Metaphysics of the Principle of Least Action

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Abstract

Despite the importance of the variational principles of physics, there have been relatively few attempts to consider them for a realistic framework. In addition to the old teleological question, this paper continues the recent discussion regarding the modal involvement of the principle of least action and its relations with the Humean view of the laws of nature. The reality of the possible paths in the principle of least action is examined from the perspectives of the contemporary metaphysics of modality and Leibniz's concept of the possibles striving from essence to existence. This paper introduces a model of a two-level modality based on an intuition that deep ontological connections exist between the possible paths in the principle of least action and possible quantum histories in the Feynman path integral. The proposed modal interpretation of the principle of least action replaces the classical representation of the system's motion along a single history in the actual modality by the simultaneous motions along an infinite set of all possible histories in the possible modality. To address the issue of necessity, I assume that the principle of least action has a general physical necessity and lies between the laws of motion with a limited physical necessity and those with a metaphysical necessity.

Keywords: principle of least action, modality, possibilia, Feynman path integral, dispositional essentialism, Leibniz

1 Introduction

The principle of least action (PLA) is one of the most general laws of theoretical physics and simultaneously one of the most philosophically conflicting laws. Over the centuries, many scientists have linked it to hopes of a universal theory, despite the related metaphysical disputes about causality. Fermat, Leibniz, Maupertuis, and Euler were sure that nature is thrifty in all its actions thanks to the perfection of God. Planck (1915, p. 68) believed that, among the more or less general laws, which manifest the achievements of physical science in the course of recent centuries, the principle of least action is probably the one, which, as regards form and content, may claim to come nearest to that final ideal goal of theoretical research.

The PLA and other variational principles provide an alternative and more efficient approach to mechanics than Newton's laws. The PLA and the calculus of variation, in

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general, are more global than local differential equations and are widely used for solving dynamic tasks in diverse fields of physics, including classical mechanics, electrodynamics, relativity theory, and quantum physics (Feynman & Hibbs, 1965; Lanczos, 1986; Landau & Lifshitz, 1975; Lemons, 1997; Goldstein et al., 2002; Yourgrau & Mandelstam, 2000; Taylor & Wheeler, 2000; Papastavridis, 2002; Sieniutycz & Farkas, 2005; Hanc & Taylor, 2004; Ogborn & Taylor, 2005).

And yet, the PLA has always been surrounded by a fog of mysticism. The system seems to "choose" the actual path along which an action is less than along other paths. It is as if the system's final state determines the path that the system takes to reach that state. On one hand, we cannot allege that an object actually "chooses" or "calculates" the path of minimal action. On other hand, it appears that the actual path is somehow connected with the future actual state or event. A general principle of causality states that a cause should always precede its effect. This view of causality is used in most of the physical laws and is consistent with the grounded belief that causal influences cannot travel backwards in time. Nevertheless, until today, we have not understood how a physical system seems to "choose" its actual path or history from all possibilities for motion or why this actual history involves minimal action. Moreover, the history of physical teleology might alternatively suggest a relationship between the PLA and the problem of determinism (Stöltzner, 2003). Besides being between teleology and determinism, the PLA does not follow from other physical laws.[2] Additionally, it appeals to a modal notion of "possibilities".

Today, in spite of Planck's hope, the PLA is generally accepted only as a mathematical tool equivalent to the differential equations of motion (Yourgrau & Mandelstam, 2000). However, some physicists have tried to clarify the foundations of the variational principles (Polac, 1959; Asseev, 1977; Lanczos, 1986; Stöltzner, 1994, 2003; Yourgrau & Mandelstam, 2000; Wang, 2008). At the same time, as Butterfield (2004a) stressed, this topic seems wholly ignored in the philosophical literature about variational principles. He assumed that only thanks to the rise of modal metaphysics in analytical philosophy the topic is nowadays plainly visible and focused almost entirely on the way specifying final conditions and teleology. Indeed, recently, some authors have examined how the PLA and other variational principles are involved in modal metaphysics (Butterfield, 2004a, 2004b; Katzav, 2004, 2005; Ellis, 2005; Bird, 2007; Terekhovich, 2013; Thebault & Smart, 2013). They have

[2] As shown, the conservation laws can be derived from the variational principles (Goldstein, et al., 2002; Hanc & Taylor, 2004; Brizard, 2008).

treated the relations of the PLA with teleology, dispositional essentialism, the Humean view of the laws of nature, and the truthmaker principle. However, the study of modality is comprehensive and concerns some other issues connected with necessity and possibility. The themes of modality and the nature of possible worlds are widely discussed in modal metaphysics (Plantinga, 1974; Adams, 1974; Kripke, 1980; Lewis, 1986; Blackburn, 1993; Chihara, 1998; Armstrong, 2004; Fine, 2005) and in relation to different physical phenomena (Swoyer, 1982; Shoemaker, 1984, Ellis, 2001; Bird, 2006).

This paper continues the recent discussion of the metaphysical issues of the PLA, especially regarding the modal involvement of the PLA. I think that Butterfield (2004a, 2004b) is right that all analytical mechanics is steeped in modality. However, unlike his position, I am sure that the most promising direction for the PLA is a metaphysical investigation of the possible paths or histories connected with the laws of quantum systems. In addition to presenting criticism of other concepts, I propose a positive solution of the metaphysical content of the PLA. First of all, I examine a question of a reality of "possible paths" or "possible histories" in the PLA, as well as how they are connected with the notion of "possible objects" or "possibilia" of contemporary metaphysics of modality and Leibniz's concept of the possibles striving from essence to existence.

This paper's proposed solution for some of the metaphysical issues of the PLA is based on the intuition that quantum mechanics might be a key to understanding the philosophical content of this principle. I assume that deep ontological connections exist between the possible paths of the PLA and quantum possible histories of the Feynman path integral formalism (FPI).[3] This paper introduces a model of a two-level modality based on a realistic approach to the possible or virtual motions in the calculus of variations. It considers the possible paths in the Feynman integral to being descriptions of similar processes taking place in the possible modality of being. Proposed in the paper the modal interpretation of the PLA replaces the classical representation of the system's motion along a single history in the actual modality by the simultaneous motions along an infinite set of all possible histories in the possible modality. To address the issue of necessity, I assume that the PLA has a general physical necessity and lies between the laws of motion with a limited physical necessity and those with a metaphysical necessity. It means that the PLA is

[3] It is known that the PLA is connected with the FPI through the notion of "action" and can be derived from the FPI as a limit on a large scale. Some authors have argued that all of classical mechanics could be represented as a short-wave approximation of quantum mechanics, and therefore, the action has the meaning of the phase of quantum amplitude (Feynman & Hibbs, 1965; Taylor, 2003; Ogborn & Taylor, 2005).

a necessary consequence of the laws with a metaphysical necessity.

The rest of this paper is structured as follows. Section 2 gives a short description of the PLA. Section 3 shows the connection between the PLA and the FPI of quantum mechanics. Section 4 briefly introduces some metaphysical difficulties of the PLA related to teleology, necessity, the truthmaker principle, and a notion of possibility. Section 5 discusses the problem of the reality of the possible histories, possible objects and possible worlds from the perspectives of various stances of modal metaphysics, including the Leibniz concept. The basic assumptions of the modal interpretation of the PLA are formulated in Section 6. The relations between the PLA and dispositional essentialism are considered in Section 7. Section 8 explains how the modal interpretation of the PLA can change the view of causality in the PLA. Section 9 compares the arguments in the discussion about the Humean and non-Humean views of the laws of nature concerning the PLA and suggests a new approach supporting the connections among the PLA, other laws of motion and FPI. Section 10 presents the conclusions.

2 Principle of least action (PLA)

Let us consider two ways how classical mechanics explains the motion of a falling apple: Newton's laws and Hamilton's principle of least action.[4]

Newton's laws of motion

Firstly Newton said: *Give me the apple's initial position and its velocity or two very nearby apple's positions*. Then Newton answered the question: *What is the position of the apple at the next instant if there is Earth's gravity or some force*. Newton postulated the first law of motion or the principle of inertia. If there were no acting forces, the apple would possess a mysterious internal tendency to continue in motion with the same velocity along a straight line. The second law of motion postulated that Earth's gravity or some force causes motion in the direction of the applied force. In other words, if the apple "perceives" at a distance the effect of the force, the apple is accelerated or changes its own the velocity. Thus, the path of the apple's actual motion is the result of the combining or summation of

[4] Here, I use the description of Feynman (1964, p. 19-2) and Hanc (2006). Hanc made it graphically for three approaches: Newton's laws, Hamilton's and Maupertuis' principles of least action.

two tendencies or “effects”: the apple’s inertial motion and the motion due to the acting force. Finally, we obtain a differential equation to calculate all positions of the apple.

Hamilton’s principle of least action

Hamilton said: *Give me both the apple’s initial and final events (positions and times) in advance*. Then Hamilton answered the question: *Which path is followed by the falling apple between the initial and final events if the apple has potential energy.* According to Hamilton’s principle, this is the path that has the least action. The action is the difference between kinetic energy and potential energy integrated over time. This difference is called the Lagrangian and appears in Lagrange’s equations of motion. In other words, the average kinetic energy less the average potential energy is as little as possible for the path of the apple going from one point to another. It means that the action reaches the minimum compared with all possible paths from the initial to the final events. Now, we do not need to know how the apple works its way from one event to another; we need to know only the initial and the final apple’s positions and times. Then, we must find all possible paths (or possibilities for moving) from the initial to the final events and by so-called Euler variational method choose the one with the minimal action. Only this path is observed as the actual one, and it exactly coincides with the path calculated by Newtonian approach. However, in Hamilton’s principle, we do not think about forces. We also do not need the fictitious inertial force, since, in the absence of the potential field, the apple’s path with the minimal action is a straight line with constant velocity.

In the present paper, the term *principle of least action* (PLA) covers not only Hamilton’s principle but all integral variational principles of physics. These are treated by mathematicians within the discipline of the calculus of variations and by physicists within analytical mechanics. The main point of each principle is to postulate the abstract space for a set of possible events, paths or histories for the system. According to the PLA, the actual history (along which the system moves from one event to another within a specified space interval in configuration space) differs from all possible histories, consistent with the given constraints, that its function, called the action, is stationary and takes an extremal value. In the calculus of variations, it is said that the variation of action upon infinitesimal variation in the history is equal to zero. In Hamilton’s form of the PLA, the action of the body along the actual path equals an integral of the difference between the average kinetic and potential energy of the body. In the general case, the action can be calculated through an integral of the state function of the system over history, time, n-dimensional volume, or four-dimensional space-time. In most cases, the action is a local minimum; however, it also may be a maximum. Moreover, for any systems, the differential equations of motion could be derived from the PLA.

The PLA is not restricted to mechanics. Historically, the PLA arose from the optical-mechanical analogy with Fermat's principle, in which the light moves along the path that takes the minimal amount of time. The PLA is used in electromagnetism, statistical mechanics, special and general relativity. According to Taylor's (2003) expression, a stone moving with nonrelativistic speed in the region of a small space-time curvature obeys nature's command: *Follow the path of least action!* The stone moving with any possible speed in curved space-time obeys nature's command: *Follow the path of maximal aging (or maximal proper time)!* Here, Taylor kept in mind the relativistic analogue of the PLA – the principle of maximal aging (Taylor & Wheeler, 2000). Taylor (2003) also proposed a scheme where the PLA, on the one hand, is a limiting case of the principle of maximal aging, on the other hand, a limiting case of the of Feynman path integral formalism where an electron obeys the nature`s command: *Explore all paths!* In other words, Newtonian mechanics becomes a limiting case and approximation of general relativity and quantum mechanics.

3 PLA and Feynman path integral (FPI)

When Butterfield (2004b) considered the variational principles of analytical mechanics throughout the philosophy of classical mechanics, he recognized the apparent fact that the actual world is quantum, not classical. The best illustration of this fact is a deep relationship between the PLA and the FPI or the sum-over-histories model (Feynman & Hibbs, 1965). Indeed, quantum electrodynamics and the majority of quantum field theories are connected with the FPI, which uses the same notion of the action that the PLA does. The FPI calculates probabilities by summing up over classical configurations of variables, assigning a phase to each configuration, which equals the action of that configuration.[5] It is assumed that a quantum system simultaneously takes an infinite set of all possible alternative paths or histories, which correspond to the boundary conditions. In our classical world, these possible paths or histories are mutually exclusive although at the

[5] This formulation of quantum mechanics is mathematically equivalent to the Heisenberg matrix method and Schrödinger wave equation, but intuitively more understandable.

quantum level these possible histories coexist. We can say that these possible histories are in quantum superposition. If the possible histories are coherent or mutually consistent (the difference between their quantum phases is constant), they say that there is a coherent quantum superposition. The probability amplitude of each possible history has an equal magnitude and varying phase, which corresponds to the classical action. The coherent or consistent histories are united due to the rule of interference. The resulting history has a maximal probability, which is given by the square of the sum of the probability amplitudes. It can be observed as the single actual history. It is important that other possible histories (called virtual or imaginary) do not disappear. They continue to be the necessary parts of the superposition though these histories are not observed because their probabilities are too small.

As I know, Feynman did not insist on some philosophical interpretation of the FPI and quantum electrodynamics. However, he used his diagrams, which were directly connected with the FPI, not only as calculation tools but also as theoretically motivated representations of physical processes occurring in space and time (Wüthrich, 2010). He always realised that these approaches yield the same results as Newtonian laws in the classical limit (Feynman et al., 1964). To understand his view, let us imagine that a classical body, as well as a photon or electron, moves simultaneously along all possible histories or world lines between the initial and final events. Whereas the phase of the quantum amplitude is very high, a set of world lines (making a significant contribution to the probability of the classical body's detection) is reduced to a narrow bundle. In the limit, the bundle shrinks to the single world-line predicted by the classical Hamilton's form of the PLA (Taylor, 2003). Thus, what Newtonian physics treats as cause and effect (force producing acceleration), the quantum "many paths" view treats as a balance of the changes in phase produced by the changes in kinetic and potential energy (Ogborn & Taylor, 2005). Thus, classical mechanics becomes a short-wave approximation of quantum mechanics, and the action has the meaning of the phase of quantum amplitude. The PLA of classical mechanics can be derived from the sum-over-histories model of quantum mechanics as a limit on a large scale. At the same time, the PLA is the limit of general relativity for low speeds and weak gravity (Taylor, 2003).

In quantum physics, it is generally accepted that possible paths or alternative virtual quantum histories in the FPI are merely formal mathematical tools for calculation, and it cannot be interpreted as implying that a quantum system actually follows one of the histories over which the FPI is computed. However, in recent years, there has been growing interest in a view that these possible histories have some grade of reality (Sharlow, 2007; Valente, 2011; Kent, 2013; Wallden, 2013; Wharton. 2013). In this paper, I do not consider any metaphysical issues of the FPI, although it would be a very exciting business. My aim is more modest – to combine the metaphysical analysis of the PLA and FPI's model of the summation of all possible histories, along which the quantum object moves simultaneously.

4 Metaphysical issues of PLA

I do not have an ambitious goal to clarify all issues of the PLA and criticise all available attempts to address them. I only present a general review of some issues related to teleology, necessity, the truthmaker principle and a notion of possibility.

4.1 Teleology and PLA

In the PLA, it appears that a physical system "foresees" in advance which path (of all possible paths for motion) will minimise an action. Feynman (1964) formulated this question the original way:

> *Is it true that the particle doesn't just "take the right path" but that it looks at all the other possible trajectories? ... The miracle of it all is, of course, that it does just that. ... It isn't that a particle takes the path of least action but that it smells all the paths in the neighborhood and chooses the one that has the least action* (Feynman, 1964, p. 19-9).

Although the various "as if" metaphors do not help us account for this old metaphysical issue of the PLA, we are compelled to do it. We cannot undoubtedly claim that natural objects "foresee", "smell", "calculate", or "choose" some histories, especially the path of minimal action. At the same time, it appears that the observed events along the actual path are somehow connected with the future actual event. Indeed, why do physical systems behave such that one of their characteristics of actual motion takes an extremal value? In the history of science, there are three opinions regarding the philosophical reasoning of the action's minimum: the perfection of God (theological view), the economy of nature (teleological view), and the economy of the human mind (instrumental view).

After Leibniz, Maupertuis, and Euler, the teleological view of the PLA or a phantom of a final cause is considered too metaphysical and mystical (Goldstine, 1980; Lanczos, 1986). Hamilton disclaimed the economy of nature referring to the action in his principle is a local minimum, but sometimes it may be maximum. D'Alamber, Lagrange, Hertz, Jacobi, and others creators of the variational principles were sure that there are not ontological foundations of the parsimony of nature. They considered the PLA only a figurative scientific model (Goldstine, 1980; Panza, 2003). Mach postulated the principle of the economy of thought, which in particular required preferring most economical, simple, and practical description of phenomena from all possible (Mach, 1907). He even argued that the variational principles of mechanics are no more than other mathematical formulations of Newtonian laws and that they do not contain anything new (Stöltzner, 2003). Following Mach, Born emphasized that extreme descriptions talk not about properties of nature but our aspiration for the economy of thinking (Born, 1963).

According to Yourgrau and Mandelstam (2000), in reality, the variational principles evince greater propinquity to derived mathematico-physical theorems than to fundamental laws. From the perspective of the absolute majority of modern physicists, the PLA is nothing but an equivalent method of mathematical description, such as differential equations. Although, as Katzav (2004) pointed out, the mere fact that the PLA can be deduced from other equations does not show that there is such an explanation, because deduction and explanation are not the same. There is also a philosophical view of teleology in the PLA as a different form, but, in fact, an equal point of view of ordinary causality of efficient causes (Plank, 1952). It means that the PLA and other variational principles of physics do not show any benefits to either determinism or teleology (Whitrow, 1980).

It is accepted that an explanation by means of the simplicity, perfection and parsimony of nature does not coordinate with empirical paradigms of science. However, the mere instrumental interpretation of the PLA does not explain a close connection between the action, energy and time as well as an analogy of the PLA with the principle of minimal effort. A clear physical interpretation of the action is not usually given in the physical theories. We only know that the action is the integral of some expression along a possible path or history of a system in a configuration space. The expression can be Lagrangian, for instance, the difference between kinetic and potential energy or, in the case of continuous fields, the Lagrangian density. The integral can be over the path, time, n-dimensional volume, or four-dimensional space-time. In the quantum field theory, the action has a meaning of the phase of quantum amplitude (Feynman & Hibbs, 1965).

Moreover, we know that it is not only as if the action "wants" to be extremal. In other variational or extremal principles, many characteristics "prefer" to be stationary, taking a minimal or maximal value from all possible values. These could include: the optical length, constraint, proper time, curvature of space-time, thermodynamic potentials (for instance, entropy) and others. It is very strange that so many different kinds of systems seem to "strive" for very similar aims. Besides, it seems that we can assign a proper number to each possible event in a configuration space and to each possible history of a quantum system. Thus, mathematics faces a modality here.

In Section 3, I mention the quantum formalism of FPI that uses the same "all at once" principle. The FPI considers as if the quantum system simultaneously takes an infinite set of all possible alternative histories from one event to another. The question is whether this coincidence is just formal and accidental or based on an unknown, metaphysically necessary law.

4.2 Necessity of PLA

Despite the significant place of the PLA among the laws of motion, philosophers have not sufficiently examined whether this principle is truly necessary or not. Many philosophers, beginning with David Hume, have argued that the laws of nature are metaphysically contingent truths. According to the Humean view (or the Regularity Theory), these laws are mere regularities, expressed by the universal quantifications that form part of a best system of law-statements (Lewis, 1986).

Other philosophers have argued that all (Shoemaker, 1998; Bird, 2005) or some (Ellis, 2001) natural laws are necessary truths, not contingent; and that physical possibility is equivalent to metaphysical possibility.[6] Consequently, according to the non-Humean view (or the Necessitarian Theory) there are necessary connections between events, and we must reject the theory of Humean Supervenience and implement a new kind of realism in philosophical analysis (Ellis, 2001).

Both opposite views raise some questions. If the Humean view is right, are the differential laws of motion (e.g., Newtonian laws) and variational laws (e.g., the PLA) metaphysically accidental to an equal degree? Then why is the PLA being considered one of the most fundamental laws, from which all other laws of motion can be derived? If the non-

[6] One of the important results of the discussion in metaphysics of modality is a distinction between kinds of possibility and necessity (Fine, 2002; Vaidya, 2011). The metaphysically possible event is possible by virtue of its own essence or true in one of the metaphysically possible worlds. A metaphysical necessity states about essence or truth in all metaphysically possible worlds. Something is considered as physically possible if it is permitted by the laws of physics. Respectively, a physical necessity directly follows from these laws. It is considered that the physical area of possible events is narrower than the metaphysical ones. The issues of physical possibility and metaphysical possibility are widely discussed (Ellis 2002; Della Rocca 2002; Mackie 2006).

Humean view is right, could we say that the PLA is a metaphysically more fundamental law governing the physical phenomena of the world? If the latter is true, we face three consequent questions.

(1) How can other laws of motion (e.g., Newtonian laws) be mathematical and logical consequences of the PLA?

(2) How does the metaphysical necessity of the PLA involve contingency of the classical system's possible histories and uncertainty of the quantum system's probability amplitudes?

(3) What is the source of the metaphysical necessity of the PLA?

In Section 9, to answer these questions, I broaden the notion of a physical necessity and suggest two new notions: the laws with a limited physical necessity and the laws with a general physical necessity.

4.3 Truthmaker principle and PLA

Butterfield (2004a) perhaps was the first, who noticed that the variational principles (including the PLA) threaten to contravene the truthmaker principle. The truthmaker is that in virtue of which something is true (Armstrong, 2004). The truthmaker principle states that any actual law is made true by only actual facts. Indeed, it seems mysterious to state an actual dynamical law by a comparison of the actual history with possible histories that do not obey this law.

To argue that there is no problem here, Butterfield compared the possible histories of the variational principles with Lewisian counterfactuals and truth-conditions (Lewis, 1973). Butterfield consider an analogue of a Lewisian world an instantaneous state that determines a history, i.e. a phase space trajectory (or even such a trajectory), of the system. Lewis (2003) defined a possible individual a to be a truthmaker for a proposition A iff every world where a exists is a world where A is true. According to Butterfield, despite Lewis' truth-conditions mention other worlds, "counterfactuals are made true by the character of the actual world—since their mention of other worlds reflects only internal relations between worlds." Thus, if a variational principle is itself an infinite conjunction of the counterfactuals, we can say that the truthmakers of Hamilton's principle (a form of the PLA) are "scattered across the worlds". In spite of the serious arguments, I think that the debate related to the PLA and the truthmaker principle is far from complete.

4.4 Possibilities and PLA

Butterfield (2004a) did not agree with philosophers, who said that the virtual displacements or variations mentioned in the variational principles have nothing to do with possibilities of the sort discussed in modal metaphysics. He was even convinced that "mechanics is up to its ears in modality, of some kind or kinds." Indeed, the problem of the reality of the possible (virtual) paths or histories in the PLA and other variational principles is not treated seriously at all. Moreover, from the point of view of physics, there is no problem, since the notion of "possible history" is no more than a heuristic and mathematical tool for writing the laws of motion.

From the perspectives of metaphysics, there is a weird state of affairs. On the one hand, all possible histories are logically possible and "exist" only in our minds. On the other hand, we might consider and calculate the possible histories in which the physical system could have evolved in reality. A philosopher faces an issue. What if the possible histories in the PLA possess some grade of reality? What if they take place in the semi-real space of a possible event? If this assumption is true, we face another set of questions:

(1) How does a multitude of possible histories turn into the actual history, or why do only some of the possible histories become actual? Does this transformation occur accidentally or by law? If this happens by law, how is the selection made?

(2) What happens to the possible histories that never become actual?

(3) Could we describe the possible histories in the PLA by using the metaphysical theories of possible worlds and possible objects?

In discussing modality in analytical mechanics, Butterfield (2004a) set aside the metaphysical debate about the nature of possibilities. On the contrary, I believe that here is the most promising direction, and perhaps, the nature of the possible histories is precisely what explains the metaphysical issues of the PLA. In Section 5, I involve some modal concepts in the discussion regarding the ontological status of PLA's possible histories. Then I use them to support my modal interpretation of this principle.

5 PLA, possible objects, possible histories, and possible worlds

There is a wide-spread opinion that any object is an actual object. From this follows the concept that non-actual possible objects are nothing. There is another conservative view that any object is an existing object. However, from a metaphysical perspective, these statements are not obvious. Moreover, the issue of the reality of possible objects (so-called *possibilia*) is one of the most difficult challenges of metaphysics.

First of all, the analytical philosophers have been paying special attention to the correlation between the *being*, *existence* and *essence* of possible objects and possible states of affairs in different possible worlds (Adams, 1974; Lewis, 1986; Fine, 1994; Armstrong, 1997; Divers, 2002). Some theories concerning possible objects do not invoke the possible worlds. For instance, *essentialism* is a doctrine that objects have essential properties in terms of an entity's *de Re*[7] modal properties (Fine, 1994), and the so-called *Meinongian approach* constructs a general theory of objects other than ordinary concrete existing objects (Zalta, 2006).

Of course, the possible histories in the PLA do not equal the possible objects. Moreover, we can imagine two types of possible histories – for both possible and actual objects.

(a) An actual object is defined by its actual states in actual space or actual events in actual space-time (we can call it the actual world). The set of such consecutive actual events forms an actual history. Accordingly, a possible object is defined by its possible events in the same or another space-time of possible events (we can call it the possible world). Therefore, the set of the consecutive possible events could be considered the *possible history of the possible object.*

(b) An actual object possesses its possible states in actual space or actual events in actual space-time. The set of such consecutive possible events could be considered the *possible history of the actual object*. It means that the same actual event could be reached by many possible histories. At the same time, many other possible histories may start from one actual event. The main restriction is that the possible histories of the actual object must be consistent with the physical laws of the actual world.

Before suggesting my approach regarding the nature of the possible histories in the PLA, I briefly review several metaphysical concepts of the possible objects (possibilia) and possible worlds.

The simplest way to address the issue of the nature of the possible objects (possibilia) is to use the metaphysical notions of *the being* and *existence*. However, we always need to

[7] According to modal logic, if a statement is true in all possible worlds, then it is necessary. A statement that is true in some possible worlds is possible. To emphasise the difference between modal logic and metaphysics of modality, the philosophers often divide modalities into two kinds: *de Dicto* and *de Re*. The second kind consists of the modalities that inherent in things and phenomena, regardless of our language.

specify where something is or exists. For instance, the radical possibilist or modal realist (Lewis, 1986) states that the possible objects and possible events have being and exist no less than the actual ones in an infinite number of possible worlds. For the classical possibilist (Russell, 1903, §427), every existing object is (ontologically) in our world, but some of them (possibilia) could only have existed there. According to the most commonsensical position, such objects are not actual and do not actually exist, but they have a certain grade of being. The actualist denies any reality of possibilia, which are mind-involving and exist only as names, fictions, "ersatz" linguistic, or theoretical constructions (Adams, 1974; Armstrong, 2004). Thus, everything that is, exists as actual thing, and physical existence equal being. Some of the actualists (Plantinga, 1974) invoke unactualised individual essences. They said that every object has an individual essence that is independent of the object that has it, whether the object is actual or non-actual.

Bird (2006), in a discussion with Armstrong (1997) about potency as an essentially dispositional property, showed that a key problem is that ontology has a number of terms to describe perhaps different and unequal kinds or degrees of being. One can say of something that it *is*, that it *exists*, that it *is real*, and that it *is actual*. One says that the merely possible objects are not real; they do not exist because possibilia (such as unrealised manifestations of potencies) are a violation of naturalism. According to Bird, a source of this mistake lies in a picture dominated by modal realism (possibilia cannot exist in the actual world but can exist in other possible worlds). Thus, we are faced with a dilemma: if we accept modal properties we accept other possible worlds. However, this seems to conflict with causal naturalism. Bird's solution is to reject modal realism. Possibilia are not things that exist (if at all) in other worlds, but not in this one; instead, they are things that have being in our world but do not exist. Thus, unrealised manifestations of possibilities are part of the world just as much as manifestations that are realised.

Let us pass from the possible objects to the possible histories. Every object can be considered a unit of two aspects: static (an event) and dynamic (a set of consecutive events or a history). Since the PLA describes the movement of the actual objects, I consider only the possible histories that comprise the set of the consecutive possible events in the actual space-time of our world.

If we apply the mentioned above metaphysical theories to the PLA, we could say that:

(modal realism) the possible history *exists* in possible worlds, but not in our actual world; in the actual world, we can observe the only history with the minimal action;

(possibilism) the possible history has *some grade of being* in our world but does *not exist*; the observed history with the minimal action has the full being and consequently exists in the actual world;

(actualism) the possible history *is* a name or fiction and does *not exist* in our world, although it can have some individual *essence*; we can observe the actual history with the minimal action because it is not a fiction and does not depend on our minds;

(dispositional essentialism) the possible history *is* in our world as unrealised manifestations of possibilities, but the possible history does *not exist* there. The observed history with the minimal action is the realised manifestation of one of possible histories and thus, it exists in the actual world. In Section 7, I consider other options of applying dispositional essentialism to the PLA.

Leibniz

Although most of the theories of possible worlds are based on the ideas of Leibniz, his metaphysical system is significantly different from almost all of modern modal theories and deserves a separate study.[8] For Leibniz, actuality is something that expresses the existence but potentiality expresses only the essence. Since Leibniz was a scientist no less than a philosopher, he was dissatisfied with the too abstract Aristotelian model of the implementation of the potentiality (*dynamis*) through activity (*energeia*) to the actuality (*entelechia*). Leibniz tried to imagine how this metaphysical process manifested itself in physical processes.

Leibniz's theory of the striving possibles (1951, pp. 347–349), distinguished between essence (the nature of a thing) and existence. He postulated that the principle of governing essences is that of possibility or non-contradiction. He suggested that each essence (each possible thing) tends of itself towards existence, but the one that will actually exist is that which has the greatest perfection or degree of essence or the greatest number of possibilities at the same time. The more perfection, the more existence. According to Leibniz, the things are incompatible with the other things; therefore, some possible things do not achieve their actualisation. From the collision of all possibilities, only those things that contain the greatest number of possibilities will be actualised. In other words, "the

[8] Leibniz argued two concepts of possibilities, which has long been rejected by most philosophers. The first was that God has an infinite number of possible worlds. We can be aware of all them because, according Leibniz, being is inherent in everything that can be thought, but not everything obtains being. Thus, by the will of God, the only most perfect world is actualised. The second was the doctrine of the striving possible. The second, however, seems plainly inconsistent with the first. See discussion about this (Blumenfeld, 1973; Shields, 1986).

possibles vie with one another for existence by combining forces with as many other essences as they are mutually compatible with" (Blumenfeld, 1973). Thus, the world arises in which the largest part of the possible things is actualised. Leibniz gave physical examples of such things: a straight line among all lines, a right angle among all angles, and a circle or a sphere among all figures as the most capacious ones.

If we expand the Leibnizian doctrine of the striving possibles and apply it to the PLA, we could say that:

(Leibnizian) every possible event or possible history has its *essence* and tends towards *existence* in our actual world. Among the infinite set of the possible histories, only the history with the minimal action can *exist* as actual because it has the highest degree of essence and combines the greatest number of possibilities at the same time.

It seems that despite many differences, the views of the possible histories based on the contemporary metaphysics of modality and the Leibnizian theory have some resemblances and analogies. Therefore, I suppose there are enough reasons to suggest a new approach regarding the nature of the possible histories in the PLA. Let us start from a two-level model of being or two realms of our world.

At the first level, there are the possible events and histories of the actual objects. The possible events and histories have essences but do not actually exist. We can call this level a possible modality of being or a possible realm of the world.

At the second level, there are only actualised events and histories of the actual objects. The event and histories have both essences and existence. Unlike the possible histories, the actual ones possess more dispositions towards existence or a higher degree of essence. We can call this level an actual modality of being or an actual realm of the world.

6 Modal interpretation of PLA

To provide a positive metaphysical solution for the issues of the PLA mentioned in Section 4, I consider a hypothesis concerning the nature of the possible histories in this principle. The hypothesis is based on the two metaphysical models: modality and combination. These models together form a modal interpretation of the PLA.

The modality model is a statement of the two-level modality of the physical system's histories. It is based on various modal approaches to the reality of possibilia in the contemporary metaphysics of modality and Leibniz's theory of the striving possibles, which are described in Section 5.

In the modality model, I do not involve the possible worlds. The possible worlds are important metaphysical notions, but even if they exist, this fact does not matter for an analysis of the PLA. Moreover, in metaphysics, Leibniz's idea of many possible worlds became an obstacle to the development of his other ideas of possibilities. Ones often connect the possibilia with possible worlds is the authority of Lewis.[9] I think that the possible objects, possible events, possible histories, and Lewisian worlds are not the same; thus, we cannot use Lewis' arguments. Here, I agree with Bird's (2006) opinion given in Section 5 that a source of mistake lies in a picture dominated by modal realism (possibilia cannot exist in the actual world but can exist in other possible worlds). Bird's solution is to reject modal realism. Possibilia are not things that exist (if at all) in other worlds, but not in this one; instead, they are things that have being in our world but do not exist. According to another possible objection, in the realm of classical physics, the possible worlds are set up to determine the actual world, while, in the quantum realm, possible worlds contribute with a certain probability. Here, the quantum object's possible histories with a certain probability are confused with the possible worlds.

According to the modality model, the possible histories, mentioned in the PLA, have essences in the possible modality but do not actually exist in the actual modality. The actual or actualised histories have existence in the actual modality. Now, I specify what essence and existence mean for the possible histories of the PLA. The absence of existence means just the absence of physical observation and interaction in the actual modality. At the same time, the possibility of the histories means their non-contradiction of the classical physical laws of the actual realm of our world. Since the possible events and histories have essences, they occur simultaneously in different possible space-times of the possible realm of our world. The properties of these possible space-times are the subject of a special investigation. The actual history is naturally consistent with the physical laws of our world and occurs in the only actual space-time.

Following Leibniz (see Section 5), I connect the essence of the possible histories with the tendency towards existence. Every possible history tends from the possible modality of being into existence in the actual modality. It can mean that a certain physical system moves from the initial actual event along an infinite set of possible histories. These

[9] Butterfield (2003) discussing various modal involvements of the variational principles tied them to David Lewis' work on modality, especially to his work on counterfactuals (Lewis, 1973). Butterfield even took an instantaneous state as the analogue of a Lewisian world.

movements occur simultaneously in different possible space-times in the possible modality. Such a picture contradicts the classical laws because the physical system's possible histories are mutually exclusive. Unexpectedly, quantum mechanics can help here.

In Section 3, I have mentioned the deep relationship between the PLA and the FPI. To calculate the probabilities of the quantum particle's history, the FPI supposes that the quantum system simultaneously takes an infinite set of all possible alternative histories corresponding to the boundary conditions. At the quantum level, these histories coexist, and physicists say that the possible histories are in quantum superposition.

Using an analogy with the FPI, the modality model claims that all of the physical system's possible histories *are* jointly in the possible modality of being or possible realm of the world. The possibility of the histories means their non-contradiction, both in the classical and quantum physical laws. In other words, the modality model replaces the classical representation of a system's motion along a single actual trajectory by a representation of simultaneous motions along an infinite set of possible histories. The possible motions occur simultaneously in the possible realm of our world.

Another part of the modal interpretation of the PLA is the combination model, which is a statement of the combination or integration of all the physical system's possible histories. This model aims to explain of how the set of the possible histories in the possible modality turns into the actual history, why only some of the possible histories become actual and how this selection occurs.

Let us remember the physical models where some actual movements are supposed to be the combination and summation of a set of the possible or virtual movements. Section 2 has shown that in the Newtonian approach, the actual motion is the result of the combination or summation of two possible motions: the inertial motion and the motion due to the acting force. In optics, Huygens' principle states that all points of a wavefront of light may be regarded as new sources of virtual secondary waves that expand in every direction. The sum of these secondary waves (or a surface tangent to them) constitutes the new actual wavefront at any subsequent time.

In the integral variational principles, the actual process or history differ from all alternative possible processes that system's functional (not only the action) is stationary and takes an extremal value. The functional is defined by the integral of a certain expression (the Lagrangian or Lagrangian density), and can be calculated over the path, time, n-dimensional volume or four-dimensional space-time. In the differential variational principles, instead of the integration ones use the summation that set equal to zero.

As discussed in Section 3, according to the FPI, the coherent histories are united due to the rule of interference or the summation of quantum phases. Each quantum phase corresponds to the classical action, and (in the general case) the resultant actual history obtains a maximal probability and minimal action. The actual history can be obtained as the limit of a narrow bundle of the possible histories significantly contributing to the quantum amplitude. It is important to emphasise that other possible histories do not disappear and continue to be the necessary parts of the quantum superposition, though they are not observed due to their very small contributions to the probability amplitude.

Some decades before Feynman introduced the path integral, Schrödinger did not agree with Heisenberg (1962) that an observed trajectory of a single particle resulted one of the possible trajectories transforming to the actual trajectory. Schrödinger explained the actual trajectory by a set or a field of all possible trajectories. According to Schrödinger, none of the infinite set of possible histories would have the advantage of being implemented in a particular case; all these are equally real. Instead of the logical opposition between an "either–or" in point mechanics, he proposed using a "both–and" in wave mechanics (Schrödinger, 1965). Instead of the implementation of only one possible entangled state, all are summed up. It occurs due to the resonance or interference of the waves (Schrödinger, 1952).

In the modal interpretation of the PLA, the combination model of the physical system's possible histories is based on an analogy with the mathematical operations of summation or integration. On the analogy of quantum superposition, we can treat the metaphysical superposition of the coherent set of the possible histories in the possible modality. Due to the combination of all the system's possible histories, only the resultant history obtains existence in the actual modality, and it is the only that becomes observable in the four-dimensional space-time of the actual realm of our world. Hence, the actual history is the necessary combination of all possible histories although, each possible history is accidental and has its own probability.

If we use the analogy between the modal interpretation of the PLA and Leibniz's scheme of the striving possibles, the action could be compared with essence. The more essence a possible history has, the less action there is. It means that every possible history has its essence and tends towards existence in the actual modality. In the infinite set of the possible histories, only the history with the minimal action can exist as actual because it has the highest degree of essence and combines the greatest number of possibilities at the same time. In the possible modality or in the possible realm of the actual world, the collision and "competition" occur between the possible histories. The result of such a "competition" has the maximal essence and is manifested in the actual existence as the unique history. Other possible histories do not disappear completely since they remain in the possible modality. Thus, the metaphysical meaning of the action is a measure of essence, which consists of the necessity of each possible history to be realised in actuality. Consequently, the sense of the PLA lies not with the mystic economy of nature, but merely in the observable effect of the combination of the possibilities to move.

As we can observe, the modal interpretation of the PLA has some relations with the approaches of modal metaphysics (Section 5). It is very different from actualism, in which alternative, unrealised possible histories are fictions or theoretical constructions only. At the same time, it is close to Plantinga's (1974) view of an individual independent essence of the non-actual object. My hypothesis also closely resembles dispositional essentialism since the possible histories can be treated as manifestations or potencies unrealised in the actual world.

7 PLA and dispositional essentialism

Recently, the relationship between the PLA and dispositional essentialism provoked rich discussions (Katzav, 2004, 2005; Ellis, 2005; Bird, 2007; Thébault & Smart, 2013). According to dispositional essentialism[10], at least some sparse, fundamental properties have objective a propensity or disposition that exist and nudge the outcomes one way or another. The world is, ultimately, merely something like a conglomerate of objects and irreducible dispositions. The dispositional properties are, unlike categorical properties, supposed to be properties that are not wholly manifest in the present; thus, they are the ultimate ontological units that explain events. Any object that possesses the dispositional essence of some potency is disposed to manifest the corresponding disposition under stimulus conditions, in any possible world (Shoemaker, 1984; Ellis, 2001; Bird, 2006, 2007).

Applying dispositional essentialism to the PLA (also see Section 5), we can translate it as follows: Each point in velocity-configuration space represents an instantaneous pattern of dispositions or dispositional and categorical property instantiations, and the history

[10] There are other attempts to consider the propensities and dispositions as something real, see, for example (Suárez, 2004, 2011).

represents the actual evolution of the system through various states (Thébault & Smart, 2013).

Katzav (2004) argued that dispositional essentialism is not compatible with the ontological presuppositions of the PLA and, ultimately, dispositionalist ontology is not able to account for the metaphysical presuppositions of science. Katzav made the assumption that the PLA suggests that the action of any given physical system could have taken various values, and, thus, that any such system could have been correctly described by the different equations of motion. The PLA allows us to derive the equations of motion of a system by comparing the various quantities of the action that the system might have had rather than by appealing to the system's actual history, it does not offer an historical explanation for why the actual equations of motion are actual. The PLA requires that dispositions do supervene on nondispositional properties taken together with something like a law, namely whatever makes the PLA true.

In reply to Katzav, Ellis (2005) argued that only a sophisticated dispositionalist can accommodate the PLA and its metaphysical necessity. He supposed that how things are disposed to behave also depends on how the kinds of things and properties are placed in the natural kinds hierarchies. Ellis claimed that the PLA is of the essence of the global kind in the category of objects or substances. Then every continuing object must be disposed to evolve in accordance with the PLA.

Thébault and Smart (2013) argued with Katzav and stated that dispositional essentialism is consistent with the PLA. One of the reasons is that there is only one metaphysically possible history in which the physical system could have evolved, but this still allows for there to be many logically possible histories. Despite all their arguments and objections, Thébault and Smart accepted that the dispositionalist has no the teleological metaphysical interpretation explaining the important and surely non-accidental PLA. Katzav, in his turn, left open the question of whether dispositionalism remains viable. He proposed that we might try to maintain dispositionalism, for example, by combining the instrumentalist view of the PLA with the realist view of the equations of motion.

I suggest reconciling the PLA with dispositional essentialism in another way based on the modal interpretation of the PLA. We might suppose that each object's possible history possesses own disposition.[11] According to Bird (2006), possibilia (such as unrealised

[11] Popper (1990), for example, had seen a world of propensities, as an unfolding process of realising possibilities and of unfolding new possibilities; and the propensities or dispositions that have not realised themselves, have their own reality. Each of these propensities has an objective measure, which can be associated with probability. Concerning quantum objects, the Popper's theses were

manifestations of potencies) are things that have being in our world but do not exist. Thus, unrealised manifestations of possibilities are part of the world just as much as realised manifestations. In Section 5, I assumed that if we apply such a view of possibilia to the PLA, we could say that the possible histories have essences and being in the possible modality as unrealised manifestations of possibilities, but the possible histories do *not actually exist* in the actual modality. Thus, the observed history with an extremal action is the realised manifestation of one of the possible histories and exists in the actual world.

According to the modality model of the PLA (Section 6), the actual or actualised histories have existence in our world due to their higher degree of essence. The dispositions of actualised histories differ by degrees of necessity in being manifested in the actual modality, and the degree of necessity can be measured by the value of the action. It seems that the dispositions also "compete" with one another. The result of this "competition" with the maximal disposition, maximal degree of necessity, and minimal action is realised in the actual history. The "competition" of the dispositions occurs simultaneously in the possible realm of our world. Other dispositions remain unrealised.

8 Modal interpretation of PLA and teleology

One of the metaphysical issues of the PLA is teleological (Section 4.1). It is as if a physical system "foresees" in advance which history (of all possible histories of motion) will minimise an action. The system seems to "choose" the actual history along which an action is less than of along other histories. It seems as if the system's final state determines the history that the system takes to reach that state.

Now I examine how the modal interpretation of the PLA can change the view of causality in the PLA. According to the modal interpretation of this principle, we replace the classical representation of a system's motion along a single actual history by a representation of simultaneous motions along an infinite set of possible histories. It means that the possible motions occur simultaneously in the possible realm of our world. It is no longer necessary to say as if the system "knows", in advance, which of its histories possesses the minimal action and thus will be the actual history. The system does not need

that the propensities are the relational properties of the quantum entities in experimental set-ups; the quantum wave function, or state, is a description of a propensity wave over the outcomes of an experimental set-up.

to "choose" anything. Rather, it merely uses or actualises the maximal number of possibilities of motion in each subsequent actual event. To achieve this aim, the system moves simultaneously along all possible histories. All systems do the same in the possible modality; they just try every possible history.

According to the combination model of the modal interpretation of the PLA, due to the combination, the maximal number of the possible histories integrates with each other into the actual history in actual modality. In moving along the actual history only, the system can take the maximal number of its possible states. Only the actual history has the highest degree of essence and combines the greatest number of possibilities at the same time. The system does not need to "calculate" the value of the action or anything else; the rule of the combination of the history's actions does so. The continuous mutual play of the system's attempts in the possible modality and the following combination of these attempts create the system's actual events and actual history and as a result, the entire actual realm of our world.

The approach to the analytical mechanics through simultaneous movements of physical systems is quite weird. However, this approach can help us with the fundamental metaphysical challenge of the way in which the universe actually works. We commonly predict a future state of a system if we know the initial state and the differential equation of the dynamic law. From this, we sometimes conclude that the universe makes the same and as if it "calculates" its own states successively, one by one. In other words, we believe that in the metaphysical law, the past causes the future. However, more often, we can predict a future state of a system by using the PLA or other principles of analytical mechanics. Wharton (2015) posed two unexpected questions. What if the universe works in a different way and does not follow Newton's rule but Hamilton's and Lagrange's? What if the universe does not "considers" its states and histories successively but all at once? Perhaps the metaphysical law states that an actual state is a result of all possible states and all possible histories connecting the past and the future. Thus, according to Wharton, without the mentality that the past "causes" the future by some algorithmic process, the question "How do objects in the universe 'know' what future boundary they are supposed to meet?" is no longer well-posed.

9 Modal interpretation of PLA and laws of motion

I believe that the modal interpretation of the PLA can both to address the metaphysical issue of the necessity of this principle (considered in Section 4.2) and provide a new view of the laws of motion. In this section, I start from the metaphysical perspective and then suggest some physical arguments in favour of my position.

9.1 Between Humean and non-Humean concepts

The Humean recognises only one actual world, and that the laws of this world are descriptions of regularities exhibited by the events in the actual history of our universe. Modalities, like possibility, necessity, and counterfactual statements are introduced as conceptual tools that enable us to deal theoretically with the actual world; they do not have an independent life of their own (Dieks, 2010).

At first sight, the PLA is the same law as all of the others; and the Humean concept of the laws of nature, without amendment or discomfort, is happily committed to the PLA being the most fundamental law of nature (Thébault & Smart, 2013). Authors have argued that the PLA is the law in virtue of the history that the physical system follows is that which extremises action. Though they have agreed that, even with regard to the Humean view, the PLA is our most fundamental law, from which all other laws of nature can be derived. Here, “the fundamental law” does not refer to some metaphysical essence; rather, it refers to the PLA having a more explanatory power. This seems to be a weak argument. The Humean view does explain neither why the most fundamental law appeals to the strange notions of the possible event and possible histories nor how these differ from the actual ones.

Thébault and Smart (2013) claimed that, from the Humean perspective, the PLA has the explanatory value. Here, they partly agree with Katzav (2004), though according to them, the real explanatory role is played by whatever makes the PLA non-accidental. Katzav showed that the explanatory force of the PLA is founded on the fact that certain quantities are extremal. It seems to imply that, if the history is actual, its actuality is not an accident; moreover, that something is not an accident enables appealing to it in explanations. Unfortunately, Katzav did not provide any positive metaphysical account of the PLA.

The arguments for the non-Humean view are insufficient as well. In Section 4.2, I assumed that if the non-Humean view is right and the PLA is metaphysically necessary truth, we face three consequent questions.

(1) How can other laws of motion (e.g., Newtonian laws) be mathematical and logical consequences of the PLA?

(2) How does the metaphysical necessity of the PLA involve contingency of the classical system's possible histories and uncertainty of the quantum system's probability amplitudes? What kind of the metaphysical mechanism manifests here?

(3) What is the source of the metaphysical necessity for the PLA?

Now, I outline a hypothesis whose answers lie between the Humean and non-Humean concepts of the nature of the PLA. To explain how it is possible, I suggest two new notions: the laws with a limited physical necessity and the laws with a general physical necessity.

First, let us return to the idea of the two-level modality considered in Section 5. The first level is the possible modality or possible realm of the world. The second level is the actual modality or actual realm of the world. According to the modal interpretation of the PLA, the possible histories have essences in the possible modality but do not actually exist in the actual modality. The actual or actualised histories have existence because, unlike the possible histories, they have more dispositions towards existence or a higher degree of essence.

This model means that, in the actual modality, the PLA is a mere regularity since the action's minimum is not a necessary reason for a history to be actual. The action could also be maximal or take a stationary value. Thus, the PLA seems to be a metaphysically contingent truth. However, in the actual modality, the PLA and other variational principles are universal tools of study regarding how various physical systems move, and they cannot be accidental. I agree with Thébault and Smart (2013) that the real explanatory role is played by something that makes the PLA non-accidental. I suppose that the unknown something that makes the PLA non-accidental is necessary and lies in the possible realm of the world. The PLA is the fundamental law that governs some kinds of physical motion, but the Newtonian laws are its mathematical consequences; however, in the possible modality, this is not the case.

Let us divide all physical laws of motion into those that are necessary only for certain kinds of physical systems and those that are necessary for most of the physical systems in our universe. Let us call the former *laws with a limited physical necessity* and the latter *laws with a general physical necessity*. The laws of motion with a limited physical necessity involve only the actual realm, and only actual motions can be described by these laws. The laws with a limited physical necessity lack any metaphysical grounds; thus, they are relative and metaphysically contingent truths. At the same time, they follow the laws of motion with a general physical necessity that also involve the possible motions in the possible realm of our world. In our case, the PLA and other variational principles are the laws of motion with a general physical necessity; their necessity is wider than that of the Newtonian and other differential laws, which work only for actual objects.

According to its modal interpretation, the PLA is a necessary consequence of two laws with a metaphysical necessity. The first law corresponds to the modality model of the PLA (Section 6) where all systems, in each actual state, tend to actualise the maximal number of their possibilities for motion. The first law calls for the actual objects to move simultaneously along all possible histories in the possible realm of our world.

The second law with a metaphysical necessity corresponds to the combination model of the PLA, where due to the combination of all the system's possible histories, only the resultant history obtains existence in the actual modality, and it is the only one that becomes observable in the four-dimensional space-time of the actual realm of our world. Thus, the second law calls for the actual history to be the sum of all possible histories.

The PLA as the law with a general physical necessity plays a unique role. It is an intermediate law between the laws with a limited physical necessity and those with a metaphysical necessity. Some laws of motion are direct consequences of the PLA, while others are not, but all are necessary consequences of the two laws with a metaphysical necessity that rule the possible histories in the possible realm of our world. In Section 9.2, I suggest some physical arguments that support my position.

9.2 Laws of motion

The first argument based on the PLA and any variational principles reveals the association of conservation laws with symmetries when the appropriate symmetries are to be found and when systems can be modeled by using a Lagrangian. According to idea of E. Noether, "any infinitesimal transformation of either the action variables, or the independent variable, which leaves the Lagrangian unchanged, leads automatically to a certain conservation law" (Lanczos, 1986, p. 386). Feynman (1994) wrote that there is a deep connection between the symmetry laws and conservation laws, but the connection requires that these laws obey the PLA. From the PLA and other variational principles we can get the differential equations of motion. Moreover, the conservation laws can be derived either from the differential equations of motion or the variational principles (Goldstein, et al., 2002; Hanc & Taylor, 2004; Brizard, 2008). Wigner (1972) called the symmetry principles "a superprinciple, which is in a similar relation to the laws of nature as these are to the events." Using the relationship between the PLA and FPI, Feynman (1994) also showed that the symmetry laws and conservation laws "satisfy a principle of least action, it turns out, because they come from quantum mechanics."

Thus, we can suppose that the laws of symmetry and conservation, as well as the PLA and FPI, can be included in a group of laws with a general physical necessity. It means that they are necessary consequences of some laws with a metaphysical necessity.[12]

The second argument – that the PLA lies between the laws of motion with a limited physical necessity and those with a metaphysical necessity – is that it and other variational principles are widely used for solving dynamic tasks in many fields of physics, without employing the differential equations of motion. As shown by Taylor (2003), the differential equations of the main theories of motion are connected with the PLA, which is the limiting case and approximation of the FPI. If we expand Taylor's scheme, the differential equations of the main theories of motion can be considered equivalent to one of the variational principles, which could be represented as limiting cases of the FPI.

For instance, Einstein's theory of relativity is connected with the relativistic principle of maximal aging (or the principle of maximal proper time) for the smooth curved space-time and the PLA for particles in the gravity field (Taylor & Wheeler, 2000). It is known that Newtonian mechanics is connected with the PLA in Maupertuis' and Hamilton's forms. The non-relativistic classical field theory is connected with the PLA for charged particles in the electromagnetic field. The differential and variational equations of the quantum field theory are connected with the FPI. The PLA, as well as Huygens' and Fermat's principles that form the basis of an optical–mechanical analogy, are the limiting cases of the FPI.

The various relationships among many differential laws of motion cannot be accidental. Something has to bind all these laws. Perhaps, it is the extremality of the system's characteristics. Indeed, in the PLA, the resultant history obtains the minimal action. In other variational principles, the resultant histories can obtain the minimal optical length, the minimal difference between kinetic and potential energy, minimal constraint, maximal proper time, the minimal curvature of space-time, and others. These characteristics are always stationary, taking either minimal or maximal values (since maximum and minimum are two undivided sides, which mathematically differ by only a minus sign). The FPI calculates probabilities by summing up over classical configurations of variables, assigning a phase to each configuration, which equals the action of that

[12] There are physicists who argue that there is not a general global energy conservation law in general relativity theory (Peebles, 1993).

configuration (Section 3). The probability amplitude of each possible history has an equal magnitude and varying phase, which corresponds to the classical action. The coherent histories are united due to the rule of interference. The resulting history has a maximal probability, which is given by the square of the sum of the probability amplitudes.

It seems that probability's maximum in the FPI somehow corresponds to the extrema of other actual physical systems' characteristics. If this intuition is true, the PLA and other variational principles could be reduced to the principle of maximal probability. Hence, to be realised in the actual realm of the world, it is enough that the history's probability be maximal, which arises from the summation of quantum probability amplitudes, as defined by the FPI.

If we continue to compare the PLA and FPI, we can associate each variation in the PLA with one of the possible histories in the FPI. It means that the variations can take place not only in the mathematician's head but also in the possible modality. Moreover, as emphasised in Section 8, the system does not "calculate" the value of every variation; the rule of the combination of the possible history's actions does this.

Summing up the answer to the question of the necessity of the PLA (Section 4.2), I suppose that this principle is a conceptual tool that enables us to deal theoretically with the actual world, and it lacks independent metaphysical essence (Humean view). At the same time, this principle is non-accidental since the source of its necessary lies in the possible realm of the world. The PLA is a necessary consequence of the two laws with a metaphysical necessity (non-Humean view). These two laws correspond to the modal interpretation of the PLA. The first law calls for the actual objects to move simultaneously along all possible histories in the possible realm of our world. The second law calls for the actual history to be the sum of all possible histories. Thus, the PLA lies between the Humean and non-Humean concepts.

This supposed approach could also explain how mechanical and geometrical descriptions of motion can be derived from the quantum behavior of systems. It can explain the efficiency of the variational principles for a description of any kind of motion, as well as the efficiency of the FPI for a description of quantum behavior. It could also explain why the calculus of variations is so widely distributed, not only in linear physics, but also in nonlinear thermodynamics, biology and theory of information. Perhaps, the simple and constant rules of the combination of possibilities constitute the reason why our universe seems to us so uniform, ordered, and harmonious. It seems that Wharton (2015) is right, and the universe does not follow Newton's rule but Hamilton's and Lagrange's. It does not "consider" its states and histories successively, but all at once.

10 Conclusion

The history of metaphysics has shown that the old metaphysical issues of the PLA cannot be solved by the old approaches. Unexpectedly, quantum mechanics can help us here, especially the FPI. Another source of inspiration can be Leibniz's concept of the possibles, which are always striving from essence to actual existence. This concept has long been rejected by most philosophers; today, however, it finds an ally in quantum behavior.

According to the proposed modal interpretation of the PLA, the classical systems, as well as quantum particles, are in the superposition of all their possible events, and they move from each initial actual event simultaneously along all possible histories under the given boundary conditions. By analogy with Leibniz's concept of the striving possibles, the action in the PLA could be compared with essence. It means that every possible history has its essence and tends towards existence in the actual modality. Among the infinite set of the possible histories, only the history with the minimal action can exist as actual because it has the highest degree of essence and combines the greatest number of possibilities at the same time. In the possible modality or in the possible realm of the actual world, the collision and "competition" occur between the possible histories. The result of such a "competition" has the maximal essence and is manifested in the actual existence as the unique history. Other possible histories do not disappear completely, since they remain in the possible modality. Thus, the metaphysical meaning of the action is a measure of essence, which consists of the necessity of each possible history to be realised in actuality. Consequently, the sense of the PLA lies not with the mystic economy of nature, but merely in the observable effect of the combination of the possibilities to move.

The modal interpretation of the PLA changes the view of causality in this principle. It is no need as if to "foresee" a final event or "calculate" a value of the action; the rule of the combination of the possible histories' actions does this. The system merely uses or actualises the maximal number of possibilities of motion in each subsequent actual event. The continuous mutual play of the system's attempts in the possible modality and the following combining of these attempts create the system's actual events and actual history. Summing up, we can assume that the totality of all possible events and possible motions of quantum objects forms the possible realm of reality, and the set of the actual events and actual motions forms the actual realm of reality. These realms are "parallel" and continuously pass into each other. Thus, the actual reality is the combination of all possible realities that coexist in a superposition in the possible realm of our world. Moreover, the actual reality constantly emerges due to the interactions of all possibilities. The general process when object's being changes from a possible to an actual modality, in some sense, seems similar Bohm's (Bohm, 1980) holomovement. This conclusion is also consistent with the common metaphysical hypothesis that quantum systems belong to the sphere of potentiality, and classical systems belong to the sphere of actuality.

The idea that the universe has multiple histories is now accepted as scientific fact, and some physicists are working to combine Einstein's general theory of relativity and Feynman's histories into a complete, unified theory that will describe everything that happens in the universe (Hawking, 2001, p. 80). I believe that modal metaphysics will be an efficient tool in this quest although the main obstacles, as always, will be our habits of thought and our common sense.

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