The Truth About the Future

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Abstract   There is a long-standing disagreement among *Branching-Time* theorists. Even though they all believe that the branching representation accurately grasps the idea that the future, contrary to the past, is open, they argue whether this representation is compatible with the claim that one among many possible futures is distinguished—the single future that will come to be. This disagreement is paralleled in an argument about the bivalence of future contingents. The single, privileged future is often called the *Thin Red Line*. I reconstruct the history of the arguments for and against this idea. Then, I propose my own version of the Thin Red Line theory which is immune to the major objections found in the literature. I argue that the semantic disagreement is grounded in distinct metaphysical presuppositions. My solution is expressed in a conceptual framework proposed by John MacFarlane, who distinguishes semantics from postsemantics. I extend his distinction and introduce a new notion of presemantics to elucidate my idea.

1 Introduction

Arthur Prior (1967, chapter 7) proposed a novel perspective on indeterminism. He suggested, probably inspired by a letter from Saul Kripke (cf. Øhrstrøm and Hasle 1995, pp. 174 ff.), that we should conceive of time as of a tree-like, branching structure. Branching proceeds into the future and never into the past; the single "trunk" of any given moment represents its unique past and each "branch" represents a possible future continuation of this moment. Our world is indeterministic if there is more than one branch on the tree of possibilities. Prior's image grasps a natural intuition that the possibilities available at different moments change. I

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could have run a marathon this year but it is no longer possible; there is too little time left for me to prepare.

The idea proposed by Prior was ably developed and clarified by Thomason (1970, 1984) who defined the tree of possibilities as a full-blown model of a tempo-modal propositional logic. For historical reasons, this model is called Ockhamist (cf. Definition 2). The formalism has proved to be very insightful. It has been used on many occasions by philosophers, logicians and even computer scientists (for historical details, see Øhrstrøm and Hasle 1995; Braüner et al. 2000). However, the interpretation of the formalism has generated controversy from the very instant of its creation. If one unpacks the details of the model and evaluation of sentences, one realizes that *to ascribe a truth value to a sentence, one needs to specify the future branch with respect to which the truth value is ascribed*. It is especially puzzling when we deal with sentences in the future tense. A sentence like "The summer will be hot" receives no interpretation whatsoever unless we specify one of the possible continuations; then, the sentence is true in the continuations in which the summer is hot, and false in those in which it is not. The formal details are outlined in Sect. 2.1.

The intuitive problem with this theory consists in the fact that there is no trace of such specification of possible continuation in our everyday talk about the future which this theory intends to model. Robert M. Adams additionally argues that such analysis "deprives prediction of its normal point" (Adams 1974, p. 219; similar arguments can be found in Malpass and Wawer 2012). Probably even Prior himself envisaged the difficulties with branch relativization since he called the branch-dependent ascriptions of truth values "prima facie" (Prior 1967, p. 126). Additionally, he devised the so-called "Peircean" interpretation of sentences in the branching model which is branch-independent. Thomason (1970) carefully defined the Ockhamist semantics but he was far from endorsing it in full generality, he just used it as a tool for his branch-independent supervaluational semantics. Recently, MacFarlane (2003, 2008) applied a relativist semantics to explain our predictions with no need of specification of a future branch. Finally, a group of philosophers and logicians, beginning with McKim and Davis (1976), has tried to extend Ockhamism. Their guiding idea was to distinguish only one of the possible futures as relevant for establishing the truth values of future contingents—the future that is going to happen. The roots of this theory are traced back to the Middle Ages (cf. Øhrstrøm 1984 who claims that Anselm of Canterbury and William of Ockham might be thought of as founding fathers and able proponents of the idea). All the conceptions which appeal to the notion of a single future within the context of Branching-Time (*BT*) are dubbed Thin Red Line (*TRL*) theories by Belnap and Green (1994) and Belnap et al. (2001).

All the opponents of Ockhamism share the intuition that branch-relativized analysis of predictions is unpalatable. They disagree about the appropriate alternative. The Peirceans believe that every future contingent is simply false (Prior 1967). The supervaluationists claim that they lack the truth value (Thomason 1970). The relativists maintain that the interpretation of a future contingent is relative to the context of assessment; it is true as assessed from some contexts, false as assessed from others and lacking the truth value from still others (MacFarlane 2003). The *TRL* theory in turn posits that every future contingent is simply either true of false.

Additionally, the truth value of such predictions is relative to neither a branch nor a context of assessment; it depends only on what the future will bring. For sources of motivation of the *TRL* theory and arguments against the alternative proposals consult (McKim and Davis 1976; Øhrstrøm 1984, 2009; Øhrstrøm and Hasle 1995; Braüner et al. 1998, 2000; Borghini and Giuliano 2011; Malpass and Wawer 2012).

It is important to stress that a Thin Red Line theorist (from now on a *TRL*-ist) is usually also an indeterminist. Consequently, he does not believe that the truth of a prediction entails its necessary truth. He apparently disagrees at this point with Aristotle, Łukasiewicz, and Thomason; at the same time he is in agreement with William of Ockham, Leibniz, and Øhrstrøm. However, the *TRL*-ist is often accused of deterministic tendencies. He is claimed to privilege the uniqueness of the future (and the bivalence of future contingents) over indeterminism.

I argue that the main incentive for the debate stems from competing conceptions of our world and the possibilities inherent in this world. On the one hand, some critics of the *TRL* embrace the metaphysical conception according to which our world itself is an immense branching object that contains all the possibilities and all the mutually incompatible events happening in them. On the other hand, the *TRL* proponents take our world to be a much more modest object which *realizes* only one of the available possibilities. The cost of such metaphysical modesty is the abandonment of the clearly realistic notion of possibility that the former conception proposes. *TRL* critics argue that one cannot have both Real Possibilities and such limited conception of our world; *TRL* supporters claim that one can.

I propose a formalism general enough to express both points of view which does not privilege either. I use this tool to outline a new approach to the problem of the Thin Red Line, and argue that it is a sane philosophical position. However, my proposal is at most conditionally accurate; it presupposes some realistic account of (temporal) possibility which is not committed to concrete existence of the possibilities.

I am going to proceed as follows: first, I will present the history of the discussion of the idea of the *TRL* in the context of Branching-Time (*BT*) semantics. If one is not interested in or familiar with the history of this idea, one can skip Sect. 2 (with the exception of Sect. 2.1 which provides general semantic definitions); it should not affect the overall clarity of the further reading. In Sect. 3, I present my own *TRL* solution to the problem of future contingents preceded by the introduction of some necessary conceptual background. Finally, Sect. 4 is devoted to answering the logical, epistemological, objective, and "actuality" objections that were famously formulated against the *TRL* theory by Belnap et al. (2001). I am not going to criticize alternative theories of future contingents but instead focus on presentation and defense of my own position. The competing approaches will be discussed only as a background to my own proposal.

2 A Report on the War: The History of the Thin Red Line

In military terminology, the concept of the Thin Red Line refers to a certain defensive complex. It consists of small in number but strategically deployed and

usually well-equipped and trained military units. Due to their localization, they can marginalize the factor of being outnumbered and defend even a large territory against the prevailing forces of the enemy.

In the context of Branching-Time semantics, the Thin Red Line acquired a new meaning, but I will use its military connotation to explicate the philosophical ideas behind this concept. We shall see that the philosophical Thin Red Line was severely bombarded, especially in 1994 and 2001, by Nuel Belnap and his allies. However, the defenders never surrendered and often returned fire. I am going to present the dialectic of the development of the concept of the *TRL* in a series of such "attacks" and "defenses". I organize these military maneuvers in logical rather than chronological fashion, which means that I describe various versions of *TRL* semantics from the simplest to the more complicated as responses to increasingly challenging counter-arguments offered by its critics. This mode of presentation might not completely coincide with the actual historical succession of ideas or intentions of the authors, but I hope that it might be found revealing. I am going to focus on the problems of the *TRL* theory which are called "logical" by Belnap et al. (2001, p. 161) and might also be called "semantic" or "linguistic". The attacks usually take the form of examples of sentences which sound intuitive and which are valid in Ockhamism, but are invalidated by one or other of the *TRL* semantics. The defenses are attempts to restore the validity of these sentences while preserving the core of the *TRL* intuition.

Finally, I will propose my own strategy of defense of the Thin Red Line (Sect. 3) I will show that my way of reinforcing the defenders helps them not only to repulse the attacks, but even to mount counter-attacks in areas where the attackers' lines are overstretched (Sect. 4).

2.1 The Battle Field: Introduction to Ockhamism

To keep the exposition simple, I will consider models for a language containing sentential variables (*Var*), truth-functional connectives ($\land, \neg$) and three modal connectives: P ("It was the case that"), F ("It will be the case that") and $\Diamond$ ("It is possible that"). All the modal connectives have natural duals: $H: = \neg P \neg$ ("It has always been the case that"), $G: = \neg F \neg$ ("It is always going to be the case that") and $\Box: = \neg \Diamond \neg$ ("It is settled that"). The connectives $\lor, \rightarrow, \leftrightarrow$ are defined in a classical manner. A well formed sentence ϕ of the language is defined by standard induction.

Definition 1 (*BT-Model*). A *BT*-model is a triple $\mathfrak{M}: = \langle M, <, V \rangle$ where:

- $M \neq \emptyset$;
- $<$ is a partial order defined on M which satisfies the conditions of backward linearity:

  - $\forall m, m_1, m_2[(m_1 < m \land m_2 < m) \rightarrow (m_1 < m_2 \lor m_2 < m_1 \lor m_1 = m_2)]$, and historical connectedness:
  - $\forall m_1, m_2 \exists m_3 \ m_3 \leq m_1 \land m_3 \leq m_2$;

- *V* is a valuation function $V: Var \mapsto \wp(M)$.[1]

Set *M* contains *possible moments* (instantaneous but spatially maximally extensive stages of the world). Therefore, it is not time but the possibilities that branch in the misleadingly named Branching-Time model. The relation $<$ should be thought of as a relation of modal-temporal precedence *earlier-possibly later*.

Finally, we need to formally define an important notion of a *history*[2] which is a maximal, linearly ordered by $<$, subset of *W*. I denote a history by the letter *h*. The set of all histories in the model is denoted by *Hist*.

There are many ways to interpret the modal connectives within such defined *BT* models. Our point of reference will be Ockhamism which is a version most commonly used in philosophical logic.[3] In this semantics the truth clauses are defined for a model $\mathfrak{M}$ and a pair *m/h* (the notation indicates that $m \in h$).

Definition 2   (*A sentence ϕ is true in a BT-model $\mathfrak{M}$, at a point m/h*).

- $\mathfrak{M}, m/h \vDash p$ iff $m \in V(p)$ for $p \in Var$;
- $\mathfrak{M}, m/h \vDash \neg\phi$ iff it is not the case that $\mathfrak{M}, m/h \vDash \phi$ ($\mathfrak{M}, m/h \nvDash \phi$);
- $\mathfrak{M}, m/h \vDash \phi \wedge \psi$ iff $\mathfrak{M}, m/h \vDash \phi$ and $\mathfrak{M}, m/h \vDash \psi$;
- $\mathfrak{M}, m/h \vDash P\phi$ iff $\exists m'(m' < m \wedge \mathfrak{M}, m'/\mathfrak{h} \vDash \phi)$;
- $\mathfrak{M}, m/h \vDash F\phi$ iff $\exists m'(m < m' \wedge m' \in h \wedge \mathfrak{M}, m'/h \vDash \phi)$;
- $\mathfrak{M}, m/h \vDash \Diamond\phi$ iff $\exists h'(m \in h' \wedge \mathfrak{M}, m/h' \vDash \phi)$.

Notice that $\mathfrak{M}$ is not an intended Kripke model for our language since the "points" at which we evaluate sentences are *m/h* pairs rather than simply moments *m* from set *M*. This fact is reflected in the definition of modal connectives as well since it is not simply $<$ but rather history-related-$<$ that represents the accessibility

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[1] Some logicians (e.g. Zanardo 1996) assume an alternative, history-dependent notion of valuation ($V: Var \mapsto \wp(M \times Hist)$) but I think that it significantly reduces the philosophical attraction of *BT* theory. If we accept the history dependent valuations in full generality, we lose most of the connection between semantics and metaphysics—the underlying branching structure is not reflected in evaluation of sentences. For example, if sentential variables like "John is meeting Paul in the main square in Warsaw" are history-dependent, they might well be meeting at *m* in history h_1 and not meeting at *m* in history h_2 even though h_1 and h_2 are literally identical up to and long after *m*. As a result, the question, "Did John meet Paul in the main square in Warsaw yesterday?" has no straightforward answer. In some histories leading to the present they did; in some others they did not. It is the case even though these histories consist of literally the same events up to the present.

The motivation for history dependent valuations is, to a large extent, of a formal nature. Namely, the respected logical rule of substitution is not valid in models with history-independent valuations. The more philosophically oriented incentive is that some of the present tense sentences have at least a "trace of futurity" (Prior 1967, p. 124). The sentences like "I am having the last cigarette," "I am choosing to stay home," or "Barack Obama is beginning a 4-year term" might be quoted as examples. To accommodate such examples Prior (1967) proposed a syntactic distinction of two types of sentences and Zanardo (1996) discusses a solution based on the structure of the histories. I agree that such phenomena should be taken into account, but I still think that the history-independent valuation is a better choice as a first semantic approximation. To simplify, I am going to think of sentential variables as being, so to say, wholly about the present.

[2] The nomenclature in the field is not homogeneous. What are called histories by some are called routes, chronicles or branches by others.

[3] Some undermine the historical accuracy of the terminology. For a historical argument that the semantics does not reflect William of Ockham's original intentions see e.g. Øhrstrøm (1984, p. 217).

relation for temporal connectives. However, I am going to stick to the standard terminology and definitions especially since, as shown in (Zanardo 1996, p. 6), any Ockhamist *BT* model can be redefined such that it becomes a standard Kripke model.[4]

The consequences of our definitions are quite satisfying from the logical standpoint. The Ockhamist models validate axioms of logic $S5$ for the purely modal fragment of our language and of temporal logic of linear time for the purely temporal fragment of the language. However many philosophers and logicians, as mentioned earlier, have found these models deficient.

2.2 The Battle Begins

2.2.1 Building the First Trenches: TRL$_1$

Ockhamism proposes a history-dependent semantics which the *TRL*-ist finds unacceptable. He grasps the nettle and attempts to provide an alternative semantics for future contingents. Since he wants to distinguish one of the possible histories, the very first proposal that comes to mind is to redefine the notion of the model by adding the information about the existence of the *TRL*. Here is the first attempt:

Definition 3 (*TRL$_1$-model*). A *TRL$_1$*-model $\mathfrak{M}$ is a quadruple $\langle M, <, TRL_h, V \rangle$ in which M, $<$ and V are used as in Definition 1 and $TRL_h \in Hist$.

Now, having incorporated the additional parameter into the structure, we can attempt to get rid of the history parameter (the h on the left-hand side of symbol $\vDash$) altogether:

Definition 4 (*Sentence ϕ is true in TRL$_1$-model $\mathfrak{M}$ at m*).

- $\mathfrak{M}, m \vDash p$ iff $m \in V(p)$ where $p \in Var$;
- Natural definitions for truth-functional connectives;
- $\mathfrak{M}, m \vDash P\phi$ iff $\exists m'(m' < m \wedge \mathfrak{M}, m' \vDash \phi)$;
- $\mathfrak{M}, m \vDash F\phi$ iff $\exists m'(m < m' \wedge m' \in TRL_h \wedge \mathfrak{M}, m' \vDash \phi)$;
- $\mathfrak{M}, m \vDash \Diamond \phi$ iff $\exists m'(m < m' \wedge \mathfrak{M}, m' \vDash \phi)$;
- $\mathfrak{M}, m \vDash \Box \phi$ iff $\forall h(m \in h \rightarrow \exists m'(m' \in h \wedge m < m' \wedge \mathfrak{M}, m' \vDash \phi))$.

We need to define $\Diamond$ and $\Box$ separately since they are no longer duals. These definitions are mimicking the idea of the Peircean sense of operators f and F (cf. Prior 1967, Ch. 7; Barcellan and Zanardo 1999, p. 3; Belnap et al. 2001, p. 161).

2.2.2 The First Shots: Truth Values Outside the TRL

The crucial objection against the *TRL$_1$* is presented in two different manners. First, Belnap and Green (1994, p. 379) write:

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[4] Since our notion of valuation differs from that of Zanardo (1996), the redefinition would render the Ockhamist model a Kripke model based on a particular general frame.

> The coin will come up heads. It is possible, though, that it will come up tails, and then later (*) it will come up tails again (though at that moment it could come up heads), and then, inevitably, still later it will come up tails yet again. The trouble is that at (*) the example says that tails will happen, not merely that it might, whereas the explanation of the future tense given above presupposed that the moment of evaluation was in the *TRL*.

and in (Belnap et al. 2001, p. 162) the attack is re-formulated:

> We have no trouble with predictions that will be or have been made, but we have no way of understanding predictions that might have been made. We have no way of getting a grip on "Had things gone otherwise, Jack would have asserted the following: 'It will (eventually) rain.'" Given the context of Jack's assertion, the *TRL* is no longer able to guide us in understanding his reference to his future.

Formally, the problem comes down to the question of interpretation of sentences beginning with the connective F at a point $m \in M$ such that $m \notin TRL_h$. Belnap and Green (1994, p. 379) write that "Branching $+ TRL$ has the defect that it gives no account of the future tense relative to moments that do not lie in the TRL_h". In fact, that is not quite right since according to the TRL_1 all the future tensed sentences evaluated outside the *TRL* are simply false.[5] Anyway, it is still a very serious objection and it was recognized as a fatal one even by the most persistent defenders of the *TRL* (see e.g. Braüner et al. 1998). As a result, they decided to reconsider the notion of the *TRL* in a way that accounts for predictions at arbitrary evaluation points of a model.

2.2.3 Battle Lines Re-Drawn: TRL Functions

Having noticed the difficulty raised in the previous section, most of the *TRL*-ists decided to reconsider their position in a way that answers the above-mentioned problem. The generally accepted strategy was to draw a thin red line for every point of the model (cf. McKim and Davis 1976; Braüner et al. 1998, 2000; Barcellan and Zanardo 1999; Øhrstrøm 2009), but there were exceptions to this strategy (e.g. Malpass and Wawer 2012). The drawing was made with the use of the *TRL*-function (*TRL_{fcn}*) which maps moments in M into histories. The function, intuitively speaking, picks for each moment in a model its actual future. Obviously, not every function $f : M \mapsto Hist$ will do. Some constraints must be put on the function for it to represent the intended idea. First of all, since the function is about to pick the actual history for a moment, the moment had better be a part of this history. So, the minimal constraint is the following:

Condition 1 $\forall m \in M \, m \in TRL_{fcn}(m)$. (McKim and Davis 1976, p. 235).

Let us try to define a new notion of a model:

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[5] It is by no means the only problem with this semantics. For a short list of non-intuitive validities and very intuitive non-validities generated by it see (appendix to Malpass and Wawer 2012).

Definition 5 (*TRL₂-model*). A *TRL₂*-model $\mathfrak{M}$ is a quadruple $\langle M, <, TRL_{fcn}, V \rangle$ in which M, $<$ and V are used as in Definition 1 and TRL_{fcn} satisfies Condition 1.

Having added the *TRL*-function, we can redefine the truth clause for the future tense operator so it behaves properly over the whole domain. (The rest of the connectives are interpreted as in Definition 4.)

Definition 6 (*Fφ is true in TRL₂ model* $\mathfrak{M}$ *at m*).

- $\mathfrak{M}, m \vDash F\phi$ iff $\exists m'(m < m' \wedge m' \in TRL_{fcn}(m) \wedge \mathfrak{M}, m' \vDash \phi)$;

This cunning maneuver sets aside one set of worries but does not solve all the problems.

2.2.4 Gaps in the Lines: Fp → FFp Fails

It turns out that this general definition of TRL_{fcn} is not completely satisfactory. One of the most striking deficiencies is the fact that under TRL_2 semantics neither of these two very intuitive sentences: $FF\phi \rightarrow F\phi$ and $F\phi \rightarrow FF\phi$ is valid. In the usual temporal logic, these sentences define transitivity and density of accessibility relation respectively. However, under TRL_2 semantics they change their usual meaning and fail for different reasons. Nonetheless, most of us would agree that those are valid principles so a respectable semantics for temporal logic should convey this view.

To see that both of the validities fail, consider the following simple TRL_2 model $\mathfrak{M} := \langle M, <, TRL_{fcn}, V \rangle$ such that:

- $M = \{m_1, m_2, m_3, m_4\}$
- $\{m_1, m_2, m_3\} = h_1$, $\{m_1, m_2, m_4\} = h_2$
- $m_1 < m_2 \wedge m_2 < m_3 \wedge m_2 < m_4 \wedge m_4 \nleq m_3 \wedge m_3 \nleq m_4$
- $TRL_{fcn}(m_1) = h_1$, $TRL_{fcn}(m_2) = h_2$
- $m_1, m_2, m_3 \notin V(p)$, $m_4 \in V(p)$

We can observe that in this model: $\mathfrak{M}, m_1 \vDash FFp$ since $\mathfrak{M}, m_2 \vDash Fp$ but $\mathfrak{M}, m_1 \not\vDash Fp$. Therefore $FFp \rightarrow Fp$ is not valid. Similarly $\mathfrak{M}, m_1 \vDash F\neg p$ and $\mathfrak{M}, m_1 \not\vDash FF\neg p$ so the converse implication is not valid either.

This consequence is due to the fact that the moments in the tree might not "accord" with respect to their *TRLs*. As visible in the foregoing example, the source of trouble is the fact that the *TRLs* of m_1 and m_2 are different even though m_2 is in the *TRL* of m_1.

2.2.5 Overzealous Defense: TRL_fcn Excludes Branching

One of the ways to cure the aforementioned flaw is to impose an additional constraint on TRL_{fcn} in order to avoid "disagreement" between moments in the tree. Belnap and Green (1994) suggested the following move:

Condition 2 $\forall m_1, m_2 \, m_1 \leq m_2 \rightarrow TRL_{fcn}(m_1) = TRL_{fcn}(m_2)$. (Belnap and Green 1994, p. 380)

Definition 7 (*TRL₃-model*). *A TRL₃-model* $\mathfrak{M}$ is a quadruple $\langle M, <, TRL_{fcn}, V \rangle$ in which M, $<$, and V are used as in Definition 1 and TRL_{fcn} satisfies Conditions 1 and 2.

The problem from the previous section disappears; both $FF\phi \rightarrow F\phi$ and $F\phi \rightarrow FF\phi$ are valid (in any densely ordered frame). Nevertheless, the price is very high. As a result of Condition 2, we exclude any branching TRL_3-models. A very easy proof is sufficient to establish it:

1. Assume that there is a branching TRL_3-model, that is, there are $m_0, m_1, m_2 \in M$ such that:

    (a) $m_1 \not\leq m_2$ and $m_2 \not\leq m_1$, and $m_1 \neq m_2$;
    (b) $m_0 \leq m_1$ and $m_0 \leq m_2$;

2. since $m_0 \leq m_1$ and $m_0 \leq m_2$, then by Condition 2, $TRL_{fcn}(m_0) = TRL_{fcn}(m_1) = TRL_{fcn}(m_2)$. Let $TRL_{fcn}(m_0)$ be a history h;
3. by Condition 1, $m_0, m_1, m_2 \in h$;
4. and by definition of a history as a linearly ordered set we have that $m_1 < m_2$ or $m_2 < m_1$ or $m_1 = m_2$ which contradicts 1(a).

A very similar proof was used by Belnap and Green (1994, p. 380) to establish the fundamental discrepancy between the idea of a single future and the branching representation of ontic indeterminism. However, Belnap and Green's response to the problems raised in the previous sections is too hasty and one should (and the *TRL*-ists did) take more moderate steps in the campaign.

2.2.6 Cautious Defense: TRL_{fcn} Allows Branching

The philosophers and logicians arguing in favor of the *TRL* acknowledged that the problems described in Sect. 2.2.4 are serious ones and should be dealt with by positing some constraints on TRL_{fcn}. Nevertheless, the Condition 2 proposed by Belnap and Green (1994) is evidently too strong. Is there a middle way that would cure the aforementioned problems and preserve an indeterministic, tree-like picture? Yes there is. In fact, it was first noticed long before Belnap and Green raised their criticism. Already in 1976, McKim and Davis proposed a weaker cousin of Condition 2 which did not force the deterministic conclusion[6].

Condition 3 $\forall m_1, m_2 \in M \big((m_1 < m_2 \wedge m_2 \in TRL_{fcn}(m_1)) \rightarrow TRL_{fcn}(m_1) = TRL_{fcn}(m_2)\big)$. (McKim and Davis 1976, p. 235).

The condition is thought as follows: if a history h is picked as the future of a given moment m_1, then every $m_2 > m_1$ which is in h must confirm m_1's "choice". Nonetheless, the moments above m_1 that are not in h are free to choose otherwise (unless their antecedents above m_1 enforce some choice upon them). Having introduced the new condition we can slightly reconstruct our *TRL*-model.

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[6] A very similar condition was proposed in a different context by Thomason and Gupta (1980).

Definition 8 (TRL_4-model). A TRL_4-model $\mathfrak{M}$ is a quadruple $\langle M, <, TRL_{fcn}, V \rangle$ in which M, $<$ and V are as in Definition 1 and TRL_{fcn} satisfies Condition 1 and Condition 3.

Using Condition 3 as a compass we managed to sail between the Charybdis of determinism and the Scylla of the previously mentioned logical difficulties. Indeed, the function can be defined with no inconsistency on branching models, the sentence $FF\phi \rightarrow F\phi$ is valid and $F\phi \rightarrow FF\phi$ is valid in every densely ordered model.

Some authors (e.g. Barcellan and Zanardo 1999; Braüner et al. 1998) considered one additional and quite natural condition on TRL_{fcn}, namely:

Condition 4 $\exists m \in M \, \forall m' \big(m' < m \rightarrow TRL_{fcn}(m') = TRL(m) \big)$

Condition 4, together with Conditions 1 and 3, guarantee that one (and only one) of the histories in the tree is special in a sense that it is picked as the actual one for every moment in it. It means that all the moments in this history are in accord with respect to their choice of actual futures: $\forall m \in h^* \, TRL_{fcn}(m) = h^*$. The history h^* is called "real" by Barcellan and Zanardo (1999) and "normal" by Braüner et al. (1998).

As we shall soon see, it is not the end of the battle. Belnap et al. (2001) have found their way across the TRL_4 trenches and attacked again.

2.2.7 Another Storm: $\phi \rightarrow HF\phi$ Fails

Belnap et al. (2001) pointed out that TRL_4 is not a foolproof tactic either. The most important disadvantage is that it fails to validate $\phi \rightarrow HF\phi$. This sentence is considered fundamental by most of the temporal logicians and is usually included into the axiomatization of the minimal temporal logic. This sentence (and its counterpart: $\phi \rightarrow GP\phi$) guarantees a certain minimal symmetry between past and future; that is, if a moment m_1 is in the past of a moment m_2, then m_2 is in the future of m_1. This very feature fails under the TRL_4 semantics. To see this, let us examine the following simple example of a model:

- $M = \{m_1, m_2, m_3\}$
- $\{m_1, m_2\} = h_1, \{m_1, m_3\} = h_2$
- $m_1 < m_2 \wedge m_1 < m_3 \wedge m_2 \not\leq m_3 \wedge m_3 \not\leq m_2$
- $TRL_{fcn}(m_1) = h_1, TRL_{fcn}(m_2) = h_1, TRL_{fcn}(m_3) = h_2$
- $m_1, m_2 \notin V(p), m_3 \in V(p)$

Now, we can easily notice that $\mathfrak{M}, m_3 \not\models p \rightarrow HFp$. Evidently $\mathfrak{M}, m_3 \models p$ but $\mathfrak{M}, m_3 \not\models HFp$ because $\exists m \, m < m_3$ such that $m \not\models Fp$, namely m_1. The bizarre nature of this consequence is well illustrated by Belnap et al.'s (2001, p. 166) natural language example, slightly adjusted to the notation of our case. Let m_1 happen at 1:00 P.M., m_2 and m_3 both happen at 2:00 P.M., and let p mean "The coin lands tails":

> Now picture Jack at the moment of use, m_3, where the coin landed tails at 2:00 P.M. It would seem that in order to speak truly at m_3, Jack would be obliged to

say "The coin has landed tails, but this is not what was going to happen at 1:00 P.M. At 1:00 P.M. the coin was going to land heads. It's just that it didn't."

Another troublesome example found out by Belnap et al. (2001) is that of a sentence $F\phi \rightarrow \Box FPF\phi$ which is valid in TRL_4 and translated by Belnap et al. into "That something will happen does indeed imply that it is inevitable that it will be true that it was going to happen" (p. 167). Which is dangerously close to the deterministic $F\phi \rightarrow \Box F\phi$ saying that whatever will happen, will happen out of necessity.

A group of *TRL* defenders, conscious of these problems, has proposed manifold strategies meant to repel this storm.

2.2.8 How to Fight?

Don't budge an inch The first of the tactics is to stick to the TRL_4 solution and somehow explain away the counter-intuitive consequences. For example Barcellan and Zanardo (1999) appeal to the research in Artificial Intelligence and, in particular, in Partial Information Reasoning. They argue that TRL_{fcn} can be viewed as a representation of a course of events which "best fits suitable criteria like *minimal change principles, probability, typicality* and others." (p. 7). Observe that the "real" history h^* is the one that fits the criteria perfectly—it is the history which develops in the best possible agreement with the criteria.[7] However, TRL_{fcn} also tells us which development of the course of events is the one best fitting the criteria in any particular situation, even outside h^*. Interestingly, in models which TRL_{fcn} satisfies Condition 4, the sentence $\phi \rightarrow HF\phi$ might serve as a test of "proper development" of a course of events since we have that $m \in h^*$ iff $\mathfrak{M}, m \vDash \phi \rightarrow HF\phi$ for an arbitrary valuation function V. So $\phi \rightarrow HF\phi$ is valid at m for any $m \in h^*$.

Another rationalization of the failure of $\phi \rightarrow HF\phi$ is proposed by Braüner et al. (2000) where they appeal to the notions of *imagination* describing scenarios other than perfect. Their argument goes as follows:

> The counter-factual assumption of q does not invalidate the truth of the past prediction $PF\neg q$. If I am awake now, it certainly was true yesterday that I was going to be awake after one day. The prediction was true (but of course not necessary) even if I now—while being awake—imagine that I were asleep. For this reason one might say, that the truth of $q \wedge PF\neg q$, where q stands for 'I am asleep,' is in fact conceivable. (Braüner et al. 2000, p. 203)

However, it is not their last word since they quickly add that "(...) this piece of argumentation is somewhat strained" (p. 203) and continue to present another proposal.

Counter-attack: Counter-factual Thin Red Lines Braüner et al. (2000) were dissatisfied with TRL_4 for a couple of reasons. First of all, it did not validate $\phi \rightarrow HF\phi$ and second, it did not provide the straightforward interpretation of modal

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[7] Another set of criteria might be Leibnizian simplicity of laws, richness in phenomena, happiness of minds, maximality of perspectives and then h^* simply becomes the best of possible worlds which God chose to actualize.

operators (presumably the authors would not be content with the account of $\square$ and $\lozenge$ proposed in Definition 4). Therefore, they decided to devise a new *TRL* semantics which would deal with these problems.

To this effect, they utilized the concept of TRL_{fcn} described so far and defined the set of counterfactual *TRLs* of moment m as follows:

Definition 9   Let $TRL_{fcn}: M \mapsto Hist$ be a function satisfying Conditions 1, 3 and 4. The set $C(m)$ of counterfactual branches of a moment m is:

$$C(m) := \{h \in Hist | m \in h \wedge \forall m' > m (m' \in h \rightarrow TRL_{fcn}(m') = h)\}$$

Due to Condition 3, $TRL_{fcn}(m) \in C(m)$ but the set might be larger. If there are many elementary possibilities open at m, then $C(m)$ might contain a single representative from each elementary possibility.[8] Braüner et al. (2000) call these histories "immediate possibilities" of m.

With the notion of counterfactual *TRLs* at our disposal, we can define new truth clauses for the connectives in the TRL_4-model.

Definition 10   (*Sentence ϕ is true in TRL$_4$ model $\mathfrak{M}$ at m/h*). Let $\mathfrak{M}$ be a TRL_4 model. Assume that for every pair m/h, $m \in h \in C(m)$, then:

- $\mathfrak{M}, m/h \vDash p$ iff $m \in V(p)$ where $p \in Var$;
- Standard definitions for truth-functional connectives;
- $\mathfrak{M}, m/h \vDash P\phi$ iff $\exists m'(m' < m \wedge \mathfrak{M}, m'/h \vDash \phi)$;
- $\mathfrak{M}, m/h \vDash F\phi$ iff $\exists m'(m < m' \wedge m' \in h \wedge \mathfrak{M}, m'/h \vDash \phi)$;
- $\mathfrak{M}, m/h \vDash \lozenge\phi$ iff $\exists h'(h' \in C(m) \wedge \mathfrak{M}, m/h' \vDash \phi)$.

Operators G, H, and $\square$ are duals of F, P, $\lozenge$ respectively.

It is easy to see that, given these new clauses, the sentence $\phi \rightarrow HF\phi$ is valid and $F\phi \rightarrow \square FPF\phi$ is not. Additionally, we have a clear interpretation of modal operator $\lozenge$ analogous to its Ockhamist interpretation (see Definition 2). Hence Braüner et al. (2000) achieved the aims they stated but this solution is not without objections.

The first of these is formulated by Braüner et al. themselves in an observation that this new semantics invalidates a sentence $F\lozenge\phi \rightarrow \lozenge F\phi$. To see that it is problematic consider the following two sentences: (1) "Tomorrow, I might have dinner out" and (2) "Necessarily, tomorrow I will stay home" According to our new semantics, (1) and (2) might well be true at the same moment m. To understand this phenomenon, one needs to keep in mind that the notion of possibility under discussion is that of "immediate possibility". So even though going out tomorrow is not among my immediate possibilities at this moment, tomorrow might bring new immediate possibilities, some of them containing my dining out. What is now

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[8] The set of elementary possibilities open at moment m is a partition of the set of histories passing through m, induced by the equivalence relation of 'being undivided at m' ($\approx_m$). $h_1 \approx_m h_2$ iff $m \in h_1 \wedge m \in h_2 \wedge \exists m' > m(m' \in h_1 \wedge m' \in h_2)$ (See Belnap 2003, p. 19, ff.). For in-depth studies of this and related notions cf. (Zanardo 1998).

impossible might become possible in the future. For a more detailed exposition cf. (Braüner et al. 1998, 2000).

Nonetheless, I believe this approach faces a more important, conceptual difficulty. It seems to betray the fundamental principles motivating the introduction of the concept of the *TRL*. First of all, TRL_{fcn} plays no exceptional role in interpreting connective F. Notice that $TRL_{fcn}(m)$ is as good as any other $h \in C(m)$ while interpreting $F\phi$ at m. The second, and even more important problem has to do with re-introduction of a history parameter into our point of evaluation. The sentences do not have history-independent truth values! Remember that securing such truth values was one of the main motivations for constructing the *TRL* semantics in the first place. The only difference between "pure" Ockhamism and this version of the *TRL* is that we introduce a slightly modified notion of possibility. I think that this was not what all the fuss with the *TRL* was about. Consequently, I do not consider this attempt by Braüner et al. to be a promising line of defense of the notion of the *TRL*.[9]

2.3 Lessons of History

As we have seen, the history of the conflict tends to take on a more and more technically oriented dimension. This process has begun when the weapon of the *TRL*-function was first introduced. It was designed to defend against the very first challenge, which was to provide the semantic treatment of sentences evaluated outside the *TRL*. The supporters of the *TRL* accepted this criticism in full generality. At this point, they agreed to leave the safely guarded Thin Red Line. Nonetheless, their situation in the open field is much more difficult. As we have seen, they need to provide more and more elaborate notions of the *TRL* to counter the formal objections that arise. At the same time the attackers can safely rest on the intuitive logical features of Ockhamism, and wait for the opponent to wave the white flag.

I believe that an unconditional acceptance of the initial argument was a mistake. It was a trap which broke the solid defenders' ranks. My piece of advice to a *TRL*-ist is to be patient and to dig his heels into the initially drawn, firm red line of defense. The details of this advice are outlined in the following sections.

3 The Thin Red Line Reconsidered

My conception of the Thin Red Line is substantially different to those discussed above. Most notably, I dispose of TRL_{fcn} altogether and I use the distinction between contexts and possible moments to express the new solution. Before I clarify the distinction and present my proposal, let me recall the fundamental desiderata which every *TRL* theory aims to satisfy. First, it intends to provide a semantic treatment of

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[9] In Braüner et al. 1998 had proposed yet another version of the *TRL* semantics. It is technically more sophisticated than the one presented in this section. In particular, they evaluate sentences at pairs m/TRL_{fcn} rather than m/h. However, in the evaluation of sentences, TRL_{fcn} plays the exact formal role that the history parameter does in the just described semantics. Consequently, the aforementioned objections apply *mutatis mutandis* to this earlier theory of Braüner et al.

future contingents such that every prediction is ascribed one of the two truth values (while by a prediction, I mean a use of a future-tensed sentence). Second, the theory does not want to appeal to the history parameter or relativize to the context of assessment while ascribing the truth value. Thirdly, it aims to avoid the deterministic conclusion that every truth about the future is a necessary truth. Finally, it needs to answer the entire list of objections raised by Belnap et al. (2001). I believe that my solution pays due attention to all the above-mentioned tasks.

I intend to follow MacFarlane's (2003) terminology and distinguish *semantics proper* from *postsemantics*. I introduce an additional notion of *presemantics* and then define and defend the theory of the *TRL* as a certain combination of presemantic, semantic, and postsemantic assumptions.

3.1 Truth at a Point of Evaluation and Truth at a Context

Semantics proper is primarily concerned with the truth value of a sentence at a given point of evaluation. The semantics proper for branching time that I am going to accept is Ockhamism, outlined in Definition 2, in which the appropriate point of evaluation of a sentence is a triple $\langle \mathfrak{M}, m/h \rangle$ (where $\mathfrak{M} = \langle M, <, V \rangle$ and $m \in h$). I chose this semantics proper mainly because it generates the acceptable set of validities and it neatly combines the ordinary temporal logic of linear time with the notion of historical possibility.[10] Another advantage of this semantics is that it can be easily fused with a variety of different postsemantics. Hereafter, if I speak simply about semantics, I mean semantics proper.

However, the semantics proper operates on a quite high level of abstraction; it deals with truth values of sentences at points of evaluation. For a more down-to-earth application of a semantic theory, we need to specify how to relate the *particular use of a sentence at a context* to the semantic interpretation of this very sentence.[11] The semantics answers whether *a sentence is true at a given point of evaluation* but it does not tackle the issue if *a sentence is true while used at a given context*. The distinction of two notions of truth which I use here is MacFarlane's (2008, p. 83). Belnap et al. (2001) stick to a single notion of truth, but they distinguish between embedded and stand-alone sentences for similar purposes.

In my opinion, the problem of future contingents is about truth values of contingent predictions—future tensed assertive *speech acts*. Consequently, it is the truth value of a sentence used at a context rather than the truth value of a sentence at a point of evaluation which is the primary object of controversy. The notion of truth at a point of evaluation is, in a sense, an auxiliary concept which helps us to

[10] However, the logical properties of the Ockhamist structures are much more complicated than one might initially expect. For example, the task of modal axiomatization of this class of structures has proved to be a very difficult one. For the latest attempt, see (Reynolds 2003).

Some logicians, like Nishimura (1979) or Øhrstrøm and Hasle (1995) believe that Ockhamism validates some counter-intuitive sentences. For this reason it should be replaced by another semantics. Other authorities (Thomason 1984; Belnap et al. 2001) think otherwise, but this is a whole new issue only remotely connected with the problem of future contingents.

[11] A use of a sentence might be thought of as an idealization of a speech act. I follow Belnap et al.'s terminology and write about *uses of sentences*. For similar purposes, Kaplan (1989b) coined a technical term, "occurrence," while MacFarlane (2003) exploits a term "utterance."

understand our use of a language (a similar view was defended by MacFarlane 2003, p. 329). It is the reason why I think that there is no need to change the Ockhamist semantics proper as long as it generates intuitive validities and accurately models the temporal notion of possibility. The proper solution to the problem of future contingents should be focused mostly on the accurate analysis of the use of a prediction at a given context and this is what I am going to investigate.

Since the notion of context is so crucial, let us spare a few words explaining its role. Kaplan (1989b) observes that the context of use of a sentence is of cardinal importance in the proper analysis of our linguistic practice for at least two reasons. First, it is essential in a proper understanding of indexical expressions like "I", "here", "now", etc. Kaplan (1989b) spends most of his effort unfolding the details of semantic treatment of indexical expressions, but he believes that the context has a "more fundamental role, a role that would be required even if the language contained no indexicals," (1989a, p. 595) which he describes as "designating" and which is then stressed and explained as "initializing" in (Belnap et al. 2001, p. 148). Consider, for example, the sentence "It is raining." Some philosophers (e.g. Twardowski 1900) have considered such sentences incomplete unless the place and date (maybe possible world as well) are explicitly specified. Others (see Prior 1967) took such sentences to be perfectly complete but evaluated in various circumstances, on different occasions of use. The appropriate circumstances of evaluation are "provided" (or designated, or initialized) by a context of use of a sentence. Since different contexts provide different circumstances, two *uses* of the same sentence might have different truth values (even though the truth value of a sentence in given circumstances do not change in time). The two notions of truth might be systematically related as follows: *a use of a sentence at a context is true iff the sentence is true in the circumstances provided by the context of use*.

Let me introduce some notation: I write $\mathfrak{M}, c \Vdash \phi$ to indicate that a use of sentence ϕ is true at context c, in a model $\mathfrak{M}$. Sometimes, I express the same idea with a phrase: sentence ϕ is true as used at context c. The notion of truth at a context ($\Vdash$) should be carefully distinguished from the notion of truth at a point of evaluation ($\vDash$) which is reserved for semantics proper. As noted, one of the major roles of the context is to initialize the process of evaluation—for a given use of a sentence, the context determines the moment of the model at which the process of evaluation of the sentence begins. Following Belnap et al. I call this role "initialization" and model it by a function I which is a map from the set of all contexts C to the set of moments of the model M.

However, to know the moment at which to start the evaluation process might not be sufficient to assess a use of a sentence. We need to relate the notion of truth at a context to the semantic notion of truth at a point of evaluation more specifically; i.e. one needs to know not only *when*, but also *how* to assess the sentences used. We shall see that no answer to any of these questions is completely uncontroversial. In the conceptual framework which I propose, I deal with these two issues separately. The first is in the domain of what I call presemantics and the second is studied by postsemantics. The more detailed proposals are developed in the next two sections.

3.2 Presemantics and the *TRL*

The novelty of my proposal consists in the introduction of an element which I call presemantics. To understand what presemantics is concerned with, we should first comprehend what the context of a use of a sentence is. I am going to think of the context as a set of circumstances that accompany the use of the sentence, e.g. the place, the time, the speaker, and the addressee of the use of the sentence. All of these are in fact "pieces of reality" surrounding the event of a use of a sentence. I share this view with Kaplan (1989a) and Belnap et al. (2001):

> The contexts (...) are metaphysical, not cognitive. (Kaplan 1989a, p. 597).
> The agent, time, and place are all *drawn from the world*. (Kaplan 1989a, p. 591, emphasis mine).
> Assignments (...) have no *fact-of-the-matter* parameter as do the pure indexicals and true demonstratives. (Kaplan 1989a, p. 593, emphasis mine)
> You can reasonably decide to treat "now" and "here" as context-dependent, but that is because there is *in fact* a time of use and a place of use. (Belnap et al. 2001, p. 148)

Since the notion of the context has this "fact-of-the-matter" character, the accepted set of contexts C and the structure of this set is a partial indicator of metaphysical presuppositions. After all, a use of a sentence (an idealization of a speech act) is a concrete event in our world, and a context consists of concrete circumstances that accompany this event. Therefore, I assume that postulating and using certain contexts in the semantic theory is tantamount to the metaphysical commitment to their *concrete* existence. The purpose of *presemantics* is to explicate, at least partially, the implicit metaphysical assumptions which are smuggled into the discussion with the notion of the context. On the formal level, this explication is performed in two steps. First, we postulate a set of accessible contexts C ordered by temporal succession relation $\preceq$. Second, we decide which possible moments should be initialized by accessible contexts (we decide on the shape of function I).[12]

The prevailing presemantic assumptions accepted, often tacitly, by the theorists in the field are that

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[12] It is important to stress that not necessarily, at every $c \in C$, someone uses a sentence. The concept of an accessible context is more extensive. For example, no-one is situated next to me, so the place and time next to me is not actually a context of use of a sentence; however, they are a part of an *accessible* context. In general, I think of a set of conditions to be an accessible context of use if there is at least time and space (and probably some matter as well), so if we added a speaker or writer, the sentence would be used. Sometimes, for stylistic reasons, I drop the adjective "accessible". Kaplan (1989b) sometimes calls such circumstances *possible contexts* (p. 523) or *possible occasions of use* (p. 494) but usually refers to them simply as contexts. I decided to use the notion of accessibility to avoid confusion with historical possibility.

My arguments can be easily rephrased if one insists on thinking of a context of use as conditions in which a sentence is actually being used. To do so, one would need to replace each subsequent appearance of a sign '$I[C] = $' with '$I[C] \subseteq$' and '$C = $' with '$C \subseteq$'. However, I believe that this mode of presentation would affect the clarity of the arguments, so I decided to coin a technical notion of accessible context.

(a) I is the identity function, that is $\forall c \in C\, I(c) = c$,
(b) $I[C] = M$, where M is a set of possible moments[13].

This approach seems to be implicitly assumed by Thomason (1970) and MacFarlane (2003, 2008), and is almost explicitly stated by Belnap et al. (2001). I will call these assumptions "branching concretism" to stress Belnap et al.'s metaphysical view which gives rise to them. The authors think of possible moments as causally ordered *concrete* events which, *in toto*, constitute our world (cf. Belnap et al. 2001, pp. 139–140, 178–181, in particular: "a moment is an instantaneous concrete event"). The presemantic assumptions (a) and (b) naturally ensue from such metaphysical vision. If we accept that possible moments are concrete entities making up the world, it is natural to think of them as accessible contexts of use of sentences.

However, branching concretism does not seem to be the obligatory option. We can negate either of the branching concretist assumptions. Let us first focus on (a). We can abandon this formally handy identification by providing an alternative metaphysical account of possibility. There is a persistent line of thought which insists that possibilities do not consist of—contrary to Belnap et al.'s assumption—concrete entities. This idea was famously, though casually, advocated in (Kripke 1980). The author presents his views discussing the example of throwing a pair of dice:

> Hence there are thirty-six possible states of the pair of dice, as far as the numbers shown face-up are concerned, though only one of these states corresponds to the way the dice actually will come up. (. . .) Now the 'actual world' in this case is the *state* of the dice that is actually realized. *Another* entity, more 'concrete' than this state, is the Lesniewskian-Goodmanian physical entity which is the 'sum' of the two dice. This complex physical entity ('the dice,' thought of as a single object) is before me on the table, after the throw, and its actual position determines the actual state of the (two) dice. But when we talk in school of thirty-six possibilities, in no way do we need to posit that there are some thirty-five *other* entities, existent in some never-never land, corresponding to the physical object before me. (. . .) The thirty-six possibilities, the one that is actual included, are (abstract) *states* of the dice, not complex physical entities.(. . .)
> 'Possible worlds' are little more than the miniworlds of school probability blown large. (. . .) 'Possible worlds' are total 'ways the world might have been', or states or histories of the *entire* world. (. . .)
> The 'actual world'—better, the actual state, or history of the world—should not be confused with the enormous scattered object that surrounds us. The latter might also have been called 'the (actual) world' (. . .). Thus the possible but not actual worlds are not phantom duplicates of the 'world' in this other sense. (Kripke, 1980, p. 16–20).

The intuition behind this quote is that possibilities (possible worlds, moments, or histories) are not—in contrast to our world—concrete. This general insight was developed in plenitude of different directions in the discussion over the accurate

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[13] The symbol $I[C]$ denotes the the image of set C through the function I; that is $I[C] := \{x | \exists c \in C$ such that $I(c) = x\}$.

interpretation of the possible worlds discourse. Some of the proposals are quite evidently inadequate for our general purpose, which is to model metaphysical indeterminism. Clearly, identifying possible moments with works of fiction (à la Rosen 1990), or mental acts (a view discussed by Rescher 1975) would not establish them as appealing foundations for ontic indeterminism. Belnap et al. (2001, p. 179) argue that entities of fundamentally 'linguistic' character, such as maximal and consistent sets of sentences (e.g. Carnap 1947), would not give ontic indeterminism its due either.

Nonetheless, there are specimens of possible worlds theories which are more likely to encompass the realist intuition about possibilities while rejecting their concrete existence. They are sometimes classified as actualist(ic) modal realism (e.g. Chihara 1998; Divers 2002). In the course of the debate over the status of possible worlds, many such conceptions were developed. Probably the best known and most influential are those which construe possible worlds as: maximal possible states of affairs (e.g. Plantinga 1970, 1974, 1987), sets of basic particular situations (Cresswell 1972), ways things might have been (understood as properties or states in Stalnaker 1976), maximal consistent sets of language independent propositions (Adams 1974), world propositions (Fine 1977), or complex structural universals (Bigelow and Pargetter 1990). Belnap et al. (2001) decided to endorse the branching concretism position, but we need to be aware that it is not the unique conception at our disposal.

I do not want to defend nor develop any of the aforementioned, actualist attempts. My intention is to generalize the branching-time formalism to allow for the distinction, fundamental for the actualists, between concretely existing chunks of physical reality and possible moments which represent reality but need not be identified with it. Therefore, I will assume in what follows that appealing to possible moments, in contrast with accessible context, *does not* bring the commitment to their concrete existence.[14]

The parameter C encodes all the accessible contexts of use which are concrete parts of the world. Therefore, it can be naturally associated with Kripke's notion of 'the (actual) world'—"the 'concrete' Lesniewskian-Goodmanian phisical entity" or "the enormous scattered object that surrounds us." (This formal paraphrase of Kripke's idea is backed by some remarks of Kaplan, e.g. "[A]n actual-world is simply the circumstance of a context of use." 1989a, p. 596.) The set M stands for possible moments—the possible states of the world, which need not be identified with concrete events and contexts. Thanks to my formal apparatus we can disentangle these two notions, reject the concretist's assumption (a) and claim, in the actualist's manner, that $C \cap I[C] = \emptyset$. The formalism itself does not favor either branching concretism or branching actualism, but it enables one to express both of them. We shall see that various views on the relation of contexts and possible moments might help one to understand the debate on the status of the *TRL*.

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[14] It is important to stress that the abstract nature of possible moments does not detach them from the reality. Nor does it imply that the branching structure of possibilities is arbitrary or independent from the features of the real objects. I am inclined to believe that the exact form of the tree of possibilities is determined by the modal aspects of our concrete world and properties of objects existing in it. However, it is a further, complex issue which I will not discuss in this paper.

If we abandond the branching concretist assumption (a), it seems that it is no longer so controversial to reject (b). The claim (b) states that $I[C] = M$ which means that every possible moment is initialized by some accessible context of use. Our view on the nature of possible moments might crucially influence our opinion in this respect. In the branching concretist picture, every context initializes a single possible moment, namely itself. However, if we consider possible moments to be *representations* of concrete reality, we need to decide which possible moment should be initialized by a given context. The most natural claim is: the one which represents the context accurately. As a result, if our physical world is not tree-shaped but its possibilities are, then not every possible moment is initialized. Consequently, if we distinguish the metaphysical status of utterances and their contexts from the status of possible worlds/moments/histories, we might be willing to differentiate among possibilities. Some of them are initialized by uses of sentences and some are not.

Thanks to the concepts of initialization (I) and accessible contexts (C) we gain a subtle tool for expressing some of the metaphysical presuppositions. Different assumptions about the structure of the world have various consequences in the accepted presemantics. Let us now set the new concepts to work and sketch a few presemantic possibilities:

Branching concretism $I[C] = M = C$. As I said, it seems to be the view most commonly held among theorists in the field. According to such view, our world is a tree-like structure.

Presentism $card(C) = 1$, $I[C] = \{m\}$, where $m \in M$. There is only one moment at which sentences can be used, because *there is only one moment*—the present moment (represented by the single $c \in C$). It is an open question whether $c = m$ or not (see the discussion of the status of TRL_h below).

Growing Block Universe There is an isomorphism between C and $GB_{m_p} = \{m \in M | m \leq m_p,$ *for a chosen* $m_p \in M\}$ and $I[C] = GB_{m_p}$.[15] GB_{m_p} represents the part of the universe that has grown until the present, which is represented by m_p. One might argue that this representation does not grasp the "dynamic" nature of the growth, in which case it should be seen as a snapshot of the growth at the moment when m_p is on the top. Again, I remain neutral whether we should identify C with GB_{m_p}.

Thin Red Line There is an isomorphism between C and a single $h \in Hist$ and $I[C] = h$. Intuitively, C is the physical, non-branching world that we live in, and h is the Thin Red Line, the possible history which is actualized; I shall call it TRL_h.

The formalism is neutral with respect to the metaphysical status of TRL_h. In particular, we do not need to identify it with C. As I argued, if we assent to some actualist realist notion of possibility it is very natural to claim that $C \cap TRL_h = \emptyset$ (after all, if actualized possible moments are abstract states, they are not identical to concrete contexts). If TRL_h is an abstract representation of the world, it is metaphysically on a par with all other possibilities; it just happens to represent the world as it actually is. Nonetheless, we can decide to metaphysically differentiate TRL_h from other histories and assume that $C = TRL_h$. However, we need to be careful since such metaphysical distinction might suggest a camouflaged form of

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[15] More precisely, an isomorphism holds between $\langle GB_{m_p}, \leq \rangle$ and $\langle C, \preceq \rangle$.

ontic determinism. At the end of the day, it is the claim that among all the histories, only TRL_h concretely exists. One might try to argue, however, that even given that $C = TRL_h$, we can avoid a deterministic conclusion. To this end, we can appeal to the conception of possibility developed by Linsky and Zalta (1994, 1996). They are concentrated on possibilia rather than possible worlds or moments but we can apply their general idea. According to this view, all the possible moments (M) actually exist. However, only a part of them (TRL_h) *concretely* exist and the rest is actual but abstract. To diminish the deterministic concern, one can argue, following the authors, that TRL_h is only *contingently* concrete and other histories are *contingently* abstract. We are tempted to derive the deterministic conclusion out of the different metaphysical status only if we believe (unjustifiably, according to the authors) that being concrete is an essential feature of concrete objects and being abstract of the abstract ones. If we dispose of this alleged prejudice, we can safely identify our concrete world with TRL_h. Such a solution would accept the concretist's assumption (a) but reject (b). Apparently, it is harder to justify the acceptance of (b) combined with the rejection of (a).

The Thin Red Line position, as I see it, is a conjunction of a couple of metaphysical claims: (1) the physical world which we inhabit is not branching (the world is represented by C linearly ordered by $\preceq$);[16] (2) every moment in this world exists in the very same way (eternalism); (3) this world *might* be different than it actually is, so it is not deterministic (the possibilities of this world, encoded by M, branch). As a result, my *TRL*-ist has a quite different stance on the status of time and possibility. On the one hand, the world has concrete, temporal parts. On the other, it does not have analogous concrete, modal parts since only one of the possible histories is realized. Belnap et al. (2001, p. 179) conjecture that such a metaphysical view has its origins in both classical and relativistic physics. However, since it was held already by medieval theologians, it must have had another source of motivation—probably in the notion of atemporal, divine knowledge.

Presemantics is an important aspect of my *TRL* theory. It will be used to provide a bivalent treatment of future contingents. To this end, we need to endow model $\mathfrak{M}$ (Definition 1) with some bit of presemantic information. Let me just add the two elements discussed so far as elements of the model, so a model $\mathfrak{M}$ becomes a quintuple $\langle M, <, V, C, I \rangle$. To specify which presemantics the model $\mathfrak{M}$ is supplied with, I will use appropriate subscripts: $\mathfrak{M}_{BC}$ for branching concretist presemantics, $\mathfrak{M}_{PRES}$ for presentist, and $\mathfrak{M}_{TRL}$ for *TRL*-ist. The addition of these elements to the model does not affect the *semantic proper* interpretation of the connectives introduced in Definition 2, i.e. $\langle M, <, V, C, I \rangle, m/h \vDash \phi$ iff $\langle M, <, V \rangle, m/h \vDash \phi$.

3.3 Postsemantics and the *TRL*

Postsemantics builds on presemantics and semantics proper to give an account of how to apply the notions of context, initialization and point of evaluation to assess

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[16] We need to remember that the idea of time which we entertain here is a prerelativistic one. Consequently, we can freely speak about events in time being linearly ordered by temporal relations. However, to adopt to a more contemporary physics, we would need to say that $\langle C \preceq \rangle$ is isomorphic to a Minkowki spacetime (special relativity) or to a differentiable manifold (general relativity).

our utterances. In postsemantics, we can finally study the concept of truth of a use of sentence ϕ at context c, in model $\mathfrak{M}$. I will describe several proposals discussed in the literature and introduce my own *TRL* postsemantics in contrast. I use the superscripts over a sign $\Vdash$ to differentiate among definitions of truth-at-the-context.

For each of the Definitions 11–14, the moment of evaluation on the right-hand side of the equivalence is called a *context-initialized* moment.

Definition 11   Supervaluationism $\mathfrak{M}, c \Vdash^{SUP} \phi$ iff $\mathfrak{M}, I(c)/h \vDash \phi$ for every h such that $I(c) \in h$.

The supervaluationist takes a use of a sentence to be true if the sentence is true at each history passing through the context-initialized moment. Therefore, one can truly use a sentence ϕ if and only if one can truly use $\Box\phi$. Interestingly, the fact that a use of a sentence is not true does not imply that the use of its negation is true. For some sentences and contexts, neither a use of the sentence nor its negation is true. For exposition and a more detailed analysis of supervaluationism in the context of *BT*, see (Thomason 1970). For a, *TRL*-friendly version of supervaluationism see (Malpass and Wawer 2012).

Definition 12   Relativism: $\mathfrak{M}, c, c_a \Vdash^{REL} \phi$ iff $\mathfrak{M}, I(c)/h \vDash \phi$ for every history h such that $h \in H_{I(c)|I(c_a)}$. (The symbol $H_{m_1|m_2}$ denotes $H_{m_1} \cap H_{m_2}$ if $m_1 < m_2$ and H_{m_1} otherwise.)

It is a generalization of supervaluational semantic. It adds another aspect to the context—a moment of a context of assessment (c_a) of a sentence used at c. For a detailed exposition and motivation of this novel idea see (MacFarlane 2003, 2008).

Definition 13   Conservatism: $\mathfrak{M}, c/h \Vdash^{CON} \phi$ iff $\mathfrak{M}, I(c)/h \vDash \phi$.

This theory simply refuses to assess the truth value of a sentence used at context c unless the history of evaluation is specified. For example, Belnap et al. (2001) argue that a use of the sentence like "The summer will be hot" is analogous to a use of the sentence "x is white." In both cases, we cannot assess if they are true, as used at context c, unless we provide some additional information. In the latter case, we need to specify the assignment of a free variable and in the former, the history of evaluation (see Belnap et al. 2001, p. 155, semantic thesis 6–6). In general, to assess if a use of a sentence at a context is true, we need to have all the information necessary to evaluate the sentence at the context-initialized moment of evaluation. In particular, we need to know with respect to which model, which moment, which assignment of free variables and which history we should evaluate a sentence. The context provides a lot of the required information but, as Belnap et al. urge, it provides neither an assignment of free variables nor a history. However, if we supplement the information provided by the context with additional data, we can ascribe the truth value to a use of a sentence.[17] For an inventive, recent fusion of supervaluationism and conservatism see (Placek 2011).

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[17] Belnap et al.'s terminology differs from mine at this point. When I write about a use of a sentence being true at a context, the authors write about a stand-alone sentence being true at a context-initialized point of evaluation.

I call this position conservatism since it preserves as much semantics as possible, including the history parameter, while shifting to postsemantics. Most of the alternative views came out of the conviction that predictive speech acts, such as "The summer will be hot," should be assessed with no resort to the history parameter. To this effect, a supervaluationist simply takes into account all the histories passing through the context-initialized moment; a relativist considers all the histories containing the context of assessment; and a *TRL*-ist evaluates with respect to a single, "red" history.

Definition 14   Thin Red Line: $\mathfrak{M}_{TRL}, c \Vdash^{TRL} \phi$ iff $\mathfrak{M}_{TRL}, I(c)/TRL_h \vDash \phi$.

A use of a sentence is true at context c if and only if the sentence is true at a moment initialized by c, at the history which corresponds to the actual course of events.

This postsemantics, combined with Ockhamist semantics and *TRL*-presemantics, guarantees exactly what was intended. On the one hand, for every sentence ϕ, including a future contingent, and every context of use c, either the sentence or its negation is true as used at c. At this point, the *TRL* position differs from supervaluationism and relativism, which accept truth value gaps for some sentences, used at some contexts. On the other hand, contrary to Belnap et al.'s conservative position, every use of a future contingent can be assessed as true or false with no specification of a history parameter. The process of evaluation of the sentences begins, by default, at the history which corresponds to our world.

Interestingly, one of the recent remarks on the *TRL* theory made by Øhrstrøm and Hasle (2011) might be seen as a *TRL*-postsemantics for branching concretist presemantics. Their idea, translated to my notation would be that $\mathfrak{M}_{BC}, c \Vdash^{TRL} \phi$ iff $\mathfrak{M}_{BC}, I(c)/TRL_{fcn}(I(c)) \vDash \phi$. To use this postsemantics, we would need to enrich the notion of the model by the addition of TRL_{fcn} which, under this new reading, can be most naturally thought of as a function assigning a future to every context of use of a sentence—*the history of a context of use*. However, this idea is mentioned only in passing in (Øhrstrøm and Hasle 2011) so it is hard to find out the authors' exact intentions behind their definition.[18]

4 Answering Objections

4.1 Logical Objections

As we have seen in Sect. 2, there were numerous attempts to prove the *TRL* theory logically faulty. I will first confront my solution with the most natural worry and then I will show that it is to a large extent immune to this sort of trouble.

4.1.1 A Simple Case

One might claim that the *TRL* theory presented here is just a version of the TRL_1 discussed in Sect. 2.2.1, that is, a theory that singles out a unique history and closely

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[18] I would like to thank Peter Øhrstrøm for drawing my attention to this recent idea and making the manuscript of the Stanford Encyclopedia of Philosophy entry on "Future Contingents" available to me.

ties the interpretation of the "It will be the case" (F) connective with this very history. It is partly right, so let me reconsider the criticism of this particular version of the *TRL* theory presented in Sect. 2.2.2.

The objection naturally splits into two, the more abstract one and the more specific which asks for interpretation of particular sentences of natural language. The latter derives from the former which I will confront first. It is phrased as follows: "Branching + *TRL* has the defect that it gives no account of the future tense relative to moments that do not lie on *TRL*" (Belnap and Green 1994, p. 379).

In the framework, we are working with, the question is ambiguous since it contains the word "moments". We need to carefully distinguish moments of context from moments of evaluation. These two notions are co-extensive for branching concretist, but might diverge in other presemantics. In general, the strength of the argument crucially depends on which of the meanings we have in mind. If "moments" refers to moments of evaluation, it is a fair objection; any respectable semantics should provide some reasonable interpretation of a language at every point of evaluation; TRL_1 does not do it, hence it is not acceptable. Observe that my version of the *TRL* theory does not face this difficulty since it utilizes a decent Ockhamist semantics which provide a (history-dependent) treatment of every sentence at every point of evaluation. However, if the word "moments" in the quote refers to moments of context of speech acts, it is by far an objection that begs the question since it presupposes branching concretism (all possible moments are accessible contexts of use) and then demands a proper analysis of speech acts being made at all possible moments.[19] This is exactly where the *TRL*-ist should object by saying that speech acts are concrete events and they happen in our world only and our world is not a branching structure.

Now, let us turn to the examples of sentences which *TRL* theory is supposed to be unable to interpret:

> The coin will come up heads. It is possible, though, that it will come up tails, and then later (*) it will come up tails again (though at that moment it could come up heads), and then, inevitably, still later it will come up tails yet again. (Belnap and Green 1994).

The sentence might be naturally translated into the formal language to $Fp \wedge \Diamond F(q \wedge \Diamond Fp \wedge F(q \wedge \Box Fq))$ (where p means "The coin is landing heads" and q means "The coin is landing tails"). Now, we can apply our *TRL* theory to evaluate this particular prediction made at a given context. The *TRL* postsemantics proposed in Definition 14 (p. 23), the presemantic assumptions made at page 21, and semantic transformations described in Definition 2 (p. 5) guarantee a very non-controversial result. (I leave to the inquisitive reader the straightforward, but rather laborious computation.) I should stress that the procedure provides very natural truth conditions for sentences evaluated in or out of TRL_h. Sentences are used in the actual world only, but the content of these sentences might appeal to various possible circumstances and we have a natural way to ascribe truth values to sentences in these circumstances.

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[19] In (Belnap et al. 2001, p. 162) the authors specify that they mean moments of context of use indeed.

Let us now consider the second example:

Had things gone otherwise, Jack would have asserted the following 'It will (eventually) rain.' Given the context of Jack's assertion, the TRL is no longer able to guide us in understanding his reference to his future. (Belnap et al. 2001, p. 162)

This one is much more demanding. The authors use a counterfactual construction to move the point of evaluation away from the context at which the initial sentence was used. Then, they quote Jack's (possible) assertion made at the switched evaluation point and ask about the interpretation of this assertion at the switched point. To give an appropriate semantic treatment of such examples of direct speech, Belnap et al. (2001, p. 174) and Belnap (2002, p. 44) devised the operators which *shift the context of use* in the process of evaluation (the operators they propose are "α asserts 'A'" and "Truly utters(t_1, t_2, 'A')" respectively).[20] In particular, the context of use might be shifted to some unactualized circumstance. This idea is perfectly consistent with branching concretism, but it conflicts with the *TRL* theory since the latter states that there are no contexts besides those which initialize moments on TRL_h.

Especially interesting cases of direct speech, in context of our discussion, are *possible predictions*. For example, even though I have not just used the sentence "I will eat dinner soon," I really could, as the *TRL*-ist is happy to admit, have used the sentence just now (e.g. to inform my wife who is sitting next to me). So far, we understand when a use of a sentence is true at a context, but *what does it mean for a possible use to be true*? Formally speaking, we need to provide some analysis of the construction "α truly uses 'ϕ' " or similar. A part of the task of such analysis is to provide the truth conditions for conditionals like: "Had things gone otherwise, I would have truly used the sentence 'I will eat dinner soon.'"[21]

Belnap et al.'s objection suggests that a correct analysis is out of range of the *TRL*-ist due to the shortage of contexts. It seems to me, however, that the authors' context-shifting technique is not the only way to understand such phrases. An alternative idea is to construct a translation ϕ^t of a possibly-used sentence ϕ such that any sentence ψ^t, used at *TRL*-acceptable context c has the same truth conditions as ψ, where ψ^t is a sentence in which every appearance of "α truly uses 'ϕ' " is replaced by ϕ^t. One needs to be very careful in this process, especially coping with indexical expressions in ϕ. However, I believe this procedure to be available even for such indexically loaded, possibly used sentences as: "I will actually talk to you tomorrow." Consequently, the project which would give account of the truth values

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[20] Technically speaking, Kaplan (1989b, p. 511) would call such operators monsters, but the analysis gives desirable results and the authors stress that "in any case, better a monster than a fog" (Belnap et al. 2001, p. 174), which I find a decent maxim.

[21] Evidently, we would need to appeal to some treatment of conditionals in *BT*. However, it is an independent project, to a large extent external to the problem discussed here. Interestingly, even though Belnap et al. use a counterfactual construction phrasing their objection, they do not propose any semantics for such connectives. However, there are some initial studies of counterfactuals in the context of *BT* (e.g. Thomason and Gupta 1980; Øhrstrøm and Hasle 1995; Placek and Müller 2007) and they can be readily applied to our semantics.

of possible uses of sentences within the *TRL* setting is not, in principle, doomed to failure. I acknowledge that it is a very important open problem and it needs to be solved for the *TRL* theory, as presented here, to be adequate. However, I am not going to tackle it at this juncture.

4.1.2 The General Case

The aim of this section is to show that the *TRL* solution that I propose is in fact *semantically equivalent* to the conservative proposal of Belnap et al. (2001). It means that the set of truths generated by both these semantics are exactly the same. As a result, no logical objection of the sort presented in Sect. 2 applies to the *TRL* theory proposed by me. The theory is simply immune to the "logical attacks."

To proceed with the process of comparison (and differentiation) of the theories, I need to introduce a few general semantic definitions. So far, the only notions we were concerned with were the truth of a sentence at a point of evaluation and the truth of a use of a sentence at a context. Since we use two parallel notions of truth ($\vDash$ and $\Vdash$), the process of generalization is also twofold. We have already seen what it means for a sentence to be true at a point of evaluation $\mathfrak{M}, m/h$ in Definition 2. Let me introduce some generalizations of $\vDash$:

Definition 15 (*Truth in a model* $\mathfrak{M}$). Let $\mathfrak{M} = \langle M, <, V, C, I \rangle$ be a model and ϕ a sentence of our language. The sentence ϕ is true in the model $\mathfrak{M}$ ($\mathfrak{M} \vDash \phi$) iff $\mathfrak{M}, m/h \vDash \phi$ for arbitrary $m \in M$ and $h \in Hist$ such that $m \in h$.

Belnap et al. (2001, p. 236) calls this notion validity in model $\mathfrak{M}$.

Definition 16 (*Truth in a structure* $\mathfrak{F}$). Let $\mathfrak{F} = \langle M, < \rangle$ be a *BT* structure and ϕ a sentence. Then, we say that ϕ is true in $\mathfrak{F}$ ($\mathfrak{F} \vDash \phi$) iff for every model $\mathfrak{M}$ based on $\mathfrak{F}$, $\mathfrak{M} \vDash \phi$.

Belnap et al. (2001, p. 236) calls this notion $\mathfrak{F}$-validity.

Definition 17 (*Ockhamist truth*). A sentence ϕ is Ockhamist true iff $\mathfrak{F} \vDash \phi$ for arbitrary *BT* structure $\mathfrak{F}$.

Very closely related notions are called *LD*-validity by Kaplan (1989b, p. 547) and logical necessity by MacFarlane (2008, p.84).

We can now introduce the parallel generalizations associated with the concept of truth at a context. I already defined what it means for a use of sentence ϕ to be true at context c, in model $\mathfrak{M}$ (Definitions 11–14). Even though the predictions differ depending on the accepted postsemantics, we can wave these differences aside in the unified postsemantic definitions:

Definition 18 (*Valid in a model* $\mathfrak{M}$). Sentence ϕ is valid in model $\mathfrak{M} = \langle M, <, V, C, I \rangle$ iff $\mathfrak{M}, c \Vdash \phi$ for every $c \in C$.

Belnap et al. (2001, p. 237) call it in-context validity in $\mathfrak{M}$. Keep in mind though, that Belnap et al.'s postsemantics require them to add a history parameter to every moment of a context of use.

Definition 19 (*Valid in a BT-structure* $\mathfrak{F}$). We say that sentence ϕ is valid in structure $\mathfrak{F} = \langle M, < \rangle$ ($\mathfrak{F} \Vdash \phi$) iff $\mathfrak{M} \Vdash \phi$ for any $\mathfrak{M}$ based on $\mathfrak{F}$.

Belnap et al. (2001, p. 237) call it in-context validity in $\mathfrak{F}$. The definition can be naturally generalized to classes of frames and all *BT* frames (MacFarlane 2008, p. 84, describes it as the logical truth).

I follow in my notation Kaplan (1989b) and generalize validity from truth at a context. By the same token, I diverge from the common usage of a concept of validity as a generalization of a notion of truth at an evaluation point. Observe, however, that the distinction between truth at a context and truth at a point of evaluation is not, for the most part, introduced in formal modal logic.

With this apparatus at hand, we can begin a comparison of logics of different theories. The first, simple but important, observation is that C and I have no impact on our semantics proper so far, they reveal their effect only in postsemantics. Consequently, we can state the following fact:

Fact 1 *For any two models* $\mathfrak{M} = \langle M, <, V, C, I \rangle$ *and* $\mathfrak{N} = \langle M, <, V, C', I' \rangle$, *any sentence* ϕ, *any* $m \in M$, *and any* $h \in Hist$:

$$\mathfrak{M}, m/h \vDash \phi \quad \text{iff} \quad \mathfrak{N}, m/h \vDash \phi$$

Proof By straightforward induction on complexity of ϕ. $\square$

This observation basically says that no matter which presemantics and postsemantics you choose, you will not notice any difference in truth values of the sentences at points of evaluation. This fact easily generalizes to a higher level of logical truth.

Corollary 2 *For any two models* $\mathfrak{M} = \langle M, <, V, C, I \rangle$ *and* $\mathfrak{N} = \langle M, <, V, C', I' \rangle$ *and any sentence* ϕ:

$$\mathfrak{M} \vDash \phi \quad \text{iff} \quad \mathfrak{N} \vDash \phi$$

The analogous equivalence holds for frames and classes of frames. It is an important result in context of the debate of the *TRL* since it might be seen as a peace treaty putting an end to the logical war I reconstructed in Sect. 2 The branching concretist model differs from the *TRL* model (as well as from growing universe and presentist models) only with respect to what is an accessible context of use of a sentence. As a result, the logic of all these models is just the same or at least a fragment of logic described in the language we are using so far. Things change when we introduce context dependent expression to our language, especially the "actually" operator (cf. Sect. 4.4). However, the fact 1 will still hold for the indexical-free part of the language.

The perspective shifts dramatically if we switch from the truth of a sentence at a point of evaluation to the truth of a use of a sentence at a context since different models might differ in how to relate contexts to elements of the model:

- First, they may disagree with respect to which possible moments are initialized by contexts. Let us say that we have three different models based on the same "core" $\langle M, <, V \rangle$: (a) a branching concretist one $\mathfrak{M}_{BC}$, where $I[C] = M = C$, (b) a *TRL*-ist model $\mathfrak{M}_{TRL}$, where $I_{TRL}[C] = TRL_h$, (c) and a presentist $\mathfrak{M}_{PRES}$, where $I_{PRES}[C] = \{m\}$. Now, let us think about a use of a sentence ϕ in a branching concretist model $\mathfrak{M}_{BC}$ at a moment c such that $I_{PRES}(c) \neq m$ and $I_{TRL}(c) \notin TRL_h$. Let us assume that $\mathfrak{M}_{BC}, c/h \Vdash^{CON} \phi$—a use of sentence ϕ at context c is true at history h. The *TRL*-ist and presentist, if they do not want to accept branching concretist metaphysics can do nothing but object that it is not an accessible context. The presentist would say that there are no accessible contexts other than the present one and the *TRL*-ist that there are no others than those in our world (which initializes moments on TRL_h). Hence there is no way to judge the truth of ϕ as used at c.
- Second, due to the differences in postsemantics, different theories might assess the very same use of a sentence at the very same context differently. If the sentence ϕ at stake is a future contingent used at c in a model $\mathfrak{M}$, then:

  - supervaluationism will simply assess both ϕ and $\neg \phi$ not true ($\mathfrak{M}, c \nVdash^{SUP} \phi$ and $\mathfrak{M}, c \nVdash^{SUP} \neg \phi$);
  - according to relativism, it depends on a context of assessment, at some future context c_{a1}, it is true ($\mathfrak{M}, c, c_{a1} \Vdash^{REL} \phi$) and at some other c_{a2}, it is not ($\mathfrak{M}, c, c_{a2} \nVdash^{REL} \phi$), it is most certainly not true as assessed at $c_a = c$; ($\mathfrak{M}, c, c \nVdash^{REL} \phi$);
  - the conservative believes that there is no point asking whether a use of a future contingent at a context is true until we specify in respect of which history the truth is stated. And there are histories h_1 and h_2 such that $\mathfrak{M}, c/h_1 \Vdash^{CON} \phi$ and $\mathfrak{M}, c/h_2 \Vdash^{CON} \neg \phi$;
  - while the *TRL*-ist will simply claim that either a use of ϕ or its negation is true ($\mathfrak{M}, c \Vdash^{TRL} \phi$ or $\mathfrak{M}, c \Vdash^{TRL} \neg \phi$). Moreover, it is assessed with resort to neither the history nor the context of assessment. Note that it is exactly what *TRL* theory was meant to achieve!

The concept of validity in a model (Definition 18) needs a few words of comment. Thanks to the introduction of the set of accessible contexts C and the initialization function I, this notion becomes a very interesting one and it might be instructive to examine it. First of all, the whole range of sentences might be valid in a model $\mathfrak{M}$ even though they are not true in it. It is not an unknown phenomenon in the philosophy of indexicals. After all, "I am here now" is true whenever used (it is called valid by Kaplan and me), but it is by no means true in every model. After all, there are lots of places and moments, such that I am not there and then. This kind of sentence might be regarded as valid due to its *linguistic* properties.

Kaplan (1989a, p. 597) distinguishes another class of sentences that might be regarded as valid due to their *relation to the contexts of use*. The example he gives is "Something exists" which is valid even though there are no indexicals in the sentence. It is valid because, for something to be a context of use of a sentence, it needs to exist.

The introduction of parameters C and I brings another interesting aspect to the notion of validity. The sentence might be valid in a model if it *contingently* happens to be true in every accessible context of its use. The presentist case provides the most evident example. Let us consider a presentist model $\mathfrak{M}$ such that $C = \{c\}$. If c is in the year 2012AD, then it is true to say at c that it is 2012AD. Since for the presentist c is the only accessible context, the sentence "It is 2012AD" is simply valid (with no further specification) in this very model or, if it is clear which model we have in mind, we can simply say that the sentence "It is 2012AD" is valid, full stop. There is no need to specify the context or the moment of evaluation. It is valid *simpliciter*.

The divergence between different presemantic theories can be observed primarily on the level of truth at a context and validity in a model. However, it is enough to generalize the notion only one level further—to the level of validity in a structure (Definition 19)—and all the differences disappear. If a sentence is valid in a structure $\mathfrak{F}$, then it is truly used at every context, in every model $\mathfrak{M}$ based on $\mathfrak{F}$. So the factor of "contingent" content of accessible contexts which influences validity in a model is canceled out if we take all the models into account.

4.2 Objective Complaint

What in the structure of our world could determine a single possibility from among all the others to be "actual"? (Belnap et al. 2001, p. 162).

The question has at least two natural readings. If it is to be read as, "What in the structure of our world *makes it necessary* that TRL_h rather than some other possible history represents the world as it actually is?", then the answer is: "Nothing!". The world is indeterministic and it can develop along any of the possible ways. It simply develops along one of them which we call TRL_h.

However, if the question is to be read: "What in the structure of the world *makes it necessary* that only one of the histories represents the world as it actually is?", then the answer is, "The structure of the world itself." According to the *TRL* view, the physical, concrete universe simply does not branch in time. It resembles something like a single spacetime and it does not contain incompatible events (e.g. if a coin is tossed in our world, then there is the coin which shows heads in our world, or there is the coin which shows tails in our world, but there is no place in our world for both of these results to occur). The histories represent all the ways the world *might* develop, but the world develops in one way only. Therefore, one (and only one) of the histories *must* represent the world as it actually develops. There is nothing deterministic about this result. The physical world "determines" the TRL_h in the very same way in which the "complex physical entity ('the dice,' thought of a a single object) (. . .) and its actual position *determines* the actual state of the (two) dice." (Kripke 1980, p. 17, emphasis mine)

To see that it is no mystery, compare Belnap et al.'s objective complaint with another puzzle. Let there be a fair lottery in which only one of the tickets is drawn and the drawn ticket wins, then we might ask:

What in the structure of the lottery could determine a single ticket from among all the others to be "the winning one."

If it means to ask, "What makes it necessary that ticket *a* rather than some other ticket wins?", the answer is "Nothing!"; after all, the lottery is fair. If, on the other hand, the question is "What makes it necessary that only one of the tickets wins?", the answer is, "The structure of the lottery itself"—we draw the tickets just once so one (and only one) of the tickets *must* win. This fact does not make the lottery deterministic.

Is it just a misunderstanding then that leads us to think that *TRL* theory supports determinism? I think that the issue is deeper than that. Notice that in the answer to the objective complaint presented above, I appealed to the distinction between our world and possibilities of this world. It is legitimate since the *TRL* theory, as I present it, is based upon actualist modal realism. The accuracy of some such understanding is assumed by the *TRL* theory outlined here.[22]

For the branching concretist, however, our world is identical to the collection of concretely existing possibilities. Given this perspective, it is difficult to think of any feature of this world which would ontologically distinguish a single branch indeed. There are qualitative differences between histories; nonetheless, it is hard to see how any such difference might make one of the histories in our world "the actual one." It seems, from the concretist perspective, that the only way to existentially distinguish one of the histories is to make it the only history and, by the same token, to accept determinism. No wonder then that the concretist treats the concept of the Thin Red Line with such distrust.

The dispute between the branching concretist and the *TRL* theorist might be seen as another manifestation of an argument between genuine (or extreme) modal realism à la David Lewis and actualist modal realism à la Alvin Plantinga. As Divers (2002, p. 300, n. 5) notices, Lewis (1986, ch. 3) argues that only genuine realism is The Realism about possibility and accuses actualist realists of being merely 'ersatzers'. Analogously, branching concretists seem to believe that only their notion of possibility is good enough to capture the Real Indeterminism. However, some actualists (Plantinga 1987) argued quite the opposite. In their opinion, it is only the actualist realism that can provide the True Realist notion of possibility and David Lewis' concretist view is some sort of 'reductionism'. The *TRL*-ist can argue along the similar lines.

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[22] Interestingly Belnap et al. do not argue at length for their notion of possibility either. Their main reason is of a pragmatic nature: "We only urge that fashioning a rigorous theory of agency and indeterminism is worthwhile, and that in doing so it is greatly useful to construe possible events as both concrete and objective. This study presupposes, but does not argue for, this point of view." (Belnap et al. 2001, p. 179.)

Moreover, in their recent paper, Placek and Belnap (2011) depart from such a concrete depiction of our world. In their "physically motivated models," they define possible moments as equivalence classes of quadruples of reals with respect to a relation induced by an appropriate distribution of properties over indexed quadruples in $\mathbb{R}^4$. Clearly, such objects are not paradigm concrete entities.

4.3 Epistemological Complaint

> [H]ow we could know whether *we* are on TRL_{abs}. How could we find out? (Belnap et al. 2001, p. 163)

If one asks oneself anything, one does it at some context. However, if our world is in fact a huge, 4-dimensional, non-branching object, then every context $c \in C$ is mapped on TRL_h (which Belnap et al. call TRL_{abs}). Consequently, whenever we ask ourselves a question whether we are on TRL_h or not, the answer is affirmative. We do not even need to investigate. At the same time, it is contingent that TRL_h rather than some other history corresponds to our world. As a result, the sentence "I am on TRL_h" is like the sentence "I exist"; they are both contingent but true whenever used, so they are known, in a sense, *a priori* (cf. Kripke 1980, pp. 54ff.).

4.4 Actuality Complaint

> The *TRL* theory also has troubles with actuality. (...) [T]his world's being the actual world does not favor it over any others, but is just a reflection of the fact that this is the world at which we are conversing. To suppose that there is one from among the histories in *Our World* that is the absolutely actual history is rather like purporting to stand outside Lewis's realm of concrete possibilia and pointing to the one that is actual. But this is wrong in both cases. (Belnap et al. 2001, p. 163).

There are two worries that might be extracted from this quote. The first one is rather similar to the objective complaint. It expresses the view that TRL_h is no different from other histories and it cannot be distinguished on the basis of being actual. I refer to Sect. 4.2 for discussion of this argument. I think that this quote also supports my claim that there is some affinity between David Lewis' and Belnap et al.'s notions of possibility.

Another worry lurking in the quoted fragment originates in the observation that the actual world is the world at which we are conversing. The authors allude to the indexical theory of actuality. Adams (1974, p. 214) traces the origins of this idea back to Leibniz. In the 20th century it was discussed and rejected by Arthur Prior (see Lewis 1970, p. 185, n. 6), then articulated and ably defended by Lewis (1970) and later formally developed by Kaplan (1989b) and applied to Ockhamism by Belnap et al. (2001). The core of the idea is that words like 'actually' or 'actual' are structurally similar to indexical expressions like 'now', 'here', 'I', etc. The distinctive feature of these words and expressions containing them is that their reference is not fixed once and for all, but changes from one context of use to another. Just as 'here' refers to different places on different occasions of use, 'actual' refers to different possible circumstances depending on the context in which it is used. This linguistic idea is quite naturally combined with the philosophical picture presented by branching concretism (and Lewisian modal realism) since according to this position, the possible circumstances have the same metaphysical status as the actual ones. The only way to distinguish the latter is by using the phrase "The circumstances I am actually in." A use of the word "actually" indicates your

exact position on the tree of possibilities (or in the space of possible worlds). This account of actuality might seem to be at odds with the *TRL* ideology since the latter suggests that the actual world is metaphysically distinguished from any possible history (including *TRL_h* for that matter). However, I am going to show that my version of the *TRL* theory is compatible with the indexical notion of actuality. It shows that the indexical nature of the word 'actually' is partly independent of the accepted metaphysics of possibility. In fact, my approach generates much better and more intuitive predictions than any other account of indexical "actually" available for *BT*.

Proceeding formally, I adopt Kaplan's (1989b) treatment of indexicals appropriately modified by Belnap et al. (2001) to fit the branching framework. The lesson from Kaplan is that to deal with the semantics of indexicals we need to take into account the context of use of a sentence in which an indexical appears. More precisely, we need to somehow "store" the information at which context sentence ϕ was used, so we can utilize this information evaluating the sub-sentence ψ of ϕ which contains an indexical expression. So far, our point of evaluation has a form $\langle \mathfrak{M}, m/h \rangle$ where $\mathfrak{M} = \langle M, <, V, I, C \rangle$. The initialization function dictates where to start the process of evaluation of the sentence used at $\langle \mathfrak{M}, c \rangle$, namely, we start at $I(c)$, but this fact is not stored in our point of evaluation. Since the used sentence may contain modal and tense operators, the sub-sentences of it might well be evaluated at a moment different from the context-initialized one. However, as Kaplan makes clear, we need to keep the context of use fixed and utilize it when the embedded indexical connective is being interpreted. For this purpose, let me add another parameter to the point of evaluation, I will just call it a context parameter. As a result, the new point of evaluation has the following form: $\langle \mathfrak{M}, c, m/h \rangle$ where $c \in C$.

Let me now phrase my semantic definition of the connective "it is actually the case that" (@) and its competitors present in the arena. Notice that all the truth clauses below do refer to the piece of information stored in the context parameter. My proposal is a very simple and natural one:

1. **TRL definition of** *Actually*:

$$\mathfrak{M}_{TRL}, c, m/h \vDash @\phi \quad \text{iff} \quad \mathfrak{M}_{TRL}, c, I(c)/TRL_h \vDash \phi$$

The presemantic structure of $\mathfrak{M}$ guarantees that for each $c \in C$, $I(c) \in TRL_h$ so the operator @ is well-defined. Let me now present the alternative definitions available at the *BT* market. All of them were designed for the branching concretist model $\mathfrak{M}$:

2. **Conservative definition of** *Actually*$_1$: $\mathfrak{M}, c, m/h \vDash @_1\phi$ iff $\mathfrak{M}, c, I(c)/h' \vDash \phi$ for every h' such that $I(c) \in h'$. (Belnap et al. 2001, p. 246).
3. **Conservative definition of** *Actually*$_2$: $\mathfrak{M}, c, m/h \vDash @_2 \phi$ iff either (a) $I(c) \in h$ and $\mathfrak{M}, c, I(c)/h \vDash \phi$ or (b) $I(c) \notin h$ and $\mathfrak{M}, c, I(c)/h' \vDash \phi$ for every h' such that $I(c) \in h'$. Belnap et al. (2001), p. 246).
4. **Supervaluationist definition of** *Actually*: $\mathfrak{M}, c, m/h \vDash @\phi$ iff $\mathfrak{M}, c, I(c)/h' \vDash \phi$ for every h' such that $I(c) \in h'$. (MacFarlane 2008, p. 99).
5. **Relativist definition of** *Actually*: $\mathfrak{M}, c, c_a, m/h \vDash @\phi$ iff $\mathfrak{M}, c, c_a, I(c)/h' \vDash \phi$, for every $h' \in H_{I(c)|I(c_a)}$. (MacFarlane 2008, p. 99).

The litmus paper that I am going to use to test the definitions is the *initial-redundancy* requirement for the actuality operator proposed by MacFarlane (2008). The requirement, appropriately modified for our notation, should be understood as a demand that for any model $\mathfrak{M}$, any sentence ϕ, and any context c:

$$\mathfrak{M}, c \Vdash \phi \quad \text{iff} \quad \mathfrak{M}, c \Vdash @\phi$$

The conservative believes that the context c is not sufficient to judge the truth value of a use of a sentence. He additionally demands to specify the history h such that $I(c) \in h$. Hence the *initial redundancy* test for the conservative is slightly different:

$$\mathfrak{M}, c/h \Vdash \phi \quad \text{iff} \quad \mathfrak{M}, c/h \Vdash @\phi$$

The initial-redundancy seems to be a reasonable demand. If it is true to say that ϕ it should be equally true to say that $@\phi$ (and *vice versa*). The addition or removal of operator $@$ simply makes no difference as far as uses of sentences are concerned.

The first victim of the test is the conservative operator $@_1$. Let us say that a sentence ϕ is contingent at moment m if there are histories h_1, h_2 such that $m \in h_1 \cap h_2$ and $\mathfrak{M}, c, m/h_1 \vDash \phi$ and $\mathfrak{M}, c, m/h_2 \vDash \neg\phi$. Let us now consider a sentence Fp contingent at $I(c)$. Let h_1 be such that $\mathfrak{M}, c, I(c)/h_1 \vDash Fp$, then by conservative postsemantics $\mathfrak{M}, c/h_1 \Vdash Fp$. However, since ϕ is contingent at $I(c)$, there is h_2 such that $\mathfrak{M}, c, I(c)/h_2 \vDash \neg Fp$ which means that, by definition of $@_1$, $\mathfrak{M}, c, I(c)/h_1 \vDash \neg@_1 Fp$ which implies in turn, by conservative postsemantics, that $\mathfrak{M}, c/h_1 \Vdash \neg@Fp$. Consequently, there is model $\mathfrak{M}$, sentence ϕ and context c such that $\mathfrak{M}, c/h \Vdash \phi$ and $\mathfrak{M}, c/h \Vdash \neg@_1\phi$ which is equivalent to the failure of the initial redundancy test. In fact, it is the case for every future contingent. Therefore, with respect to some histories, we can truly use the sentence, "There will be a sea battle tomorrow even though actually there will be none."

Belnap et al.'s (2001) alternative proposal ($@_2$) does pass the initial redundancy test but is not acceptable for independent reasons. It generates a number of counter-intuitive results, for example:

- If sentence ϕ is evaluated at a point m/h such that $I(c) \in h$, then $@_2$ loses a part of its indexical nature. MacFarlane (2008, p. 99) even claims that *Actuality$_2$* is simply redundant at such points. This is not quite accurate since *Actuality$_2$* retains a part of its indexical function even at these points, it just functions as another indexical—*Now* (as defined by Belnap et al. 2001, p. 246). However, MacFarlane is partially right: at such points *Actuality$_2$* loses its indexical nature as a *modal* operator. It is particularly visible at context-initialized moments of evaluation where the sentence, "The future might be different from what it actually will be" ($\Diamond(Fp \wedge \neg@_2 Fp)$) is always false. Similarly, the sentence "Necessarily, there will be a sea battle if and only if there actually will be a sea battle" ($\Box(Fp \leftrightarrow @_2 Fp)$) is always true at such points.
- For some model $\mathfrak{M}$, there is a sentence ϕ and a context-initialized point of evaluation $\langle \mathfrak{M}, c, I(c)/h \rangle$ such that the following is true $\mathfrak{M}, c, I(c)/h \vDash P\Diamond F(\phi \wedge \neg@_2\phi) \wedge @_2\phi$. So we can truly say "It might have been the case that there would be a sea battle and actually there would be none (even though there actually will be one)." Such oddities result from the fact that

even if we use $@_2$ twice in one sentence, it might behave as a modal indexical for the first time and not as one on the other occasion.

I find these reasons decisive in abandoning $@_2$ as a candidate for a proper analysis of the word 'actually'. Interestingly, there is a straightforward and formally neat way to solve all the problems of the conservative—it is to include "the history of a context of use" (h_c) as an element of a context and bind the interpretation of the operator @ with this aspect of the context in the following way: $\mathfrak{M}, c, h_c, m/h \vDash @\phi$ iff $\mathfrak{M}, c, h_c, I(c)/I(h_c) \vDash \phi$. However, Belnap et al. (2001, p. 151) very strongly object to the idea of the history of the context of use. I find their arguments persuasive (contrary to e.g., Borghini and Giuliano 2011) and it is important to stress at this point that my TRL_h should not be thought of as the history of the context of use which changes from one context to another. The history TRL_h is "initialized" by the world itself and its structure and content rather than the context.

We have eliminated both the conservative definitions of the operator @ as candidates for an analysis of the indexical meaning of "actually". However, the remaining three proposals seem to be on a par. To differentiate between them, we need to devise a test stronger than the initial redundancy. One reasonable strengthening is a demand that not only uses of ϕ and $@\phi$ should be co-true at every context (as MacFarlane 2008 insists), but uses of their negations also should. Formally, the stronger requirement is that both equivalences ($\mathfrak{M}, c \Vdash \phi$ iff $\mathfrak{M}, c \Vdash @\phi$) and ($\mathfrak{M}, c \Vdash \neg\phi$ iff $\mathfrak{M}, c \Vdash \neg@\phi$) are satisfied. It sounds reasonable since "actually" is a *modal* indexical, so it should be not only initially redundant but redundant also in scope of extensional connectives such as negation. It amounts to the demand that at a given context c, we can truly say that it is not the case that there will be a sea battle if and only if we can truly say that it is not the case that there actually will be a sea battle.

This stronger test is failed by some sentences at some points of evaluation under every definition of "actually" in the *BT* setting that I am aware of, except my *TRL* definition of @ presented above. In particular, the supervaluationist and relativist definitions of @ fail:

Supervaluationism For every contingent sentence ϕ used at c: $\mathfrak{M}, c \Vdash \neg@\phi$ and $\mathfrak{M}, c \not\Vdash \neg\phi$ (even though it is not true either, i.e. $\mathfrak{M}, c \not\Vdash \phi$);

Relativism For every sentence ϕ used at c and still contingent while assessed at c_a: $\mathfrak{M}, c, c_a \Vdash \neg@\phi$ and $\mathfrak{M}, c, c_a \not\Vdash \neg\phi$ (even though it is not true either, $\mathfrak{M}, c, c_a \not\Vdash \phi$).

The *TRL* definition of 'actually' that I proposed satisfies this stronger test in full generality. In fact, operators $\neg$ and @ are mutually "transparent," i.e. the equivalence $\neg@\phi \leftrightarrow @\neg\phi$ is true (not only valid) in every $\mathfrak{M}_{TRL}$ model.

We can propose an even stronger, and yet still quite natural version of the initial-redundancy requirement, i.e. we can demand for every context c and every sentence ϕ that $\mathfrak{M}, c \Vdash \phi \leftrightarrow @\phi$. We just express the metalinguistic version of the initial-redundancy test in the object language. It simply means that at any context, one is guaranteed to say the truth, claiming that there will be a sea battle if and only if there actually will be a sea battle. MacFarlane (2008) explicitly rejects this strengthening but he agrees that we need to "get over our qualms" to do so (p. 99 ,

n. 22). Again, all the treatments of 'actually' discussed in the literature, besides otherwise faulty conservative $@_2$, would falsify this equivalence whenever ϕ is a contingent sentence. At the same time, the equivalence $\phi \leftrightarrow @\phi$ is valid (but not true) in every *TRL*-model.

It is easy to understand why the *TRL* model constitutes such a friendly environment for the operator 'actually', while branching concretism is so hostile to it. In the *TRL* theory, there is exactly one point of evaluation initialized by any use of a sentence. Importantly, this point contains a specific history (TRL_h) as its element. It is quite evident that this very point should be utilized for interpretation of the operator @. The *TRL*-ist sharply distinguishes the actual from the possible so the interpretation of @ is quite straightforward.

The branching concretist on the other hand, denies distinguishing any 'actual' history. However, to retain the indexical meaning of @, he needs to tie it to some feature of the context. The only available item seems to be "(...) a unique causal past, and a unique future of possibilities, the whole of which is summed up by the moment of use" (Belnap et al. 2001, p. 226). As a result, he tends to identify actuality with necessity. Belnap et al. (2001) even propose an intended reading of $@_1A$ to be: "It is settled true at this actual moment that *A*" (2001, p. 153). Consequently, the concretist (no matter whether conservative, supervaluationist, or relativist) usually takes the sentence $\Box\phi \leftrightarrow @\phi$ to be valid—true whenever used. In particular, we are semantically guaranteed to be right when saying: "If only it actually will rain tomorrow, it is settled that it will." Such observations further reinforce the concretist conviction that actuality is only a camouflaged form of necessity and whoever talks about the actual future is a determinist in disguise.

The *TRL*-ist intends to disentangle the notions of actuality and necessity and he seems to be successful in his attempt. In particular, it is not difficult to find a sentence ϕ and a context at which a use of $@\phi \leftrightarrow \Box\phi$ is false. A similar implication, $@\phi \rightarrow \Box@\phi$, is true but it does not doom us to determinism. (Just as the truth of, "If it is raining now, then it will always be the case that it was raining now," does not doom us to a flood.) It just witnesses to the indexical nature of the operator @.

To sum up, it appears that the *TRL* combination of presemantics, semantics, and postsemantics presented here generates the most intuitive predictions for the behavior of the indexical operator "actually". I take this fact as another argument for the more general thesis that the *TRL* theory is metaphysically underpinned by some actualist notion of possibility.

5 Conclusions and Perspectives

In this paper, I proposed a new approach to the problem of the Thin Red Line. The reconstruction of the history of the debate suggests that none of the existing *TRL* theories is completely free of formal or conceptual worries. I decided not to elaborate a modification of some previous solution but to rethink the basis of the issue. My diagnosis is that the predominant impetus for the argument is metaphysical in character; different semantic judgments naturally ensue. I argue

that the major discrepancy consists in the contrasting notions of the world and possibility presumed by the parties.

In my opinion, the *TRL* theory is a specimen of the actualist modal realism in philosophy of possibility combined with eternalism in philosophy of time. If we look at the Thin Red Line from this perspective, it is much easier to understand the motivation behind this view and to defend it against the attacks. In my attempt to secure the Thin Red Line presented above, I have presumed some actualist interpretation of possibility, but have not endorsed any one in particular. It would be a very interesting enterprise to investigate which (if any) of the actualist conceptions is best suited for the general project of modeling temporal possibility and metaphysical indeterminism.

I proposed a theory-neutral formalism which explicates the semantic impact of various metaphysical beliefs. In this framework, I expressed the most important theories of future contingents elaborated in the context of Branching-Time. To this effect, I adapted and extended the terminology of MacFarlane (2003, 2008) and divided the semantic labor between presemantics, postsemantics and semantics proper. The first notion is particularly important since it enables one to clearly distinguish the *TRL*-ist's position from the alternatives. Formally speaking, the *TRL* view comes down to three assumptions:

- the presemantic one: $I[C] = TRL_h$;
- the semantic one: Ockhamism is the accurate semantics for our tempo-modal language;
- the postsemantic one: $\mathfrak{M}_{TRL}, c \Vdash^{TRL} \phi$ iff $\mathfrak{M}, c, I(c)/TRL_h \vDash \phi$.

The essential commitment of this construed *TRL* theory is that the contexts are linearly ordered so no context initializes a possible moment outside TRL_h. One of the consequences of this commitment is that the theory cannot afford the operators which shift the context parameter to the positions which initialize possible moments outside TRL_h. As I mentioned, such operators are sometimes used in the context of *BT* to give an account of speech reports. An interesting future project would be to provide an analysis compatible with *TRL* metaphysics.

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