Intentional time inconsistency

(For the full text view of final published version, please use https://rdcu.be/7XGc)

Agah R. Turan

November 22, 2018

Abstract

We propose a theoretical model to explain the usage of time inconsistent behavior as a strategy to exploit others when reputation and trust have secondary effects on the economic outcome. We consider two agents with time-consistent preferences exploiting common resources. Supposing that an agent is believed to have time-inconsistent preferences with probability p, we analyze whether she uses this misinformation when she has the opportunity to use it. Using the model originally provided by Levhari and Mirman (Bell J Econ 11(1):322-324,1980), we determine the optimal degree of present bias the agent would like to have while pretending to have time-inconsistent preferences and we provide the range of present bias parameter under which deceiving is optimal. Moreover, by allowing the constant relative risk aversion class of utility form, we characterize the distinction between pretending to be naive and sophisticated.

Time-inconsistent preferences Hyperbolic discounting Dynamic game Common property resources Perfect Bayesian equilibrium

1 Introduction

The notion of time inconsistency which is characterized by a preference reversal from a larger but later reward to an imminent one as the delays to both rewards decrease has long been recognized by the economists and has been frequently documented by psychologists in the delay discounting literature.[1] Consider this as an example of preference reversal: a majority of people say that they would say they prefer a \$100 check that can be immediately cashed in over a \$200 check that can be cashed in after 2 years. The same people do not prefer a \$100 check that can be cashed in after 6 years over a \$200 check that can be cashed in after 8 years, even though this is the same comparison with a 6-year delay (Ainslie and Haslam 1992). The dependence of decisions on time distance creates dynamic inconsistency, meaning that the individual's future plan will

[1] For the insights established by Adam Smith and David Hume, see Palacios - Huerta (2003) and for a review of studies providing evidence of preference reversal, see Green and Myerson (2004, 2010).

be inconsistent with his current optimal plan. A rational agent may restrict his future options to compensate for these inconsistencies. While limiting his future options, an individual can engage in social commitments as well as individual commitments.[2]

Social relations based on trust and reputation can make a rational agent conform to his original plan (Benabou and Tirole 2004; Bryan et al. 2010). However, social relations may not always resolve the self-control problem; furthermore, as Battaglini et al. (2005) demonstrated, they can even aggravate the problem of self-regulation. Can social interaction itself be a source of dynamic inconsistency? Can people without self-control problems actually behave as if they had self-control problems within their social relationships? For example, a person who fails to stick to family budget despite all promises to his/her spouse gets rid of self-control problem once they are divorced. As another example, a colleague who acts as if he has a severe procrastination problem is actually simply shirking work. In this paper, we propose a theoretical model to explain the usage of time-inconsistent behavior as a strategy to exploit others when reputation and trust have secondary effects on the economic outcome.

Consider two agents with time-consistent preferences exploiting common resources. Assume that one of them wrongly believes that the other agent has time-inconsistent preferences. We have examined the following questions in an analytically solvable model: Does the agent who is believed to have time-inconsistent preferences, use this misinformation as an advantage? What is the degree of time inconsistency she would like to pretend? How does it depend on her own and the other agent's impatience?

Supposing that an agent is believed to have time-inconsistent preferences with probability p, we analyze whether she uses this misinformation when she has the opportunity to use it. Since this is a game with observable actions, we use perfect Bayesian equilibrium as an equilibrium notion. We characterize the pooling equilibrium where she plays with time-inconsistent preferences irrespective of her type. This proves that if an agent has created a perception that he might have problems with self-control, she prefers to act accordingly. Hence the choices seen as a result of the self-control problem can actually be intentional.

We use a quasi-hyperbolic discounting structure (Phelps and Pollack 1968) to represent the consumption-saving decisions of an agent with time-inconsistent preferences. There are emotion-based and various cognitive mechanisms shown as the driver of time inconsistency[3]. In the emotion-based mechanism, time-inconsistent behavior has often been represented as a result of conflict between two different decision-making systems, the current and future self, which have narrow and wide temporal perspectives, respectively (Thaler and Shefrin 1981;

[2] Consider a person with a bad habit of staying up too late. Every morning, he promises to go to bed early, but at night, he always goes to bed later than he intended. By knowing that he breaks his promises so many times, a rational agent may pre-commit his future behavior. For example, he can say his spouse that he feels tired and sluggish so that he should go to bed early. If you are married, then you know that your spouse will make you go to bed early either by kindness or by force.

[3] For a comprehensive review on psychological determinants of intertemporal preferences, see Urminsky and Zauberman, 2015

Metcalfe and Mischel 1999). This idea provides the rationale for the quasi-hyperbolic discounting function which can be decomposed into two distinct processes: one that captures the extra weight given to immediate rewards and another that discounts exponentially (McClure et al. 2007).

Using the analytically tractable version of dynamic fishery model originally provided by Levhari and Mirman (1980), we determine the optimal degree of present bias that the agent would like to have while pretending to have time-inconsistent preferences. Next, we consider the constant relative risk aversion (CRRA) class of utility form to characterize the distinction between pretending to be naive and sophisticated. While the sophisticates are aware of their self-control problem, the naifs are not. By assuming that output elasticity is one, we prove that there exists equilibrium under strategies linear in stock and that exploitation of the resources increases when agent pretends to have time-inconsistent preferences. We show that, the decision to pretend to have time-inconsistent preferences and the preference between pretending to be naive and sophisticated is sensitive to the each of these parameter values: present bias parameter, discount rate and the degree of concavity of the utility function. We analyze how the decision to pretend to have time-inconsistent preferences and the preference between pretending to be naive and sophisticated depends on the parameters of our model by providing results derived from calculations based on our characterization of the equilibrium.

There has been extensive research, started by Strotz (1955) and accelerated by Laibson (1994, 1997, 1998) studying the consumption-saving decisions of an agent with time-inconsistent preferences (see also Harris and Laibson 2001; Krusell et al. 2000 and 2002; Krusell and Smith 2003) This interest has recently shifted to environmental models with imperfect intergenerational altruism (see Karp, 2005; Haurie, 2005 and 2006; Di Corato, 2012).

Our paper complements the literature that studies the effects of time preference on the exploitation of common resources. The fishery model has been used as a metaphor for any kind of renewable resource on which the property rights are not well defined (For recent surveys on this topic, see Van Long 2011 and Jorgensen et al. 2010). Levhari and Mirman (1980) and Van Long et al. (1999) analyze the game between time-consistent agents having different discount rates. Nowak (2006), analyze this model by assuming that agents are hyperbolic players whose preferences change over time. Haan and Hauck (2014) consider a common pool problem and propose a solution concept for games that are played among hyperbolic discounters that are possibly naive about their own, or about their opponent's future time inconsistency. We consider the game between time-consistent and (seemingly) time-inconsistent agents and extend the analysis to CRRA class of utility form.

The article is organized as follows. The next section introduces the model and characterizes the equilibrium conditions of the game. In Sect. 3, we define the perfect Bayesian equilibrium in which the agent who is believed to have time-inconsistent preferences and use this misinformation as an advantage. Two different versions of dynamic fishery model are introduced and the main results of the paper are proven in Sects 4 and 5, respectively. Section 6 concludes.

2 The model

In this section, we analyze three different cases. In case 1, time-consistent agent, let us call agent A, will play the game with an agent who has the same preferences with him. In case 2, his opponent has time-inconsistent preferences and she is unaware of her inconsistency problem. In case 3, his opponent has time-inconsistent preferences and she is aware of her problem. From now on, we will call the agent in case 2 "agent N", as a sign of her naivety and call the agent in case 3 "agent S", as a sign of his sophistication.

Consider that the dynamics of a common property renewable resource is governed by

$$x_{t+1} = f\left(x_t - \sum_{i=1}^{2} c_t^i\right),\ t = 0, 1, , ...,$$

where c_t^i is the consumption by player $i \in \{1, 2\}$ at period t with

$$c_t^i \geq 0,\ \forall i \text{ and } x_{t+1} \geq 0,\ \forall t \geq 0.$$

The payoff of agent A can be written as

$$\sum_{t=0}^{\infty} \delta^t u(c_t) \tag{1}$$

where u denotes the instantaneous utility function and $0 < \delta < 1$ denotes the discount factor.

Now consider the problem of an agent who has time-inconsistent preferences:

$$U_t = u(c_t) + \beta \sum_{\tau=t+1}^{\infty} \delta^t u(c_\tau)$$

Quasi hyperbolic model, which is also referred as $\beta - \delta$ model, differs from the standard exponential model when the imperfect altruism or present bias parameter, β, is less than one. The discount rate applied between current and the immediate future period is $\frac{1-\beta\delta}{\beta\delta}$ whereas per-period discount rate for the future periods is $\frac{1-\delta}{\delta}$. Note that the per-period discount rate for a given time period changes as that period approaches. For example, when $t = 0$, the per-period discount rate for period 1 (the discount rate applied between period 1 and 2) is $\frac{1-\delta}{\delta}$ but when $t = 1$, the per-period discount rate for period 1 is $\frac{1-\beta\delta}{\beta\delta}$. Laibson (1997) applied this model to explore the implications of time-inconsistent preferences on consumption-saving decisions. Quasi-hyperbolic discounting structure of the model formulates the behavior of an agent who has time-inconsistent preferences in a simple and highly tractable way (For a literature survey on models of hyperbolic discounting, see Frederick et. al. 2002).

We make the following assumptions regarding the properties of the utility and the production functions.

Assumption 1 $u : R_+ \to R_+$ is continuous, twice continuously differentiable, strictly increasing, strictly concave, and $u'(0) = +\infty$. $f : R_+ \to R_+$ is continuous, twice continuously differentiable, strictly increasing and satisfies $f(0) = 0$.

At each period, agents decide on their planned consumption a_t^i, Their actual consumption is given by the rule below:

$$c_t^i = \left\{ \begin{array}{ccc} a_t^i & if & \sum\limits_{i=1}^{2} a_t^i < x_t \\ \frac{x_t}{2} & if & \sum\limits_{i=1}^{2} a_t^i \geq x_t \end{array} \right\}$$

In a single agent optimization problem, when the utility function satisfies Inada condition, consumption in any period satisfies the nonnegativity constraint. This is not held in a noncooperative game setup. There is a noncooperative equilibrium where both agents set their planned consumption to the available stock and all available stock get exhausted right away. Using this option, we can define the incentive compatibility constraint. An agent can always set her planned consumption to the available stock and end the game. We consider this situation as an exit strategy where the agent's utility is defined as follows:

$$IC\left(x\right) = u\left(\frac{x}{2}\right) + \frac{\delta}{1-\delta} u\left(0\right)$$

In any equilibrium, the equilibrium payoff should be greater than the payoff that an agent can get using exit strategy. We write and solve the model in terms of actual consumption c_t^i as if the agent does not choose exit strategy. After characterizing the candidates of equilibrium, we check whether they satisfy the incentive compatibility constraint and constitute an equilibrium. When the utility function is unbounded from below, all candidates satisfy the incentive compatibility constraint as $\lim_{x \to 0} IC\left(x\right) = -\infty$. When the utility is bounded from below, candidates of equilibrium may not constitute an equilibrium as we show in Sect. 5.1.

The sets of feasible strategies available to the players are interdependent and in addition, the agents' choices in the current period affect the payoffs and their choice sets in the future. A Markov strategy for an agent is a function that defines consumption decision of an agent for all possible values of the stock of the common resource. In Markov strategies, all the past influences current play only through its effect on a state variable.

For a given strategy of the other agent satisfying $c\left(x\right) < x$, best response function of agent A is defined with the following problem:

$$V^A\left(x\right) = \max_{\mathbf{d} \leq x - c(x)} u\left(\mathbf{d}\right) + \delta V^A\left(f\left(x - c\left(x\right) - \mathbf{d}\right)\right) \tag{2}$$

Using the first order condition and the envelope theorem, we get

$$\begin{aligned} u'\left(d\left(x\right)\right) = {} & \delta f'\left(x - c\left(x\right) - d\left(x\right)\right) u'\left(d\left(f\left(x - c\left(x\right) - d\left(x\right)\right)\right)\right) \\ & \left(1 - c'\left(f\left(x - c\left(x\right) - d\left(x\right)\right)\right)\right) \end{aligned} \tag{3}$$

where $d\left(x\right) \in \arg\max_{\mathbf{d} \leq x - c(x)} u\left(\mathbf{d}\right) + \delta V^A\left(f\left(x - c\left(x\right) - \mathbf{d}\right)\right)$.

The equilibrium condition changes according to the type of player that he faces.

2.1 Case 1: the strategic interaction with the time-consistent agent

Using the conditions 2 and 3, the symmetric Markovian perfect Nash equilibrium (MPNE), $g_{tc}()$, and the corresponding value function $V_{tc}()$ are defined by the following:

$$u'(g_{tc}(x)) = \delta f'(x - 2g_{tc}(x))\, u'(g_{tc}(f(x - 2g_{tc}(x)))) \\ (1 - g'_{tc}(f(x - 2g_{tc}(x)))) \quad (4)$$

$$V_{tc}(x) = u(g_{tc}(x)) + \delta V_{tc}(f(x - 2g_{tc}(x))) \quad (5)$$

2.2 Case 2: The strategic interaction with the naive agent

The agent N, who has time-inconsistent preferences and is unaware of her inconsistency problem, solves the problem below:

$$\max_{\mathbf{c} \leq x - d(x)} u(\mathbf{c}) + \beta\delta V_{tc}(f(x - \mathbf{c} - d(x))) \quad (6)$$

for a given $d(x)$ where $d(x) < x$ represents agent A's strategy. Since the naive agent having time-inconsistent preferences believes that her future self will act like the other agent who has time-consistent preferences, continuation value function represented with V_{tc}. The conditions of MPNE, where $g_N()$ denotes equilibrium consumption of agent N, and $g_{A_N}()$ denotes equilibrium consumption of agent A are defined with:

$$u'(g_N(x)) = \beta\delta f'(x - g_N(x) - g_{A_N}(x))\, V'_{tc}(f(x - g_N(x) - g_{A_N}(x))) \quad (7)$$

and

$$u'(g_{A_N}(x)) = \delta f'(x - g_N(x) - g_{A_N}(x))\, u'(g_{A_N}(f(x - g_N(x) - g_{A_N}(x)))) \\ (1 - g'_N(f(x - g_N(x) - g_{A_N}(x)))) \quad (8)$$

2.3 Case 3: The strategic interaction with the sophisticated agent

The agent S, who has time-inconsistent preferences and is aware of her inconsistency problem, solves the problem below:

$$\max_{c \leq x - d(x)} u(\mathbf{c}) + \beta\delta V_S(f(x - \mathbf{c} - d(x))) \quad (9)$$

for a given $d(x)$ where $d(x) < x$ represents agent A's strategy. The continuation payoff, $V_S()$, satisfies:

$$V_S(x) = u(c(x)) + \delta V_S(f(x - c(x) - d(x))) \quad (10)$$

where $c(x) \in \arg\max_{\mathbf{c} \leq x-d(x)} u(\mathbf{c}) + \beta\delta V_S (f(x - \mathbf{c} - d(x)))$ .The conditions of MPNE, where $g_S()$ denotes equilibrium consumption of agent S, and $g_{A_S}()$ denotes equilibrium consumption of agent A are defined by the following:

$$u'(g_S(x)) = \beta\delta f'(x - g_S(x) - g_{A_S}(x))\, u'(g_S(f(x - g_S(x) - g_{A_S}(x)))) \left[\left(1 - \frac{1}{\beta}\right) g'_S(f(x - g_S(x) - g_{A_S}(x))) + \frac{(1 - g'_{A_S}(f(x - g_S(x) - g_{A_S}(x))))}{\beta}\right]. \quad (11)$$

and

$$u'(g_{A_S}(x)) = \delta f'(x - g_S(x) - g_{A_S}(x))\, u'(g_{A_S}(f(x - g_S(x) - g_{A_S}(x)))) (1 - g'_S(f(x - g_S(x) - g_{A_S}(x)))). \quad (12)$$

3 Pretending to have time-inconsistent preferences

Suppose that agent 1 is believed to have time-inconsistent preferences with probability p. We will analyze whether she uses this misinformation when she has the opportunity to use it. It is common knowledge that agent 2 has time-consistent preferences, so that he is the agent A in section 2.

If agent 1, who has time-consistent preferences pretend to be a sophisticated agent with time-inconsistent preferences and use $g_S(x)$, agent 2's best reply will be $g_{A_S}(x)$. In this case, agents' corresponding utilities denoted with $V_S(x), V_{A_S}(x)$ are defined as follows:

$$V_S(x) = u(g_S(x)) + \delta V_S(f(x - g_S(x) - g_{A_S}(x)))$$
$$V_{A_S}(x) = u(g_{A_S}(x)) + \delta V_{A_S}(f(x - g_S(x) - g_{A_S}(x)))$$

Similarly, if Agent 1, who has time-consistent preferences use $g_N(x)$, agent 2's best reply will be $g_{A_N}(x)$. In this case, agents' corresponding utilities denoted with $V_N(x), V_{A_N}(x)$ are defined as follows:

$$V_N(x) = u(g_N(x)) + \delta V_N(f(x - g_N(x) - g_{A_N}(x)))$$
$$V_{A_N}(x) = u(g_{A_N}(x)) + \delta V_{A_N}(f(x - g_N(x) - g_{A_N}(x)))$$

Recall that, $g_{tc}()$ denotes the symmetric MPNE of the game. The value function $V_{tc}()$ under symmetric equilibrium is defined as:

$$V_{tc}(x) = u(g_{tc}(x)) + \delta V_{tc}(f(x - 2g_{tc}(x)))$$

It is common knowledge that agent 2 has time-consistent preferences. Agent 1, however, has private information about its preferences. Agent 2 believes that player 1 has time-inconsistent preferences with probability p and time-consistent preferences with probability $(1-p)$. As a simplifying assumption, we let the agent with time-inconsistent preferences be sophisticated. Our analysis can easily be applied to the case where she is naive or she can choose between pretending to be naive or sophisticated. We will show that if $V_{A_S}(x)$ and $V_S(x)$ satisfy the incentive compatibility constraint and if $V_S(x)$ is greater than $V_{tc}(x)$, there exists a perfect Bayesian equilibrium where the agent 1 pretends to have time-inconsistent preferences.

Let the history at period t be $h_t = \left\{c_k^1, c_k^2, x_k\right\}_{k=0}^t$ where the c_k^1 and c_k^2 are consumptions of agent 1 and agent 2 and x_k is resource stock at period k. Let $A(h_t) = [0, x_t]$ denote agents' feasible actions at period t+1 when the history is h_t. Agent 1 has a type θ in a set $\Theta = \{tco, tin\}$ where tco represent time-consistent preferences and tin represent time-inconsistent preferences. Agent 2's belief about agent 1's type after the history h_t is represented with $\mu(h_t)$. A pure strategy $s^1 = \left\{s_k^1\right\}_{k=0}^t$ for agent 1 is a map from the set of possible histories and types into feasible actions (so that it satisfies $s_k^1(h_k, \theta) \in A^1(h_t)$) and similarly a pure strategy s_2 for agent 2 is a map from the set of possible histories into feasible actions. The next proposition defines the pooling equilibrium where the agent 1 plays with time-inconsistent preferences irrespective of her type.

Proposition 1 *Consider the Markov strategies $g_S(x), g_{A_S}(x), g_{tc}(x)$ and value functions $V_S(x), V_{A_S}(x), V_{tc}(x)$. If $V_S(x) \geq V_{tc}(x)$ and $V_I(x) \geq IC(x)$ where $I \in \{S, A_S\}$, there is a perfect Bayesian equilibrium $\left(\left(\tilde{s}^1(h,\theta), \tilde{s}^2(h)\right), (\mu(h))\right)$ such that*

$$\tilde{s}_t^1(h_t, \theta = tco) = \tilde{s}_t^1(h_t, \theta = tin) = g_S(x)$$
$$\tilde{s}_t^2(h_t) = \left\{ \begin{array}{ll} g_{A_S}(x) \;\; and \;\; \mu(h_t) = p & if \;\; h_t = \left\{g_S(x_k), c_k^2, x_k\right\}_{k=0}^t \\ g_{tc}(x) \;\; and \;\; \mu(h_t) = 0 & if \;\; h_t \neq \left\{g_S(x_k), c_k^2, x_k\right\}_{k=0}^t \end{array} \right\}$$

Proof. By definition $g_S(x) \in \arg\max_{\mathbf{c}} u(\mathbf{c}) + \beta\delta V_S(f(x - \mathbf{c} - g_{A_S}(x)))$ and $g_{A_S}(x) \in \arg\max_{\mathbf{d}} u(\mathbf{d}) + \delta V_{A_S}(f(x - \mathbf{c} - g_S(x)))$. We have to show that agent 1 does not deviate from her strategy even if she has time-consistent preferences. This requires that:

$$V_S(x) \geq \max_{\mathbf{c}} u(\mathbf{c}) + \delta V_{tc}(f(x - \mathbf{c} - g_{A_S}(x)))$$

The right-hand side of the equation defines the utility of agent 1 if she deviates from her strategy and left-hand side of the equation defines the utility of agent 1 if she keeps using $g^S(x)$. The inequality follows from the fact that, for any arbitrary $\mathbf{c}$, we have:

$$u(\mathbf{c}) + \delta V_{tc}(f(x - \mathbf{c} - g_{A_S}(x))) \leq u(\mathbf{c}) + \delta V_S(f(x - \mathbf{c} - g_{A_S}(x))) \leq V_S(x)$$

so that

$$\max_{\mathbf{c}} u(\mathbf{c}) + \delta V_{tc}(f(x - \mathbf{c} - g_{A_S}(x))) \leq V_S(x).$$

Because of the assumption of the proposition, the agents do not use the exit strategy and $\left(\left(\tilde{s}^1(h,\theta), \tilde{s}^2(h)\right), (\mu(h))\right)$ constitutes an equilibrium. ■

In this equilibrium, agent 1 plays with $g_S(x)$ irrespective of her type or history and agent 2 plays with $g_{A_S}(x)$ as long as agent 1 play with $g_S(x)$. If Agent 1 plays with a strategy different than $g_S(x)$, agent 2 updates his belief and plays with $g_{tc}(x)$.

Here we have two properties assuring the existence of pooling equilibria: Agent 2's utility does not depend on agent 1's type but on her strategy and agent 2 can not damage agent 1's dishonesty unless he damages himself. Because of the first property, agent 2's prior belief does not change as long as agent 1 act as if she had time-inconsistent preferences. By the second property, agent 2 can not force agent 1 to play symmetric MPNE even if he is sure that agent 1 has time-consistent preferences.[4][5]

Next, we analyze two different versions of dynamic fishery model. The first model let us examine the optimal degree of time inconsistency she would like to pretend? And the second model let us make the distinction between acting naive and sophisticated.

4 The optimal degree of time-inconsistency

This analytically tractable version of dynamic fishery model is originally provided by Levhari and Mirman (1980) and has been used broadly to analyze common property resource games. This is the simplest setup to show that social relations could be a source of dynamic time inconsistency.

Assumption 2 $u(c) = log\,(c)\,;\; f\,(x) = x^{\alpha}\;, 0 < \alpha < 1.$

[4] Note that, even if both players know that they have time consistent preferences, we might have a subgame perfect equilibrium where the consumption path coincides with the one defined in Proposition 1: The agent 1 continues to play $g_S(x)$ and agent 2 continues to play $g_{A_S}(x)$ as long as no agent deviates from this strategy. If at least one player deviates from this strategy, they both play $h\,(x) = x$ and the resources are exhausted. For a set of payoffs to be supportable in discounted dynamic programming, see Fudenberg and Tirole (1991).

[5] The pooling equilibrium that we define resembles the well-known variant of the chain-store game in which there is a small probability p that the monopolist is "tough" and prefers fight rather than cooperate if there is an entry to the market. In the original chain-store game, a monopolist plays against a succession of K potential competitors. In each period one of the potential competitors decide whether or not to compete with the monopolist. If it decides to enter then the monopolist chooses either to cooperate or to fight. Each potential competitor prefers to stay out rather than entering and being fought, but prefers the most when it enters and the monopolist does not fight. If a competitor enters, the monopolist prefers to cooperate rather than fight, but it prefers the most if there is no entry. In the unique subgame perfect equilibrium of the game, each potential competitor chooses to enter and the monopolists always chooses to cooperate (Selten 1978). Kreps and Wilson (1982) shows that the regular monopolist turns the failure of correct common knowledge about its payoff into an advantage by acting like a tough one and preserves its reputation at least until the horizon gets close. Similarly, we show that agent 1 turns the failure of correct common knowledge about its preferences into an advantage and acts as if he might have problems with self-control.

Using the conditions 4, 7,8, 11,12 and by postulating an equilibrium with strategies linear in stock, we characterize $g_{tc}(x), g_S(x), g_{A_S}(x), g_N(x), g_{A_N}(x)$. Since the period utility is unbounded from below, the incentive compatibility constraint is automatically satisfied as $\lim_{x\to 0} IC(x) = -\infty$.

Proposition 2 *Under Assumption 2, for all games defined in case 1,2 and 3, there exists a MPNE where $g_{tc}(x), g_S(x), g_{A_S}(x), g_N(x), g_{A_N}(x)$ defined as follows:*

$$
\begin{aligned}
g_{tc}(x) &= \left(\frac{1-\delta\alpha}{2-\delta\alpha}\right)x \\
g_N(x) &= \frac{1-\delta\alpha}{1+\beta-\alpha\delta}x \text{ and } g_{A_N}(x) = \frac{\beta(1-\delta\alpha)}{1+\beta-\alpha\delta}x \\
g_S(x) &= \frac{1-\delta\alpha}{1+\beta-\alpha\delta}x \text{ and } g_{A_S}(x) = \frac{\beta(1-\delta\alpha)}{1+\beta-\alpha\delta}x
\end{aligned}
$$

Proof. See Appendix. ■

Note that in this setup, the equilibrium does not depend on the distinction between acting naive or sophisticated. This result is not surprising as Pollak (1968) showed this property in a continuous-time model with logarithmic utility.

Corollary 3 *Under Assumption 2, exploitation rate of the resources is higher when agent 1 has or acts as if she had time-inconsistent preferences.*

Proof. See Appendix. ■

By knowing that agent's true preferences are represented by Eq. (1), we can discuss if she gets more utility by acting as if she had time-inconsistent preferences. Moreover, we can characterize the optimal value of β if it were to be chosen. Under Assumption 2, whenever we have an equilibrium under linear strategies, agent 1's utility is given by the following:

$$V^1(x) = \frac{\log x}{1-\delta\alpha} + \frac{\log c + \frac{\delta\alpha}{1-\delta\alpha}\log 1 - c - d}{1-\delta}$$

where c and d represent the equilibrium consumption rates of the agents, i.e. $c = \frac{g_N(x)}{x} = \frac{g_S(x)}{x}$ and $d = \frac{g_{A_N}}{x} = \frac{g_{A_S}}{x}$. Agent 1 finds the optimal value of β by solving the trade off between her consumption rate and combined investment rate governed by the following maximization problem:

$$\max_{\beta} \log c + \frac{\delta\alpha}{1-\delta\alpha}\log 1 - c - d \tag{13}$$

Since α affects the common resource exponentially, $\frac{\delta\alpha}{1-\delta\alpha}$ can be thought as a modified discount rate of an agent having time consistent preference. Note that, as β decreases, the consumption rate of the agent, $\frac{1-\delta\alpha}{1+\beta-\alpha\delta}$, increases, whereas the combined investment rate, $\frac{\beta\delta\alpha}{1+\beta-\alpha\delta}$, decreases. Even if β is not under her control, she may prefer acting like a player with time-inconsistent preferences. Next, we characterize the optimal value of β and provide the range of β under which deceiving is optimal.

Proposition 4 *a) If agent 1 can choose β, She chooses $\beta = \alpha\delta$. b) If agent can not choose β, she acts as if she had time-inconsistent preferences when* $\beta \geq \min\left(\alpha\delta, \left(\frac{1}{2-\delta\alpha}\right)^{\frac{1}{\delta\alpha}}\right)$.

Proof. See Appendix. ■

The reason for finding such a simple solution for the optimal level of β is the logarithmic utility function. It assures the existence of equilibrium under linear strategies and let the trade-off between consumption and combined investment rate be defined under the additively separable form. The relation between the optimal value of β and discount factor can be analyzed further by differentiating the discount rates of the agents. Let us assume that agents can have different discount factors denoted with δ_1 and δ_2. By applying the methods used in the proof of Proposition 2, one can characterize the equilibrium values of consumption rates and show that the optimal value of β is equal to $\alpha\delta_2$. Interestingly, while equilibrium consumption rates depend on both δ_1 and δ_2, the optimal value of β does not depend on agent 1's own discount rate.[6] This result is specific to the logarithmic utility. Whenever finding the optimal degree of time inconsistency problem reduces to a trade-off between consumption rate and combined investment rate, the objective function can be represented with $h\left(c\left(\beta\right), 1 - c\left(\beta\right) - d\left(\beta\right)\right)$ .The first order condition of the problem is as follows:

$$\frac{h_1\left(c\left(\beta\right), 1 - c\left(\beta\right) - d\left(\beta\right)\right)}{h_2\left(c\left(\beta\right), 1 - c\left(\beta\right) - d\left(\beta\right)\right)} c'\left(\beta\right) - \left(c'\left(\beta\right) + d'\left(\beta\right)\right) = 0. \tag{14}$$

where h_i represents the partial derivative with respect to the ith term. Under logarithmic utility, this condition reduces to $\beta c'\left(\beta\right) - \left(c'\left(\beta\right) + d'\left(\beta\right)\right) = 0$ and since $\frac{d'(\beta)}{c'(\beta)}$ is independent of δ_1, the optimal value of β does not depend on her own discount factor. In general, $\frac{h_1(c(\beta),1-c(\beta)-d(\beta))}{h_2(c(\beta),1-c(\beta)-d(\beta))}$ and $\frac{d'(\beta)}{c'(\beta)}$ depends on the (modified) discount factors of both agents and optimal degree of time inconsistency characterized by the nonlinear interaction of them.

While the relation between the optimal value of β and the agent 1's own discount factor is not necessarily monotonic, in Proposition 3, we show that the optimal value of β is increasing in the discount factor of her rival. For the model introduced in the next section, we plot the optimal level of β of the Naive agent for multiple cases by freeing the modified discount factor of the agents one at a time. This analysis, too, suggests that the optimal value of β is increasing in the discount factor of her rival. The basic reasoning behind it is as follows: Let us consider the optimal value of β and equilibrium consumption rates under discount factors δ_1 and δ_2^L denoted by β^L, c^L,and d^L. Let us fix the value of β and δ_1, and consider what happens when δ_2 increases from δ_2^L to δ_2^H. In the new equilibrium pair, (c, d), the agent 1's consumption rate increases and agent 2's consumption rate decreases. This result follows from two facts: First, for a given

[6] With heterogeneous discount factors, we get $c = \frac{(1-\delta_1\alpha)\delta_2\alpha}{\delta_2+\beta\delta_1-\alpha\delta_1\delta_2}$, $d = \frac{(1-\delta_2\alpha)\beta\delta_1\alpha}{\delta_2+\beta\delta_1-\alpha\delta_1\delta_2}$and $1 - c - d = \frac{\beta\alpha\delta_1\delta_2}{\delta_2+\beta\delta_1-\alpha\delta_1\delta_2}$.

consumption of the agent 1, the agent 2's consumption rate is decreasing in her own discount factor. Second, the agents' consumption decisions are strategic substitutes; when one agent consume more, the other agent replies this with consuming less. Since both agent 1's consumption rate and combined investment rate have changed, β^L may not be optimal anymore. Agent 1 adjusts to the behavioral change of her rival by increasing the level of β and thus by decreasing her equilibrium consumption rate. Let us denote the optimal value of β and equilibrium consumption rates under discount factors δ_1 and δ_2^H by β^H, c^H and d^H. Our reasoning implies that when δ_2 increase from δ_2^L to δ_2^H we have $\beta^H > \beta^L$ and $c^L < c^H < c$. This is an interesting result as it suggests that agent 2's patience can mitigate the self-control problem of the (seemingly) time inconsistent agent.

Note that our game is dynamic i.e. if the agent 2 observed information that contradicts his belief, he would react. However, as we show in Proposition 4, $V^1(x) > V_{tc}(x)$. By the Proposition 1, there is a pooling equilibrium under which agent 1 acts as if she had time-inconsistent preferences so that agent 2 does not observe any contradicting information to update his initial belief.

Our analysis can be easily extended to the case where both agents have time-consistent preferences but there is uncertainty about the agent 1's discount factor. In the Appendix, we characterize the pooling equilibrium and the optimal value of the discount factor. Our analysis shows that the agent 1 may pretend to be less patient when there is uncertainty about her true discount factor. While solving the optimal value of the discount factor, she considers the same trade-off between her consumption rate and combined investment rate defined in (13). In sum, uncertainty in both the present bias and the discount factor can be used as an advantage. However, there is a fundamental difference between them in terms of the nature of the strategic interaction. While, in the former, agent 1 acts with time-inconsistent preferences, in the latter she acts with time-consistent preferences and agents differ only in terms of discount factors. As the nature of the strategic interaction changes, we see a qualitative difference even under the simplest model we define in this section. While the optimal level of present bias is independent of the agent 1's own discount factor, as we show in the Appendix, the optimal level of the agent 1's discount factor depends on her true discount factor.

5 Distinction between pretending to be naive and sophisticated

In the previous setup, the equilibrium does not depend on the distinction between acting naive or sophisticated. In this section, we use CRRA class of utility form to characterize this distinction. To have an equilibrium under strategies linear in stock, we assume that output elasticity is one.

Assumption 3 $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$; $f(x) = Ax$, $1 < A^{1-\sigma} < \frac{1}{\delta}$, $\sigma \in (0, \infty)$.

Using the conditions 4, 7,8, 11,12, and by postulating an equilibrium with strategies linear in stock, we can characterize $g_{tc}(x), g_S(x), g_{A_S}(x), g_N(x), g_{A_N}(x)$. By Proposition 1, together with $V_S(x) \geq V_{tc}(x)$, the incentive compatibility condition, $V_I(x) > IC(x)$ where $I \in \{S, A_S\}$, should be met in order for $\{g_S(x), g_{A_S}(x)\}$ to constitute an equilibrium. Similarly, together with $V_N(x) \geq V_{tc}(x)$, $V_I(x) > IC(x)$, where $I \in \{N, A_N\}$ should be met in order for $\{g_N(x), g_{A_N}(x)\}$ to constitute an equilibrium.

Proposition 5 *Under Assumption 3,*
a) There exist a symmetric MPNE where $g_{tc}(x)$ and corresponding value function $V_{tc}(x)$ defined as follows:

$$g_{tc}(x) = a_{tc}x \text{ and } V_{tc}(x) = u(a_{tc}x)\frac{1}{1-\delta\left(A\left(1-2a_{tc}\right)\right)^{1-\sigma}}$$

where $a_{tc} \in \left(0, \frac{1}{2}\right)$ solves $h_{tc}(c) = (1-2c)^{\sigma} - \delta A^{1-\sigma}(1-c) = 0$.
b) If $h_N\left(\frac{1}{1+B^{1/\sigma}}\right) > 0$ where $B = \frac{\beta\delta A^{1-\sigma}a_{tc}^{1-\sigma}}{1-\delta A^{1-\sigma}(1-2a_{tc})^{1-\sigma}}$ and $h_N(c) = Bc^{\sigma} - \delta A^{1-\sigma}(1-c)$, $g_N(x)$ and $g_{A_N}(x)$ defined as follows:

$$g_N(x) = a_n x \text{ and } g_{A_N}(x) = \left(1 - a_n\left(1 + B^{1/\sigma}\right)\right)x$$

where $a_n \in \left(a_{tc}, \frac{1}{1+B^{1/\sigma}}\right)$ solves $h_N(c) = 0$.
c) $g_S(x)$ and $g_{A_S}(x)$ defined as below:

$$g_S(x) = a_s x \text{ and } g_{A_S}(x) = \beta a_s x$$

where $a_s \in \left(a_{tc}, \frac{1}{1+\beta}\right)$ solves $h_S(c) = (1-c-\beta c)^{\sigma} - \delta A^{1-\sigma}(1-c) = 0$.

Proof. See Appendix. ■

Note that, for the game between agent N and agent A, we need an additional condition for the existence of MPNE with strategies linear in stock. The following proposition shows that the degree of concavity, σ, is less than or equal to one, is sufficient to satisfy this condition. There could be an MPNE under linear strategies for $\sigma > 1$.For example, it has been satisfied for $\sigma = 1.5$ and $\sigma = 2$ for all parameter values that we consider in Sect. 5.2.

Proposition 6 *If $\sigma \leq 1$, we have $h_N\left(\frac{1}{1+B^{1/\sigma}}\right) > 0$, i.e., there exists a unique $\{g_N(x), g_{A_N}(x)\}$.*

Proof. When $\sigma \leq 1$,we have

$$h_N\left(\frac{1}{1+B^{1/\sigma}}\right) = \left(\frac{B^{1/\sigma}}{1+B^{1/\sigma}}\right)^{\sigma} - \delta A^{1-\sigma}\frac{B^{1/\sigma}}{1+B^{1/\sigma}} > 0$$

which follows from the fact that $\frac{B^{1/\sigma}}{1+B^{1/\sigma}}$ and $\delta A^{1-\sigma}$ are smaller than 1 by Proposition 5-a and by Assumption 3. $h_N\left(\frac{1}{1+B^{1/\sigma}}\right) > 0$ together with $h^N(0) < 0$

and $h'_N(c) > 0$, implies that there exists unique c such that $h_N(c) = 0$. By Proposition 5-b, there exists a unique $\{g_N(x), g_{A_N}(x)\}$. ■

Under Assumption 2, we have proved that exploitation rate is higher when agent 1 has or acts as if she had time-inconsistent preferences. Although we can not fully characterize the equilibrium strategies in open form for some values of σ, the next corollary to Proposition 5 proves that the same relation holds under Assumption 3.

Corollary 7 *Under Assumption 3, the rate of exploitation is higher when agent 1 has or acts as if she had time-inconsistent preferences.*

Proof. See Appendix. ■

Consider the case that agent 1 has time-consistent preferences but she is believed to have time-inconsistent preferences with probability p. For an initial level of available resource stock x, when she acts with her true preferences, her payoff will be $V_{tc}(x)$. As we show in Proposition 1, she might pretend to have time-inconsistent preferences. By acting like a naive player, she gets $V_N(x)$ and by acting like a sophisticated player, she gets $V_S(x)$. Since we focus on an equilibrium with strategies linear in stock[7], any payoff $V(x)$ can be defined as $Vx^{1-\sigma}$ *i.e.*, we can compare $V_{tc}(x), V_N(x)$ and $V_S(x)$ independently of available resource stock simply by comparing V_{tc}, V_N and V_S.

5.1 Example: the degree of concavity is less than one

To analyze the case where the degree of concavity is less than one, we consider $\sigma = 0.5$. We plot the preference over acting like a naive player, acting like a sophisticated player and acting with time-consistent preferences in Fig. 1. It is worthwhile emphasizing that the results that we provide are not derived from numerical simulations, but are calculations based on our characterization of the equilibrium. We characterize V_{tc}, V_N, and V_S in terms of two parameters, β and the composite parameter $\delta A^{1-\sigma}$. Because of the structure of our model, $\delta A^{1-\sigma}$ represents the modified discount factor of the agents. By assumption, both present-bias parameter and modified discount factor are less than one. To focus on plausible values of discount factor, we let the modified discount factor be greater than half.

(Figure 1 will be inserted here)

On the left panel, we show the greatest among $V_{tc}(x)$, $V_N(x)$, $V_S(x)$. The green region represents the parameter set where $V_N(x)$ is greater than both $V_{tc}(x)$ and $V_S(x)$. The blue region represents the parameter set where $V_S(x)$ is the greatest of them and the red region represents the parameter set where $V_{tc}(x)$ is the greatest.

On the right panel, we characterize the equilibrium for different value of the parameters. The green region represents the equilibrium where agent 1 act like a

[7] We restrict ourselves to linear strategies to obtain definite results. By relaxing the assumption on output elasticity, one can show numerically that the decision to pretend to have time−inconsistent preferences and the preference between naive and sophisticated behavior may depend on the available resource stock

naive player and use strategy $g_N(x)$ and agent A use strategy $g_{A_N}(x)$. Similarly, the blue region represents the equilibrium where agent 1 act like a sophisticated player and use strategy $g_S(x)$ and agent A use strategy $g_{A_S}(x)$. The red region represents the equilibrium where agent 1 act with time consistent preferences and both agent use strategy $g_{tc}(x)$.

The difference between the left panel and the right panel demonstrates the effects of incentive compatibility. The area where red replaces green represents the parameter set where $V_{A_N}(x) < IC(x)$. In this region, since agent A will use exit strategy, $\{g_N(x), g_{A_N}(x)\}$ does not constitute an equilibrium. Similarly, the area where red replaces blue represents the parameter set where $V_{A_S}(x) < IC(x)$.

We also plot the optimal level of β if the agent 1 can decide. The coordinates of the circles represent the optimal level of β for a given level modified discount factor, $\delta A^{1/2}$. The color of a circle represents who makes this decision. When $\delta A^{1/2}$ is greater than $\widetilde{a}$, the agent 1 act with time consistent preferences no matter the value of the present-bias parameter is, i.e. the optimal level of $\beta = 1$. When $\delta A^{1/2}$ is less than $\widehat{a}$, the agent 1 act like a naive player as long as she could choose present-bias. In the mid-region represented with $(\widehat{a}, \widetilde{a})$, she switches between acting like a naive and sophisticated player as the modified discount factor increases.[8]

5.2 Example: the degree of concavity is higher than one

To analyze the case where the degree of concavity is greater than one, we consider two cases: $\sigma = 1.5$ and $\sigma = 2$. Note that, when the degree of concavity is greater than one the period utility is unbounded from below, i.e. incentive compatibility constraint is automatically satisfied as $\lim_{x\to 0} IC(x) = -\infty$.

We characterize V_{tc}, V_N, and V_S in terms of two parameters, β and the composite parameter $\delta A^{1-\sigma}$. In Fig. 3, we plot the preference over acting like a naive player, acting like a sophisticated player and acting with time-consistent preferences when the degree of concavity is one and a half. The upper side of the figure is blue, implying that, when β is greater than $\widetilde{\beta} \approx 0.66$, the agent 1 pretend to be a sophisticated player no matter the value of $\delta A^{-1/2}$ is. When the present-bias is less than $\widetilde{\beta}$, the preference over acting like a naive player, acting like a sophisticated player and acting with time-consistent preferences depends on the value of $\delta A^{-1/2}$. Whenever the agent 1 can choose the optimal level of present-bias, she switches between acting like a naive and sophisticated player as the modified discount factor increases.

(Figure 2 will be inserted here)

[8] As we did in Sect. 4, one can solve the model for heterogeneous discount factors. While an MPNE in linear strategies does not exist when agent 1 pretend to be sophisticated, it still exists when both agents act with time consistent preferences or when agent 1 pretend to be naive. For the naive player, we plot the optimal level of β for multiple cases by freeing modified discount factor of agents one at a time. Our analysis confirms the discussion in Sect. 4 that the optimal level of β depends on the nonlinear interaction of agent 1's own discount rate and the discount rate of the agent 2.

Finally, we analyze the case where the degree of concavity is two. In this case, the agent 1 acts like a sophisticated player when β is greater than $\widehat{\beta} \approx 0.47$ no matter the value of modified discount factor, $\frac{\delta}{A}$ is. By comparing Figs. 2 and 3, we can see that blue region enlarges as the degree of concavity increase. Recall that, the blue region represents the parameter set where acting like a sophisticated agent is the dominant strategy. At the bottom right corner, there is a red region where the agent prefers acting with time consistent preferences. However, it is dominated by the time inconsistent preferences when the agent can choose the optimal level of β.

(Figure 3 will be inserted here)

6 Discussion

The emergence of time-inconsistent behavior is considered as a consequence of human nature (Ainslie and Haslam, 1992) while the consistent behavior is a skill to be acquired. Ainslie (1992) argues that "[i]t is just as supportable, however, to say that living mostly for the present moment is our natural mode of functioning, and that consistent behavior sometimes acquired, to a greater or lesser extent, as a skill. Some philosophers have even suggested that we should not acquire such a skill-that we would be happier if we abandoned our complex ways of banking on the future and lived for the present instant."

Although this argument explains the endogenous formation of time-consistent behavior, it can also be used to discuss why the endogenous formation of time-inconsistent behavior can arise. Sigmund et. al. (2001) show that reputation is necessary for fostering social behavior among selfish agents and punishment works much better than rewarding in promoting cooperative behavior. If time inconsistency is socially accepted as a natural mode of functioning then the time-inconsistent behavior is rarely punished. Moreover, we have shown that the bad reputation in time consistency can improve agent's welfare by letting him exploit the other agent. These two arguments suggest that agents may act as if they have time inconsistency problem since it may bring economic benefit without social cost. This means that together with the endogenous formation of time-consistent behavior, endogenous formation of time-inconsistent behavior is also possible.

While the self control problem is presented as a conflict between the immediate rewards and the long-term interests, social dilemmas arise from the conflict between the individual and the collective interests. The correspondence between self control on the dimension of time and social cooperation on the dimension of social space has been pointed out by a number of authors (see Rachlin, 2000; for the formalization of the social cooperation problem in the same way as the self-control problem). The studies on time preference that address the relationship between these two concepts report that self control reduces the over-exploitation of the common property resources and benefits cooperation (Fehr and Leibbrandt, 2011; Burks et al., 2009; Houser et al., 2012). We consider the opposite relation and analyze how social dilemma may result in time-inconsistent behavior.

Frederick et al. (2002) document the lack of agreement among studies measuring time preference and argue the existence of a discrepancy between the pure time preference and the elicited measures. There are many contextual factors affecting elicited time preference and it is important to isolate the contextual factors that are central to the actual time preference from the remaining factors that distort the link between them (Soman et al., 2005; Urminsky and Zauberman, 2015). The social environment may affect elicited time preferences differently, depending on whether or not the social relationships are based on trust and reputation. Further experimental and correlational analyses are needed to increase our understanding of the temporal or permanent effect of social environment on time preference.

In this paper, our concern is the consumption-saving decisions of people. A similar analysis can be made for the decision of when to do a task while one of the agents is wrongly believed to have a tendency to procrastinate (see O'donoghue and Rabin, 1999; for procrastination under quasi-hyperbolic discounting). As an illustration, consider an experiment in which some participants in a group are misrepresented as having procrastination problems to the rest of the group and that they are aware of this misrepresentation. There are three ways how these participants might behave: (1) they might behave according to their actual time preferences, (2) they might behave according to the perceptions of others about them, or (3) they might make extra effort to destroy this perception. Our results suggest that the second option is possible when reputation and trust have secondary effects on the economic outcome. For example, in a workplace where personal qualifications and work ethics are not the primary cause of promotion and they bring nothing but workload, the agent who is believed to have time-inconsistent preferences may use this misinformation as an advantage. Studying the reasons behind the possible behavioral differences will allow us to understand project team or group study procrastination better.

Finally, we can interpret our game as the strategic interaction between countries exploiting common resources. By construction, discount rates are subjective parameters. When resources are exploited by countries, time preference is represented with social discount rates that are generally not observable or common knowledge. Our analysis reveals that it is not the actual time preference, but the others' perception of you that is important and countries might use misinformation about their social discount rates. Soman et al. (2005) quote the news commenting that "some emerging countries like China and Mexico have such a strong desire to make progress in the present that they consume resources at a rate that is detrimental to the future progress". They ask if it makes sense to think of discount factors for nations and if these nations display high discount rates. In their words, these are intriguing questions without a perfect answer. However, one thing we can say for sure is that there is a public opinion that the time preferences of developing countries are different from the developed countries. Our model implies that as long as the emerging countries believed to have high discount rates, whenever there is a divergence between their national interests and global collective ones, their acts will be consistent with the public perception of them.

7 References

Ainslie, G. (1992), *Picoeconomics: The strategic interaction of successive motivational states within the person.*, Cambridge: Cambridge University Press.

Ainslie, G., & Haslam, N. (1992). Hyperbolic discounting. In G. Loewenstein and J. Elster (Eds.), *Choice over time* (pp. 57-92). New York: Russell Sage Foundation.

Amir, R., & Nannerup N. (2006). Information structure and the tragedy of the commons in resource extraction. *Journal of Bioeconomics, 8*(2), 147-165.

Bryan, G., Karlan, D., & Nelson, S. (2010). Commitment devices. *Annual Review of Economics, 2*(1), 671-698.

Battaglini, M., Benabou R., & Tirole J. (2005). Self-control in peer groups. *Journal of Economic Theory, 123*, 105-134.

Benabou R., & Tirole J. (2004). Willpower and personal rules., *Journal of Political Economy, 112*(4), 848-886.

Burks, S. V., Carpenter, J. P., Goette, L., & Rustichini, A. (2009). Cognitive skills affect economic preferences, strategic behavior, and job attachment. *Proceedings of the National Academy of Sciences, 106*, 7745-7750.

Di Corato, L. (2012). Optimal conservation policy under imperfect intergenerational altruism. *Journal of Forest Economics, 18*(3), 194-206.

Fehr, E., & Leibbrandt, A. (2011). A field study on cooperativeness and impatience in the tragedy of the commons. *Journal of Public Economics, 95*, 1144-1155.

Frederick, S., Loewenstein, G., & O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature, 40*(2), 351-401.

Fudenberg, D., & Tirole, J. (1991). *Game theory.* Cambridge: MIT Press.

Green, L., & Myerson, J. (2004). A discounting framework for choice with delayed and probabilistic rewards. *Psychological Bulletin, 130*(5), 769-792.

Green, L., & Myerson, J. (2010). Experimental and correlational analyses of delay and probability discounting. In G. J. Madden and W. K. Bickel (Eds.), *Impulsivity: The behavioral and neurological science of discounting* (pp. 67-92). Washington, DC: American Psychological Association.

Harris, C., & Laibson, D. (2001). Dynamic choices of hyperbolic consumers. *Econometrica, 69*(4), 935-957.

Haurie, A. (2005). A multigenerational game model to analyze sustainable development. *Annals of Operations Research, 137*(1), 369-386.

Haurie, A. (2006). A Stochastic multi-generation game with application to global climate change economic impact assessment. *Annals of the International Society of Dynamic Games, 8*, 309-332.

Haan, M., & Hauck, D. (2014). Games with possibly naive hyperbolic discounters *Mimeo.*

Houser, D., Montinari, N., & Piovesan, M. (2012). Private and public decisions in social dilemmas: Evidence from children's behavior. *PLoS ONE, 7*(8), e41568.

Jorgensen, S., Martin-Herran, G., & Zaccour, G. (2010). Dynamic games in the economics and management of pollution. *Environmental Modeling and Assessment 15*, 433-467.

Karp, L. (2005). Global warming and hyperbolic discounting. *Journal of Public Economics, 89*(2), 261-282.

Kreps, D.M., Wilson, R., (1982). Reputation and imperfect information. *Journal of Economic Theory, 27*(2), 253-279.

Krusell, P., & Smith, A. A. (2003). Consumption-savings decisions with quasi-geometric discounting. *Econometrica, 71*(1), 365-375.

Krusell, P., Kuruscu, B., & Smith, A. A. (2002). Equilibrium welfare and government policy with quasi-geometric discounting. *Journal of Economic Theory 105*(1), 42-72.

Krusell, P., Kuruscu, B., & Smith, A. A. (2000). Tax policy with quasi-geometric discounting. *International Economic Journal, 14*(3), 1-40.

Laibson, D. (1994). Essays in hyperbolic discounting. Ph.D. dissertation, MIT.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics, 112*, 443-77.

Laibson, D. (1998). Life-cycle consumption and hyperbolic discount functions. *European Economic Review, 42*(3), 861-871.

Levhari, D., & Mirman, L. (1980), The Great Fish war: An example using a dynamic Cournot-Nash solution. *The Bell Journal of Economics, 11*(1), 322-334.

McClure, S. M., Ericson, K. M., Laibson, D. I., Loewenstein, G., & Cohen, J. D. (2007). Time discounting for primary rewards. *The Journal of Neuroscience, 27*(21), 5796-5804.

Metcalfe, J., & Mischel, W. (1999), A hot/cool system analysis of delay gratification: Dynamics of willpower. *Psychological Review, 106*(1), 3-19.

Nowak, A. (2006). A multigenerational dynamic game of resource extraction. *Mathematical Social Sciences 51*, 327-336.

O'Donoghue, T., & Rabin, M. (1999). Doing it now or later. *American Economic Review, 89*(1), 103-124.

Palacios-Huerta, I. (2003). Time-inconsistent preferences in Adam Smith and David Hume. *History of Political Economy 35*(2), 241-268.

Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *The Review of Economic Studies, 35*(2), 185-199.

Pollak, R. A. (1968). Consistent planning. *The Review of Economic Studies, 35*(2), 201-208.

Rachlin, H. (2000). *The science of self-control*. Cambridge: Harvard University Press.

Selten, R. (1978). The chain store paradox. *Theory and Decision, 9*(2), 127-159.

Sigmund, K., Hauert, C., & Nowak, M. A. (2001). Reward and punishment. *Proceedings of the National Academy of Sciences, 98*(19), 10757-10762.

Soman, D., Ainslie, G., Frederick, S., Li, X., Lynch, J., Moreau, P., Mitchell, A., Read, D., Sawyer, A., Trope, Y., Wertenbroch, K. & Zauberman, G. The psychology of intertemporal discounting: Why are distant events valued differently from proximal ones? *Marketing Letters, 2005, 16*, 347-360

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *The Review of Economic Studies, 23*(3), 165-180.

Thaler, R. H. & Shefrin, H. (1981). An economic theory of self control. *Journal of Political Economy, 89*(1), 392-406.

Urminsky, O. & Zauberman, G. (2015). The Psychology of intertemporal preferences. In G. Wu and G. Keren (Eds.), *Blackwell Handbook of Judgment and Decision Making* (pp. 141-181).

Van Long N. (2011). Dynamic games in the economics of natural resources: A survey. *Dynamic Games and Applications, 1*(1), 115-148.

Van Long N., Shimomura, K., & Takahashi, H. (2011), Comparing open-loop with Markov equilibria in a class of differential games. *The Japanese Economic Review 50*, 457-469.

8 Appendix

8.1 The proof of proposition 2

a) Follows from Amir and Nannerup (2006).
b) From part a, one can compute that:

$$V_{tc}(x) = \frac{\log x}{1-\delta\alpha} + \frac{\log\left(\frac{1-\delta\alpha}{2-\delta\alpha}\right) + \frac{\delta\alpha}{1-\delta\alpha}\log\frac{\delta\alpha}{2-\delta\alpha}}{1-\delta}$$

Under Assumption 2, by 7 and 8, we have

$$g_N(x) = \frac{(1-\delta\alpha)(x - g_{A_N}(x))}{(1-\delta\alpha+\beta\delta\alpha)}$$

$$\frac{g_{A_N}(y)}{g_{A_N}(x)} = \delta f'(x - g_N(x) - g_{A_N}(x))(1 - g_N'(y)) \text{ where } y = (f(x - g_N(x) - g_{A_N}(x)))$$

By using the linearity of $g_N()$ and $g_{A_N}(x)$, we find $g_N(x) = \frac{1-\delta\alpha}{1+\beta-\alpha\delta}x$ and $g_{A_N}(x) = \frac{\beta(1-\delta\alpha)}{1+\beta-\alpha\delta}x$.
c) Under Assumption 2, by 11 and 12, we have

$$\frac{g_S(y)}{g_S(x)} = \beta\delta\alpha(x - g_S(x) - g_{A_S}(x))^{\alpha-1}\left[\left(1-\frac{1}{\beta}\right)g_S'(y) + \frac{(1-g_{A_S}'(y))}{\beta}\right]. \tag{15}$$

$$\frac{g_{A_S}(y)}{g_{A_S}(x)} = \delta f'(x - g_S(x) - g_{A_S}(x))(1 - g_S'(y)) \text{ where } y = (f(x - g_S(x) - g_{A_S}(x)))$$

Using the linearity of $g_S()$ and $g_{A_S}(x)$, we find $g_S(x) = \frac{1-\delta\alpha}{1+\beta-\alpha\delta}x$ and $g_{A_S}(x) = \frac{\beta(1-\delta\alpha)}{1+\beta-\alpha\delta}x$.

8.2 The Proof of Corollary 3

It follows from Proposition 2, as we have

$$2g_{tc}(x) = \frac{2(1-\delta\alpha)}{2-\delta\alpha}x < \frac{(1+\beta)(1-\delta\alpha)}{1+\beta-\alpha\delta} = g_N(x) + g_{A_N}(x) = g_S(x) + g_{A_S}(x)$$

8.3 The Proof of Proposition 4

a) Under case 2 and 3, agent 1's utility is given by the following:

$$V^1(x) = \log(g(x)) + \delta V^1\left(\left(x - g(x) - g^A(x)\right)^\alpha\right)$$

$$\text{where } g(x) = g^S(x) = g^N(x) = \frac{1-\delta\alpha}{1+\beta-\alpha\delta}x \text{ and}$$

$$g^A(x) = g^A_S(x) = g^A_N(x) = \frac{\beta(1-\delta\alpha)}{1+\beta-\alpha\delta}x$$

By guessing that the value function has the form $A\log x + B$, we compute

$$V^{1}(x) = \frac{\log x}{1-\delta\alpha} + \frac{\log \frac{1-\delta\alpha}{1+\beta-\alpha\delta} + \frac{\delta\alpha}{1-\delta\alpha}\log \frac{\beta\delta\alpha}{1+\beta-\alpha\delta}}{1-\delta}$$

This implies that utility maximizing β solves the following problem:

$$\max_{0<\beta\leq 1} \log \frac{1}{1+\beta-\delta\alpha} + \delta\alpha\log\beta$$

From the first order condition we get $\beta = \delta\alpha$. Note that objective function is increasing when $\beta < \delta\alpha$ and decreasing when $\beta > \delta\alpha$ i.e. $\beta = \delta\alpha$ is the unique maximization point.

b) If the agent cannot choose β, she gets $V^{1}(x)$ if she acts as if she had time-inconsistent preferences and gets $V_{tc}(x)$ if she acts truly. Note that $V^{1}(x) > V_{tc}(x) \iff \log\frac{1}{1+\beta-\delta\alpha} + \delta\alpha\log\beta > \log\frac{1}{2-\delta\alpha}$. By part (a), we know that left-hand side of the equation is decreasing in β when β is greater than $\delta\alpha$. Since they are equal to each other for β equals to 1, the left-hand side is greater than the right-hand side when $\beta \in [\delta\alpha, 1)$. Let us consider the case, such that $\left(\frac{1}{2-\delta\alpha}\right)^{\frac{1}{\delta\alpha}} < \alpha\delta$. Our result follows from the fact that, for any $\beta \in \left[\left(\frac{1}{2-\delta\alpha}\right)^{\frac{1}{\delta\alpha}}, \alpha\delta\right)$, we have $\log\frac{1}{1+\beta-\delta\alpha} + \delta\alpha\log\beta > \delta\alpha\log\beta \geq \log\frac{1}{2-\delta\alpha}$.

8.4 The proof of proposition 5

a) Under Assumption 3, by 4, we have

$$\left(\frac{g_{tc}(y)}{g_{tc}(x)}\right)^{\sigma} = \delta\left(1 - g'_{tc}(y)\right) \text{ where } y = f\left(x - 2g_{tc}(x)\right)$$

By imposing that $g_{tc}(x) = a_{tc}x$ we get $(1-2a_{tc})^{\sigma} - \delta A^{1-\sigma}(1-a_{tc}) = 0$. Since $h_{tc}(0) > 0, h_{tc}\left(\frac{1}{2}\right) < 0$ and $h_{tc}(c)$ is continuous in c implies that there exist a_{tc} $\in \left(0, \frac{1}{2}\right)$ such that $h_{tc}(a_{tc}) = 0$. By 5, we get:

$$V_{tc}(x) = u(a_{tc}x)\frac{1}{1-\delta\left(A\left(1-2a_{tc}\right)\right)^{1-\sigma}} \tag{16}$$

b) Under Assumption 3, by 7 and 16, we have

$$\frac{\left(\left(x - g_N(x) - g_{A_N}(x)\right)\right)^{\sigma}}{\left(g_N(x)\right)^{\sigma}} = B$$

$$\left(\frac{g_{A_N}(y)}{g_{A_N}(x)}\right)^{\sigma} = \delta\left(1 - g'_N(y)\right) \text{ where } y = f\left(x - g_N(x) - g_{A_N}(x)\right)$$

By imposing that $g_N(x) = a_n x$, and $g_{A_N}(x)$ is linear in x, we get $g_{A_N}(x) = \left(1 - a_n\left(1 + B^{1/\sigma}\right)\right)x$ and

$$Ba_n^{\sigma} - \delta A^{1-\sigma}(1-a_n) = 0$$

Consider function $h_N()$. In equilibrium, we must have $x - g_N(x) - g_{A_N}(x) > 0$ i.e. $a_n < \frac{1}{1+B^{1/\sigma}}$.Since $h_N(a_{tc}) = Ba_{tc}^{\sigma} - \delta A^{1-\sigma}(1-a_{tc}) = \frac{\delta A^{1-\sigma}(\beta-1)a_{tc}}{1-\delta A^{1-\sigma}(1-2a_{tc})^{1-\sigma}} < 0$, $h_N'(c) > 0$, and by assumption of the proposition $h_N\left(\frac{1}{1+B^{1/\sigma}}\right) > 0$, there exists unique $a_n \in \left(a_{tc}, \frac{1}{1+B^{1/\sigma}}\right)$ such that $h_N(a_n) = 0$.
c) Under Assumption 3, by 11, we have

$$\left(\frac{g_S(y)}{g_S(x)}\right)^{\sigma} = \beta\delta A\left[\left(1-\frac{1}{\beta}\right)g_S'(y) + \frac{\left(1-g_{A_S}'(y)\right)}{\beta}\right].$$
$$\left(\frac{g_{A_S}(y)}{g_{A_S}(x)}\right)^{\sigma} = \delta A\left(1-g_S'(y)\right) \text{ where } y = \left(f\left(x - g_N(x) - g_{A_N}(x)\right)\right)$$

By imposing that $g_S(x) = a_s x$, and $g_{A_S}(x)$ is linear in x, we get $g_{A_S}(x) = \beta a_s x$ and

$$(1 - a_s - \beta a_s)^{\sigma} - \delta A^{1-\sigma}(1-a_s) = 0$$

In equilibrium, we must have $x - g_S(x) - g_{A_S}(x) > 0$ i.e. $a_s < \frac{1}{1+\beta}$.Consider function $h_S()$. Since $h_S(a_{tc}) > h_{tc}(a_{tc}) = 0$, $h_S\left(\frac{1}{1+\beta}\right) < 0$ and $h_S(c)$ is continuous in c, there exists $a_s \in \left(a_{tc}, \frac{1}{1+\beta}\right)$ such that $h_S(a_s) = 0$.

8.5 The proof of corollary 7

Since the equilibrium strategy of a naive and a sophisticated agent does not necessarily coincide, we have two different cases to consider. Let us consider the strategic interaction with a naive agent first. We have

$$h_N(a_n) = Ba_n^{\sigma} - \delta A^{1-\sigma}(1-a_n) = (1-c_n)^{\sigma} - \delta A^{1-\sigma}(1-a_n) = 0$$

where $c_n x = g_N(x) + g_{A_N}(x) = a_n x + \left(1 - a_n\left(1 + B^{1/\sigma}\right)\right)x$.
Since $(1-2a_{tc})^{\sigma} - \delta A^{1-\sigma}(1-a_{tc}) = 0$ we have the relation that:

$$a_n > a_{tc} \Leftrightarrow c_n > 2a_{tc}$$

By proposition 5-b, we conclude $g_N(x) + g_{A_N}(x) > 2g_{tc}(x)$.
Similarly, we have

$$h_S(a_s) = (1-c_s)^{\sigma} - \delta A^{1-\sigma}(1-a_s) = 0$$

where $c_s x = g_S(x) + g_{A_S}(x)$.This implies that:

$$a_s > a_{tc} \Leftrightarrow c_s > 2a_{tc}$$

By Proposition 5-c, we conclude $g_S(x) + g_{A_S}(x) > 2g_{tc}(x)$.

8.6 The pooling equilibrium and optimal value of discount factor when there is an uncertainty about agent 1's discount factor

Suppose that agent 1's true discount factor is δ_1, while agent 2 believes that her discount factor is $\hat{\delta}_1$ with probability p. Consider the games with discount rate pairs $\left(\hat{\delta}_1, \delta_2\right)$ and (δ_1, δ_2) .Since both agents have time consistent preferences, MPNE of the games can be found by following the steps we define in Sect. 2.1. Let us denote MPNE by $\hat{g}^1_{tc}(x), \hat{g}^2_{tc}(x)$ and value functions by $\hat{V}^1_{tc}(x), \hat{V}^2_{tc}(x)$ for the game with discount rate $\left(\hat{\delta}_1, \delta_2\right)$. Similarly denote MPNE of the game with discount rate (δ_1, δ_2) by $g^1_{tc}(x), g^2_{tc}(x)$ and the corresponding value functions by $V^1_{tc}(x), V^2_{tc}(x)$. We can define the pooling equilibrium as below:

If $\hat{V}^1_{tc}(x) \geq V^1_{tc}(x)$ and $\hat{V}^i_{tc}(x) \geq IC\,(x)$, where $i \in \{1, 2\}$, there is a perfect Bayesian equilibrium $\left(\left(\tilde{s}^1(h, \theta), \tilde{s}^2\,(h)\right), (\mu\,(h))\right)$, such that

$$\tilde{s}^1_t(h_t, \theta = tco) = \tilde{s}^1_t(h_t, \theta = tin) = \hat{g}^1_{tc}(x)$$

$$\tilde{s}^2_t(h_t) = \left\{ \begin{array}{ll} \hat{g}^2_{tc}(x) \text{ and } \mu\,(h_t) = p & if\ h_t = \left\{\hat{g}^1_{tc}(x_k), c^2_k, x_k\right\}^t_{k=0} \\ g^2_{tc}(x) \text{ and } \mu\,(h_t) = 0 & if\ h_t \neq \left\{\hat{g}^1_{tc}(x_k), c^2_k, x_k\right\}^t_{k=0} \end{array} \right\}$$

Under Assumption 2, by Levhari and Mirman (1980), we have:

$$\hat{g}^1_{tc}(x) = \left(\frac{\delta_2\left(1 - \hat{\delta}_1\alpha\right)}{\hat{\delta}_1 + \delta_2 - \hat{\delta}_1\delta_2\alpha}\right) x \text{ and } g^2_{tc}(x) = \left(\frac{\hat{\delta}_1\,(1 - \delta_2\alpha)}{\hat{\delta}_1 + \delta_2 - \hat{\delta}_1\delta_2\alpha}\right) x$$

Agent 1 finds the optimal value of $\hat{\delta}_1$ by solving the trade off between her consumption rate and combined investment rate governed by the following maximization problem:

$$\max_{\hat{\delta}_1} \log c + \frac{\delta_1\alpha}{1 - \delta_1\alpha} \log 1 - c - d$$

where c and d represent the equilibrium consumption rates of the agents, i.e. $c = \frac{\hat{g}^1_{tc}(x)}{x}$ and $d = \frac{\hat{g}^2_{tc}(x)}{x}$. From the first order condition, we get $\hat{\delta}_1 = \frac{\alpha\delta_1\delta_2}{1-\alpha\delta_1+\alpha^2\delta_1\delta_2}$.Note that objective function is increasing when $\hat{\delta}_1 < \frac{\alpha\delta_1\delta_2}{1-\alpha\delta_1+\alpha^2\delta_1\delta_2}$ and decreasing when $\hat{\delta}_1 > \frac{\alpha\delta_1\delta_2}{1-\alpha\delta_1+\alpha^2\delta_1\delta_2}$ i.e. $\hat{\delta}_1 = \frac{\alpha\delta_1\delta_2}{1-\alpha\delta_1+\alpha^2\delta_1\delta_2}$ is the unique maximization point.