THREE PROBLEMATIC THEORIES OF CONDITIONAL ACCEPTANCE

Michael J. SHAFFER

ABSTRACT: In this paper it is argued that three of the most prominent theories of conditional acceptance face very serious problems. David Lewis' concept of imaging, the Ramsey test annd Jonathan Bennett's recent hybrid view all face viscous regresses, or they either employ unanalyzed components or depend upon an implausibly strong version of doxastic voluntarism.

KEYWORDS: conditionals, Ramsey test, imaging, doxastic voluntarism, hypothetical belief

One of the most plausible suggestions concerning how the probabilities of conditionals ought to be construed is that the probability of a conditional should be interpreted as the conditional probability of the consequent given the antecedent. This comports well with the widely held view that the acceptability of a proposition goes by high probability. So,

$P(A > B) = P(B \mid A)$ for all A, C in the domain of P with P(A) greater than 0,

and,

$P(B \mid A) = P(BA)/P(A)$ provided $P(A) \neq 0$.

Alan Hàjek has proposed the acronym 'CCCP' to refer to this account (the conditional construal of conditional probability). Unfortunately, as David Lewis and others demonstrated, CCCP cannot be correct on pain of triviality. Based on some rather minimal assumptions, Lewis showed that any language having a universal probability conditional is a trivial language, and, hence, by reductio CCCP must be rejected.[1] Furthermore, CCCP was proved to be trivial under

[1] In David Lewis, "Probabilities of Conditionals and Conditional Probabilities," *Philosophical Review* 85 (1976): 297-315.

considerably weaker assumptions than those originally made in Lewis and so the result has proven to be resilient.[2]

So, subsequent to rejecting CCCP, Lewis suggested that probability conditionals should be understood as policies for *feigned* minimal belief revision, and that the probability of such a conditional should be understood to be the probability of the consequent given the minimal revision of P(·) that makes the probability of the antecedent of the conditional equal to 1. Formally, imaging is defined as follows:

$P(A > B) = P'(B)$, if A is possible.

In this expression P′(·) is the minimally revised probability function that makes P(A) = 1. Lewis tells us that we are to understand this expression along the following lines. P(·) is to be understood as a function defined over a finite set of possible worlds, with each world having a probability P(w) Furthermore, the probabilities defined on these worlds sum to 1, and the probability of a sentence, A for example, is the sum of the probabilities of the worlds where it is true. In this context the image on A of a given probability function is obtained by 'moving' the probability of each world over to the A-world closest to w. Finally, the revision in question is supposed to be the minimal revision that makes A certain. In other words, the revision is to involve all and only those alterations necessary for making P(A) = 1.[3] So is Lewis' concept of imaging then the correct way to interpret the acceptability conditions of conditionals? The answer suggested here is that it is not.

First what are we to make of the expression P′(B)? Normal probability functions are defined over a set of literal beliefs about what is possible. But what then is the meaning of a probability one *would* assign to the consequent after making the minimal revision of one's beliefs needed to make the probability of the antecedent equal to one? It is not obviously a probability assignment relative to what one actually believes. Such probabilities seem rather to be probability assignments defined over what the agent *might or would believe*. How such hypothetical probabilities are to be epistemically interpreted is not at all clear.

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[2] See Alan Hájek, "Probabilities of Conditionals Revisited," *Journal of Philosophical Logic* 18 (1989), 423-428.

[3] See Peter Gärdenfors, "Imaging and Conditionalization," *The Journal of Philosophy* 79 (1982): 747-760, Donald Nute, *Topics in Conditional Logic* (Dordrecht: D. Reidel, 1980), and Charles B. Cross, "A Characterization of Imaging in Terms of Popper Functions," *Philosophy of Science* 67 (2000): 316-338, for some extensions of the concept of imaging.

This worry arises chiefly because the revision in terms of which P'(B) is defined *does not actually occur*—as *ex hypothesi*—it is only a feigned revision. Such revisions only occur counterfactually and it is not clear how exactly we are to interpret counterfactual probability functions. They have something to do with probability assignments over beliefs an agent would have where she to fully believe the antecedent of the relevant conditional and this has something to do with what those beliefs would be in a minimally revised state relative to the agent's initial belief state. But, this formal answer does little to help us understand the *epistemic* nature of such hypothetical probabilities. Moreover, this is complicated by the fact that what counts as a minimal revision has not been satisfactorily fleshed out in the literature, and so, in any case, we appear to be at a loss to actual employ Lewis' solution in practice.[4] Nevertheless, one might still wish to maintain that imaging is the correct *formal* account of the acceptance conditions for conditionals even if we are at something of a loss to epistemically interpret hypothetical probabilities defined over possible belief states composed of beliefs we don't actually hold.

More interestingly, however, Lewis' suggestion places us in a position that appears to involve a viscous infinite regress and this has apparently gone unnoticed in the discussion of conditionals and their probabilities since Lewis introduced the concept of imaging in 1976. The regress arises as follows. In order to assess the numerical value associated with the image on A of P(·) we must accept another conditional concerning what we would believe if we were certain of A. Again, this is because the belief revision is not an actual belief revision. So, in order to accept an expression of the form A > B we would need to assign a probability to the conditional "If I was certain of A (if it were the case that P(A) = 1), then my beliefs would be K," where K is the set of my minimally revised beliefs and probability ascriptions on those beliefs. Presumably, *this new conditional* about what I would believe if I were certain of *A* must itself be interpreted in terms of imaging as well, for it is not a proposition about which we are certain and—following Lewis—the acceptability of a proposition goes by high subjective probability. So, we must presumably employ imaging *again* in order to accept this conditional about the feigned revision. In order to do this we will have to perform

[4] See Peter Gärdenfors, *Knowledge in Flux* (Cambridge: The M.I.T. Press, 1988), Isaac Levi, *For the Sake of the Argument: Ramsey test Conditionals, Inductive Inference, and Nonmonotonic Reasoning* (Cambridge: Cambridge University Press, 1996), Sven Ove Hansson, "Formalization in Philosophy," *The Bulletin of Symbolic Logic* 28 (2000): 162-175, for a discussion of the problems with the notion of minimal revision.

another feigned revision and so on. Let us consider a simple example. Consider the following set of simple propositions and relevant belief(s):

R: It is raining.

G: The ground is wet.

Belp: x believes that P(p) = 1.

K: x's standing system of beliefs.

According to imaging, in order to accept R > G one must feign a revision in order to assign a value to P′(G) and be able to assess whether to accept BelR > K. But obviously this is itself a conditional and so in order to accept R > G, if we are to avoid viscous circularity, we must be able to assign a probability to BelR > K and thus to P′′(K). By imaging this requires assessing whether to accept Bel(BelR) > K′ but this requires being able to determine the value of P′′(K′) and so the viscous regress begins.

A bit more formally, this problem arises as follows. If P(A > B) = P′(B) by imaging, then to assess the numerical value of P′(B) so that the agent can accept A > B (to the degree of belief that it should be accepted) without succumbing to viscous circularity the agent must accept the conditional P(A) = 1 > K, where K is that agent's minimally revised set of beliefs and probability distribution over those beliefs. Again, to accept P(A) = 1 > K—by Lewis' own admission—is to assign a (high) probability to that sentence, so the agent must be able to evaluate P(P(A) = 1 > K) if the agent is to be able to assess P(A > B). But, by imaging, P(P(A) = 1 > K) = P′′(K), where P′′(K) is the agent's minimally revised beliefs and probability distribution on those beliefs were the agent certain that P(A) = 1 > K or P(P(A) = 1 > K) = 1. Again, according to the definition of the concept of imaging this is itself also only a feigned revision. So, in order to assign a numerical value to P′′(K) the agent must accept a conditional about what that agent would believe if he was certain that if he was certain that A, then B or P(P(A) = 1 > K) = 1 > K′ (where K′ is that agent's suitably revised beliefs and his probability distribution on those beliefs). So, the agent must assign a numerical value to P(P(A) = 1 > K) = 1 > K′ and by imaging P(P(A) = 1 > K) = 1 > K′ = P′′′(K′). But the same line of reasoning applies to this conditional and so on ad infinitum and there does not seem to be any obvious, non-ad hoc, way to stem this regress that results from the nature of imaging qua its being hypothetical. So, for this reason, even if we can make sense of probability distributions over hypothetical beliefs, it does not appear as if imaging will allow us to clearly specify a well-defined prior probability for

conditionals. However, imaging is not the only account of the acceptance conditions for conditionals.

Carlos Alchourrón, Peter Gärdenfors, and David Makinson developed the AGM theory of belief revision in the 1980s and a number of related theories have arisen as a consequence.[5] These theories are fundamentally based on the concept of a belief state, belief set or a corpus of beliefs, K, typically satisfying the following minimal conditions (where it is assumed that belief states are given a representation in some language L):

> (Df BS) A set of sentences, K, is a belief state if and only if (i) K is consistent, and (ii) K is objectively closed under logical implication.

The content of a belief state is then defined as the set of logical consequences of K (so $\{b: K \in b\} =_{df.} Cn(K)$). Given this basic form of epistemic representation, the AGM-type theories are intended to be a normative theory about how a given belief state which satisfies the definition of a belief state is related to other belief states satisfying that definition relative to: (1) the addition of a new belief b to K_i, or (2) the retraction of a belief b from K_i, where $b \in K_i$. Belief changes of the latter kind are termed *contractions*, but belief changes of the former kind must be further sub-divided into those that require giving up some elements of K_i and those that do not. Additions of beliefs that do not require giving up previously held beliefs are termed *expansions*, and those that do are termed *revisions*.[6] Specifically, for our purposes here it is the concept of a revision that is of crucial importance to the issue of providing an account of rational commitment for conditionals. In any case, given AGM-style theories the dynamics of beliefs will then simply be the epistemically normative rules that govern rational cases of contraction, revision and expansion of belief states.

The fundamental insight behind these theories is then that belief changes that are contractions should be fundamentally conservative in nature. In other words, in belief changes one ought to make the minimal alterations necessary to incorporate new information and to maintain or restore logical consistency. This fundamental assumption is supposed to be justified in virtue of a principle of informational economy. This principle holds that information is intrinsically and

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[5] See Carlos E. Alchourrón, Peter Gärdenfors and David Makinson, "On the Logic of Theory Change: Partial Meet Functions for Contraction and Revision," *Journal of Symbolic Logic* 50 (1985): 510-530, Gärdenfors, *Knowledge in Flux*, and Levi, *For the Sake of the Argument*.

[6] In point of fact the AGM theory really only holds that there are two dynamical operations on belief states, because revision is defined in terms of expansion and contraction.

practically valuable and so we should retain it at all costs unless we are forced to do otherwise. So, while the details are not important here, the revision operations on belief states are restricted so as to obey a principle of minimal mutilation.

What is important to the topic of this paper is that on the basis of such theories of belief revision, the defenders of this approach to belief dynamics have also proposed that one could also give a theory of rational conditional commitment.[7] The core concept of this theory is the Ramsey Test:[8]

> (RT) Accept a sentence of the form 'If p, then q' in the state of belief K if and only if the minimal change of K needed to accept p also requires accepting q.

Even in this quasi-formal form we can see what the AGM and other theorists have in mind. The Ramsey Test requires that we modify our beliefs by accepting p into our standing system of beliefs and then see what the result is.[9] What this theory then requires of us is either (1) that our actual system of beliefs must be altered in order to believe a conditional or (2) that we hypothetically modify our beliefs by hypothetically accepting p in order to accept a conditional.[10] So, there are at least two main possible interpretations of the Ramsey Test. However, there are serious problems with this theory of conditional endorsement given either interpretation.

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[7] See Peter Gärdenfors, "An Epistemic Approach to Conditionals," *American Philosophical Quarterly* 18 (1981): 203-211, and Gärdenfors, *Knowledge in Flux*.

[8] See Frank Plumpton Ramsey, "Law and Causality," in *F.P. Ramsey: Philosophical Papers*, ed. David Hugh Mellor (Cambridge: Cambridge University Press, 1990), 140-163.

[9] David H. Sanford, *If P, then Q*, Second edition (New York: Routledge, 2003) objects that in many cases where the antecedent of such a conditional is a radical departure from what we believe to be the case, we cannot in fact employ the Ramsey test because we do not know what would be the case if we believed such an antecedent. So, he claims that many conditions are simply void, rather than true or false. It is worth pointing out here that Sanford's criticism is weak at best. It simply does not follow that because we cannot always clearly determine what would be the case if we were to believe some claim, a conditional with such an antecedent has no truth value. See Timothy Williamson, *The Philosophy of Philosophy* (Blackwell: Oxford, 2007), chapters 5 and 6, for discussion of one suggestion for how such knowledge might be obtained.

[10] Jonathan Francis Bennett favors a version of the former interpretation (see his *A Philosophical Guide to Conditionals* (Oxford: Oxford University Press, 2003), 28-30), but his denials that Ramsey did not intend 2 and that 2 is an incorrect interpretation of the Ramsey Test are not especially convincing and there is little in the way of textual evidence to support this claim because of the brevity of Ramsey's comments on the matter. Both Gärdenfors ("An Epistemic Approach to Conditionals," and *Knowledge in Flux*) and Levi (*For the Sake of the Argument*) seem to endorse interpretation 2 and to my mind this is the more common interpretation.

First, while the details of the various theories of belief revision are not at issue here, it has proved to quite difficult to define an acceptable account of a minimal belief revision.[11] More worrisome yet, given interpretation (1), is the fact that the RT theory of conditionals appears to depend essentially on the truth of doxastic voluntarism—the view that we can change our beliefs at will. The truth of doxastic voluntarism is of course a matter of serious contention, but we need not delve too deeply into the debate about doxastic voluntarism here in any case to see that problems arise for the Ramsey test.[12] This is because the Ramsey test theory of conditionals depends on the truth of the least plausible version of doxastic voluntarism, what we might call *unrestricted doxastic voluntarism*. This is just the view that beliefs are totally, completely and directly under our control. But this is utterly and irreparably unrealistic from both the psychological and epistemological perspectives. On this interpretation of the Ramsey test, we must literally *believe* the antecedent of a conditional in order to apply the test at all. This is true for *every* conditional and thus requires that we be able to voluntarily believe *any* proposition, because any proposition can be the antecedent of a conditional. This includes propositions like "I can walk through the wall of my office," "6 + 3 = 11" and even perhaps "It is raining and it is not raining." It is not clear that it is possible to do this. In part this seems to be the case because belief seems to be intrinsically evidential in nature. But the Ramsey test then appears to assume the falsity of evidentialism and so is problematic from an epistemological perspective.[13] But even if evidentialism is false the Ramsey test is still problematic because of the psychological implausibility of unrestricted doxastic voluntarism and it is quite easy to verify this. Simply consider the following conditional: "If I could fly at will, then I would go to Paris." On this interpretation on the Ramsey test we would have to be able to literally form the belief that we can fly at will in order to see if the conditional is acceptable and this would be to directly form and adopt a contra-evidential belief. It is *manifestly* clear that we cannot adopt just any old belief like this at will. One might of course *claim* to be able to do so, but this illusion can be easily be dispelled by examining behaviors—the real indicators of true belief. Given unrestricted doxastic voluntarism and interpretation (1) it would have to be the case that in applying the Ramsey test to our example I would

[11] See Hansson, "Formalization in Philosophy."

[12] See Matthias Steup, "Doxastic Voluntarism and Epistemic Deontology," *Acta Analytica* 15 (2000): 25-56, and Matthias Steup (ed.), *Knowledge, Truth and Duty* (Oxford: Oxford University Press, 2001), for discussion of doxastic voluntarism view.

[13] See Earl Brink Conee and Richard Feldman, *Evidentialism* (Oxford: Oxford University Press, 2004), for a thorough discussion of evidentialism.

have to willfully formulate a belief that would entail my not being bothered by leaping off sky-scrapers and so on. But this is not really the case for obvious reasons.

Second, given interpretation (2) of the Ramsey test suffers from a problem much like that which we saw arising with respect to imaging. If we take interpretation (2) of the Ramsey test to mean that in considering whether to accept A > B we should *hypothetically add* A to our standing system of beliefs K, make the appropriate revisions in terms of the AGM postulates (or other similar postulates) and then see if B is in the resulting system of beliefs, then in order to accept A > B we must accept the following additional conditional: "if I were to add A to my standing belief system K, then I would believe K′." However, in order to accept this conditional we must apply the Ramsey test again, and thus to avoid viscous circularity we are faced with another viscous infinite regress like that that arises in the case of imaging. If we take interpretation (2) of the Ramsey test to instead mean that in order to see if we should accept A > B we must add the *hypothetical belief* A, then we are owed an account of what hypothetical beliefs are, how they interact with ordinary beliefs and how we can assess conditionals using them without introducing the sort of viscous infinite regress noted here. But, no such account has been offered. There is however one other important version of the Ramsey test worth examining.

Jonathan Bennett's particular interpretation of the Ramsey test is a version of interpretation (1) and it also shares more in common with imaging than typical versions of the Ramsey test. Bennett is careful to take the term 'test' in Ramsey test quite literally and so favors (1) because he alludes to some of the sorts of problems that have been raised here with respect to the hypothetical nature of the revisions involved in imaging and the Ramsey test given interpretation (2).[14] His formulation of the Ramsey test is basically as follows:

> (RT′) To evaluate A > C, (a) take the set of probabilities that constitutes my present belief system K, and add to it P(A) = 1; (b) revise the standing system of beliefs K to accommodate P(A) = 1 in the most natural and conservative way; and (c) see whether K includes a high probability for C.

So, (a) is a step in the direction of imaging, but the essence of RT′ is still the Ramsey test as described by Ramsey given interpretation (1) because of (b) and (c). Of course, Bennett's view depends on being able to articulate an adequate notion of a minimal revision, but there are other serious problems that afflict his view

[14] See Bennett, *A Philosophical Guide to Conditionals*, 29.

that are shared with imaging and RT. First and foremost, because his view involves the literal revision of one's standing system of belief in the sense of interpretation (1), Bennett's view also illicitly assumes the truth of unrestricted doxastic voluntarism. As we have seen, this assumption is both epistemically and psychologically problematic and it is not any less problematic when it comes to changing partial beliefs than when it comes to the cases of changes of full belief discussed above. It is one thing to say that we can change our probability assignments at will, but it is quite another to actually do so. This is the sort of thing that would require our seeing substantive behavioral changes (e.g. in terms of betting behaviors), but this dose not happen at will and it does not actually happen in cases of applying RT or RT′. In trying to see, for example, whether I should accept "If I were the President of the United States, then I would withdraw all troops from Iraq" I do not seem to actually assign a probability of 1 to the proposition that "I am the President of the United States," at least not if I am of sound mind. Finally, if such probability revisions are not *hypothetical revisions*, but revisions that involve adding hypothetical probabilities or partial beliefs to our initial doxastic states, then we are owed an account of hypothetical probabilities or partial beliefs. But, we have been provided with no such thing. As a result, as with imaging there are serious problems with the Ramsey test—interpreted either as a hypothetical or literal test—and so neither account is an adequate account of the acceptance of conditionals.