SPOTLIGHT ON: THE NATURE OF SCIENTIFIC CHANGE

In Pursuit of Conceptual Change: the Case of Legendre and Symmetry

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Scientific knowledge is in constant flux: sometimes the change is fundamental, sometimes it is incremental; despite the important differences between these two kinds of changes, we find consistent features in the way the new knowledge relates to the antecedent state of a discipline. Logically, generating knowledge requires a fixed set of presuppositions, anchored in a given conceptual framework. The practitioner may or may not be aware of all the elements that are involved in the process of generating knowledge but, whether the elements are assumed explicitly or implicitly, they have to be fixed for the production of knowledge to be coherent. In other words, the scientist determines the relevant background and keeps it fixed throughout the episode during which he or she seeks to contribute to some aspect of scientific knowledge.

There is thus a variety of background knowledge and, generally, we distinguish between two sets of elements of knowledge, which we call a 'baseline' and a 'snapshot'. A baseline captures scientific knowledge at a certain time and it is relatively stable for some given duration. The baseline represents the sum of what is, in principle, available to the community of practitioners in the field. Hence, this kind of background knowledge has no nuances and exhibits no preferences, for it is just an inventory of elements. In contrast, a snapshot is personal, that is, it is the result of applying some rules of selection to the baseline, separating the wheat from the chaff as seen in the context of a specific conceptual framework and metaphysical outlook. A snapshot is directly related to a baseline but it is not simply a subset since it includes, in addition to the selected elements, individual assessments of the elements; such assessments may not be found in the standard works of the relevant field, for they reflect the idiosyncratic view of a practitioner. Evaluations, which are personal to a large extent, create a tension, or a problem, which the scientist then seeks to address. In sum, the baseline is public and more or less explicit: what all practitioners are expected to know in a given domain. On

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the other hand, the snapshot is unique to the individual scientist and often it was not fully articulated by the practitioner; rather, it is frequently the case that the historian (or philosopher) identifies implicit elements of the snapshot that were taken for granted by the scientist.

To write a history of scientific change, and more specifically, conceptual change, the historian of science must establish a set of references. Change is relative, for it can be determined only through comparison. So what methodology facilitates such a comparison? In other words, how does the historian obtain the baseline of the domain under discussion at a particular period, and the snapshot for a specific practitioner? The availability of dictionaries is useful, but key classical texts, encyclopedias, and review articles in learned journals are the best sources of evidence for the historian who wishes to determine the relevant baseline (e.g. Diderot *et al.*, 1751–1765). In fact, scientists themselves discovered the importance of establishing a baseline, and on several occasions in the modern era a group of scientists committed themselves to a comprehensive recording of the edifice of knowledge of the discipline under scrutiny (see e.g. the case of pure and applied mathematics in von Dyck, 1904). By its very nature, as personal and idiosyncratic, a snapshot is much more difficult to determine than the relevant baseline. Here the historian has to exercise a great measure of ingenuity and to engage sympathetically with the author in question, seeking in the writings of the scientist clues for new elements and assessments. Some historians turn to private records (e.g. laboratory notebooks), others seek published commentaries which throw light on the novel contribution, still others search for autobiographical notes and reminiscences (including those by colleagues), or even circumstantial evidence that may reveal the scientist's sources and assessments at the initial stages of research. But one should not underestimate the vast amount of information that can be gleaned from the very publications of the practitioner under consideration.

In our work on the making of the concept of symmetry as a revolutionary scientific concept, we addressed the fundamental change that the French mathematician, Adrien-Marie Legendre (1752–1833), introduced into solid geometry. The baseline was Euclidean geometry as it had been formulated in the early modern era and addressed by the leading mathematicians of the time. Euclidean geometry was of course well established in the public domain in the early 1790s. However, as historians of science we sought to take a snapshot of Legendre's epistemological commitments when he drafted his monumental, *Éléments de géométrie, avec des notes* (Legendre, 1794). Indeed, the notes proved to be most useful since they directed us to the evaluation of the work of his predecessors that Legendre had (presumably) carried out before composing his *Éléments*. One of our goals, then, was to appreciate the difficulty of the problem that Legendre faced from the perspective of his time, a problem whose solution was quickly embedded in the baseline of early 19th-century mathematics.

Legendre informed his readers of a problem that Robert Simson (1687–1768) had identified which, according to Legendre, was not solved satisfactorily. Simson realized that Euclid's extrapolation from the plane to the three-dimensional case of similarity and equality—in the definitions at the beginning of *Elements*, Book XI—had failed on the key issue of solid angles. Simson focused on the arrangement of the plane angles that form a solid angle and noted that two solid angles could be unequal even though all the plane angles that comprise one of them are equal to those that comprise the other. This difficulty was removed by insisting that, for two solid angles to be equal, the same order of the equal plane angles is required. He therefore suggested a more restrictive definition of similarity and equality in solid geometry than that of Euclid to avoid ambiguous cases (Hon and Goldstein, 2008, § 8.2).

Simson's difficulty was right there in the public domain, part of the baseline of solid geometry, but it is to Legendre's credit that he recognized the importance of the issue. Legendre then evaluated the problem in a new way and saw that a satisfactory solution required an entirely novel concept in solid geometry. In Legendre's view, two solid angles whose plane angles are equal can be related in two ways: if the order in both cases is the same, the solid angles are equal and therefore superposable; but, if one order is the inverse of the other, superposition is not possible. Legendre called the latter relation *symmetrical*. This new concept of symmetry was then extended to polyhedra (composed of solid angles), thereby capturing a new relation between two non-superposable solids where the sides of one solid are equal and similar to the sides of the other. This is undoubtedly a prime example of a conceptual change which, interestingly enough, has neither been discussed in the historical literature nor has it been acknowledged in philosophical analyses.

Why did Legendre call the new concept, *symmetry*? He was, of course, aware that this term occurs in Greek mathematics (see e.g. Euclid's *Elements*, Book X) in the sense of 'commensurable'. But, although this meaning was widely known in the 18th century, it was not in current mathematical usage (Hon and Goldstein, 2008, chs. 2 and 8). Clearly, we had to examine another baseline, that of the concept of symmetry itself. For this purpose we posed the question: what were the standard usages of the term, *symmetry*, at the time Legendre composed his revision of Euclid's geometry? The *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers* by Denis Diderot, Jean le Rond d'Alembert, and their group of dedicated scholars, offers a superb baseline of the contemporary edifice of knowledge both in aesthetics and in geometry. Such a baseline allowed us to fulfill one of the necessary requirements of scholarship which is, as Truesdell formulated it, 'the capacity to make an informed judgment of the contemporary meanings of words' (Truesdell, [1973] 2008, p. 29 n. 1). A perusal of the *Encyclopédie* shows that the concept of symmetry was understood at the time in two senses: (1) in the traditional Vitruvian sense, that is, a set of proper proportions of the parts of a building with each other and with the whole, and (2) in the contemporary sense in France, as it was rendered by Claude Perrault (1613–1688) in his French translation and commentary on Vitruvius's *De architectura* (1673), that is, symmetry is the equality and similarity of columns, pilaster, doors, windows, etc. on one side of the façade of a building to those on the other side, with respect to some central feature of the façade. The entry, *symmetry*, in the *Encyclopédie* (Diderot *et al*., 1751–1765) gives us confidence that our conclusion is correct, namely, Legendre introduced a completely new meaning for an old term, *symmetry*, which at the time was used principally in architecture in an aesthetic, rather than scientific, sense (Hon and Goldstein, 2008, ch. 4). Here philosophy assists history in the form of a distinction between the property of a single entity and the relation of two entities. The distinction throws light on the different usages of the concept of symmetry and facilitates accounting for the complex history of this concept which has both aesthetic and scientific aspects.

Like scientists, historians of science also rely essentially on two different kinds of background knowledge in order to call attention to the changes that have taken place in the edifice of scientific knowledge. In our case, the baseline consists of data related to domains where the concept of symmetry has been thought to be applicable, in antiquity, medieval times, and the early modern period up to the time of Legendre at the end of the 18th century. In parallel we have our own snapshot which includes, among other things, the principles we have adopted for evaluating historical data. We hold that words count—they are, after all, markers of ideas. When analyzing a concept historians of science are all too often accustomed to make the comment (implicitly), 'What's in a word?' It is quite common for historians to point to the application of some concept in a scientific text even when no specific term is attested for it. We take a different approach which emphasizes the usages of terms in their specific contexts and the changes in their meanings over time. Hence, to trace the invocation of symmetry (both as a term and as a concept) and to determine its various meanings, we first identify the passages where the term was introduced and then discuss the contexts in which we find it. In general, we seek to assess the usages of certain technical terms in a particular domain—as indicative of the application of the corresponding concepts—and to follow their migration from one domain to another. The result of this approach is striking: our snapshot provides tools that, when applied systematically and consistently, yield a completely different story from what has been told heretofore. Again, we have the same baseline as other historians and philosophers of science who have dealt with the history of the concept of symmetry. But our search to locate the changes in a key concept using our own method of assessing the data, the so-called snapshot, leads to a finding that comes with a surprise: bilateral and rotational symmetry are entirely modern concepts that appear near the end of the 18th century (Weyl, 1952, pp. 47ff; see also Hon and Goldstein, 2004; Goldstein and Hon, 2007; Hon and Goldstein, 2008, chs. 6 and 9). Other modern senses of symmetry that relate to group theory only appeared much later.

In fact, the application of this methodology disclosed a complex story, for the 'life-history' of the concept of symmetry has a long evolutionary phase and a surprising revolutionary turn in Legendre's *Elements* of 1794. To recognize this intriguing life-history, a vast array of evidence had to be examined. A consistent approach was taken in determining baselines, which required an effort to avoid the bias that would be introduced by assuming that current connotations of the term were already invoked in the past. We located usages of the term, *symmetry*, in well known texts from antiquity to early modern times and examined such usages in their specific contexts. The application yielded two trajectories which we have called the 'mathematical path' and the 'aesthetic path'. It became apparent that, despite the variety of usages in many different domains, there is a conceptual unity underlying the invocation of symmetry in the period from antiquity to the 1790s which is distinct from the scientific usages of this term that emerged at the end of the 18th century. On the mathematical and scientific side, we considered such figures as Euclid and Archimedes in antiquity, Boethius (480–524) and Oresme (1323–1382) in the Middle Ages, and Copernicus (1473–1543) and Galileo (1564–1642) in early modern times. On the aesthetic side we investigated the changes in aesthetic sensibilities, recognized in the Renaissance, starting with Leon Battista Alberti (1404–1472), and implemented in a new style of French architecture, that called for new senses of symmetry (Hon and Goldstein, 2008, Part I). We argued that the aesthetic usage of symmetry in the 17th and 18th centuries, notably by Perrault and Montesquieu (1689–1755), yielded insights into the contrast between Legendre's felicitous choice in 1794 of the term, *symmetry*, as a scientific concept, and Kant's failed attempt in 1768 to establish the concept of *incongruent counterparts* (Hon and Goldstein, 2008, Part II).

The methodology that facilitates such a coherent account depends in large measure on an essential distinction between the analyst and the actor. In historical writing on scientific matters the analyst is the modern author and the actor is the scientist under consideration. If the analyst does not adhere to a strict distinction between his or her categories and those of the actor, a coherent story may emerge but one that does justice neither to the actor nor to history. The distinction between the categories of the analyst and those of the actor requires an exercise in defamiliarization, that is, the analyst has to suppress temporarily much of what he or she knows of subsequent developments in order to treat the subject matter sympathetically, allowing the context of the time to dominate the analysis. After all, the actor looks back to his or her own past, not to the future where the analyst is situated. History of science would not be a part of the discipline of history if it considered scientific problems as eternal and unchanging, for it is the attempt to ascertain what answers have been given to an evolving set of questions and in what order. Specifically, the history of a concept should reflect its meanings at each stage as the application of the concept changes over time. In different contexts, different problems arise and, in turn, different answers are given. The analyst can be expected to focus on this transition in meanings and to portray the actor in his or her own context, rather than in the analyst's contemporary milieu.

We have described the first phase of a historical analysis of conceptual change, namely, the establishment of baselines and snapshots, consistent with the data obtained from the period under consideration and the actor's own work. The evolution of meanings of existing concepts as well as the making of new concepts constitute in many ways engines of change in scientific knowledge. To be sure, more steps have to be taken in order to obtain a comprehensive view of scientific change, but we leave the details for another day.

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