Found Sci
DOI 10.1007/s10699-017-9544-9

Found Sci
DOI 10.1007/s10699-017-9544-9

Why Zeno's Paradoxes of Motion are Actually About Immobility.

Maël Bathfield

Abstract Zeno's paradoxes of motion, allegedly denying motion, have been conceived to reinforce the Parmenidean vision of an immutable world. The aim of this article is to demonstrate that these famous logical paradoxes should be seen instead as paradoxes of immobility. From this new point of view, motion is therefore no longer logically problematic, while immobility is. This is convenient since it is easy to conceive that immobility can actually conceal motion, and thus the proposition "immobility is mere illusion of the senses" is much more credible than the reverse thesis supported by Parmenides. Moreover, this proposition is also supported by modern depiction of material bodies: the existence of a ceaseless random motion of atoms – the 'thermal agitation' – in the scope of contemporary atomic theory, can offer a rational explanation of this 'illusion of immobility'. Our new approach to Zeno's paradoxes therefore leads to presenting the novel concept of 'impermobility', which we think is a more adequate description of physical reality.

Keywords Zeno's paradoxes · illusion · motion · immobility · thermal agitation

Acknowledgements The author would like to thank the reviewers of this manuscript for their judicious remarks and constructive comments. Dr. Celine Scornavacca is also greatly thanked for her careful reading and helpful comments.

✉ mbathfield.science@gmail.com

Published online: 8 November 2017

"*Zeno's arguments about motion, which cause so much disquietude to those who try to solve the problems that they present, are four in number*" wrote Aristotle approximately twenty-five centuries ago (*Physics* VI:9, 239b10)[1].

Nowadays, Zeno of Elea is best known for expressing four arguments against motion – usually titled 'The Dichotomy', 'Achilles', 'The Arrow' and 'The Stadium' – and classically mentioned as Zeno's paradoxes of motion. Zeno was a Greek philosopher and member of the Eleatic School founded by Parmenides (5th century BC). According to Parmenides' philosophy, reality is one, immutable, and unchanging, and all plurality, change, and motion are mere illusions of the senses. As a zealous disciple, Zeno is believed to have written a book of paradoxes to defend Parmenides' philosophy. This book, however, has not survived, and what we know of his arguments against motion comes from Book VI of *Physics* by Aristotle (4th century BC)[2]. In *Physics*, Aristotle gave his own arguments with the aim of explaining why "*Zeno's reasoning is fallacious*" (*Physics* VI:8, 239b5) and why "*Zeno's argument makes a false assumption*" (*Physics* VI:2, 233a22); therefore, he did not present Zeno's arguments against motion as paradoxes. Aristotle's opinion was widely accepted until the end of the 19th century (Dowden 2017), when it started being questioned[3]. This led to Zeno's arguments finally being introduced as paradoxes. Since then, Zeno's paradoxes have been the topic of several monographs, e.g., W.C. Salmon (1970), F.A. Shamsi (1973) and J.A. Faris (1996)[4] in which extensive bibliographic references can be found. Over the last decades, they have been a regular subject of discussion in journals or books[5] (see for example Harrison 1996; Papa-Grimaldi 1996; Alper and Bridger 1997; Lynds 2003; Antonopoulos 2004; Hasper 2006; Peijnenburg and Atkinson 2008; Łukowski 2011; Laraudogoitia 2013; Reeder 2015; Ardourel 2015).

Since Zeno's paradoxes were first presented, many solutions for them have been proposed, but none have succeeded in resolving all paradoxical aspects of Zeno's arguments against motion: either the proposed solution solved one (or several) arguments however at least one was always left unsolved (the most recent example: Ardourel 2015), or the proposed solution led to other unsolved problems (for example, the "standard mathematical solution" leading to the supertask's problem, see section IV.1). Another possibility is that the proposed solution simply did not lead to a consensus (Aristotle's solution for example).

The aim of this article is to demonstrate that Zeno's reasoning relies on false assumptions. The demonstration is based on a new approach to Zeno's arguments, which consists in not seeing them as paradoxes of motion, but instead as *paradoxes of immobility*. From this new

[1] In this article, all translations of Aristotle's quotations come from R. P. Hardie and R. K. Gaye (1930).

[2] As an additional statement for 'The Arrow' argument (see section II), the sentence "But it moves neither in the place in which it is, nor in the place in which it is not" (by Diogenes Laertius, *Vitae Philos.* 9, 72) can also be attributed to Zeno with the help of other primary sources (see Vlastos 1966).

[3] For example, Aristotle's answer to 'The Dichotomy' argument (i.e. distinction between '*infinite in respect of divisibility*' and '*infinite in respect of their extremities*'; *Physics* VI:2, 233a25) is considered as "philosophically unsatisfactory" by G.S. Kirk and J.E. Raven (1957).

[4] These three monographs also treat the paradoxes of plurality (attributed to Zeno as well). The relation between the paradoxes of motion and those of plurality is discussed in detail in a recent article by P.S. Hasper (2006). The paradoxes of plurality are also described by N. Huggett (2010) and by B. Dowden (2017). The present article focuses only on the paradoxes of motion.

[5] This is true even without taking into account the intense literature about the problems derived from Zeno's paradoxes (as, for example, the problem of the supertasks see section IV.1).

point of view, the concept of motion is no longer problematic, and the paradoxical aspect of Zeno's arguments only relies on the concept of immobility. This permits us to disprove Zeno's philosophy using his own arguments, by proving that immobility is a mere illusion of the senses.

In the first four sections of this paper, the four arguments reported by Aristotle are presented and discussed in order to show that the logical structure of the paradoxes depends only on the concept of immobility. The last section is dedicated to showing that the proposition "immobility is mere illusion of the senses" is not only more easily acceptable than the Parmenidean reverse thesis, but it is also supported by modern depiction of material bodies: by the existence of thermal agitation of atoms in the scope of contemporary atomic theory, no physical object should be declared totally motionless[6].

It has been suggested that 'Achilles' and 'The Dichotomy' are designed to refute the doctrine that space and time are continuous ('infinitely divisible system'), while the two other arguments are intended to refute the idea that space and time have an atomic/discrete structure ('atomistic system') (Kirk and Raven 1957; Owen 1957; Grünbaum 1967). These two different hypotheses about the *physical nature* of space and time will be mostly referred to as *continuous framework* or *discrete*[7] *framework* in the rest of the paper. Since modern physics describes matter in terms of discrete entities (molecules, atoms and all its components) that are able to move in a *continuous framework*[8], discussing the *discrete framework* may appear unnecessary. But we are convinced that the arguments of 'The Arrow' and 'The Stadium' are instructive in terms of the logical premises on which they are based and that they may yield to a better understanding of the two other arguments. This is why, in this article, the order of introduction of the four arguments against motion does not follow the one by Aristotle.

The aim of the next four sections is systematic rather than historical: the original citations from Aristotle are examined in terms of logical issues in order to abstract the dialectical structure of Zeno's arguments *(English translation of the original texts is in italics in the following sections)*.

---

[6] This conclusion is independent of the existence of any inertial frame of reference or of Galileo's equivalence.

[7] In the entire paper, the word discrete is a synonym of 'atomic' and so, *does not* refer to any mathematical definition related to ℕ, the set of natural numbers.

[8] Even for quantum physics: quantum discontinuity applies only to energy levels (and to other quantum properties as the spin) of matter components (electrons, neutron, proton, quarks…) but not to space or time (see for example Grünbaum 1967; Grünbaum 1970). One of the cornerstones of the quantum theory is the Schrödinger equation, which is a partial differential equation where time and space coordinates are variables, and such an equation needs a *continuous* framework to be well defined.

I The Stadium[9]

The fourth argument is that concerning the two rows of bodies, each row being composed of an equal number of bodies of equal size, passing each other on a race-course as they proceed with equal velocity in opposite directions, the one row originally occupying the space between the goal and the middle point of the course and the other that between the middle and the starting-post. This, he [Zeno] thinks, involves the conclusion that the half a given time is equal to double that time. ***The fallacy of the reasoning lies in the assumption that a body occupies an equal time in passing with equal velocity a body that is in motion and a body of equal size that is at rest; which is false.***

[Aristotle, *Physics* VI:9 239b33- 240a8; Translated by R. P. Hardie and R. K. Gaye (1930)]

(Aristotle's explanation of the argument in fragments 240a8 to 240a19, see Appendix 1, for a reconstruction of his explanation)

This argument against motion is probably the most difficult to understand directly from the text of Aristotle, and it is the least known and discussed among Zeno's arguments. In his attempt to solve the paradox, Aristotle argued (cf. sentence in bold) that the velocity $V_{B/A}$ of a body B relative to a body A at rest, cannot be the same as the velocity $V_{B/C}$ of B relative to another body C, which is also in motion at speed $V_{C/A}$ with respect to A. This is trivial in a *continuous framework* since the notion of relative speed exists and we have the relation $V_{B/A}= V_{B/C} + V_{C/A}$, which simply expresses the transitivity of Galilean relativity ; so, if $V_{C/A}\neq 0$, then $V_{B/A} \neq V_{B/C}$ and Aristotle's reasoning holds. But the notion of relative speed can be proved to be incompatible with the *discrete framework*. Thus, in this framework, the argument leads to the conclusion that, if A is at rest, then $V_{B/C}=0$ and $V_{B/A}= V_{C/A}=$ one 'unit of speed' (a thorough explanation is provided in Appendix 1). In others words, in a *discrete framework*, an absolute frame of reference must exist (the one of A), and only one velocity is possible for any one body in motion with respect to this absolute frame of reference.[10] This is in contrast with our everyday experience of motion. So, in the *discrete framework*, 'The Stadium' argument actually leads to a paradoxical situation, even if it does not deny the existence of motion.

It must be noted that, because the only basic assumption of 'The Stadium' argument is that space and time have an atomic/discrete structure,[11] the only way to bypass the paradox is Aristotle's solution (*Physics* VI:9 240a18): denying such an assumption and considering the reverse hypothesis, which is that space and time are infinitely divisible (the *continuous framework*).

---

[9] Also sometimes called 'The moving rows' argument (see, for instance, Kirk and Raven 1957 or Dowden 2017).

[10] According to E.W. Beth (1946), such a conclusion, not mentioned by Aristotle, is also included in a work of Diodorus Cronus (4th–3rd century BC), part of which was divulgated by Sextus Empiricus (160 – 210 AD). The same conclusion is also directly attributed to Sextus by M.J. White (1982).

[11] It is possible to reach the same conclusion directly from the hypothesis of discretization of space and time (see, for example, chapter 5 of the monograph by J.A. Faris (1996)).

II The Arrow

(Part I) *Zeno's reasoning, however, is fallacious, when he says that (a)* ***if everything when it occupies an equal space is at rest, and*** *(b)* ***if that which is in locomotion is always occupying such a space at any moment,*** *(c)* ***the flying arrow is therefore motionless.*** *(d) This is false, for time is not composed of indivisible moments any more than any other magnitude is composed of indivisibles.*
(Part II) *The third is that already given above, to the effect that* ***the flying arrow is at rest,*** *which result follows from the assumption that time is composed of moments: if this assumption is not granted, the conclusion will not follow.*
[Aristotle, *Physics* VI:9, 239b5 (for part I), 239b30 (for part II); Translated by R. P. Hardie and R. K. Gaye (1930)]. (Labels (a), (b), (c) and (d) are added to help further comments)

Philologists are not all in agreement concerning the interpretation of Aristotle's original text: does the Greek term 'toi nun' (translated above by 'moments') mean 'atomic duration' or 'temporal point' in the context of this argument?[12] (Vlastos 1966; Faris 1996). In consequence, some authors explain 'The Arrow' argument in the *discrete framework* (see for instance, Kirk and Raven 1957; Owen 1957; Salmon 1970; Russell 1970; Faris 1996; Huggett 2010), while many others comment on it in a *continuous framework* (see for instance, Bergson 1907;[13] James 1911; Vlastos 1966; Lear 1981; White 1982; Faris 1996; Harrison 1996; Smith 2003; Reeder 2015). Even if the interpretation of 'The Arrow' argument relies strongly on the choice of the spatial and temporal framework, we will now show below that both versions rely on an identical fallacious hypothesis.

II.1 'The Arrow' argument in the discrete framework:[14]

In such a framework, the main argument (the sentences in bold) can be easily interpreted as:
(a) → **(a_d)** When a body occupies a defined and invariable number of indivisible elements of space, it is at rest.
(b) → **(b_d)** During any indivisible moment of time, a moving body occupies a defined and invariable number of indivisible elements of space.
(c) → **(c_{d1})** Thus, during any indivisible moment of time, a moving body is at rest.
(c_{d2}) Because the whole duration of motion is composed of indivisible moments of time,
(c_{d3}) the moving body is at rest during the whole duration of its motion.

[12] If the latter is chosen, it could seem redundant to refer to 'indivisible temporal point' in proposition (d). But according to G. Vlastos (1966), such odd terms (as indivisible point) are quite usual in Aristotle's texts.

[13] But curiously, the 'cinematographic vision of motion' depicted by Bergson (in a *continuous framework*) is frequently used by other authors to describe the motion of the flying arrow in the *discrete framework* (see also footnote 17).

[14] According to Kirk and Raven (1957), the choice of the discrete framework permits us to get a consistent overall picture of the four arguments: for each space-time framework, one of the arguments leads to a paradoxical situation for the motion of a single body ('The Dichotomy' in the *continuous framework* and 'The Arrow' in the *discrete framework*), while another one leads to a paradox when considering the relative motion of two bodies (the 'Achilles' in the *continuous framework* and 'The Stadium' in the *discrete framework*). So, if in each space-time framework the concept of motion is inconsistent, motion should be impossible.

Sentence (d) and Part (II) can be interpreted as: (a_d) and (b_d) are false because space and time are continuous – rejection of the *discrete framework* – thus (c_{d3}) cannot be a valid conclusion. But, according to Aristotle (see Part II of the argument), (c_{d3}) may hold in the *discrete framework*. Is this truly the case? Proposition (c_{d1}), which concerns what happens during an indivisible moment of time, is deducted from premise (b_d), which also concerns what happens during an indivisible moment of time, and from premise (a_d), in which no temporal reference is given. But, actually, two other premises (a_{d+}) and (b_{d+}) need to be added to logically lead to proposition (c_{d1}):

(a_{d+}) Premise (a_d) is also true during an indivisible moment of time.

(b_{d+}) The number of elements of space occupied by a body is the same, whether being in motion or being at rest.[15]

However, from (a_d) plus (a_{d+}) and from (b_d) plus (b_{d+}), we logically obtain a 'proposition of indistinguishability': during any indivisible moment of time, both a body in motion and a body at rest occupy a defined and invariable number of indivisible elements of space. So, why should we conclude that 'the flying arrow is motionless' (and therefore motion is impossible) rather than that 'an arrow being at rest is therefore still in motion' (and therefore immobility is illusory)? The answer is simply that, because there is no change in the spatial position during any indivisible moment of time, one tends to accept more easily the implicit premise (^{i}p), rather than its opposite:

(^{i}p) The primordial category of being is 'being in a state of rest' and not 'being in a state of motion'[16].

But, such a reasoning involving (^{i}p) actually relies on the definitions of motion and immobility of the *continuous framework* and not on those of the *discrete framework*. Indeed, while defining immobility during any period of time as the 'absence of change in spatial position during this period of time' is sufficient in the *continuous framework*, an adequate definition for the *discrete framework* needs an additional specification about duration: the length of this period of time has to be strictly larger than the length of an indivisible moment of time. The reason for this specification is that: (1) as reminded by Aristotle, "*our use of the phrase 'being at rest' also implies that the previous state of a thing is still unaltered, not one point only but two at least being thus needed to determine its presence*" (*Physics* VI:8, 239a15), and (2) the counterparts of 'temporal points' from the *continuous framework* are 'indivisible moments of time' in the *discrete framework*. Therefore, 'being at rest' cannot be defined during an indivisible moment of time[17], so consequently, the hidden premise (a_{d+}) is false. Premise (a_d) could be corrected as follows:

---

[15] Premise (b_{d+}) may seem obvious but, historically, the hypothesis of a real length contraction of moving body was proposed by George Francis FitzGerald in 1889, and theorized by Hendrik Antoon Lorentz in 1892, for accounting for the null-results of the famous Michelson–Morley experiment (1887). This hypothesis did not survive long; it had been forgotten by the time of Albert Einstein's view on relative motion (see the history and context of birth of the special theory of relativity for more details).

[16] These two expressions come from F. A. Shamsi (1973).

[17] Actually, the term 'during an indivisible moment of time' is not adequate as it assumes that some change could be performed during this period of time. But, by definition of the *discrete framework*, no change can occur 'during an indivisible moment of time'. A more adequate term should be simply '**at** an indivisible moment of time', but such a term was not used previously in order to retain the reader in the 'classical' fallacious reasoning. Note that describing motion as 'a succession of rest' or quoting the famous Bergson's sentence "movement is made of immobilities" (the 'cinematographic vision of motion', see also footnote 13)

(a'_d) When a body occupies a defined and invariable number of indivisible elements of space *during at least two consecutive indivisible moments of time*, it is at rest (during the corresponding number of indivisible moments of time).
But (a'_d) cannot be jointed with premises (b_d) and (b_{d+}) to lead to (c_{d1}) anymore. Eventually, the illusory consistency of 'The Arrow' argument in the *discrete framework* relies on a definition of immobility illegally imported from the *continuous framework*.

However, even if 'The Arrow' argument is invalided, the description of motion in the *discrete framework* is still troublesome. Let us consider two consecutive indivisible moments of time. For a body being at rest, the occupied indivisible elements of space are the same at two successive indivisible moments of time, while the elements of space occupied by a body in motion change. But, when should this change be performed since there is no time left between two consecutive indivisible moments of time? Changes cannot occur during any indivisible moment of time, and thus, they seem to occur out of time![18] This fact, combined with the true paradox of 'The Stadium' (see previous section I), seems suspicious enough to finally reject the hypothesis of an atomic structure for time and space (rejection of the *discrete framework*),[19] as Aristotle did.

II.2 'The Arrow' argument in the *continuous framework*:

In such a framework, the main argument (the sentences in bold) is actually more difficult to interpret in a consistent way than in the *discrete framework*, leading to various reconstructions of the argument in the literature.[20] Here we present a new reconstruction that stays close to Aristotle's text and is thus similar to the one proposed for the *discrete framework*. Our reconstruction falls within the framework of classical mechanics, and more specifically within the framework of the 'mass point particle mechanics', in which real-world bodies are modelled as point particles (with the help of the concept of 'center of mass')[21]:
(a) → **(a_c)** When a body is at a determinate point in space, it is at rest.
(b) → **(b_c)** At any temporal point during its motion, a moving body is also located at a determinate point in space.
(c) → **(c_{c1})** So, at any temporal point during its motion, a moving body is at rest.
(c_{c2}) Because the whole duration of the motion is only composed of (an infinite number of) temporal points,
(c_{c3}) the moving body is at rest during the whole duration of its motion.
Sentence (d) and Part (II) can be interpreted as: Aristotle argues that (c_{c3}) is not an admissible conclusion because premise (c_{c2}) is supposed to be false.

in the *discrete framework* is surely, but erroneously, assuming that the term 'being at rest' is defined at an indivisible moment of time.

[18] In such a context, the Diogenes Laertius's sentence attributed to Zeno "But it moves neither in the place in which it is, nor in the place in which it is not" (see also footnote 2) appears much more understandable.

[19] This also rejects potential future forms of the standard quantum theory incorporating minimal distances ('hodons') and times ('chronons') (Grünbaum 1967).

[20] See, for instance, the very different reconstructions from James (1911), Lear (1981), Faris (1996), and Harrison (1996).

[21] See section V.4 for more details about some principles of classical mechanics.

Actually, whatever the soundness of premise (c_{c2}),[22] the intermediate conclusion (c_{c1}) can already sound odd since "its motion becomes nothing but a sum of rests, for it exists not out of any point; and *in* the point, it doesn't move" (James 1911). This is typically the 'cinematographic vision of motion' described by Henri Bergson, for whom it is an "absurd proposition, that movement is made of immobilities" (Bergson 1907).[23] Actually, such an absurd proposition logically arises from the implicit premise already mentioned in the previous section:

(^{i}p) The primordial state of being should be 'being in a state of rest' and not 'being in a state of motion'.

Once one accepts the fact that 'to be at a point' should primarily be a synonym of 'to be at rest at this point',[24] one must accept proposition (a_c). But because 'being in motion' is still a manner of 'being', which also requires a specific location in space, one must accept proposition (b_c). In the frame of (^{i}p), (a_c) combined with (b_c) leads logically to (c_{c1}). But *without* (^{i}p), (a_c) combined with (b_c) leads to another (obvious) conclusion: a body is at a determinate point in space whatever its state of motion. So, without the supposed ontological primacy of 'being at rest' upon 'being in motion' (i.e. rejection of premise (^{i}p)), proposition (c_{c1}) is false, and thus the paradoxical situation vanishes.

In other words, the fact that a body can be at a determinate point in space at a 'temporal point' does not inform us about its state of motion, simply because "*rest and motion are always in a period of time*" (*Physics* VI:8, 239a21). However, modern kinematics (as a branch of classical mechanics) permits to describe motion of body not only during periods of time but also at specific durationless instants. To be able to do that, physicists use an additional parameter, called the 'instantaneous velocity' (speed at a 'temporal point'), to fully cinematically describe the motion of a body. In articles and books that comment on 'The Arrow' argument (in the *continuous framework*), the notion of 'instantaneous velocity' is, almost systematically, introduced as the standard solution to the paradox. Indeed, it is an adequate answer because this concept permits to conceptualize motion at a 'temporal point' (i.e. it is possible for a body to be both at a point and be moving), but it does not consist in the

---

22 In the history of mathematics, questions about validity of premise (c_{c2}) led to the 'Metrical paradox of extension', which can be briefly summarized by the following question: Does an extended line consist of unextended points? Actually, this questioning is much more related to the 'paradox of infinite divisibility' (one of the paradoxes of plurality) than to the paradoxes of motion; the metrical paradox of extension questions only the consistency of Cantor's continuum in a mathematical framework: there is no notion of motion in such a framework. According A. Grünbaum, the problem is solved: "The set-theoretical analysis of the various issues raised or suggested by Zeno's paradoxes of plurality has enabled me to give a consistent metrical account of an extended line segment as an aggregate of unextended points" (Grünbaum 1967). Unfortunately, time *is not* a mathematical concept. So, the solution of this paradox does not really help to define an ontological nature for the concept of time. However, the solution permits, for practical purposes (e.g. in classical kinematics), that periods of time can be described in a consistent way as composed of temporal points. In a similar way, by shooting a picture with a digital camera, reality can be described with the help of colored pixels, but reality is not actually made of pixels.

23 Bergson was not really interested in explaining why it is absurd. He simply used the paradox to support its pre-established philosophical positions (i.e. claiming that immediate experience and intuition are more significant than rationalism and science for understanding reality) (Tooley 1988).

24 Such a synonymy is striking in Bergson's comments: "Yes again, if the arrow, which is moving, ever *coincides with a position, which is motionless*" or "If it had *been there*, it would have *been stopped there*" (italics are added) (Bergson 1907). It is also found in Harrison's interpretation of 'The Arrow' argument: "How can a particle be at a point and also be moving at that point?" (Harrison 1996).

real logical resolution of the paradox. Actually, the logical resolution is that using the notion of ‘instantaneous velocity’ (whatever its strict mathematical or physical definition)[25] implicitly rejects premise (^{i}p). In other words, the philosophical statement underlying the notion of ‘instantaneous velocity’ is the following: the state of motion *by default* of a body is ‘being in motion *or* being at rest’ (and not simply ‘being at rest’ anymore).

In conclusion, ‘The Arrow’ argument (interpreted in the *continuous* or in the *discrete framework*) contains a circular reasoning: the paradoxical facet of the argument relies on the fact that the primordial state of being is considered by Zeno to be ‘being at rest’ (= premise (^{i}p)). But such a premise is a logical consequence of a stronger proposition stating that ‘being’ can be only ‘being at rest’. This proposition is logically equivalent to Parmenides’ position rejecting motion as real (see **Fig. 1**).

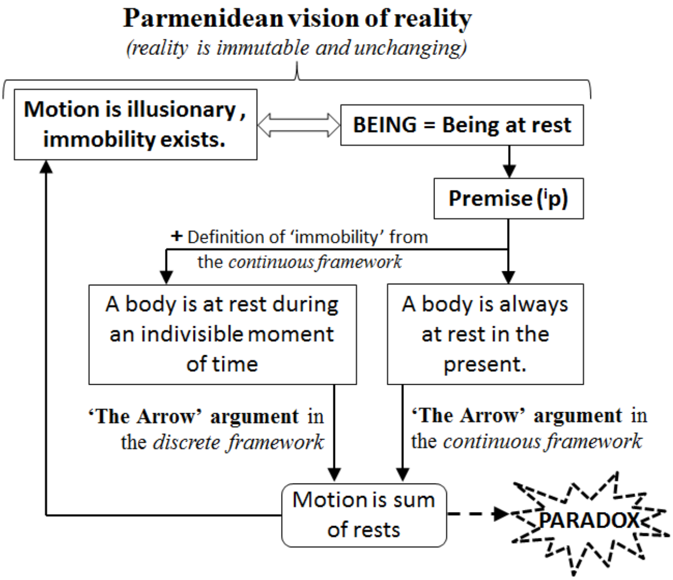

Fig. 1 ‘The Arrow’ argument and Zeno’s circular reasoning.

[25] Current apparent philosophical problems raised by a specific definition of the ‘instantaneous velocity’ (see Arntzenius 2000, Carroll 2002, Smith 2003, Meyer 2003, Lange 2005) will be discussed in a forthcoming short article.

III The runners/ Achilles

The second is the so-called 'Achilles', and it amounts to this, that (a) ***in a race the quickest runner can never overtake the slowest, (b) since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.*** *(c) This argument is the same in principle as that depends on bisection ['The Dichotomy' argument, see next section], though it differs from it in that the spaces with which we successively have to deal are not divided into halves. (d) The result of the argument is that the slower is not overtaken: but it proceeds along the same lines as the bisection-argument [...], so that the solution must be the same.*

[Aristotle, *Physics* VI:9, 239b15-239b25; Translated by R. P. Hardie and R. K. Gaye (1930)]

(Labels (a), (b), (c) and (d) are added to help further comments)

This argument (sentences (a) and (b) in bold) is called 'Achilles' according to the fact that Achilles, the swift warrior from Greek mythology, was chosen as a character in it. This argument requires a slower runner beginning to move some distance ahead of a faster runner; it is also frequently named 'Achilles and the Tortoise', even if the 'tortoise' is a later commentator's addition. The argument suggests that the race will never be finished since, even if the distance separating the two competitors becomes increasingly small, it will never be equal to zero (see **Fig. 2**).

Indeed, if r is the ratio of the velocity of Achilles (V) to the one of the 'tortoise' (v) (thus r = V/v > 1), and d is the initial distance between the two competitors, it can be easily shown that the distance still separating the two characters at each 'step' (l_n) is expressed by: $l_n = d/r^n$ (where l_n *never* equals 0). From the point of view of the 'tortoise', it is at rest and Achilles is coming closer and closer to the (supposedly unattainable)[26] goal, which is the moment/distance when/where he catches up with it.

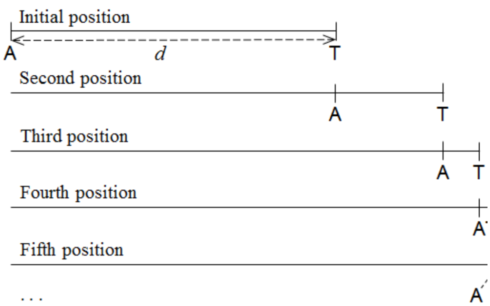

Fig. 2 The 'Achilles and the Tortoise' argument. d = initial distance between the two competitors; Example in which Achilles **A** is three times faster than the 'tortoise' **T** (r=3).

As remarked by Aristotle (in sentences (c) and (d)), this situation is identical to the one depicted in the *progressive version* of 'The Dichotomy' argument presented in the next section, and so the same (mathematical) 'classical solution' can be used here.[27]

[26] We know from experimental evidence that Achilles will eventually overtake the 'tortoise'.

[27] However, there also exists a specific physical solution for the 'Achilles' argument: a runner will cover a finite distance in a finite number of steps, as noticed already by H. Bergson (1969), J.O. Wisdom (1970) and M. Kline (1980). By considering the itinerary of the runners in terms of steps (or more precisely in terms of jumps for Achilles and in terms of tiny steps for 'the tortoise'), it is easy to identify the last step before Achilles overtakes: when the distance separating the two competitors becomes less than the length of Achilles' jump + length of the tortoise's step. Thus, the race will finish at Achilles' next jump. To avoid such an obvious solution of the 'runners-paradox', it would be more judicious to exemplify the argument

But, before trying to solve the paradox, we had better check the internal consistency of the argument: does the argument really deny motion? From sentence (b), we can conclude that Achilles can reach the initial starting point of the ‘tortoise’. But the initial distance *d* is not specified (e.g., it is not restricted to any values) and therefore Achilles is virtually able to reach any point of space. So, how can Zeno conclude from this premise that Achilles will not be able to catch the ‘tortoise’? Simply *via* a dialectic that would not put an illusionist to shame: his argumentation forces our attention to focus on a restricted region of space, in which the overtaking will never occur.[28] Indeed, with sentence (b), Zeno strongly urges us to think repeatedly about the point where the ‘tortoise’ is located when Achilles performs his next step (see **Fig. 2**); and this point is always some (tiny) distance ahead from Achilles’ current position. But the overtaking of the ‘tortoise’ by Achilles will not occur at such a point: it will obviously occur at a point where, at the same time, both Achilles and the ‘tortoise’ are located. So, Zeno’s argumentation diverts our attention from the existence of such a meeting point… in order to conclude the non-existence of this point (in sentence (a))! So, the important hidden point that creates the illusion of a paradox is that *the conclusion is actually not logically connected to the propositions of the argumentation (since sentence (b) tells us nothing about the meeting point)*.

Also, there is again a kind of circular reasoning here: Zeno first assumes that the overtaking will not occur, confuses us with a pseudo-argument, and finally concludes that the overtaking will not occur. As Aristotle already noticed,[29] it is easy to break the circle by asking the following question: ‘what distance does the tortoise travel when Achilles catches up to it?’ Resolving the equation d+v.t = V.t, we found that this distance is v.t= $d/(r-1)$. For example, if Achilles is three times faster than the ‘tortoise’ (r=3), the ‘tortoise’ travels the distance $d/2$ at the end of the race (and so Achilles travels the distance $3d/2$). The fact that Achilles needs to travel through an infinite number of points before reaching the ‘tortoise’ (that is simply a logical consequence of any motion considered in the *continuous framework*) does not impede the existence of the meeting point.[30] Apparently, Zeno was a very good illusionist for making it disappear.

---

with a sailing race: imagine two sailboats driven by a stable wind on a quiet sea. One of them has a sail five times larger than the other one, which is one mile ahead. The ‘sailboats-paradox’ is then expressed by replacing the word ‘runner’ by the word ‘sailboat’ in Aristotle’s original sentence. With this new illustration of the argument, we are faced with a more continuous motion, which is more in conformity with the continuous framework of space and time chosen to create the paradox.

[28] Obviously, the tricky point is that this region of space is nevertheless defined and limited by the point where the overtaking occurs (in the same manner that the open set [0,1) is defined by the number 1 but this number 1 is not part of the set). This is why Peijnenburg and Atkinson (2008) wrote that “Any difficulty we have with the Achilles lies in the concept of an open set [a,b)”.

[29] He wrote “*but it [the slowest runner] is overtaken nevertheless if it is granted that it traverses the finite distance prescribed*” (*Physics* VI:9, 239b28).

[30] Since, we repeated it, there is no logical connection between the proposition ‘space and time are continuous’ and the proposition ‘Achilles can/cannot catch the tortoise’. They are logically independent.

IV The Dichotomy[31]

The first asserts the non-existence of motion on the ground that ***which is in locomotion must arrive at the half-way stage before it arrives at the goal.***
[Aristotle, *Physics* VI:9, 239b11; Translated by R. P. Hardie and R. K. Gaye (1930)]

'The Dichotomy' is called as such because it uses an infinite sequence of halves or midpoints that can be identified in any line or distance using a repeated division by two. This is possible as we naturally place the argument in an "infinitely divisible system" for space (*continuous framework*). Not all commentators are in agreement concerning the interpretation of Aristotle's original text, and it results in two versions of this paradox (see **Fig. 3** below):

- *Progressive version:* to reach the goal located at a specific distance, one must firstly travel half this distance, and then half the remaining distance, and again half of what remains, and so on. Thus, even if one gets closer to the destination, there is always some distance left: this distance is expressed by the formula: $l_n = 1/2^n$(where l_n *never* equals 0).[32] Consequently, motion seems impossible due to the fact that the final point can never be reached.
- *Regressive version:* before the second half of the distance can be covered, one must cover the first half. But before doing so, the first quarter must be completed. However, before this can be done, one must traverse the first eighth, and so on to infinitum. Consequently, motion seems impossible because it is not possible to get started.

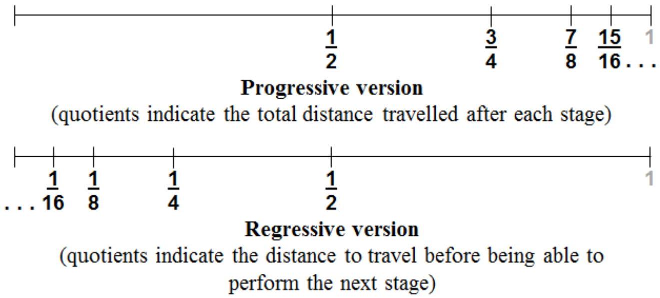

Fig. 3 The two versions of 'The Dichotomy' argument

[31] In G. S. Kirk and J. E. Raven's book (1957) and in M.J. White's article (1982), this argument is called 'The Stadium'!

[32] It is obviously the same formula as the one for the 'Achilles' argument presented in the previous section, with *d*=1 and *r*=2.

IV.1 Limits of the *progressive version* of 'The Dichotomy':

In the past, the *regressive version* of the argument was preferred to the *progressive* one (Hasper 2006; Blay 2010). But since the 19th century, when the 'standard mathematical resolution' based on the convergence of power series was proposed (see below), the *progressive version* became more and more popular. Consequently, most of the contemporary essays by mathematicians evoking Zeno's dichotomy argument claim that the problem is solved since the construction of the theory of infinite series in the 19th century.[33] Indeed, the *progressive version* of 'The Dichotomy' argument seems easily solvable since the overall distance travelled after n steps can be expressed as the partial sum of power series: $\boldsymbol{D_n} = \sum_{x=1}^{n} \mathbf{1/2^x}$. According to the theory of infinite series, all power series $\sum_{x=1}^{n} 1/q^x$ with $q > 1$ are convergent and thus admit a sum. The formula of the sum for this geometric series is $\sum_{x=1}^{\infty} 1/q^x = 1/(q-1)$. In the case of 'The Dichotomy', we have that $\sum_{x=1}^{\infty} \mathbf{1/2^x} = 1$. This mathematical result is generally considered as a solution to the paradox.

However, such a formula does not consist in a logical resolution of the paradox since it consists only in a *mathematical reformulation* of the initial problem: *by definition*, the sum of an infinite series is the limit of its partial sums: $\sum_{x=1}^{\infty} \mathbf{1/2^x} = \lim_{n\to\infty} \sum_{x=1}^{n} (\mathbf{1/2})^x$. It is useful here to recall the definition of a limit: we call $\boldsymbol{L}$ the limit of the sequence $\boldsymbol{D}_n$ if the following condition holds: for each real number $\boldsymbol{\varepsilon} > 0$, there exists a natural number N such that, for every n>N, we have $|\boldsymbol{D}_n - \boldsymbol{L}| < \boldsymbol{\varepsilon}$. In other words, the limit of a sequence is the value that the terms of a sequence 'tend to': the difference between the terms of the series and its limit can be as small as we want (we just have to use an appropriate large value of n) … but it is never null ($\boldsymbol{\varepsilon} \neq 0$)! This is exactly the problem depicted by Zeno in 'The Dichotomy' argument in its *progressive version* (and in the 'Achilles' argument). Thus, this mathematical reformulation does not solve the paradox.[34]

Actually, the *progressive version* of 'The Dichotomy' argument does not suggest that any motion needs an infinite time to be performed, but rather that an infinite number of acts have to be carried out in order to complete the motion. Thus, the following philosophical problem arises: how is it possible that an infinite number of acts can be performed within a finite interval of time? This problem is known as the problem of the 'supertask', a term coined by James Thomson (1954). A 'supertask' is defined as a quantifiably infinite number of operations that occur sequentially within a finite interval of time. The time needed to perform one operation is constantly decreasing (usually, the duration of the $(n+1)^{th}$ operation is half of the duration of the previous one n). Thus, the total length of the 'supertask' can be expressed as an infinite sum. Obviously, the mathematical solution (the one stating that it is possible for infinite series to converge to a finite number) ensures that the entire duration of the 'supertask' has a finite value, but is not sufficient to ensure that one is able to complete it, mainly because there is no operation which can be identified as the last one (i.e. the one terminating the task). The question is now: Is a supertask (logically) possible? Such a

[33] However, as one counter-example given by current mathematicians aware of the limit of calculus to solve the paradox, we can quote Joseph Mazur (2007): "I'll show that while this may seem to be the case on the surface, the math in question—basic algebra—does nothing to address the underlying phenomeno-logical problem that the paradox drives at."

[34] Note that the only useful information coming from the convergence of the infinite series depicted by Zeno is that the time needed to cross a finite distance is finite, but such information was already available from our usual experience of motion.

question[35] is actually beyond the scope of this paper, mainly because motion, being "just one task performed in the physical world" (Romero 2014), cannot truly be considered as a supertask (contrary to Zeno's suggestions).[36]

Furthermore, because of the logical equivalence with the 'Achilles' argument, the internal consistency of 'The Dichotomy' arguments in its *progressive version* can also be questioned: it accepts the existence of motion (up until some specific points of the journey) to ultimately refute it just because the final point cannot be reached. This is why Maurice Caveing (1982) noted that, if a moving body can travel the first half-distance, what impedes it from running through the second one? So, he simply argued that, in order to have a truly paradoxical situation, "progression involves regression". Indeed, the *regressive version* of 'The Dichotomy' argument seems much more convincing than the *progressive* one, and it leads to a true philosophical problem.

IV.2 Philosophical problem of the *regressive version* of 'The Dichotomy':

Let us consider proposition (q): a body is moving in a straight line from point A to point B separated by a non-null distance. The *regressive version* of the argument states the following series of propositions (p_i): (p_1) before the second half of the distance AB can be covered, one must first cover the first half. (p_2) But before doing so, the first quarter must be completed. (p_3) Before this can be done, one must traverse the first eighth, and then p_4, p_5, p_6… then so on to infinitum…. What can we conclude from this series of propositions (p_i)? Actually nothing, because this infinite regression of the argument tells us nothing about what happens in point A. The classical conclusion (*CC)* is that 'motion is impossible because it is not possible to get started'; such a conclusion is actually not logically connected to neither proposition (q) nor to the series of propositions (p_i).[37] Conclusion *CC* relies only on the primary hypothesis (h) that '*the body is at rest in A'*, which is not stated in proposition (q). If the body is already in motion in A, then 'The Dichotomy' argument is unable to impede its motion. So, it is only if one accepts hypothesis (h) that the argument leads to the incapacity to detect the very beginning of the motion.

Actually, when interpreted in the general context of the 'paradox of plurality' (see footnote 4), A. Papa-Grimaldi (1996) argues that "the problem which was at the heart of Zeno's formulation of his paradoxes is the impossibility to conceptualize the passage from One to Many". Thus, in the present context, 'The Dichotomy' argument against motion

[35] Note that there is no agreement in the philosophical community on the question, and the philosophical problem of the 'supertask' led to intense discussions in the literature. For example, four of the eleven articles compiled by W.C. Salmon (1970) about Zeno's paradox are dedicated to this problem ("Tasks, Super-Tasks, and the Modern Eleatics" by P. Benacerraf; "Tasks and Super-Tasks" and "Comments on Professor Benacerraf's Paper" by J. Thomson, and "Modern Science and Zeno's Paradoxes" by A. Grünbaum). Moreover, new articles or books dedicated to the problem are regularly published: some examples are Laraudogoitia 1996, Atkinson 2007, Peijnenburg and Atkinson 2008, Lee 2011, Romero 2014. A forthcoming short article will be specifically dedicated to this problem.

[36] This proposition (i.e. motion is not a supertask) would be very probably supported by Aristotle and by H. Bergson, since both authors claim (in their own style) that, even if motion can be potentially decomposed in an infinite number of parts, it is not actually composed of parts.

[37] It is the same logical fallacy noticed already for 'The Achilles' argument (see above in section III), and it was also already noticed by P. Benacerraf (1970).

enlightens the following philosophical problem (*PP*): *How to conceptualize the passage from stillness to motion* (*or vice versa)?*[38] So, at the end, the *regressive version* of 'The Dichotomy' does nothing else than point out the insuperable difficulties to conceptualize the passage from immobility to motion, which basically comes from their respective 'exclusive definition'; Each term is simply defined as the negation of the other one, and consequently being 'half in motion' or 'half at rest' does not make sense: "Vague expressions dominate natural language. Almost every expression of natural language, which does not have a mathematical sense, is vague. […] One can show that names such as 'motion', 'rest', 'being' and 'non-being' are precise (non-vague) terms of natural language" (Łukowski 2011).

IV.3 Logical conclusion of 'The Dichotomy' argument: [39]

Being aware of the existence of the primary hypothesis (*h*) and of the philosophical problem (*PP*), it can be claimed that the logical conclusion of the *regressive version* of 'The Dichotomy' should be the following proposition (*P1)*: 'if a body is currently at rest, it will never move in the future'. This can also be expressed by the following contraposition (*C1)*: 'if a body is currently moving, it was never at rest in the past'. Moreover, the 'symmetry' of the philosophical problem (*PP*) strongly suggests that proposition (*P2*) 'if a body is currently moving, it will never stop moving in the future' should also be valid; that leads to contraposition (*C2)*: 'if a body is currently at rest, it was never moving in the past'. Then after combining (*P1*) + (*C2*) and (*P2*) + (*C1*), we can conclude with the following statement (*LC*): *reality should be composed of 'bodies being at rest' with other 'bodies being in motion' where no bodies are able to change state*.

From 'The Dichotomy' argument, Zeno concluded that motion is impossible, i.e. bodies should always be at rest. So, one can easily see that Zeno only reached half of the above logical conclusion (*LC*). This is due to the fact that he again assumed the implicit premise (^{i}p) (already mentioned for 'The Arrow' argument, see section II: *the primordial category of being should be the one of 'being in a state of rest' and not the one of 'being in a state of motion'*), and such a premise impeded him to consider propositions (*P2*) and (*C1*). Finally, Zeno engaged again in a circular reasoning (see **Fig. 4** below).

[38] European thinkers focused on such a philosophical aspect of Zeno's paradoxes during the 18th century, and the *regressive version* of the argument was much more popular than the *progressive version* at that time (Blay 2010). But the problem is still considered intriguing nowadays: see, for instance, Medlin 1963, Hamblin 1969, Priest 1985, Mortensen 1985, Jackson and Pargetter 1988, Smith 1990, or the recent book by Łukowski re-expressing the problem: "In any mental experiment, it is impossible to move imperceptibly from motion to rest or vice versa by some 'small steps'. Moreover, it is difficult to know how those 'small steps' — leading us through intermediary states — should be understood. So, in contrast with various 'hues' of being a heap, bald or red, there are no 'hues' of being [in] motion. Either something is in motion or not" (Łukowski 2011).

[39] For each proposition or conclusion (concerning rest and motion) in this sub-section, the phrase 'In a given frame of reference,…' should be implicitly added.

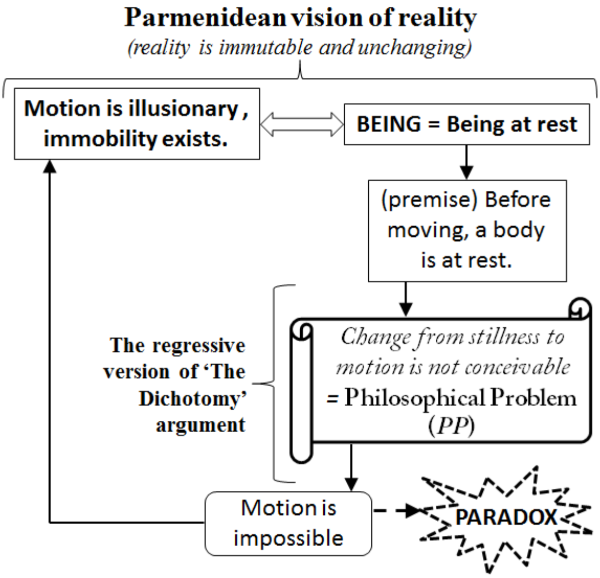

Fig. 4 Zeno's circular reasoning (within 'The Dichotomy' argument) involving paradoxical propositions and an unsolved philosophical problem (see also **Fig. 1**).

An important concluding remark: it is worth noting that the only two arguments that pretend to deny the existence of motion (i.e. 'The Arrow' in both *discrete* and *continuous framework*, and the *regressive version* of 'The Dichotomy'), actually contain hidden and implicit premises about rest/stillness (see summary **Table 1** below).

Table 1 Summary of the analysis of Zeno's arguments considered as *paradoxes of immobility*

Physical framework	*Discrete framework* (Atomic structure for space and time)		*Continuous framework* (Space and time are continuous)			
Argument	**The Stadium**	**The Arrow**		**Achilles**	**The Dichotomy – progressive version**	**The Dichotomy – regressive version**
See section	I	II.1	II.2	III	IV.1	IV.2
Implicit premise / remarks	none	***Rest is defined during one indivisible moment of time***	***Being = being primarily at rest***	none / arguments logically identical		***Before moving, a body is at rest***
Logical conclusion of the argument	Only one speed is possible	A moving body is at rest	Motion is sum of rests	Motion of a body cannot finish		Motion of a body cannot start
Strictly deny existence of motion?	*No*	***Yes***	***Yes***	*No*	*No*	***Yes***
Observed physical situation	Any speed seems possible	Motion seems to exist		*Logical fallacy: no logical connection between initial propositions and final conclusion*	*Same logical fallacy as the 'Achilles' (so 'progression involves regression')*	Motion can start
*Paradoxical?**	*Yes*	*Yes*	*Yes*			*Yes*
Solution (of the paradoxical situation)	Rejection of the discrete framework	Use an adequate definition of rest for the *discrete framework*	Use of the notion of 'instantaneous velocity' = Rejection of the implicit premise (to avoid **circular reasoning**, see **Fig. 1**)			Rejection of the implicit premise (to avoid circular reasoning, See **Fig. 4**)
Other remaining problems	-	Change occurs out of time!	-	Derived question: logical problem of the 'supertask' (see section IV.1)		Philosophical problem *PP* (see section IV.2)

* i.e. is the logical conclusion of the argument in disagreement with the observed physical situation? If yes, it is considered as paradoxical.

V Immobility as an illusion of the senses

Roughly speaking, in the previous sections we saw that:

1. In the *discrete framework*, while ‘The Stadium’ argument is truly paradoxical, the paradox of the ‘The Arrow’ can be overcome; but description of motion in such a framework is still problematic. So, to overcome these difficulties, we need to place ourselves in the *continuous framework*.
2. In the *continuous framework*, two of Zeno’s arguments against motion (‘Achilles’ and the *progressive version* of ‘The Dichotomy’) are logically inconsistent.
3. The reasoning of ‘The Arrow’ argument and the *regressive version* of ‘The Dichotomy’ are valid but contain a circular reasoning (see **Fig. 1** and **Fig. 4** above).

What we missed, as we were most likely distracted by the dialectic ‘tour de force’ of Zeno, is that only one of the four possible situations depicted in **Table 2** (the first one) is considered by Zeno.

Table 2 The four possible combinations of hypothesis about the physical status of motion and immobility.

Case label	**1**	**2**	**3**	**4**
Physical status of immobility	*Real*	*Real*	*Illusory*	*Illusory*
Physical status of motion	*Illusory*	*Real*	*Illusory*	*Real*
Corresponding philosophical position	**Parmenides’ philosophy**	**Intuitive philosophy**	**Solipsism**	**Philosophy of ontological non-immobility**
	Being = being at rest	Being = being at rest OR in motion	The whole of reality is an illusion	Being = being in motion
Logical consequences	Circular reasonings (see **Fig. 1**, and **Fig. 4**)	Conclusion *LC* (see section IV.3) and philosophical problem *PP* (see section IV.2)	Neither science nor philosophy of science possible	Immobility = imperceptible motion (see section V.2)
Compatible with physical experience?	NO (Zeno’s paradoxes)	NO	(Not debated here)	YES (thermal agitation, see section V.3)

The second case could be seen as a ‘natural’ or ‘intuitive’ metaphysical position about immobility and motion. This possibility was already examined when proposing a logical conclusion of the *regressive version* of ‘The Dichotomy’ argument (see section IV.3), and it led to the conclusion: *Reality should be composed of ‘bodies being at rest’ with other ‘bodies being in motion’ (where no bodies are able to change state)*. Like Zeno’s paradoxes, this is manifestly not in agreement with our physical experience of motion and rest. Moreover, it is in this situation that the philosophical problem *PP (How to conceptualize the passage from stillness to motion* (*or vice versa)?)*[40] is more prominent.

[40] See section IV.2.

The third case is quite close to the philosophical position of ‘epistemological solipsism’: because all bodies are either in motion or at rest, if both notions of motion and rest are assumed illusory, the whole of reality is an illusion. This case will not be examined further in this paper since we will see shortly that the fourth case permits us to stay in the frame of a ‘philosophical realism’.

The fourth case corresponds to a philosophical position that is the opposite of the one that Zeno and Parmenides defended, stating that only motion is possible in the real world. The next sub-sections will be devoted to showing that considering immobility as an illusion is much easier and more consistent with reality than supporting the Parmenidean reverse position, and that the philosophical problem *PP* vanishes in such a situation.

V.1 A choice of philosophy

According to Parmenides' philosophy, reality is one, immutable, and unchanging; and all plurality, change, and motion are mere illusions of the senses. A contemporary of Parmenides, Heraclitus of Ephesus, built a totally opposed vision of the world, known as the ‘philosophy of becoming’. One of the features of his philosophy that is particularly relevant for our purposes can be found in a text of Plato quoting Heraclitus: "All entities move and nothing remains still" (Plato, *Cratylus 401 section d line 5*). From a first glance at this sentence, we could believe that Heraclitus considered immobility as illusory. Let’s name this philosophical position the pseudo-Heraclitean philosophy (it corresponds obviously to the fourth case of **Table 2**). One could wonder if both philosophies are epistemologically equivalent: does the term ‘illusory’ refer to the same concept in Parmenides and pseudo-Heraclitean philosophies? The answer is negative.

It is easy to conceive that a 3-dimensional material body, perceived as at rest, can actually be moving: examples of moving objects that appear to be at rest can be found in everyday life and can be caused, for example, by optic-geometrical effects and/or physiological limitations of our perception (see Appendix 2 for more details). In all these ordinary cases, illusion of immobility is easily explainable in a rational manner: there is really some movement, but it is imperceptible. In other words, because different magnitudes for motion exist, the illusion of immobility can be easily conceptually constructed from the concept of motion by invoking a magnitude that cannot be detected.

On the other hand, there are no different magnitudes possible for immobility. So, at a conceptual level, can the illusion of motion be rationally constructed from the concept of immobility in Parmenides' philosophy? We do not think so. Moreover, more practically, it is also very difficult to conceive that motion can conceal immobility: there is no example of an ordinary situation in which a 3-dimensional material body, perceived as in motion in a period of time by a specific observer, finally reveals to have been at rest during the same period of time (and with respect to the same observer).[41] Thus, in Parmenides' philosophy, the illusions capable of concealing motion with immobility were probably ‘magical’. This is not in contradiction with ancient times when divine entities were usually part of the philosophical

[41] Of course, we do not consider here possible temporary troubles in perception and/or psychics that may cause hallucinations. Moreover, by specifying “3-dimensional material body”, we exclude here both the cases of some optical illusions, in which a still image seems to move, and digital videos/motion pictures, in which some tricks can be used to induce an illusion of motion. Both cases concern 2-dimensional pictures that are representations of reality rather than the reality itself.

principles of thinkers.[42] So, Parmenides' philosophy seems consistent only in an 'enchanted world', while the pseudo-Heraclitean philosophy remains consistent in a more rational world, a world 'disenchanted' by modern science and philosophy.

Actually, the pseudo-Heraclitean philosophy is not really faithful to the 'philosophy of becoming' since, according to A. Nehamas (2002), Heraclitus and Parmenides' views have much more in common than is generally recognized. In particular, Heraclitus' philosophy cannot be reduced to a philosophy simply opposing Parmenides' views: it does not only focus on change and motion but it also focuses on what is stable and what remains identical in a constantly changing world.[43] This predominant role of the concept of stability in the pre-Socratic thought (and even perhaps in the subsequent Western thought) supports Henri Bergson's view: "Of immobility alone does the intellect form a clear idea" (Bergson 1907). But Bergson's assumption may no longer hold if we consider the possibility of a philosophical position that totally rejects the classical notion of immobility. This classical notion assumes (using modern kinematic concepts) that, in a frame of reference distinct from the body in question,[44] the instantaneous velocity of a body can be strictly equal to zero during a non-null period of time ΔT (i.e. $\forall\, t \in \Delta T$, $v(t) = 0$, so $dv(t)/dt = 0$).

Here, we propose a new philosophical position rejecting the concept of 'strict' immobility as pertaining to reality assuming that, in any frame of reference distinct from a specified body, the instantaneous velocity of this body cannot be equal to zero during a non-null period of time ΔT (i.e. $\forall\, t \in \Delta T$, $v(t) \neq 0$ OR if $v(t) = 0$ so $dv(t)/dt \neq 0$)[45]. We call this new position the *'philosophy of ontological non-immobility'*. The logical consequence of this premise is that all bodies should be continuously in motion, which can sound very odd with respect to our ordinary usage of the terms 'motion' and 'immobility' to describe reality. This is why the embracement of the *'philosophy of ontological non-immobility'* requires both a more consistent use[46] for the word 'immobility' and the introduction of a new concept.

V.2 The new concept of 'impermobility'

The term 'immobility' (and all its synonyms: stillness, being at rest, being immobile, being motionless, being stationary, etc…) has always been supposed to be ontologically different to the terms 'motion' or 'movement': for instance, when F.A. Shamsi analyzed the

[42] This is the case with Parmenides and Heraclitus.

[43] There are two quotations that support this idea: "All Presocratic thinkers were struck by the dominance of change in the world of our experience. Heraclitus was obviously no exception, indeed he probably expressed the universality of change more clearly than his predecessors; but for him it was the obverse idea of the *measure* inhering in change, the stability that persists through it, that was of vital importance" (Kirk and Raven 1957, *italic* is from the quotation), and "It is that some things stay the same only by changing. One kind of long-lasting material reality exists by virtue of constant turnover in its constituent matter. Here constancy and change are not opposed but inextricably connected" (Graham 2015).

[44] This specification of the frame of reference is important since, in the context of modern kinematics, any motion of any object can be cancelled by studying the system in the reference frame attached to this object; so using this *arbitrary* procedure, any object is by definition at rest ($v = 0$ and $dv/dt = 0$) in its own reference frame.

[45] I.e. the instantaneous velocity is, most of the time, not equal to strictly zero, but it can be equal to strictly zero at some durationless instants.

[46] We could argue that the current use of 'immobility' is not consistent since it induces problems as soon as we consider immobility as real: see first and second cases in **Table 2**.

philosophical consequences of Zeno's paradox, he came to the conclusion that: "In the infinitely divisible system, there is a fundamental disjunction between 'being in a state of motion' and 'being in a state of rest'." Or in other words: "…it obliges us to postulate a special category of being, *viz*, being in a state of motion, which is in no way reducible to the other category, that of being in a state of rest" (Shamsi 1973). As we have already discussed in section IV.2, Zeno's paradoxes do nothing else than point out the insuperable difficulty of conceptualizing the passage from immobility to motion or vice-versa (= philosophical problem *PP*), which basically comes from the respective 'exclusive definitions' of these two terms.

But, if examined in the context of linguistic philosophy (i.e. philosophy of language can help to solve philosophical problems),[47] a solution for the philosophical problem *PP* can be easily pointed out: deny the ontological difference between immobility and motion by simply defining the former as an imperceptible form of the latter. Indeed, *if with the word 'immobility' we intend imperceptible motion*, there would be only a change in spatial magnitude (and not a change from one ontological state to another) when something, being 'at rest', begins to move. In this context, a body at rest is not a body that does not move, but it is only a body that seems not to be moving. Obviously, radically changing the meaning of such a common word is probably the best way to produce both impenetrable propositions and misunderstanding. A better way to linguistically solve problem *PP* is to propose a new word to replace the word 'immobility' when describing reality. We thus introduce here the word 'impermobility' (and its associated adjective 'impermobile') derived from the contraction of its definition: imperceptible mobility/motion/movement.[48]

With the help of this new concept, problem *PP* (How to conceptualize the passage from immobility to motion (or vice versa)?) turns into *PP'* (How to conceptualize the passage from impermobility to motion (or vice versa)?). Now, the answer to the latter problem has been given above by means of changes in magnitude of motion. Of course, it can be argued that problem *PP* is still unresolved. But by embracing the '*philosophy of ontological non-immobility*', this problem does not need to be solved anymore: is it really needed to conceptualize the passage from a concept that does not pertain to reality to another concept that is pertaining to reality?

The notion of 'immobility' should not be seen as a concept intrinsic to reality but rather as a notion that *emerges from our perception of the reality* via human senses (sometimes by means of physical instruments). But, because both senses and observational technical devices are limited in their capacity of description (due to detection threshold, specific sensitivity and measurement variability), they cannot catch all types of motion that exist; so, the word 'immobility' should refer more to a 'weakness' of our description of reality than to an actual feature of reality (considered as ontologically independent of our conceptual schemes in a context of 'philosophical realism'). So, while the reality of the detection threshold and specific sensitivity (as a technical limitation of measurement devices) does not exclude the actual existence of strictly null magnitude, experimental measurements cannot offer any evidence of such an existence. And thus, from an experimentalist's point of view, the new concept of 'impermobility' − introduced to solve the logico-philosophical problem *PP* (see

[47] Most of Zeno's paradoxes of plurality (see footnote 4) are solved by an approach of linguistic philosophy.

[48] The terms 'impermobility/impermobile' can also be constructed by inserting the prefix 'per' — here meaning deviation or destruction — into the words 'immobility/immobile'.

above) − may be more adequate than the term ‘immobility’ for the characterization of the kinematic state of material bodies.

Therefore, the *‘philosophy of ontological non-immobility’* proposes the replacement of the two circular reasonings of Zeno depicted in **Fig. 1** and in **Fig. 4** by a consistent reasoning based on four logically equivalent propositions: “Being = being in motion” ⇔ “Immobility is not real” ⇔ “Immobility is illusion of the senses” ⇔ “The notion of ‘impermobility’ (= imperceptible motion) should be used to describe a body what seems not moving” (see **Fig. 5** next page). In such reasoning, there is no longer any paradox and, more importantly, the philosophical problem *PP* is solved.

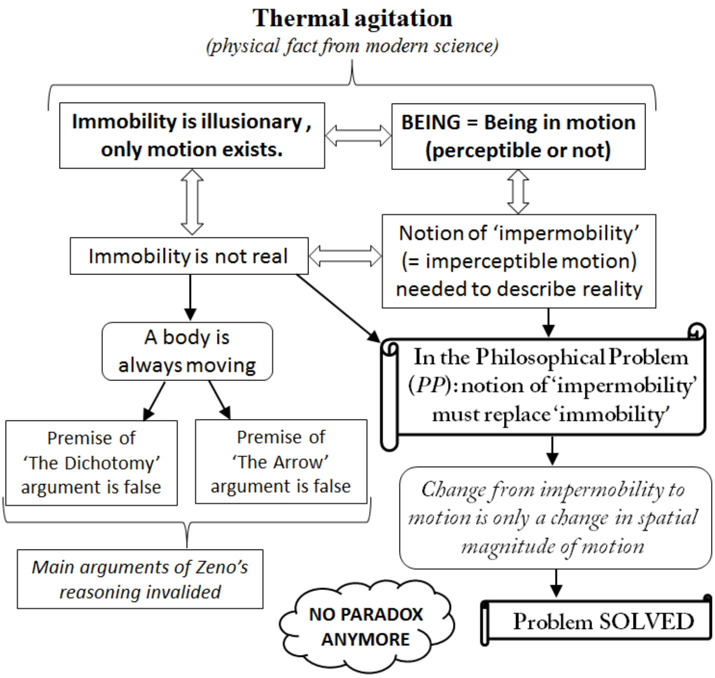

Fig. 5 Logical consequences (in the *continuous framework*) of a *‘philosophy of ontological non-immobility’* compatible with the existence of thermal agitation: we have no paradoxical situation anymore and the philosophical problem is solved.

Finally, the concept of ‘impermobility’ does nothing else than pointing out that ‘strict immobility’ is always an illusion of the senses, even for bodies where we cannot guess from our immediate experience that they are actually moving. So, to complete a rational vision of reality that should be in agreement with the *‘philosophy of ontological non-immobility’*, we need to go beyond our immediate experience and immediate thought. According to G. Bachelard (1934), this is one of the principal aims of ‘modern science’. In the present case, current science provides us with a theory that is fully compatible with our new philosophy. Indeed, in the light of ‘thermal agitation’− one of the fundamental assumptions pertaining to modern atomic theory − immobility seems truly to be very hard to find in the microscopic behavior of material bodies, as we shall see in the next section.

V.3 Thermal agitation[49]

As noticed already above, modern science describes matter in terms of discrete entities (atoms and all their components) but this matter is nevertheless able to move in a *continuous* space-time framework. The birth of the idea of atomism is usually attributed to Democritus and Leucippus (contemporaries of Zeno of Elea). But it was only during the 19th century that diverse scientific fields of investigation such as chemistry, crystallography and thermodynamics (with the kinetic theory of gases) converged on the modern notion of atoms theorizing the existence of different atomic elements constituting all materials. A few historical milestones from this century should be mentioned: John Dalton built the first atomic theory in chemistry (1808); Robert Brown described for the first time (1827) the continuous jittery motion of particles inside grains of pollen (today denoted by Brownian motion); Albert Einstein published a paper explaining in detail how the motion that Brown had observed was a result of the particles being moved by individual water molecules (1905). In 1906, independently of Einstein, Marian Smoluchowski came to the same conclusion. This explanation of Brownian motion by thermal agitation served as a definitive confirmation that atoms and molecules actually exist, and was further verified experimentally by Jean Perrin in 1908.

Thermal agitation is the ceaseless random motion of atoms and molecules constituting matter. This motion is always present, whatever the state of matter (solid, liquid and gas), but it is faster and more energetic when the temperature is high. Theoretically, thermal motion is supposed to stop at the temperature of zero Kelvin (-273.15°C) because the thermal energy of matter vanishes (in the classical non-quantum interpretation). However, the laws of thermodynamics state that absolute zero cannot be reached using only thermodynamic means. This limit of temperature can thus be theoretically defined but it is practically unattainable. *Consequently, every single atom and molecule of the universe is ceaselessly moving*. Obviously, the spatial magnitude of this thermal motion is so tiny that it cannot be directly perceived by our senses or by any optical microscope. Only some physical processes can indirectly reveal the existence of thermal motion. Historically, Brownian motion was the first evidence for thermal motion, but the much more common (non-convective) diffusion process in fluids (liquids or gases) offers another indirect proof of it. For example, a glass of water into which an ink drop is added will spontaneously (without mixing) homogenize: the diffusion of the ink molecules in the entire volume of water is driven by thermal agitation. Seeing this homogenization process, we easily conceive that the ink molecules can freely travel within the space delimited by the glass of water. Actually, the movement of molecules continues after the homogenization and occurred before adding the ink, but at these instants there is no visual effect of thermal agitation. In the solid state, atoms and molecules are not free to move around everywhere within the volume of the solid. Indeed, solids do not deform spontaneously. But thermal agitation exists in the form of vibrating oscillation of the atoms around equilibrium points. This is the foundation of the theory of thermal conduction in solids: for an electrically non-conducting solid, heat conduction is attributed to atomic activity in the form of lattice vibration. Moreover, within a molecule (and independently of the state of matter), atoms constituting the molecule are subject to vibrational oscillations; in

[49] This sub-section does not contain any bibliographic references since it exposes only encyclopedic knowledge, which is not subject to debate in the scientific community.

the context of analytical chemistry, some spectroscopy techniques −like infrared and Raman spectroscopy− are specially based on the existence of such a vibrational motion of atoms within the molecules.[50]

It is worth noting that thermal agitation is immune from Zeno's paradoxes: the two consistent arguments against motion in the *continuous framework* are 'The Arrow' and the *regressive version* of 'The Dichotomy' (see **Table 1**), but their consistency is circular and needs introducing to the notion of immobility (see **Fig. 1** and **Fig. 4**). Now, this notion is not needed when defining thermal agitation, where all atoms or molecules are really ceaselessly moving in a temporally infinite motion (i.e. without any conceivable beginning or ending in time).

Next, a recall about how classical mechanics describes motion is the final step to rationally understanding how thermal agitation (a microscopic-scale phenomenon) can affect the state of motion of a macroscopic material body.

V.4 Motion in classical mechanics

Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects) and systems of bodies (groups of objects), without questioning the causes of motion. Put simply, classical mechanics often models real-world objects as point particles, i.e. objects with negligible size. The motion of a point particle is characterized by a small number of parameters: its position, mass, and the forces applied to it. In reality, the kinds of objects that classical mechanics describes always have a non-zero size. However, the results for point particles can be used to study such objects by treating them as composite objects, made up of a large number of interacting point particles. Indeed, the *center of mass* of a composite object behaves like a point particle: the center of mass (or barycenter) of a distribution of mass in space is defined as the unique point where the weighted relative position of the distributed mass sums to zero. The distribution of mass is balanced around the center of mass and the average of the weighted position coordinates of the distributed mass defines its coordinates. Calculations in mechanics are often simplified when formulated with respect to the center of mass. In the case of a system of N particles P_i ($i = 1, \ldots, N$), each with mass m_i located in space with coordinates $\mathbf{r}_i$ ($i = 1, \ldots, N$), the coordinates $\mathbf{R}$ of the center of mass satisfy the condition: $\sum_{i=1}^{N} m_i.(\mathbf{r}_i\text{-}\mathbf{R})=0$. Solving this equation for $\mathbf{R}$, we obtain the formula: $\mathbf{R}=\frac{1}{M}\sum_{i=1}^{N} m_i.\mathbf{r}_i$, where M is the sum of the masses of all of the particles. The center of mass being only a mathematical point in space, is a 'point representing a physical object' just like the one that Zeno actually used in his arguments.

Considering that each solid body is composed of atoms and that these atoms are ceaselessly vibrating (random fluctuations), the center of mass of the object is also vibrating. Obviously, because of statistical compensation during the addition of the random fluctuations, the amplitude of motion of the center of mass is several orders of magnitude lower than the one of an individual atom. Actually, regardless of the real physical magnitude (10^{-20}, 10^{-50} or 10^{-500} m, no matter), the important fact is that, even if extremely small, this spatial amplitude is never strictly null (during a non-null period of time). Furthermore, the non-

[50] As an example of academic reference see Sherwood (1972). Generally, the existence of 'atomic and/or molecular dynamics' is part of the current scientific paradigm of most chemists. For them, the proposed *'philosophy of ontological non-immobility'* will probably appear quite obvious and natural.

perception/detection of motion does not always involve slow motions, since fast motions restricted in tiny spatial domains can also be imperceptible or undetectable. So, the instantaneous velocity of the center of mass does not need to be extremely low to restrict its motion within an extremely tiny area: the velocity vector just needs to ceaselessly change direction (and optionally change magnitude) in a random manner; and the feature of being random is exactly an essential part of the definition of thermal agitation.

Finally, the existence of thermal agitation can act as a physical and rational assistance for the proposed conceptual change: replacement of the word ‘immobility’ by the new concept of ‘impermobility’ when describing the kinematic state of any material body. While there is, by definition, an ontological difference between motion and immobility, there is no ontological difference between motion and ‘impermobility’. The difference is only in the order of magnitude of the two kinds of motion; this difference is obviously very large (probably a large gap of 15 to 20 orders of magnitude) between the motion of the center of mass of an ‘impermobile’ object and the discernible macroscopic motion of the same object.[51]

Conclusion

In this paper we showed that, among Zeno’s arguments against motion reported by Aristotle, only ‘The Stadium’ argument is truly paradoxical. But, since this paradox only concerns the *discrete framework* (atomic structure for time and space), the solution is simply to reject such a framework in favor of the *continuous framework* (an infinitely divisible system for time and space). We also showed that, in the latter framework, the ‘Achilles’ and the *progressive version* of ‘The Dichotomy’ arguments are logically equivalent but they fail to deny motion because of their internal inconsistency. Additionally, we proved that consistent arguments can be found in ‘The Arrow’ and in the *regressive version* of ‘The Dichotomy’, but their consistency relies on a hidden premise (the primordial state of being is ‘being in a state of rest’ and not ‘being in a state of motion’) leading to a circular reasoning. Finally, we showed why Zeno’s arguments against motion should be seen as *paradoxes of immobility*. In such a kind of paradox, motion is logically paradoxical because immobility is at first supposed to exist. Thus, trying to identify logical issues in Zeno’s arguments in order to solve the paradoxes is not necessary anymore, since simply denying that ‘immobility is pertaining to reality’ protects the concept of motion from any logical problems.

In fact, Zeno’s arguments simply illustrate the unresolved logico-philosophical problem *PP* consisting in attempting to conceptualize the passage from stillness to motion (or vice versa). The circularity of the arguments highlights both the strength and weaknesses of Zeno’s dialectic: his argumentations are very convincing when considered in the context of a Parmenidean vision of the world because they are fully consistent within it, but they are also weak since they cannot resist opposite philosophical positions. In particular, the ‘*philosophy of ontological non-immobility*’ introduced in this paper — in which the new concept of ‘impermobility’ (simply meaning imperceptible motion) replace the word immobility (and all its synonyms) when describing reality — is immune from Zeno’s paradoxes. According to such a philosophy, immobility is merely an illusion of the senses, and reality includes only

[51] Both motions are considered in the same reference frame, but both the reality and the conceptualization of thermal agitation are actually not dependent on the specification of any Galilean reference frame.

motion. The main advantages of this philosophical position are: (1) the self-evident resolution of the above-mentioned philosophical problem *PP*, (2) a complete compatibility with the description of material bodies depicted by modern science (by the mean of thermal agitation and kinematics). The undeniable non-intuitive aspect is perhaps its only minor drawback.

After all, the current situation related to the consistency of some of Zeno's arguments ('The Arrow' and the *regressive version* of 'The Dichotomy') still offers us the following 'personal' choice: (1) If you think that immobility must exist in the real world, you must logically accept that motion is an illusion; otherwise you are stuck into a paradoxical situation; (2) If you think that immobility does not exist in the real world, there is no problem to consider motion as real anymore, and the above-mentioned logico-philosophical *PP* problem is solved. Actually, by the mere fact that such a choice exists, it reveals an important epistemological conclusion: the real existence of immobility (i.e. as being more than a human-imagined concept) should be considered as an *a priori* belief for which we may search some experimental evidences. Contemporary science strongly suggests that there are no such evidences, but it does not impede anybody from looking after them.

Finally, the present article does not pretend to exhaust all philosophical aspects or epistemological consequences of this new vision of Zeno's paradoxes, and it should rather be seen as a new starting point to continue the long debate about the nature of motion and stillness. In particular, since classical notion of immobility is based on our immediate experience and since immediate experience is one of the 'epistemological obstacles' identified by Gaston Bachelard,[52] we can legitimately wonder: should the notion of immobility also be a stealthy 'epistemological obstacle'? If yes, we hope that the present article will be of help in overcoming it.

[52] According to G. Bachelard (1934), epistemological obstacles are all representations that are blocking or hindering scientific progress.

Appendix 1: Explanation of 'The Stadium' argument

Because this explanation is largely inspired from the one of J.A. Faris (1996), the translation of Aristotle 's text by this author is given below:
(Part I) *The fourth is the one about the two equal rows of bodies that move past each other in opposite directions at equal speeds, the one row from the end of the stadium, the other from the mid-point; in this argument Zeno thinks it follows that the half of a given time is equal to its double.*
(Part II) *For example, let AA be the stationary bodies, BB those, equal to these in size and number, starting from the mid-point, and CC those starting from the end, equal in size and number to the As and in speed to the Bs.*
[Aristotle, *Physics* VI:239b33-240a1 (for part I); 240a4-240a18 (for part II) ; Translated by J.A. Faris (1996)]

The fourth argument against motion is probably the most difficult to understand directly from the text of Aristotle. Like the 'Arrow' argument, a *discrete space and time framework* have to be used to correctly interpret it… but it is still not obvious to see how Zeno attains the strange conclusion "*the half of a given time is equal to its double*". One explanation to get to the same paradoxical conclusion that Zeno propounded is the following:

(a) The first B in the time of its movement has passed 1 A (e.g. B_3 was in front of A_2 in the initial position and is in front of A_3 in the final position, see **Fig. 6**).

(b) The first C in the time of its movement has passed 2 B's (e.g. C_1 passes B_2 and B_3).

(c) The time taken by the first C to pass a B = the time taken by the first B to pass an A.

(d) The first C in the time of its movement has also passed 1 A (C_1 was in front of A_2 in the initial position and is in front of A_1 in the final position).

Therefore:

From a,b,c: The first B's time = half the first C's time (1).

But, from a,d: The first B's time = the first C's time (2).

Therefore, from (1) and (2): Half the first C's time = the first C's time (i.e. half of a given time is equal to the whole).

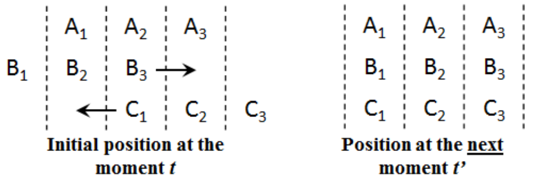

Fig. 6 Schematic representation of 'The Stadium' argument (when the cardinality of each group is 3) (Moments ***t*** and ***t'*** are two consecutive indivisible moments of time)

The corner stone of Zeno's argumentation is obviously proposition (c). Indeed, this one can seem quite dubious at first glance, and it would be more natural to state the following: the time taken by the first C to pass a B = *half* the time taken by the first B to pass an A. But such a proposition is a true proposition only in a 'continuous world': the Stadium argument is trivial in a *continuous framework,* as the notion of relative speed exists; unfortunately, it is not valid in a discrete space and time framework in which intermediate positions are not allowed (see **Fig. 7**). Thus, proposition (c) is a direct consequence of discretization of time and space.

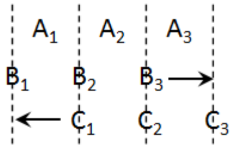

Fig. 7 Non-existing position in a discrete space and time framework

As noticed already by Adolf Grünbaum (1967, 1970), another way to run into a paradox is to consider the same maneuver but in the reference frame of the B's instead of the reference frame of the A's: in this case, the A's and the C's are moving in the same direction, and problems occur when considering the speed of these rows. Of course, the A's are moving at the Bs' previous rate, but what is the speed of the C's now? Let's suppose that we find ourselves looking again at the rows all in the same position (i.e. final position of **Fig. 6**) but in this case, C_1 goes to B_1 from B_3 without ever passing in front of B_2 (remember that there is no time left between two consecutive indivisible moments of time). So, C_1 should perform a kind of teleportation! To avoid that, C_1 should be in front of B_2 after this motion… but in this case, we can remark that the row of C's is now moving at the same rate as the A's i.e. it is not moving any faster than the A's (see **Fig. 8**).

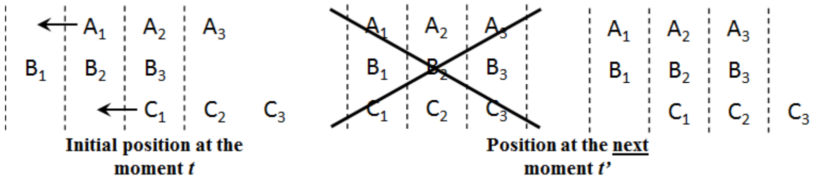

Fig. 8 Schematic representation of 'The Stadium' argument in the reference frame of the B's (Moments ***t*** and ***t'*** are two consecutive indivisible moments of time)

Finally, the argument reveals that the hypothesis of an atomic structure of space and time implies automatically that firstly, an absolute reference frame must exist (in the 'The Stadium' argument, this absolute reference frame is of course the A row) and secondly, in this absolute reference frame, motion can be carried out at only one speed (change of only 1 element of space between two consecutive indivisible moments of time). The huge problem is that such a situation does not allow the existence of relative motion at any specific speed. So, 'The Stadium' argument does not explicitly ban the existence of motion but forbids the existence of relative motion. Of course, such a forbidding remains in discrepancy with our classical physical experience, so the paradoxical situation remains (in the *discrete framework)*.

Appendix 2: Common situations in which movement is not perceived

Because it is so usual to experience movement in everyday life, we generally do not realize that such a perception of motion requires four conditions, which are depicted in

Fig. 9. In a given inertial frame, where an object is physically in motion, the lack of one of these conditions does frequently induce an illusion of immobility (in the same Galilean reference frame).

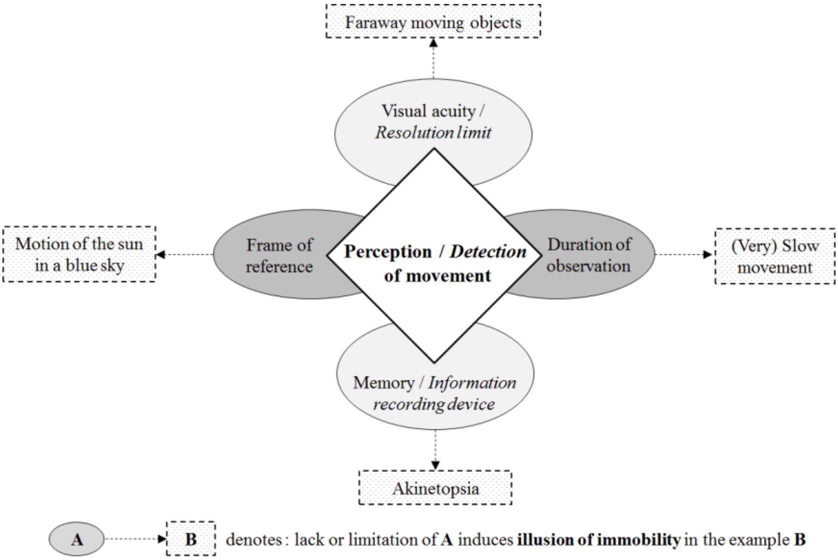

Fig. 9 Schematic representation of the four requirements needed for the perception (by human or animal) or the detection (by any instrument) of motion

One of the simplest examples of such an illusion is the motion of the sun during a very sunny day. When looking at the sun in a perfect blue sky (without any clouds) for a few minutes around noon, one may have the impression that the sun is not moving. But if one stays in the sun, motionless, near a tree and a few minutes later, is in the shadow of the tree, one can easily infer that the sun is moving. In this case, thanks to the tree used as a reference frame, motion can be quickly/easily detected. Without this (local) reference frame, we can still define the position of the sun according to its elevation above the horizon, but without any measurement device to precisely record the azimuth value, we would generally need a few hours to detect actual changes in its position (and if you are less hurried and prepared to extend the duration of observation, waiting for sunset is obviously also evidence of the apparent movement of the sun). So, depending on presence (or absence) of a reference frame, and on relative speed of the motion, the minimum duration of the observation has to be adjusted to correctly perceive the movement and avoid the illusion of stillness.

Moreover, the detection of motion always implies a comparison of two different positions at two different times. This operation of comparison requires having recorded the first position to memory (brain for animals or a specific device for machines).[53] Such an operation is extremely natural for most people but, unfortunately, not for people affected by akinetopsia (motion blindness). Patients with akinetopsia cannot perceive motion in their visual field, despite being able to see stationary objects. For them, the world is devoid of motion. Even for healthy people, physiological limits of vision can induce illusion of immobility: as the 'normal' visual acuity is around 1 arcminute (1' = 1/60°), any movement of 9 m of spatial largeness cannot be detected if observed from more than 30 km. Moreover, direction of motion in comparison to the axis of observation could also affect the perception of actual motion: let us imagine someone on a beach, bordered by a coast where a lighthouse is located. At a distance of 3000 m there is a 15 m-high sailing boat travelling at 10 knots (~5.1 $m.s^{-1}$) towards the lighthouse (see **Fig. 10**). For the observer, the boat travels at 5.9 $'.s^{-1}$ and consequently the observer can detect the motion of the boat in less than one second. In this motion the lighthouse acts as local reference frame and its apparent distance from the sailing boat is decreasing quite quickly; but if the boat faces the observer, the apparent distance between the lighthouse and the boat does not change so rapidly anymore. Only changes in the apparent size (or angular diameter) of the boat can give an indication of the actual movement. With the considered configuration (see **Fig. 10**), a modification of this apparent size of more than 1' is performed after 32 s. Before this time elapses, the observer has the illusion that the moving boat is actually still.

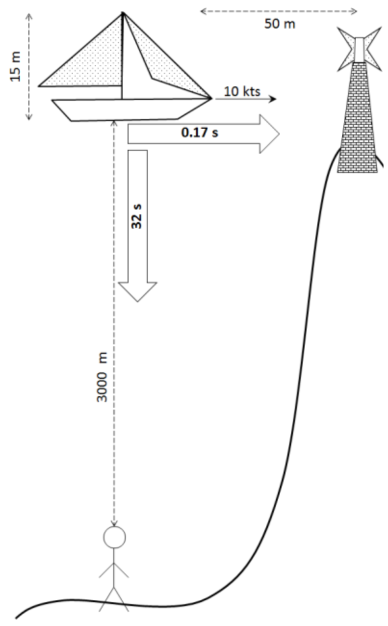

Fig. 10 Schematic representation of an example of geometric effect on the duration of observation to be able to perceive motion.

Physiological limitations of human eyes not only restrict the vision of remote objects but also that of close ones. The least distance of distinct vision is the smallest distance at which someone with 'normal' vision can comfortably look at something. This distance is typically about 25 cm for a 'normal' subject. Combined with the value of 'normal' visual acuity, this means that we cannot directly see something smaller than ~0.07 mm. However, motion on a microscopic scale exists and anybody can use a microscope to be convinced of that.

Thus, in some common situations, it is easy to be persuaded that stillness is illusory and optico-geometrical effects can mask actual motion.

[53] This is in agreement with Michael Tooley's view: "For it seems to me that perceptual knowledge that something is moving involves unconscious use of short-term memory" (Tooley 1988).

References

Alper, J. S., & Bridger, M. (1997). Mathematics, Models and Zeno's Paradoxes. *Synthese,* 110, 143–166.

Antonopoulos, C. (2004). Moving without being where you're not; A non-bivalent way. *Journal for General Philosophy of Science,* 35, 235–259.

Ardourel, V. (2015). A discrete solution for the paradox of Achilles and the tortoise. *Synthese*, 192, 2843-2861.

Arntzenius, F. (2000). Are there really instantaneous velocities? *The Monist*, 83(2), 187-208.

Atkinson, D. (2007). Losing energy in classical, relativistic and quantum mechanics. *Studies in History and Philosophy of Modern Physics,* 38, 170–180.

Bachelard, G. (1934). *La formation de l'esprit scientifique*. (5th ed., Bibliothèque des textes philosophiques). Paris: Librairie philosophique J. VRIN.

Benacerraf, P. (1970). Tasks, Super-Tasks, and the Modern Eleatics. In W. C. Salmon (Ed.), *Zeno's paradoxes* (pp. 103-129). Indianapolis: Bobbs-Merrill.

Bergson, H. (1907). *L'évolution créatrice* (86th ed.). Paris: Les Presses universitaires de France. [Relevant part in English translation of this Bergson's book is also presented as chapter 3 in the monograph by Salmon (1970)]

Bergson, H. (1969). *La pensée et le mouvant. Essais et conférences (1903 à 1923)*. Paris Les Presses universitaires de France.

Beth, E. W. (1946). Historical studies in traditional philosophy. *Synthese*, 5(5-6), 258-270.

Blay, M. (2010). *Penser avec l'infini : La fécondité d'une notion mathématique et philosophique, de Giordano Bruno aux Lumières*. Paris: Vuibert/Adapt-Snes.

Carroll, J. W. (2002). Instantaneous Motion. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition*, 110(1), 49-67.

Caveing, M. (1982). *Zénon d'Élée, prolégomènes aux doctrines du continu: étude historique et critique des fragments et témoignages*. Paris: Vrin.

Dowden, B. (2017). Zeno's Paradoxes. In *The Internet Encyclopedia of Philosophy*, ISSN 2161-0002, http://www.iep.utm.edu/zeno-par/ (no publication date available); Accessed Sept. 10th 2017.

Faris, J. A. (1996). *The paradoxes of Zeno.* Aldershot (Hants., England) ; Brookfield (Vt., USA): Avebury.

Graham, D.W. (2015), Heraclitus. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Vol. Fall 2015). https://plato.stanford.edu/archives/fall2015/entries/heraclitus/ ; Accessed Sept. 10th 2017.

Grünbaum, A. (1967). *Modern Science and Zeno's Paradoxes*. Middletown, Conn.: Wesleyan University Press.

Grünbaum, A. (1970). Modern Science and Zeno's Paradox of Motion. In W. C. Salmon (Ed.), *Zeno's paradoxes* (pp. 200-250). Indianapolis: Bobbs-Merrill.

Hamblin, C. L. (1969). Starting and stopping. *The Monist*, 53(3), 410-425.

Hardie, R. P., & Gaye, R. K. (1930). *Physica* by Aristotle (translation). In J.A. Smith and W.D. Ross (Ed.), *The Woks of Aristotle*, vol. II, Oxford: At the Clarendon Press. Avialable online at: https://archive.org/details/workstranslatedi02arisuoft ; Accessed Sept. 10th 2017.

Harrison, C. (1996). The Three Arrows of Zeno : Cantorian and Non-Cantorian Concepts of the Continuum and of Motion *Synthese*, 107, 271-292.

Hasper, P. S. (2006). Zeno Unlimited. *Oxford Studies in Ancient Philosophy,* 30, 49-85.

Huggett, N. (2010). Zeno's Paradoxes. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Vol. Winter 2010). http://plato.stanford.edu/archives/win2010/entries/paradox-zeno/ ; Accessed Sept. 10th 2017.

Jackson, F., & Pargetter, R. (1988). A Question about Rest and Motion. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition*, 53(1), 141-146.

James, W. (1911). *Some problems of philosophy: A beginning of an introduction to philosophy*. New York: Longmans, Green and Co.

Kline, M. (1980). *Mathematics: The Loss of Certainty*. New York: Oxford University Press.

Kirk, G. S., & Raven, J. E. (1957). *The Presocratic Philosophers: A Critical History with a Selection of Texts*. Cambridge, UK: Cambridge University Press. Avialable online at: https://archive.org/details/presocraticphilo033229mbp ; Accessed Sept. 10th 2017.

Lange, M. (2005). How Can Instantaneous Velocity Fulfill Its Causal Role? *The Philosophical Review*, 114(4), 433-468.

Laraudogoitia, J. P. (1996). A Beautiful Supertask. *Mind,* 105, 81-83.

Laraudogoitia, J.P. (2013). Zeno and flow of information. *Synthese*, 190, 439-447.

Lear, J. (1981). A note on Zeno’s arrow. *Phronesis*, 26(2), 91–104.

Lee, C. (2011). Nonconservation of momentum in classical mechanics. *Studies in History and Philosophy of Modern Physics,* 42, 68–73.

Łukowski, P. (2011). *Paradoxes* (Vol. 31, Trends in Logic). Netherlands: Springer.

Lynds, P. (2003). Time and Classical and Quantum Mechanics: Indeterminacy vs. Discontinuity. *Found. Phys. Lett.,* 16(4), 343-355.

Mazur, J. (2007). *The Motion Paradox: The 2,500-year-old Puzzle Behind All the Mysteries of Time and Space*. New York: Dutton.

Medlin, B. (1963). The Origin of Motion. *Mind*, 72(286), 155-175.

Meyer, U. (2003). The Metaphysics of velocity. *Philosophical Studies*, 112, 93-102.

Mortensen, C. (1985). The limits of change. *Australasian Journal of Philosophy*, 63(1), 1-10.

Nehamas, A. (2002). Parmenidean Being/ Heraclitean Fire. In V. Caston and D. W. Graham (eds.), *Presocratic Philosophy: Essays in Honour of Alexander Mourelatos* (pp. 45-64). Aldershot: Ashgate.

Owen, G. E. L. (1957). *Zeno and the Mathematicians* (Vol. LVIII, Proceedings of the Aristotelian Society). N.S.

Papa-Grimaldi, A. (1996). Why Mathematical Solutions of Zeno's Paradoxes Miss the Point: Zeno's One and Many Relation and Parmenides' Prohibition. *The Review of Metaphysics,* 50(2), 299 - 314.

Peijnenburg, J., & Atkinson, D. (2008). Achilles, the Tortoise, and Colliding Balls. *History of Philosophy Quarterly,* 25, 187-201.

Priest, G. (1985). Inconsistencies in Motion. *American Philosophical Quarterly*, 22(4), 339-346.

Reeder, P. (2015). Zeno’s arrow and the infinitesimal calculus. *Synthese*, 192(5), 1315-1335.

Romero, G. E. (2014). The Collapse of Supertasks. *Foundations of Science*, 19(2), 209-216.

Russell, B. (1970). The Problem of Infinity Considered Historically. In W. C. Salmon (Ed.), *Zeno's paradoxes* (pp. 45-58). Indianapolis: Bobbs-Merrill.

Salmon, W. C. (Ed.). (1970). *Zeno's paradoxes*. Indianapolis: Bobbs-Merrill.

Shamsi, F. A. (1973). *Towards a definitive solution of Zeno's paradoxes*. Karachi: The Times Press.

Sherwood, P. M. A. (1972). *Vibrational Spectroscopy of Solids*. Cambridge: University Press.

Smith, J. W. (1990). Time, change and contradiction. *Australasian Journal of Philosophy*, 68(2), 178-188.

Smith, S. R. (2003). Are instantaneous velocities real and really instantaneous?: an argument for the affirmative. *Studies in History and Philosophy of Modern Physics,* 34, 261–280.

Thomson, J.F. (1954). Tasks and Super-Tasks. *Analysis*, 15(1), 1-13.

Tooley, M. (1988). In Defense of the Existence of States of Motion. *Philosophical Topics*, 16(1), 225-254.

Vlastos, G. (1966). A note on Zeno's arrow. *Phronesis*, 11(1), 3-18.

White, M. J. (1982). Zeno's arrow, divisible infinitesimals, and Chrysippus. *Phronesis*, 27(3), 239-254.

Wisdom, J. O. (1970). Achilles on a Physical Racecourse. In W. C. Salmon (Ed.), *Zeno's paradoxes* (pp. 82-88). Indianapolis: Bobbs-Merrill.

Maël Bathfield graduated in chemistry from the University Claude Bernard Lyon 1 (UCBL), France, in 2003, before undertaking a PhD in polymer chemistry. He defended his thesis in 2006 and since then he occupied several post-doctoral positions at Ecole Nationale Supérieure de Chimie de Montpellier, France. During his PhD, a scientific controversy in a specific polymerization process led him to be interested, first, in history of chemical kinetics theories. Then, since chemistry and physics merge together at the fundamental level, he was rapidly interested in the construction of the modern atomic theory. He is mostly self-educated in history and philosophy of sciences, but in 2010 he attended the courses of the "History and Philosophy of Sciences" Master, jointly organized by UCBL and University of Montpellier 2, France. His personal research focuses on the implicit foundations of modern physics (thermodynamics, special/general relativity, and quantum physics) in connection with its current fundamental difficulties (interpretation of quantum theory, incompatibility between general relativity and quantum physics, dark matter/ dark energy).