

The Multiple-Computations Theorem and the Physics of Singling Out a Computation

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ABSTRACT

The problem of multiple-computations discovered by Hilary Putnam presents a deep difficulty for functionalism (of all sorts, computational and causal). We describe in outline why Putnam's result, and likewise the more restricted result we call the Multiple-Computations Theorem, are in fact *theorems* of statistical mechanics. We show why the mere interaction of a computing system with its environment cannot single out a computation as the preferred one amongst the many computations implemented by the system. We explain why nonreductive approaches to solving the multiple-computations problem, and in particular why computational externalism, are dualistic in the sense that they imply that nonphysical facts in the environment of a computing system single out the computation. We discuss certain attempts to dissolve Putnam's unrestricted result by appealing to systems with certain kinds of input and output states as a special case of computational externalism, and show why this approach is not workable without collapsing to behaviorism. We conclude with some remarks about the nonphysical nature of mainstream approaches to both statistical mechanics and the quantum theory of measurement with respect to the singling out of partitions and observables.

1. INTRODUCTION

A very influential theory of mind in contemporary philosophy as well as science is the computational theory of the mind, according to which the relations between the mind and the brain is one of software to hardware, i.e., of a computation and the physical system on which it is implemented. If the mind (or certain parts of it) is a computation, the task of science is to discover what is the computation and how precisely it is implemented on the brain. A well-known problem faced by this view is (what we call in this paper) “*the multiple-computations theorem*,” according to which any relevant degree of freedom in the brain—down to its microscopic physical details, as it evolves according to the physical equations of motion during a given time interval—implements more than one computation, and the task for the computational theory of the mind is to find out which of them is the mind. We examine

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several proposals for solving this problem, show why they are not satisfactory, and propose our own solution to it. We analyze the notion of computation in terms of physics, within a reductive physicalist framework, and our solution is given in purely physical terms.

Hilary Putnam was perhaps the first thinker who proposed what is called today the functionalist theory of the mind, namely the idea that mental states are functional states, in particular computationally characterized states (see [Putnam 1967](#); for a general overview, see [Rescorla \[2017\]](#)). Putnam proposed this view as an alternative to the so-called identity theory according to which mental states are identical with physical-chemical states,¹ which he thought was wrong because it is quite implausible. But twenty years later [Putnam \(1988\)](#) realized that his functionalist approach to the mind faces a serious, perhaps devastating, difficulty:

The difficulty with this claim, and with all such claims, is not that physically possible organisms don't have functional organizations, but that they have too many. A theorem proved in the Appendix to this book shows that there is a sense in which everything has every functional organization. When we are correctly described by an infinity of logically possible "functional descriptions," what is the claim supposed to mean that one of these has the (unrecognizable) property of being our "normative" description? Is it supposed to describe, in some way, our very essence?" ([Putnam 1988](#), xiv–xv)

Here Putnam talks about what we call "*the multiple-computations theorem*," according to which our brain, as it evolves, implements during the same time interval more than a single computation. Since according to the computational theory of the mind our mind is a computation implemented by the brain, the theorem might entail that we have many minds (see, e.g., [Shagrir \[2012\]](#)); We call this "*the many-minds problem*." To avoid the many-minds problem one needs to show that only one of the implemented computations is preferred in that it is the mental one: the theory has to offer a criterion for *singling out that computation*. Now, the computational-functionalist view is usually taken to be a version of nonreductive *physicalism*, according to which everything is *fundamentally* physical; and then the question arises whether preference of one computation as the mental one can be done on the basis of physics. This is the question addressed in this paper.

The paper is structured as follows. In Section 2 we describe in a bit more detail the multiple-computations theorem and the problem of singling out a computation. In Section 3 we give in outline our argument explaining why the multiple-computations theorem is a theorem of physics, being a consequence of statistical mechanics. In Section 4 we argue that the view called computational externalism, according to which it is the physical interaction of a computing system with its environment that singles out one computation as preferred, fails because it leads to a regress that cannot be halted. In Section 5 we discuss attempts to dissolve Putnam's unrestricted version of the theorem by appealing to systems with certain kinds of input and output states, and we argue that this view is a special case of computational externalism and that, as noted by Putnam himself, it implies behaviorism. We conclude in Section 7 with some

remarks about the nonphysical nature of some mainstream approaches to both classical statistical mechanics and quantum mechanics with respect to the singling out of partitions and observables, and discuss what a physical solution of the problem should look like.

2. THE PROBLEM OF SINGLING OUT A COMPUTATION

According to Putnam's most influential and strong formulation of the idea of multiple-computations (Putnam 1988, 121): "[e]very ordinary open system is a realization of every abstract finite automaton."² Putnam illustrated this very strong theorem as follows. Consider a system that, following the laws of physics, evolves from time 12:00 to time 12:07, changing its *physical state* every minute, such that its states at these moments are $S_0, S_1, S_2, S_3, S_4, S_5, S_6$ and S_7 . If one *assigns the value* "1" to the disjunction S_0 or S_2 or S_4 or S_6 and the value "0" to the disjunction S_1 or S_3 or S_5 or S_7 , then the sequence of physical states may be seen as implementing the sequence of symbols 10101010. To see the nature of this theorem, notice that a different *value assignment*, in which the value "1" is assigned to the disjunction S_0 or S_1 or S_2 or S_3 and the value "0" to the disjunction S_4 or S_5 or S_6 or S_7 , results in the system implementing the sequence of symbols 11110000, as it undergoes *exactly the same physical evolution* as before. As we can see from the above quotation, Putnam's idea is that the value assignment is not dictated by the system's sequence of states, and in this sense as far as the computing system is concerned all of the value assignments are on a par, and none is preferred, that is: by looking at the computing system itself, in all of its details, it is impossible to single out any one computation as preferred. None is more actual or more natural (etc.) than the others; all the computations, corresponding to all the value assignments, exist simultaneously: not only potentially but in actuality.

Putnam's example may give the impression that the number of computations that may be simultaneously implemented by a given dynamical system is finite, but if we emphasize its physical basis, we can see that this is not the case. The states S_i are coarse-grained states and since the state space of a physical system is *continuous*, there are infinitely many ways of partitioning it to coarse-grained states such as the S_i sets, and therefore, even a simple mechanical system, consisting of a single particle constrained to a finite region of physical space and to a finite time interval, is subject to Putnam's strong result.³ Unless additional constraints on the notion of physical implementation are introduced, the system implements infinitely many computations corresponding to the infinitely many partitions of the state space into macrostates and corresponding value assignments. Searle's (1992) famous example of the wall behind him implementing the "WordStar" software is a point concerning this general applicability of Putnam's result.

The multiple-computations theorem raises two closely related problems. One is the many-minds problem, mentioned above. Since a single brain undergoing some specific microphysical process during some specific period of time implements several computations during that time interval, and since each of these different computations is associated with some different cognitive process, the same brain seems to have several minds. But empirically it seems that we have a single whole mind. How

can this be, and how can this be accounted for in terms of *physics*? Notice that the result is that there are several distinct *whole* minds implemented by the *whole* brain; this is not the unproblematic case in which different computations are implemented by different parts of the brain.

The other (closely related) problem concerns physical implementations of computations on systems that we use as computers, a paradigmatic case being the laptop on which this paper is being written. It is a fact that we perceive this physical system as carrying out one specific computation, for example some particular word processing, rather than any other computation that is actually being carried out on the same system at the same time, according to the multiple-computations theorem. How do we come to perceive the particular computation that we do? How can we account for this on the basis of *physics*?

Moreover, the two problems can be combined to form a third problem. Since without adding constraints, almost every physical process implements every finite computation (see [Godfrey-Smith 2009](#); [Schuetz 2012](#)), systems that we normally do *not* take to be cognitive systems at all seem to carry out, amongst the computations that they implement, also computations that are associated with cognition.

To solve the three problems, we focus here on the following question that gives rise to all of them. Since (according to the theorem) any physical system can be said to implement simultaneously infinitely many computations, can one single out on the basis of physics a single computation that is actually implemented? In other words, what are the *physical* facts that determine which computation (if any) is actually implemented? This is first and foremost a question of *fact* concerning the way in which physical computations come about.

The attempts in the literature to solve these problems are along two lines of thinking:

1. One is to object to the multiple-computations theorem and argue that it is mistaken in the sense that most (except perhaps one) of the processes are not “computations,” if one understands this concept properly.
2. The other line of thinking is to concede the theorem, and find criteria for preferring one computation as the one that is associated with the mind or with the computation implemented on this laptop.

Let us start with the first line of thinking; in the rest of the paper we shall focus on the second. Critics of Putnam’s result have argued that Putnam’s notion of the physical implementation of a computation is much too liberal, and that constraints have to be imposed on the physical-to-computational mapping if we are to say that a system implements a computation. For example, in order for a system to implement a computation it is argued that the system has to be associated with a certain causal structure; see [Chrisley \(1994\)](#), [Melnyk \(1996\)](#), [Chalmers \(1996, 2011, 2012\)](#); others have argued that the system has to satisfy some dispositional constraints, see [Klein \(2008\)](#); yet others have put forward mechanistic conditions, see [Piccinini \(2008, 2015\)](#); [Milkowski \(2013\)](#); for modal constraints, see [Chalmers \(1996, 2011, 2012\)](#);

Copeland (1996); and for pragmatic constraints, see Egan (2012). For an overview of this issue, see Piccinini (2017).

Since our aim is to address the second line of thinking, we do not purport to defend any of these different notions of physical computation. As a *working hypothesis* we shall use the so-called “simple mapping” account of computation which figures in Putnam’s (1988) analysis. In the context of the multiple-computations theorem, even if one adds constraints on the mapping, there are examples in the literatures showing that relatively simple dynamical systems that are nevertheless complex enough for implementing computations implement simultaneously more than one computation as they undergo some microdynamical process (according to any of the above-mentioned or other criteria; see, e.g., Copeland [1996]; Shagrir [2001, 2012]; Sprevak [2010]; Piccinini [2015]; and our [2019a, 2021a]). For our purpose in this paper, this modest version is all we need. Hereafter, we use the name the “*Multiple-Computations Theorem*” to refer to this modest version of Putnam’s result. We take this result to be uncontroversial.⁴ And so we turn to study the above-mentioned second line of thinking that aims to solve the problems raised by this theorem.

In the context of this second line of thinking, our aim in this paper is two-fold.

1. We first wish to generalize Putnam’s result and show in outline that the multiple-computations theorem is a theorem of physics, specifically: it is a theorem of statistical mechanics (classical or quantum).
2. Secondly, we will offer a solution to the problem of how to single out a computation based on the physicalist type-identity theory of mind and brain. In particular, we will show that the externalist approach for trying to do so, by which the physical interaction of a computing system with its environment selects a computation, doesn’t work since it is subject to an infinite regress. We will argue that the only way to avoid the regress and single out a computation on the basis of physics is to adopt the psychophysical identity theory.

One remark before we proceed. The problem of singling out a computation should be clearly distinguished from two other issues concerning the functional theory of the mind that Putnam was worried about. (1) One is Putnam’s argument that apparently a given *mental* kind may be realized by heterogeneous *functional* kinds, a result that eventually led Putnam to reject the functional theory of the mind (see Shagrir 2005).⁵ (2) Another is the argument that the same functional kind can be multiply realized by heterogeneous *physical* kinds. By contrast to the previous argument, this one was seen by Putnam (1967) as correct and as one of the strongest arguments in favour of functionalism, and remains so to this day (see Polger and Shapiro 2016). We have argued in detail elsewhere (2019b, 2021a) that multiple realizability of mental kinds by physical kinds is *incompatible* with physicalism, since it implies that each and every mental state (token state) has *nonphysical* features.⁶ We set this point aside in the present paper.

From now on we focus on the question of how to single out a computation on the basis of physics alone, and we will show that this can be done only if mental kinds are strictly identical with physical kinds (so that multiple realizability is ruled out).

3. MULTIPLE COMPUTATIONS AND THE PHYSICS OF COMPUTATION

Let us start by specifying explicitly our working hypotheses.

Working Hypothesis 1: As stated above, in what follows we shall work with the so-called “simple mapping” account of computation which figures in [Putnam’s \(1988\)](#) analysis.

Working Hypothesis 2: As noted above, some complex enough systems that are suitable for implementing computation (according to any of the above-mentioned accounts of computation) implement more than one computation as they undergo some microdynamical process.

Working Hypothesis 3: Our third hypothesis is that everything is fundamentally physical. In particular, computations are implemented by physical processes as described by contemporary physics (classical or quantum).⁷

We will now show that the multiple-computations theorem is a special case of a more general theorem of statistical mechanics, in which a similar problem arises of whether and how there are preferred partitions of the states of a physical system to macrostates. We will first describe the origin of the partitions problem in statistical mechanics and then make the connection with computations.

One of the core ideas of statistical mechanics is this. A system is, at each moment, in a given *microstate*, which is its precise state, and this microstate evolves according to the equations of motion. Every microstate is different, but different microstates can be partially identical, that is, they can be identical in some aspect of them (in some so-called *macrovariable*) given by their partial description. A set of microstates that share the same macrovariable, that are an equivalence set relative to that macrovariable, is called a *macrostate*. Consider for example macrovariables that are energy distributions: the microstates of a system are partitioned into sets in each of which the energy is distributed among the particles in a different way. A famous macrostate is the Maxwell-Boltzmann energy distribution (see [Sklar 1993](#); [Albert 2000](#); [Uffink 2007](#); [2017](#); and our [\[2012\]](#)). What makes it special? It happens to be the case that if the energy is distributed among the particles of an ideal gas in that way, then the gas satisfies certain regularities, for example the ideal gas law. But the states of a physical system can be partitioned into macrostates in infinitely many ways (actually a continuous infinity) corresponding to infinitely many possible partial descriptions of the microstates. For example, the microstate of a system *S* can be measured by various measuring devices, each of which is sensitive to a different *aspect* (or *macrovariable*) of *S*’s microstates, and (correspondingly) provides a different partial description of it. Each such partial description brings about a different partition of the microstates of *S* into macrostates. One device can measure (for example) the position of *S*’s particles (or the volume of *S*) while another device can measure the voltage, and the outcome of each measurement will entail that the same microstate belongs to a different equivalence set of microstates. All of those macrostates-sets and corresponding macrovariables, are on a par: as it were, they all “exist” in the same sense, since all of them are

nothing but aspects (given by partial descriptions) of the microstate, which according to physicalism is all there is in the fundamental reality. Since all of these sets are brought about by partial descriptions of the same microstate, this same microstate belongs to all of these sets simultaneously. Moreover: as the microstate of the system evolves according to the laws of nature, its different macrovariables (aspects) evolve too, and some of them may exhibit regularities.

Despite all of this, it is a fact that *we* experience only some of the macrovariables of the microstates of our environment, including some that exhibit interesting and useful regularities. How is this fact to be explained? In a physicalist framework the explanation is quite simple: We are physical systems that interact physically with our environment, and are therefore physically sensitive to certain macrovariables of our environment and not others (See the “Ludwig scenario” in our [2016]).

Some of the macrovariables to which we (as physical system) are sensitive, exhibit regularities. This in itself doesn’t make them more “real” or more “natural” than other macrovariables. It may be that other macrovariables, to which we are not sensitive, exhibit regularities as well, and we are not aware of those regularities precisely because we are not sensitive to those macrovariables. As our science evolves we may come to conjecture that some macrovariables of our environment, to which we are not sensitive, behave regularly and may therefore be useful to us, and we may build measuring devices that are sensitive to them and “translate” them to macrovariables to which we are sensitive.

The fact that the choice of a preferred partition that has physical significance depends on which macrovariables *we* happen to be able to measure or sense (via our sense organs) may raise the suspicion that a nonphysical element of an “agent” or a “mind” or a “self” etc. is being introduced into the theory. Is that the case? If one accepts the physicalist approach one is *committed* to the answer in the negative. And then the only way to account physically for the fact that a partition is preferred relative to an observer is to accept a fully physical account of the observer, that is, to accept type-identity reductive physicalism of the mind. (We have shown elsewhere that nonreductive approaches are committed to token dualism, see our [2019a, 2019b, 2021a, 2021c]).⁸

As we said, these ideas come from statistical mechanics. Here are their consequences for the theory of physical computation.

According to the minimal “simple mapping” account of computation, a computation is fixed by two requirements: dynamics plus value assignment (see our [2019a, 2021a]).

The *dynamics* is, in terms of physics, the evolution of the complete microstate of the computing system, according to the equations of motion. Building a physical computer means building a system that behaves in a certain way.

Value assignment is, in terms of physics, focusing our attention on certain macrovariables of that microstate and calling them “1” and “0”; in other words, a value assignment consists in (i) *selecting as preferred* a partitioning of the set of microstates of the computing system into certain sets, in each of which the microstates share a certain aspect; (ii) a mapping from one such set of microstates (that share a certain

aspect) to the value “1” and from another such set of microstates (that share another aspect) to the value “0”.

If we wish to implement a computation of a certain logical function, we need to make sure that there is a *harmony* between the dynamics and the value assignment, and here there are two ways to go. (1) We can start by identifying a certain microdynamics of our computing system and then we choose certain macrovariables as representing the computational states 0 or 1 in such a way that the computation we are interested in will be carried out as the microstate evolves. (2) The other way is to start with a value assignment, namely, with a mapping between the 0 and 1 and certain macrovariables of the computing system, and then construct a dynamical process that the system will undergo such that as those mapped macrovariables evolve they will give us the desired computation. This second way is perhaps better suited to describe the construction of standard computers as we know them. The macrovariables are selected so that we, humans, are sensitive to them, for example: a part of the computing system consists of the output device in which pixels are of the size and colors to which our eyes are sensitive.

In sum: A physical implementation of a given computation by a given physical system is a particular *harmony* between the *microdynamical evolution* of the system and a *partition* of the microstates of the system into macrostates, such that if those macrostates are associated with computational states then the dynamics yields a sequence of macrostates that corresponds to the desired sequence of computational states, i.e., to the desired computation.

In statistical mechanics such harmony is the way to describe all macroscopic evolutions. Our result is that this idea of harmony between dynamics and partition to macrostates is the physical underpinning of Putnam’s (1988) theorem. According to our analysis, the *value assignment* in implementing a certain desired computation is nothing but a *partition of the states of the computing system into the “right” macrostates*: these macrostates are the “right ones,” since given the dynamics they yield the sequence of computational states that fits the desired computation. Conversely, given a certain partition, a microdynamics will be the “right one” if it yields the right sequence of computational states.

4. THE PHYSICS OF SINGLING OUT A COMPUTATION

The fact that we experience certain macrovariables but not others is explained, then, by our physical interaction with the environment. But this is not the end of the story because this idea raises several problems that need to be addressed.

We just mentioned that, despite the multiplicity of macrovariables of each microstate and hence the multiplicity of the partitions to macrostates, the observer may be sensitive to only one of those macrovariables, ignoring the rest for all practical purposes at that moment, and so that macrovariable is preferred relative to that observer. Talk about observers may have nonphysical connotations that we want to and can avoid. To do so it is convenient to start by thinking in terms of *measuring devices* (and address human observers later). Along the lines mentioned above, each possible measuring device is sensitive to only one macrovariable of the measured system, and therefore *objectively and physically relative to that device* that macrovariable is preferred.

Of course, *other* measuring devices will “see” *other* macrovariables as preferred, and that too is an objective physical fact. (In the context of value assignment, as we shall see, this means that every measuring device would fix a value assignment relative to it.)

However, here the following problem arises: a measuring device does *not* select a preferred macrovariable of the measured system, for the following reason. (In the context of value assignment, this will have implications for the externalist approach to the multiple-computations problem.)

We start (in this paragraph) with the most general and abstract argument, and then (in the following paragraphs) describe the idea in more concrete but perhaps a bit more restricted context. Once the measuring device and the measured system are coupled by the interaction they form a unified system, and from then on the trajectory of its microstate is best described in the state of the combined system. And now the problem addressed above reappears: the microstate of the combined system has infinitely many macrovariables, corresponding to infinitely many ways to partition the (combined) state space into macrostates-sets. Under some of these descriptions the measuring device will appear to have the properties and regularities we are used to seeing in our experience, and these may nicely correspond to the relative macrovariables that we expect to see in the measured system. Different descriptions, however, that is, different partitionings of the combined system, will not yield such nice-looking properties and regularities, and will not reveal these facts of which we are familiar from experience. The properties and behaviors when seen under these partitionings will look cumbersome. As far as physics is concerned, this fact does not make them less “real” nor less “existent” nor even less “natural” than the nice-looking partitionings; so nothing in the physical underpinning of measurement yields a criterion for preferring one partitioning over others (or one value assignment over others). And this fact is precisely what we want to explain: *why* is that that we see the world in terms of the “nicely behaving” macrovariables and not the “cumbersome” ones, given that from the perspective of fundamental physics all of the macrovariables are equally real.⁹ Can physics provide a criterion for such preference?

Here is a more concrete (but perhaps more limited) way of thinking about the same problem. A measuring device is part of the environment of the measured system; for simplicity, let us assume for a moment that the measuring device is the entire environment of the measured system. The idea that the interaction with the environment selects the preferred computation (or partition) is sometimes called in the literature *computational externalism* (see, e.g., Harbecke and Shagrir [2019]; Shagrir [2018]; Piccinini [2015]). The idea is that the environment is sensitive to certain properties (aspects, macrovariables) of the computing system and not to others; and so relative to that environment these macrovariables are preferred and give rise to the value assignment, and hence select one computation as preferred, thus solving the multiple-computations problem. The environment of this laptop, for example, is sensitive to certain properties (macrovariables) of the laptop and not to others; and so relative to this environment, those properties are preferred in that they determine the computation that the laptop actually carries out (that it carries out Word rather than Searle’s WordStar, for example). A particularly important part

of this laptop's environment is the human observer. The engineers build our laptops with our physical making in mind, so that the way that we interact with the laptop induces a value assignment that, given the dynamics—that is built to be in harmony with this value assignment!—the desirable computations appears. And this seems to be an objective-physical basis for preferring a certain computation over others!

Unfortunately, this idea of how to single out a computation by means of a physical interaction with the environment doesn't work. To see why, we need to think for a moment about the physical nature of measuring devices (which is the environment). Consider a measuring device *E*. This measuring device, being itself a physical system, which is in some physical microstate at each moment, and its microstate has its own macrovariables. In fact, when our measuring device *E* interacts with our system of interest *S* (thus allegedly fixing one of its macrovariables as preferred relative to *E*), it is not the *entire* microstate of *E* that interacts with *S*: only some macrovariables of the microstate of *E* get correlated with some macrovariables of the microstate of *S*. Suppose that, as *E* interacts with *S*, the macrovariable *ME1* of *E* gets correlated with macrovariable *MS1* of *S*. The environment *E* has other macrovariables as well, and it may be (and is often the case!) that another macrovariable of *E*'s microstate, say *ME2*, interacts with a different macrovariable of *S*'s microstate, say *MS2*. For example, the electromagnetic interaction between *E* and *S* brings about correlations between certain macrovariables of their microstates, and the gravitational interaction between *E* and *S* brings about correlation between other macrovariables of these microstates. And so, in virtue of these two simultaneous interactions (of *ME1* with *MS1* and of *ME2* with *MS2*) the *same* environment *E* picks out *two* distinct macrovariables of *S*; and again, none of them is preferred over the other. In order to prefer *MS1* over *MS2* (or the other way around of course), we need to prefer *ME1* over *ME2* (or vice versa). But nothing in *E*, nor in *S*, dictates this preference. Needless to say, introducing another environment in order to prefer *ME1* over *ME2* leads to a useless (possibly infinite) regress (see figure 1). And so, we are back to square one: the multiple-computations problem cannot be solved by introducing the interaction with the environment, because we now have a *multiplicity* of system-environment correlations.

Here is another way to look at this. The laptop on which this paper is written implements more than a single computation, and the fact that you see it as implementing one particular computation (e.g., word processing) is relative to the fact that a particular macrovariable in you (call it *ME1*, alluding to the previous discussion) interacts with a particular macrovariable of the laptop (call it *MS1*, alluding to the previous discussion). This solves only part of the problem, but the main problem remains: what selects within your sense organs or brain or both the macrovariable *ME1* rather than *ME2*? Both exist in your brain, so it might have been the case that the interaction between macrovariable *ME2* (rather than *ME1*), with its interaction with the laptop's macrovariable *MS2* (rather than *MS1*), would fix the computation that you experience in the computer. The problem is that, as far as we could see until now, there is no physical fact that prefers *ME1* over *ME2* within your brain, without repeating the problem by some sort of an infinite regress.¹⁰ (Additionally, it also seems problematic to think that your experience is fixed by something or somebody

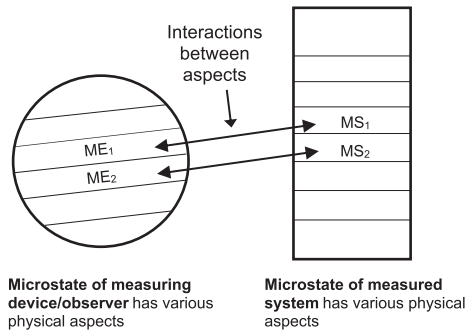


Figure 1. Regress due to multiple system-environment correlations.

observing you, so that if that observer is replaced or disappears your experience would change or disappear. But we skip this problem now.)

Nevertheless, despite the above regress, it seems to us that this externalist approach is in the right direction, although it misses a crucial point, as we shall immediately see. Our conjecture is that the regress can be stopped by a *further special* kind of step along the chain of interactions.

The idea is this (see figure 2): the fact that we have a single mind, as well as the fact that we see our laptop as implementing a certain particular computation, are facts about our experience of ourselves and of our environment. And the solution is to understand this experience within reductive type-identity physicalism, as follows. Among the many macrovariables of the observer, one is special in that it is *identical with* the observer's mental experience (see figure 2). The fact that, whenever the observer is in a microstate that has macrovariable ME2 (for example, as in figure 2), the observer is in a certain mental state, regardless of the environment in which this observer is situated and regardless of the fact that the observer's brain has other macrovariables (such as ME1, in figure 2); those macrovariables exist, no less than ME2, and qua macrovariables of the microstate of the observer they are on a par with the macrovariable ME2; the only difference is that macrovariable ME2 is *identical* with the observer's mental experience. And this is the reason why in the experience of this observer there is preference to ME2 and correspondingly to MS2 of the environment.¹¹

In this way the mind-brain identity theory stops the regress. If one accepts an identity theory, it follows that there is a fact of the matter concerning which macrovariables of the brain (perhaps together with our sense organs and body) are identical with which mental states. The central hypothesis of this theory is that every mental state of every mental type M at any time *t* just is (*identical to*) a state in which some macrovariable M of the brain obtains at *t*. If this hypothesis is true, the regress along the chain of interactions is stopped whenever the mental state of an observer becomes correlated with a certain partition of the states of the computing system.

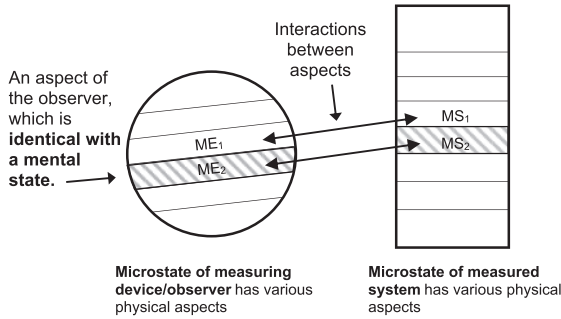


Figure 2. Regress halted by observer's preferred mental states.

5. COMPUTATIONAL EXTERNALISM AND BEHAVIORISM

Up to now we have argued that the interaction with the environment is not enough to select a preferred partition (or computation), and therefore that computational externalism faces a regress. In particular we argued that the only way to stop the regress and select a computation is by *identifying* certain physical *internal* states of the system which are identical to an observer's mental states. We shall now argue, following Putnam but within our present framework, that any attempt to solve the multiple-computations problem and stop the regress by appealing to some external input and output states lead to straightforward behaviorism.

The externalist view (to which we object here) is sometimes motivated by observations such as the following.¹² We know from observations that many living organisms respond to external stimulations by moving their bodies. And we know from brain science that such motion is a result of muscle contractions and relaxations, which are brought about by signals received from nerve endings at neuromuscular junctions. In short, spike trains are the main triggers of behavior. The thought is that the neuromuscular junction is the (part of the) local environment of the computing system (which is mostly inside the brain), and this local environment fixes the relevant neurocomputational states in the computing system. Let us describe this thought in terms that are more closely connected to our present discussion. The behavior is the output of the computing system, and therefore neuromuscular junctions are the relevant environment, which selects the preferred computation as the one which is identical to our mental states. In other words: *because* the behavior—which is the evolution of a *certain* set of macrovariables in the body (that is, in the neuromuscular junctions environment)—is the output, the *relative* macrovariables of the computing system in the brain are the ones that determine the computation. A different set of macrovariables of the computing system in the brain would not only determine a different computation, but would also fix a different set of macrovariables in the body, which are not the “behavior” as we experience it and seek to explain it. Let us call the set of macrovariables of the neuromuscular junctions that are correlated with standard notions of “behavior” and are correlated with certain macrovariables in the brain “the *nice* set of macrovariables”; and other sets are “*cumbersome*” since they

correspond to facts in the world that we cannot clearly call “behavior,” and that in general we don’t even perceive as “kinds” since they are not correlated with the macrovariables to which our sense organs are sensitive and that appear in our theories. The question now is this: what sort of fact makes it the case that the “nice” set of macrovariables are “real,” so that this is “really” the computation carried out in the brain? In particular, is this sort of fact *physical*? This is the problem parlayed into the terms of our present discussion.

As already argued by Putnam (1988, Appendix), the multiple-computations theorem itself entails that if one fixes (for whatever reasons) certain input and output states of a desired computation, then the interaction between the systems entails that there are relative physical states of the computing machine that will be mapped to the logical states of the computation and will stand in the ‘right’ causal relations to one another and to the input and the output states. We join Putnam in concluding that if this argument stops the regress it does so at the cost of straightforward behaviorism:

Imagine, however, that an object S which takes strings of “1”s as inputs and prints such strings as outputs behaves from 12:00 to 12:07 exactly as if it had a certain description D. That is, S receives a certain string, say “111111,” at 12:00 and prints a certain string, say “11,” at 12:07, and there “exists” (mathematically speaking) a machine with description D which does this (by being in the appropriate state at each of the specified intervals, say 12:00 to 12:01, 12:01 to 12:02, . . . , and printing or erasing what it is supposed to print or erase when it is in a given state and scanning a given symbol). In this case, S too can be interpreted as being in these same logical states A, B, C, . . . at the very same times and following the very same transition rules; that is to say, we can find physical states A, B, C, . . . which S possesses at the appropriate times and which stand in the appropriate causal relations to one another and to the inputs and the outputs. The method of proof is exactly the same as in the theorem just proved (the unconstrained case). Thus we obtain that the assumption that something is a “realization” of a given automaton description (possesses a specified “functional organization”) is equivalent to the statement that it behaves as if it had that description. In short, “functionalism,” if it were correct, would imply behaviorism! If it is true that to possess given mental states is simply to possess a certain “functional organization,” then it is also true that to possess given mental states is simply to possess certain behavior dispositions!” (Putnam 1988, 124–25)

In the above externalist motivation, the preference of certain sets of macrovariables (for what we called the “nice” set of macrovariables) is due to their being *behavioral* states, since it is only facts accessible to us about the external behavior of the organism that filter out all other computations that are equally implemented in the brain according to the theorem.

Thus, we can phrase our argument in the form of a dilemma for computational externalism and the computational theory of the mind: Given the multiple-

computations theorem, there are only two (exclusive) ways to single out a preferred computation:

Horn 1: Adopt computational functionalism, and fix the computation by the interaction of the computing system with its environment, thus endorsing behaviorism, as Putnam says;¹³

Horn 2: Adopt a full-fledged type-identity theory in which the internal macrovariables of the brain identical to mental states fix the preferred computation.

In Horn 2 the account of the mental is type-identical, and not computational. One might, for reasons of convenience, choose to describe the evolution of the macrovariables that are identical with the mental states and processes in terms of computation, and to the extent that this is convenient we have no objection to it; but then the computational description is not significant, and *multiple realization is ruled out*. The fact that the mind is described as implementing a computation is secondary and not essential to the fact that some physical states of our brain (and perhaps body) are (identical with) mental states. What makes physical kinds mental is the type-identity of mind and brain, rather than some computational identity. This is the internal feature that stops the regress and breaks the symmetry of all simultaneously implemented computations implied by Putnam's theorem.¹⁴ (It also fixes the computation by this laptop as the computation that we physically see due to the interaction between our physical[-mental] macrovariables and those of the laptop.)

6. CONCLUSION

We have shown in this paper that the only way to solve the multiple-computations problem and single out a computation is by appealing to the brain states of observers. Here it is instructive to cite von Neumann, who wrote about the quantum-mechanical theory of measurement:

[W]e must always divide the world into two parts, the one being the observed system, the other the observer. In the former, we follow up all physical processes (in principle at least) arbitrarily precisely. In the latter, this is meaningless. The boundary between the two is arbitrary to a very large extent That this boundary can be pushed arbitrarily deeply into the interior of the body of the actual observer is the content of the principle of the psychophysical parallelism—but this does not change the fact that in each method of description the boundary must be put somewhere, if the method is not to proceed vacuously, i.e., if a comparison with experiment is to be possible. Indeed, experience only makes statements of this type: an observer has made a certain (subjective) observation; and never any like this: a physical quantity has a certain value. (von Neumann 1955, ch. 6, 420)

Applying von Neumann's line of thought to the computational theory of the mind and the problem of selecting the "mental" computation, *which is our mind* according to this

theory, the many-minds problem is solved by realizing that the preferred macrovariable, that gives rise to a preferred partition to macrostates is the one *identical* with our relevant mental states. This preferred partition, in turn, leads to the preference of certain macrovariables in our environment, for example of this laptop, and hence to the experience that the laptop carries out one computation rather than another.

It is crucial to note that here identity physicalism is absolutely necessary for solving the problem on the basis of physics. One can easily see from our argument that if the mental states of observers are not strictly identical (in the sense of type-identity) to certain macrovariables of the brain, then von Neumann's division of the world into observer and observed along the measurement chain will introduce into the chain *nonphysical* facts. In this sense, only identity physicalism accounts for *both* the environment and our mental states by the *same* physics, that is by stipulating that the mental states of observers are nothing but physical states, using the same physics in which one describes the observed parts of the world.

Indeed, it seems to us that our proposal of type-identity physicalism is a necessary condition for solving not only the multiple-computations problem, but also the measurement problem in quantum mechanics. In the quantum case, one can show that the objective identification of mental states of observers with features of the observers' brains plays a crucial role in solving the quantum version of the multiple-computations problem in the two major theories currently on the table in the foundations of (nonrelativistic) quantum mechanics: Bohm's pilot-wave (1952) theory (see our [2013]) and the theory of spontaneous state reduction by Ghirardi, Rimini, and Weber (1986). Both are correctly taken to be theories of "quantum mechanics *without* an observer," since in both theories mental states are not added to the primitive ontology, but rather they are identical with features of the brain that these theories attempt to describe (see Allori 2013). However, we showed elsewhere (see our [2020b, 2021b]) that our solution is *not* open for the Everett (1957) (many-worlds) interpretation of quantum mechanics. The reason is that in the Schrödinger evolution—which is all there is according to this interpretation!—there are no physically preferred bases unless one adds something to the Hilbert space structure: in particular, in states of quantum entanglement, nothing makes the brain states of observers—which by stipulation are identical to mental states in the *absence of entanglement*—physically preferred. This holds even if the entangled brain states of the observers are subject to decoherence.¹⁵

NOTES

1. See different presentations of reductive type identity physicalism in e.g., Smart (2017); Papineau (1993, 2002); Hemmo and Shenker (2019a, 2019b, 2021a, 2021c); and Shenker (2017c); see Stoljar (2021) for an extensive overview and discussion.
2. Requiring certain physical input and output adds some constraints, but even in that case Putnam's claim is quite strong; see Section 4 about this issue.
3. On coarse graining of the state space, see Sklar (1993); Albert (2000); Uffink (2007); Frigg (2008); Hemmo and Shenker (2012); Shenker (2017a, 2017b); Goldstein et al. (2020); on the multiplicity of partitions in the classical state space, see our (2016) ("Ludwig's problem"). On applying this result for the multiplicity of value assignments see our (2019a, 2021a).

4. Moreover, we have shown (see our [2019a, 2021a]) that according to statistical mechanics a physical system undergoing a given microdynamical evolution can compute at the same time interval two different computations: one of these computations is *necessarily thermodynamically minimally dissipative* according to Landauer's Principle in physics, whereas the other computation need not be dissipative (on Landauer's Principle, see Landauer [1961, 1992]; Bennett [1982, 2003]; on Landauer's Principle and physical computation, see Ladyman [2009]; and our [2019a, 2021a]).
5. Multiple realization seems to be compatible with supervenience of mental kinds on functional kinds (or on physical kinds). But the multiplicity of functions (e.g., computations) given a single physical process seems to be incompatible with supervenience (see, e.g., the many-minds problem mentioned above).
6. Here is the argument in outline. Consider a microstate M of the entire universe. What facts make it the case that M realizes (or is a token of) one high-level property L rather than another \acute{L} ? If these facts are in M (or about M) by assumption, multiple realizability means that there is no physical fact in M (or about M , i.e., there is no macrovariable of M ; see below), that can provide an answer to our question. The only remaining option is that there is some list of all the physical microstates that realize the high-level kind L , so that an item that belongs to the list is of kind L . Here is a dilemma. Either the list is part of physics (horn 1), or it isn't (horn 2).

Horn 1: If the list is physical, then we should add it to the microstate M , in which case multiple realization no longer holds with respect to the full state (call it M^*), since now (after adding the list to M) there is a physical fact about M^* that answers our question.

Horn 2: If the list is not physical, then the *nonphysical fact* of the existence of the list must hold in each and every microstate M , so as to determine whether M belongs to L or \acute{L} . And so multiple realization, if genuine, is a case of token dualism.

Another way to see this is by invoking Laplace's Demon. Suppose that the Demon has access to all possible and actual fundamental physical facts and only to physical facts. If so, the Demon can know whether M is of kind L only if the fact that makes M be of the kind L is a macrovariable of M . But if, as in Horn 2 of the dilemma, the fact that makes M a case of L is not in M , then it is not accessible even to Laplace's Demon, and this is a case of token dualism. Note that the same argument applies (*mutatis mutandi*) if M and M^* are replaced with temporal sequences of microstates (for more details, see e.g., Hemmo and Shenker [2019b]).

7. We don't address here Hempel's dilemma concerning the empirical adequacy and meaning of the hypothesis of physicalism; see our view on this issue in Firt, Hemmo and Shenker (2021).
8. The *same* argument (*mutatis mutandi*) applies in quantum statistical mechanics. Moreover, in pure Hilbert space quantum mechanics there is an additional problem of multiplicity, which does not arise in classical mechanics, namely the problem of how to single out a preferred basis in Hilbert space; see our (2020b, 2021b).
9. A counterpart of this idea undermines the Many Worlds interpretation of quantum mechanics even when there are decoherence interactions with the environment (see our [2020b, 2021b]).
10. The same regress arises in quantum statistical mechanics for essentially the same reason, where in quantum mechanics there is an additional problem of introducing new physical facts that should break the basis-symmetry of Hilbert space; see our (2020b, 2021b).
11. Nonreductive approaches reject identity and accept instead a metaphysical dependence relation of supervenience, or realization or grounding, etc. We show elsewhere (see our [2019a, 2019b, 2021a]) that all these approaches entail dualism, because the metaphysical relation that fixes which physical kinds (or states) belong to which mental kinds adds *nonphysical* facts into reality. For a nonreductive approach to statistical mechanics, see Albert (2000, 2014); Loewer (2020); and our criticism in (2021e). In Hilbert space quantum mechanics an additional problem arises because of the basis symmetry of Hilbert space with respect to the equation of motion and the physical state of the system; see our (2020b, 2021b).
12. We thank an anonymous reviewer for raising this point.
13. The multiple-computations theorem and all of our above arguments equally apply to the causal version of functionalism.
14. To single out a computation on the basis of physics alone, other proposals for singling out a computation (e.g., Piccinini [2015]; Coelho Mollo [2018, 2019, 2020]; Dewhurst [2018a, 2018b]; Schweizer [2016, 2019]; Millhouse [2019]; Shagrir [2018, 2021]; Fresco and Milkowski [2020]) should be compatible with type-identity of physical kinds and mental kinds. We argued elsewhere that any theory which is

committed to mind-brain supervenience but allows multiple realizability is *token-dualist* in the sense that it is committed to the existence of *nonphysical* facts on every occasion in a which there is a fact about the mental (see our [2015, 2019a, 2019b, 2021a, 2021c, 2021d]; Shenker [2017c]).

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