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Some non-revisionist solutions to some semantic antinomies

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Introduction

In this paper it will be shown that Russell's Paradox can be solved without advocating the Theory of Types and also that it can be solved without augmenting, regimenting, or otherwise revising natural language. It will also be shown that, in addition to being unnecessary, the Theory of Types is demonstrably false. Finally, it will be shown that the Epimenides (or "Liar's Paradox") can be solved in much the same way as Russell's Paradox.

1.0 Russell's Paradox

Let K be the class of all classes that are not members of themselves, i.e. that don't have the property of being self-members. In that case,

(K_1) "K is a self-member (i.e. K has the property of being a member of itself)

entails, and is entailed by,

(K_2) "K is not a self-member.

By the Law of Excluded Middle (LEM), *at least* one of those statements affirms a truth (a true proposition). By the Law of Non-Contradiction, *at most* one of those affirms a truth. But if the one is true, so is the other, in which case a given proposition is both true and false, which is impossible.

According to Russell (1902,1903,1908), the just-stated argument demonstrates the falsity of the Axiom of Comprehension (*for any property P, there is a class C such that C contains every instance of P and nothing else).* This contention of his is widely accepted.[1]

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[1] Some-e.g. Graham Priest (2006a, 2006b)—take it to show that there are true contradictions. Priest (correctly) believes that Ki *is* meaningful and, on that basis, infer (understandably but erroneously) that it *ipso facto* affirms some proposition. Priest knows that *if* K_1 affirms a proposition, that proposition is both true and false. So Priest holds that some propositions are

2.0 Why Russell has failed to identify an actual counterexample to the Axiom of Comprehension

The statement:

(S_1) "The class of all spoons has the property of not being member of itself

is merely an abbreviation for:

(S_2) "The class of spoons isn't a spoon."

And the statement:

(T_1) "The class of non-spatiotemporal entities has the property of being member of itself

is short for:

(T_2) "The class of non-spatiotemporal entities is non-spatiotemporal."

In general, when used meaningfully, the expression "property of (not) being a member of itself is eliminable. Why? Because "itself isn't to be defined denotatively, i.e. it isn't to be defined by identifying its referent. That expression *perse* no more has a referent than any other pronoun. "He," the expression-type, has no referent; it is *occurrences* of "he" that have referents (cf. "John has a lot money, but *he* doesn't have a car"). Similarly, "the property of (not) being identical with itself does not, in and of itself, have a referent; its *occurrences* have referents. The occurrences of it in S_1 and T_1 refer, respectively, to the property of being a spoon and the property of being non-spatiotemporal. (S_2 is equivalent with "the class of spoons doesn't have the property of being a spoon," the same thing *mutatis mutandis* holding of T_2.)

With these points in mind, let's take another look at K_1. The property that a thing must have to be a member of the class of spoons is that of being a spoon. What is the property a thing must have to be a member of K? In other words, what does "itself refer to in K_1?

"The answer is clear," it will be said. "It refers to the property of *not* being a member of itself. Since K is the class of all classes that aren't members of themselves, to say that K belongs to itself is to say that K doesn't belong to itself."

But this answer is circular. We wanted to know what "the property of being a member of itself referred to, and the answer we're given is: "the property of *not* being a member of itself," which leaves us with the same question *mutatis mutandis* as before.

We ask: "What property must a thing must have to belong to K? What does 'the property of being a member of K' refer to?" We are told: "the property of not being a

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both true and false. Since, as we'll now see, K_1 affirms nothing, this line of reasoning is spurious. (See Brown (2006) for a helpful discussion of Priest's work.)

member of itself." We then ask: "What does 'the property of not being a member of itself refer to?" We are told: "The property of being a member of K." Our questions haven't been answered.

Thus, "the property of (not) being a member of itself *doesn't refer to anything.* It's a free variable; it's like the occurrence of "he" in a token of "he has no friends" that occurs in a context where it is neither assigned a referent (as it would be if, while pointing to John, I said *"he* is a nice person") nor appropriately bound (as in, "if a man envies you, *he* is someone you should fear"). *Therefore "the property of (not) being a member of itself' doesn't have a referent and Ki therefore doesn't express a proposition.* Since K_1 it doesn't express a proposition, it isn't true or false; and since "the property of not being a member of itself doesn't pick anything out, we don't have a counterexample to the Axiom of Comprehension on our hands. So to the extent that the motivation for TT lies in the contention that K_1 *is* such a counterexample, there is no motivation for TT.

"You are guilty of a serious oversight," it might be said. "Though it doesn't refer to anything, the expression 'the property of not being a member of itself has a sense, like "the man on the moon," and K_1 therefore *does* express a proposition, just like 'the man on the moon plays the violin."'

First of all, if the expression "the property of not being a member of itself doesn't refer to anything, then we have made our case: we have shown that we don't have in Ki a counterexample to the Axiom of Comprehension. That said, if that expression has a sense, then (arguably[2]) we were wrong to say that

K_1 expresses no proposition. But this brings us to the second point: the "the property of not being a member of itself does not have a sense; it just appears to have a sense, however paradoxical this may sound. Let's suppose for argument's sake that it *does* have a sense. In that case, Ki is equivalent (though not necessarily synonymous) with:

K_{1S}: "There exists something that is uniquely a set of all sets that aren't members of themselves; moreover, that thing is a member of itself."[3]

In K_1, we said, the occurrence of "itself doesn't refer to anything, a consequence

[2] I say "arguably" in acknowledgement of the fact that Frege (1892) believed that sentences containing sense- bearing, but non-referring terms to express no propositions. He believed, for example, that "the man on the moon is a violinst" fails to affirm any proposition. But this view of his is inconsistent with his view, which he advocates in the very same paper the view just mentioned, that "the phi is psi" is equivalent with "there exists something that is uniquely phi and any such thing is psi."

[3] Strictly speaking, the correct paraphrase is: "There exists something that is uniquely a set of all sets that aren't members of themselves; and any such thing is a member of itself." By replacing the occurrence in K_{1S} of "that thing" with "any such thing," we sidestep the need to answer the question: "to what does the aforementioned occurrence refer?"

being that K_1 fails to encode a proposition. To the extent that those arguments were cogent, they also, very plainly, show that the occurrence in K_{1S} fails to refer. The expression "the man whose best friend *that woman* is" does not have a sense *unless the occurrence of "that woman" picks someone out.* If that occurrence is empty, then that expression has no sense; for, under that circumstance, there is no condition that it expresses such that any object, actual or possible or even impossible, could possibly satisfy that condition. The sense of a sense-bearing expression is nothing other than a condition that a thing must satisfy to be picked out by an expression. The sense of "the tallest spy" is the condition of uniquely being a spy who is taller than any other. If Smith satisfies that condition, then tokens of "the tallest spy" refer to him; otherwise, they don't. Nothing is picked out by the occurrence of "that woman" in a token, occurring in a woman-free context, of "the man whose best friend *that woman* is." For that reason, in such a context, there is no condition such that, if a thing satisfies that condition, it is the referent of that definite description. In objection to this, one might say that *there is* such a condition, albeit one that is varies, i.e. one that varies depending on whether it is Jane or Sally or Ethel (or no one) who is picked out by the occurrence of "that woman." But that only proves our point; for variables *ipso facto* don't refer. If "the occurrence of "x" in "x is even" picked something out, then "x is even" would say the same thing as "2 is even" or "three is even" or some other *bona fide* sentence. But "x is even" doesn't attribute the property of being even to anything, the reason being that "x" doesn't refer, the reason being "x" is a variable., and neither does the variable-like occurrence of "that woman" in the aforementioned definite description.

And, very obviously, the occurrence of "x" in the "the man who is x's best friend" doesn't refer, for which reason that definite description doesn't have a sense. (It *does* have a sense if, but *only* if, the occurrence of "x" is replaced with a constant, e.g. "Betty" or "Jane." Pending such a replacement, it has a "variable" sense, which means that it has no sense. By obvious extensions of this reasoning, the occurrence of "itself in K_1 lacks a sense, and so does the corresponding occurrence in K_{1S}. So the hypothetical objector's point is doubly misguided: even if it's correct, "the property of being a set of all sets that aren't members of themselves" doesn't pick out a property, and our central thesis still stands; and that point *isn't* correct, so that, just as we said, K_1 doesn't express a proposition, the same being true of K_{1S}, given that latter is equivalent with K_1. To sum up, "the property of being the set of sets that aren't members of themselves" picks nothing out, and that property, being non-existent, isn't a counterexample to the Axiom of Comprehension or, indeed, to any proposition.

Notice that the rules of English semantics are not at fault here. Those rules are unambiguous as to how anaphoric terms are to be interpreted. Those rules make it clear that, because it contains a free variable, "x is a mad man" affirms no proposition; and those rules make it clear that, for much the same reason, sentence(-tokens) containing free occurrences of pronouns *do not affirm propositions.* So the semantic rules constitutive of the English language assign neither truth *nor* falsity to K_1; so they don't

assign *both* truth *and* falsity to K_1. So they *don't* countenance any violation of LCN, at least not to the extent that, relative to them, K_1 violates some logical principle. So those rules do not, at least not to that extent, need to be modified in accordance with TT or otherwise logically reconstructed. So TT is unnecessary.

2.1 The eliminability of extensional occurrences of anaphoric pronouns

The fact that occurrences of "itself are always eliminable is a consequence of a more general principle, namely: the term "self is *always* eliminable. (I am referring to the term "self *qua* suffix, as in "himself," not *qua* garden-variety noun, as in "people have selves, whereas lizards do not." All references to "self are to be thus disambiguated.[4]) In fact, *all* pronouns are eliminable. This point is subject to three qualifications, none of which redounds to the credit of Russell's analysis of K_1.

First, when the word "self occurs in contexts governed by terms denoting propositional attitudes (e.g. "believes," "hopes"), it is not eliminable, at least not in any straightforward way. For example,

(L_1) "Larry believes himself to be a great pianist,"

isn't shorthand for:

(L_2) "Larry believes Larry to be a great pianist."[5]

Larry might have amnesia and not *know* that he's Larry but still believe that Larry—with whom, unbeknownst to himself, he is identical— to be a great pianist. The reason L_1 and L_2 aren't equivalent is that, in L_1, "self' falls within the scope of an expression ("believes") that denotes a propositional attitude. But in K_1 "self doesn't occur within the scope of "believes" or indeed *any* intensional operator: it is functioning extensionally. So this qualification is irrelevant to our critique of Russell.

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[4] Let's start with a purely clarificatory point. The word "self has two meanings or functions. Sometimes it can function as a garden-variety noun. That's how it's functioning in "Hume denied that he had a self." And sometimes "self functions as a way of constructing 'terms of laziness,' to adapt an expression of Geach's, viz. expressions that are useful, because conducive to expedience, but technically eliminable. That's how it functions in
(J_1) John punched himself,
which is short for:
(J_2) John punched John.
When functioning denotatively (as in "my self (in other words, my soul or psychological essence, or some such) is something that underlies and unifies the various fleeting occurrences and states constitutive of my consciousness") isn't eliminable (at least not in any relevant sense). But (so I am trying to show) when functioning *anaphorically,* it *is* eliminable (cf. J_1 and J_2). Henceforth, all references to the term "self are to the anaphoric device.
Now the requisite qualifications of our contention (that "self is always eliminable):

[5] Perry (1977) makes this point.

Second, when a pronoun functions as a bound-variable, as opposed to a *bona fide* device of reference, it is not eliminable, at least not in any straightforward way. The sentence:

($\$_1$) "If someone gives you $1,000,000 and asks for nothing in return, that person has a good heart."

is not equivalent with:

($\$_2$) "If someone gives you $1,000,000 and asks for nothing in return, someone has a good heart."

This is because ($\$_1$)'s meaning is given by a sentence in which "that person" is replaced by a bound variable:

($\$_3$) "for any persons x and y, if x gives y $1,000,000 and asks for nothing in return from y, then x has a good heart,"

But this is irrelevant to our critique of Russell's point, since the occurrence of "itself in K_1 clearly doesn't occur as a bound variable. K_1 is a singular statement, not a quantified generalization. It says of some (putative) object K that K isn't a member of K.

3.0 A positive argument against TT

TT says that no meaningful sentence concerns *every* sentence.

But TT is given by a sentence that violates that very stricture, and is thus self-defeating, since it's given by the sentence: "All meaningful sentences concern no sentences other than those that belong to orders (or types) lower than those to which they themselves belong."[6]

4.0 Generalizing these points so as to solve another, similar antinomy

Let tf be a token of the sentence-type:

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[6] James Cargile (1980) makes points that, although capable of interpretations, are not unreasonably interpreted as constituting a version of the argument just put forth. I believe that Russell (1908) himself anticipates it. (Russell makes a series of points—e.g. that, if the Theory of Types is correct, then there is no one Law of Excluded Middle, but one such law for first-level statements, an analogue of that law for second-level statements, etc.—which converge around the point that, if the Theory of Types is correct, then there is *ipso facto* no *one* Theory of Types, but one such theory for first-level statements, an analogue thereof for second-level statements, etc. But he doesn't go so far as to make that point; and given that it obviously wasn't a lack of acuity on Russell's part that held him back from making this point, we must assume it to be an awareness, if only an unconscious or vague one, of the fact that he couldn't have made it without *ipso facto* undermining the very theory he was advocating.

(TF) "This sentence is false."

According to the conventional wisdom:

(CW) tf is false if it's true and true if it's false. So, if it's either true or false, it's *both* true *and* false. Which is impossible. So it's neither. Therefore there is no proposition that it express.

Therefore, it is meaningless. But—it is usually said—it is a *bona fide* sentence-token.[7]

4.1 Why CW is a spurious argument

It is in virtue of what a given sentence-token *affirms* that it is true or false. Describing a sentence-token as "true" ("false") is an abbreviated way of saying that it affirms a true (false) proposition.[8] If it affirms anything, a given token of "this *sentence* is true (false)" affirms that some *proposition* is false. Technicalities aside, "this sentence is true (false)" means "this *proposition* is true (false)."

Suppose that the occurrence of "this sentence" in a token of "this sentence is true" refers to the sentence "snow is white." In that case, what is being said is an abbreviation for:

(TP) "The *proposition* meant by this sentence is false."

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[7] Advocates of CW (e.g. Russell (1908), Tarski (1930), and Kripke (1975)) take it to be to the discredit of the English language that the rules constitutive of it permit the construction of such a sentence. But, those rules being what they are, it is indeed a *bona fide* sentence-token of English.

[8] *A defense of this claim:* No series of noises is a sentence-fype: an ink-deposit is, at most, a sentence-token. (In this context, "burst of noise" is short-hand for "anything (be it an ink-deposit, a pattern of light on a monitor, etc.) that might token an expression, the same qualification *mutatis mutandis* holding of related terms (e.g. "acoustic"). But, given some ink-deposit, *which* sentence-type it tokens is a function of what it means. A series of noises *acoustically* just like utterances of the English sentence "snow is white" could mean anything. Let L be a language such that, for some sentence S that belongs to L, S means chickens can fly" but utterances of S are *acoustically* indistinguishable from utterances of the English sentence "snow is white." Obviously utterances of S are not identical with utterances of "snow is white", and tokens of S are not identical with tokens of "snow is white," a corollary being that S and "snow is white" are different sentence-fypes. So sentence-types are not individuated by the physical properties of their tokens; and, what obviously follows, it isn't solely in virtue of a given object's physical (e.g. acoustical or morphological) properties that a given token is an instance of this as opposed to that sentence-type. Given some physical object x, is in virtue, not just of what x's physical properties are, but also of what it means, that x tokens this as opposed to that sentence-type. If a burst of noise acoustically just like a token of "snow is white" were produced a million years ago by some volcano, it would token no expression. Since meaning doesn't supervene on acoustics—since, in other words, two acoustically indistinguishable entities needn't coincide in meaning—no burst of noise *by itself* constitutes an expression-token.

The semantic rule for TP is:

(STF) If, in context C, there is some uniquely salient sentence S whose meaning is some proposition P, then, in C, a token t of TF is true exactly if P is true; and if, in C, there is no such sentence and therefore no such proposition, then, in C, a token t of TF is neither true nor false, as there is no proposition affirmed by such a token.

The proposition, supposing there to be one, affirmed by a given sentence-token is a function of *inter alia* what its parts refer to. This is a consequence of the "principle of compositionality"[9] namely:

(CM) What is meant (referred to) by a given expression is determined by what its parts mean (refer to)."[10]

Let 0 be the occurrence of "the proposition meant by this sentence" in a token t of TP. If 0 refers to nothing, t is neither true nor false. What t affirms depends on what 0 refers to. 0 must have a referent *independently* of what t affirms. Otherwise t won't affirm anything.

This is easily shown. I'm having an exchange with my friend Jerry. I say: *"He* is *her* father." Since the context doesn't supply either one of the italicized terms with a referent, Jerry asks me who "he" refers to. I say *"her* father." Jerry then asks me who *"her"* refers to. I say *"his* daughter." If the answers I've given to Jerry are the only viable ones, then there is *no* viable answer to either question, in which case neither italicized terms refer to anything and I've therefore affirmed nothing. In an email exchange with Jerry, I say: *"the proposition meant by this sentence* is false." Jerry asks: "Which proposition is that?" I say: "The one meant by the underlined sentence." Jerry then asks: "Which proposition is meant by the underlined sentence?" I say: "the one meant by the italicized expression." If the answers I've given are the only viable ones, there is *no* viable answer to either question, in which case neither the underlined term nor the italicized terms refer to anything and I've therefore affirmed nothing.

4.2 What we are to conclude from the points made in Section 4.1

Tokens of TP are neither true nor false. So such tokens are not both true *and* false. So, assuming the correctness of the Strawson-Kaplan view of indexicals, the semantic rules of English *do not* permit the construction of sentences that violate LNC. (Of course, in this context, "English" could just as well refer to any natural language.) Many a logician-philosopher has argued English (and every other natural language)

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[9] Which was enunciated by Frege (1892). (Some of its more important consequences are clearly drawn in Barwise and Perry (1983).) I don't know whether Frege was the first to state it. (I doubt that he was.)

[10] The order in which those parts occur is also relevant, that being why "the man Smith saw standing next to Jones" doesn't have the same referent as "the man Jones saw standing next to Smith."

was defective. Often, that logician's reasoning was as follows:

> The semantic rules of English are propositions (to the effect that such and such symbols (or symbol-tokens) have thus and such meanings (or satisfaction-conditions). There exist sentences, such as "K is a self-member" and "this sentence is false," such that (i) relative to those rules those sentences (or their tokens) qualify as meaningful, and such that (ii) the propositions meant by those sentences are, if existent, violations of LCN. Therefore, a logical falsehood is among the logical consequences of those semantic rules. So, if we take the primitive semantic rules (e.g. "Smith" refers to Smith) of English (for example) as axioms, and the derived rules (e.g. "Smith runs fast" is true exactly if Smith runs fast") as theorems—and this is how they are to be taken—then we find there to be violations of LCN, and thus logical falsehoods, among the theorems in question. And no axiom-set can be accepted if it has a logical falsehood as a consequence.
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> At least one reason for this, it is to be noted, is that there is nothing that *doesn't* follow from such an axiom-set. So the axiom-set in question (the least inclusive set comprising the primitive semantic rules of English) no less has the consequence that "Smith runs fast" is true iff Richard Nixon was born on Mars than it does that "Smith runs fast" is true if Smith runs fast. This means that, relative to those rules, anything means everything: given any sentence, there is no proposition that it doesn't mean. And this in turn means, for all intents and purposes, that, relative to those rules, nothing means anything. So until we so revise the semantic rules of English that logical falsehoods are not among their consequences, any given English sentences means everything and therefore rules nothing out and therefore, in effect, means nothing. Thus, until we so revise those rules, there is, in effect, no English language. Thus we must revise them.[11]

In this paper, we have seen the semantic rules of English *not* to be guilty of at least two of the violations of the laws of logic of which they have been alleged to be guilty. And there is reason to believe that the considerations on the basis of which we established this can so generalized as to acquit the semantic rules of English of many other, similar charges of non-compliance with the laws of (classical) logic.

[11] See Russell (1908) and Tarski (1983).

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