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A Classical Logic of Existence and Essence

Abstract. The purpose of this paper is to provide a new system of logic for existence and essence, in which the traditional distinctions between essential and accidental properties, abstract and concrete objects, and actually existent and possibly existent objects are described and related in a suitable way. In order to accomplish this task, a primitive relation of essential identity between different objects is introduced and connected to a first order existence property and a first order abstractness property. The basic idea is that possibly existent objects are completely determinate and that essentially identical objects are just different individuations of the same individual essence. Accordingly, essential properties are defined as properties that are invariant with respect to this kind of identity, while abstract objects are determined by being characterized by essential properties only. Once such ideas are implemented, a number of classical intuitions about objects, their essence, and their way of existence can be consistently interpreted.

Keywords: essence; existence; qualitative identity; essential identity; abstract objects; possible worlds semantics; quantified modal logic

Introduction

Let P be a property that is not instantiated at the actual world and suppose we are considering the possibility of the existence of something which is P. We may think that it is possible that there be something which is P because it is possible for some existing thing to be P, or because it is possible for some counterpart of some existing thing to be P, or because it is true that some possible thing is P. Arguably, a classical philosopher, following the tradition of Aristotle and Aquinas, would say that it is possible that there be something which is P because a certain essence which can exist has the potentiality of being P. Such a position builds on a rich metaphysics in which a number of crucial dis-

tinctions about modal notions can be expressed. The need for clarifying such distinctions and to represent them within a unique modal logic is at the origin of the present work. Thus, in this paper we introduce a novel modal system, **CML** for classical modal logic, which in our opinion comes closest to capturing crucial modal ideas distinctive of classical metaphysics.[1]   In particular, we are interested in providing a unified framework enabling us (1) to flesh out classical intuitions concerning the notions of individual essence, an essential identity and a existence, (2) to specify the distinctions between mere and real possibility, essential and potential property, an abstract and a concrete object, and (3) to state some fundamental theses of classical metaphysics involving these notions and distinctions. In fact, to the best of our knowledge, no such framework is currently available.

The paper is organized as follows. Section 1 is a brief introduction to the general concepts exploited in our system of classical modal logic. In Section 2 we develop the system axiomatically and provide its semantic characterization. In Section 3 we highlight some of the distinctions that can be expressed within the system and some of the most important classical theorems that can be derived in it. Finally, in Section 4 we sketch a comparison with the principal systems of interpreted modal logic on the market and highlight the originality of our system and some of its benefits.

1. Classic modal framework

In this section we will sketch the metaphysical framework underlying our system of classical modal logic **CML**.

1.1. Objects as individual substances

The individual variables of **CML** range over objects intended as *individual substances*, namely individual and ontologically independent objects.[2] An individual substance is a complete object, i.e., an object that

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[1] Here the expression 'classical modal logic' is only used to refer to system **CML**, which is largely inspired by the metaphysical tradition stemming from Aristotle, Aquinas, and Suarez. Hence, classical modal logic is not to be identified with basic quantified modal logic **QML**. See (Menzel, 1991) for a discussion of different interpretation of **QML**.

[2] This idea is related to the division of beings proposed by Aristotle in the second chapter of the Categories: "Of things there are: (a) some are said of a subject but are

is completely characterized with respect to all properties and relations.[3] In addition, an individual substance is what it is in virtue of its *individual essence*, which is the source of its *essential identity*. In more detail, we say that two objects are essentially identical precisely when they are different individuations of a unique individual essence. Thus, Socrates, who is sitting at a certain moment, is essentially identical with Socrates, who is standing at that very moment, since Socrates is characterized by a unique individual essence. Accordingly, we will say that Socrates, who is sitting at a certain moment, and Socrates, who is standing at that very moment, are particular *individuations*, or particular *individual variants*, of a unique individual essence, and that these variants are essentially identical even if

1. they are not identical, since Socrates, who is sitting at a certain moment, is characterized by a property that is not possessed by Socrates, who is standing at that very moment;
2. they do not exist in the same possible world, since it is impossible for Socrates, who is sitting at a certain moment, to be standing at the same moment at the same possible world.

In speaking of the different individual variants of the same individual essence, we are assuming a distinction between accidental and essential properties: any object is characterized by a set of essential properties

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not in any subject. For example, man is said of a subject, the individual man, but is not in any subject. (b) Some are in a subject but are not said of any subject. (By 'in a subject' I mean what is in something, not as a part, and cannot exist separately from what it is in.) For example, the individual knowledge-of-grammar is in a subject, the soul, but is not said of any subject; and the individual white is in a subject, the body (for all colour is in a body), but is not said of any subject. (c) Some are both said of a subject and in a subject. For example, knowledge is in a subject, the soul, and is also said of a subject, knowledge-of-grammar. (d) Some are neither in a subject nor said of a subject, for example, the individual man or individual horse (for nothing of this sort is either in a subject or said of a subject). Things that are individual and numerically one are, without exception, not said of any subject, but there is nothing to prevent some of them from being in a subject (the individual knowledge-of-grammar is one of the things in a subject)" (Cat. 1a 20-1b 10). In addition, resting on the assumption that a relation of attribution reflects a relation of ontological dependence, Aristotle can state his well-known conclusion: "Thus all the other things are either said of the primary substances as subjects or in them as subjects. So if the primary substances did not exist it would be impossible for any of the other things to exist" (Cat. 2b 3-7).

[3] See (Gracia, 1994), for a discussion of the concepts of individual object and individuation in medieval metaphysics, and the way in which the idea of individuation through complete characterization emerges in Suarez.

plus a set of accidental properties. We also assume that the accidental properties of an individual substance are the result of actualizing potentialities rooted in the essence of the substance itself, e.g., the potentiality of standing up, whereas the essential properties belong to all individuations of the same individual essence. Still, the distinction between essential and accidental properties does not coincide with the distinction between necessary and non-necessary properties, since both essential and accidental properties of an individual substance are assumed to be necessary. For instance, the standing Socrates is characterized in all the possible worlds both by the essential property of being-human and by the accidental property of being-standing at a certain moment. This is why we will think of our objects as elements of a *constant* domain of possible entities having the same properties in all possible worlds, with the exception of the property of existence.

1.2. Existence and possibility

In classical metaphysics, two fundamental distinctions are thoroughly investigated: the first is the distinction between existent and non-existent objects; the second is the distinction between two notions of possibility: potentiality and logical possibility. Let us consider both of them in turn.

1.2.1. Essence and existence

The distinction between existent and non-existent objects rests on the distinction between essence and existence, which is one of the main metaphysical issues in medieval philosophy. Intuitively, the essence of an object is what grounds its essential identity and it is characterized in terms of its essential properties, which are properties that cannot possibly vary through the change of the object and that one has to conceive in order to have the right concept of it. By contrast, existence is a special property, which is not involved in the conception of the object.[4] Accordingly, we introduce a first order predicate "**E**" of existence, to denote the first order

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[4] In a well-known passage, Aquinas states: "Everything that does not belong to the concept of an essence or quiddity comes to it from the outside and enters into composition with the essence, because no essence can be understood without its parts. Now every essence or quiddity can be understood without knowing anything about its being. From this it is clear that being is other than essence or quiddity (*esse est aliud ab essentia*), unless perhaps there is a reality whose quiddity is its being" (Thomas Aquinas, De Ente et Essentia, IV, p. 55)

property of existence, and we assume that the extension of that property varies across worlds, since different objects can exist in different worlds.[5] In addition, we will assume that no two essential identical objects can exist at the same world, in accordance with the principle that what exists is completely determinate, with respect to all its properties.

The predicate of existence plays a very different role from that performed by the other predicates. In fact, the *real* predicates — to use Kant's terminology — are used to state how objects are determined, while the existence predicate is used to state whether it exists in a certain world. Consequently, even though all objects possess the same real properties in all worlds, they can be existent in one world and non-existent in others.

1.2.2. Potentiality and possibility

The distinction between potentiality and logical possibility was fully developed in late scholasticism and rests on two elements of classical metaphysics, already present in Aristotle: a formal element consisting of the fact that what is possible is non-contradictory; a causal element consisting of the the fact that what is really possible is what can be produced by virtue of some productive power.[6] This distinction can be introduced in our framework by combining the use of the existential quantifier and the predicate of existence. Thus, we interpret "$\exists x(x = y)$" as stating that y is a logically possible object and "$\Diamond \mathbf{E}(y)$" as stating that y is a potentially existent objects, i.e., that it can actually exist. As a consequence, our framework allows both for a distinction between actually existing objects and merely possibly existing objects, expressed by

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[5] See (Zalta, 1988; Williamson, 1998; Stalnaker, 2003, part I; McDaniel, 2009; Shaffer, 2009; Berto, 2012, part II) for different defences of the distinction between existence and actual existence and of the assumption of different ways of existing. In particular, the classical point of view seems to be best captured by a variant of the position proposed by Shaffer, according to which what exists is what is grounded in what actually exists. Hence, non-actual possible objects can be viewed as objects whose existence is either rooted in some productive power or not excluded by some preventive power.

[6] See Suarez: "The aptitude to existence that characterizes what is possible consists, as far as the object is concerned, in the fact that it is not absurd for it to exist and, as far as its cause is concerned, in the fact that this cause has the power to produce it". (*Aptitudo obiectiva rerum possibilium ad existendum non est ex parte illarum, nisi non repugnantia quaedam, et ex parte causae denotat potentiam ad illas producendas*) (Disputationes Met. VI.4.9).

(i) $\mathbf{E}(y)$
(ii) $\Diamond\mathbf{E}(y) \wedge \neg\mathbf{E}(y)$

and for a distinction between possibly existing objects and merely logi-
cally existing objects, expressed by

(iii) $\Diamond\mathbf{E}(y)$
(iv) $\exists x(x = y) \wedge \neg\Diamond\mathbf{E}(y)$

In the system we are going to propose, the foregoing intuitions are
captured by appropriate axioms. In particular, we introduce a specific
axiom stating that all properties possessed by an object x, with the
exception of properties involving existence, necessarily characterize x,
and a set of axioms characterizing the relation of essential identity as an
equivalence relation. This relation will allow us to articulate the distinc-
tion between essential and non-essential properties. As a consequence,
we will end up with three kinds of properties:

1. *non-real properties*, like existence;
2. *real essential properties*, like being a man;
3. *real non-essential properties*, like being white.

As we will see, we do not exclude the possibility of the property
of existence as being essential to at least some entity, thus capturing
some interesting classical theses on the ontological status of an entity
that exists by virtue of its very essence. Finally, we will also introduce a
predicate "$\mathbf{A}$" for the property of being abstract, and so develop a sketch
of a theory of abstract objects.

2. System CML

In this section we introduce our system of classical modal logic. The
language of **CML** is an extension of the basic modal first-order language
with identity. The new elements are given by two predicates, which
will be used to express the distinction between existent and non-existent
objects and the distinction between concrete and abstract objects, and
an equivalence relation of essential identity.

DEFINITION 1 (language of **CML**). The set of formulas of the language
$\mathcal{L}(\mathbf{CML})$ of **CML** is defined according to the following rules:

$$x = y \mid P_i^n(\mathbf{x}) \mid x \approx y \mid \mathbf{E}(x) \mid \mathbf{A}(x) \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid \forall x\varphi$$

where P_i^n is the i-th n-place predicate and $\mathbf{x}$ is a sequence of n variables.

In what follows we will use: x, y, a, ..., as names for individual variables.

2.1. The axiomatic system

CML is an extension of the basic modal first-order predicate logic with identity. Consider the following groups of axioms and rules.

Group I: modal axioms and necessitation rule.

Axiom T: $\Box\varphi \to \varphi$
Axiom 5: $\neg\Box\varphi \to \Box\neg\Box\varphi$
Axiom E: $\exists x \mathbf{E}(x)$
Axiom NE: $\varphi \to \Box\varphi$, provided $\mathbf{E}$ is not in φ
Rule NR: if $X \vdash \varphi$, then $\Box X \vdash \Box\varphi$, where $\Box X = \{\Box\varphi \,|\, \varphi \in X\}$

Axiom E states that in all possible worlds there is some existent object, thus excluding the possibility of empty worlds. The intuitive sense of this axiom is discussed below. Axiom NE is a principle of necessitation for real properties. According to NE, if a group of objects is determined by the property expressed by φ and if this property is indifferent with respect to existence, because the existence predicate does not occur in it, then the property necessarily characterize that group of objects. This is due to the assumption, made explicit at the semantic level, that every object is completely characterized in any possible world, so that it possesses the same real properties at all possible worlds.

Group II: general axioms on essential identity.

Axiom C1: $x = y \to x \approx y$
Axiom C2: $x \approx y \to y \approx x$
Axiom C3: $x \approx y \wedge y \approx z \to x \approx z$
Axiom CE: $\mathbf{E}(x) \wedge \mathbf{E}(y) \wedge x \approx y \to x = y$
Axiom CP: $\mathbf{E}(x) \wedge x \approx y \to \Diamond\mathbf{E}(y)$

Axiom C1 states that identity is sufficient for essential identity. Besides, identity implies qualitative identity also with respect to non-essential properties. Given axiom C1, it follows from axioms C2 and C3 that essential identity is an equivalence relation, while axiom CE states that no two different but essentially identical objects can exist. The intuition here is that, once a world is selected, only one among all individuations of an object can exist in that world. Finally, axiom CP states that different

but essentially identical objects are on a par with respect to their modal ontological status: if an object can exist, all its variants can exist.

Group III: axioms on abstract objects.

Axiom A1: $\mathbf{A}(x) \to \mathbf{E}(x)$
Axiom A2: $\mathbf{A}(x) \to \Box\mathbf{A}(x)$
Axiom A3: $\mathbf{A}(x) \wedge x \approx y \to x = y$

According to a classical viewpoint, abstract objects are existing and unchanging objects. Therefore, axiom A1 states that all abstract objects are existent, whereas axioms A2 and A3 ensure that abstract objects cannot possibly change. To be sure, A2 states that no abstract object can change with respect to the very property of being abstract, while A3 states that no abstract object has more than one individuation, so that there is no possible world where an abstract object can be qualitatively different.

DEFINITION 2 (**CML** and $\vdash_{\mathbf{CML}}$). **CML** is the smallest set of formulas of $\mathcal{L}(\mathbf{CML})$ which contains all instances of axioms of first order logic with identity of groups I–III and is closed under the rules of first order logic with identity and the rule of necessitation. Derivability in **CML**, $\vdash_{\mathbf{CML}}$, is defined as usual.

2.2. Some basic theorems

It is not difficult to see that the following theorems are derivable in **CML**.

T1: $\vdash_{\mathbf{CML}} \forall x\Box\varphi \to \Box\forall x\varphi$　　　　　　　　　　(Barcan Formula BF)
　　(in terms of $\exists$, $\Diamond\exists x\varphi \to \exists x\Diamond\varphi$)
T2: $\vdash_{\mathbf{CML}} \Box\forall x\varphi \to \forall x\Box\varphi$　　　　　(Reverse Barcan Formula RBF)
　　(in terms of $\exists$, $\exists x\Diamond\varphi \to \Diamond\exists x\varphi$)

These theorems reflect the fact that the domain of quantification contains conceivable objects. Since all conceivable objects are present at all possible worlds, the domain of quantification is constant.

T3: $\vdash_{\mathbf{CML}} x = y \to \Box(x = y)$　　　　　　　　　　(necessity of identity)
T4: $\vdash_{\mathbf{CML}} x \neq y \to \Box(x \neq y)$　　　　　　　　(necessity of non-identity)
T5: $\vdash_{\mathbf{CML}} x \approx y \to \Box(x \approx y)$　　　　　(necessity of essential identity)
T6: $\vdash_{\mathbf{CML}} x \not\approx y \to \Box(x \not\approx y)$　　　(necessity of essential non-identity)

Necessity of identity and non-identity is standard. Necessity of essential identity and its negation is a consequence of the interpretation of essential identity as a relation linking objects that share the same individual essence and the assumption that any object is necessarily characterized by its essence.

T7: $\vdash_{\textbf{CML}} \textbf{A}(x) \to \Box\textbf{E}(x)$

T7 is an immediate consequence of A1 and A2: since it is necessary that abstract objects are existent, necessarily abstract objects are necessarily existent; but all abstract objects are necessarily abstract; therefore, all abstract objects are necessarily existent.

2.3. The semantic system

The central concept of **CML** semantics is that of classical frame.

DEFINITION 3 (Classical frame for $\mathcal{L}(\textbf{CML})$). A classical frame for $\mathcal{L}(\textbf{CML})$ is a tuple $\mathfrak{F} = \langle W, D, A, \xi, \delta \rangle$, where

1. W is the set of metaphysically possible worlds
2. D is the set of conceivable objects
3. $A \subseteq D$ is the set of metaphysically abstract objects
4. $\xi \subseteq \wp(D)$ is a set of sets of essentially identical objects
5. $\delta : W \to \wp(D)$ is a function assigning to each world its inner domain

In addition, the elements of a classical frame are required to satisfy the following conditions:

C1:	$\bigcup \xi = D$
C2:	for all $C_1, C_2 \in \xi$, $C_1 \cap C_2 \neq \check{R} \Rightarrow C_1 = C_2$
C3:	for all $C \in \xi$, $C \cap \delta(w) \neq \check{R} \Rightarrow C$ is a singleton
C4:	for all $C \in \xi$, $C \cap \delta(w) \neq \check{R} \Rightarrow C \subseteq \bigcup_{w \in W} \delta(w)$
C5:	for all $w \in W$, $A \subseteq \delta(w)$
C6:	for all $C \in \xi$, $C \cap A \neq \check{R} \Rightarrow C$ is a singleton
C7:	for all $w \in W$, $\delta(w) \neq \check{R}$

We remark that ξ play the same role of a set of individual essences. Indeed, since every element C of ξ can be viewed as a set of objects sharing the same individual essences, C itself can be viewed as the extensional counterpart of an individual essence. That said, conditions *C1*–*C3* ensure that ξ is a partition of D and that no variant of a certain object can exist at a world where that object exists. Intuitively, every

object in D has an individual essence, so that every object in D is in some $C \in \xi$, and no two essentially different objects share the same essence, so that no two $C_1, C_2 \in \xi$ can overlap. Condition $C4$, in accordance with our assumption on possible objects, states that every variant of a potential object, namely an object that can exist, is a potential object as well. Conditions $C5$ and $C6$, in accordance with our assumptions on abstract objects, state that every abstract object exists in every possible world and that no abstract object can have variants. Condition $C7$, in accordance with our assumption on possible worlds, states that the inner domain of a possible world is always non empty. Finally, we want to stress that no assumption is made as to the ontological status of abstract objects. To be sure, no condition is introduced to the effect that the set A is non empty. Thus, abstract objects can be viewed either as abstractions, i.e., epistemic objects, or as independently existing objects.

DEFINITION 4 (Model for $\mathcal{L}(\textbf{CML})$). A model for $\mathcal{L}(\textbf{CML})$ is a pair $\mathfrak{M} = \langle \mathfrak{F}, \mathfrak{I} \rangle$, where $\mathfrak{F}$ is a frame for $\mathcal{L}(\textbf{CML})$ and $\mathfrak{I}$ is a valuation function such that

1. $\mathfrak{I}(x) \in D$;
2. $\mathfrak{I}(P_i^n) \subseteq D^n$, for each i.

It is worth noting that both variables and predicates are treated as rigid designators. This is a consequence of the fact that D is a set of possible individuals intended as individual substances, not as individual essences or individual substrata.

DEFINITION 5 (Variant model for $\mathcal{L}(\textbf{CML})$). A variant $\mathfrak{M}_d^x$ of a model $\mathfrak{M} = \langle \mathfrak{F}, \mathfrak{I} \rangle$ relative to x is the model $\mathfrak{M} = \langle \mathfrak{F}, \mathfrak{I}_d^x \rangle$, where

1. $\mathfrak{I}_d^x(y) = d$, if $x = y$;
2. $\mathfrak{I}_d^x(y) = \mathfrak{I}(y)$, if $x \neq y$.

Hence, $\mathfrak{I}_d^x$ agrees with $\mathfrak{I}$ on every value, except possibly the value of x.

DEFINITION 6 (Truth). The concept of truth is recursively defined as follows:

$\mathfrak{M}, w \vDash x = y \Leftrightarrow \mathfrak{I}(x) = \mathfrak{I}(y)$
$\mathfrak{M}, w \vDash P_i^n(x_1, \ldots, x_n) \Leftrightarrow \langle \mathfrak{I}(x_1), \ldots, \mathfrak{I}(x_n) \rangle \in \mathfrak{I}(P_i^n)$
$\mathfrak{M}, w \vDash x \approx y \Leftrightarrow \mathfrak{I}(x), \mathfrak{I}(y) \in C$ for some $C \in \xi$
$\mathfrak{M}, w \vDash \textbf{E}(x) \Leftrightarrow \mathfrak{I}(x) \in \delta(w)$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Leftrightarrow \mathfrak{I}(x) \in A$
$\mathfrak{M}, w \vDash \neg\varphi \Leftrightarrow \mathfrak{M}, w \nvDash \varphi$
$\mathfrak{M}, w \vDash \varphi \wedge \psi \Leftrightarrow \mathfrak{M}, w \vDash \varphi$ and $\mathfrak{M}, w \vDash \psi$
$\mathfrak{M}, w \vDash \forall x\varphi \Leftrightarrow \mathfrak{M}_d^x, w \vDash \varphi$ for all $d \in D$
$\mathfrak{M}, w \vDash \Box\varphi \Leftrightarrow \mathfrak{M}, v \vDash \varphi$, for all $v \in W$

DEFINITION 7 (validity). 1. φ is valid in $\mathfrak{F}$: $\mathfrak{F} \Vdash \varphi$ iff $\langle\mathfrak{F}, \mathfrak{I}\rangle, w \vDash \varphi$, for all w and $\mathfrak{I}$.

2. φ is valid: $\Vdash \varphi$ iff $\mathfrak{F} \Vdash \varphi$, for all $\mathfrak{F}$.

Finally, $X \Vdash \varphi$ iff $\langle\mathfrak{F}, \mathfrak{I}\rangle, w \vDash X \Rightarrow \langle\mathfrak{F}, \mathfrak{I}\rangle, w \vDash \varphi$, for all w and $\langle\mathfrak{F}, \mathfrak{I}\rangle$, where $\langle\mathfrak{F}, \mathfrak{I}\rangle, w \vDash X$ iff $\langle\mathfrak{F}, \mathfrak{I}\rangle, w \vDash \psi$, for all $\psi \in X$.

2.4. Proof of soundness

We have to prove that $X \vdash_{\mathbf{CML}} \varphi \Rightarrow X \Vdash \varphi$. As usual, the proof is by induction on the length of a derivation. We only consider axioms that are proper to **CML**. The proof for the rule **RN** of necessitation is standard.

Axiom **NE**: $\Vdash \varphi \to \Box\varphi$, provided **E** does not occur in φ

It suffices to prove that, for all φ in which **E** does not occur, $\mathfrak{M}, w \vDash \varphi \Rightarrow \mathfrak{M}, w \vDash \Box\varphi$. The proof is by induction on the length of formulas not including **E**. The base case follows from the rigidity of the predicates other than **E**. The other non-modal cases are straightforward. The modal case follows from the modal fact that $\mathfrak{M}, w \vDash \Box\varphi \Rightarrow \mathfrak{M}, w \vDash \Box\Box\varphi$.

Axiom **E**: $\Vdash \exists x\mathbf{E}(x)$
        for all $w \in W$, $\delta(w) \neq \check{\mathrm{R}}$, by $C7$
        for all $w \in W$, $d \in \delta(w)$ for some $d \in D$
        $\mathfrak{M}, w \vDash \exists x\mathbf{E}(x)$ iff $\mathfrak{M}_d^x, w \vDash \mathbf{E}(x)$ for some $d \in D$
        $\mathfrak{M}, w \vDash \exists x\mathbf{E}(x)$ iff $d \in \delta(w)$ for some $d \in D$
        $\mathfrak{M}, w \vDash \exists x\mathbf{E}(x)$, whence the conclusion

Let us consider the axioms concerning essentially identical objects.

Axiom C1: $\Vdash x = y \to x \approx y$
        $\mathfrak{M}, w \vDash x = y \Rightarrow \mathfrak{I}(x), \mathfrak{I}(y) \in C$, for some $C \in \xi$, by $C1$
        $\mathfrak{M}, w \vDash x = y \Rightarrow \mathfrak{M}, w \vDash x \approx y$, by def. $\vDash$
Axiom C2: $\Vdash x \approx y \to y \approx x$
        $\mathfrak{M}, w \vDash x \approx y \Rightarrow \mathfrak{I}(x), \mathfrak{I}(y) \in C$, for some $C \in \xi$, by def. $\vDash$
        $\mathfrak{M}, w \vDash x \approx y \Rightarrow \mathfrak{M}, w \vDash y \approx x$, for some $C \in \xi$, by def. $\vDash$

Axiom C3: $\Vdash x \approx y \wedge y \approx z \to x \approx z$

$\mathfrak{M}, w \vDash x \approx y \Rightarrow \Im(x), \Im(y) \in C_1$, for some $C_1 \in \xi$, by def. $\vDash$

$\mathfrak{M}, w \vDash y \approx z \Rightarrow \Im(y), \Im(z) \in C_2$, for some $C_2 \in \xi$, by def. $\vDash$

$\Im(y) \in C_1$ and $\Im(y) \in C_2 \Rightarrow C_1 = C_2$, by $C2$

$\mathfrak{M}, w \vDash x \approx y \wedge y \approx z \Rightarrow \Im(x), \Im(z) \in C$, for some $C \in \xi$

$\mathfrak{M}, w \vDash x \approx y \wedge y \approx z \Rightarrow \mathfrak{M}, w \vDash x \approx z$, by def. $\vDash$

Axiom CE: $\Vdash \mathbf{E}(x) \wedge \mathbf{E}(y) \wedge x \approx y \to x = y$

$\mathfrak{M}, w \vDash \mathbf{E}(x) \wedge \mathbf{E}(y) \Rightarrow \Im(x), \Im(y) \in \delta(w)$, by def. $\vDash$

$\mathfrak{M}, w \vDash x \approx y \Rightarrow \Im(x), \Im(y) \in C$, for some $C \in \xi$, by def. $\vDash$

$\mathfrak{M}, w \vDash \mathbf{E}(x) \wedge \mathbf{E}(y) \wedge x \approx y \Rightarrow \Im(x), \Im(y) \in C \cap \delta(w)$

$\mathfrak{M}, w \vDash \mathbf{E}(x) \wedge \mathbf{E}(y) \wedge x \approx y \Rightarrow \Im(x) = \Im(y)$, by $C3$

Axiom CP: $\Vdash \mathbf{E}(x) \wedge x \approx y \to \Diamond\mathbf{E}(y)$

$\mathfrak{M}, w \vDash \Diamond\mathbf{E}(x) \Rightarrow \Im(x) \in \delta(v)$, for some v, by def. $\vDash$

$\mathfrak{M}, w \vDash x \approx y \Rightarrow \Im(x), \Im(y) \in C$, for some $C \in \xi$, by def. $\vDash$

$\mathfrak{M}, w \vDash \Diamond\mathbf{E}(x) \wedge x \approx y \Rightarrow \Im(x) \in C \cap \delta(v)$, for some v

$\mathfrak{M}, w \vDash \Diamond\mathbf{E}(x) \wedge x \approx y \Rightarrow \Im(y) \in \delta(v)$, by $C4$

Finally, let us consider the axioms concerning abstract objects.

Axiom A1: $\Vdash \mathbf{A}(x) \to \mathbf{E}(x)$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \Im(x) \in A$, by def. $\vDash$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \Im(x) \in \delta(w)$, by $C5$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \mathfrak{M}, w \vDash \mathbf{E}(x)$, by def. $\vDash$

Axiom A2: $\Vdash \mathbf{A}(x) \to \Box\mathbf{A}(x)$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \Im(x) \in A$, by def. $\vDash$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \mathfrak{M}, v \vDash \mathbf{A}(x)$, for all $v \in W$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \mathfrak{M}, w \vDash \Box\mathbf{A}(x)$, by def. $\vDash$

Axiom A3: $\Vdash \mathbf{A}(x) \wedge x \approx y \to x = y$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \Rightarrow \Im(x) \in A$, by def. $\vDash$

$\mathfrak{M}, w \vDash x \approx y \Rightarrow \Im(x), \Im(y) \in C$, for some $C \in \xi$, by def. $\vDash$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \wedge x \approx y \Rightarrow \Im(x) \in C \cap A$

$\mathfrak{M}, w \vDash \mathbf{A}(x) \wedge x \approx y \Rightarrow \Im(x) = \Im(y)$, by $C6$

2.5. Proof of completeness

As usual, it suffices to prove that every **CML**-consistent set X of formulas is satisfiable. The proof begins with a standard extension of X to a maximally **CML**-consistent and witnessed set of formulas w^* and then exploits a canonical model construction.[7] In what follows w, v, etc. vary

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[7] See (Hughes and Cresswell, 1996, ch. 14) for an introduction.

on maximally **CML**-consistent witnessed sets of formulas. In addition, we will use this notation:

- $w/\square := \{\varphi \,|\, \square\varphi \in w\}$,
- $[x] := \{y \,|\, x = y \in w^*\}$,
- $[x]_C := \{[y] \,|\, x \approx y \in w^*\}$.

The following facts are crucial in order to carry out the proof.

FACT 1: for all x, $[x] \in [x]_C$.

Straightforward, by the definition of $[x]$ and C1.

FACT 2: for all x and y, $[x] = [y]$ iff $[x] \cap [y] \neq \check{R}$.

Straightforward, by the definition of $[x]$ and the standard axioms on $=$.

FACT 3: for all x and y, $[x]_C = [y]_C$ iff $[x]_C \cap [y]_C \neq \check{R}$.

Suppose $[x]_C = [y]_C$. Since $x \approx x \in w^*$, by C1, $[x]_C = [x]_C \cap [y]_C \neq \check{R}$. Suppose now $[x]_C \cap [y]_C \neq \check{R}$ and let $[z] \in [x]_C \cap [y]_C$. Since $[z] \in [x]_C$ and $[z] \in [y]_C$, $x \approx z \in w^*$ and $y \approx z \in w^*$. Hence, $x \approx y \in w^*$ and $y \approx x \in w^*$, by C2 and C3. Therefore $[z] \in [x]_C$ iff $x \approx z \in w^*$ iff $y \approx z \in w^*$ iff $[z] \in [y]_C$.

FACT 4: $w/\square \subseteq w^* \Rightarrow w/\square = w^*/\square$.

Suppose $\varphi \in w/\square$. Then $\square\varphi \in w$. Thus $\square\square\varphi \in w$, by the axioms of group I, and so $\square\varphi \in w/\square$. Therefore $\square\varphi \in w^*$, so that $\varphi \in w^*/\square$. Suppose now $\varphi \notin w/\square$. Then $\neg\square\varphi \in w$, by the maximality of w. Thus $\square\neg\square\varphi \in w$, by the axioms of group I, and so $\neg\square\varphi \in w/\square$. Therefore $\neg\square\varphi \in w^*$, so that $\varphi \notin w^*/\square$.

FACT 5: for all $C \in \xi$, y_1, y_2, if $[y_1], [y_2] \in C$, then for all w, $y_1 \approx y_2 \in w$.

Suppose $C \in \xi$ and $[y_1], [y_2] \in C$, and let $C = [x]_C$. Then y_1 and y_2 are such that $x \approx y_1 \in w^*$ and $x \approx y_2 \in w^*$. Thus, $y_1 \approx y_2 \in w^*$, by C2 and C3, so that $\square(y_1 \approx y_2) \in w^*$, by **T5**. Hence, $y_1 \approx y_2 \in w$, by fact 4 and axiom **T**.

DEFINITION 8 (Canonical model for $\mathcal{L}(\textbf{CML})$ induced by w^*). The canonical model for $\mathcal{L}(\textbf{CML})$ induced by w^* is the model $\mathfrak{M}^* = \langle W, D, A, \xi, \delta, \mathfrak{I}\rangle$, where

1. $W = \{w \,|\, w/\square \subseteq w^*\}$;
2. $D = \{[x] \,|\, x = x \in w^*\}$;
3. $A = \{[x] \,|\, \textbf{A}(x) \in w^*\}$;

4. $\xi = \{[x]_C \mid x = x \in w^*\}$;
5. δ is such that $\delta(w) = \{[x] \mid \mathbf{E}(x) \in w\}$, for each $w \in W$;
6. $\Im$ is such that $\Im(x) = [x]$ and
$$\Im(P_i^n) = \{\langle[x_1]\dots[x_n]\rangle \mid P_i^n(x_1,\dots,x_n) \in w^*\}.$$

Note that $\Im(P_i^n)$ is well-defined in virtue of axiom **NE**.

Theorem 1. *The frame of the canonical model is a classical frame.*

It is a direct consequence of definition 8 that $A \subseteq D$, $\xi \subseteq \wp(D)$, and $\delta : W \to \wp(D)$. Hence, we only have to check that condition *C1–C6* are satisfied.

C1: $\bigcup \xi = D$

Suppose $d \in \bigcup \xi$. Then $d \in C$, for some $C \in \xi$. Let $C = [x]_C$. Then $d = [y]$ for some y such that $x \approx y \in w^*$, and so $d \in D$. Suppose now $d \in D$. Then $d = [x]$ for some x. Since $x \approx x \in w^*$, $d \in [x]_C$, and so $d \in \bigcup \xi$.

C2: $C_1, C_2 \in \xi, C_1 \cap C_2 \neq \check{\mathrm{R}} \Rightarrow C_1 = C_2$

Suppose $C_1, C_2 \in \xi$ and $C_1 \cap C_2 \neq \check{\mathrm{R}}$. Then, there is a $d \in C_1 \cap C_2$. Let $C_1 = [x]_C$ and $C_2 = [y]_C$, for some x and y. The conclusion is a consequence of fact 3.

C3: for all $C \in \xi$, $C \cap \delta(w) \neq \check{\mathrm{R}} \Rightarrow C$ is a singleton

Suppose $C \in \xi$ and $C \cap \delta(w) \neq \check{\mathrm{R}}$. Suppose there are $d_1, d_2 \in C \cap \delta(w)$. Let $C = [x]_C$, $d_1 = [y_1]$, and $d_2 = [y_2]$ for some y_1 and y_2. Then $y_1 \approx y_2 \in w$, by fact 5. Since $d_1, d_2 \in \delta(w)$, $\mathbf{E}(y_1), \mathbf{E}(y_2) \in w$, so that $y_1 = y_2 \in w$, by CE. Hence, $y_1 = y_2 \in w^*$, by **T3**, and so $[y_1] = [y_2]$.

C4: for all $C \in \xi$, $C \cap \delta(w) \neq \check{\mathrm{R}} \Rightarrow C \subseteq \bigcup_{w \in W} \delta(w)$

Suppose $C \in \xi$ and $C \cap \delta(w) \neq \check{\mathrm{R}}$. Suppose $d_1 \in C \cap \delta(w)$ and $d_2 \in C$. Let $C = [x]_C$, $d_1 = [y_1]$, and $d_2 = [y_2]$ for some y_1 and y_2. Then $y_1 \approx y_2 \in w$, by fact 5. Since $d_1 \in \delta(w)$, $\mathbf{E}(y_1) \in w$, so that $\Diamond \mathbf{E}(y_2) \in w$, by CP. Hence, there is a v such that $d_2 \in \delta(v)$, so that $d_2 \in C \cap \delta(v)$.

C5: for all $w \in W$, $A \subseteq \delta(w)$

Suppose $d \in A$ and let $d = [x]$. Since $[x] \in A$, $\mathbf{A}(x) \in w^*$, so that $\Box \mathbf{E}(x) \in w^*$, by T7. Hence, $\mathbf{E}(x) \in w$, and so $[x] \in \delta(w)$.

C6: for all $C \in \xi$, $C \cap A \neq \check{\mathrm{R}} \Rightarrow C$ is a singleton

Suppose $C \in \xi$ and $C \cap A \neq \check{\mathrm{R}}$. Suppose there are $d_1, d_2 \in C \cap A$. Let $C = [x]_C$, $d_1 = [y_1]$, and $d_2 = [y_2]$ for some y_1 and y_2. Then

$y_1 \approx y_2 \in w^*$, by fact 5. Since $d_1, d_2 \in A$, $\mathbf{A}(y_1), \mathbf{A}(y_2) \in w^*$. Hence, $y_1 = y_2 \in w^*$, by A3, and so $[y_1] = [y_2]$.

$C7$:   for all $w \in W$, $\delta(w) \neq \check{\mathrm{R}}$

By axiom $\mathbf{E}$, $\vdash_{\mathbf{CML}} \exists x \mathbf{E}(x)$, and so $\vdash_{\mathbf{CML}} \Box \exists x \mathbf{E}(x)$, by necessitation. Hence, $\Box \exists x \mathbf{E}(x) \in w^*$, so that $\Box \exists x \mathbf{E}(x) \in w$, since $w/\Box = w^*/\Box$, and $\exists x \mathbf{E}(x) \in w$, by axiom $\mathbf{T}$. Therefore, $\mathbf{E}(y) \in w$, for some witness y, since w is $\forall$-complete, and so $[y] \in \delta(w)$.

THEOREM 2 (truth lemma). $\mathfrak{M}^*, w \vDash \varphi \iff \varphi \in w$.

PROOF. By induction on the length of a formula.

For $x = y$:

$\mathfrak{M}^*, w \vDash x = y \Leftrightarrow \Im(x) = \Im(y)$, by def. $\vDash$

$\mathfrak{M}^*, w \vDash x = y \Leftrightarrow [x] = [y]$, by def. $\Im$

$\mathfrak{M}^*, w \vDash x = y \Leftrightarrow x = y \in w^*$, by def. $[x]$

$\mathfrak{M}^*, w \vDash x = y \Leftrightarrow x = y \in w$, by $\mathbf{T3}$, def. W, $\mathbf{T}$

For $P_i^n(x_1, \ldots, x_n)$:

$\mathfrak{M}^*, w \vDash P_i^n(x_1, \ldots, x_n) \Leftrightarrow \langle \Im(x_1), \ldots, \Im(x_n) \rangle \in \Im(P_i^n)$, by def. $\vDash$

$\mathfrak{M}^*, w \vDash P_i^n(x_1, \ldots, x_n) \Leftrightarrow \langle [x_1], \ldots, [x_n] \rangle \in \Im(P_i^n)$, by def. $\Im$

$\mathfrak{M}^*, w \vDash P_i^n(x_1, \ldots, x_n) \Leftrightarrow P_i^n(x_1, \ldots, x_n) \in w^*$, by def. $\Im$

$\mathfrak{M}^*, w \vDash P_i^n(x_1, \ldots, x_n) \Leftrightarrow P_i^n(x_1, \ldots, x_n) \in w$, by NE, def. W, T

For $x \approx y$:

$\mathfrak{M}^*, w \vDash x \approx y \Leftrightarrow \Im(x), \Im(y) \in C, C \in \xi$, by def. $\vDash$

$\mathfrak{M}^*, w \vDash x \approx y \Leftrightarrow [x], [y] \in C, C \in \xi$, by def. $\Im$

$\mathfrak{M}^*, w \vDash x \approx y \Leftrightarrow x \approx y \in w^*$, by def. ξ

$\mathfrak{M}^*, w \vDash x \approx y \Leftrightarrow x \approx y \in w$, by $\mathbf{T5}$, def. W, $\mathbf{T}$

For $\mathbf{E}(x)$:

$\mathfrak{M}^*, w \vDash \mathbf{E}(x) \Leftrightarrow \Im(x) \in \delta(w)$, by def. $\vDash$

$\mathfrak{M}^*, w \vDash \mathbf{E}(x) \Leftrightarrow [x] \in \delta(w)$, by def. $\Im$

$\mathfrak{M}^*, w \vDash \mathbf{E}(x) \Leftrightarrow \mathbf{E}(x) \in w$, by def. δ

For $\mathbf{A}(x)$:

$\mathfrak{M}^*, w \vDash \mathbf{A}(x) \Leftrightarrow \Im(x) \in A$, by def. $\vDash$

$\mathfrak{M}^*, w \vDash \mathbf{A}(x) \Leftrightarrow [x] \in A$, by def. $\Im$

$\mathfrak{M}^*, w \vDash \mathbf{A}(x) \Leftrightarrow \mathbf{A}(x) \in w^*$, by def. A

$\mathfrak{M}^*, w \vDash \mathbf{A}(x) \Leftrightarrow \mathbf{A}(x) \in w$, by A2, def. W, $\mathbf{T}$

The cases concerning connectives and quantifiers are standard. $\dashv$

3. Kinds of properties and objects

In this section we develop a theory of the distinction between different kinds of properties and objects based on the framework introduced above. As a first step, the concepts of invariant and essential property are introduced, exploiting the idea that a property is invariant to an individual precisely when it is necessarily shared by all of its qualitatively different individuations, while it is essential to an object if and only if it is both invariant and non-universal. Hence invariant properties are defined in terms of necessity and essential identity. As a second step, the concepts of invariant and necessary object are introduced, exploiting the idea that an object is invariant when it is identical with all its qualitative individuations, so that it is uniquely individuated, and it is necessary if and only if it necessarily exists. Finally, we show how some fundamental theorems of classical metaphysics can be captured within the present logical framework.

3.1. Some basic kinds of properties

We now define the concepts of invariant and essential property, assuming that a property is referred to by an expression like $\lambda x.\varphi$, where x is the only free variable which occurs in φ and it is different from a (note that $\lambda x.\varphi$ is not an expression of the modal language).

DEFINITION 9 (Invariant and possible properties).

1. invariant property: $\mathbf{IP}(a, \lambda x.\varphi) := \forall x \Box (a \approx x \to \varphi)$,
2. possible property: $\Diamond(a, \lambda x.\varphi) := \exists x \Diamond (a \approx x \land \varphi)$

A property is invariant with respect to a just in case it is invariantly shared by all qualitatively different individuations of a. According to a classical view, there are two kinds of invariant properties. The first kind is constituted by the so called transcendental properties, i.e., properties that characterize every possible object without exception. The second kind is constituted by the invariant properties which characterize only some objects. In what follows, properties of this second kind are identified with the essential properties in a strict sense. The basic idea here is that an essential property is a property which both flows from the essence of a, so that every object having the same essence as a is characterized by it, and individuates a, that is, it is differentiates a with respect to

other entities. It is then evident that any differentiating property is not shared by all objects.[8]

DEFINITION 10 (Essential properties).
$$\mathbf{EP}(a, \lambda x.\varphi) := \forall x \square (a \approx x \to \varphi) \land \exists x \neg \varphi$$

A property is essential when *it is a non-universal invariant property*. The difference between invariant and essential properties can be illustrated by considering the following conditions.

1. $\lambda x.(x \approx x)$, i.e., being essentially identical with itself
2. $\lambda x.(x \approx a)$, i.e., being essentially identical with a
3. $\lambda x.(x = x)$, i.e., being identical (and thus qualitatively identical) with itself
4. $\lambda x.(x = a)$, i.e., being identical (and thus qualitatively identical) with a

Every object is both identical and essentially identical with itself, so that both $\forall x \square (x = x)$ and $\forall x \square (x \approx x)$ are logically valid. Hence, both $\forall x \square (a \approx x \to x = x)$ and $\forall x \square (a \approx x \to x \approx x)$ are logically valid, and so $\lambda x.(x = x)$ and $\lambda x.(x \approx x)$ are invariant, but not essential, properties. In general, all universal properties are invariant, but no such property is essential. By contrast, let us consider the two other properties: $\lambda x.(x = a)$ and $\lambda x.(x \approx a)$. It is not difficult to see that

$\lambda x.(x = a)$ is not universal
$\lambda x.(x \approx a)$ is not universal
$\forall x \square (a \approx x \to x \approx a)$ is logically valid
$\forall x \square (a \approx x \to x = a)$ is not logically valid

The following propositions are then straightforward corollaries of the Definitions 9 and 10.

PROPOSITION 1: $\square(a, \lambda x.(x \approx x))$.
PROPOSITION 2: $\mathbf{EP}(a, \lambda x.(x \approx a))$.
PROPOSITION 3: $\square(a, \lambda x.(x = x))$.
PROPOSITION 4: $\neg\mathbf{EP}(a, \lambda x.(x = a))$.

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[8] This move is similar to the one proposed in (Zalta 2006, Section 3), where a distinction is introduced between necessary, weakly essential, and strongly essential properties. According to Zalta, a property is strongly essential to a just in case it is a non-necessary property exemplified by a in every world in which a exists. In our framework every real property exemplified by a is necessarily exemplified by it, so that the non-necessity condition is replaced with the non-universality condition.

Finally, note that, in the present framework, a property of an object can be said to be possible in a number of different ways. To illustrate:

- possible property: $\exists x \Diamond (a \approx x \wedge \varphi)$.
- potential property: $\exists x \Diamond (\mathbf{E}(x) \wedge a \approx x \wedge \varphi)$.
  All the properties of existent objects are potential, and so essential properties of existent objects are potential.
- accidental property: $\exists x \Diamond (a \approx x \wedge \varphi) \wedge \exists x \Diamond (a \approx x \wedge \neg\varphi)$.
  Not all the properties of existent objects are accidental, since essential properties are not accidental.

3.2. Fine's problem

One of the most interesting consequences of the definition of an essential property is that it provides a plain solution to Fine's challenge concerning the relation between objects and their singletons. As is known, according to Fine, any definition of essential property should avoid the conclusion that being the unique member of the corresponding singleton is an essential property of an object. Now, the question as to whether it is essential for a to be the unique element of $\{a\}$ can be interpreted in two different ways. According to a first interpretation, the relevant property is $\lambda x.(x \in \{a\})$. In this case, we can conclude that it is not the case that $\mathbf{EP}(a, \lambda x.(x \in \{a\}))$, since $\forall x \Box (a \approx x \rightarrow x \in \{a\})$ is not logically valid, as witnessed by any object with at least a qualitative variant. According to a second interpretation, the relevant property is $\lambda x.(x \in \{x\})$. In this case, we can conclude that, while being an invariant property, $\lambda x.(x \in \{x\})$ is not an essential property of a, since $\lambda x.(x \in \{x\})$ is a universal property.

3.3. Theorems about essential properties

Invariant properties are characterized by the following traits. Note that propositions correspondent to E1–E8 are true with respect to essential properties.

E1: $\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \varphi_a^x$.
E2: $\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \Box\varphi_a^x$.
E3: $\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \Box\mathbf{IP}(a, \lambda x.\varphi)$.
E4: $\neg\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \Box\neg\Box\mathbf{IP}(a, \lambda x.\varphi)$.
E5: $\mathbf{IP}(a, \lambda x.\varphi) \wedge a \approx b \rightarrow \mathbf{IP}(b, \lambda x.\varphi)$.

E6: $\neg\mathbf{IP}(a, \lambda x.\varphi) \wedge a \approx b \rightarrow \neg\mathbf{IP}(b, \lambda x.\varphi)$.
E7: $\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \mathbf{IP}(a, \lambda y.\mathbf{IP}(y, \lambda x.\varphi))$.
E8: $\neg\mathbf{IP}(a, \lambda x.\varphi) \rightarrow \mathbf{IP}(a, \lambda y.\neg\mathbf{IP}(y, \lambda x.\varphi)$.
E9: $\forall x \Box \varphi \rightarrow \mathbf{IP}(a, \lambda x.\varphi)$.
E10: $\mathbf{IP}(a, \lambda x.\varphi) \wedge \forall x \Box(\varphi \rightarrow \psi) \rightarrow \mathbf{IP}(a, \lambda x.\psi)$.

E1–E10 state some basic and intuitive facts concerning invariant properties. According to these propositions, any object is characterized and necessarily characterized by all its invariant properties. Furthermore, properties that are invariant to an object are necessarily invariant to it, are invariant for all its variants, and so it is an invariant property of a to be characterized by its invariant properties. Similarly, properties that are not invariant to an object are necessarily not invariant to it, are not invariant for all its variants, and so it is an invariant property of a not to be characterized by its non-invariant properties. Finally, every universal necessary property is invariant to any object and if something is invariantly characterized by a certain property, then it is invariantly characterized by any property strictly implied by that property.

Let us prove E3 and E5 (the other proofs are left as an easy exercise).

Proof of E3:

$\mathbf{IP}(a, \lambda x.\varphi) \vdash \Box(a \approx x \rightarrow \varphi)$ by def. $\mathbf{IP}$
$\mathbf{IP}(a, \lambda x.\varphi) \vdash \Box\Box(a \approx x \rightarrow \varphi)$ by modal logic
$\mathbf{IP}(a, \lambda x.\varphi) \vdash \forall x \Box\Box(a \approx x \rightarrow \varphi)$ by logic
$\mathbf{IP}(a, \lambda x.\varphi) \vdash \Box\forall x \Box(a \approx x \rightarrow \varphi)$ by modal logic
$\mathbf{IP}(a, \lambda x.\varphi) \vdash \Box\mathbf{IP}(a, \lambda x.\varphi)$ by def.$\mathbf{IP}$

Proof of E5:

$\mathbf{IP}(a, \lambda x.\varphi), a \approx x \vdash \varphi$ by def.$\mathbf{IP}$
$\mathbf{IP}(a, \lambda x.\varphi), a \approx b, b \approx x \vdash \varphi$ by C3
$\mathbf{IP}(a, \lambda x.\varphi), a \approx b \vdash b \approx x \rightarrow \varphi$ by logic
$\mathbf{IP}(a, \lambda x.\varphi), a \approx b \vdash \Box(b \approx x \rightarrow \varphi)$ by $\mathbf{NE}$ and E3
$\mathbf{IP}(a, \lambda x.\varphi), a \approx b \vdash \mathbf{IP}(a, \lambda x.\varphi)$ by logic and def. $\mathbf{IP}$

E5 is worth noting, because it allows for the substitution of essentially identical objects with respect to essential properties. Hence, it is possible to introduce a substitution rule for essentially identical objects that parallels the standard rule of substitution:

Standard Substitution ($=$)	Essential Substitution ($\approx$)
$X \vdash \varphi$	$X \vdash \mathbf{IP}(a, \lambda x.\varphi)$
$X \vdash a = b$	$X \vdash a \approx b$
$X \vdash \varphi_b^a$	$X \vdash \mathbf{IP}(b, \lambda x.\varphi)$

Hence, E5 implies that two individuations of the same object can only differ with respect to non-essential properties.

3.4. Some basic kinds of objects

Having defined the most interesting kinds of properties, let us now move to the definitions of the most interesting kinds of objects.

DEFINITION 11 (invariant and variant objects).

1. a is invariant: $\mathbf{IO}(a) := \mathbf{IP}(a, \lambda x.a = x)$, i.e., $\forall x \Box (a \approx x \to a = x)$,
2. a is variant: a is not invariant $:= \Diamond(a, \lambda x.a \neq x)$, i.e., $\exists x \Diamond (a \approx x \wedge a \neq x)$.

According to the definition, an object is variant if there is a qualitative variant of it that is different from it, whereas it is invariant if no such variant exists. Traditionally, invariant objects are just unchangeable objects, i.e., entities that do not alter their qualitative identity, whereas variant objects are changeable ones.

DEFINITION 12 (necessary and contingent objects).

1. a is necessary $:= \Box \mathbf{E}(a)$,
2. a is contingent $:= \mathbf{E}(a) \wedge \neg \Box \mathbf{E}(a)$.

A necessary object is an object that necessarily exists, while a contingent object is an existent, but not necessarily existent, object.

3.5. Theorems on the kinds of objects

Let us start by proving that every invariant object is invariantly characterized by all its real properties.

THEOREM 3. *Let* $\lambda x.\varphi$ *be a real property. Then*

$$\mathbf{IO}(a) \wedge \varphi_a^x \vdash \mathbf{IP}(a, \lambda x.\varphi).$$

PROOF. Suppose $\lambda x.\varphi$ is a real property, so that $\forall x(\varphi \to \Box\varphi)$.

$\varphi_a^x, a = x \vdash \varphi$	by logic
$\mathbf{IO}(a), a \approx x \vdash a = x$	by def. $\mathbf{IO}$
$\mathbf{IO}(a) \wedge \varphi_a^x, a \approx x \vdash \varphi$	by logic
$\mathbf{IO}(a) \wedge \varphi_a^x \vdash \Box(a \approx x \to \varphi)$	by $\mathbf{NE}$, E3, and $\lambda x.\varphi$ real
$\mathbf{IO}(a) \wedge \varphi_a^x \vdash \mathbf{IP}(a, \lambda x.\varphi)$	by logic and def. $\mathbf{IP}$ $\dashv$

Theorem 3 is the *First fundamental theorem* of classical modal logic. It captures the classical intuition according to which, since an unchangeable object is an object with no unactualized potentiality, no property is accidental to it. In other words, if an object is purely actual, then all its properties, more precisely real properties, are essential to it.

THEOREM 4. *Every necessary object is invariant.*

PROOF. Suppose a is a necessary object.

$\mathbf{E}(a), \mathbf{E}(x), a \approx x \vdash a = x$	by CE
$\mathbf{E}(a), a \approx x, a \neq x \vdash \neg\mathbf{E}(x)$	by logic
$\Box\mathbf{E}(a), \Box(a \approx x), \Box(a \neq x) \vdash \Box\neg\mathbf{E}(x)$	by $\mathbf{RN}$
$\Box\mathbf{E}(a), \Box(a \approx x), \Diamond\mathbf{E}(x) \vdash \Diamond(a = x)$	by logic
$\Box\mathbf{E}(a), \Diamond\mathbf{E}(x), a \approx x \vdash a = x$	by $\mathbf{NE}$
$\mathbf{E}(a), a \approx x \vdash \Diamond\mathbf{E}(x)$	by CP
$\Box\mathbf{E}(a), a \approx x \vdash a = x$	by logic
$\Box\mathbf{E}(a) \vdash \Box(a \approx x \to a = x)$	by logic
$\Box\mathbf{E}(a) \vdash \mathbf{IO}(a)$	by logic and def. $\mathbf{IO}$ $\dashv$

COROLLARY 1. *Every necessary object is invariantly existent.*

PROOF. we have seen that $\Box\mathbf{E}(a), a \approx x \vdash a = x$. Hence

$\Box\mathbf{E}(a), a \approx x \vdash \Box\mathbf{E}(a)$	by logic
$\Box\mathbf{E}(a), a \approx x \vdash \Box\mathbf{E}(x)$	by logic
$\Box\mathbf{E}(a) \vdash \Box(a \approx x \to \Box\mathbf{E}(x))$	by logic
$\Box\mathbf{E}(a) \vdash \mathbf{IP}(a, \lambda x.\mathbf{E}(x))$	by logic and def. $\mathbf{IP}$ $\dashv$

Since every object that exists by essence is necessary, we obtain:

COROLLARY 2. *Every object that essentially exists is invariant and invariantly existent.*

Since $\mathbf{A}(a) \vdash \Box\mathbf{E}(a)$, so that $\mathbf{A}(a) \vdash \mathbf{IO}(a) \wedge \mathbf{IP}(a, \lambda x.\mathbf{E}(x))$, we get.

COROLLARY 3. *Every abstract object is invariant and invariantly existent.*

Theorem 4 is the *second fundamental theorem* of classical modal logic. It links invariance and existence and captures the classical intuition according to which what is necessarily existent is purely actual, and so not subject to change. In addition, on the assumption that there are abstract objects, we conclude that all abstract objects exist by essence. As we will see what exists by essence, being necessary, is *in actu* all that has the power of being: *actus purus*.

THEOREM 5. *If all objects exist, then all objects are unchangeable.*

$\forall x \mathbf{E}(x) \vdash \mathbf{E}(a)$	by logic
$\mathbf{E}(a), \mathbf{E}(x), a \approx x \vdash a = x$	by CE
$\forall x \mathbf{E}(x), a \approx x \vdash a = x$	by logic
$\forall x \mathbf{E}(x) \vdash \mathbf{IO}(a)$	by logic and def. **IO**

This conclusion is in accordance with the fact that changeable objects have potential variants, i.e., variants which do not actually exist.

3.6. The relation of exemplification

The predicate of existence allows us to introduce a distinction between characterization and exemplification.

DEFINITION 13 (Exemplification). $\mathbf{EX}(a, \lambda x.\varphi)$ if and only if $\mathbf{E}(a) \wedge \varphi_a^x$.

A property is exemplified just in case it characterizes an existent object. As straightforward consequences of the definition we obtain:
- $\mathbf{EX}(a, \lambda x.\varphi) \rightarrow \varphi_a^x$
- $\mathbf{EX}(a, \lambda x.\varphi) \rightarrow \mathbf{E}(a)$

According to the definition, only existent objects can exemplify properties, so that the thesis characterizing serious actualism is recovered with respect to the exemplification relation.

DEFINITION 14 (Potential Exemplification).
$\mathbf{PEX}(a, \lambda x.\varphi)$ if and only if $\mathbf{E}(a) \wedge \exists x(a \approx x \wedge \varphi)$.

A property is potentially exemplified just in case it characterizes a variant of an existent object. The connection between actual exemplification and potential exemplification is intuitive: $\mathbf{EX}(a, \lambda x.\varphi) \rightarrow \Diamond(a, \lambda x.\varphi)$. Indeed, given $\mathbf{E}(a) \wedge \varphi_a^x$, we get $\mathbf{E}(a) \wedge a \approx a \wedge \varphi_a^x$, and so $\mathbf{E}(a) \wedge \exists x(a \approx x \wedge \varphi)$. Moreover, assuming that something is *actus purus* just in case it exemplifies all its potential properties, we can conclude that every unchangeable object is *actus purus*.

THEOREM 6. *Every invariant object is actus purus.*

PROOF. Suppose that $\lambda x.\varphi$ is a real property. Then
$\mathbf{IO}(a) \wedge \mathbf{PEX}(a, \lambda x.\varphi) \vdash \mathbf{EX}(a, \lambda x.\varphi)$.

$\mathbf{IO}(a), a \approx x \vdash a = x$	by def. $\mathbf{IO}$
$\mathbf{IO}(a), \mathbf{E}(a), a \approx x \wedge \varphi \vdash \mathbf{E}(a) \wedge \varphi_a^x$	by logic
$\mathbf{IO}(a), \mathbf{E}(a), \exists x(a \approx x \wedge \varphi) \vdash \mathbf{E}(a) \wedge \varphi_a^x$	by logic
$\mathbf{IO}(a), \mathbf{PEX}(a, \lambda x.\varphi) \vdash \mathbf{EX}(a, \lambda x.\varphi)$	by logic and def. $\mathbf{PEX}$, $\mathbf{EX}$ $\dashv$

As a corollary, we conclude that *every necessary object is actus purus*, which is one of the most important proposition in classical metaphysics. The other crucial proposition in classical metaphysics, the one according to which there is only one necessary object, is not deducible within the present framework. Hence, the possibility is open for more than an object to be necessarily existent[9].

4. Comparison with current modal frameworks

The aim of this section is to briefly highlight the position of the system proposed above within the context of the modal frameworks currently available.

4.1. The central tenets of classical modal logic

In a sense, classical modal logic can be viewed as based on (1) an analysis of a threefold interpretation of the word "is" and (2) the introduction of three corresponding distinctions:

"is" of existence	general *vs* actual existence
"is" of identity	complete *vs* essential identity
"is" of attribution	accidental *vs* essential attribution

(i) The distinction between general and actual existence is construed as a distinction between the sense of existence captured by the existential quantifier, whose domain consists of all the objects admitted in our

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[9] The impossibility of the existence of more than one necessary object might be derived in a more powerful theoretical context, where a relation of ontological dependence is introduced together with an axiom stating that, if a necessary object a exists, then every other object is ontologically dependent, with respect to its existence, by a. This axiom, however, is not uncontroversial. Indeed, e.g., in the platonic tradition, it is often assumed that there is more than one necessarily existing entity.

ontology, and the sense of existence captured by a predicate of existence, which restricts the existential quantifier to the domain of actually existing objects, i.e., things having the primitive non-relational property of actual existence.

(ii) The distinction between identity and essential identity is construed as a distinction between the sense of identity captured by a primitive relation, which is characterized by the principle of the general indiscernibility of identical objects, and the sense of identity captured by a primitive relation, which is characterized by the principle of essential indiscernibility of the essential identical objects, where it is possible for two qualitatively distinct objects to be essentially indiscernible, i.e., essentially identical.

(iii) The distinction between accidental and essential attribution is construed as a distinction between the sense of attribution according to which the attributed property actually characterizes an object and the sense of attribution according to which the attributed property characterizes all the possible variants of the same object. However, both the accidental and the essential properties are, in a specific sense, necessary properties. Hence, if an object is P, then that very object, *the object which is P*, cannot possibly lack this property, even if that very object, *the object* which is P, can lack this property.

Let us then sketch how classical modal logic is related to the main conception according to which the quantifiers range over a unique fixed domain, which includes both actual and possible objects. In particular, we will consider Meinongianism, Leibnizianism, Lewisianism and the simplest modal logic.

4.2. Comparison with Meinongianism

Let us define Meinongianism as the position holding (1) that impossible objects, i.e., objects characterized by incompossible properties, can exist and (2) that the distiction between being and existence coincides with the distinction between the sense of existence rendered by the existential quantifier and the sense of existence rendered by a first order predicate of existence.[10] Then, Meinongianism agrees with classical modal logic in assuming that possible objects exist and exist in a sense that is not the one in which actual objects exist. However, Meinongianism and clas-

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[10] See (Berto, 2012, ch. 6) for an introduction to different kinds of Meinongianism.

sical modal logic disagree on some substantial points. First, according to classical modal logic both actual and possible objects are completely determinate, whereas Meinongianism is consistent with the assumption that both actual and possible objects are completely non-determinate. As a consequence, properties, except existence, are not relative to possible worlds. Indeed, it is a tenet of classical modal logic that any object necessarily possesses every real property it possesses, whilst in the framework of restricted Meinongianism no contradiction arises if one assumes that no object necessarily possesses any real property.[11] Secondly, according to classical modal logic there is only one kind of properties and there is only one kind of predication, whereas it is a typical trait of Meinongianism to distinguish either between different kinds of properties or between different kinds of predications. Still, two crucial kinds of predications are definable in the classical modal framework: essential predication (related to invariant properties) and exemplification (related to actually possessed properties). In addition, predication with respect to abstract objects allows us to cope with encoding, since we can define encoding in terms of exemplification by an abstract object. Finally, in classical modal logic logically impossible objects are not in the domain of quantification, even if actually impossible objects are admitted, while a further typical trait of Meinongianism is to allow for the existence of contradictory, and so logically impossible, objects.[12]

4.3. Comparison with Leibnizianism

Leibniz defines the concept of substance, or individual object, as a complete concept of an object, which is a concept from which all the attributes that characterize the object can be deduced. Hence, a substance is something complete, i.e., something including, for every predicate P, P or its negation.[13] Leibniz' conception of individual object seems to imply the following principles.

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[11] But note that in Zalta's view objects necessarily encode the properties they encode.

[12] It is true that the present framework might be extended by introducing logically impossible worlds, but in classical metaphysics logically impossible worlds are weird entities, since the things that exist, in the sense of the existential quantifier, are precisely the things whose existence is free from contradiction.

[13] See (Nachtomy, 2007, ch. 2 and ch. 4) for an introduction to the concept of an individual object in Leibniz.

- The principle of Leibnizian possibility.
  LP: $\forall x \Diamond \mathbf{E}(x)$.
- The principle of Leibnizian essentialism, or super-essentialism.
  LE: $\forall x, y \Box(x \approx y \to x = y)$, i.e., $\forall x \nabla(x)$.

Accordingly, Leibniz's system is a specification of the classical modal system, where the distinction between potentiality and logical possibility is neglected, so that all the logically possible objects are actually possible, and the relation of essential identity coincides with the relation of identity. As a consequence, Leibniz's system allows us to derive the following crucial propositions.

Proposition 5. *No object can possess an accidental property.*

Proof. $P(a) \vdash x = a \to P(x)$
  $P(a) \vdash x \approx a \to P(x)$
  $P(a) \vdash \Box(x \approx a \to P(x))$
  $P(a) \to \mathbf{Ess}(a, P(x))$ $\dashv$

Hence, every real property is an essential property.

Proposition 6. *No object can exist in more than one world.*

To say that an individual a exists in more than one world is equivalent to saying that there is a variant of a that exists in a different world: $\exists x \Diamond(x \approx a \land x \neq a \land \mathbf{E}(x))$, which implies $\exists x \Diamond(x \approx a \land x \neq a)$, contradicting LE. Hence, individual objects are world-bound.

4.4. Comparison with Lewisianism

There are several similarities between Lewisianism and classical modal logic.[14] On the one hand
- possible objects exist,
- possible objects are completely determinate,
- properties, except existence, are not relative to possible worlds,
- there is only one kind of properties and there is only one kind of predication.

On the other hand, the extension of the property of existence changes across worlds and the relation of across worlds identity might be inter-

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[14] See (Lewis, 1986, ch. 1) for a first hand presentation.

preted as a relation of counterparthood, since objects are world-bound.[15]
That being said, it is not difficult to point out some significant intuitive
differences between the two standpoints, even if it is more difficult to
identify them at the semantic level. In fact, in contrast to Lewisianism,
classical modal logic assumes (1) that the predicate of existence is not re-
ferred to a relational property, such as the property of being co-habitant
in a certain world, but to a primitive property of existence and (2) that
the relation of essential identity is not instantiated by objects that are
counterparts of each other, but by qualitatively different variants of an
essentially identical object. The first difference is not reflected in our
semantics as it stands, and so it is left to our intuition. The second
difference is indeed reflected by two facts:

- the relation of essential identity is an equivalence relation, while the
  counterpart relation is a similarity relation, since it depends on the re-
  spect that one assume in order to compare objects in different worlds;
- there is a unique relation of essential identity in our framework, while
  there is no limit to the number of counterpart relations that can be
  introduced in a Lewisian framework.

It is worth noting that, while the first fact can be bypassed, by assuming
strong counterpart relations that are symmetric and transitive,[16] the
second one marks a central distinction between the two approaches. To
be sure, the selection of a specific counterpart relation among all the
possible relations of this kind can only be based on some real character-
istic of the related object, and this real characteristic is precisely what
in the classical framework plays the role of the essence.

A final difference between Lewisianism and classical modal logic is
due to the fact that classical modal logic allows for the existence, in the
sense of the existential quantifier, of some logically possible objects that
cannot exist at a certain world. In fact, a sentence like $\exists x(x = a) \wedge
\neg \Diamond Ea$ can be true in our logic, while it should be rejected in a Lewisian
framework, in light of the fact that possible existence is existence in a
different possible world. Indeed, if possible existence is existence in a
different possible world and everything that exists, in the sense of the
existential quantifier, exists at some world, then everything that exists,

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[15] For an analysis of similarities and dissimilarities between Lewisianism and
Meinongianism (see Linsky and Zalta, 1991).

[16] The counterpart relation allows for different interpretations. See (Fara, 2008)
for a discussion.

in the sense of the existential quantifier, is possibly existent, in the sense of the predicate of existence, at any world.

4.5. Comparison with simplest modal logic

In (Linsky and Zalta, 1994) it is argued that the simplest quantified modal logic can be indifferently interpreted both in the light of possibilism and in the light of actualism. To be sure, while possibilists assume an original difference between possible but non-existent objects and existent objects, actualists can assume an original distinction between actual but non-concrete objects and concrete objects.[17] According to the new actualism, the distinction between being abstract and being concrete is not absolute, so that abstract objects are not essentially abstract and concrete objects are not essentially concrete. As a consequence, even though all objects are actual, it is possible for an abstract object to become concrete and for a concrete object to become abstract. As we can see, Linsky and Zalta's concepts of abstractness and concreteness are definable in the present framework:

1. $\mathbf{C}_z(x) := \mathbf{E}(x) \wedge \neg \mathbf{A}(x)$
2. $\mathbf{A}_z(x) := \mathbf{A}(x) \vee \neg \mathbf{E}(x)$

Hence, classical modal logic is compatible with Linsky and Zalta's interpretation of the predicate of existence under the condition that two senses of abstractness are introduced: (1) abstractness as a condition of existence, e.g., of mathematical objects, and (2) abstractness as a condition of non-existence, e.g., of physical objects. However, contrary to Linsky and Zalta's interpretation, in classical modal logic predication is not relative to possible worlds, and this is a crucial difference between the two approaches.

References

Adams, R. M., 1981, "Actualism and thisness", *Synthese* 49: 3–41.

Berto, F., 2012, *Existence as a Real Property*, Dordrecht: Springer.

Chalmers, D., D. Manley, and R. Wasserman, 2009, *Metametaphysics*, Oxford: Clarendon Press.

Fara, D. G., 2008, "Relative-sameness counterpart theory", *The Review of Symbolic Logic* 1: 167–189. DOI: 10.1017/S1755020308080155

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[17] On contingently non-concrete objects, see also (Linsky and Zalta, 1996).

Fine, K., 1994, "Essence and modality", *Philosophical Perspectives* 8: 1–16.

Fine, K., 1995, "Ontological dependence", *Proceedings of the Aristotelian Society* 95: 269–290.

Fitting, M., and R. Mendelsohn, 1998, *First-Order Modal Logic*, Dordrecht: Spinger.

Gracia, J., 1994, *Individuation in Scholasticism*, Albany, NY: State University of New York Press.

Hughes, G., and M. Cresswell, 1996, *A New Introduction to Modal Logic*, London: Routledge.

Jager, T., 1982, "An actualist semantics for quantified modal logic", *Notre Dame Journal of Formal Logic* 23: 335–349.

Kripke, S., 1980, *Naming and Necessity*, Cambridge: Harvard University Press.

Lewis, D., 1986, *On The Plurality of Worlds*, Oxford: Blackwell.

Linsky, B., and E. Zalta, 1991, "Is Lewis a Meinongian?", *Australasian Journal of Philosophy* 69: 438–453.

Linsky, B., and E. Zalta, 1994, "In defense of the simplest quantified modal logic", *Philosophical Perspectives* 8: 431–458.

Linsky, B., and E. Zalta, 1996, "In defense of the contingently non-concrete", *Philosophical Studies* 84: 283–294.

Linsky, B., and E. Zalta, 2006, "Essence and modallity", *Mind* 115: 659–693.

Loux, M., 1979, *The Possible and the Actual*, Ithaca: Cornell.

Lowe, E. J., 1998, *The Possibility of Metaphysics*, Oxford: Clarendon Press.

Lowe, E. J., 2006, *The Four-Category Ontology*, Oxford: Clarendon Press.

McDaniel, K., 2009, "Ways of being", pages 290–319 in (Chalmers et al., 2009).

Menzel, C., 1990, "Actualism, ontological commitment, and possible worlds semantics", *Synthese* 58: 355–389.

Menzel, C., 1991, "The true modal logic", *Journal of Philosophical Logic* 20: 331–374. DOI: 10.1007/BF00249434

Nachtomy, O., 2007, *Possibility, Agency, and Individuality in Leibniz's Metaphysics*, Dordrecht: Springer.

Plantinga, A., 1974, *The Nature of Necessity*, Oxford: Oxford University Press.

Plantinga, A., 1976, "Actualism and possible worlds", *Theoria* 42: 139–160.

Plantinga, A., 1983, "On existentialism", *Philosophical Studies* 44: 1–20.

Shaffer, J., 2009, "On what grounds what", pages 347–383 in (Chalmers et al., 2009).

Stalnaker, R., 2003, *Ways a World Might Be*, Oxford University Press: Oxford.

van Inwagen, P., 2009, "Being, existence, and ontological commitment", pages 472–506 in (Chalmers et al., 2009).

Williamson, T., 1998, "Bare possibilia", *Erkenntnis* 48: 257–273.

Zalta, E., 1988, *Intensional Logic and the Metaphysics of Intentionality*, Cambridge: MIT Press.

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