George G. Pennacchi (University of Illinois), and I contributed 50% of this work.

Abstract

This thesis examines monetary policy in the presence of digital currencies issued by firms, as well as the effectiveness and credibility of recent bank recapitalisation reforms.

The first chapter discusses the consequences for monetary policy arising from currencies issued by firms—such as Facebook's Libra—in order to generate seignorage revenues and information on consumers. The paper develops a benchmark with two important results. First, information shapes the degree of currency competition: firms do not accept their competitors' currencies. Second, the central bank loses its policy autonomy. Profit-maximising firms implement a variant of the Friedman rule. As a result, public currency is unable to compete unless the central bank sets their nominal interest rate to zero, resulting in deflation. Importantly, private currency market power breaks this benchmark: inflationary pressures arise if firms form currency consortia, but decision powers and seignorage claims are concentrated in the hands of one firm.

The second chapter (with Alkiviadis Georgiadis-Harris, LSE) evaluates the effectiveness and market disciplining effects of bail-ins. During a bail-in, the government mandates equity write-downs and debt-to-equity swaps with the goal of recapitalising failing banks. We develop a model of asymmetric information on asset returns in which banks issue debt in order to finance projects. In equilibrium, the maturity of bail-in debt shortens if the government intends to impose losses on bank creditors. Runs ensue in the anticipation of bail-ins, rendering such policies ineffective. Controlling the maturity structure of debt has two benefits. First, it allows the government to avoid bailouts. Second, long-term bail-in debt leads to an increase in market discipline. The model provides an explanation why regulators impose a minimum maturity of one year for bail-in debt, and a motivation to treat short-term debt preferentially during intervention.

The third chapter (with George Pennacchi, University of Illinois) empirically investigates the credibility of bank recapitalization reforms using a structural model similar to Merton (1974, 1977). In the data, credit spreads on bank debt are valued as the product of market-perceived 'no-bailout' probabilities and expected no-bailout loss rates. Thus, no-bailout probabilities are estimated by regressing credit spreads on model-implied no-bailout loss rates. Before the Lehman bankruptcy, we find significantly higher bailout probabilities for US banks, relative to non-financial firms. Since then, relative bailout probabilities have clearly declined.

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1 Money Talks: Information and Seignorage

1.1 Introduction

The last decade has seen large-scale innovation in the realm of digital currencies. The most prominent example is Bitcoin, a private, decentralised digital currency (PDDC) for which transactions are verified using cryptographic technologies, and which has been issued in large amounts. Not yet in existence but already the subject of research and discussion are central bank digital currencies (CBDC), to be issued by monetary authorities and complementing banknotes, coins and bank reserves. This paper discusses a third type of digital currency: private, centralised digital currencies (PCDC) issued and operationally managed by firms or groups of firms that produce consumption goods. With over a billion users of Alibaba's Alipay and Tencent's WeChat Pay each, they are already much more important in number of transactions than Bitcoin. With the announcement of Libra—now renamed Diem and to be issued by a consortium of 100 firms lead by Facebook—they are perceived as a serious rival to central bank currency in the future.[1]

Unlike decentralised digital currencies, PCDC do not offer anonymity. The owner of the technology knows the identity of the consumer and verifies transactions centrally. The collected transaction data have great value in understanding consumer tastes, raising the profits of the seller. Therefore, introducing a centralised digital currency brings one benefit: it generates information rents. Unlike CBDC, PCDC generate further income that stays in private hands and is not rebated to the fiscal authorities. Issuing zero-interest currency backed by interest-paying assets, the firms obtain a second benefit: seignorage revenues. This paper studies how these two benefits affect the issuance of PCDC. It also studies their effect on monetary policy, both private and public, once the PCDC is widely used.

To this end, the paper develops a benchmark which characterises conditions such that firms

[1]Libra is clearly designed to be a currency as users will hold tokens in their digital wallets. These tokens can be exchanged for other currencies at the prevailing exchange rate. However, Alipay and WeChat Pay have features of both currencies and payment technologies which simply access bank accounts. On the one hand, users can hold tokens in their digital wallets. On the other hand, when making payments using the app, users can also debit their bank accounts. In this case, Alipay and WeChat Pay resemble payment technologies such as debit cards, rather than currencies.

introduce PCDC and in doing so endanger the central bank's policy autonomy. Once a single firm has introduced their PCDC, I find that the central bank is forced to implement the Friedman rule of zero nominal interest rates if two conditions are met. First, firms—best thought of as platforms, conglomerates or firm consortia which supply the entire range of consumption goods—have market power in product markets and charge prices that are independent of information. Second, consumer holdings of a firm's PCDC correspond one-for-one to their consumption expenditure with said firm. This paper then breaks the benchmark by introducing private currency market power. In particular, I show that central bank policy autonomy is reinstated if firms form currency consortia for which decision powers and seignorage claims are concentrated in the hands of one firm.

Building towards the benchmark model, the paper first develops a partial equilibrium framework with information frictions. Consumers value consumption heterogeneously, but their types and purchases are unobservable initially. Firms can only identify consumers after introducing a payment technology that generates transaction data. The model thus contains a notion of information based on past purchasing behaviour. Furthermore, competition among firms is imperfect. Consumers are subjected to search frictions and must engage in directed, sequential search for price offers in the spirit of Diamond (1971). This gives rise to market power and firms charge prices as if they were monopolists. Importantly, I make assumptions such that these monopoly prices are independent of consumer types, and thus also of information. Instead, information is useful since firms compete for the most profitable consumers' attention through advertising which directs the consumers' search for price offers.

The paper then develops a general equilibrium framework which fully nests the partial equilibrium model. The payment technology is modelled as money: consumers face a cash-in-advance constraint following Lucas and Stokey (1987), forcing them to hold currency exactly proportional to their consumption expenditure. The government issues public currency. Firms pay a fixed cost to introduce a private currency which brings two important benefits. First, usage of their currency generates information rents. As in the partial equilibrium model, the issuer learns their customers' types and exploits this information to increase profits. Second, firms obtain seignorage revenues proportional to the demand for their currency. Private currency does not pay interest and firms invest the proceeds from issuing money into interest-bearing bonds.[2] Finally, I endogenise the firms' currency acceptance. Thus, the money in circulation could be public, private, or both.

The benchmark model yields two important results. First, information shapes the degree of currency competition: firms do not accept competitor currencies. Information breaks the firms' indifference between the currencies in circulation since transaction data help to profitably allocate scarce advertising capacity. The model's prediction is consistent with empirical observations

[2]Private currencies are thus modelled as stablecoins.

from the world of digital platforms and payment technologies. In China, Alipay and WeChat Pay dominate the market for digital payments. However, customers of Alibaba cannot use WeChat Pay to purchase goods. Similarly, Amazon does not accept Google Pay and PayPal, which until recently belonged to eBay. Given the model's prediction and above observations, I anticipate that firms such as Amazon, Apple and Google will not accept Facebook's Libra in the future.

The second result is about the effect of optimal private monetary policy on public monetary policy. In models of money as the medium of exchange—such as the one presented in this paper—seignorage revenues correspond to a tax on consumption. Holding money is necessary to consume but it incurs an opportunity cost: it does not pay interest and therefore yields a lower return than bonds. It follows that the nominal interest rate of the economy can be interpreted as a tax rate. Consumption expenditure is proportional to money holdings and forms the tax base. Relative to an economy with zero nominal interest rates, firm profits and consumer utility are reduced. However, if the medium of exchange is public currency, positive interest rates generate seignorage revenues for the government which saves on interest expenses. When consumers hold PCDC in order to transact, the associated seignorage revenues do not accrue to the government but to an agent directly involved in the transaction: the firm. Given the first result, consumers can only use a firm's PCDC with said firm. If a consumer's holdings of a PCDC equal their consumption expenditure with the issuing firm, then this firm perfectly internalises the effect of positive interest rates on money holdings, consumption and thus profits. In order to maximise the sum of product profits and seignorage revenues, they fully remove the opportunity cost of money, either through product discounts or by setting interest rates in the private currency to zero.

Although competition among firms is imperfect, profit-maximising private monetary policy restricts public monetary policy, and vice versa. Consumers do not hold a currency if it is associated with a higher total price of consumption goods, combining the product price and the currency's opportunity cost. Thus, whenever interest rates in the public currency are larger than zero, competitor firms which only accept this public currency are unable to compete. Consumers do not demand public currency, and the central bank is forced to implement a zero interest rate policy. While limiting the central bank's policy autonomy, the introduction of private currency disciplines the public currency: implementing the Friedman rule of zero nominal interest rates improves welfare, and only the firms' market power prevents efficiency. However, while zero interest rates are desirable in a model of money as medium of exchange, they are associated with deflation—an outcome that may be undesirable for reasons outside of this model.[3]

I highlight a first mover advantage in introducing PCDC. Since strictly positive interest rates act as a consumption tax and reduce profits, all firms in the economy benefit from the privately-enforced Friedman rule. Private seignorage revenues therefore impose a positive externality on

[3]One example includes economies with nominal wage rigidities. The central bank may want to use inflation as a tool to reduce real wages in response to a negative productivity shock.

competing firms. Information rents however impose a negative externality. The issuer of private currency is able to identify the more profitable customers and improves their customer base at the expense of the competitor firms. The first mover therefore trades off the total gains, arising from both seignorage and information, against a fixed cost of introducing the private currency. After the introduction of one private currency, competitors only trade off the information gains against the fixed cost. If information gains in the economy are sufficiently large, each firm therefore forms a digital currency area as introduced by Brunnermeier et al. (2019): in order to maximise information rents, firms only accept their own PCDC. The public currency loses its role as medium of exchange.

Breaking the benchmark. The paper's third main result concerns the industrial organisation of currency consortia. If firms form currency consortia, but decision powers and seignorage claims are concentrated in the hands of one firm, inflationary pressures arise. The dominant firm obtains seignorage revenues corresponding to transactions with other consortium member firms. By implementing positive nominal interest rates—associated with inflation in the private currency—the consortium leader taxes other firms' transactions. Given such private monetary policy, the central bank's autonomy is reinstated. In fact, I show scenarios in which the public currency disciplines the private currency. One interpretation of this result is that the central bank regains policy autonomy as the private currency becomes more widely used in the economy. It also explains why the Libra consortium does not plan to design their currency as interest-bearing: doing so would reduce the profits of the leader of the consortium.

Policies to escape the benchmark. The consequences for monetary policy outlined in the benchmark are stark: firms perfectly internalise the opportunity cost of money and set the economy on a deflationary path. The paper therefore analyses whether the government can implement policies that allow them to escape the privately-enforced zero interest rate environment. First, I consider interest-bearing CBDC, and find that the central bank indeed regains autonomy as long as consumers are fully compensated for any foregone interest: digital public currency cannot yield a lower return than bonds.

This paper then evaluates a frequently made argument for the success of public currencies: governments can force firms to pay taxes in the public currency. In doing so, the government may indeed escape the zero interest rate environment. However, this policy gives rise to a capital gains motive: via high inflation, issuers of private money devalue their liabilities relative to the public-denominated assets. Usually in models of money as medium exchange, high inflation—associated with high nominal interest rates—leads to welfare losses. Surprisingly, welfare is improving in private currency inflation as it allows firms to circumvent the efficiency wedge introduced by the sales tax. Importantly, product discounts compensate consumers for the loss of value in their money holdings. Lastly, this paper considers macroprudential policies of private currencies. The exact same equilibrium logic applies if firms are required to invest parts of their proceeds in government currency: devaluing the private currency and giving product discounts achieves welfare-improving capital gains.

Contribution to the literature. This paper is the first to formally discuss private digital currencies as an information-generating device. It is also the first to discuss private, profit-maximising monetary policy conducted by producers of consumption goods and the arising consequences for monetary policy. The paper therefore primarily speaks to the literature on the digitalisation of money and currency competition.

Brunnermeier et al. (2019) discuss various aspects of the digitalisation of money. They describe, albeit without a model, how the introduction of digital currency areas helps promote platform cohesion and information collection. Central bank policy autonomy may be under pressure if public currencies lose their role as medium of exchange. This paper develops a formal model to endogenise the issuance of PCDC in order to achieve information and seignorage rents. I find that the introduction of one private currency already limits central bank autonomy. However, this result does not hold if the private currency is widely used in exchange.

The monetary policy consequences described in this paper resonate with the findings of Benigno et al. (2019). In their paper, a global private currency is used in two countries together with the local public currencies. By a no-arbitrage argument, all exchange rates are fixed in equilibrium, and monetary policies become synchronised. Their paper covers the special case of Libra as interest-bearing currency: central banks must set interest rates on bonds to zero for public currency to be able to compete. This paper obtains this result more generally when considering the nature of the issuer of currency, even if Libra does not pay interest. It also establishes that Libra-issuers have no incentive to pay interest if the currency is widely used.

Many more papers discuss digital currencies. However, they either analyse the effect of other types of digital currency on public monetary policy, or they analyse PCDC but not the consequences for monetary policy. In Fernández-Villaverde and Sanches (2019), entrepreneurs issue PDDC such as Bitcoin which compete with public currency. The entrepreneurs obtain seignorage revenues by expanding the money supply and may frustrate the government's attempts to implement the Friedman rule. Skeie (2019) discusses digital currency runs when a PDDC competes with public currency experiencing high inflation rates. Fernández-Villaverde et al. (2020) analyse an economy in which the central bank issues CBDC and invests in real assets. As a result, it may face a trade-off between price stability and excessive liquidation of its investments.

In Chiu and Wong (2020), a digital platform faces the choice between introducing private token money and accepting government currency. Issuing tokens is costly but generates transaction fees. However, the monopolist platform is fixed in size. Thus, it does not fully compete with other platforms or non-platform firms that accept government currency. This paper endogenises platform size. Private currency competes with government currency through the prices faced by consumers in each denomination. Gans and Halaburda (2015) discuss PCDC as a customer retention device in a partial equilibrium setting. Li and Mann (2018), Catalini and Gans (2018), Prat et al. (2019), Rogoff and You (2020) and Cong et al. (2020), among others, analyse PCDC with a focus on optimal design to finance at low interest rates, rather than to compete with government fiat money. Keister and Monnet (2020) discuss how CBDC can generate information for central banks over the quality of banks' balance sheets. While sharing the notion of information and digital currency, this paper discusses how PCDC generates information for firms over consumer tastes.[4] Lastly, Garratt and van Oordt (2021) and Garratt and Lee (2021) discuss how payment data collection leads to welfare losses due to price discrimination and monopoly formation. CBDC preserve privacy and help increase consumer welfare. However, the technology through which firms collect information—PCDC in this paper—is not directly modelled, and these papers do not analyse the consequences for monetary policy.

Information and privacy in the digital economy are being tackled from many other directions; see Goldfarb and Tucker (2019) and Bergemann and Bonatti (2019) for extensive surveys. Customer recognition by imperfectly competing firms has been addressed in Villas-Boas (1999) and Fudenberg and Tirole (2000).[5] In more recent work, Bonatti and Cisternas (2020) investigate price discrimination by short-lived monopolists based on a consumer score that aggregates information on past purchases. Modelling information in a general equilibrium framework, this paper endogenises the introduction and acceptance of digital currencies by firms in order to discuss consequences for monetary policy.

This paper develops a dynamic variant of the Diamond sequential search model in order to create a tractable framework, incorporating a notion of information based on past purchase behaviour into a monetary model. In reality, the search process is noisy: see Burdett and Judd (1983), as well as Burdett et al. (2017) and Bethune et al. (2020) in a monetary context. In this paper, search frictions are merely a useful modelling tool: the model yields closed form expressions for equilibrium prices, profits and thus the equilibrium value of information even in the monetary general equilibrium framework.[6]

Organisation of this paper. Section 1.2 presents the partial equilibrium model containing notions of imperfect competition and information based on past purchasing behaviour. Section 1.3 combines this building block with a general equilibrium framework in which firms issue PCDC,

[4]For further papers discussing pricing of cryptocurrencies, see, among others, Athey et al. (2016), Chiu and Koeppl (2017), Biais et al. (2018), Prat and Walter (2018), Budish (2018), Schilling and Uhlig (2019), Sockin and Xiong (2020) and Choi and Rocheteau (2020).

[5]See also Acquisti and Varian (2005), and Fudenberg and Villas-Boas (2012) for a survey of this literature.

[6]In a similar vein, this paper does not offer a comprehensive theory of advertising and product pricing. Advertising serves as a tool to exploit information. With a focus on the search process and pricing of consumption goods, advertising for attention has been explored by, among others, Armstrong and Zhou (2011) and Haan and Moraga-González (2011).

and discusses consequences for monetary policy. The effects of currency consortium ownership structures on inflation outcomes are developed in Section 1.4. Section 1.5 discusses interest-bearing CBDC and policies that force firms to hold public currency. Section 1.6 concludes.

1.2 Partial equilibrium model

This section introduces a two-period partial equilibrium model with notions of imperfect competition and information based on past purchasing behaviour. Firms do not know their customers and cannot observe their types until they introduce a private payment technology that reveals consumer purchases. The paper employs search frictions in the spirit of Diamond (1971) as a useful modelling device. In particular, they allow for an analytical characterisation of equilibrium prices and thus the value of information. Given this characterisation, the paper endogenises the introduction and general acceptance of firms' payment technologies. Importantly, the partial equilibrium model is flexible enough to serve as a building block for the general equilibrium framework presented in Section 1.3, in which the payment technology is modelled as money.

1.2.1 Environment

1.2.1.1 Consumers

There is a continuum of consumers j on the unit interval. Consumers differ in their taste of the consumption good. Their type θ_j is both private information and constant over time. There are two periods: $t \in \{0, 1\}$. Consumer j's period utility derived from consumption is given by:

$$u_j(c_t) = \theta_j^{1-\alpha} c_t^{\alpha} - p_t c_t \qquad \alpha \in (0, 1) \tag{1.1}$$

where p_t denotes the price of the consumption good. At the beginning of the game, each consumer draws their type from a publicly known, common binary distribution: $\theta_j \in \{\theta_L, \theta_H\}$, $P[\theta_j = \theta_H] = q$, with $\theta_H > \theta_L \geq 0$.[7]

Firms, introduced below, cannot transmit any information to consumers, neither about their own price nor their competitor's price. Consumers thus have to search for price offers. I assume directed, sequential search with perfect recall. Each period features two sub-periods, day and night. There is no discounting between sub-periods. During the day, firms set their prices for the period, and consumers obtain an initial price quote by visiting one firm. Having learnt one price, consumers have the option to visit a second firm at night, and learn about their price,

[7] The exponent on θ_j is included for exposition purposes and without loss of generality. As a result, consumer demand schedules and equilibrium firm profits are linear in consumer types.

before making their final consumption decision. The first search is free, the second search costs $S > 0$.[8]

1.2.1.2 Firms

The model features two firms, $i \in \{f, g\}$, which supply a homogeneous non-durable good. They set their period prices during the day. Firms seek to maximise profits and produce at constant marginal costs mc. In the second period, firms have the ability to send an advertising message to a limited fraction $\xi < 1$ of consumers, at zero cost.[9] They cannot advertise to any consumers beyond this fraction.[10,11] Advertising is the mechanism through which firms exploit information on consumer types. In particular, I make a behavioural assumption affecting the initial search decision: consumers visit the more heavily advertising firm first, as long as they expect this firm to charge a weakly lower price than its competitor. In equilibrium, all firms charge the same price to all consumers, regardless of type and time period. Unless firms can direct the consumers' initial search towards their firm, they have no means of making use of information. Let firm i's decision to send an advertising message to consumer j be denoted by $a^{i,j} \in \{0, 1\}$. Importantly, firms can neither observe consumer types nor competitor strategies, and thus have to form beliefs. Denote the set of firm i's strategies and beliefs by

$$\sigma_0^i = p_0^i, \quad \sigma_1^i = \left(p_1^i, a^i\right), \qquad \text{and} \qquad \mu_t^i\left(\sigma_t^{-i}, \theta\right), \quad t \in \{0, 1\} \tag{1.2}$$

1.2.1.3 Payment technologies

At the beginning of the first period, firms may pay a fixed cost to introduce a private payment technology that reveals otherwise unobservable first period purchases. This generates information, as firms identify their customers and back out their types θ. In the absence of a private payment technology, I assume that an unmodelled neutral payment technology is used.[12] In this case, firms do not learn consumer identities or types. Firms decide whether to accept the competitor's technology. For a given transaction, only one payment technology can be used: if a consumer of firm i uses the payment technology of firm $-i$, then the information accrues to

[8] As is common in the literature, the first search is free in order to prevent the market from breaking down. Note that for sufficiently small yet strictly positive initial search costs, all consumers enter the market due to the concavity of the utility function.

[9] I assume that $\xi > q/2$. If firms split the market equally, a firm sells to a measure $q/2$ of high valuation consumers. This assumption implies that their advertising capacity exceeds the number of their high valuation customers.

[10] I implicitly assume that reaching the first fraction ξ of consumers is costless, reaching any consumers beyond the initial fraction ξ is infinitely costly. This is an extreme version of the cost function considered in Grossman and Shapiro (1984) and Tirole (1988).

[11] I could in principle allow firms to advertise in the first period but—without any information on consumer types—they cannot do better than randomising.

[12] From Section 1.3 onwards, public currency plays the role of this neutral payment technology.

firm $-i$. Let the corresponding strategy set be denoted by

$$\Gamma^i = \left(\gamma^{i,int}, \gamma^{i,acc}\right), \text{ with } \gamma^{i,int}, \gamma^{i,acc} \in \{0,1\} \tag{1.3}$$

I assume that firm strategies Γ are fully observable by all agents in the economy.

1.2.2 Consumer strategies in the final period

Consider the consumer's decision problem at time-1. In the final period of the game, decisions do not affect any future payoffs. Omitting time subscripts for readability, consumer j's demand schedule conditional on purchasing from firm i charging price p^i is given by

$$c(p^i, \theta_j) \; = \; arg\,\underset{c}{max}\,\left(\theta_j^{1-\alpha} c^{\alpha} - p^i c \right) \; = \; \theta_j \left(\frac{\alpha}{p^i} \right)^{\frac{1}{1-\alpha}} \tag{1.4}$$

Given price p^i, consumers simply demand quantities that maximise utility. Their types θ shift their demand schedules: the higher the consumer's valuation, the larger the quantities that they purchase. Clearly, consumers prefer to be charged low prices, but they cannot observe a firm's pricing strategy until they obtain a price offer. Since firms cannot transmit any information, consumers have to form beliefs. Let $\mu^j(p)$ denote such beliefs of consumer j, and let $\psi^j(\mu^j(p), a^j)$ denote their initial search strategy.[13] Consumers first visit the firm which they expect to charge the lower prices. If they expect both firms to charge the same price, the relative advertising intensity determines the first search given the behavioural assumption. Importantly, advertising breaks the consumers' indifference in equilibria in which all firms charge the same price. Only if consumers expect both firms to charge the same price, and both firms have advertised with the same intensity, consumers randomise.

Having obtained one price offer p^i, consumers need to decide whether to search again. Let the continuation search strategy be denoted by $\omega^j(p^i, \mu^j(p^{-i}))$. Consumers trade off the expected gains due to a lower price offer against the fixed cost of searching $S > 0$. This gives rise to a cut-off rule: consumers are only willing to pay the fixed cost if the other firm's expected price $\mathbb{E}\left[p^{-i}|\mu^j\right]$ is sufficiently lower than the first price offer p^i.

1.2.3 Characterising the monopoly price

Given the consumer demand schedule, profits selling to a consumer of type θ are given by

$$\Pi(p, \theta) \; = \; (p - mc)\, c(p, \theta) \tag{1.5}$$

[13] A formal description of the consumer's search strategies is provided in Appendix 1.7.1.1.

The profit function $\Pi(p, \theta)$ is continuous and concave in p for all $p > 0$, and there exists a unique profit-maximising price $p(\theta)$ given by:

$$p(\theta) \;=\; arg\,\max_{p}\, \Pi(p, \theta) \;=\; \frac{mc}{\alpha} \;\equiv\; p^{mon} \tag{1.6}$$

The profit-maximising monopoly price is given by a constant mark-up over marginal costs and is thus independent of the consumer's type θ. Two assumptions yield this result: θ does not affect the price elasticity of consumption; and marginal costs are constant. It follows that monopoly profits are linear in the consumer type:

$$\Pi(\theta) \;=\; \kappa\theta \tag{1.7}$$

where κ is a constant. Note that the profit function $\Pi(p, \theta)$ is strictly increasing in p for all $p < p^{mon}$, and strictly decreasing in p for all $p > p^{mon}$.

1.2.4 Equilibrium definition and prices

I now proceed to define the equilibrium and to solve for equilibrium strategies. First, I show that firms charge the monopoly price p^{mon} in equilibrium.[14] Equilibrium profits on a given consumer increase linearly in type θ, and advertising thus aims to maximise the average type of a firm's customer base.

Equilibrium definition. *The Perfect Bayesian Equilibrium (PBE) of this game is given by the set of firm strategies* $\left(\Gamma^i, \sigma^i_t\right)_{t\in\{0,1\},\, i\in\{f,g\}}$ *that solve the firms' profit maximisation problems, given beliefs* $\mu^i_t(\sigma^{-i}_t, \theta)$*; and the set of consumer strategies* $\left(\psi^j_t, \omega^j_t, c^j_t\right)_{t\in\{0,1\},\, j\in[0,1]}$ *that solve the utility maximisation problem, given beliefs* $\mu^j_t(p_t)$*. Beliefs are formed rationally and updated according to Bayes' Law.*

Lemma 1.1 (Dynamic Diamond Paradox)**.** *For any $S > 0$, both firms charge the monopoly price in both time periods in equilibrium:*

$$p^i_t(\theta) \;=\; p^{mon} \quad \textit{for all} \;\; i, t \tag{1.8}$$

The proof is provided in Appendix 1.7.1.2. Intuitively, having obtained a price offer, consumers operate a cut-off rule in the final period. They compare the additional gains from searching—potentially obtaining a lower price offer—against the fixed cost of searching. This gives market

[14] Throughout this paper, I focus on equilibria in which $p^i_t \leq p^{mon}$ for all i, t. It is easy to see that one firm charging a price $p^i = p^{mon}$ and the other firm charging a price $p^{-i} > p^{mon}$ constitutes an equilibrium, with consumer beliefs formed rationally and thus consistent with above prices. All consumers visit firm i. Neither firm has an incentive to deviate from above prices. Firm i charges the profit-maximising monopoly price and has no incentive to increase the price. Lowering the price does not attract any additional consumers. Firm $-i$ cannot attract any consumers by lowering their price, given consumer beliefs.

power to firms. Conditional on being visit by a consumer, they face a profitable upwards deviation in the price—unless firms already charge the monopoly price. Given the assumptions of constant price elasticities and constant marginal costs, the monopoly price is independent of consumer types. In equilibrium, consumer beliefs about firm pricing must be correct, which is only true if all firms charge the monopoly price in the final period and consumers expect them to do so. Thus, all consumers face the same price in the final period. Giving away information in the first period therefore does not harm consumers. Consumers again only consider product prices when deciding on their initial search and operate a cut-off rule having obtained a price offer. This leads to equilibrium monopoly pricing also in the first period.

Given the search frictions and assumptions on utility and marginal costs, firms obtain full market power, but do not price discriminate. Thus, high valuation consumers do not have any incentives to mimic the low valuation consumers' actions in equilibrium. The advantage is model tractability: the framework includes notions of imperfect competition and information, and is flexible enough to be built into a standard monetary framework. The disadvantage is that the model is silent on strategic consumer behaviour (see i.e. Bonatti and Cisternas, 2020). In future work, I plan to explore the interaction between information-generating private currencies and price discrimination, with a focus on private monetary policy and arising consequences for public monetary policy.

1.2.5 Equilibrium advertising strategies

Having characterised equilibrium prices allows us to characterise equilibrium advertising strategies. Since all firms charge the monopoly price, profits on a consumer of type θ are given by $\Pi(\theta) = \kappa\theta$. When devising an advertising strategy, firms aim to maximise the value of their customer base: profits are increasing in the average type of their customers. The firms' relative levels of information thus determine the breakdown of profits.

Consider first a firm that has not introduced a payment technology, $\gamma^{i,int} = 0$. This firm neither knows their customers' identity, nor their type θ, and thus randomly advertises among the general population of consumers. Consider next a firm that has obtained information on their customer base and is able to distinguish consumers of types θ_L and θ_H. The firm advertises to consumers of known high valuation, and uses the remaining advertising capacity to advertise to consumers for which they did not obtain any information.[15]

[15] To see that this strategy is optimal for a given firm i, consider first their randomising competitor $-i$. The probability of double-advertising is equal for all consumers, and thus the informed firm aims to advertise to as many high type consumers as possible. If the competing firm is also informed, consider the choice between advertising to a consumer of known low type, and a consumer of unknown type. Begin with a consumer of unknown type to which firm i advertises. Since the competitor $-i$ is informed, they know their type exactly. If $\theta = \theta_L$, the competitor does not advertise and firm i sells to this consumer with probability one. If $\theta = \theta_H$, the competitor $-i$ advertises with probability one, and firm i wins this customer with probability $\frac{1}{2}$. Fixing firm $-i$'s advertising strategy for a previous customer of firm i, switching to a consumer of unknown type reduces the

1.2.6 Payment technology introduction and acceptance strategies

Having solved for equilibrium prices, advertising strategies and thus the value of information, this subsection characterises the firms' payment technology introduction and acceptance strategies in equilibrium.

Proposition 1.1 (No interoperability). *Firms do not accept their competitor's payment technology:*

$$\gamma^{i,acc} \; = \; 0, \quad i \in \{f, g\} \tag{1.9}$$

The proof is provided in Appendix 1.7.1.3. Intuitively, more information is always better, as it helps to improve the customer base at the expense of the competitor firm. Therefore, firms have no incentive to generate information for other firms. I strongly expect this result to hold up in more general settings. A firm should not accept a competitor payment technology in any environment in which the usage of this technology generates a direct benefit to the competitor.

The model therefore predicts that digital giants including Amazon, Apple and Google should not accept Libra as a payment technology upon its introduction. This prediction is consistent with observations from the real world. In China, the market for digital payments is dominated by Alipay and WeChat Pay. The social media platform WeChat resembles Facebook, and their payment technology cannot be used to purchase goods with Alibaba, the owner of Alipay. In other parts of the world, Amazon does not accept payments with Google Pay and PayPal, which until recently belonged to eBay.

Lemma 1.2 (Symmetric payment technology introduction decision). *Consider a fixed cost $k > 0$ to introduce the payment technology. Let the value of information in the partial equilibrium model be denoted by Δ. Then both firms introduce the payment technology if $\Delta \geq k$; otherwise no firm does so.*

The proof is provided in Appendix 1.7.1.4. By Lemma 1.1, industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$. Whenever firms have equivalent sets of information—either because no firms have generated transaction data, or both have generated equal amounts—the expected payoffs for the two firms equal. Firms evenly divide the market in half. It follows that any information gains are mere redistributions from one firm to another. When firm i introduces the technology, their profits increase at the expense of their competitor. When the competitor also introduces their own technology, both firms again obtain information of the same value: firm i loses exactly those profits initially gained from firm $-i$.

probability of being matched with the known low type by $\frac{1}{2}$. At the same time, it increases the probability of being matched with an unknown type by $\frac{1}{2}$. The proof is completed by realising that the unknown type has a higher expected valuation $\mathbb{E}[\theta] > \theta_L$.

Lemma 1.2 states that there is no first mover advantage in introducing payment technologies. The result resonates with the observation that Alipay and WeChat Pay share the market for digital payments in China. With the imminent introduction of Libra, all of the digital giants Apple, eBay, Google and Facebook will have introduced payment technologies. However, this result is less likely to hold up in more general environments than the one presented in this paper. Having to handle an increasing number of payment technologies, or currencies in Section 1.3, could become increasingly costly for consumers. General adoption also depends on network effects: consumers only want to use a payment technology (hold a particular currency) if they expect others to accept it. The number of equilibrium payment technologies may therefore well be limited once these considerations are included in the framework. In a similar vein, Section 1.3 presents a first mover advantage arising due to the second benefit of introducing a PCDC: seignorage revenues.

1.3 General equilibrium: Monetary framework

This section develops a general equilibrium framework which perfectly nests the partial equilibrium model of the previous section. The payment technology is specified as money which consumers need to hold in order to transact. The government supplies public currency. Firms choose whether to introduce a private currency, and whether to accept the public and competitor private currencies. Crucially, currencies compete. Consumers hold the currency in which they face the lowest total cost of purchasing consumption goods. In the general equilibrium framework, this total cost consists of two objects: the price charged by firms and the opportunity cost of holding money. Money does not pay interest, implying that consumers have to forego income. Thus, the opportunity cost of a particular currency is given by the nominal interest rate of bonds in this denomination. Importantly, the central bank conducts monetary policy by setting the nominal interest rate—and hence affects the opportunity cost of holding public currency. At the same time, firms issuing private currency maximise the sum of product profits and seignorage revenues. They jointly choose their product price and implement a private monetary policy that achieves this goal. Since currencies compete, the existence of a private currency may have stark consequences for the central bank which finds itself unable to implement their desired interest rates. In fact, this section shows that optimal private monetary policy forces the central bank to set interest rates to zero, leading to deflation.

1.3.1 Environment

1.3.1.1 Households

The model features overlapping generations (OLG) of consumers who live for three periods and discount the future at rate β. They consume a credit good C^c and a money good C^m, and supply labour N_t. At each point in time, three cohorts born in three different periods co-exist. Their age is denoted by $A \in \{y, mid, o\}$. Period utility for consumer j is given by

$$U_{A,j} \; = \; U(C^c) \; + \; \theta_{A,j}^{1-\alpha} \left(C^m\right)^{\alpha} \; - \; N \tag{1.10}$$

The market for the money good corresponds to the market of Section 1.2. Consumers derive utility from money good consumption for the first two periods of their life during which they value consumption heterogeneously according to their type θ_j. They do not value consumption of the money good in the final period of their life:

$$\left(\theta_{y,j},\, \theta_{mid,j},\, \theta_{o,j}\right) \; = \; \left(\theta_j,\, \theta_j,\, 0\right) \tag{1.11}$$

The two firms $i \in \{f, g\}$ introduced in Section 1.2 supply the money good and charge a price p_t^i. Consumers again search sequentially for price offers within a given period. To simplify without loss of generality, I assume that the first search is free, and the second search is infinitely costly. In the second period of their lives, consumers receive an advertising message sent by firms, as before. Consumers also decide which payment technology, now modelled as money, to use at a given firm. This decision is explained in detail in subsection 1.3.1.3.

The OLG structure is useful to replicate the setting of the previous section: it cuts off the purchase history and thus limits the degree of learning to the first period of consumer lives; it also allows the game among firms and consumers to be solved backwards from the final period in which consumers derive utility from money good consumption. Yet the model requires an infinite horizon for money to achieve its equilibrium value (see subsection 1.3.1.3 for details).

Turning to the credit good, I assume that the Inada conditions hold for the utility function, $U(C^c)$. This implies the existence of a consumption level C^* satisfying $U'(C^*) = 1$.[16] The market for the credit good is a useful modelling device. First, the credit good serves as the numeraire. Consumers supply labour and are compensated with a real wage w_t which allows them to purchase exactly as many units of the credit good. Money good producers charge real prices p_t in units of the credit good. Second, the market for the credit good pins down

[16] The consumer's period utility corresponds to a buyer's period utility in Lagos and Wright (2005). In their paper, sellers in the decentralised market pay a utility cost for every unit produced. Instead, I assume that firms produce according to a production function outlined below, using labour which is supplied by the household. As in their paper, these assumptions on the utility function generate tractability.

the real wage w_t, described in detail in the following subsection. Third, the separability and quasi-linearity of the utility function ensure that credit good consumption is pinned down in equilibrium. Intuitively, consumers can always supply an additional unit of labour at constant marginal disutility in order to purchase w_t more units of the credit good. This pins down the real interest rate. Effectively, the credit side of the economy is super-neutral with respect to monetary policy which allows me to focus on its direct effect on the levels of money good consumption—and thus on the role of money as medium of exchange.

Consider consumer j who has visited firm i and learnt their price p_t^i. This consumer forms a portfolio consisting of money and bonds in three currencies. Let the public currency be denoted by $M^{\$}$. Firm f issues private currency $M^{\approxeq}$, firm g issues currency $M^{\mathbb{G}}$. Going forward, I refer to the public currency as the *Dollar* and to firm $f's$ private currency as *Libra*. Omitting all (A, j)-subscripts to indicate individual decision and state variables of consumer j aged A, let their total money holdings in Dollar values be denoted by $e_t M_t$:

$$e_t M_t \;=\; M_t^{\$} \;+\; e_t^{\approxeq} M_t^{\approxeq} \;+\; e_t^{\mathbb{G}} M_t^{\mathbb{G}} \tag{1.12}$$

where the exchange rates $(e_t^{\approxeq}, e_t^{\mathbb{G}})$ denote the price of private currencies in terms of the Dollar. The price level P_t is the price of the Dollar in terms of the credit good; dividing nominal money balances in Dollar values by P_t thus yields real money balances in units of the numeraire. Consumers face non-negativity constraints on their real money balances for each currency. Turning to bonds, the Dollar value of total bond holdings, $e_t Q_t B_t$, is given by:

$$e_t Q_t B_t \;=\; Q_t^{\$} B_t^{\$} \;+\; e_t^{\approxeq} Q_t^{\approxeq} B_t^{\approxeq} \;+\; e_t^{\mathbb{G}} Q_t^{\mathbb{G}} B_t^{\mathbb{G}} \tag{1.13}$$

where Q_t denotes the prices of bonds issued at time-t, to mature in the following period. Bond prices are inversely related to the interest rate prevailing in the respective currencies:

$$Q_t^x = \frac{1}{1 + i_t^x} \qquad x \in \{\$, \approxeq, \mathbb{G}\} \tag{1.14}$$

In sum, consumer j's budget constraint, all in terms of the credit good, is then given by:

$$C_t^c \;+\; p_t^i C_t^m \;+\; \frac{e_t M_t}{P_t} \;+\; \frac{e_t Q_t B_t}{P_t} \;\leq\; w_t N_t \;+\; \frac{e_t M_{t-1}}{P_t} \;+\; \frac{e_t B_{t-1}}{P_t} \;+\; T_t \tag{1.15}$$

where T_t denotes the total real lump-sum transfer from firms and government to consumer j. I assume that firms and governments transfer all proceeds to the young in equal proportions.[17]

[17] If households receive large transfers when they are old, they may want to supply negative amounts of labour in order to achieve the optimal level of credit good consumption. As long as C^* is sufficiently high, this assumption ensures that the labour supply is always positive, without affecting any other equilibrium outcomes.

1.3.1.2 Firms

There are two sectors, the credit good and the money good sector. All firms maximise lifetime profits.[18] The firms in the money good sector are the ones introduced in Section 1.2: two firms $i \in \{f, g\}$ compete for consumer demand, choose a price p^i, and send an advertising message $a^{i,j}$ to middle-aged consumers. They decide whether to introduce and accept payment technologies, outlined in detail in the subsection 1.3.1.3.

In the general equilibrium framework, each sector produces given production functions that are linear in the only input factor labour:

$$Y_t^c \ = \ N_t^c \qquad\qquad Y_t^m \ = \ N_t^m \tag{1.16}$$

I assume perfect competition in the credit good market. Given the linearity of the production function, the real wage and thus real marginal costs for all firms in this economy are given by

$$w_t = mc_t = 1 \qquad\quad \text{for all } \ t \tag{1.17}$$

1.3.1.3 Payment technologies: Money

The payment technology of Section 1.2 is now modelled as money: consumers need to hold currency in order to facilitate transactions of the money consumption good. In particular, they face a cash-in-advance (CIA) constraint. The timing assumption is that of Lucas and Stokey (1987): the "cash market" opens after the "credit market". Search takes place when the credit market is open, and consumers choose their cash balances in order to purchase goods from the firms they have visited. Consumption finally takes place at the end of the period.[19]

Firm f's currency introduction and acceptance decision is given by

$$\Gamma^f \ = \ \left(\gamma^{\text{≋}}, \gamma^{f,\$}, \gamma^{f,\mathbb{G}}\right) \tag{1.18}$$

Conditional on all private currencies being introduced, the CIA constraint faced by consumer j at firm f is therefore given by

$$p_t^f C_t^{m,f} \ \leq \ \gamma^{f,\$} \frac{M_t^{\$}}{P_t} \ + \ \frac{e_t^{\text{≋}} M_t^{\text{≋}}}{P_t} \ + \ \gamma^{f,\mathbb{G}} \ \frac{e_t^{\mathbb{G}} M_t^{\mathbb{G}}}{P_t} \tag{1.19}$$

with a corresponding currency decision and resulting CIA constraint for firm g. One unit of real money allows for one unit of real money good expenditure. For a given currency, this is only true if the firm also accepts this currency. As an example, if $\gamma^{f,\$} = 0$, then Dollar holdings do

[18] In equilibrium, consumer discount factors are equalised. Firms use this representative discount factor.
[19] An alternative timing specification is presented in Appendix 1.7.3.1. The results are unchanged.

not enable consumption purchases with firm f. The CIA constraint above imposes that firms always accept their own private currency as means of payment.

Assumption 1.3.1. *Consumers perfectly anticipate which currencies each firm accepts.*

This assumption ensures that firms do not forgo any revenues by not accepting a particular currency. Since PCDC are newly being introduced, this assumption initially boils down to the public currency being widely used as an alternative form of payment. Casual empiricism from the world of digital giants, as discussed in the previous section, further suggests that the network effects of not accepting PCDC are not reducing revenues to the extent that firms want to accept competitor technologies. Perhaps more controversially, this assumption also implies that firms are more willing to form digital currency areas by not accepting the public currency. I certainly plan to revisit the question of such network effects, particularly the effects of not accepting PCDC on smaller firms, in future work.

Given this setup, the OLG structure helps address a feature of the cash-in-advance model: consumers need to hold real money balances proportional to their consumption expenditure but there is no direct exchange of money and goods. Consumers enter the following period with exactly the same amount of nominal money even if they have consumed. Including a third period in which money holdings unravel is useful since money has no value to consumers when they have died, and they would otherwise strategically reduce their money good consumption when middle-aged. Furthermore, the model requires an infinite horizon for money holdings of old consumers to be valued: younger generations need to demand money in order to purchase consumption goods.

1.3.2 Money balances and money good demand schedules

Given the problem's set-up, this section provides a summary and intuitive discussion of the consumer optimality conditions. A formal, step-by-step solution to the consumer problem, including the full set of optimality conditions, is presented in Appendix 1.7.2. The equilibrium of this economy is defined in Appendix 1.7.2.1.

Importantly, the monetary framework nests the equilibrium mechanism of Section 1.2. As in the partial equilibrium model, firms have market power due to the search frictions and charge monopoly prices which are independent of consumer types—all consumers face the same prices in all time periods. Hence, the consumers' decisions when young do not affect future payoffs. It follows that consumers maximise present utility in each time period when determining money balances and interacting with producers of the money good.

First, the optimality conditions for credit good consumption establish the equilibrium relationship between nominal and real interest rates as well as the inflation rate. Assuming separability and quasi-linearity of the utility function, combined with constant wages $w_t = 1$, renders the credit side of the economy super-neutral with respect to monetary policy. It follows that credit good consumption is equal for all time periods t and consumer ages A:

$$C^c_{A,t} \; = \; C^* \tag{1.20}$$

Intuitively, given the unit real wage, consumers can always purchase one more unit of the credit good by supplying an additional unit of labour. The real interest rate of the economy is then pinned down by the time rate of preference: $1+r_t = \beta^{-1}$. For all currencies $x \in \{\$, \approxeq, \mathbb{G}\}$, define the inflation rate π^x_{t+1} as the change in their price relative to the credit good over time. The first order conditions for bonds for all consumers, regardless of their type and asset holdings, simplify to the Fisher equation (here expressed in bond prices):

$$Q^x_t \; = \; \beta(1 + \pi^x_{t+1})^{-1} \tag{1.21}$$

The price of bonds needs to compensate for the fact that consumers discount the future and that nominal bonds change real value over time, captured by the inflation rate. Going forward, I thus refer to bond prices Q^x_t as *inverse inflation rate* in currency x: the higher the inflation rate, the lower the inverse inflation rate, and the higher is the opportunity cost of holding money.

Second, money is dominated by bonds in terms of returns whenever $Q^x_t < 1$. Intuitively, consumers only hold money in order to enable consumption purchases. Thus, their real money balances do not exceed their real money good expenditure. Consumers do not hold currencies that are not accepted by firm i; if multiple currencies are accepted, they hold the one with the highest inverse inflation rate, denoted by $\tilde{Q}^i_t$. Appendix 1.7.2.2 formally shows that, whenever $\tilde{Q}^i_t < 1$, the CIA constraint holds with equality:

$$m_t \; = \; p^i_t \, C^m_t(\theta_{A,j}, p^i_t, \tilde{Q}^i_t) \tag{1.22}$$

where m_t denotes the equilibrium real money portfolio of the consumer at time-t. If $\tilde{Q}^i_t = 1$, there is no opportunity cost of holding money, and the CIA constraint may be slack. The solution also implies a zero lower bound on nominal interest rates:

$$Q^x_t \; \leq \; 1 \qquad \Leftrightarrow \qquad i^x_t \; \geq \; 0 \tag{1.23}$$

for all currencies $x \in \{\$, \approxeq, \mathbb{G}\}$. For negative interest rates, markets for bonds and money do not clear: consumers want to borrow infinite amounts at negative rates to purchase money which pays zero interest.

Finally, the demand schedule for the money good that maximises present money good utility is then given by

$$C_t^m(\theta_{A,j}, p_t^i, \tilde{Q}_t^i) \;=\; \theta_{A,j}\left[\frac{\alpha}{p_t^i(2-\tilde{Q}_t^i)}\right]^{\frac{1}{1-\alpha}} \tag{1.24}$$

Compared to Section 1.2, the demand schedule is now a function of the *inflation-adjusted price*: firms charge a price p^i which is scaled up by the opportunity cost of having to hold a currency that is accepted in exchange. If $\tilde{Q}_t^i = 1$, bonds and money have the same return. There is no opportunity cost of money and consumers pay the real price only once. If $\tilde{Q}_t^i < 1$, consumers pay the full price once to firms, and another $(1 - \tilde{Q}_t^i)$-times to the issuer of currency. As $\tilde{Q}_t^i$ approaches zero—or equivalently inflation in this currency approaches infinity—consumers have to hold an asset that fully loses its real value in the process, and thus pay the full price a second time. The expression for the consumer's demand schedule stresses how seignorage revenues act as a tax on consumption. Like a tax rate, the difference in returns to bonds and money scales up the firms' prices.

Consumers visit the firm which they expect to charge the lowest inflation-adjusted price. This is true for any age. When consumers are middle-aged, they receive an advertising message from firms. For equal inflation-adjusted prices, they visit the more heavily advertised firm. Otherwise they randomly choose a firm, as before. When consumers are young, they do not receive an advertising message and randomise immediately for equal inflation-adjusted prices. Due to the assumption of infinite search costs, consumers only ever visit one firm to obtain a price offer in a given period.

1.3.3 Producers issuing PCDC remove the opportunity cost of money

Given search frictions and resulting market power, firms set their prices in each period as if they were monopolists. In contrast to Section 1.2, the model now contains two types of (money good) producers: those that do not issue private currency and need to accept the public currency; and those that have introduced a PCDC. The consumer demand schedule of Equation (1.24) reveals that consumers consider two factors: the real price p_t^i in terms of the numeraire, and the opportunity cost of money captured by $(2 - \tilde{Q}_t^i)$. For a firm without private currency, the firm sets the price and the central bank implements a particular inverse inflation rate. A product producer that issues private currency chooses both: it charges a price and controls monetary policy for the currency used in the transaction. This subsection first characterises monopoly pricing for firms that transact in the public currency. It then jointly characterises product pricing and private monetary policy for firms that transact in their private currency.

Consider firm i that does not obtain seignorage revenues on a given transaction with consumer

j. The firm's corresponding profits are given by

$$\Pi_t^{m,i} \;=\; \left(p_t^i - 1\right) C_t^m \left(\theta_{A,j}, p_t^i, Q_t^{\$}\right) \tag{1.25}$$

Firms charge the same price as in the partial equilibrium Diamond search game, but profits are distorted by the inverse inflation rate. Taking public monetary policy as given, firms maximise profits. Importantly, firms optimally do not internalise the opportunity cost of money that issued by a third party:

$$p^{mon} \;=\; arg\, \underset{p_t^i}{max}\;\Pi_t^{m,i} \qquad \Rightarrow \qquad \Pi_t^{m,i}\big(\theta_{A,j}, Q_t^{\$}\big) \;=\; \kappa\theta_{A,j}\big(2 - Q_t^{\$}\big)^{\frac{1}{\alpha-1}} \tag{1.26}$$

Next consider firm f which issues Libra.[20] Libra is modelled as a stablecoin: it is backed with Libra-denominated bonds that have been issued by the household.[21] For every unit of money issued, they purchase a unit of bonds.[22] The firm's total seignorage revenues, $s_t^{\approxeq}$, are then given by

$$s_t^{\approxeq} \;=\; (1 - Q_t^{\approxeq})m_t^{\approxeq} \tag{1.27}$$

This expression is very intuitive: firm f sells money at price one, but purchases bonds at price $Q_t^{\approxeq}$. This generates real revenues on every unit of real Libra balances, denoted by $m_t^{\approxeq}$. Consider now a transaction with a given consumer j of age A, again omitting subscripts for readability. Positive seignorage revenues require $Q_t^{\approxeq} < 1$, implying a binding CIA constraint:

$$m_t^{\approxeq} \;=\; p_t^f C_t^m(\theta_{A,j}, p_t^f, Q_t^{\approxeq}) \tag{1.28}$$

Total profits for a Libra transaction with this consumer j are then given by the sum of product profits and seignorage revenues. The profit maximisation problem becomes

$$\underset{p_t^f, Q_t^{\approxeq}}{max} \;\; \Pi_t^{m,f} + s_t^{\approxeq} \;=\; \left(p_t^f(2 - Q_t^{\approxeq}) - 1\right)\theta_{A,j}\left[\frac{\alpha}{p_t^f(2 - Q_t^{\approxeq})}\right]^{\frac{1}{1-\alpha}} \tag{1.29}$$

The following proposition jointly characterises private monetary policy and product pricing:

[20] A full derivation of the profit maximisation problem is provided in Appendix 1.7.2.5.

[21] Section 1.5.2 discusses an economy in which only Dollar-denominated bonds exist.

[22] Note that households are perfectly happy supplying bonds in exchange for money as long as the real interest rate on bonds does not exceed $1 + r_t = \beta^{-1}$. Since $w_t = 1$, every unit of interest payments will have to be made up by supplying one unit of labour in the future, but the disutility from supplying labour is discounted by the rate of time preference β.

Proposition 1.2 (Profit-maximising private monetary policy). *A producer of the consumption good, who also supplies the money used in the transaction and controls the associated inflation rate, chooses an inflation-adjusted price satisfying*

$$p_t^f(2 - Q_t^{\approx}) \; = \; p^{mon} \tag{1.30}$$

While continuing to charge an inflation-adjusted price corresponding to the monopoly price, the firm optimally removes the opportunity cost of money. It does so by providing product discounts, pursuing a private monetary policy of $Q^{\approx} = 1$, or implementing a combination of the two.

Firm f charges a real price p_t^f. At the same time, consumers need to hold money to purchase consumption goods and are subjected to the associated opportunity cost. This is captured by $(2 - Q_t^{\approx})$. Consumers pay the real price once directly to firms, and another $(1 - Q_t^{\approx})$-times indirectly to the issuer of currency as seignorage tax. Importantly, if a transaction takes place in private currency, the tax rate is also set by firm f. As the supplier of Libra and therefore recipient of seignorage revenues, firm f maximises the total sum of producer profits and consumption tax income. In doing so, they perfectly internalise any distortionary equilibrium effects. If demand is price elastic, which is true here, taxes are associated with deadweight losses. Thus, the firm optimally implements a PCDC variant of the Friedman rule, removing the tax income altogether to avoid such deadweight losses. It does so either by providing product discounts, or by implementing a monetary policy that does not entail an opportunity cost of money in the first place, setting $Q_t^{\approx} = 1$. Effectively, firms obtain a degree of freedom: they can implement any private monetary policy and then set prices accordingly. Firm f charges exactly that inflation-adjusted price—consisting of a real product price and the opportunity cost of money—which maximises total profits. The breakdown between product profits and seignorage revenues is irrelevant to firm f.

The result of Proposition 1.3 appears surprising in the context of proposed regulation. Regulators worry that issuers of Libra inflate away the value of their currency in order to increase profits, hurting consumers in the process. The model suggests that the equilibrium effect goes in the exact opposite direction: firms remove the opportunity cost of holding money, which in the case of $Q_t^{\approx} = 1$ is associated with deflation (Equation 1.21).

Corollary 1.1 (Currency design equivalence). *If firms were allowed to pay interest on private currency in the model, Proposition 1.2 would extend to interest payments. Firms are indifferent between implementing $Q_t^{\approx} = 1$, or fully compensating for inflation using either price discounts or interest payments on currency.*

Given the private currencies' digital nature, it is technologically feasible to pay interest on money holdings. Corollary 1.1 helps explain why the Libra White Paper v2.0 (April 2020) proposes to design Libra as non-interest-bearing currency: consumers can be incentivised to hold Libra through different measures other than interest payments. Considering the regulators' worries about the consequences arising from the introduction of PCDC, this result is again surprising. One suggested policy to mitigate the consequences is to prevent issuers of PCDC from paying interest. The model suggests that such a policy does not have any bite. It prevents issuers of PCDC from adding a feature to their currency which they do not need to add. It also cannot avoid the consequences for monetary policy outlined below.

Crucially, the results obtained in this section rely on the fact that the issuer of PCDC obtains seignorage revenues corresponding exactly to its product sales. Consumers only hold Libra in order to transact with firm f. This section should therefore be considered a benchmark. Section 1.4 discusses currency consortia more generally in which seignorage revenues may not be distributed according to sales shares. Section 1.5 introduces policies that affect seignorage revenues, i.e. through macroprudential policies which limit how firms can invest the proceeds from issuing money.

1.3.4 The presence of a PCDC pushes nominal interest rates to zero

Having characterised the inflation-adjusted prices that firms charge in different currencies, I am now ready to derive the consequences for private and public monetary policy. Suppose only firm f has introduced their private currency Libra. Since I fix the firms' introduction decisions, let me call this scenario *partial equilibrium*. The partial equilibrium is interesting to discuss for various reasons. First, understanding partial equilibrium payoffs is required to fully characterise the best response of a competing firm upon the introduction of a private currency. Second, if seignorage does not provide an advantage over the competitor in equilibrium (which I show below), then counter-innovations in payment technologies need not take the form of currencies. Lastly, there may be first mover advantages outside of the model: examples include network effects, or the regulator's lack of appetite for another digital currency run by some digital conglomerate in the future.

Corollary 1.2 (Search and choice of currency). *Suppose $Q_t^{\$} < 1$. Consumers only visit firms which have introduced their own private currencies. Whenever these firms accept the Dollar and their private currency, consumers prefer to transact using the private currency.*

Proposition 1.3 (Monetary policy consequences for one-sided introduction). *Suppose the government supplies a positive amount of money, $M_t^{\$,S} > 0$. Then money markets only clear if*

$$Q_t^{\$} = 1 \qquad \Leftrightarrow \qquad i_t^{\$} = 0 \tag{1.31}$$

Given the pricing formula for bonds, this policy is associated with deflation: $\pi_{t+1}^{\$} = \beta - 1 < 0$.

Consumers rationally form beliefs about firms' prices. Given optimal product pricing combined with private monetary policy, and unless $Q_t^{\$} = 1$, purchasing at firm f using Libra is less costly than a) using the Dollar at firm f, and b) purchasing at firm g. Suppose $Q_t^{\$} < 1$. No consumer visits firm g's store, and thus aggregate consumption of the money good provided by firm g is zero. Since no consumer uses the Dollar at firm f, the non-negativity constraint binds, and $M_t^{\$} = 0$. Therefore money markets cannot clear if the government supplies a positive Dollar supply, $M_t^{\$,S} > 0$, unless $Q_t^{\$} = 1$. Seignorage revenues accruing to the currency-supplying firm lower their inflation-adjusted prices; these prices are only matched by the competing firm in the absence of any government seignorage revenues. That is, the central bank's interest rate needs to satisfy $i_t^{\$} = 0$.

Upon the introduction of a PCDC, the obtained consumption levels are those that would be achieved in a pure Dollar economy in which the central bank follows the Friedman rule. Here, in equilibrium, this Friedman rule is privately enforced. The PCDC disciplines the public currency, and the central is forced to remove the opportunity cost of holding money by setting nominal interest rates to zero. However, the allocations are not efficient due to the search frictions, giving rise to monopoly pricing. Firms still sell their product at prices exceeding marginal costs.

The mechanism described in this paper differs from the one in Benigno et al. (2019). In their paper, the currency consortium may pay out part of their seignorage proceeds generated on Dollar bond holdings as interest payments, and thus all consumers hold Libra—unless there are no seignorage proceeds because the central sets the nominal interest rate to zero. In this paper, firms optimally reduce their inflation-adjusted price. Even without paying interest on Libra, firms only accepting the Dollar are put out of business whenever the nominal interest rate is larger than zero.

Furthermore, and in contrast to the above paper, I find that the firms' optimal currency acceptance decisions ensure that the Dollar-Libra exchange rate is determined:

Corollary 1.3 (Acceptance of public currency). *Issuers of PCDC do not accept the public currency in order to maximise information rents:*

$$\gamma^{f,\$} = 0 \tag{1.32}$$

Corollary 1.4 (Exchange rate determination). *In the partial equilibrium scenario in which firm f has introduced Libra, the Dollar-Libra exchange rate is determined.*

In models of currency competition, such as Schilling and Uhlig (2019) and Fernández-Villaverde and Sanches (2019), total real money balances are determined in equilibrium. This is does not apply to the portfolio breakdown among the competing currencies. Consumers are only willing to hold all currencies in circulation if no single currency dominates the others in terms of returns.[23] Since consumers are indifferent between all currencies, there are not enough equilibrium conditions to pin down individual currency balances and thus the exchange rates between currencies. This result was initially obtained by Kareken and Wallace (1981). In this paper, consumers compare the inflation-adjusted product prices in the currencies in circulation. Effectively, the model lifts the currency indifference result obtained in Kareken and Wallace (1981) from the currency level to the product level. Firm f only accepts Libra, firm g only accepts the Dollar. Given equilibrium consumption levels for each firm, the equilibrium real balances of both Dollar and Libra are pinned down. The model features market clearing conditions for each currency, not just one for money balances as a whole. Thus, the Dollar-Libra exchange rate is determined.

1.3.5 Both firms introduce a private currency if information rents are large

Using the results of subsection 1.3.4, I now allow firms to counter-innovate by also introducing a private currency, thus calling this scenario *general equilibrium*. I highlight a first mover advantage in the monetary framework: while information rents reduce the competing firm's profit, seignorage revenues accrue to both firms in equilibrium. The second mover therefore may not find it profitable to also introduce a PCDC.

To demonstrate, note that the first mover gains can be neatly decomposed into two components: information rents in a zero interest rate environment, and seignorage revenues. Prior to the introduction of a private currency, firms split the market equally. By Equation (1.26), the opportunity cost of holding Dollars acts as a tax on consumption, lowering profits for all firms in every time period. Proposition 1.3 showed that this opportunity cost is fully removed for all currencies upon the introduction of one private currency. Both firms benefit equally and achieve the seignorage gains, denoted by $\Delta^{\$}\left(\left\{Q^{\$}_{t+s}\right\}_{s\geq 0}\right)$.

Turning to information, the firms' advertising decision is unchanged relative to Section 1.2. For each level of the equilibrium opportunity cost of money, consumer types θ still shift consumer demand schedules and profits (see Equation 1.26). It follows that firms advertise to high valuation consumers, and prefer to advertise to consumers of unknown type rather than to consumers of known low type. Section 1.3.4 established that issuers of PCDC force the central bank to implement zero interest rates. Thus, let Δ^I denote the lifetime information rents for an economy with zero interest rates.[24] The first mover trades off the total gains from introducing a private currency against a fixed cost of doing so. The second mover then trades off only the information gains Δ^I against the fixed cost.

[23] Maintaining equal levels of non-return benefits, i.e. arising due to a CIA constraint.

[24] Let Δ denote the period information gain for an economy with zero interest rates. It corresponds to the one-time information gain in Section 1.2. Firms discount future profits using the household's real discount factor, given by β in equilibrium. Since the firm benefits from information for the first time in the period after introducing the currency, the total lifetime information gain is given by $\Delta^I = \frac{\beta}{1-\beta}\,\Delta$.

Proposition 1.4 (First mover advantage). *If the fixed cost of introducing the currency is smaller than the lifetime information gain, $k \leq \Delta^I$, then both firms introduce private digital currencies. Neither accept the Dollar nor their competitor's currency. The Dollar loses its role as medium of exchange, and thus money market clearing for a positive supply $M_t^{\$,S} > 0$ again requires $Q_t^{\$} = 1$.*

For an intermediate cost k, with $\Delta^I < k \leq \Delta^I + \Delta^{\$}\left(\{Q_{t+s}^{\$}\}_{s\geq 0}\right)$, only one firm introduces a private currency. The competitor firm does not accept it. The first mover achieves both information and seignorage rents. While information rents impose a negative externality on the competitor firm, forcing the central bank to implement the Friedman rule imposes a positive externality.

Whether the public currency loses its role as medium of exchange depends on the size of the economy's information rents. For sufficiently large information rents, all firms form digital currency areas as introduced by Brunnermeier et al. (2019): although transactions take place within one economy, they are settled using different currencies in different marketplaces. Information generated by purchases is valuable, and thus firms aim to maximise their own information set while minimising that of the competitor. This is achieved if each firm only accepts their private currency. It follows that firms prefer not to transact in the Dollar even for a central bank monetary policy of zero interest rates. However, given the privately-enforced Friedman rule result, payment technologies introduced by second movers need not take the form of private currencies. One currency already disciplines the government, and firms can rely on other technologies to generate transaction data.

Proposition 1.4 also shows that it is more profitable to introduce a PCDC in countries with high inflation. Incentives are higher if the central bank is ill-disciplined and monetary policy effectively taxes transactions between firms and consumers. Even in the absence of any PCDC, public currency interest rates are capped. As $i_t^{\$}$ increases—or equivalently, as $Q_t^{\$}$ decreases—profits decrease. Unless the cost of introducing a PCDC approaches infinity, there exists a threshold level of public currency ill-discipline at which one firm introduces a PCDC, disciplining the government forever. This finding resonates with the observed dollarisation in economies which experienced high inflation rates.

Finally, the model's first mover advantage also resonates with frequently made arguments for the clear dominance of one currency in the majority of economies.[25] In all likelihood, handling multiple currencies is mentally costly, and the marginal cost of holding another currency is increasing. General adoption also depends on network effects: consumers only want to hold a particular currency if they expect others to accept it. Interestingly, the People's Bank of China is planning on issuing CBDC very shortly. One doubt about its future success is whether the

[25] One argument for the success of public currencies is that the government can force agents to pay taxes in the public currency. I evaluate the equilibrium effects of such a policy in Section 1.5.3.

CBDC can compete with Alipay and WeChat Pay which have successfully established themselves economy-wide. The example of China shows that two digital payment technologies can co-exist, but the number of currencies that can circulate in an economy may well be limited. Such considerations strengthen this paper's result. Together, they help explain why Facebook is pushing ahead with their currency project, although they have been unable to convince the desired amount of 100 firms to join their consortium so far.

1.4 Industrial organisation of currency consortia

In the benchmark of Section 1.3, firms internalised the opportunity cost of holding money due to seignorage revenues. Monetary policy was forced to follow suit and implement a zero interest rate policy. Inspired by the institutional set-up of Libra as a currency consortium consisting of multiple firms, I now consider seignorage dividend structures more generally, and discuss their effect on equilibrium inflation outcomes. I highlight inflationary pressures if Libra becomes more widely used: the Libra issuer benefits from non-zero interest rates as they correspond to a tax on other firms' transactions. Central bank policy autonomy is reinstated. I show scenarios in which the Dollar disciplines Libra.

In this section, consumption utility is derived from *two* money goods:

$$U_{A,j,t} \;=\; U(C_t^c) \;+\; \left(C_t^{m,1}\right)^{\alpha} \;+\; (\chi\theta_{A,j})^{1-\alpha}\left(C_t^{m,2}\right)^{\alpha} \;-\; N_t \tag{1.33}$$

For simplicity, and to isolate the mechanism, I assume that firm f supplies the first money good, $C^{m,1}$, as a monopolist. The market for the second money good, $C^{m,2}$, mirrors the money good market of the previous section, and so do the resulting optimality conditions. Two firms (f^*, g^*) produce the second money good, and consumers have to sequentially search to obtain price offers from these firms. Information is valuable as it allows firms to identify the high valuation consumers of type θ_H. In this spirit, let the average customer base of firm f^* be denoted by θ^*. The parameter χ determines the relative size of the two money good markets.

Consider the *partial equilibrium* scenario in which firms (f, f^*) have formed a currency consortium that issues Libra, the only private money in the economy. Firm g^* accepts the Dollar but does not accept Libra. Importantly, the information rents break the equivalence between the two currencies (as in Propositions 1.1 and 1.4).

I assume that firm f leads the consortium, deciding on the Libra inverse inflation rate $Q_t^{\approx}$. One possible interpretation is that the leading firm determines the initial private currency set-up, including private monetary policy, and the second firm joins the currency consortium afterwards taking this set-up as given. The parameter ζ characterises the dividend structure: firm f receives a share ζ of the total Libra seignorage revenues. In equilibrium, the consumers' CIA constraints hold with equality if seignorage revenues are positive, and thus real Libra balances correspond to real consumption expenditure in Libra:

$$s_t^{\approx} = (1 - Q_t^{\approx}) m_t^{\approx}, \quad \text{where } \; m_t^{\approx} = p_t^f C_t^{m,1,f} + p_t^{f^*} C_t^{m,2f^*} \tag{1.34}$$

$C_t^{m,1,f}$ denotes the aggregate consumption of the first money good with firm f. Similarly, $C_t^{m,2,f^*}$ denotes aggregate consumption of the second money good supplied by firm f^*. Total profits of firm f are given by the sum of product profits and seignorage dividends:

$$\Pi_t^f \;=\; (p_t^f - 1)\, C_t^{m,1,f} \;+\; \zeta\,(1 - Q_t^{\approx})\Big[p_t^f C_t^{m,1,f} \;+\; p_t^{f^*} C_t^{m,2,f^*}\Big] \tag{1.35}$$

with a corresponding profit function for firm f^*. Appendix 1.7.4 formally derives equilibrium pricing strategies as a function of the Libra inverse inflation rate and given parameters $(\zeta,\, \chi)$, yielding an expression for firm f's equilibrium profits $\Pi_t^f\big(Q_t^{\approx}, \zeta, \chi\big)$.

In this partial equilibrium analysis, firm f^* only produces positive amounts of the consumption good—and thus prefers to be part of the currency consortium—if they charge a weakly lower inflation-adjusted price than their competitor g^*. It follows that the Libra inverse inflation rate is bound from below:

$$Q_t^{\approx} \;\geq\; \underline{Q}(Q^{\$}, \zeta) \tag{1.36}$$

Given this set-up, I now analyse the relationship between dividend structures and the equilibrium Libra inflation rate. To illustrate, Figure 1.1 plots the unconstrained profit function and the lower bound.[26] The lower bound here is constructed for a desired central bank policy of $Q_t^{\$} = 0.95$.

Corollary 1.5 (Benchmark). *By Proposition 1.2, total currency consortium profits—the sum of product profits and seignorage revenues—are maximised when*

$$p^f\big(2 - Q^{\approx}\big) \;=\; p^{f^*}\big(2 - Q^{\approx}\big) \;=\; p^{mon} \tag{1.37}$$

[26]Parameter values are given by $\alpha = 0.9$, $\eta = 0.5$, $\chi = 2$ and $\theta^* = 0.75$.

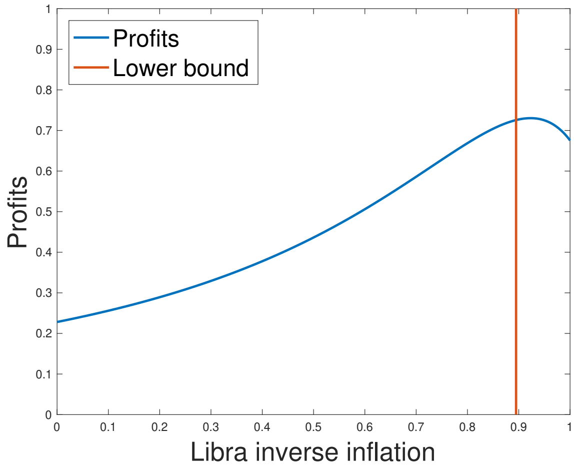

Figure 1.1: Profits of consortium leader and lower bound on private inverse inflation

Proposition 1.5 (Concentrated ownership leads to inflationary pressures). *The benchmark outlined in Corollary 1.5 is only achieved for a Libra monetary policy of* $Q_t^{\approx} = 1$*. The consortium leader only implements this policy if their dividend share does not exceed their share in Libra transactions:*

$$\zeta \leq \frac{1}{1 + \chi\theta^*} \qquad \Rightarrow \qquad Q_t^{\approx} = 1 \tag{1.38}$$

Concentrated ownership creates inflationary pressures in Libra:

$$\zeta > \frac{1}{1 + \chi\theta^*} \qquad \Rightarrow \qquad Q_t^{\approx} < 1 \tag{1.39}$$

As the share of seignorage dividends increases, firm f is increasingly willing to give product discounts compensating for the opportunity cost of holding Libra. At the same time, the temptation to increase Libra inflation is increasing as firm f receives a larger share of seignorage revenues generated by transactions with firm f^*. The balance of purely internalising the opportunity cost through private monetary policy tips in favour of partial internalisation through prices and obtaining seignorage revenues from other Libra transactions. As the relative size of the second money good market, captured by χ, increases, ownership becomes more concentrated. One interpretation of Proposition 1.5 is that inflationary pressures increase as the private currency becomes more commonly used. The focus of the decision-making firm f shifts away from its own product profits to seignorage revenues generated by other firms' transactions. Similarly, as the average customer base of firm f^*, θ^*, improves, ownership also becomes more concentrated. This suggests that inflationary pressures increase in the value of information generated in markets that use Libra.

To demonstrate, Figure 1.2 shows basic comparative statics for the dividend share parameter ζ

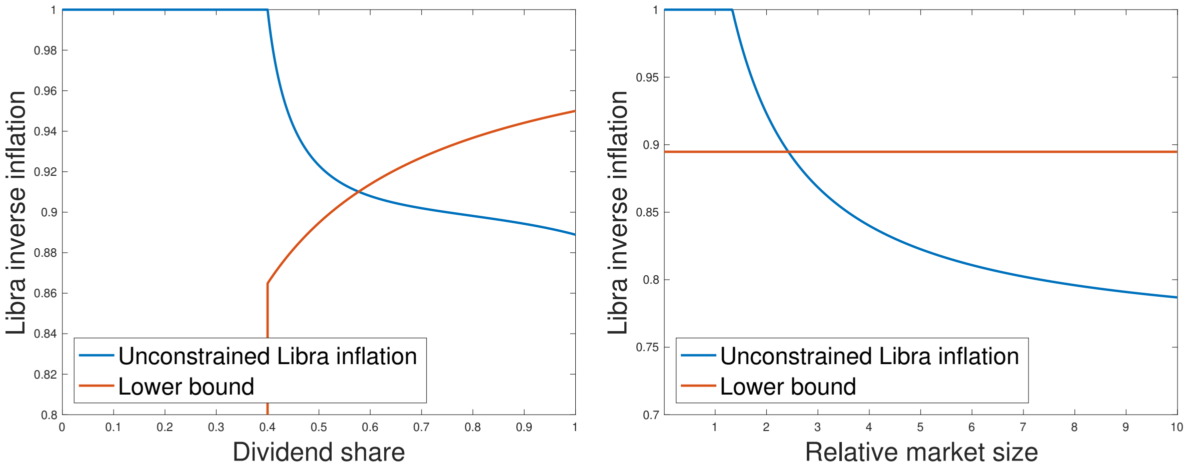

Figure 1.2: Private inflation as a function of ownership concentration and relative market size

and the relative market size parameter χ, leading to different values for the unconstrained Libra inflation rate and the lower bound due to Dollar pricing. In Section 1.3, private currency issuers perfectly internalised the opportunity cost of holding money, disciplining the government's currency. This is not always true when one firm dominates the currency consortium in terms of decision-making powers and claims to seignorage dividends. For low values of ownership concentration and relative market size, the currency consortium disciplines the central bank which finds itself unable to implement their desired policy in equilibrium. As the degree of concentration and the relative market size increase further, it is the central bank that disciplines the currency consortium: if Libra inflation ($Q_t^{\approx}$) were to lie above (below) the lower bound, then firm f would not obtain any seignorage payments from the second money good market, as firm f^* is priced out by their competitor g^*.

The fact that the leading firm benefits from private currency inflation has implications for the optimal design of Libra. In particular, the model offers an explanation why the Libra consortium are planning on designing their currency as non-interest-bearing:

Corollary 1.6 **(No currency design equivalence).** *The consortium leader benefits from private currency inflation, and therefore does not want to internalise the opportunity cost of holding money through interest payments.*

In sum, this section demonstrates that inflationary pressures arise—in the partial equilibrium of a one-sided introduction of private currencies—if many firms combine to form a digital currency area, but ownership of seignorage dividend shares is concentrated in the hands of one firm. Another interpretation is that the desired Libra inflation rate increases as it becomes more widely used. The general equilibrium concerns remain: in partial equilibrium, firm f optimally maintains non-zero levels of the opportunity cost of holding money. The counter-innovation incentives for firm g^* therefore increase relative to Section 1.3.

1.5 Extensions

This section presents a variety of further extensions to the benchmark outlined in Section 1.3. First, paying interest on public currency allows the central bank to escape the zero interest rate environment. Second, regulators fear that issuers of PCDC may be tempted to inflate away the value of their currency if it is backed with assets denominated in another currency. I show that firms cannot increase profits through such capital gains. Lastly, the paper evaluates a frequent argument for the success of public currencies: the government can force firms to pay sales taxes in the public currency. Similarly, as seen in China for Alipay and WeChat Pay, the regulator could impose macroprudential policies which force firms to invest proceeds from issuing private currency in central bank reserves.

1.5.1 Interest-bearing CBDC

In Section 1.3, the central bank was forced to set a zero nominal interest rate. Here I show that central bank digital currency, paying an interest rate i_t^m, allows central banks to internalise the opportunity cost of holding public money without giving up central bank autonomy.

Proposition 1.6. *The central bank can escape the zero lower bound by issuing interest-bearing digital currency. The interest rate on digital currency must match the interest rate paid on bonds:*

Appendix 1.7.5.1 presents two variants of the model which formally prove Proposition 1.6. For a two-sided introduction of private currency, the Dollar loses its role as medium of exchange. However, the central bank can convince consumers to hold Dollars even though it does not facilitate transactions. As long as the interest paid on bonds does not exceed the interest received on money, consumers are happy to scale up their balance sheets by purchasing government currency. For a one-sided introduction, consumers only hold Dollars if accepting firms do not charge strictly higher inflation-adjusted prices. Since issuers of private currency internalise the opportunity cost of holding money, the central bank of Section 1.3 was forced to follow suit and set $Q_t^{\$} = 1$. Equivalently, the central bank can compensate consumers by paying sufficient interest on money.

1.5.2 Capital gains due to higher private currency inflation

Previously I assumed that the assets held by firms, household-issued bonds, were of the same denomination as their liabilities, private currency. In this subsection, I assume that bonds may only be issued in Dollars. This could give rise to a motive for higher inflation on the private currency: firms can effectively force consumers to hold the private currency by not accepting the Dollar; then generating a high inflation rate on private currency can lead to an appreciation of the Dollar-denominated assets relative to the liabilities denominated in the private currency. The firm obtains capital gains. This section establishes that issuers of PCDC cannot increase their profits relative to the benchmark using capital gains.

Proposition 1.7. *Suppose the firm only holds Dollar-denominated assets. Then, in equilibrium and relative to the benchmark of Proposition 1.2, the firm cannot increase profits through capital gains resulting from higher inflation on its own currency.*

The formal proof is provided in Appendix 1.7.5.2. Intuitively, total profits are broken down into three components: product profits, direct seignorage revenues, and indirect capital gains. However, the profit function mirrors the one in Equation (1.29). Thus, the result on the jointly optimal private monetary policy and product pricing of Proposition 1.2 still holds.

$$\begin{aligned} \Pi_t^f &= \underbrace{\Pi_t^{f,m}\big(p_t^f, Q_t^{≋}\big)}_{\text{product profits}} + \underbrace{(1 - Q_t^{\$})m_t^{≋}}_{\text{seignorage}} + \underbrace{\big(Q_t^{\$} - Q_t^{≋}\big)m_t^{≋}}_{\text{capital gains}} \\ &= \Pi_t^{f,m}\big(p_t^f, Q_t^{≋}\big) + (1 - Q_t^{≋})m_t^{≋} \end{aligned} \tag{1.40}$$

To deepen the intuition, note that seignorage revenues are governed by the difference in the price of money and the price of Dollar bonds, $Q_t^{\$}$. Capital gains however are governed by the difference in the Dollar and Libra bond prices: for every unit of real Libra balances the firm holds real Dollar bonds, and their relative value increases proportionally in the Libra inverse inflation rate. On the consumer's side, money holdings are determined by the opportunity cost of holding Libra. The breakdown between direct seignorage revenues and indirect capital gains, captured by the Dollar bond price, is irrelevant for consumers. Thus, profits are only affected by the Libra inverse inflation rate and the firm implements a private monetary policy as characterised in Proposition 1.2.

The general equilibrium consequences for monetary policy are unchanged. Firm f perfectly internalises the opportunity cost of holding Libra. Firm g is priced out of the market unless $Q_t^{\$} = 1$, forcing the central bank to follow suit.

1.5.3 Forcing firms to hold public currency

A popular argument for the success of government fiat currency is the fact that governments can demand taxes to be paid in public currency. In this section firms need to pay a fraction τ of their sales in tax, payable in Dollars:

$$\tau p_t^f C_t^{m,f} \leq \frac{M_t^{f,\$}}{P_t} \tag{1.41}$$

where $M_t^{f,\$}$ denotes the Dollar holdings of firm f. It is immediately clear that such a policy leads to positive demand for Dollars whenever firm f sells any goods to consumers, even if they only accept Libra. This section establishes intuitively that sales taxes payable in Dollars lead to high levels of private currency inflation which, surprisingly, are welfare-improving. A formal derivation of the result is presented in Appendix 1.7.5.3.

Consider a binding CIA constraint for which real expenditure on consumption goods with firm f correspond to the economy's real Libra balances: $p_t^f C_t^{m,f} = m_t^{\approx}$. Firms prefer to hold interest-bearing bonds over money, and thus the constraint in Equation (1.41) binds:

$$\tau m_t^{\approx} = \frac{M_t^{f,\$}}{P_t} \qquad (1-\tau)m_t^{\approx} = \frac{B_t^f}{P_t} \tag{1.42}$$

where B_t^f denotes firms f's Dollar bond holdings.[27] Total firm profits are then given by:

$$\Pi_t^f \ = \ \underbrace{\Pi_t^{f,m}\big(p_t^f, Q_t^{\approx}, \tau\big)}_{\text{product profits}} \ + \ \underbrace{(1-\tau)(1-Q_t^{\$})m_t^{\approx}}_{\text{seignorage}} \ + \ \underbrace{\big(Q_t^{\$} - Q_t^{\approx}\big)m_t^{\approx}}_{\text{capital gains}} \tag{1.43}$$

Forcing firms to hold a part of their asset portfolio in public currency acts as a tax on direct seignorage revenues, breaking the one-for-one relationship with capital gains of the previous subsection. The firm collects interest only on its bond holdings, but the capital gains accrue for both money and bond holdings. The balance of internalising the opportunity cost of money either through discounts or through deflation tips in favour of high inflation and corresponding product discounts, in order to achieve capital gains.

Proposition 1.8. *Suppose the government forces the firm to pay sales taxes in Dollars. Then both firm profits and consumer welfare are increasing in private currency inflation, and are maximised as*

$$Q^{\approx} \ = \ 0 \qquad \Leftrightarrow \qquad \pi^{\approx} \ \to \ \infty \tag{1.44}$$

It follows that private monetary policy is characterised by infinite inflation, associated with higher consumption and thus higher consumer welfare. Consumers have to hold an asset that fully loses its real value in order to purchase a consumption good; however, this good is sold at a low price, possibly below marginal costs, to compensate for the inflation losses. Effectively, capital gains allow the firm to circumvent both the sales tax and the enforced opportunity cost of holding Dollars. Both consumers and firm benefit.

General equilibrium. The option to circumvent the sales tax through capital gains combined with product discounts has stark consequences for the competitor firm g, who is fully priced out

[27] Without loss of generality, I maintain the assumption that all household bonds are issued in Dollars.

of the market (see Appendix 1.7.5.3). Thus, the incentive to counter-innovate and also introduce a digital private currency is extremely high. Since the government forces firms to hold Dollars to pay taxes, the central bank does not need to set nominal interest rates to zero. While public money may lose its role as medium of exchange between firms and consumers, it finds a new role as medium of exchange between firms and government.

Macroprudential policy. Regulating agencies in Europe and the US are in the process of drawing up a rule book for companies issuing stablecoins such as Libra, including restrictions on asset investments. In the model, the equilibrium logic of this subsection is unchanged when firms are forced to hold government currency as a macroprudential measure. Issuers of private currency aim to circumvent the policy by inflating away the value of their currency, achieving capital gains; consumers are compensated with discounts. Profits and welfare are increasing in private currency inflation.[28]

Macroprudential policy for a widely used PCDC. Alipay and WeChat Pay are subjected to very tight macroprudential regulation imposed by the People's Bank of China (PBoC). In particular, they need to hold 100% of their proceeds as reserves with the PBoC.[29] The model helps explain why the value of private currency—the tokens held in digital wallets—is pegged to the value of the public currency, the Renminbi (RMB). Both Alipay and WeChat Pay are widely used with third-party firms. These firms do not give product discounts as they do not receive any seignorage revenues or capital gains. The issuers of PCDC therefore trade off the increase in capital gains against the seignorage and information rents obtained when other firms transact using their technologies. If the latter outweigh the former, then firms prefer to also implement the public interest rate set by the PBoC and peg the value of their tokens to the value of the RMB. If capital gains are sufficiently large, then these firms would prefer to inflate away the value of their currency. In that case, no transactions with other third-party firms would take place using their technologies.

1.6 Conclusion

This paper is the first work to formally analyse complementarities between information and private digital currency, deriving theoretical predictions squaring with casual empirical observations from the world of digital platforms. It is also the first to discuss monetary policy conducted by product producers. The model highlights three important mechanisms. First, information shapes the degree of currency competition. Firms aim to draw up digital currency areas, either by themselves or as part of a consortium, in order to maximise information rents. They do not

[28]See Appendix 1.7.5.4 for a formal derivation of the result.
[29]Source: General Office of the People's Bank of China (2018), issue no. 114.

accept currencies run by their competitors. Second, seignorage revenues accrue to the issuer of private currency. However, the resulting private and public inflation outcomes depend on how widely a currency is being used. If only the issuing firm accepts their currency, then they remove the opportunity cost of money; the central bank is forced to implement a zero interest rate policy, leading to deflation. Inflationary pressures arise when a private currency is widely used in transactions with firms that do not obtain seignorage revenues. The model explains the Libra consortium's plans not to pay interest on their currency. It shows that an interest-bearing CBDC reinstates central bank autonomy in an environment that would otherwise require zero interest rates. Surprisingly, this tool is not required if Libra should become widely used—until private currencies become so widely used that the public currency loses its role as medium of exchange.

This paper also highlights important and fruitful avenues of future research: the interaction of private, profit-maximising monetary policy and information generation in the presence of price discrimination, and the arising consequences for monetary policy; and further analysis on the formation of currency consortia and digital currency areas in the presence of network effects.

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1.7 Appendix

1.7.1 Appendix to Section 1.2

1.7.1.1 Consumer strategies in full formality

Given the demand schedule $c(p, \theta_j)$, let $V_j(p)$ denote the value function of a consumer of type θ_j:

$$V_j(p) \;=\; u_j\big(c(p, \theta_j)\big) \tag{1.45}$$

Consumers shop at the firm where they have obtained the lowest price offer. Since time-1 is the final period, decisions have no consequences for any future payoffs. Thus, consumers first visit the firm which they expect to charge lower prices. Let $\mu^j(p)$ denote the beliefs of consumer j over the firms' prices, and $\psi^j(\mu^j(p))$ their initial search strategy. If they expect both firms to charge the same price, the relative advertising intensity determines the first search; otherwise consumers randomise:

$$\psi^j(a^j, \mu^j(p)) \;=\; \begin{cases} i & \text{if } \;\mathbb{E}\Big[p^i|\mu^j\Big] \;<\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big] \\ i & \text{if } \;\mathbb{E}\Big[p^i|\mu^j\Big] \;=\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big], \text{ and } a^{i,j} \;>\; a^{-i,j} \\ \begin{cases} i & \text{with prob. } \frac{1}{2} \\ -i & \text{with prob. } \frac{1}{2} \end{cases} & \text{if } \;\mathbb{E}\Big[p^i \mid \mu^j\Big] \;=\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big], \text{ and } a^{i,j} \;=\; a^{-i,j} \end{cases} \tag{1.46}$$

Let $\omega^j(p^i, \mu^j(p^{-i}))$ denote the consumer's continuation search decision, having learnt about firm i's price. Consumers decide not to search whenever they believe that obtaining another price offer is not profitable. Optimally, they follow a cut-off rule, trading off paying the additional search cost S and achieving higher utility due to a lower price offer:

$$\omega^j(p^i, \mu^j(p^{-i})) \;=\; \begin{cases} 0, & \text{if } \;V_j(p^i) \;\geq\; \mathbb{E}\big[V_j(p^{-i}) \mid \mu^j(p^{-i})\big] \;-\; S \\ 1, & \text{o/w} \end{cases} \tag{1.47}$$

1.7.1.2 Proof of Lemma 1.1

First, I show that charging the monopoly price is indeed the unique equilibrium outcome in the final period (again omitting time subscripts for readability). Since the firm's profit function has a unique maximum at p^{mon} for all consumer types θ, firms have no incentive to increase the price. Given consumer beliefs $\mu(p) = p^{mon}$, the consumers' initial search strategy is either decided through advertising, or via a coin toss. Thus firms also have no incentive to reduce the price to attract more consumers. Consumers, having obtained one monopoly price quote, have no incentive to search any further, given their beliefs to obtain the exact same price quote a second time. Therefore no agents have any incentive to deviate unilaterally.

Now consider an equilibrium candidate for which firm i charges a price $p^i < p^{mon}$, and the competitor $-i$ charges a price weakly larger than firm i: $\mu(p^i) = p^i \leq p^{-i} = \mu(p^{-i})$. According to the consumers' initial search strategies, and given consumer beliefs, all consumers visit firm i first if the above inequality is strict. By the law of large numbers, half of the measure of consumers visit firm i if the inequality holds with equality. Consider next the consumers' cut-off rule $\omega\big(p^i, \mu(p^{-i})\big)$. Let $\bar{p}$ denote the price that makes the consumer exactly indifferent between obtaining another price quote or purchasing from firm i, satisfying:

$$V^j(\bar{p}) \;=\; \mathbb{E}[V^j(p^{-i}) \mid \mu(p^{-i})] \;-\; S \tag{1.48}$$

Note that $V^j(p^i)$ is continuous in p^i. Further note that the profit function $\Pi(p^i, \theta)$, which captures profits conditional on selling, is also continuous in p^i. It then follows that there exists an $\varepsilon > 0$ for every $p^i < p^{mon}$ such that charging a price $\tilde{p} = p^i + \varepsilon \leq min\{\bar{p}, p^{mon}\}$ yields strictly higher profits: $\Pi(\tilde{p}, \theta) > \Pi(p^i, \theta)$. Firm i thus unilaterally deviates, and there can be no equilibrium for which $p^i \leq p^{-i} \leq p^{mon}$, with at least one inequality strict.

It follows that consumers of both types anticipate to face the same price p^{mon} at both firms in the final period. Therefore their time-0 strategies cannot affect future utility, removing all strategic considerations other than searching for the lower price. The conditional demand schedules and the continuation search strategies at time-0 mirror those of time-1. The initial search strategy is given by

$$\psi_0^j\big(\mu^j(p_0)\big) = \begin{cases} \begin{cases} i & \text{with prob. } \frac{1}{2} \\ -i & \text{with prob. } \frac{1}{2} \end{cases} & \text{if } \mathbb{E}\Big[p_0^i \mid \mu_0^j\Big] \;=\; \mathbb{E}\Big[p_0^{-i} \mid \mu_0^j\Big] \\ \\ i & \text{if } \mathbb{E}\Big[p_0^i \mid \mu_0^j\Big] \;<\; \mathbb{E}\Big[p_0^{-i} \mid \mu_0^j\Big] \end{cases} \tag{1.49}$$

Following the same steps as above, I find that both firms charge the monopoly price at time-0, and thus in all time periods.

1.7.1.3 Proof of Proposition 1.1

In the initial period, given that consumers randomise between firms in equilibrium, firm f is visited by exactly half of the measure of consumers, with the average type of their customers equalling the population average type. Assume that all consumers use firm f's technology at firm f (without loss of generality). With a payment technology generating information, firm f thus learns that $\frac{q}{2}$ consumers have high valuations, while $\frac{1-q}{2}$ consumers have low valuations. If $\xi > \frac{q}{2}$, some advertising capacity remains after firm f has advertised all of its previous high valuation customers. Suppose that firm g accepts their competitor's technology. Let the share of purchases with firm g conducted using firm f's technology be denoted by $\rho > 0$. Since $\xi > \frac{q}{2}$, it must be that $\xi > \frac{(1+\rho)q}{2}$ for some $\rho > 0$. Information accrues to the owner of the technology used to transact. Not accepting the competitor technology, $\gamma^{g,acc} = 0$, corresponds to a share of $\rho = 0$. Effectively, I compare payoffs for firm g for these two values of ρ.

Consider first a firm g that does not operate a payment technology. This firm cannot match consumer ID and purchase, and therefore does not know who their previous customers are. Thus, it randomises among the general population of consumers when advertising. If $\rho = 0$, firm f optimally advertises to a fraction $\frac{q}{2}$ of all consumers, and optimally does not advertise to a fraction of $\frac{1-q}{2}$. The remaining capacity is used for a total measure of $\xi - \frac{q}{2}$ of consumers that were firm g's customers in the previous period. Thus firm f has no information on their types: some low valuation consumers receive firm f advertising. Now suppose $\rho > 0$. Firm f learns the valuation for an additional $\frac{\rho}{2}$ measure of consumers, and therefore identifies an additional $\frac{\rho q}{2}$ measure of high valuation types and directs advertising towards them. Given the randomisation strategy of firm g, the probability of double advertising to a previous customer of g's with a high valuation increases. The probability of double advertising to a low type decreases in exactly the same proportion as it increases for high types. Overall, the expected match quality for firm g decreases as ρ becomes strictly positive, and so do profits.

The above reasoning is demonstrated algebraically below for the case in which $\xi > \frac{(1+\rho)q}{2}$. Firm g randomises, advertising to each consumer with a probability of ξ. Thus, the matching probability conditional on firm f's advertising decision is given by

$$P\big[\psi^j = f | a^{f,j} = 1\big] = (1-\xi) + \frac{\xi}{2} \qquad P\big[\psi^j = f | a^{f,j} = 0\big] = \frac{1-\xi}{2} \tag{1.50}$$

Consumers with known type generate profits of $\kappa\theta_H$ and $\kappa\theta_L$, respectively; consumers with unknown type generate $\kappa\mathbb{E}[\theta]$ in expectation. Firm f identified a measure $\frac{(1+\rho)q}{2}$ of consumer to be the high type, and a measure of $\frac{(1+\rho)(1-q)}{2}$ of consumers to be the low type. Since $\xi > \frac{(1+\rho)q}{2}$, firm f advertises to all high types, and uses the remaining capacity for consumers that have not been f's customers in the previous period. Expected profits of firm f are thus given by:

$$\begin{aligned}
\Pi^f \;=\; & \frac{(1+\rho)q}{2}\left[(1-\xi)+\frac{\xi}{2}\right]\kappa\theta_H \;+\; \frac{(1+\rho)(1-q)}{2}\left[\frac{1-\xi}{2}\right]\kappa\theta_L \\
& + \left[\xi-\frac{(1+\rho)q}{2}\right]\left[(1-\xi)\;+\;\frac{\xi}{2}\right]\kappa\mathbb{E}[\theta]\;+\;\delta\left[\frac{1-\xi}{2}\right]\kappa\mathbb{E}[\theta] \qquad (1.51) \\
& \text{where}\quad \delta = 1-\frac{(1+\rho)(1-q)}{2}-\xi
\end{aligned}$$

$$\frac{\partial\Pi^f}{\partial\rho} \;=\; \frac{\kappa}{4}q(1-q)\big[\theta_H-\theta_L\big] > 0 \qquad (1.52)$$

The derivative of the profit function with respect to ρ yields that firm f's profits are increasing in ρ. Given that equilibrium industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$, it must be that firm g's profits are decreasing in ρ. It follows that $\gamma^{g,acc} = 0$.

If $\xi < \frac{(1+\rho)q}{2}$, the relevant comparison of payoffs is for the case where firm g does not accept firm f's payment technology ($\rho = 0$); and the case where firm f identifies more high valuation types than they can advertise to, corresponding to a maximum value of ρ given by $\bar{\rho}$, satisfying $\xi = \frac{(1+\bar{\rho})q}{2}$.

The algebraic proof for a firm that has also introduced a payment technology is not presented here. Noting that the information loss is even stronger if firm g is collecting information themselves, the claim immediately follows. Above I showed that more competitor information and a constant information for firm g negatively affected firm g's profits. With a technology, not only does firm g generate information for competitor firm f when accepting their technology, but they also directly reduce the information they themselves collect. The losses in profits are even stronger for a firm that has introduced a payment technology and thus $\gamma^{g,acc} = 0$.

1.7.1.4 Proof of Lemma 1.2

The first mover gains in profits from introducing the payment technology, denoted by Δ^{1st}, are given by

$$\Delta^{1st} = \Pi^i\big[(1,0),(0,0)\big] - \Pi^i\big[0,0\big] \qquad (1.53)$$

Since industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$ in any scenario, and the firms' payoffs always equal for fully symmetric strategies and equivalent levels of information, it must be that[30]

$$
\begin{aligned}
\Delta^{2nd} &= \Pi^i\big[(1,0),(1,0)\big] - \Pi^i\big[(0,0),(1,0)\big] \\
&= \Pi^i\big[(1,0),(1,0)\big] - \Big[\Pi^i\big[0,0\big] - \Delta^{1st}\big]\Big] \\
&= \Delta^{1st}
\end{aligned}
\tag{1.54}
$$

First and second mover gains, denoted by Δ^{2nd}, are equal. If one firm finds it profitable to introduce the payment technology, so does the other: $\Delta = \Delta^{1st} = \Delta^{2nd} \geq k$. Any potential advantages that the payment technology delivers are only redistributions of equilibrium profits from one firm to another. When firm i introduces the technology, their profits increase at the expense of their competitor. When the competitor also introduces their own technology, the game is symmetric again: firms have information of equal value, profits are shared equally, and firm i loses exactly those profits initially gained from firm $-i$.

1.7.2 Appendix to Section 1.3

1.7.2.1 Equilibrium definition

The competitive equilibrium of this economy is given by

1. Set of firm strategies that solve the profit maximisation problems, given beliefs μ_t^i

$$\Big(\Gamma^i, p_t^i, a_t^i, Q_t^x\Big)_{i\in\{f,g\},\, x\in\{≋,\mathbb{G}\},\, t\geq 0}$$

2. Set of consumer strategies that solve the utility maximisation problem, given beliefs μ_t^j

$$\Big(\psi_{j,y,t}, \psi_{j,mid,t}, C^m_{j,A,t}, C^c_{j,A,t}, M^x_{j,A,t}, B^x_{j,A,t}, N_{j,A,t}\Big)_{j\in[0,1],\, A\in\{y,mid,o\},\, x\in\{\$,≋,\mathbb{G}\},\, t\geq 0}$$

3. Set of prices $\Big(w_t, P_t, Q_t^{\$}, e_t^x\Big)_{x\in\{≋,\mathbb{G}\},\, t\geq 0}$

such that the markets for labour, the credit good, the money good, bonds and money clear. Beliefs are formed rationally, and updated according to Bayes' Law.

[30]Information is not symmetric because firms will always obtain information on different individual consumers. However it is equivalent if both firms have collected information on half of the population, where each half is of the same average type $\mathbb{E}[\theta]$.

1.7.2.2 The middle-aged consumer's maximisation problem

Consider the second period of consumer j's life, born at time $t-1$, having visited firm f's shop and learnt their price p_t^f. The consumer maximises remaining lifetime utility subject to budget constraints when middle-aged and old (Equation 1.15) and the CIA constraint when middle-aged (Equation 1.19). As in the main body of text, drop all j-subscripts for readability. Consumer j's total money holdings in Dollar values is denoted by $e_t M_t$:

$$e_t M_{mid,t} \;=\; M^{\$}_{mid,t} \;+\; e^{\approxeq}_t M^{\approxeq}_{mid,t} \;+\; e^{\mathbb{G}}_t M^{\mathbb{G}}_{mid,t} \tag{1.55}$$

where the exchange rates $(e^{\approxeq}_t, e^{\mathbb{G}}_t)$ denote the price of private currencies in terms of the Dollar. Formally, the non-negativity constraints on their real money holdings reads:

$$\frac{M^{\$}_{mid,t}}{P_t},\; \frac{e^{\approxeq}_t M^{\approxeq}_{mid,t}}{P_t},\; \frac{e^{\mathbb{G}}_t M^{\mathbb{G}}_{mid,t}}{P_t} \;\geq\; 0 \tag{1.56}$$

The Dollar value of total bond holdings, $e_t Q_t B_t$, is given by:

$$e_t Q_t B_{mid,t} \;=\; Q^{\$}_t B^{\$}_{mid,t} \;+\; e^{\approxeq}_t Q^{\approxeq}_t B^{\approxeq}_{mid,t} \;+\; e^{\mathbb{G}}_t Q^{\mathbb{G}}_t B^{\mathbb{G}}_{mid,t} \tag{1.57}$$

where Q_t denotes the prices of bonds issued at time-t, to mature in the following period. Bond prices are inversely related to the interest rate prevailing in the respective currencies:

$$Q^x_t = \frac{1}{1+i^x_t} \qquad x \in \{\$, \approxeq, \mathbb{G}\} \tag{1.58}$$

Lastly, summarise money holdings accepted in exchange by firm f's as

$$\gamma^f \,\frac{e_t M_{mid,t}}{P_t} \;=\; \gamma^{f,\$}\frac{M^{\$}_{mid,t}}{P_t} \;+\; \frac{e^{\approxeq}_t M^{\approxeq}_{mid,t}}{P_t} \;+\; \gamma^{f,\mathbb{G}}\,\frac{e^{\mathbb{G}}_t M^{\mathbb{G}}_{mid,t}}{P_t} \tag{1.59}$$

The above imposes that firm f accepts their own currency Libra.

The utility maximisation problem is then given by

$$\max_{\substack{\{C^c_{mid,t}, C^c_{o,t+1}, C^m_{mid,t}, N_{mid,t},\\ N_{o,t+1}, B_{mid,t}, M_{mid,t}\}}} U(C^c_{mid,t}) \;+\; \theta_j^{1-\alpha}(C^m_{mid,t})^\alpha \;-\; N_{mid,t} \;+\; \beta\Big[U(C^c_{o,t+1}) - N_{o,t+1}\Big]$$

$$\begin{aligned}
\text{s.t.}\quad C^c_{mid,t} + p^f_t C^m_{mid,t} + \frac{e_t(M_{mid,t} + Q_t B_{mid,t})}{P_t} \;&\leq\; N_{mid,t} + \frac{e_t(M_{y,t-1} + B_{y,t-1})}{P_t} \\
C^c_{o,t+1} \;&\leq\; N_{o,t+1} + \frac{e_{t+1}(M_{mid,t} + B_{mid,t})}{P_{t+1}} \\
p^f_t C^m_{mid,t} \;&\leq\; \gamma^f\,\frac{e_t M_{mid,t}}{P_t} \\
\frac{M^{\$}_{mid,t}}{P_t},\ \frac{e^{\approxeq}_t M^{\approxeq}_{mid,t}}{P_t},\ \frac{e^{\mathbb{G}}_t M^{\mathbb{G}}_{mid,t}}{P_t} \;&\geq\; 0
\end{aligned} \tag{1.60}$$

The first order conditions (FOCs) are then given by

$$C^c_{mid,t}:\quad U'(C^c_{mid,t}) \;=\; \lambda_{mid,t} \tag{1.61}$$

$$C^c_{o,t+1}:\quad U'(C^c_{o,t+1}) \;=\; \lambda_{mid,t+1} \tag{1.62}$$

$$C^m_{mid,t}:\quad \alpha\theta_j^{1-\alpha}(C^m_{mid,t})^{\alpha-1} \;=\; p^f_t(\lambda_{mid,t} + \nu_{mid,t}) \tag{1.63}$$

$$N_{mid,t}:\quad 1 \;=\; \lambda_{mid,t} \tag{1.64}$$

$$N_{o,t+1}:\quad 1 \;=\; \lambda_{o,t+1} \tag{1.65}$$

$$B^x_{mid,t}:\quad Q^x_t \;=\; \beta\,\frac{\lambda_{o,t+1}}{\lambda_{mid,t}}\,\frac{e^x_{t+1}}{e^x_t}\,\frac{P_t}{P_{t+1}} \tag{1.66}$$

$$M^x_t:\quad 1 \;=\; \beta\frac{\lambda_{o,t+1}}{\lambda_{mid,t}}\frac{e^x_{t+1}}{e^x_t}\frac{P_t}{P_{t+1}} \;+\; \gamma^{f,x}\nu_{mid,t} \;+\; \rho^x_{mid,t} \tag{1.67}$$

for all currencies $x \in \{\$, \approxeq, \mathbb{G}\}$. The Lagrange and Kuhn-Tucker multipliers of the budget and CIA constraints are denoted by $\lambda_{A,t}$ and $\nu_{mid,t}$. The Kuhn-Tucker conditions for the CIA and the non-negativity constraints are given by

$$\nu_{mid,t}\left(\gamma^f\,\frac{e_t M_{mid,t}}{P_t} - p^f_t C^m_{mid,t}\right) = 0 \qquad \text{and} \qquad \nu_{mid,t} \geq 0 \tag{1.68}$$

$$\rho^x_{mid,t}\,\frac{e^x_t M^x_{mid,t}}{P_t} = 0 \qquad \text{and} \qquad \rho^x_{mid,t} \geq 0 \qquad \forall\, x \tag{1.69}$$

Combining FOCs for consumption of the credit good and labour supply immediately yields that $C^c_{mid,t} = C^c_{o,t+1} = C^*$. Because consumption of the credit good is equal across time, the real interest rate of the economy is pinned down by the discount factor β. Defining inflation as

$1 + \pi^x_{t+1} = \frac{P^x_{t+1}}{P^x_t}$, the FOCs for bonds in all currencies simplify to

$$Q^x_t = \beta(1 + \pi^x_{t+1})^{-1} \tag{1.70}$$

Using this expression for bond prices, money FOCs become

$$\gamma^{i,x}\nu_{mid,t} = 1 - Q^x_t - \rho^x_{mid,t} \tag{1.71}$$

Consumers only hold currencies accepted by firms, and only hold the one with the lower inflation rate (higher bond price) if multiple currencies are accepted. Consider first the case where firm f only accepts one currency: $\gamma^f = (\gamma^{f,\$}, \gamma^{f,\approx}, \gamma^{f,\mathbb{G}}) = (0, 1, 0)$. The LHS of the above is zero, requiring $(\rho^\$_t, \rho^{\mathbb{G}}_t) > 0$ whenever $(Q^\$_t, Q^{\mathbb{G}}_t) < 1$; it follows that $M^\$_{mid,t} = M^{\mathbb{G}}_{mid,t} = 0$. Consider next a firm that accepts multiple currencies, i.e. the Dollar and Libra. Combining the two FOCs for $M^\$$ and $M^\approx$ shows that whenever $Q^\$_t < Q^\approx_t$, it must be that $\rho^\$_{mid,t} > \rho^\approx_{mid,t}$; since $\rho^\approx_{mid,t} \geq 0$, this requires $\rho^\$_{mid,t} > 0$, yielding $M^\$_{mid,t} = M^{\mathbb{G}}_{mid,t} = 0$.

The FOC for money also implies a zero lower bound on the nominal interest rate. All of $\gamma^{f,x}$, $\nu_{mid,t}$ and $\rho^x_{mid,t}$ are non-negative, and hence Q^x_t can take a maximum value of one (i^x_t can take a minimum value of zero).

Denote the highest bond prices corresponding to currencies accepted by firm f by $\tilde{Q}^f_t$. The FOC for this money then reads

$$\nu_{mid,t} = 1 - \tilde{Q}^f_t$$

which implies that $\nu_{mid,t} > 0$ whenever $\tilde{Q}^f_t < 1$. Then the CIA constraint holds with equality:

$$m_{mid,t} = p^f_t C^m_{mid,t}(p^f_t, \tilde{Q}^f_t)$$

where $m_{mid,t}$ denote real money balances of currencies held by the consumer. Turning to the FOC for money good consumption, the demand schedule is given by

$$C^m_{mid,t}(p^f_t, \tilde{Q}^f_t) = \theta_j \left[\frac{\alpha}{p^f_t(2 - \tilde{Q}^f_t)}\right]^{\frac{1}{1-\alpha}} \tag{1.72}$$

Combining all of the equilibrium conditions—see Appendix 1.7.2.4 for a full derivation—I can write down the middle-aged household's value function at time-t as

$$V^j_{mid,t}(M_{y,t-1}, B_{y,t-1}) = \bar{V}_{mid} + \frac{e_t(M_{y,t-1} + B_{y,t-1})}{P_t} + \theta_j\tilde{\kappa}\Big[p^f_t(2 - \tilde{Q}^f_t)\Big]^{\frac{\alpha}{\alpha-1}} \tag{1.73}$$

where $\tilde{\kappa} > 0$ is a constant, and function of α only. Utility derived from the money good consumption at time-t is only affected by the price that consumers face at the firm they visit, and by the inflation rate on the money that they need to hold. Clearly, consumers visit the firm which they expect to charge the lowest inflation-adjusted real price. For equal inflation-adjusted prices, they visit the more heavily advertised firm:

$$\psi^j_{mid,t}\big(a^j_t, \mu^j_t(p_t), Q_t\big) = \begin{cases} i & \text{if} \quad \mathbb{E}\Big[p^i_t(2-\tilde{Q}^i_t) \mid \mu^j_t\Big] \; = \; \mathbb{E}\Big[p^{-i}_t(2-\tilde{Q}^{-i}_t) \mid \mu^j_t\Big], \text{ and } a^{i,j}_t \; > \; a^{-i,j}_t \\ i & \text{if} \quad \mathbb{E}\Big[p^i_t(2-\tilde{Q}^i_t) \mid \mu^j_t\Big] \; < \; \mathbb{E}\Big[p^{-i}_t(2-\tilde{Q}^{-i}_t) \mid \mu^j_t\Big] \end{cases} \tag{1.74}$$

If both inflation-adjusted price and advertising intensity equal, consumers randomly choose a firm.

1.7.2.3 The young consumer's maximisation problem

The middle-aged consumer's value function is independent of any consumer's decisions taken when young, apart from asset holdings. Consider a consumer born at time t. Given the equilibrium pricing by a producer issuing private currency, utility is affected by the currency introduction decisions Γ. In the absence of private currencies, Dollar inflation also affects utility:

$$V^j_{mid,t+1} = \bar{V}_{mid}\big(\Gamma, Q^{\$}_{t+1}, \theta_j\big) + \frac{e_{t+1}(M_{y,t}+B_{y,t})}{P_{t+1}} \tag{1.75}$$

Having visited firm i's shop and learnt their price p^i_t, the young consumer's utility maximisation problem is given by

$$\begin{aligned} \max_{\{C^c_{y,t}, C^m_{y,t}, N_{y,t}, B_{y,t}, M_{y,t}\}} \quad & U(C^c_{y,t}) \; + \; \theta_j^{1-\alpha}(C^m_{y,t})^{\alpha} \; - \; N_{y,t} \; + \; \beta\, V^j_{mid,t+1}(M_{y,t}, B_{y,t}) \\ \text{s.t.} \quad C^c_{y,t} + p^i_t C^m_{y,t} + \frac{e_t(M_{y,t}+Q_t B_{y,t})}{P_t} &\leq N_{y,t} \; + \; T_{y,t} \\ p^i_t C^m_{y,t} &\leq \; \gamma^i \, \frac{e_t M_{y,t}}{P_t} \\ \frac{M^{\$}_{y,t}}{P_t}, \; \frac{e^{\approxeq}_t M^{\approxeq}_{y,t}}{P_t}, \; \frac{e^{\mathbb{G}}_t M^{\mathbb{G}}_{y,t}}{P_t} &\geq \; 0 \end{aligned} \tag{1.76}$$

The resulting equilibrium conditions below, together with the budget and CIA constraint holding with equality, mirror those for the middle-aged consumer:

$$C^c_{y,t} = C^* \tag{1.77}$$

$$Q^x_t = \beta(1+\pi^x_{t+1})^{-1}, \qquad x \in \{\$, \approxeq, G\} \tag{1.78}$$

$$C^m_{y,t}(\theta_j, p^i_t, \tilde{Q}^i_t) = \theta_j \left[\frac{\alpha}{p^i_t(2-\tilde{Q}^i_t)}\right]^{\frac{1}{1-\alpha}} \tag{1.79}$$

Combining all of the above, fully derived in Appendix 1.7.2.4, the value function of consumer j is given by

$$V_{y,t}^{j} = \bar{V}_y \ + \ T_{y,t} \ + \ \theta_j \tilde{\kappa} \Big[p_t^i (2 - \tilde{Q}_t^i) \Big]^{\frac{\alpha}{\alpha-1}} \ + \ \beta \, \bar{V}_{mid}\big(\Gamma, Q_{t+1}^{\$}, \theta_j\big) \tag{1.80}$$

Since firms do not advertise in the consumer's initial period, consumers simply visit firms seeking to minimize the inflation-adjusted cost of purchasing the money good, and randomise if indifferent:

$$\psi_{y,t}^{j}\big(\mu^j(p_t), Q_t\big) = \ i \qquad \text{if} \quad \mathbb{E}\Big[p_t^i (2 - \tilde{Q}_t^i) \mid \mu_t^j \Big] \ < \ \mathbb{E}\Big[p_t^{-i} (2 - \tilde{Q}_t^{-i}) \mid \mu_t^j \Big] \tag{1.81}$$

1.7.2.4 Deriving the consumer's value functions

The value function of the middle aged consumer j at time-t is given by

$$V_{mid,t}^{j} = U(C^*) \ + \ \theta_j^{1-\alpha} (C_{mid,t}^{m,i})^{\alpha} \ - \ N_{mid,t} \ + \ \beta \big[U(C^*) - N_{o,t+1} \big] \tag{1.82}$$

where

$$N_{mid,t} = C^* + p_t^i C_{mid,t}^{m,i} + \frac{e_t(M_{mid,t} + Q_t B_{mid,t})}{P_t} - \frac{e_t(M_{y,t-1} + B_{y,t-1})}{P_t} \tag{1.83}$$

$$N_{o,t+1} = C^* - \frac{e_{t+1}(M_{mid,t} + B_{mid,t})}{P_{t+1}} \tag{1.84}$$

$$C_{mid,t}^{m,i} = \theta_j \left[\frac{\alpha}{p_t^i (2 - \tilde{Q}_t)} \right]^{\frac{1}{1-\alpha}} \tag{1.85}$$

Plugging in and rearranging, the expression becomes

$$V_{mid,t}^{j} \ = \bar{V}_{mid} \ + \ \theta_j \left[\frac{\alpha}{p_t^i (2 - \tilde{Q}_t)} \right]^{\frac{\alpha}{1-\alpha}} \ - \ p_t^i \theta_j \left[\frac{\alpha}{p_t^i (2 - \tilde{Q}_t)} \right]^{\frac{1}{1-\alpha}} \tag{1.86}$$

$$- \ \frac{e_t(M_{mid,t} \ + \ Q_t B_{mid,t})}{P_t} \ + \ \frac{e_t(M_{y,t-1} \ + \ B_{y,t-1})}{P_t} \ + \ \beta \frac{e_{t+1}(M_{mid,t} \ + \ B_{mid,t})}{P_{t+1}} \tag{1.87}$$

By the nominal stochastic discount factors, I know that

$$\frac{e_t^x Q_t^x}{P_t} = \beta \frac{e_{t+1}^x}{P_{t+1}} \tag{1.88}$$

for all $x \in \{\$, \approx, \mathbb{G}\}$. Therefore $B_{mid,t}$ cancels out, and the expression for $M_{mid,t}$ simplifies substantially:

$$V^j_{mid,t} = \bar{V}_{mid} + \theta_j \left[\frac{\alpha}{p^i_t(2-\tilde{Q}_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t \theta_j \left[\frac{\alpha}{p^i_t(2-\tilde{Q}_t)}\right]^{\frac{1}{1-\alpha}} \tag{1.89}$$

$$- (1-Q_t)\frac{e_t M_{mid,t}}{P_t} + \frac{e_t(M_{y,t-1}+B_{y,t-1})}{P_t} \tag{1.90}$$

Optimally consumers only bring currencies accepted by the firm, choose the one subjected to the lower inflation rate than other currencies accepted. The cash-in-advance constraint is holding with equality. It follows that

$$p^i_t C^m_{mid,t} = \frac{e_t M_{mid,t}}{P_t} \tag{1.91}$$

and the expression for the value function becomes

$$V^j_{mid,t} = \bar{V}_{mid} + \theta_j \left[\frac{\alpha}{p^i_t(2-\tilde{Q}_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t(2-\tilde{Q}_t)\theta_j \left[\frac{\alpha}{p^i_t(2-\tilde{Q}_t)}\right]^{\frac{1}{1-\alpha}} + \frac{e_t(M_{y,t-1} + B_{y,t-1})}{P_t} \tag{1.92}$$

Combining the two middle terms yields the expression as in Appendix 1.7.2.2, with $\tilde{\kappa} = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) > 0$.

The value function of the young consumer j at time-t is found in analogy to the middle age value function above. Starting point is the middle-age value function:

$$V^j_{mid,t+1} = \bar{V}_{mid}\big(\Gamma, Q^{\$}_{t+1}, \theta_j\big) + \frac{e_{t+1}(M_{y,t}+B_{y,t})}{P_{t+1}} \tag{1.93}$$

Given the time-$(t+1)$ equilibrium outcomes for consumption of the credit and money goods, time-t decisions only affect future utility through asset holdings: for every unit of real assets in the next period, consumers need to supply one unit less labour.

$$V^j_{y,t} = U(C^*) + \theta_j^{1-\alpha} C^m_{y,t} - N_{y,t} + \beta\left[\bar{V}_{mid}\big(\Gamma, Q^{\$}_{t+1}, \theta_j\big) + \frac{e_{t+1}(M_{y,t}+B_{y,t})}{P_{t+1}}\right] \tag{1.94}$$

where

$$N_{y,t} = C^* + p^i_t C^m_{y,t} + \frac{e_t(M_{y,t} + Q_t B_{y,t})}{P_t} - T_{y,t} \tag{1.95}$$

Plugging in the expression for $N_{y,t}$ and $C^m_{y,t}$, and again making use of the expression for the nominal stochastic discount factor and equilibrium money holdings gives

$$V_{y,t}^j = \bar{V}_y + \theta_j \left[\frac{\alpha}{p_t^i(2-\tilde{Q}_t)}\right]^{\frac{\alpha}{1-\alpha}} - p_t^i(2-Q_t)\theta_j \left[\frac{\alpha}{p_t^i(2-\tilde{Q}_t)}\right]^{\frac{1}{1-\alpha}} + T_{y,t} + \beta \bar{V}_{mid}\big(\Gamma, Q_{t+1}^{\$}, \theta_j\big) \tag{1.96}$$

As for the middle age value function, combining the middle terms yields the expression as in Appendix 1.7.2.3.

1.7.2.5 Deriving the expression for seignorage revenues

This section of the appendix formally derives the expression for seignorage revenues (Equation 1.27). Consider firm f that is conducting a transaction in its private currency. The firm generates profits from selling the money good, $\Pi_t^{m,f}$, transfers resources to the young households, issues money and holds assets in the form of bonds. The flow budget constraint at time-t is given by

$$\Pi_t^{m,f} + \frac{e_t^{\approxeq}\big(M_t^{S,\approxeq} - M_{t-1}^{S,\approxeq}\big)}{P_t} = T_t^f + \frac{e_t\big(Q_t B_t^f - B_{t-1}^f\big)}{P_t} \tag{1.97}$$

where $M_t^{S,\approxeq}$ denotes the Libra supply.[31] Libra is modelled as stablecoin: it is backed with Libra-denominated bonds that have been issued by the household.[32,33] For every unit of money issued, the firm purchases a unit of nominal bonds: $M_t^{S,\approxeq} = B_t^{f,\approxeq}$. Jumping ahead to impose the market clearing condition, $M_t^{S,\approxeq} = M_t^{\approxeq}$, and defining $m_t^{\approxeq} = \frac{e_t^{\approxeq} M_t^{\approxeq}}{P_t}$, the firm's flow budget constraint simplifies to

$$\Pi_t^{m,f} + (1 - Q_t^{\approxeq}) m_t^{\approxeq} = T_t^f$$

where $(1 - Q_t^{\approxeq}) m_t^{\approxeq}$ are the firm's seignorage revenues.

1.7.3 Alternative specifications of the CIA model

1.7.3.1 Alternative timing assumption

For simplicity, and without loss of generality, assume that the firm is a monopolist producer of the money good and also the sole supplier of money in this economy. Ignore the OLG structure and any consumer heterogeneity: a representative consumers purchases consumption goods from a monopolist and holds their money Libra. In the full model as developed in the main text, firms issuing PCDC charge inflation-adjusted prices as if they were monopolists. Here I demonstrate

[31] Such a budget constraint is relevant if firms also provide currency. Otherwise they simply transfer profits to the household.

[32] An alternative backing of the currency is discussed in Appendix 1.7.3.2. The results are unchanged.

[33] I discuss an economy in which only Dollar-denominated bonds exist in Section 1.5.2.

that this monopoly price is unchanged in the alternative CIA specification.

Suppose the money market opens before the credit market. Money then enables transactions with a one-period delay:

$$p_t C_t^m \;\leq\; \frac{M_{t-1}^{\approx}}{P_t^{\approx}} \tag{1.98}$$

where $P_t^{\approx}$ is the price of Libra in terms of the numeraire. The consumer's utility maximisation problem becomes

$$\begin{aligned}\mathcal{L}_0 \;=\; \max_{\{C_t^c, C_t^m, N_t, B_t^{\approx}, M_t^{\approx}\}_{t\geq 0}} \; \sum_{t=0}^{\infty} \Bigg\{ & U(C_t^c) \;+\; (C_t^m)^{\alpha} \;-\; N_t \\ & + \lambda_t \Bigg(N_t \;+\; \frac{B_{t-1}^{\approx}}{P_t^{\approx}} + \frac{M_{t-1}^{\approx}}{P_t^{\approx}} \;+\; T_t^f \;-\; C_t^c \;-\; p_t C_t^m \;-\; \frac{M_t}{P_t^{\approx}} \;-\; \frac{Q_t^{\approx} B_t^{\approx}}{P_t^{\approx}} \Bigg) \\ & + \nu_t \Bigg(\frac{M_{t-1}^{\approx}}{P_t^{\approx}} \;-\; p_t C_t^m \Bigg) \Bigg\}\end{aligned} \tag{1.99}$$

where λ and ν denote the Lagrange and Kuhn-Tucker multipliers corresponding to the budget and cash-in-advance constraints.[34] The first order conditions are unchanged relative to the full model, with the exception of (the now only type of) money $M_t^{\approx}$:

$$\lambda_t \frac{1}{P_t^{\approx}} \;=\; \beta \frac{1}{P_{t+1}^{\approx}} \big[\lambda_{t+1} \;+\; \nu_{t+1}\big] \tag{1.100}$$

Rearrange, and use $\lambda_t = 1$ and $Q_t^{\approx} = \beta(1+\pi_{t+1}^{\approx})^{-1}$, to find that

$$\nu_{t+1} \;=\; \frac{1}{Q_t^{\approx}} \;-\; 1 \tag{1.101}$$

The bond price is inversely related to the nominal interest rate: $Q_t^{\approx} = \frac{1}{1+i_t^{\approx}}$. The first order condition for consumption of the money good is given by

$$u'(C_t^m) \;=\; p_t(\lambda_t + \nu_t) \;=\; p_t(1+i_{t-1}) \qquad \Rightarrow \qquad C_t^m \;=\; \left[\frac{\alpha}{p_t(1+i_{t-1})}\right]^{\frac{1}{1-\alpha}} \tag{1.102}$$

Comparing the demand schedule to the one obtained in the main body of the paper, the opportunity cost of having to hold Libra is now captured by the interest rate of the previous period. Turning to the firm's problem, write the flow budget constraint as

$$\Pi_t^m \;+\; \frac{M_t^{\approx} \;-\; M_{t-1}^{\approx}}{P_t^{\approx}} \;=\; T_t^f \;+\; \frac{B_t^{f,\approx} \;-\; (1+i_{t-1}^{\approx})B_{t-1}^{f.\approx}}{P_t^{\approx}} \tag{1.103}$$

[34] For simplicity, I omit a money non-negativity constraint which never binds in equilibrium.

The currency is backed according to $M_t^{\approxeq} = B_t^{f,\approxeq}$. Define $m_t^{\approxeq} = \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}}$ to find

$$\Pi_t^m + i_{t-1} m_t^{\approxeq} = T_t^f \tag{1.104}$$

Consider a binding cash-in-advance constraint: $m_t^{\approxeq} = p_t C_t^m(p_t, i_{t-1})$. Plugging this expression into the flow budget constraint shows that total profits are given by

$$\Pi_t = \left[p_t(1+i_{t-1}^{\approxeq}) - 1\right] C_t^m(p_t, i_{t-1}^{\approxeq}) = \left[p_t(1+i_{t-1}^{\approxeq}) - 1\right] \left[\frac{\alpha}{p_t(1+i_{t-1}^{\approxeq})}\right]^{\frac{1}{1-\alpha}} \tag{1.105}$$

Firms choose an optimal inflation-price tuple (p_t^*, i_{t-1}^*) satisfying

$$p_t^*(1+i_{t-1}^*) = p^{mon} \tag{1.106}$$

This variant of the model delivers a solution that is equivalent to the monopoly solution in the main model: firms perfectly internalise the opportunity cost of holding money. Note that seignorage discounts here correspond to the previous period's interest rate. Importantly, firms can internalise the effect of non-zero interest rates when setting their price, thus manipulating consumer demand. If firms would be unable to do so, they can always implement a zero interest rate policy which, combined with real monopoly prices for the consumption good, also satisfies the above.

1.7.3.2 Backing the currency according to $M_t = Q_t B_t$

As in Appendix 1.7.3.1, consider the model of a representative consumer and a firm that is the only supplier of the money good and currency. Begin with the flow budget constraint, but assume that the stablecoin is backed according to $M_t^{\approxeq} = Q_t^{\approxeq} B_t^{f,\approxeq}$. Effectively, firms spend all of the funds raised by issuing money in financial markets, and receive interest gains from bond purchases in the following period. Plugging in and rearranging, the flow budget constraint becomes

$$\Pi_t^m + \left[\frac{1}{Q_{t-1}^{\approxeq}} - 1\right] \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}} = T_t^f \tag{1.107}$$

Rearranging and using $m_{t-1}^{\approxeq} = \frac{M_{t-1}^{\approxeq}}{P_{t-1}^{\approxeq}}$, the expression is given by

$$\Pi_t^m + \left[\frac{1 - Q_{t-1}^{\approxeq}}{Q_{t-1}^{\approxeq}}\right] \frac{m_{t-1}^{\approxeq}}{1+\pi_t^{\approxeq}} = T_t^f \tag{1.108}$$

The firm's problem becomes dynamic, but only in the sense that some revenues accrue with a one-period delay (see also Section 1.5.2 and Appendix 1.7.5.2). Firms discount future profits at rate β, the appropriate discount factor implied by the consumer maximisation problem.[35] Since $Q_t^{\approx} = \beta(1+\pi_{t+1}^{\approx})^{-1}$, the profit maximisation problem can be written as

$$\max_{p_t, i_t^{\approx}} \; (p_t - 1)\, C_t^m(p_t, Q_t^{\approx}) \; + \; (1 - Q_t^{\approx}) m_t^{\approx} \tag{1.109}$$

Plugging in the demand curve for C_t^m and the CIA constraint holding with equality, the problem becomes

$$\max_{p_t, Q_t^{\approx}} \; (p_t(2 - Q_t^{\approx}) - 1) \left[\frac{\alpha}{p_t(2 - Q_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{1.110}$$

Clearly, the solution perfectly matches the solution of the main specification.

1.7.4 Appendix to Section 1.4

Consider firm f^*'s profit function:

$$\Pi_t^{f^*} \; = \; (p_t^{f^*} - 1)\, C_t^{m,2,f^*} \; + \; (1-\zeta)\,(1 - Q_t^{\approx})\Big[p_t^f C_t^{m,1,f} \; + \; p_t^{f^*} C_t^{m,2,f^*}\Big] \tag{1.111}$$

The demand schedules, conditional on purchase, match the demand schedules of the previous sections:

$$C_t^{m,1,f}(p_t^f, Q_t^{\approx}) \; = \; \left[\frac{\alpha}{p_t^f(2 - Q_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{1.112}$$

$$C_t^{m,1,f^*}(\theta_{A,j}, p_t^{f^*}, Q_t^{\approx}) \; = \; \theta_{A,j} \left[\frac{\alpha}{p_t^{f^*}(2 - Q_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{1.113}$$

Note that there is no heterogeneity among consumers in the first money good market. The firms' first order conditions then reveal that firms charge the following prices in Libra:

$$p_t^f(\zeta, Q_t^{\approx}) = \frac{p^{mon}}{1 + \zeta(1 - Q_t^{\approx})} \qquad p_t^{f^*}(\zeta, Q_t^{\approx}) = \frac{p^{mon}}{1 + (1-\zeta)(1 - Q_t^{\approx})} \tag{1.114}$$

The equilibrium profit function of the consortium-leading firm f is then given by:

$$\Pi_t^f\left(Q_t^{\approx}, \zeta\right) = \kappa \left[\frac{1 + \zeta(1 - Q_t^{\approx})}{2 - Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \; + \; \zeta\,\left(1 - Q_t^{\approx}\right) \chi\, \theta^* \frac{\kappa}{1-\alpha} \left[\frac{\left[1 + (1-\zeta)(1 - Q_t^{\approx})\right]^{\alpha}}{2 - Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{1.115}$$

where the first term captures firm f's product profits and seignorage revenues due to its own transactions; the second term captures the seignorage revenues generated by firm f^* which

[35] Firms are owned by the household and thus use their discount factor.

accrue to firm f. Firm f^*'s customer base is denoted by θ^*.

To derive the lower bound on the Libra inverse inflation rate, begin by noting that firm f^* only wants to remain part of the consortium if they pay weakly lower inflation-adjusted prices than their competitor—otherwise they do not sell any goods and prefer to only accept the Dollar.

$$p_t^{f^*}(2 - Q_t^{\approx}) \;\leq\; p^{mon}(2 - Q_t^{\$}) \tag{1.116}$$

where the above expression imposed that firm g^* charges a real product price of p^{mon} as by Equation (1.26). Given firm f^*'s pricing strategy as above, rearrange to find

$$Q_t^{\approx} \;\geq\; \frac{Q_t^{\$} - (2 - Q_t^{\$})(1-\zeta)}{1 - (2 - Q_t^{\$})(1-\zeta)} \tag{1.117}$$

whenever this expression's numerator is weakly greater than zero. If the numerator is strictly less than zero, Libra inflation is unconstrained:

$$Q_t^{\approx} \;\geq\; 0 \tag{1.118}$$

1.7.5 Appendix to Section 1.5

1.7.5.1 Formal model demonstrating Proposition 1.6: CBDC

Consider the following variant of the model, demonstrating how the central bank can escape the zero interest rate environment if the public currency has lost its role as medium of exchange. A representative consumer derives period utility according to

$$U(C_t^c) + \left(C_t^m\right)^{\alpha} - N_t \tag{1.119}$$

The model features two currencies: the Libra issued by firm f, and the Dollar issued by the central bank. The Dollar here is a digital currency that pays interest at rate i_t^m. The budget constraint is thus given by

$$C_t^c + p_t C_t^m + \frac{Q_t^{\$} B_t^{\$}}{P_t} + \frac{e_t^{\approx} Q_t^{\approx} B_t^{\approx}}{P_t} + \frac{M_t^{\$}}{P_t} + \frac{e_t^{\approx} M_t^{\approx}}{P_t} \leq w_t N_t + \frac{B_{t-1}^{\$}}{P_t} + \frac{e_t^{\approx} B_{t-1}^{\approx}}{P_t} + \frac{(1+i_{t-1}^m) M_{t-1}^{\$}}{P_t} + \frac{e_t^{\approx} M_{t-1}^{\approx}}{P_t} \tag{1.120}$$

Firms produce according to the linear production functions as in the main body of the text. The credit market is perfectly competitive, and the real wage is again pinned down as $w_t = 1$. Firm f supplies the money good as a monopolist at price p_t and does not accept the Dollar in exchange. The consumer thus faces a CIA constraint as below:

$$p_t C_t^m \leq \frac{e_t^{≋} M_t^{≋}}{P_t} \tag{1.121}$$

Real money balances in both currencies need to be weakly positive at all points in time:

$$\frac{e_t^{≋} M_t^{≋}}{P_t}, \frac{M_t^{\$}}{P_t} \geq 0 \tag{1.122}$$

The resulting optimality conditions are largely unchanged. The first order condition for Dollar bonds simplifies to

$$Q_t^{\$} = \beta(1 + \pi_{t+1}^{\$})^{-1} \tag{1.123}$$

The simplified first order condition for Dollar balances is given by

$$1 \ = \ \beta \frac{1 + i_t^m}{1 + \pi_{t+1}^{\$}} \ + \ \rho_t^{\$} \tag{1.124}$$

where $\rho_t^{\$}$ is the multiplier of the non-negativity constraint on real Dollar balances. Combining the two equations yields

$$1 \ = \ Q_t^{\$} \, (1 + i_t^m) \ + \ \rho_t^{\$} \tag{1.125}$$

The Kuhn-Tucker conditions for Dollar balances imply that

$$\frac{M_t^{\$}}{P_t} \, \rho_t^{\$} \ = \ 0 \tag{1.126}$$

And hence positive real money balances, for which $\rho_t^{\$} = 0$, require that

$$Q_t^{\$} \, (1 + i_t^m) = 1 \qquad \Rightarrow \qquad i_t = i_t^m \tag{1.127}$$

For a one-sided introduction, the main body of text established that the issuer of PCDC charges an inflation-adjusted price satisfying

$$p^f (2 - Q_t^{≋}) \ = \ p^{mon} \tag{1.128}$$

While still achieving monopoly rents, the firm fully removes the opportunity cost of holding money. A firm that transacts in the public currency does not internalise the opportunity cost of holding Dollars and charges a real price $p^g = p^{mon}$. Thus, no consumer visits firm g if the opportunity cost of holding Dollars is positive. In Section 1.3, the central bank did not have the option to pay interest on its money and therefore was forced to set the nominal interest rate on bonds to zero. Consider now the problem for the consumer as in subsections 1.3.1.1 and 1.3.2 but with two currencies in circulation, the Dollar and Libra. The Dollar pays interest, and the period budget constraint is given by Equation (1.120). Firm g only accepts the Dollar, and so $(\gamma^{g,\$}, \gamma^{g,≋}) = (1,0)$. Having visited their shop, the first order condition for Dollar holdings for a middle-aged consumer becomes

$$1 \;=\; \beta\frac{(1+i_t^m)P_t}{P_{t+1}} \;+\; \nu_{mid,t} \;+\; \rho^{\$}_{mid,t} \tag{1.129}$$

where ν and ρ denote the Kuhn-Tucker multipliers of the CIA and non-negativity constraints. Having visited firm g, the consumer can only purchase goods from this firm and thus holds positive real Dollar balances. Combining with the first order condition for Dollar-denominated bonds, the above becomes

$$1 - Q_t^{\$}(1+i_t^m) \;=\; \nu_{mid,t} \tag{1.130}$$

First of all, note that $Q_t^{\$}(1+i_t^m) = 1$ already implies that $\nu_{mid,t} = 0$. The CIA constraint does not bind: there is no opportunity cost of money and the constraint forcing money holdings is slack. Combining Equation (1.130) and the first order condition for money good consumption (Equation 1.61), the expression becomes

$$C^m_{mid,t}(p_t^g, Q_t^{\$}, i_t^m) = \theta_j\left[\frac{\alpha}{p_t^g\Big(2 - Q_t^{\$}(1+i_t^m)\Big)}\right]^{\frac{1}{1-\alpha}} \tag{1.131}$$

The inflation-adjusted price face by consumer j at firm g is thus given by $p_t^g\Big(2 - Q_t^{\$}(1+i_t^m)\Big)$. Consumers visit firm g's store, leading to positive demand for Dollar goods in equilibrium, if

$$p_t^g\Big(2 - Q_t^{\$}(1+i_t^m)\Big) \;=\; p^{mon}\Big(2 - Q_t^{\$}(1+i_t^m)\Big) \;\leq\; p^f(2 - Q_t^{≋}) \;=\; p^{mon} \tag{1.132}$$

which requires $Q_t^{\$}(1+i_t^m) = 1$, or equivalently, $i_t^{\$} = i_t^m$.

1.7.5.2 Derivations for subsection 1.5.2: Capital gains

Consider the flow budget constraint of a firm that issues Libra currency, $M_t^{≋}$, and holds household bonds denominated in the Dollar:

$$\Pi_t^m \;+\; \frac{e_t^{≋}(M_t^{≋,S} \;-\; M_{t-1}^{≋,S})}{P_t} \;=\; T_t^f \;+\; \frac{Q_t^{\$}B_t^f \;-\; B_{t-1}^f}{P_t} \tag{1.133}$$

As before, there is full backing of the currency: $e_t^{≋}M_t^{≋,S} = B_t^f$. Plugging in the market clearing condition for Libra, $M_t^{≋,S} = M_t^{≋}$, the Libra exchange rate, $e_t^{≋} = \frac{P_t}{P_t^{≋}}$, and the definition of real

Libra balances, $m_t^{\approxeq} = \frac{e_t^{\approxeq} M_t^{\approxeq}}{P_t}$, the flow budget constraint becomes:

$$\Pi_t^m \; + \; (1 - Q_t^{\$}) m_t^{\approxeq} \; + \; \left[1 - \frac{e_t^{\approxeq}}{e_{t-1}^{\approxeq}}\right] \frac{e_{t-1}^{\approxeq} M_{t-1}^{\approxeq}}{P_t} \; = \; T_t^f \tag{1.134}$$

By the definitions of the exchange rate, the inflation rates in both currencies and real Libra balances, the above becomes

$$\Pi_t^m \; + \; (1 - Q_t^{\$}) m_t^{\approxeq} \; + \; \left[1 - \frac{1 + \pi_t^{\$}}{1 + \pi_t^{\approxeq}}\right] \frac{m_{t-1}^{\approxeq}}{1 + \pi_t^{\$}} \; = \; T_t^f \tag{1.135}$$

From consumers first order conditions, I obtain $Q_t^{\$} = \beta(1 + \pi_t^{\$})^{-1}$. Since this economy does not feature Libra-denominated bonds, define $Q_t^{\approxeq} = \beta(1 + \pi_{t+1}^{\approxeq})^{-1}$. Effectively, if a Libra bond would exist, its price would account for the time rate of preferences and the change in the value of Libra relative to the numeraire. Then I finally obtain

$$\Pi_t^m \; + \; (1 - Q_t^{\$}) m_t^{\approxeq} \; + \; \frac{Q_{t-1}^{\$} - Q_{t-1}^{\approxeq}}{\beta} m_{t-1}^{\approxeq} \; = \; T_t^f \tag{1.136}$$

where the $\frac{Q_{t-1}^{\$} - Q_{t-1}^{\approxeq}}{\beta}$ captures the capital gains due to inflation rate differentials on the two currencies. Note how higher inflation on Libra, or equivalently $Q_{t-1}^{\$} > Q_{t-1}^{\approxeq}$, yields capital gains in the following period. The firm's profit maximisation problem is now dynamic: a choice of $(Q_t^{\approxeq}, p_t^f)$ affects both profits today and tomorrow. Since there is no meaningful economic connection between time periods other than the fact that some profits only accrue tomorrow, I can write the firm's profit maximisation problem as follows:

$$\begin{aligned}
\underset{p_t^f, Q_t^{\approxeq}}{max} \; \Pi_t \; &= \; \underset{p_t^f, Q_t^{\approxeq}}{max} \; \Pi_t^m \; + \; (1 - Q_t^{\$}) m_t^{\approxeq} \; + \; \beta \frac{Q_t^{\$} - Q_t^{\approxeq}}{\beta} m_t^{\approxeq} \\
&= \; \underset{p_t^f, Q_t^{\approxeq}}{max} \; \Pi_t^m \; + \; (1 - Q_t^{\approxeq}) m_t^{\approxeq} \\
&= \; \underset{p_t^f, Q_t^{\approxeq}}{max} \; \left(p_t^f (2 - Q_t^{\approxeq}) - 1\right) \left[\frac{\alpha \theta^{1-\alpha}}{p_t^f (2 - Q_t^{\approxeq})}\right]^{\frac{1}{1-\alpha}}
\end{aligned} \tag{1.137}$$

where tomorrow's profits are discounted at rate β. The problem and also its solution are exactly as before.

1.7.5.3 Derivations for subsection 1.5.3: Sales taxes payable in public currency

Consider firm f's flow budget constraint:

$$\Pi_t^m \; + \; \frac{e_t^{\approxeq}(M_t^{S,\approxeq} - M_{t-1}^{S,\approxeq})}{P_t} \; = \; T_t^f \; + \; \frac{Q_t^{\$} B_t^f - B_{t-1}^f}{P_t} \; + \; \frac{M_t^{f,\$} - M_{t-1}^{f,\$}}{P_t} \tag{1.138}$$

Libra is backed using dollar-denominated assets: $e_t M_t^{S,\approx} = B_t^f + M_t^{f,\$}$. Since the firm achieves seignorage returns when holding bonds, the cash-tax constraint always binds. Thus, the firm holds government currency and bonds according to

$$\tau \frac{e_t^{\approx} M_t^{S,\approx}}{P_t} \;=\; \frac{M_t^{f,\$}}{P_t} \qquad (1-\tau)\frac{e_t^{\approx} M_t^{S,\approx}}{P_t} \;=\; \frac{B_t^f}{P_t} \tag{1.139}$$

The firm's profit function is derived in analogy to Appendix 1.7.5.2. Plugging in the firm's government currency and bond holdings, using definitions as above and rearranging, the flow budget constraint becomes:

$$\Pi_t^m \;+\; (1-\tau)(1-Q_t^{\$})m_t^{\approx} \;+\; \left[1-\frac{e_t^{\approx}}{e_{t-1}^{\approx}}\right]\frac{e_{t-1}^{\approx}M_{t-1}^{\approx}}{P_t} \;=\; T_t^f \tag{1.140}$$

Direct seignorage revenues now only accrue on a fraction $(1-\tau)$ of Libra balances, since a fraction τ need to be held in non-interest-bearing public currency. Following the same steps as before, the firm's flow budget constraint becomes

$$\Pi_t^m \;+\; (1-\tau)(1-Q_t^{\$})m_t^{\approx} \;+\; \frac{Q_{t-1}^{\$}-Q_{t-1}^{\approx}}{\beta}\, m_{t-1}^{\approx} \;=\; T_t^f \tag{1.141}$$

Again, the firm's profits are only dynamic in the sense that some profits accrue with a one-period delay. The profit function is thus given by

$$\Pi_t \;=\; \left[p_t^f(1-\tau)-1\right]C_t(\theta_{A,j},p_t^f,Q_t^{\approx}) \;+\; \Big[(1-\tau)\big(1-Q_t^{\$}\big) \;+\; \big(Q_t^{\$}-Q_t^{\approx}\big)\Big]p_t^f C_t(\theta_{A,j},p_t^f,Q_t^{\approx}) \tag{1.142}$$

which yields the expression as in the main body of text. The profit maximisation problem in its simplified form is then given by

$$\underset{p_t^f,Q_t^{\approx}}{max} \;\; \left[p_t^f\left[(2-Q_t^{\approx})(1-\tau)+\tau(Q_t^{\$}-Q_t^{\approx})\right]-1\right]\theta_{A,j}\left[\frac{\alpha}{p_t^f(2-Q_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{1.143}$$

Firm f optimally charges a real price given by

$$p_t^f = \frac{1}{\alpha[(2-Q_t^{\approx})(1-\tau)+\tau(Q_t-Q_t^{\approx})]} \tag{1.144}$$

Plugging this expression into the profit function and the consumer's money good consumption function, reveals both are decreasing in $Q_t^{\approx}$, thus increasing in $\pi_{t+1}^{\approx}$:

$$\Pi^f \;=\; \kappa\theta_{A,j}\left[(1-\tau)+\tau\,\frac{Q_t-Q_t^{\approx}}{2-Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \qquad C_t^m \;=\; \hat{\kappa}\theta_{A,j}\left[(1-\tau)+\tau\,\frac{Q_t-Q_t^{\approx}}{2-Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{1.145}$$

where κ is constant as in the main body of text, and $\hat{\kappa}$ is a positive constant and function of α only. It follows that Libra monetary policy is characterised by the corner solution $Q_t^{\approx} = 0$, corresponding to infinite inflation.

The competitor is fully priced out of the market. Given the tax, firm g charges an inflation-adjusted price satisfying

$$p^g(2 - Q_t^{\$}) \;=\; \frac{2 - Q_t^{\$}}{\alpha(1-\tau)} \tag{1.146}$$

As $Q_t^{\approx} \to 0$, firm f charges an inflation-adjusted price of

$$p_t^f(2 - Q_t^{\approx}) \;\to\; \frac{2}{\alpha[2(1-\tau) + \tau Q_t]} \tag{1.147}$$

Comparing the two expressions reveals that firm g can only compete if two conditions are met: a) in the absence of a tax ($\tau = 0$); and b) if $Q_t^{\$} = 1$. Effectively, there cannot be an opportunity cost to hold Dollars, and there cannot be capital gains discounts that firm f provides but firm g does not.

1.7.5.4 Derivations for subsection 1.5.3: Macroprudential policies

Libra firm f faces a macroprudential constraint:

$$\tau e_t^{\approx} M_t^{S,\approx} \;\leq\; M_t^{f,\$} \tag{1.148}$$

The firm again prefers to invest in bonds rather than currency, and the macroprudential constraint holds with equality (see Equation 1.139). Manipulating the flow budget constraint (Equation 1.138) following exactly the same steps as before, the profit maximisation problem is given by

$$\underset{p_t^{\approx}, Q_t^{\approx}}{max} \;\; [p_t^f - 1]C_t(p_t^f, Q_t^{\approx}) + (1-\tau)(1 - Q_t^{\$})\, p_t^f C_t(p_t^f, Q_t^{\approx}) + \left(Q_t^{\$} - Q_t^{\approx}\right) p_t^f C_t(p_t^f, Q_t^{\approx}) \tag{1.149}$$

Relative to the case of sales taxes payable in Dollar, profits are untouched. However, direct seignorage revenues are again reduced by a factor of τ which introduces a capital gains motive, as in the previous appendix subsection. Given optimal product discounts, profits and consumption are increasing in Libra inflation:

$$\Pi_t^f \;=\; \kappa\theta\left[1 - \tau\,\frac{1 - Q_t^{\$}}{2 - Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \qquad C_t^m \;=\; \hat{\kappa}\theta\left[1 - \tau\,\frac{1 - Q_t^{\$}}{2 - Q_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{1.150}$$

Capital gains and corresponding product discounts allow firms to circumvent the enforced opportunity cost of holding public currency.

2 On the Equilibrium Maturity of Bail-in Debt

2.1 Introduction

During the Great Financial Crisis (GFC), many banks experienced runs on their short-term debt liabilities due to an investor fear of losses on the asset side. As a consequence, governments were forced to conduct bailouts to prevent the largest banks of the economy from failing. Bailouts however are not only very costly to taxpayers, they also disincentivise banks to internalise asset losses during crises. As a response, regulators around the globe have introduced bail-ins as part of the post-crisis reforms package.[1] During a bail-in, the government enforces equity write-downs and debt-to-equity swaps of unsecured and uninsured bank debt in order to recapitalise banks. While not intended as the silver bullet to end all bank-related financial crises, bail-ins are an essential part of the reforms aimed at stabilising the financial system while committing not to conduct bailouts.

Regulation seems to have leapfrogged ahead of theory, as bail-ins have not yet received considerable attention in academic writing. Many papers in the literature employ frameworks which presuppose the type of bail-in debt that banks issue (see i.e. Farhi and Tirole, 2020). Other papers analyse bail-in policies for a given balance sheet composed of different types of bail-in debt (see i.e. Walther and White, 2020).[2] Yet the composition of bank liabilities is endogenous to the government's intended recapitalisation policy. In this context, we endogenise the maturity of bank debt and explore how it responds to the introduction of bail-ins. Given this response, can bail-ins successfully recapitalise failing banks and induce socially optimal behaviour as intended by the reforms?

Model outline. We develop a model of asymmetric information on banks' asset returns. There are two types of bankers: some with safe and high returns, others with risky and low returns in expectation. However, banks need to issue debt to finance the asset. Part of the financing can be raised via insured deposits, but we assume that banks are required to sell bail-in-able

[1] In the US, bail-ins have been introduced under Title II of the Dodd-Franck Act, whereas the EU has passed the Bank Recovery and Resolution Directive (BRRD).

[2] A notable exception is Clayton and Schaab (2020a).

debt to investors. The paper develops a benchmark in which the maturity of all bail-in debt—determined endogenously—is either short-term or long-term, and assets can be liquidated prematurely without cost. Importantly, a non-contractible signal reveals the type of the banker; short-term debt investors then decide whether they want to rollover their debt.

The model further contains a scope for government intervention: at a later stage, banks have an investment opportunity but invest inefficiently when they face a debt overhang. In the absence of a bail-in policy, the model thus features the usual time inconsistency problem on the government's side which gives rise to bailouts.

When the government's preferred form of bank recapitalisation is bail-ins, we show that banks issue short-term debt in equilibrium. Banks with high quality assets attempt to distinguish themselves by shortening the maturity of their debt, leading to an overall contraction in the maturity for all banks. Then in the run up to government intervention for banks with low quality assets, investors decide not to rollover their debt. This forces the banker to liquidate their assets and, eventually, to default. Thus, the government again requires bailouts to prevent the bank from failing and to induce efficient investment.

We then develop a notion of constrained efficiency: only banks with positive asset and investment NPV should enter financing markets; all banks invest efficiently post-intervention; and the government is not required to use public funds to achieve the efficient investment. Short-term debt leads to inefficiencies for multiple reasons. Absent of bailouts, the bank defaults and therefore does not invest. Hence the government is required to use public funds on top of conducting bail-ins. Due to these partial bailouts, negative NPV bankers find it profitable to enter financing markets ex-ante. Bail-ins do not fully achieve their desired disciplining effects.

We show that long-term bail-in debt achieves efficiency. Bail-in debt holders cannot force the bank into liquidating its assets in the run up to an intervention. Hence bail-ins can successfully induce efficient levels of investment. Since the government is not forced to conduct partial bailouts, only positive NPV projects enter the market. In this sense long-term debt has stronger disciplining effects on financial markets than short-term debt. This is in contrast with a large body of the literature (see i.e. Calomiris and Kahn, 1991), in which the threat of liquidation associated with short-term debt disciplines banks. From a policy perspective, these findings give rise to a long-term bail-in debt requirement.

We then extend the model by allowing banks to issue an endogenous mixture of short-term and long-term debt. We show that any equilibrium again features runs on bail-in debt. Long-term debt requirements are necessary to ensure bail-in effectiveness. However, if short-term debt is treated preferentially during a bail-in, the government needs to impose weaker long-term debt requirements.

Finally, we also discuss costly liquidation of assets and show that it leads to an increase in market discipline through short-term debt. However, this ex-ante disciplining effect only arises because this liquidation is inefficient ex-post.

Literature review. First, the paper relates to the literature on bank recapitalisation using bail-ins. Mendicino et al. (2017) and Tanaka and Vourdas (2018) are concerned with optimal capital regulation and conduct numerical exercises for bail-in debt and equity requirements. Dewatripont and Tirole (2018) discuss liquidity support in a bail-in environment. In Gimber (2012), bail-ins act as a commitment device not to conduct bailouts because they reduce the cost of liquidating assets. In Keister and Mitkov (2021), banks do not impose bail-ins if they anticipate bailouts, leading to runs and even larger required bailouts. In our paper, anticipated future bail-ins imposed by the government lead to an equilibrium contraction in the bail-in debt maturity structure. Runs ensue, leaving the government with no option but to bailout. Walther and White (2020), starting from a fixed balance sheet, find that the government may conduct bail-ins that are too small when they are read as a signal indicating weak bank fundamentals. Runs ensue after the government's intervention. Our paper endogenises the bank balance sheet, giving investors the option to act upon the realisation of a signal but before the government. We find that bail-ins become ineffective unless the government restricts the maturity of bail-in debt. Clayton and Schaab (2020a) discuss bail-ins and bailouts in the presence of a bank monitoring problem. Banks issue too little (long-term) bail-in debt—which can be written down and helps to avoid costly liquidation—because they do not internalise a firesale externality. If the amount of bail-in debt is sufficient, bailouts are fully replaced. Our paper points out that the nature of bail-in debt contracts responds to policy: unless the government requires bail-in debt to have long maturity, bail-ins are rendered ineffective and do not fully replace bailouts.[3,4]

Second, the paper relates to the literature on short-term debt and fragility. In Diamond and Dybvig (1983), banks provide liquidity insurance when issuing deposits while generating high returns from illiquid investments. This liquidity mismatch creates fragility.[5] In our paper, banks create a maturity mismatch (leading to a liquidity mismatch) in order to distinguish themselves from low quality projects—it does not provide liquidity services but shields investors from losses imposed by the government. In Calomiris and Kahn (1991), short-term debt disciplines banks through its threat of liquidation. Our paper points out that short-term bail-in debt leads to pre-mature liquidations followed by government bailouts while failing to fully align incentives of banks and society. Long-term debt avoids such liquidations, ensures bail-in effectiveness

[3]See Bolton and Oehmke (2019) and Clayton and Schaab (2020b) for bail-ins of investors of multinational banks.

[4]See also Pandolfi (2021) where bail-ins are ex-post efficient but lead to a breakdown of financing markets ex-ante. A combination of bail-ins and bailouts are the optimal policy. In our paper, bail-ins do not lead to a breakdown of financing markets, but the maturity structure of debt shortens to render bail-ins ineffective, leaving the government with no option but to conduct bailouts.

[5]See further canonical papers on liquidity creation and fragility by Gorton and Pennacchi (1990), Diamond and Rajan (2001) and Goldstein and Pauzner (2005).

and thus corrects ex-ante incentives. Brunnermeier and Oehmke (2013) and He and Milbradt (2016) feature a shortening of debt maturity since long-maturity creditors fear dilution by future short-maturity creditors. Our paper finds a shortening of the bail-in debt maturity structure as response to the government's proposed recapitalisation mechanism.[6]

Third, the paper relates to the literature on debt in the presence of asymmetric information. In Flannery (1986), firms signal quality of their projects through their maturity choice. In Diamond (1991), borrowers decide between issuing long-term and short-term debt, trading off liquidity risk against future improvements in their credit ratings associated with lower financing costs. Nachman and Noe (1994) derive conditions under which issuing debt is the unique equilibrium in an environment of ex-ante information asymmetries about return distributions. Tirole (2012) discusses how to optimally intervene in financing markets ex-post to overcome asymmetric information. Our paper is concerned with the anticipated response to a shift in ex-post government intervention from bailouts to bail-ins, also in the presence of asymmetric information on returns. We show that high return banks render bail-ins of low return banks ineffective in a futile attempt to distinguish themselves.

Organisation of paper. Section 2.2 introduces the model environment. Section 2.3 presents a simple notion of bailouts. Turning to bail-ins, the equilibrium debt contract and its consequences for bail-in effectiveness are derived in Section 2.4. Section 2.5 discusses efficiency properties. Mixed maturity bail-in debt is considered in Section 2.6, while Section 2.7 revisits the question on the market disciplining effect of short-term debt. Section 2.8 discusses extensions to the baseline model and Section 2.9 concludes.

2.2 Environment

The model features four different types of agents: bankers who have access to an asset but require initial financing; investors of bail-in debt; depositors who supply funding but cannot be bailed in; and the government which recapitalises banks.

The model contains four time periods. Banks decide whether to enter financing markets at time-0 before learning their type. Having learnt their type, they then seek financing at time-1. At time-2, a signal reveals the type of the banker to financial markets. Depending on the maturity of bail-in debt, investors have the option not to roll over their debt. The government then decides whether to recapitalise banks for reasons introduced below. The game ends at time-3 when all uncertainty is resolved, and funds are distributed among bankers, investors and depositors.

[6]See also He and Xiong (2012) and Diamond and He (2014).

2.2.1 Bankers

2.2.1.1 Asset

At time-1, a mass of bankers have access to an asset which generates returns at time-3.[7] There are two types of bankers, $\theta \in \{\theta^L, \theta^H\}$. A bank's type is unobservable to investors. Let the fraction of high types, θ^H, in the economy be given by μ_0. Their asset return, denoted by X^H, is deterministic. The asset return of the low type, θ^L, is drawn from a distribution with CDF $F(x)$. Let X^L denote the mean of the distribution and let $\underline{x} > 0$ denote its lower bound. We refer to this lower bound as the asset's safe return.

At time-2, the period after initial financing, a signal perfectly reveals the type of the banker (but not the return of the low type). Importantly, we assume that the signal is not contractible: agents cannot write contracts contingent on its realisation at time-2.[8]

Lastly, assets can be liquidated pre-maturely without cost. The liquidation value of the low type's asset is thus given by its expected return.[9]

2.2.1.2 Scope for intervention: Ex-post continuation investment

In the model, banks face an investment opportunity at time-2. This ex-post investment generates a scope for intervention: banks underinvest when they face a debt overhang. The return of the continuation investment i is increasing in effort e in a first-order stochastic dominance sense:

$$i(e) = \int_0^{\bar{\iota}} y \, dG(y|e) \tag{2.1}$$

where $G(y|e)$ denotes the CDF of investment returns, satisfying $G(y|e) > G(y|e')$ if $e' > e$ for all $y \in \big(0, \bar{\iota}\big)$. However, effort incurs a private cost $c(e)$ to the banker who is initially running the bank.[10] Let z denote the expected continuation investment returns net of effort cost: $z(e) = i(e) - c(e)$. We assume that $z(e)$ is strictly concave and twice continuously differentiable. It follows that the socially optimal effort level, denoted by e^*, satisfies

$$z'(e^*) = 0 \tag{2.2}$$

Going forward, let z^* denote the efficient expected net investment return: $z^* = z(e^*)$.

[7] We endogenise the size of the mass of bankers in Section 2.5.

[8] We discuss a contractible signal in Section 2.8.2, and show that it leads to multiplicity: banks either finance using short-term debt, or using CoCo bonds that fully dilute the banker upon conversion.

[9] We discuss costly liquidation in Section 2.7.

[10] Looking ahead, bail-ins involve equity write-downs and debt-to-equity swaps, and hence affect who is running the bank and thus facing the private effort cost.

When choosing effort levels, banks maximise their expected payoffs. Importantly, they are protected by limited liability. The optimal effort choice by the low type is characterised by

$$e(\hat{B}) \; = \; arg\,\max_{e} \int_{0}^{\bar{\iota}} \int_{\underline{x}}^{\infty} max\{x + y - \hat{B}, 0\} \, dF(x) \, dG(y|e) \; - \; c(e) \tag{2.3}$$

where $\hat{B}$ denotes the level of debt outstanding. Effort is inefficiently low if—with positive probability—some of the investment proceeds accrue to outside creditors and not the banker who is bearing the effort cost. It follows that banks exert too little effort whenever the asset return does not fully cover all debt outstanding with probability one. Turning to the high type, they exert efficient levels of effort if their deterministic return covers all debt outstanding: $X^H \geq \hat{B}$.

Assumption 2.2.1. *We make the following assumptions on asset and investment returns:*

1. *The total expected returns generated by bankers of the low type, even at the efficient level of expected investment gains, do not cover the initial required financing cost:* $X^L + z^* \; < \; 1$.
2. *The asset returns generated by bankers of the high type are sufficiently high such that financing is always achieved in equilibrium:* $X^H \; > \; \frac{1}{\mu_0}$.

The first assumption implies that the asymmetric information on asset returns is sufficiently severe to create adverse selection: low type bankers cannot achieve financing if they are identified as such. Hence, runs on identified low types occur for fundamental reasons in equilibrium. The second assumption implies that the high type's asset return is sufficiently high such that—even if the low type does not repay anything to bail-in investors who provide unit financing—the high type can repay all debt obligations with probability one. As a consequence, the financing market does not collapse. This assumption also implies that high types always invest efficiently.

2.2.1.3 Financing

Financing of the asset is obtained from investors of bail-in debt and depositors. We assume that bankers need to raise at least δ units of bail-in debt. Without loss of generality, we assume that this constraint binds. As a benchmark, we further assume that all bail-in debt is either short-term or long-term debt. Banks thus choose a financing strategy $\sigma \in \{d, D\}$, where d denotes short-term debt financing and D denotes long-term debt financing.[11] Short-term debt issued at time-1 matures at time-2. Importantly, short-term debt matures after the signal realisation but before the government can intervene.[12] Long-term debt does not need to be rolled over but instead matures at time-3.

[11] We endogenise the breakdown of short and long maturity bail-in debt in Section 2.6, and allow banks to issue equity in Section 2.8.1.

[12] Any short-term debt maturing after the government intervention corresponds to long-term debt in our model.

Assumption 2.2.2. *We make the following assumptions on deposit and bail-in debt financing relative to asset returns:*

1. *Bankers need to raise $\delta > X^L$ units in bail-in debt.*
2. *In principle, bail-ins are an effective tool to recapitalise banks since the low type's safe asset return covers deposits: $\underline{x} \geq (1 - \delta)$.*

The first assumption implies that the face value of bail-in debt exceeds the liquidation value of the low type's assets. In any equilibrium featuring short-term bail-in debt, such debt is risky since some investors cannot be repaid in full when choosing not to roll over.[13] The second assumption implies that bail-ins are effective absent of runs. This is the most interesting case economically. The point of this paper is to show how bail-ins create fragilities which render bail-ins ineffective and lead to an erosion of market discipline. Without this assumption, banks can never be recapitalised using bail-ins, no matter the debt maturity structure. More interestingly, we ask whether agents write debt contracts that render bail-ins ineffective in an environment in which they generally can be.

2.2.2 Investors

A unit mass of investors purchase the bail-in debt issued by bankers. They are risk-neutral, are endowed with a unit of capital at time-1, and do not discount the future. Importantly, banks have complete market power: they post a financing contract which is then priced competitively by the market, subject to the investors' participation constraint. Investors form beliefs about the type of a bank given their financing decision; upon the realisation of a given signal, investors update their beliefs according to Bayes' rule. These prior and posterior beliefs determine the required interest rate such that investors are indeed willing to supply funding and rollover their maturing claims.

We assume that all investors of the same class of debt face the same losses, either during a run, or when they are imposed on them by the government. Importantly, runs in the paper arise for fundamental reasons, not due to investor miscoordination.

2.2.3 Depositors

Depositors supply funds to the bank. Importantly, depositors in this paper are the simplest way of introducing bank liabilities that cannot be bailed in. They have no features typical of

[13] Alternatively, we could assume that only part of the asset can be liquidated as short-term bail-in debt investors withdraw; or we could include costs that reduce the liquidation value of assets.

deposits, i.e. liquidity shocks the spirit of Diamond and Dybvig (1983). Instead, they are fully insured by government. Given this insurance, they demand an interest rate of one. Depositors never withdraw their funds at time-2.

2.2.4 Government

The government chooses its desired recapitalisation strategy at the beginning of the game and implements it at time-2. In Section 2.3, recapitalisation is achieved through bailouts. In Section 2.4, the government imposes bail-ins; if they do not induce efficient investment levels, bail-ins are followed by bailouts.

2.3 A simple notion of bailouts

We introduce a very simple notion of *bailouts* which fully captures the government's time inconsistency problem at the heart of the analysis on banks that are considered 'too big to fail'.

Assumption 2.3.1. *Public funds are free at the time of intervention, but the government needs to raise distortionary taxes at the beginning of the game. The government is unable to commit not to conduct bailouts by not raising taxes initially.*[14]

As a result, the government fully removes the debt overhang at time-2. This is achieved by using public funds to decrease outstanding debt, $\hat{B}$, to match the minimum asset return. Thus, the low type receives a bailout of $b = \hat{B} - \underline{x}$. Combined with our assumption on the deterministic return of the high type, Assumption 2.3.1 ensures that all banks can always repay all debt. It follows that $\hat{B} = 1$, as all interest rates are equalised to one. Anticipating full repayment due to the government's guarantees, investors never have any incentives to withdraw funds from the bank. Since the government fully removes the debt overhang, the low type's expected payoff corresponds to the risky component of the asset, yielding $X^L - \underline{x}$ in expectation, and the expected net investment gain z^*. Given the the lowest possible debt burden of $\hat{B} = 1$, the high type achieves their maximum payoff:

$$V^{L,bailout} = x^L + z^* - \underline{x} \qquad V^{H,bailout} = V^{H,max} = x^H + z^* - 1 \tag{2.4}$$

[14] Alternatively, bailouts could be modelled as in Clayton and Schaab (2020a) where a proportional administration cost $1 > \kappa > 0$ accrues at the time of bailouts, and a proportional cost of distortionary taxation $\tau > 1$ accrues at time-1 (rather than time-2 when bailouts occur). This generates a cut-off rule for bailouts where the marginal gain of bailouts is traded off against their marginal cost, κ. Importantly, if the cost of bailouts is too large at the time of intervention, then this cost would act as a credible commitment device not to bail out the banker, and the model would never feature bailouts.

Anticipating the equilibrium section, any maturity structure of debt can be supported in equilibrium as both types achieve the same payoffs for any financing decision. Investors of short-maturity debt are willing to rollover even when they learn to have lent to the low type, as the government ensures full repayment when inducing efficient investment levels.

2.4 Bail-ins

In this section, suppose the government's preferred recapitalisation strategy are *bail-ins.* During this type of intervention, all bail-in debt claims are converted into equity in order to induce efficient investment. Before proceeding to compute equilibrium payoffs for different financing strategies under a bail-in policy, we provide some details on how we model bail-ins.

Assumption 2.2.2 ensures that bail-ins—if no assets are liquidated—fully remove the debt overhang and enable efficient investment. The definition for bail-in ineffectiveness follows naturally:

Definition 2.4.1 (Bail-in ineffectiveness)**.** *Bail-ins are ineffective whenever they are unable to induce efficient levels of the continuation investment:* $z^{\text{bail-in}} < z^*$.

Secondly, we need to specify how the government acts if bail-ins are indeed unable to induce efficient investment behaviour. Continuing the simple notion of bailouts introduced in the previous section, bailouts follow ineffective bail-ins: whenever bail-ins do not induce the socially optimal level of investment, the government uses bailouts of the minimum size required to fully remove the remaining debt overhang.

Thirdly, we need to specify the distribution of resources among investors and the banker during a bail-in. This boils down to determining the post-intervention equity shares. For this purpose, let $\underline{V}^\theta$ denote the virtual value of the banker's equity claim absent of intervention, anticipating future insolvency conditional on low investment and asset returns. Let V^θ denote the banker's payoff post-intervention. Motivated by existing bail-in regulation we consider the following condition:

Definition 2.4.2 (No creditor worse off (NCWO) condition)**.** *Any government intervention cannot make any agent—neither the banker nor investors nor depositors—strictly worse off. With regards to the banker, it must be that*

$$V^\theta \;\geq\; \underline{V}^\theta \tag{2.5}$$

Assumption 2.4.1. *The government converts bail-in debt to equity at a conversion rate such that the banker's NCWO condition as outlined in Equation (2.5) holds with equality.*

Existing bail-in regulation requires that any bank shareholders or creditors cannot be worse off under resolution than under a no-intervention policy. Since equity has a strictly positive virtual value absent of intervention—even with insolvency on the horizon for low investment and asset returns—the banker must not be fully diluted during a bail-in.[15] In the model, the NCWO condition can be derived from an intervention participation constraint that needs to be satisfied for each agent. Whenever the government intervenes and leaves any agent strictly worse off, this agent has the option of suing the government and forcing them to raise taxes to generate the no-intervention payoff. When honouring the NCWO condition during intervention, the government can avoid any such legal costs.

Before discussing the implications of the NCWO condition for our results, we introduce a notion of default which may be arising as a result of withdrawing creditors with maturing debt claims:

Definition 2.4.3 (Default)**.** *Whenever a debtor is unable to repay a creditor with a maturing debt claim, the debtor is defaulting. In this state, they receive a payoff of zero.*

Importantly for the results of the paper, the banker—having issued long-term debt—must not be fully diluted in all states of the world during a bail-in. At full dilution, the low type always achieves a zero payoff. The crucial difference between short-term and long-term debt is that short-term debt experiences runs in anticipation of losses which may not even realise. These runs lead to default. Thus, avoiding runs through long-term debt financing preserves value to low type bankers.[16]

2.4.1 Long-term debt

Suppose banks issue long-term debt: $\sigma = D$. While markets price bank debt competitively, the investors' participation constraint needs to be satisfied. Given Assumption 2.2.1 on the model's parameters, the low type does not generate returns that cover the initial cost of financing. Hence there exists a threshold belief $\underline{\mu}^D$ such that investors do not supply funding for all $\mu < \underline{\mu}^D$. All banks receive a zero payoff for such beliefs.

Consider an arbitrary belief $\mu \geq \underline{\mu}^D$ of lending to the high type for which banks do achieve financing. By the nature of long-term debt, it only matures at the end of the game and thus investors cannot respond to the signal. Once the government identifies the low type bankers, they bail in all long-term debt.[17] The investor post-intervention equity share is denoted by γ.

[15] See e.g. the *Bank of England's approach to resolution*, October 2017, based on the European Commission's Bank recovery and resolution directive (BRRD), Article 73.

[16] Equivalently, we could assume that uncertainty over returns is realised before the investment decision takes places and thus before the government intervenes. If the banker has issued long-term debt, avoiding runs and early liquidation, and invests efficiently with some strictly positive probability, avoiding a bail-in, they receive a strictly positive expected payoff.

[17] We could equivalently assume that investors are only bailed in to the degree that the debt overhang is exactly removed. Bail-in creditors receive all of the asset's safe return in excess of deposits, plus a share of equity returns

The participation constraint for a given investor is given by

$$\delta \;\leq\; \mu R_1 \delta \;+\; (1-\mu)\gamma\big[X^L + z^* - (1-\delta)\big] \tag{2.6}$$

where R_1 denotes the long-term interest rate. Bankers raise δ units of bail-in debt. Given their belief, investors lend to a high type who is able to fully repay all debt at the promised long-term interest rate with probability μ. Investors believe to be bailed in with probability $(1-\mu)$, and to receive a share γ of equity returns. Clearly, the higher the rate of debt-to-equity conversion—captured by the equity share γ—the lower is the debt burden on the high type who needs to compensate investors for losses arising when lending to the low type.

This rate of conversion is determined according to the banker's NCWO condition. Consider a scenario without any intervention. Having issued long-term debt, and given deposit insurance that prevents depositors from withdrawing, the low type does not experience runs despite being identified as such upon the realisation of the low signal. As long as the cumulative upper bound of the asset and investment return distributions is sufficiently high, they face a strictly positive probability of being able to fully repay all debt. Hence, their expected payoff is strictly positive:

$$\underline{V}^L(D,\mu) \;=\; \max_e \int_0^{\bar{\iota}} \int_{\underline{x}}^{\bar{x}} max\big\{x + y - \hat{B}(\mu), 0\big\}\, dF(x)\, dG(y|e) \;-\; c(e) \;>\; 0 \tag{2.7}$$

where $\hat{B}(\mu)$ denotes the debt burden in the absence of intervention at belief μ. By Assumption 2.4.1, the government dilutes the banker such that their NCWO condition binds:

$$V^L(D,\mu) \;=\; (1-\gamma)\big[X^L + z^* - (1-\delta)\big] \;=\; \underline{V}^L(D,\mu) \tag{2.8}$$

Bankers issue debt at an interest rate R_1 such that Equation (2.6) holds with equality. Combining this with Equation (2.8), the payoff to the high type is then given by:

$$\begin{aligned} V^H(D,\mu) \;&=\; X^H \;+\; z^* \;-\; (1-\delta) \;-\; R_1\delta \\ &=\; X^H \;+\; z^* \;-\; 1 \;-\; \frac{1-\mu}{\mu}\Big[\underline{V}^L(D,\mu) + 1 - X^L - z^*\Big] \end{aligned} \tag{2.9}$$

The high type's payoff is a combination of the asset and net investment returns, minus the unit financing cost and the investor loss of lending to the low type for which each high type banker needs to compensate. Note that $V^H(D,\mu)$ is strictly increasing in μ: the higher are investor beliefs to be lending to a high type, the lower is the required interest compensation for the loss arising due to the presence of the low type. The high type's payoff reaches their maximum value $V^{H,max}$ as $\mu \nearrow 1$.

post-intervention that is lower than the one in the main body of text. Given our two assumptions that a) banks issue debt with market power, only honouring the creditors' participation constraint, and that b) bail-ins dilute previous equity holders to the degree that their NCWO constraint binds, both models yield the same payoff to investors and bankers.

2.4.2 Short-term debt

Suppose that banks issue short-term debt: $\sigma = d$. As for long-term debt, there exists a threshold belief $\underline{\mu}^d$ such that financing is not achieved and banks receive a zero payoff for all $\mu < \underline{\mu}^d$. Conditional on achieving initial financing at time-1, we solve the game backwards from time-3 to establish that investors—in anticipation of government bail-ins and being subordinated to depositors—do not want to roll over once they identify the low type at time-2, leading to runs on bail-in debt.

Lemma 2.1. *When issuing short-term bail-in debt, runs ensue, rendering bail-ins ineffective.*

At time-2, investors update their beliefs about the banker's type according to Bayes' rule. Given the signal structure, the type of the banker is perfectly revealed. Short-term bail-in debt matures, and investors decide whether to withdraw or rollover their debt. Afterwards, the government intervenes to induce efficient investment levels.

Let the short-term interest rate at time-1 be denoted by r_1. The face value of short-term debt is thus given by $r_1\delta$. Suppose investors learn of having lent to a high type banker. Our parametric assumptions ensure that identified high type bankers are always able to rollover their maturing short-term bail-in debt. Given the banks' market power in financing markets, the required interest rate on newly issued short-term debt is determined according to the investor participation constraint. Lending to the identified high type is safe and the rollover interest rate is given by one. Investors can withdraw the face value of their debt today, or rollover and receive the face value tomorrow. Thus, conditional on a high signal, investors receive a payoff of $r_1\delta$.

Suppose investors learn of having lent to a low type banker. Suppose they roll over their maturing debt. Given the debt overhang $\hat{B} > \underline{x}$, bankers invest inefficiently, and the government bails in investors. The bank now invests efficiently, and the total expected net returns to be distributed among depositors, investors and bankers are given by $X^L + z^* < 1$. Depositors hold the most senior claims on bank returns. At the same time, the government does not fully dilute the banker to comply with the NCWO condition, in analogy to Equation (2.8). It follows that the expected payoff to investors at time-3 given the anticipated bail-in must be strictly less than the face value of debt maturing at time-2:

$$X^L + z^* - (1-\delta) - \underline{V}^L(d,\mu) \; < \; \delta \; \leq \; r_1\delta \tag{2.10}$$

Therefore, all investors choose to withdraw at face value upon the realisation of a low signal. The bank is forced to liquidate its asset. Since $X^L < \delta$, the banker cannot repay all investors, even at an interest rate of one. It follows that the banker defaults and receives a zero payoff for all beliefs μ:[18]

$$V^L(d,\mu) \;=\; 0 \tag{2.11}$$

Having established that the bank is forced into default by withdrawing investors, suppose that the government intervenes subsequently by bailing in those investor claims that have not been repaid. The previous banker is fully diluted, given their default payoffs of zero. Since all of the assets have been depleted to serve withdrawals, but the deposits remain in place, the new bank equity holders exert sub-optimal levels of effort:

$$arg\,\max_e \int_0^{\bar{\iota}} max\big\{y-(1-\delta),0\big\}\,dG(y|e) \;-c(e) \;<\; e^* \tag{2.12}$$

It follows that the government conducts bailouts to induce efficient investment levels. In particular, the government fully removes the debt overhang: $b=(1-\delta)$. As a result, the expected net gains from investing to the new equity holders—the previous investors—is given by z^*, leading to a total expected payoff of low type investors of X^L+z^*.

Having characterised payoffs of the low type banker and their investors, we can now turn to the payoffs of high type bankers and their investors. Given belief μ of facing a high type, the investor participation constraint is given by

$$\delta \;\leq\; \mu\, r_1\delta \;+\; (1-\mu)\,(X^L+z^*) \tag{2.13}$$

Banks issue short-term debt with interest rate r_1 such that this condition holds with equality. The payoff to the high type is then given by

$$\begin{aligned} V^H(d,\mu) \;&=\; X^H+z^*-(1-\delta)-r_1\delta \\ &=\; X^H+z^* \;-\; 1-\frac{1-\mu}{\mu}\big(\delta-X^L-z^*\big) \end{aligned} \tag{2.14}$$

As for long-term debt, $V^H(d,\mu)$ is strictly increasing in μ, and it reaches its maximum possible value, $V^{H,max}$, at $\mu=1$. Notably, the payoff now depends on the amount of bail-in debt financing δ. Runs force the government to bailout depositors, but losses are still inflicted on investors as asset return and net investment gain do not cover their initial financing. The higher δ, the larger is the shortfall for investors for which the high type needs to compensate.

[18] We cannot compute the low type's payoff when financing with short-term debt for a belief of $\mu=1$ since the occurrence of a low signal is a zero probability event, and Bayes' rule is not defined. We assume that the low type achieves a payoff of zero for such a belief, as they do for all other beliefs $\mu\in[0,1)$ where Bayes' rule is defined.

2.4.3 Equilibrium

The subgame beginning at time-1 only admits pooling equilibria. If the low type is identified as such when seeking financing, no investor would purchase any of their bail-in debt: even at the efficient investment level, the total returns cannot cover the cost of financing in expectation: $X^L + z^* < 1$. It follows that the low type always mimics the high type, and any equilibrium of this game must be a pooling equilibrium. The share of high types in the economy, μ_0, is then the equilibrium prior belief of investors to be lending to a high type.

With that in mind, we begin by establishing that financing with short-term debt is equilibrium dominated by financing with long-term debt for the low type; and that no financing strategy is equilibrium dominated for the high type.

Definition 2.4.4 (Equilibrium dominance)**.** *An equilibrium strategy σ dominates strategy σ' for a given type θ if the payoff for strategy σ at the equilibrium prior belief μ_0 is strictly larger than maximum payoff for strategy σ' at the best possible belief:*

$$V^{\theta}(\sigma, \mu_0) \;>\; \underset{\mu}{sup}\, V^{\theta}(\sigma', \mu) \;=\; V^{\theta}(\sigma', 1) \tag{2.15}$$

The (weakly) highest expected payoff when financing using a particular class of debt is achieved whenever markets believe that a banker is the high type with probability one. Accordingly, the high type achieves the maximum payoff $V^{H,max}$ for both short-term and long-term debt if $\mu = 1$. This maximum payoff is strictly larger than the payoff, as characterised by Equations (2.9) and (2.14) for long-term or short-term debt, at the equilibrium prior μ_0. Thus, neither strategy is equilibrium dominated. Turning to the low type, they achieve a zero payoff when financing using short-term debt due to the runs that ensue upon the realisation of a low signal. Given the government's bail-in policy, they achieve a strictly positive equilibrium payoff $V^L(D, \mu_0) > 0$ when financing using long-term debt, as by Equation (2.7). It follows that financing using short-term debt is equilibrium dominated for the low type.

Assumption 2.4.2 (Equilibrium refinement: Intuitive criterion)**.** *Consider a financing strategy σ that equilibrium dominates strategy σ' for type θ but not for type θ'. Then if investors observe a deviation from strategy σ to strategy σ', they must believe that it is type θ' who is deviating:*

$$\mu(\theta'|\sigma') \;=\; 1 \tag{2.16}$$

We are now ready to define the equilibrium of the sub-game beginning at time-1:

Equilibrium definition. *A Perfect Bayesian Equilibrium of the sub-game beginning at time-1 is given by a set of financing strategies $\sigma \in \{d, D\}$ and an investor belief system μ. The financing strategies maximise the banks' expected payoffs for both types $\theta \in \{\theta^L, \theta^H\}$, given* *investor beliefs. In equilibrium, the investors' prior beliefs for each form of financing must be correct and are updated according to Bayes' Law whenever possible. Off-equilibrium beliefs are refined according to the intuitive criterion.*

Proposition 2.1. *In the unique equilibrium of the sub-game starting at time-*1*, both firms finance using short-term debt. Runs occur upon the realisation of the low signal, rendering bail-ins ineffective.*

The result of Proposition 2.1 follows from above discussion. Consider first an equilibrium candidate in which both types pool on financing using long-term debt. According to the refinement of off-equilibrium beliefs, investors believe that any deviation to short-term debt must be coming from the high type. It follows that the high type achieves their maximum payoff when financing using short-term debt given this belief, and thus faces a profitable deviation. This rules out pooling equilibria on long-term debt.

Next, we show that pooling on short-term debt indeed constitutes an equilibrium. The strategy of financing using short-term debt does not equilibrium dominate financing using long-term debt for either type: markets are free to form any off-equilibrium belief for a deviation to long-term debt. Then, for any off-equilibrium belief that assigns sufficiently high probability to a deviation to long-term debt to the low type, neither type has an incentive to deviate to long-term debt as they face highly unfavourable credit conditions, or worse, cannot achieve financing altogether. Thus, pooling on short-term debt is indeed an equilibrium strategy.

Intuitively, the short-term debt equilibrium arises not because it is necessarily associated with higher expected payoffs for the high type. Instead, the high type attempts to distinguish themself from the low type by choosing a financing strategy that is very painful to the low type. Pooling on short-term debt induces frequent runs on the low type and this emergent fragility is exactly what renders bail-ins ineffective.

2.5 Entry into financing markets and efficiency

This section models the bankers' entry decision into financing markets and develops a notion of efficiency. At time-2, there is a unit mass of bankers who only learn their type once they have entered financing markets. Each banker has the same probability μ_0 to be the high type. Applying the law of large numbers, μ_0 is thus the investors' equilibrium prior belief of facing a high type at time-1.

However, entering financing markets is costly. Each banker draws their fixed cost k^i from a continuous distribution over the support $[\underline{k}, \bar{k}]$. Thus, banker i enters if

$$\mu_0 V^H(\sigma, \mu_0) \ + \ (1-\mu_0)V^L(\sigma, \mu_0) \ \geq \ k^i \tag{2.17}$$

where V^H and V^L denote equilibrium payoffs for high and low types at the equilibrium financing strategy σ and prior belief μ_0.

Once the bankers' types have been determined, the regulator would like to prevent the low types from financing. We assume that this is not possible ex-post. Instead, we define a notion of ex-ante constrained efficiency:

Definition 2.5.1 (Constrained efficiency)**.** *Three conditions need to be satisfied for efficiency:*

1. *Banks of both types of asset returns invest efficiently post-intervention.*
2. *The fixed cost of the marginal type that is indifferent between entering or not entering into financing markets is given by k^* which satisfies*

$$\underbrace{\mu_0 \, X^H \ + \ (1-\mu_0) \, X^L \ + \ z^*}_{\textit{expected (net) returns}} \ = \ \underbrace{1 \ + \ k^*}_{\textit{total cost}} \tag{2.18}$$

   This implies that only (weakly) positive NPV projects enter financing markets.

3. *The government is not forced to use public funds, neither for bailouts, nor for deposit insurance.*

Applying this constrained efficiency condition to the equilibrium described in Proposition 2.1 yields the paper's second main result:

Proposition 2.2. *Forcing banks to issue long-term bail-in debt achieves constrained efficiency. The market equilibrium short-term debt contract induces excessive entry into financing markets and does not lead to socially optimal investment unless the government conducts bailouts.*

When the government recapitalises banks using bail-ins and banks are forced to finance themselves using long-term debt, then all criteria for efficiency are met. First, bail-ins achieve the socially efficient investment for both high and low types. Second, combining Equations (2.7) and (2.9), and verifying against Equation (2.18), shows that the marginal type that enters financing markets pays a fixed cost of k^*. Third, the government is not required to use any public funds.

Bail-ins in the presence short-term debt lead to inefficiencies for two reasons. First, the government uses public funds to achieve the efficient continuation investment. Second, entry into financing markets is inefficiently high. To show the latter, combining Equations (2.11) and (2.14) yields the following expression for the average expected payoff from entering financing markets:

$$\begin{aligned} V^{bail-in}(d,\mu_0) &= \mu_0\, V^H(d,\mu_0) \;+(1-\mu_0)\, V^L(d,\mu_0) \\ &= V^* \;+\; (1-\mu_0)\,(1-\delta) \end{aligned} \tag{2.19}$$

where V^* denotes the expected average payoff from entering associated with efficiency. Entry is inefficiently high whenever $V^{bail-in}(d,\mu_0) > V^*$. The term $(1-\mu_0)\,(1-\delta) > 0$ captures the government subsidy to banks due to the bailouts conducted after bail-ins. The government subsidises the new bank equity holders, the previous bail-in debt holders, by removing the debt overhang to induce efficient levels of investment. This in turn reduces the debt burden of the high types, increases their payoffs and leads to excessive incentives to enter financing markets.

Lastly, and for completeness, bailouts lead to inefficiently high entry into financing markets. In particular, the expected payoff from entering financing markets in the presence of bailouts satisfies

$$V^{bailout}(\sigma,\mu_0) \;=\; V^* \;+\; (1-\mu_0)\, b \tag{2.20}$$

for all financing strategies σ, and where $b = 1 - \underline{x}$. Additionally—while efficient levels of the continuation investment are achieved—bailing out the low type bankers requires the use of public funds.

2.6 Mixed maturity debt and long-term debt requirements

A brief inspection of balance sheets reveals that banks issue a mixture of long-term and short-term debt that is not collateralised or protected by deposit insurance. Additionally, banks face long-term debt requirements. In the case of the UK, bail-in debt under the minimum requirement for own funds and eligible liabilities (MREL) requires a minimum maturity of one year.[19] Nevertheless, losses can be imposed on unsecured debt beyond MREL during a resolution. This section points out how run incentives and consequences in terms of bail-in effectiveness depend on whether short-term and long-term debt are treated differently during intervention. To demonstrate, we consider bail-in debt with mixed maturities for two policies: one in which short-term and long-term debt experience the same loss rates (or equivalently, are ranked *pari passu*); and one in which long-term debt is subordinated to short-term debt.

[19] See the *The Bank of England's approach to setting a minimum requirement for own funds and eligible liabilities (MREL)*, p.7, Article 5.2.

2.6.1 Pari passu

We extend the model to allow for mixed maturity debt which is ranked pari passu. Banks issue both short-term debt, $d > 0$, and long-term debt, $D > 0$, to finance the required δ units.

Proposition 2.3. *In equilibrium, banks issue a combination of short-term and long-term debt that induces runs and thus renders bail-ins ineffective.*

The equilibrium logic of Section 2.4 is unchanged. To illustrate, consider two candidate debt combinations (d, D) and (d', D'). Once investors learn that they have lent to the low type upon the realisation of a low signal, short-term investors withdraw their funds. In expectation, the low type is unable to repay its total debt obligations even after bail-ins induce efficient investment since $X^L + z^* < 1$. Since short-term debt is ranked pari passu with long-term debt, investors prefer not to roll over. The two debt combinations differ in terms of their ability to fully serve all withdrawing investors:

$$d' \ > \ X^L \ > d \tag{2.21}$$

It follows that a debt combination with short-term debt exceeding the value of assets conditional on a low signal induces default, yielding a zero payoff to the low type. Whenever banks are not forced into default, part of the asset remains on their balance sheet which, combined with the possibility of high investment outcomes even for low levels of effort, yields a strictly positive expected payoff to equity. Since the government intervenes but honours the banker's NCWO condition, they achieve a strictly positive payoff after a bail-in. It follows that financing using d units of short-term debt equilibrium dominates financing using d' units of short-term debt for the low type:

$$V^L(d, \mu_0) \ > V^L(d', 1) \tag{2.22}$$

Thus, any equilibrium contract with mixed maturity debt and short-term debt ranked pari passu must induce defaults due to runs on short-term bail-in debt, rendering bail-ins ineffective and leading to inefficiently high entry into financing markets.

Long-term debt requirements and bail-in effectiveness. Bail-ins are effective whenever they are able to fully remove the debt overhang. This requires that—after a bail-in—the safe return of any non-liquidated share of the asset exceeds the deposits outstanding. All short-term investors withdraw upon the realisation of a low signal. The bank liquidates assets at price X^L in order to repay d units of debt. It follows that the non-liquidated share of the asset is given by $\left(1 - \frac{d}{X^L}\right)$.

Then bail-ins are effective whenever the amount of short-term bail-in debt, d, satisfies

$$\underbrace{\left(1-\frac{d}{X^L}\right)\underline{x}}_{\text{safe asset return}} \;\geq\; \underbrace{(1-\delta)}_{\text{deposits}} \tag{2.23}$$

In this context, let us interpret δ as capital requirement with the goal of achieving bail-in effectiveness. If the regulator values deposits (for reasons of liquidity and maturity transformation along the lines of Diamond and Dybvig (1983) but unmodelled here), then they would set δ such that bailing in equity and investor debt exactly removes the debt overhang: $(1-\delta)=\underline{x}$. If short-term and long-term bail-in debt rank pari passu, then all bail-in debt must be long-term debt in order not to render bail-ins ineffective.

2.6.2 Subordination of long-term debt to short-term debt

In this subsection, we discuss an alternative bail-in policy in which long-term debt is subordinated to short-term debt.[20] As in the previous subsection, we show that there exists a level of short-term debt inducing runs. All debt combinations with short-term debt exceeding this level are equilibrium dominated by other debt combinations for the low type, and thus any equilibrium must feature runs.

Suppose all short-term debt holders roll over in anticipation of a bail-in during which they are treated preferentially. On expectation, total returns of X^L+z^* are distributed among banker, investors and depositors after the government has conducted a bail-in. Depositors are still served first, receiving full repayment of $(1-\delta)$. Although long-term debt and equity are subordinated to short-term debt, both must receive strictly positive expected payoffs by the NCWO policy (Assumption 2.4.1). Let the payoff to long-term debt denoted by $\underline{D}(d,\mu_0)$. As before, the payoff of the banker is denoted by $\underline{V}^L(d,\mu_0)$. It follows that short-term debt holders are willing to rollover into a bail-in if

$$r_1 d \;\leq\; X^L+z^*-(1-\delta)-\underline{D}(d,\mu_0)-\underline{V}^L(d,\mu_0) \tag{2.24}$$

If this condition holds, then it must be that $r_1=1$. Both for the low and the high type, short-term creditors rollover in future; this is only true if they expect to be repaid at the end of the game (in expectation), and face no credit risk at time-1. If Equation (2.24) does not hold, and the level of short-term debt exceeds the liquidation value of the asset ($d>X^L$), then short-term debt commands an interest rate $r_1>1$. It follows that short-term creditors a) do not want to

[20] In principle, there are many ways in which long-term debt could become subordinated to short-term debt. In terms of regulation, subordination of long-term debt boils down to preferential treatment of short-term debt during insolvency.

rollover, and b) force the banker into default if

$$d \;>\; max\Big\{X^L + z^* - (1-\delta) - \underline{D}(d,\mu_0) - \underline{V}^L(d,\mu_0),\; X^L\Big\} \tag{2.25}$$

Importantly, such a contract (D, d) exists given the parameter space considered in this paper. To illustrate, consider a combination of debt with $d \nearrow \delta$. By Assumptions 2.2.1 and 2.2.2, there exists a level of short-term debt for which Equation (2.25) is satisfied.

Long-term debt requirements and bail-in effectiveness. This section highlights that if regulators want to achieve bail-in effectiveness while minimizing the required amount of long-term bail-in debt (relative to short-term bail-in debt), then subordinating long-term debt to short-term debt becomes a useful policy tool. As long as Equation (2.24) is satisfied, such a policy helps to avoid any liquidation of assets even though some bail-in debt has short maturity.[21,22] Bail-in effectiveness is preserved. This is in stark contrast to the scenario in which all bail-in debt claims rank pari passu and short-term investors always withdraw.

2.6.3 Discussion of current UK policy

The UK's insolvency creditor hierarchy, to which the resolution authority adheres, is laid out in the *Bank of England's approach to resolution* (2017).[23] Deposits below the insurance threshold of GBP 85,000 are protected by the FSCS deposit insurance scheme. Further deposits by individuals and SMEs in excess of the threshold are ranking in the creditor hierarchy above senior unsecured and thus above designated bail-in debt. However, deposits in excess of the insurance threshold, which have not been deposited by individuals or SMEs, are ranked pari passu with other unsecured bail-in debt. The model predicts that the latter deposits should be prone to runs in anticipation of bail-ins, leading to outflows out of banks in the run-up of intervention. If these outflows are sufficiently large, then bail-ins are rendered ineffective.

2.7 Disciplining effect of short-term debt

Section 2.5 demonstrated that short-term debt financing leads to excessive entry into financing markets. In this version of the model, we show that costly liquidation of assets may prevent positive NPV projects from entering. In other words, disciplining of financial markets may be excessive.

[21] Any runs that ensue for pure liquidity reasons as $d > X^L$ but Equation (2.24) is satisfied can be avoided by a lender of last resort which is never forced to extend any emergency loans in equilibrium.

[22] Costly liquidation of assets is considered in Section 2.7.

[23] Source: link, page 18.

Suppose that the low type, once identified as such, can liquidate their assets at rate $\ell^L = \ell < 1$. Thus, liquidation is costly. However, we assume that there is no liquidation cost for the high type, $\ell^H = 1$. This avoids illiquidity issues for identified high types and rules out the need for intervention.[24,25] Since pre-mature liquidation of the asset now implies a waste of resources, we need to expand the notion of efficiency by one condition:

Definition 2.7.1. *Additional efficiency condition: No liquidation of assets in equilibrium.*

Adjusting the analysis of Sections 2.4 and 2.5 to incorporate the cost of liquidation, yields the paper's next result:

Proposition 2.4. *Market discipline increases if liquidation is costly. However, this disciplining effect a) is excessive whenever the liquidation loss exceeds the government's bailout, and positive NPV projects do not enter the financing market; and b) comes at the cost of ex-post inefficiencies.*

Suppose banks issue short-term debt. Since runs occurred without liquidation cost, they must also occur whenever liquidation is costly: $\ell X^L + z^* < X^L + z^* < \delta$. The low type again defaults. As before, the high type is always able to rollover short-term debt. The required short-term interest rate is then derived from the investor participation constraint as in Equation (2.13), adapted to incorporate the liquidation cost and again holding with equality:

$$r_1(\mu_0|\ell < 1) \;=\; \frac{1}{\mu_0} - \frac{1-\mu_0}{\mu_0}\frac{\ell X^L + z^*}{\delta} \;>\; r_1(\mu_0|\ell = 1) \tag{2.26}$$

Effectively, the high type now also needs to compensate for the liquidation loss which is further reducing the total repayment by the low type. Using the expression for the short-term interest rate, the payoff to the high type when financing using short-term debt is given by

$$V^H(d, \mu_0|\ell < 1) \;=\; X^H + z^* \;-\; \left[1 + \frac{1-\mu_0}{\mu_0}\left(\delta - \ell X^L - z^*\right)\right] \;<\; V^H(d, \mu_0|\ell = 1) \tag{2.27}$$

The properties of V^H are unchanged but there is a level reduction due to the increase in interest payments on each unit of short-term bail-in debt. This increase in the cost of borrowing also manifests itself in an increase in market discipline. Given Equation (2.27), the expression for the average expected payoff from entering financing markets becomes

$$V^{bail-in}(d, \mu_0|\ell < 1) \;=\; V^* \;+\; (1-\mu_0)\left[(1-\delta) \;-\; (1-\ell)X^L\right] \tag{2.28}$$

As before, the term $(1-\mu_0)\,(1-\delta)$ captures the government subsidy to banks due to bailouts. The higher the subsidy, the higher are incentives to enter financing markets. This subsidy is

[24]The liquidation cost can be microfounded as in Clayton and Schaab (2020a): in their model, arbitrageurs purchase liquidated assets but need to hold costly capital to buffer against potential losses. Since the identified high type never generates losses, no costly capital is required, and there is no liquidation cost.

[25]If liquidation of high type assets is costly, a lender of last resort solves the miscoordination problem and prevents any liquidations without ever having to extend any emergency loans.

now traded off against the liquidation loss, captured by the term $(1-\mu_0)\,(1-\ell)X^L$, which discourages entry.

2.8 Extensions

2.8.1 Equity

In this section, we expand the contract space and allow banks to issue equity: $\sigma \in \{d, D, e\}$. We treat all inside and outside equity equally: both banker and outside equity holders bear the cost of effort. As for long-term debt, equity holders cannot run. Since there is no debt overhang, the bank always invests efficiently.

Suppose bankers issue equity with the share of outside ownership given by γ^e. The investors' participation constraint is given by:

$$\delta \;\leq\; \gamma^e \Big\{ \mu \big[X^H + z^* - (1-\delta) \big] + (1-\mu) \big[X^L + z^* - (1-\delta) \big] \Big\} \tag{2.29}$$

Conditional on being able to achieve financing without requiring full outside ownership ($\gamma^e < 1$), it follows that the low type achieves strictly positive payoffs:

$$V^L(e,\mu) \;=\; (1-\gamma^e) \big[X^L + z^* - (1-\delta) \big] \;>\; 0 \tag{2.30}$$

In equilibrium, financing is always achieved at $\gamma^e < 1$ given our assumption on the high type's returns. It follows that equity financing again equilibrium dominates short-term financing for the low type:

$$V^L(e,\mu_0) \;>\; V^L(d,1) \;=\; 0 \tag{2.31}$$

Thus, in the unique equilibrium, banks again issue short-term debt.

Efficiency. Equity financing has the same efficiency properties as long-term debt financing: all banks invest efficiently, the government is not forced to use public funds, and only positive NPV projects enter financing markets. Importantly, debt does not feature positive incentive effects in our framework as in Innes (1990), or more recently, as in Clayton and Schaab (2020a) with regards to bail-ins. Those models feature a classic monitoring problem and debt induces bankers to exert costly yet socially desirable effort.

2.8.2 Market completeness

In this section, we assume that the signal is contractible. Bankers are then able to issue contingent convertible debt contracts (CoCos) which are triggered by a low signal realisation. The expanded contract space is thus given by $\sigma \in \{c, d, D\}$.

Proposition 2.5 (Multiple equilibria for a contractible signal)**.** *If the signal is contractible, the model features multiple equilibria. Additional to short-term debt, contingent convertible debt that fully writes down all equity and converts bail-in debt to equity can also be sustained in equilibrium. The equilibrium in which banks issue CoCos is constrained efficient.*

Consider a debt contract which converts into equity upon the arrival of a low signal. Such a contract is characterised not only by its rate of interest, but also by its rate of debt-to-equity conversion. Suppose that investors hold γ^c equity shares after the contract clause has been triggered. Note that a full dilution of the banker, $\gamma^c = 1$, corresponds to a conversion rate of infinity, as all of the previous equity holder's shares have a relative value of zero. Given the rate of conversion and prior belief μ, the investors' participation constraint is given by:

$$\delta \;\leq\; \mu R_1^c \;+\; (1-\mu)\gamma^c\big[X^L + z^* - (1-\delta)\big] \tag{2.32}$$

The bankers' expected payoffs are then given by:

$$\begin{aligned} V^H(c,\mu) \;&=\; X^H \;+\; z^* \;-\; R_1^c(\mu,\gamma^c)\,\delta \;-\; (1-\delta) \\ V^L(c,\mu) \;&=\; (1-\gamma^c)\,\big[X^L \;+\; z^* \;-\; (1-\delta)\big] \end{aligned} \tag{2.33}$$

where the interest rate $R_1^c(\mu,\gamma^c)$ is determined by the binding investor participation constraint.

Compare two different CoCo contracts: one with partial dilution, $\gamma^c < 1$, and one with full dilution, $\gamma^c = 1$. The low type achieves a zero payoff for the latter CoCo contract. It follows that such a contract is equilibrium dominated by any CoCo contract with a lower rate of conversion as well as the long-term debt contract. Therefore, investors believe that any deviation to a fully dilutive CoCo contract or to a short-term debt contract must be coming from the high type. Since the infinite dilution CoCo contract does not equilibrium dominate any other contract for either type, markets are again free to form off-equilibrium beliefs, and both short-term debt and infinite dilution CoCo financing can be supported as pooling equilibria.

This section highlights an *important similarity* between short-term debt and CoCo financing: both have the potential to fully wipe out of low type bankers, although the mechanism vastly differs. With short-term debt, investors liquidate their assets, forcing the bank into default. With CoCos, all contracts are automatically converted into equity contracts for low signal realisations at a conversion rate of infinity. This closely resembles the government's bail-in policy of long-term debt but without having to honour the NCWO condition: agents cannot sue the government because a contract clause—on which both parties have mutually agreed at the financing stage—has been activated. However, we do not observe banks issuing CoCo bonds with a conversion rate of infinity.

The *key difference* between these two types of contracts—and currently unmodelled here—is that short-term debt allows individual creditors to also act on their private information. In the version of the model above, both short-term debt and CoCos respond only to public news. If short-term debt issued by the low type experiences runs more frequently, then financing using short-term debt is more 'punishing' to low type bankers. This should help to rule out CoCos as equilibrium contracts which we do not observe in reality. This suggests that not only public information, but also private information has a role in creating fragilities in financial markets. We certainly aim to address this question in future work.

2.9 Conclusion

This paper demonstrates that bail-in debt is prone to runs in the presence of asymmetric information on asset returns. Requiring banks to issue long-term bail-in debt leads to an efficiency improvement since it allows governments to avoid costly bailouts and sets the socially appropriate incentives to enter financing markets. The model provides a motive to subordinate long-term debt to short-term debt: it prevents liquidation of assets in the run up to intervention and allows for weaker long-term debt requirements to achieve bail-in effectiveness.

The paper also revisits a well-established claim that short-term debt disciplines markets through its treat of early liquidation. We show how the disciplining effect of short-term debt depends on the efficiency loss associated with liquidation and the subsequent actions of a government that lacks commitment.

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3 Measuring Bail-in Credibility

3.1 Introduction

Since the Great Financial Crisis (GFC), policymakers around the world introduced reforms intended to resolve failing banks in an orderly manner and without government bailouts. These reforms have especially targeted banks that some considered "too big to fail" (TBTF). Bailouts of such banks would be avoided by requiring their subordinated and senior debt holders to bear any losses in excess of the bank's equity capital, a process referred to as "bail-in." The aim of this project is to investigate the credibility of these reforms by examining whether investors in bank debt perceive a lower probability of being bailed out, and therefore more likely to absorb losses, when a bank fails.

We estimate investor perceptions of a bailout for the United States (US) based on a structural model similar to Merton (1974, 1977). A bank's liabilities are contingent claims on its assets so that bank equity and debt can be valued as options on the bank's asset value. However, we also allow for the possibility that investors in the bank's debt might avoid losses if a government bails them out at the time of failure. As a result, credit spreads on a bank's subordinated and senior debt can be valued as the product of the probability that the debt is not bailed out and the expected loss rates on these debts in the absence of a bailout.

To estimate investor perceptions of a bailout, we first calculate what would be the expected loss rates on a bank's subordinated and senior debt if there were no bailout. The calculation uses information on the market value and volatility of the bank's shareholders equity as well as balance sheet data on the bank's liability structure. Then we examine how spreads on credit default swap (CDS) contracts written on the bank's subordinated and senior debt relate to these model-implied expected loss rates. The probability of not being bailed out is estimated as the coefficient of a regression of CDS spreads on the model-implied expected loss rates in the absence of a bailout. The intuition for this estimation technique is that if investors believe a bailout will never occur, then CDS spreads will vary one-for-one with the model-implied no-bailout expected loss rates. Instead, if investors believe that a bailout is highly likely, then CDS spreads will be insensitive to the no-bailout expected loss rates.

Our results indicate that market-perceived bailout probabilities have declined for US banks. During a pre-crisis period defined as prior to the 2008 Lehman Brothers bankruptcy, we find that bailout probabilities were perceived to be higher for banks, particularly for G-SIBs, relative to non-financial firms.[1] Since the introduction of resolution reforms, bailout probabilities have declined. In our latest subsample considered, ranging from May 2016 to July 2019, we find that 'no bailout' probabilities for banks to be at least as large as those for non-financial firms.

We then conduct a robustness analysis for UK banks and non-financial firms. We do not find a clear decline in the probability of bank bailouts, but our results may be affected by data quality that varied over our sample periods. Prior to the September 2008 Lehman insolvency, the results show that banks were less likely to be bailed out compared to non-financial firms. However, we only have 13 data points available on all UK banks during this period. We also consider a post-crisis period broken up into two subsamples: a period ranging from 2010 until 2015 and another period beginning in January 2016 when the Bank of England as UK resolution authority was granted bail-in powers. Credit risk, as captured by model-implied CDS spreads absent of bailouts, does not appear to have any explanatory power on banks CDS spreads between 2010 and 2015. Since the model explains a large share of non-financial firm CDS spreads during this subsample period, the results are consistent with full bank bailout expectations post-crisis. After 2016, we find that bank CDS spreads do respond to credit risk, but possibly to a lower degree than for non-financial firms. The difference in responsiveness between non-financial firms and banks is statistically significant when using vendor data for firm's liability structures but is statistically insignificant when we use UK regulatory data for banks' liability structures.

Related Literature. There is a large literature that examines whether investors require lower yields on the debt of banks considered TBTF due to the possibility of implicit government guarantees. Siegert and Willison (2015) offer a useful survey of the literature examining implicit funding subsidies. They categorize papers into three groups, and we follow their lead by briefly mentioning a few related studies in each group.

One group of papers in this literature studies market price reactions to events that are likely to affect investors' beliefs about banks' TBTF status, such as regulatory actions. O'Hara and Shaw (1990) find positive price reactions of the stocks of banks designated as TBTF following the Continental Illinois bailout. Schäfer et al. (2017) show that bank stock prices declined and CDS spreads increased in reaction to European bail-in events, particularly after the bail-in of Cypriot bank investors. Moreover, reactions were stronger for countries with limited fiscal capacity. Acharya et al. (2016) examine the reactions of credit spreads on large banks' bonds during the Bear Stearns rescue and the Lehman Brothers bankruptcy, finding that spreads declined for the former event and increased for the latter one. They find no significant credit spread reactions in response to the passage of the Dodd-Frank Wall Street Reform Act.

[1] G-SIBS are *Globally Systemically Important Banks*, as categorised by the Financial Stability Board (FSB).

A second group consists of cross-sectional studies that measure the funding cost advantage of TBTF banks. Consistent with a TBTF bank subsidy, Santos (2014) finds that yields on bonds issued by large firms are lower than those of similarly-rated small firms, but the large firm yield discount is greater when the firms are banks versus when they are non-banks. Acharya et al. (2016) find that bond yields of small banks, but not the largest ones, are sensitive to various measures of credit risk such as distance-to-default. This contrasts with non-financial firms' yields that are sensitive to risk measures for firms of all sizes. Gandhi and Lustig (2015) find that larger banks have significantly lower risk-adjusted returns than smaller banks, uncovering a size factor that measures exposure to bank-specific tail risk. In a similar vein, Gandhi et al. (2020) show that the largest financial institutions benefit from low excess returns on their equity financing. This benefit increases with size and interconnectedness of the financial sector, and decreases with tighter capital regulation and reduced fiscal capacity of the government.[2] Each of these paper's findings are consistent with implicit government guarantees for large banks.

A third group of papers uses structural models of banks to estimate the value of government guarantees or the likelihood of a bailout. Schweikhard and Tsesmelidakis (2011) and Merton and Tsesmelidakis (2012) employ information from a bank's market equity value to construct theoretical values of its CDS spreads based on a structural model characterized by a stochastic default barrier. Comparing actual CDS spreads to the model-implied ones allows them to calculate the present value of government guarantees on bank debt. Atkeson et al. (2019) use information in the market value of bank equity and estimates of a bank's loan and deposit cash flows to conclude that there has been a recent decline in the value of government guarantees.

Using the structural model concept that bank equity is a call option on the bank's asset value, Hett and Schmidt (2017) show that the sensitivity of bank debt relative to equity is declining in the probability of a bailout. They use this fact to measure changes in the probability of bailouts across different US financial institutions and in response to different crisis-related events. Berndt et al. (2020) also estimate bailout probabilities for US banks during the periods before and after the September 2008 Lehman Brothers bankruptcy, concluding that the likelihood of a bailout has declined. Based on an extension of the structural model of Leland (1994), they calculate a bank's distance-to-default which allows them to extract bailout probabilities from a banks' CDS spreads on senior debt.

Our paper is closely-related to Berndt et al. (2020) in that it takes a structural approach and uses information in equity values and balance sheet data to derive the theoretical values of CDS spreads and infer bailout probabilities from actual CDS spreads. Like them, we also benchmark banks' bailout probabilities relative to non-financial firms. However, we differ in using the structural model of Merton (1974, 1977) and, rather than focusing on the distance-to-default,

[2]Further studies include Araten and Turner (2013), Noss and Sowerbutts (2012), Ueda and Di Mauro (2013), Balasubramnian and Cyree (2014), and Schich et al. (2014). Anginer and Warburton (2014) analyze sectors other than the financial sector.

we estimate risk-neutral expected default losses for CDS contracts. Our much simpler approach has the advantage that it allows us to obtain point estimates of bail-in probabilities for different types of firms and subsamples, i.e. pre- and post-crisis. We can thus conclude whether bail-in probabilities differ for non-financial firms, banks and G-SIBs.[3]

Organisation of this paper. The remainder of this paper is structured as follows. Section 3.2 introduces the modelling framework. Section 3.3 presents the estimation method. The data sources and sample are described in Section 3.4. Section 3.5 presents the regression specifications and results. The robustness analysis for the UK is conducted in Section 3.6. Section 3.7 concludes.

3.2 Modelling framework

Consider a bank with a date-t market value of assets equal to A_t. Its liabilities are classified into four categories that are ordered by seniority from highest to lowest: deposits, senior debt, subordinated debt, and shareholders' equity.[4] As in Merton (1977), bank regulators are assumed to audit the bank at some future date $T > t$ at which time they determine the bank's solvency and decide on a resolution if necessary.[5] A resolution could involve imposing losses on subordinated and senior creditors or bailing them out by recapitalizing the bank without these creditors absorbing losses.

[3]Our estimated risk-neutral expected default losses implicitly combines distance to default (or expected default frequency) and loss given default, LGD, in a single measure. By solely estimating distance to default, Berndt et al. (2020) calculate pairs of pre- and post-Lehman Brothers bankruptcy bailout probabilities for different assumed LGDs. Our approach does not require assuming a particular level of LGD, though our implicit LGD embedded in risk-neutral expected default losses is likely to be more model-dependent.

[4]Note that in our implementation of the model, we allow for a richer liability structure and other features not contained in the model descriptions in this section. In particular, we account for liabilities more junior than subordinated debt and for stock dividend payouts. See Appendix 3.8.2 for details of these model and estimation adjustments.

[5]Alternatively, we could model regulators intervening when the market value of the bank's assets reaches a lower threshold, as in Black and Cox (1976), which is similar to the structural model in Schweikhard and Tsesmelidakis (2011) and Merton and Tsesmelidakis (2012). In that case, government guarantees take the form of down-and-out barrier options. For concreteness, we follow the Black-Scholes-Merton approach where government guarantees take the form of European put options, which is also the spirit of the approach in Hett and Schmidt (2017).

Table 3.1: Bank balance sheet

Assets	Liabilities
Assets, A_t	Deposits, D_t
	Senior Debt, N_t
	Subordinated Debt, B_t
	Equity, E_t

Deposits are government-insured and have a date-t book value equal to D_t. The bank pays a deposit interest rate of r_d and an additional government deposit insurance rate of p at the time it is audited. Define $\tau = T - t$ as the time until the next audit. Then the bank's date-T promised payment on deposits, including its deposit insurance premium, is

$$D_T \;=\; D_t\, e^{(r_d+p)\tau} \tag{3.1}$$

Senior debt has a date-t book value equal to N_t and is assumed to be a zero-coupon bond with a new issue yield of s_n above the risk-free interest rate of r. Thus, the bank's promised payment on this debt at date-T is

$$N_T \;=\; N_t\, e^{(r+s_n)\tau} \tag{3.2}$$

Subordinated debt has a similar zero-coupon structure with a date-t book value equal to B_t and a new issue yield of s_b above the risk-free interest rate of r. The bank's promised payment on subordinated debt at date-T is

$$B_T \;=\; B_t\, e^{(r+s_b)\tau} \tag{3.3}$$

We can now calculate the value of explicit and implicit government guarantees on the bank's debt. As in Merton (1977), the value of a government deposit insurance guarantee is analogous to a put option written on the bank's assets with an exercise price equal to the bank's promised payment on deposits. Denoting the date-t value of this deposit guarantee as $G_{D,t}$, it equals

$$G_{D,t} \;=\; Put(A_t, D_T, \tau) \tag{3.4}$$

where $Put(A_t, X, \tau)$ is a European put option written on an asset with current value A_t, exercise price X, and time until maturity τ.

Next, consider a possible implicit guarantee on the bank's senior debt. Suppose with risk-neutral probability p_N that the government fully bails out the bank's senior debt-holders rather than require them to absorb losses during a resolution. The value of this implicit guarantee on senior debt, $G_{N,t}$, equals

$$G_{N,t} \; = \; \big[Put(A_t, D_T + N_T, \tau) - Put(A_t, D_T, \tau)\big] p_N \tag{3.5}$$

The intuition for Equation (3.5) is that the value of a guarantee on senior debt equals the difference between the value of a guarantee on both senior debt and deposits, $Put(A_t, D_T + N_T, \tau)$, and the value of a guarantee on just deposits, $Put(A_t, D_T, \tau)$, times the probability of a bailout of senior debt, p_N.

Similarly, suppose with risk-neutral probability p_B that the government fully bails out the bank's subordinated debt-holders rather than require them to absorb losses during a resolution. The value of this implicit guarantee on subordinated debt, $G_{B,t}$, equals

$$G_{B,t} \; = \; \big[Put(A_t, D_T + N_T + B_T, \tau) - Put(A_t, D_T + N_T, \tau)\big] p_B \tag{3.6}$$

Let us next consider the market values of the bank's senior debt, subordinated debt, and shareholders' equity. If senior debt investors perceive the implicit bailout guarantee in Equation (3.5), then the date-t market value of senior debt, $V_{N,t}$, equals

$$\begin{aligned} V_{N,t} \; &= \; e^{-r\tau} N_T - \big[Put(A_t, D_T + N_T, \tau) - Put(A_t, D_T, \tau)\big] + G_{N,t} \\ &= \; e^{-r\tau} N_T - \big[Put(A_t, D_T + N_T, \tau) - Put(A_t, D_T, \tau)\big](1 - p_N) \end{aligned} \tag{3.7}$$

In the first line of Equation (3.7), the first two terms on the right-hand side equal the value of default-risky senior debt if there were no government guarantee. The first term is the value of default-free debt while the second term is the difference in put options that reflect the value of default losses to senior debt-holders. Combining these terms with the bailout guarantee leads to the second line. In a similar manner, the market value of subordinated debt, $V_{B,t}$, is

$$V_{B,t} \; = \; e^{-r\tau} B_T - \big[Put(A_t, D_T + N_T + B_T, \tau) - Put(A_t, D_T + N_T, \tau)\big](1 - p_B) \tag{3.8}$$

Since put options are decreasing functions of a bank's asset value and $\underset{A_t \to \infty}{lim} Put(A_t, X, \tau) = 0$, the differences in the put option values in Equations (3.7) and (3.8) decline as the market value of the bank's assets and capital grows. But the second terms on the right-hand sides of Equations (3.7) and (3.8) also decline when the probability of bailouts, p_N and p_B, increase. Thus, both a higher bank asset value and a higher bailout probability can raise the value of bank debt. Our estimation technique must distinguish between these two effects in order to isolate only the bailout probabilities.

Finally, note that limited liability implies that the bank's shareholders' equity is analogous to a call option written on the bank's assets.[6] Thus, its value, E_t, equals

$$E_t \;=\; Call(A_t, D_T + N_T + B_T, \tau) \tag{3.9}$$

Shareholders' equity benefits from government guarantees because the call option's exercise price, $D_T + N_T + B_T$, will be lower in equilibrium when investors perceive these guarantees. Specifically, the government's explicit or implicit guarantees against default lead to lower values of r_d, s_n, and s_b that reduce the bank's cost of funding.

For purposes of estimating bailout probabilities, we derive the date-t credit spreads on the bank's senior and subordinated debts implied by their market values in Equations (3.7) and (3.8). Thus, let $S_{N,t}$ be the difference between the senior debt's continuously-compounded promised yield to maturity and the default-free yield, r. This credit spread equals

$$S_{N,t} \;=\; \frac{1}{\tau}\, ln(N_T/V_{N,t}) \;-\; r \tag{3.10}$$

Similarly, let the corresponding credit spread for subordinated debt be given by

$$S_{B,t} \;=\; \frac{1}{\tau}\, ln(B_T/V_{B,t}) \;-\; r \tag{3.11}$$

Let $\Lambda_{N,t}$ and $\Lambda_{B,t}$ denote the senior and subordinated debt's risk-neutral expected loss rate in the absence of a bailout.

Proposition 3.1. *Credit spreads can be approximated as the product of the risk-neutral expected loss rates and 'no-bailout' probabilities for each class of debt:*

$$S_{N,t} \;\approx\; (1 - p_N)\, \Lambda_{N,t} \tag{3.12}$$

$$S_{B,t} \;\approx\; (1 - p_B)\, \Lambda_{B,t} \tag{3.13}$$

[6] Since our actual implementation of the model allows for interim dividend payments to shareholders, the value of shareholders' equity will also reflect those payments. See Appendix 3.8.2 for details.

where

$$\Lambda_{N,t} = \frac{1}{\tau}\frac{\left[Put(A_t, D_T + N_T, \tau) - Put(A_t, D_T, \tau)\right]}{e^{-r\tau}N_T} \tag{3.14}$$

$$\Lambda_{B,t} = \frac{1}{\tau}\frac{\left[Put(A_t, D_T + N_T + B_T, \tau) - Put(A_t, D_T + N_T, \tau)\right]}{e^{-r\tau}B_T} \tag{3.15}$$

Proof. Combine Equations (3.7) and (3.10) for senior debt, and combine Equations (3.8) and (3.11) for subordinated debt. Finally, use the approximation $ln(1 + x) \approx x$ and simplify.

Assumption 3.1. *The bank's assets follow a risk-neutral distribution that is lognormal.*

Under this assumption, put and call option values can be computed as

$$Put(A_t, X, \tau) = e^{-r\tau}X\Phi(-d_2^X) - A_t\Phi(-d_1^X) \tag{3.16}$$

$$Call(A_t, X, \tau) = A_t\Phi(d_1^X) - e^{-r\tau}X\Phi(d_2^X) \tag{3.17}$$

where $\Phi(.)$ is the standard normal cumulative distribution function and d_1^X and d_2^X are given by

$$d_1^X = \left[ln(A_t/X) + \left(r + \frac{1}{2}\sigma^2\right)\tau\right]/\left(\sigma\sqrt{\tau}\right) \tag{3.18}$$

$$d_2^X = d_1^X - \sigma\sqrt{\tau} \tag{3.19}$$

Given Assumption 3.1, we are now ready to compute the risk-neutral expected loss rates by combining Equation (3.14) for senior debt and Equation (3.15) for subordinated debt with Equations (3.16), (3.18) and (3.19).[7]

3.3 Estimation methodology

Our estimation strategy is based on the senior and subordinated credit spread Equations (3.12) and (3.13). It involves regressing senior or subordinated credit default swap spreads on estimates of the corresponding no-bailout expected loss rates, $\Lambda_{N,t}$ and $\Lambda_{B,t}$. These expected loss rates are functions of a bank's amounts of deposits, senior debt, and subordinated debt, D_T, N_T, and B_T, respectively, which we calculate from the bank's financial statements. However, these expected loss rates are also functions of the market value and volatility of the bank's assets, A_t

[7] See Appendix 3.8.1 for the fully detailed functional forms of Equations (3.14) and (3.15).

and σ, respectively, which are not directly observable. We follow the literature that began with Marcus and Shaked (1984) by inferring these values from the market value of equity, E_t, and the volatility of equity returns, σ_E. Substituting Equation (3.17) into Equation (3.9) yields:

$$E_t = A_t \, \Phi\Big(d_1^{D_T+N_T+B_T}\Big) - e^{r\tau}\Big(D_T + N_T + B_T\Big)\,\Phi\Big(d_2^{D_T+N_T+B_T}\Big) \tag{3.20}$$

$$\sigma_E = \frac{A_t}{E_t}\Phi\Big(d_1^{D_T+N_T+B_T}\Big)\,\sigma \tag{3.21}$$

Given data on the market value of the bank's equity, E_t, an estimate of the volatility of its stock returns, σ_E, the default-free interest rate, r, and balance sheet data on its current liabilities grossed up over the same horizon, $D_T + N_T + B_T$, equations (18) and (19) are two nonlinear equations in the two unknown variables A_t and σ.

The value of assets and their volatility are identified as long as the government does not provide guarantees to equity. That is, shareholders of banks are not direct recipients of bailouts but only benefit indirectly through the implied subsidy on bank debt.

We solve this two-equation system numerically. In all cases, the solutions for A_t and σ quickly converge. The value σ_E is calculated as the historical standard deviation of daily stock returns over the 6 months.[8] $D_T + N_T + B_T$ can be estimated by Equations (3.1), (3.2) and (3.3) based on the book values of current deposits, senior debt, and subordinated debt liabilities grossed up by current interest expense at the horizon τ.[9] The default-free rate, r, over the horizon τ is obtained from government bond yields. We use this default-free rate to proxy for interest expense on deposits. For senior and subordinated debt, we employ the current CDS spreads on top of the default-free rate.[10] The time horizon is five years, as this corresponds to the maturity of CDS contracts with the highest liquidity.

The frequency of balance sheet data is quarterly. Since stock market and CDS data are available at a higher frequency, we estimate a time series of A_t and σ from Equations (3.20) and (3.21) for each bank on a monthly basis by interpolating each bank's balance sheet data between quarters.[11] The output of this first step is a panel of observations composed of a cross-section of banks' monthly times series of estimated A_t and σ values. In turn, these estimates allow us to compute a time series of no-bailout expected loss rates, $\Lambda_{N,t}$ and $\Lambda_{B,t}$, for each bank.

These estimated no-bailout expected loss rates for each bank can then be related to the bank's actual senior and subordinated CDS spreads based on Equations (3.12) and (3.13). Theoretically, CDS spreads approximately equal a bond's credit spread over the default-free rate. We also allow

[8] It could equivalently be estimated as the implied volatility from options on the bank's stock.

[9] We treat all liabilities senior to senior debt as deposits.

[10] Using a moving average of CDS spreads, to account for fixed interest expenses on past debt issues, does not change any of the results presented below.

[11] We only interpolate whenever the difference between quarters is relatively small: the value of a given liability cannot increase or decrease by more than 25%.

for the possibility that there might be a (constant) liquidity premium contained in market CDS spreads. This logic leads to the following regression equation:

$$S_{j,i,t} \ = \ l_j \ + (1 - p_j)\, \Lambda_{j,i,t} \ + \ \varepsilon_{j,i,t} \tag{3.22}$$

where $S_{j,i,t}$ is the CDS spread on debt of type $j \in \{N, B\}$ for bank i at date t; l_j is the liquidity premium for debt of type j; p_j is the bailout probability for debt of type j; $\Lambda_{j,i,t}$ is the value of default losses on debt of type j of bank i at date t; and $\varepsilon_{j,i,t}$ is a regression error term. We estimate Equation (3.22) using OLS, allowing both the no-bailout probability and the liquidity premium to differ across types of firms (banks, G-SIBs and non-financial firms) and across subsamples.[12]

3.4 Data and sample

We include all US banks for which the data requirements, described below, are fulfilled. Table 3.11 in Appendix 3.8.3 lists the banks in our initial sample, broken down by G-SIBs and other banks. All our data are sourced from S&P Capital IQ (2019). It contains each bank's quarterly balance sheet data and its corresponding CDS spreads which allows easy matching of these time series.

In general, our CDS spread data consider contracts written on both senior debt and subordinated debt.[13] Also, because CDS contracts can differ in how credit events are defined, we use spreads for those CDS contracts with the most common credit event definition. CDS spreads used in our main empirical work for the US are based on the Ex-Restructuring (XR) definition of a credit event, which is the most common CDS contract covering US firms. This contract clause would cover default losses due to FDIC resolutions, as was the case in the largest U.S. bank failure, Washington Mutual Bank.[14]

Our estimation method considers several time-period subsamples. During each subsample period, a 'no bailout' probability is assumed to be fixed. By breaking up the sample into subsamples, we can trace out the evolution of probabilities through time. The ranges of these subsamples are determined by the Lehman Brothers bankruptcy and subsequent reform and

[12]Berndt et al. (2020) derive and estimate a regression equation similar to Equation (3.22). They differ in using a Leland (1994)-type model, rather than our Merton (1977)-type model, and split up a bank's risk-neutral expected loss rate, $\Lambda_{j,i,t}$, as the product of the bank's probability of insolvency (distance to default) and the bank debt's loss given default. Like them, we assume p_j is the same across banks of a given type over a given time period. Berndt et al. (2020) assume the bailout probability is the same for all U.S. G-SIBs prior to the Lehman bankruptcy event and then changes to a different constant level after the Lehman bankruptcy. Similarly, other large banks have a constant bailout probability prior to the event and change after the event. Non-SIB non-financial firms are assumed to always have a zero probability of a bailout.

[13]For US banks, CDS data for subordinated debt is only available for G-SIBs.

[14]Losses from the FDIC's proposed single point of entry resolution are also covered since this resolution is technically not a bail-in and bank debt is not expropriated.

resolution milestones. We define a pre-crisis estimation period ranging from January 2004 to the beginning of September 2008, just before the Lehman Brothers failure. We consider three post-GFC subsamples: a) January 2010 until September 2013, just before the first release of public sections of the largest banks' resolution plans in October 2013;[15,16] b) October 2013 to April 2016 just before the publication of Agency Feedback Letters for the 2015 Resolution Plan submissions of the largest banks, on April 12, 2016;[17] and c) May 2016 to July 2019.

3.5 Regression specifications and results

3.5.1 Baseline specification

Our baseline regression is given by:

$$S_{j,i,t} \;=\; \alpha_j \;+\; \beta_j \Lambda_{j,i,t} \;+\; \gamma_j^{GSIB} \mathbb{1}_{\{GSIB\}} \;+\; \beta_j^{\Delta GSIB} \mathbb{1}_{\{GSIB\}} \Lambda_{j,i,t} \;+\; \varepsilon_{j,i,t} \tag{3.23}$$

where $j \in \{N, B\}$ denotes the capital instrument for bank i. $\mathbb{1}_{\{GSIB\}}$ is a dummy variable that takes the value of one for G-SIBs. Our estimate of the coefficient β_j provides our estimate of the bail-in probability for non-G-SIB banks; our estimate of $\beta_j^{\Delta GSIB}$ provides the estimated difference in bail-in probabilities between G-SIBs and other banks. All of our specifications are estimated using OLS. Standard errors are clustered, with the clusters being given by each type of firm—G-SIB or bank—and month. This allows the errors to be correlated in any given month among firms of the same type. Before discussing the results, we should note a caveat in the implementation of regression Equation (3.23). Because the expected loss in the absence of a bailout, $\Lambda_{j,i,t}$, is an estimate of the capital instrument's true expected losses when a bailout does not occur, it is measured with error. It is well-known that a linear regression where the explanatory variable is measured with error results in a coefficient estimate that is downward biased.[18] Due to such attenuation bias, our estimates of the level of bail-in probabilities will tend to be below their true values. However, if the magnitudes of Λ estimation errors for GSIBs and other firms are similar, the coefficient $\beta_j^{\Delta GSIB}$ is a statistically meaningful measure of the difference in GSIBs' bail-in probability relative to other benchmark firms.

[15] We only restart the model at January 2010 to avoid the crisis period when bank stocks suffered huge losses, severely affecting our estimates of the distribution of asset values based on historical equity returns. Furthermore, the crisis period involved many distressed bank mergers that would complicate estimation during this period.

[16] Eleven institutions were required to submit Living Wills (or Resolution Plans): Bank of America, Bank of New York Mellon, Barclays, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, JPMorgan Chase, Morgan Stanley, State Street, and UBS. Source: Federal Reserve Board, Press release October 03, 2013, link.

[17] Agency Feedback Letters were published on that date for the following firms: Bank of America, Bank of New York Mellon, Citigroup, Goldman Sachs Group, JPMorgan Chase, Morgan Stanley, State Street, and Wells Fargo. Source: Federal Reserve Board, link.

[18] For example, see Hausman (2001).

Table 3.2: Regression results, senior debt, US

	Pre-Lehman	2010 – 09/2013	10/2013 – 04/2016	From 05/2016
Loss rates	0.668***	0.155***	0.298	1.264***
	(0.07)	(0.03)	(0.24)	(0.20)
Loss rates, $\Delta GSIB$	-0.458***	0.086	0.403	-0.567*
	(0.07)	(0.05)	(0.31)	(0.22)
constant	0.002***	0.008***	0.008***	0.007***
	(0.00)	(0.00)	(0.00)	(0.00)
constant, $\Delta GSIB$	0.000	0.006***	-0.001*	-0.001***
	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.78	0.44	0.02	0.17
N	147	351	325	474

Note: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$. SEs are clustered by type of firm and month.

Table 3.2 provides the regression results for senior debt in the US.[19] The reported coefficient is our estimate of the bail-in probability of senior unsecured debt for non-G-SIB banks. The coefficient reported one field below, labelled 'Loss rates, ΔGSIBs', is the estimated difference in bail-in probabilities for G-SIBs relative to other banks. Comparing the pre-Lehman and latest subsamples, bail-in probabilities have increased since the GFC. Before the GFC, financial markets expected senior bank debt to face losses with a probability of 66.8%. The corresponding probability for G-SIBs is given by 21%, with the difference being strongly statistically significant. Through the lens of the model, financial markets today expect that holders of bank senior will be bailed with probability one (126%). This probability is still lower for G-SIBs, with a statistically significant difference of 56.7 percentage points. Notably, bail-probabilities were perceived to be lowest in the transition period from the GFC to a well-established resolution regime in the latest subsample.

Table 3.3 provides the results for subordinated debt. Unfortunately, CDS spreads data on subordinated debt are only available for US G-SIBs. Bail-in probabilities are found to be much larger in the later samples than before the failure of Lehman Brothers. Bail-in probabilities peaked in our third subsample (Oct 2013 – April 2016). Through the lens of the model, subordinated debt faces a probability of bail-in of roughly 60% today.

[19] The resulting estimates of bail-in probabilities for each group, including non-financial firms, are presented below in Table 3.5.

Table 3.3: Regression results, subordinated debt, US

	Pre-Lehman	2010 – 09/2013	10/2013 – 04/2016	From 05/2016
Loss rates, GSIB	0.060***	0.283***	0.969***	0.589***
	(0.01)	(0.03)	(0.20)	(0.07)
constant, GSIB	0.002***	0.007***	0.006***	0.004***
	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.27	0.46	0.19	0.25
N	123	236	192	265

Note: $^{*}\, p < 0.05$; $^{**}\, p < 0.01$; $^{***}\, p < 0.001$. SEs are clustered by month.

3.5.2 Benchmark against non-financial firms

In this section, we augment the previous sample by constructing a control group of non-financial firms.[20] Given that non-financial firm CDS data are available only for senior unsecured debt, our focus is confined to this class of debt. Again, we regress the CDS spreads on our model-implied expected loss rates, allowing bail-in probabilities to differ for G-SIBs, other banks and non-financial firms. The control group regression specification is:

$$\begin{aligned} S_{j,i,t} \;=\; \alpha_j \;+\; \beta_j \Lambda_{j,i,t} \;+\; \gamma_j^{bank} \mathbb{1}_{\{bank\}} \;+\; \beta_j^{\Delta bank} \mathbb{1}_{\{bank\}} \Lambda_{j,i,t} \\ +\; \gamma_j^{GSIB} 1_{\{GSIB\}} \;+\; \beta_j^{\Delta GSIB} \mathbb{1}_{\{GSIB\}} \Lambda_{j,i,t} \;+\; \varepsilon_{j,i,t} \end{aligned} \tag{3.24}$$

where $S_{j,i,t}$ and $\Lambda_{j,i,t}$ denote the CDS spread and the expected loss on senior unsecured debt for firm i at time t, respectively. $\mathbb{1}_{\{bank\}}$ is a dummy variable that takes the value of one for banks. Our estimate of β is our estimate of the market-perceived bail-in probability of creditors to non-financial firms; $\beta^{\Delta bank}$ and $\beta^{\Delta GSIB}$ capture estimated differences for all banks, including G-SIBs, and an additional difference for G-SIBs.

The advantage of this augmented specification is the following: any model misspecification that biases probability estimates downwards should also affect non-financial firms in the same manner. Even if the level estimate of bail-in probabilities is not as informative as we would hope it is, we can still use the estimated probability difference to infer about differences in bailout expectations.

The control group contains all S&P 500 firms for which all data required are available on S&P Capital IQ. The combined sample contains 226 firms, including 15 banks.

Table 3.4 provides the regression results. Table 3.5 presents the estimated bail-in probabilities: the first row provides the bail-in probability estimate for non-financial firms ($\hat{\beta}$), the second

[20]Insurance firms are excluded from the control group.

Table 3.4: Regression results incl. non-financial firms, senior debt, US

	Pre-Lehman	2010 – 09/2013	10/2013 – 04/2016	From 05/2016
Loss rates	0.925***	0.841***	0.976***	0.460***
	(0.05)	(0.08)	(0.14)	(0.03)
Loss rates, Δbank	-0.256**	-0.686***	-0.679*	0.804***
	(0.08)	(0.08)	(0.28)	(0.20)
Loss rates, ΔGSIB	-0.458***	0.086	0.403	-0.567*
	(0.07)	(0.05)	(0.31)	(0.22)
constant	0.004***	0.009***	0.009***	0.007***
	(0.00)	(0.00)	(0.00)	(0.00)
constant, Δbank	-0.002***	-0.001***	-0.001	0.000
	(0.00)	(0.00)	(0.00)	(0.00)
constant, ΔGSIB	0.000	0.006***	-0.001*	-0.001***
	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.37	0.40	0.05	0.31
N	4260	7703	5919	8022

Note: $^{*}\ p < 0.05$; $^{**}\ p < 0.01$; $^{***}\ p < 0.001$. SEs are clustered by type of firm and month.

Table 3.5: Bail-in probabilities, senior debt, US

	Pre-Lehman	2010 – 09/2013	10/2013 – 04/2016	From 05/2016
Non-financial firms	92.5%	84.1%	97.6%	46.0%
Banks	66.8%	15.5%	29.8%	126.4%
G-SIBs	21.0%	24.1%	70.1%	69.7%

row provides our estimate for banks ($\hat{\beta}^{bank} = \hat{\beta} + \hat{\beta}^{\Delta bank}$), and the third for G-SIBs ($\hat{\beta}^{GSIB} = \hat{\beta}^{bank} + \hat{\beta}^{\Delta GSIB}$). We find that the bail-in probability for the control group varies substantially across samples. According to the model and with the caveat that our bail-in estimates are likely to be downward biased, senior unsecured creditors to non-financial firms were expected to be bailed-in with a 92.5% probability pre-crisis, with an 84.1% probability during 2010-09/2013, with 97.6% during 10/2013 – 04/2016, and finally with a 46% probability in our latest sample. Interestingly, we find that the responsiveness of bank CDS spreads relative to CDS spreads of non-financial firms has changed over time. Before Lehman, the estimated bail-in probability is significantly lower than that of our benchmark. G-SIBs faced an even lower bail-in probability. The difference between G-SIBs and other large banks is also estimated to be statistically significant.[21]

Post-crisis, the perceived bail-in probability is still significantly lower for banks than for non-financial firms—the difference estimated at minus 70 percentage points—although we no longer find any significant differences between different groups of banks. For our latest sample, covering the time period after the publication of feedback on resolution letters for the largest banks, we find that bank bail-in probabilities appear larger than for non-financial firms. This would suggest that bail-ins of creditors are now perceived to be at least as likely for banks as for non-financial firms.

The results for the United States support our initial hypothesis: We estimate bail-in probabilities as statistically significantly lower for banks, and even more so for G-SIBs, pre-crisis. The introduction of TBTF reforms, including the creation of new resolution tools, seem to have (more than) closed the gap in investor loss participation between banks and non-financial firms.

The results also indicate the presence of model misspecification that leads to estimation error in expected default losses and, in turn, attenuation bias. Clearly, we expect the bail-in probability for non-financial firms to be higher than 50%. Nonetheless, incorporating the control group mitigates the effects of the estimation error and allows us to draw conclusions on relative probabilities.

[21] The US does not formally designate D-SIBs, so instead we compare G-SIBs and other large banks for which data are available.

3.6 Robustness: UK analysis

3.6.1 Data and sample

We obtain the data from S&P Capital IQ, as above. We further complement these data with Bank of England regulatory data on UK banks' liability structures.[22] Regulatory data are likely to be of higher quality but they only become available in Q3 2014. Thus, for UK banks we estimate bailout probabilities in two ways: using only Capital IQ data and also using a combination of Capital IQ data and Bank of England regulatory data whenever the latter are available.

All required data are available for five banks: Barclays, HSBC, Lloyds, RBS, and Standard Chartered. Currently, Her Majesty's Treasury (HMT) is still the largest shareholder of RBS. HMT also held shares in Lloyds until May 17, 2017. We therefore omit all data on RBS, and only retain data on Lloyds from June 2017 onwards. This leaves four banks in the sample. Three of those—Barclays, HSBC and Standard Chartered—are all G-SIBs. Given the few data points on Lloyds, we are unable to distinguish between G-SIBs and other banks. The augmented sample, also including non-financial firms, contains a total of 52 firms.

For the UK, the vast majority of CDS contracts are based on the Modified-Modified Restructuring (MMR) contract clause definition. As discussed by Nolan (2014) and Neuberg et al. (2018), this type of CDS contract explicitly covered government-imposed bail-ins starting in October 2014. This October 2014 International Swap Dealers Association (ISDA) change to cover bail-ins was largely motivated by the 1 February 2013 bail-in of SNS Reaal Bank's subordinated debt. Investors were surprised that CDS contracts on this debt did not fully cover the debtholders' losses under the MMR credit event clause.[23] Therefore, our empirical work omits observations on UK banks from March 2013 to October 2014 when CDS spreads may not have fully-reflected default losses from bail-in credit events.[24]

Lastly, to account for the fact that the resolution milestones differ across jurisdictions, we con-

[22]The regulatory data are taken from firms' FINREP reports. These reports contain financial information that certain UK firms, including all banks, are required to report to the Bank of England (Prudential Regulation Authority) under EU law. The main difference between FINREP and firms' published accounts is that FINREP reports exclude subsidiaries that are not part of firms' "prudential consolidation groups". These groups form the basis for calculating firms' capital requirements and resources, and we would not ordinarily expect the resolution authority to bail-in debt outside such groups.

[23]The bail-in involved the Dutch government's expropriation of SNS bank's subordinated bonds, leaving none available for an auction that normally determines the cash settlement price for CDS contracts. Reuter's Christopher Whittall describes the Dutch government action as a "shock decision" (link). See also Financial Times (March 11^{th}, 2013) for evidence that the SNS bail-in raised market uncertainty regarding CDS settlement (link). Later bail-ins during the period prior to the 2014 ISDA change include Bankia (Spain) in 2013 and Espirito Santo (Portugal) in 2014.

[24]We retain observations for February 2013. The decision to expropriate bondholders was announced on February 1^{st}, and investors learned only days later that their claims were unprotected.

Table 3.6: Regression results, UK, Capital IQ data

	Senior debt			Subordinated debt		
	Pre-Lehman	Post-Crisis	Post-BRRD	Pre-Lehman	Post-Crisis	Post-BRRD
Loss rates	0.541***	-0.055	0.170***			
	(0.01)	(0.05)	(0.02)			
Loss rates				-0.04	0.732	0.165***
				(0.04)	(0.45)	(0.02)
constant	0.001***	0.009***	0.009***	0.001**	0.013***	0.016***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.96	0.00	0.17	0.01	0.06	0.25
N	13	42	87	13	42	87

Note: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$. SEs are clustered by month.

Table 3.7: Regression results, UK, regulatory data

	Senior debt			Subordinated debt		
	Pre-Lehman	Post-Crisis	Post-BRRD	Pre-Lehman	Post-Crisis	Post-BRRD
Loss rates	0.541***	-0.072	0.239***			
	(0.01)	(0.04)	(0.03)			
Loss rates				-0.04	0.180	0.140***
				(0.04)	(0.22)	(0.02)
constant	0.001***	0.009***	0.009***	0.001**	0.014***	0.016***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.96	0.00	0.17	0.01	0.01	0.26
N	13	56	148	13	56	148

Note: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$. SEs are clustered by month.

sider two post-crisis subsamples for the UK. The first one, labelled 'Post-Crisis', runs from January 2010 to December 2015. The later one runs from January 2016 to July 2019. This split reflects the granting of bail-in powers to the Bank of England in January 2016, by the UK's adoption of the BRRD regulation. Hence, we label this subsample 'Post-BRRD'.

3.6.2 Results—Baseline specification

Tables 3.6 and 3.7 provide the regression results for the UK for both sources of balance sheet data.[25] The results for the UK yield a rather mixed picture. It is worth keeping in mind that

[25] As explained above, regulatory balance sheet data only become available from Q3 2014 onwards. The regressions below thus use S&P Capital IQ data until and including Q2 2014.

our pre-Lehman sample only contains 13 data points, potentially explaining the very counter-intuitive result that subordinated debt was less likely to be subjected to loss participation than senior unsecured debt. Our coefficients estimated for the post-crisis sample are statistically insignificant; the explained variation in observed CDS spreads is very low, with an R^2 of 0.00 and 0.01, respectively. These findings could be attributed to model misspecification, but they are also entirely consistent with a view of the world that investors expected the government to fully bailout any further failing banks. Overall, the results motivate the augmented regression specification in subsection 3.6.3 below, which includes non-financial firms as a control group, addressing potential model misspecification. Notably, the results for the two different data sources are very similar post-BRRD. However, they do substantially differ for subordinated debt in the post-crisis sample.

3.6.3 Results—Benchmark against non-financial firms

The results of our regressions are presented in Tables 3.8 and 3.10. Table 3.9 presents the estimated bail-in probabilities for the regression using only Capital IQ data. The market-perceived 'no bailout' probability is estimated to be lower for banks for all subsamples, but going against our initial hypothesis, significantly so only for the post-reform period. The results suggest the presence of data quality issues before the Lehman event. Given our results for the US and the findings of the large literature on TBTF, we should expect lower bail-in probabilities for banks than for non-financial firms. For the post-crisis sample, we obtain a 'no bailout' probability for non-financial firm investors of 90%. These results speak against model misspecification but suggest that investors believed that bank debt was fully guaranteed by the government.

In the latest subsample, the difference in bail-in probabilities between banks and non-financial firms is statistically significant for Capital IQ data, but insignificant for regulatory data. Interestingly, and corresponding to our findings for the US, the model finds a much lower than expected 'no bailout' probability for non-financial firms for this latest subsample. This observation hints at some general issues with CDS markets for corporate debt in prolonged low interest rate environments, especially when considering that the model predicts 'no bailout' probabilities approaching one in previous subsamples.

Table 3.8: Regression results incl. non-financial firms, senior debt, UK, Capital IQ data

	Pre-Lehman	Post-Crisis	Post-BRRD
Loss rates	0.299***	0.869***	0.357***
	(0.04)	(0.10)	(0.07)
Loss rates, $\Delta bank$	0.242***	-0.924***	-0.187**
	(0.04)	(0.11)	(0.07)
constant	0.004***	0.012***	0.008***
	(0.00)	(0.00)	(0.00)
constant, $\Delta bank$	-0.003***	-0.003***	0.001
	(0.00)	(0.00)	(0.00)
R^2	0.24	0.28	0.24
N	215	878	1069

Note: $^{*}\ p < 0.05$; $^{**}\ p < 0.01$; $^{***}\ p < 0.001$.

SEs are clustered by type of firm and month.

Table 3.9: Bail-in probabilities, senior debt, UK, Capital IQ data

	Pre-Lehman	Post-Crisis	Post-BRRD
Non-financial firms	29.9%	86.9%	35.7%
Banks	54.1%	-5.5%	17.0%

Table 3.10: Regression results incl. non-financial firms, senior debt, UK, regulatory data

	Pre-Lehman	Post-Crisis	Post-BRRD
Loss rates	0.299***	0.869***	0.357***
	(0.04)	(0.10)	(0.07)
Loss rates, $\Delta bank$	0.242***	-0.942***	-0.118
	(0.04)	(0.11)	(0.07)
constant	0.004***	0.012***	0.008***
	(0.00)	(0.00)	(0.00)
constant, $\Delta bank$	-0.003***	-0.003***	0.001*
	(0.00)	(0.00)	(0.00)
R^2	0.24	0.28	0.24
N	215	892	1130

Note: $^{*}\ p < 0.05$; $^{**}\ p < 0.01$; $^{***}\ p < 0.001$.

SEs are clustered by type of firm and month.

3.7 Conclusion

This paper presented work measuring market-perceived bail-in probabilities. Using an adaption of the Merton (1974, 1977) model, we compute a hypothetical fair price for CDS spreads, absent of government bailouts. We determine the co-movement between this model-implied fair price and market prices of CDS and interpret the degree of co-movement as bail-in probability. For the US, the results suggest that 'no bailout' probabilities have clearly increased and are perceived to be as high for banks as for non-financial firms today. The results for the UK are somewhat mixed. The low number of pre-crisis data points limits the degree to which we can draw conclusions on bailout expectations. After the crisis, there is little co-movement between banks' model-implied fair spreads and actual CDS spreads. These finding are consistent with full bailout expectations in the aftermath of the GFC. Unfortunately, the data do not allow for robust conclusions for the time period after the introduction of bail-in powers in 2016.

Going forward, we would like to explore whether our findings, especially those for the US, hold true during the Covid-19 crisis.

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3.8 Appendices

3.8.1 Appendix to Section 3.2

The risk-neutral expected loss rates for each class of debt can be computed as

$$\begin{aligned}\Lambda_{N,t} \;=\; \frac{1}{\tau}\Bigg[&\Big(1+\frac{D_T}{N_T}\Big)\,\Phi\big(-d_2^{D_T+N_T}\big) \;-\; \frac{A_t}{e^{-r\tau}N_T}\,\Phi\big(-d_1^{D_T+N_T}\big)\\ &-\;\frac{D_T}{N_T}\,\Phi\big(-d_2^{D_T}\big)+\frac{A_t}{e^{-r\tau}N_T}\,\Phi\big(-d_1^{D_T}\big)\Bigg]\end{aligned} \tag{3.25}$$

$$\begin{aligned}\Lambda_{B,t} \;=\; \frac{1}{\tau}\Bigg[&\Big(1+\frac{D_T+N_T}{B_T}\Big)\,\Phi\big(-d_2^{D_T+N_T+B_T}\big) \;-\; \frac{A_t}{e^{-r\tau}B_T}\,\Phi\big(-d_1^{D_T+N_T+B_T}\big)\\ &-\;\frac{D_T+N_T}{B_T}\,\Phi\big(-d_2^{D_T+N_T}\big)+\frac{A_t}{e^{-r\tau}B_T}\,\Phi\big(-d_1^{D_T+N_T}\big)\Bigg]\end{aligned} \tag{3.26}$$

3.8.2 Appendix to Section 3.3

3.8.2.1 Accounting for preferred stock and CoCos

Some banks/firms have issued preferred stock or Additional Tier 1 (AT1) contingent convertibles (CoCos). These liabilities can be considered to be senior to the bank's common shareholders equity, E, but junior to the bank's subordinated debt. For simplicity, the previous analysis has left out these liabilities from the model. However, we account for the presence of these liabilities in banks' capital structure by making the following approximate adjustments. Denote P_t as the date-t as the sum of the bank's book values of preferred stock and CoCos.

When applying the Marcus and Shaked (1984) estimation method in equations (3.20) and (3.21), we include all of the non-common stock liabilities of the bank as the analogous "exercise price" for the options. Thus, when estimating the bank's values of A_t and σ at each point in time, we employ the adjusted equations:

$$E_t \;=\; A_t\,\Phi\Big(d_1^{D_T+N_T+B_T+P_T}\Big) \;-\; e^{r\tau}\Big(D_T+N_T+B_T+P_T\Big)\,\Phi\Big(d_2^{D_T+N_T+B_T+P_T}\Big) \tag{3.27}$$

$$\sigma_E \;=\; \frac{A_t}{E_t}\Phi\Big(d_1^{D_T+N_T+B_T+P_T}\Big)\,\sigma \tag{3.28}$$

3.8.2.2 Accounting for dividends paid on common shareholders' equity (stock)

In our empirical implementation of the model, we recognize that dividends are a payout from the bank's assets that leave less assets available to the bank's other creditors. Assuming that a proportion of assets, denoted by δ, are paid out per unit time in the form of common stock dividends, allows us to adjust the previous formulas. The parameter δ can be estimated as the long-run ratio of annual common stock dividends to (book) assets. In our estimations, we distinguish two different periods of long-run dividend to assets payout ratios: pre-crisis, and post-crisis.

Adjusting formulas for dividend payments is as follows. For equity, E and σ_E, which is used in the Marcus and Shaked (1984) approach, we have

$$E_t = A_t \left[1 - e^{-\delta\tau}\Phi\Big(-h_1^{D_T+N_T+B_T+P_T}\Big)\right] - e^{r\tau}\Big(D_T + N_T + B_T + P_T\Big)\,\Phi\Big(h_2^{D_T+N_T+B_T+P_T}\Big) \tag{3.29}$$

$$\sigma_E = \frac{A_t e^{-\delta\tau}}{E_t}\Phi\Big(h_1^{D_T+N_T+B_T+P_T}\Big)\,\sigma \tag{3.30}$$

where

$$h_1^X = \left[ln(A_t e^{-\delta\tau}/X) + \Big(r + \frac{1}{2}\sigma^2\Big)\tau\right]/\Big(\sigma\sqrt{\tau}\Big) \tag{3.31}$$

$$h_2^X = h_1^X - \sigma\sqrt{\tau} \tag{3.32}$$

For all of the other formulas, such as Equations (3.25) and (3.26), replace A_t with $A_t e^{-\delta\tau}$, d_1^X with h_1^X, and d_2^X with h_2^X.

Importantly, none of our conclusions change when setting $\delta = 0$ for all firms.

3.8.3 Additional Tables

Table 3.11: Banks in US sample

G-SIBs	Other Banks
Bank of America, Citigroup, JPMorgan Chase, Morgan Stanley, Bank of New York Mellon, Goldman Sachs, Wells Fargo	Ally Financial, American Express, Capital One Financial Corporation, Merrill Lynch (pre-Lehman), MetLife, SunTrust Banks, PNC Financial Services, U.S. Bancorp