Author's Accepted Manuscript

Against 'Interpretation'

Quantum Mechanics Beyond Syntax and Semantics

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This is a post-peer-review, pre-copyedit version of an article published in Axiomathes. The final authenticated version is available online at: http://dx.doi.org/10.1007/s10516-021-09579-y.

Accepted: June 27, 2021

Abstract The question "what is an interpretation?" is often intertwined with the perhaps even harder question "what is a scientific theory?". Given this proximity, we try to clarify the first question to acquire some ground for the latter. The quarrel between the syntactic and semantic conceptions of scientific theories occupied a large part of the scenario of the philosophy of science in the 20th century. For many authors, one of the two currents needed to be victorious. We endorse that such debate, at least in the terms commonly expressed, can be misleading. We argue that the traditional notion of "interpretation" within the syntax/semantic debate is not the same as that of the debate concerning the interpretation of quantum mechanics. As much as the term is the same, the term "interpretation" as employed in quantum mechanics has its meaning beyond (pure) logic. Our main focus here lies on the *formal* aspects of the solutions to the measurement problem. There are many versions of quantum theory, many of them incompatible with each other. In order to encompass a wider variety of approaches to quantum theory, we propose a different one with an emphasis on pure formalism. This perspective has the intent of elucidating the role of each so-called "interpretation" of quantum mechanics, as well as the precise origin of the need to interpret it.

Keywords Interpretation of scientific theories · Non-relativistic quantum mechanics · Semantic view · Syntactic view.

Mathematics Subject Classification (2000) MSC 03A10 · MSC 81P05

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1 Introduction

If one desires to inquire about what quantum *theory* is, the availability of a plethora of its "interpretations" seems to be a good place to start. Notably, the specificities of what the term "interpretation" means in the particular context of the cluster of quantum theories deviate significantly from what traditionally was meant in the debate revolving around the syntactic and semantic characterizations of scientific theories. Roughly speaking, the mainstream debate unfolds as follows: in syntactic approaches, scientific theories are identified as sets of formalized sentences and some "connection" rules; in semantic approaches, scientific theories are identified as classes of models. The main question of the traditional debate is: which is more appropriate? or, perhaps, none of them?

The problem of interpretation of non-relativistic quantum mechanics (QM) played a major role and, in fact, shaped the roads paved by the philosophy of science since its birth in the 20th century. The debates in philosophy of science revolving around the characterization of *what scientific theories are* were concurrent with the first steps of the development of quantum theory. Perhaps this is one reason why philosophers found it difficult to incorporate it with extant concepts. The newly introduced complexities and the strangeness of this new "theory" also contributed to his problematic reception. These difficulties, of course, were not particular to the philosophers. Not only then, but also today, it's unclear how to globally interpret its results also to the working/field scientist. Even worse: it isn't clear what is to "interpret" QM!

We will address all of these issues here, and (hopefully) settle *some* of them. The structure of this paper is as follows. In sections 2 and 3, respectively, we present the classic syntactic and semantic approaches to scientific theories. In section 4, we discussed arguments for the unification of the two approaches, taking into account the recent literature on the subject. In section 5, we discuss the problems that arise when trying to understand the current scientific enterprise in this traditional scheme, using QM as an example. In particular, we analyze textbook and set-theoretical formulations of quantum theory to illustrate the fact that these formulations are formulations of an already interpreted quantum theory, therefore not being the theory that is interpreted by so-called interpretations of quantum mechanics. In section 6, we present a concrete proposal for the unification of the syntactic and semantic approach, called QM^{bas}. In addition to illustrating a point of convergence between the syntactic and semantic views, the QM^{bas} is presented as a structure with postulates sufficiently general not to commit itself in advance to any interpretation; the price, however, is to deprive it of the status of "scientific theory", since it is a purely mathematical formulation, with no mention of physical systems or any other terms that refer to the realm of experience. To solve this problem, which we call the "isolation problem", the general postulates of QM^{bas} must be instantiated in physical theories, with specific axioms that refer to the physical domain — this is what we call "interpreting" quantum mechanics, being the subject of section 7, we where present three applications of our proposal, instantiating QM^{bas} purely theoretical postulates in specific axioms: on quantum theory with collapse, one with branching, and one with hidden variables. We conclude in section 8.

2 Interpreting scientific theories I: syntactical approach

Those who first tried to incorporate quantum theory within the philosophy of science were the logical positivists and empiricists. Their preferred approach was to formalize [or, at least, provide a formalized sketch] the products of scientific discourse and reconstruct the somewhat convoluted forms, on which scientific theories were generated, more systematically. Historically, the syntactic view ascended along with this program. But since the decline of logical positivism, semantic views — that are based on revisions of the initial syntactical program — are, then, considered to be the current *new orthodoxy* (cf. Contessa, 2006).

Unfortunately, the whole endeavor of the syntactical approach was often identified by a straw man version of it. The “Received View on scientific theories”, as termed by Putnam (1966, p. 240), became a sort of *automatic identification* of any formal approach to the syntactic approach. The semantic approach also, as we will see, meets a similar fate. Therefore, distinguishing between straw man aspects of each one and their fundamental characteristics seems to pose as a good warm-up for this discussion.

Among the main tenets of the syntactical approach is the possibility of a formal treatment of scientific theories as an axiomatic calculus. The broader sense of the syntactical approach, however, can be characterized in numerous ways. We will distinguish between the “radical” and “moderate” versions.

2.1 The radical syntactical approach

The syntactical approach has often been wrongly identified exclusively with regards to the *radical* syntactic approach in the past. Mainly as a package rooted in the theses put forth by several members of the Vienna Circle at different times, the radical syntactic approach to scientific theories is commonly characterized by five main requirements, succinctly presented by Krause and Arenhart (2016, p. 4):

Language: the non-logical vocabulary of the scientific formal language is divided in two parts: the *theoretical terms* (V_T) and the *observational terms* (V_O), being the latter closely related to empirical phenomena and the former *prima facie* not.

Logic: the formal language was constructed by a set of logical axioms, which gives rise to its underlying logic.

Theoretical postulates: a set of V_T sentences that are taken as postulates.

Semantics for V_O: observational terms are indirectly connected to V_T, which is related to a theory of meaning based on a version of the criterion of verifiability.

Correspondence rules: set of sentences that connects V_T to V_O, that is, relates purely theoretical parts of scientific theories with empirical data, given an interpretation to it.

From this perspective, the radical syntacticist answer to the question of "what is a scientific theory?" is (in a few words): *an axiomatic calculus with theoretical axioms connected to empirical observations through correspondence rules*. This also suggests how a theory can be interpreted (and named as such): to *interpret*, then, is to connect the theory with objects in the world according to specific rules. As the name suggests, the *theoretical* vocabulary V_T relates to possible observable events, while the *observational* vocabulary V_O relates to objects that can be directly observed by an experimental arrangement. This is, perhaps, the most intuitive way to see the distinction within the use of a formal language: whereas the objects designated by V_O can, at least in principle, be *observed*, the objects designated by V_T can only be obtained by means of a calculus. Consider, for example, the concept of "density" (D): in basic chemistry textbooks, it is said that *density is the ratio of the mass (m) to the volume (v) of a given material (solid, liquid or gaseous)*, yielding the following equation: $D = \frac{m}{v}$. Notice that the physical quantities m and v can be obtained employing measuring devices, thus counting as *observational terms*,[1] whereas D can be indirectly obtained through the equation, therefore constituting an example of a *theoretical term*.

These features were subject to criticism, inasmuch as the radical syntactic approach has historically been closely linked to the Logical Positivist program. Suppe (2000, S103) forged a very succinct list of some reasons that led to the decline of the radical syntactic view. According to Suppe, the two most serious objections related to the view are the untenability of its observational-theoretical distinction and a "[...] confusion of meaning relationships, experimental design, measurement, and causal relationships some of which are not properly parts of theories" concerning the correspondence rules.

The most criticized, if not the most controversial, aspect of the radical syntactic approach lies in the proposed division of vocabulary between observable V_O and non-observable (theoretical) V_T terms, a fundamental point for the effective application of correspondence rules. As pointed out by Dalla Chiara and di Francia (1979, pp. 148–149), a simple observation of a fish through an aquarium, for example, may be considered a *theoretical term* to some extent, inasmuch the lens precludes a *direct observation* required for V_O.[2] Similarly, Krause and Arenhart (2016, p. 5) point out that a purely theoretical term, such as "electric current", could be, in principle, directly experienced simply by putting one's finger inside an electric socket. Therefore, this distinction was viewed to be highly implausible, and as such does not seem fit to serve (in a non-arbitrary way) as a foundation for scientific theories.[3]

Such aspects, responsible for bringing issues to the syntactic approach, are entirely *exclusive* of the radical approach, while also being somewhat disposable (or circumvented). Thus, it can be said that they do not configure essential traits of the syntactic view *per se*. However, as one of the main tenets of syntactic approaches is the formalization of scientific theories according to a formal language, there is a line of criticism that encompasses the syntactic view as a whole: the restrictions made upon the employment of formal languages as an appropriate tool to capture what a scientific theory is.

Such aforementioned *formal language* may be considered to be a *first-order* language.[4] This point is easily accepted to what concerns *formal theories* such as mathematics or logic, however, seems inadequate when considering formal description of scientific theories (cf. Suppes, 2002, p. 4), given that most contemporary scientific theories involve more than simple relations among elements of the domains (cf. Krause and Arenhart, 2016, sec. 6.1). Generally, order-1 structures encompass a non-empty domain D and distinguished elements of D, n-ary relations on D and n-ary functions on D. An order-n structure, $n > 1$, may encompass elements, relations and operations of higher orders, not only dealing with the individuals of D, but also with collections or properties of such individuals, and so on.

Moreover, the attempt of applying this kind of formalization to relatively complicated scientific theories would be, at very best, impractical. Consider, for example, a scientific theory such as QM: before formalizing the theory itself, one would first need to formalize several general ideas related to set theory, as well as the required mathematics (such as several topics of standard functional analysis, e.g., Hilbert spaces, differential calculus, etc.). This kind of criticism makes the ideal of formalization virtually useless for actual scientific practice — which is a serious constraint.

Nevertheless, as Lutz (2015) pointed out, there seems to be *no* textual evidence that the proponents of the radical syntactic approach restricted the language of formalization to order-1 structures. In addition, the alleged obligatory requirement of preliminary formalization from scratch seems not to exist — notably, there is no documentation of such requirement made by proponents of the radical syntactic approach. Therefore, this criticism seems to configure yet another straw man attack on the radical syntactic approach (Lutz, 2012).

Lastly, it is relevant to address the criticism of the radical syntactic approach termed the "individuation problem". This objection states that the radical syntactic approach *identifies* a theory with regard to its *linguistic formulation*. But if theories are not [only] linguistic entities, the conclusion that follows is that theories are individuated incorrectly by the radical syntactic view. Similar to several of the criticisms briefly presented in this section, the individuation problem would represent a severe drawback of the radical syntactic approach, were it legitimate, since it would render impossible to formulate, for example, the *same theory* using different vocabularies.

This, however, appears not to be the case, insofar as there seems to be, again, *no evidence* that the radical syntactic approach is committed to this sort of individuation of theories based on a specific formal vocabulary. It seems, in fact, that the radical syntactic approach does not necessarily promise to provide an explication of the concept of "scientific theory" in this general sense, and even refraining to talk about it in general, focusing on particular examples of formalizations. For Rudolf Carnap, for example, this is particularly misleading. Given that his whole approach to the formalization of theories was relying precisely on allowing the tentative employment of more than one linguistic framework.

This seems to be yet another widespread misunderstanding concerning both syntactic and semantic approaches to characterization of scientific theories (Krause and Arenhart, 2016, p. 8). If it were to do so, such attempt would be made to fall since *not every scientific theory* is well [or completely] developed — and perhaps will never be[5] — to the point that allows it to be axiomatized as a (purely) formal system.

Following Lutz (2015, sec. 5) and Krause and Arenhart (2016, p. 9), proposals of the radical syntactic approach should be seen as a rational reconstruction of particular scientific theories — thus avoiding (parts of) the individuation problem. This formal reconstruction, in turn, should be understood as an *horizon* rather than as a *criterion* of which theories *ought* to meet in order to be considered as such scientific theories.

The preliminary discussion here presented allows us to proceed to a more sensible conception of syntactical approaches, which we will call the "moderate syntactical approach".

2.2 The moderate syntactical approach

As pointed out by Krause and Arenhart (2016, p. 6), it is possible to refrain from most controversial aspects of the radical syntactical approach without forfeiting the syntactical approach: one is not obliged to adhere, say, to a verificationist theory of meaning (Hempel, for example, never accepted it and still maintained the syntactical approach; Carnap also revised his initial formulations, distancing from verificationism, but never abandoned the syntactical approach completely). The same goes for accepting the relation between theories and experience via specific rules of correspondence or the division of vocabulary between V_T and V_O in order to employ a syntactic view of scientific theories. Perhaps a division is indeed necessary, but one can assume the use of a distinction as a provisional methodological commitment without necessarily committing to a particular distinction in the sense that it is more ontologically correct or more fundamental than others. Maxwell (1962, p. 13), for example, employed this strategy and favoring a division between theoretical vocabulary and "quickly decidable" vocabulary. There does not seem to be a restriction on the terms of V_O in relation to any origin or epistemic priority; thus, a fundamental (essentialist) distinction between two types of language, V_O and V_T, is not required.

The "demystification" of the syntactic view is not, however, widely known. The effort to rule out the most problematic features of the radical syntactic approach, while sticking to basic tenets of the syntactic view is precisely what constitutes the "moderate syntactical approach". Following Lutz (2015, p. 5) and Krause and Arenhart (2016, p. 10), the main traits of the view can be presented as follows:

Formal language: A scientific theory can be presented in a formal language of order-n.

Theoretical equivalence: An equivalence between theories can be put forth so that different formulations could count as the same theory.

Partial Interpretation: Some sentences of the formal language must be *partially interpreted* in the sense that the theoretical language of the theory refers to empirical objects, thus granting that the theory is an empirical theory.

The theoretical equivalence feature seems to act as a clarification of the individuation problem allegedly presented in the radical syntactic view. Accepting the commitment to the individuation of a theory as formulated in one unique language makes it possible to accept equivalent formulations of a theory (cf. Halvorson, 2012, p. 191).

Before advancing in direction to our main goal, however, it is fit to digress towards the notion of "axiomatization", which plays a central role within this discussion. As pointed out by Suppe (1977, p. 110), the conflation between axiomatization and formalization can lead to several misunderstandings. "Axiomatization" may be defined as an organization of a body of knowledge (such as a scientific theory) to clarify its structure, by singling out certain concepts as primitive (or undefined) and others as defined or derived. The main point achieved by such definition is the possibility of presenting the theory as a deductive system, in which certain propositions singled out as axioms may provide the deductive derivation of all other propositions. It should be remarked that the matter of axiomatization is marked by several misunderstandings concerning its corrigibility. In the traditional sense, named "concrete axiomatization", axioms are taken to be *sacred truths*, immutable by their own nature. Fortunately, this is not relevant to the current situation of mathematical development, which conversely defines axioms as expressions of tacit assumptions, in order to make them explicit. Quoting Dalla Chiara and di Francia (1979, p. 134): "[t]o *axiomatize* is not to *dogmatize*!". Once a theory is properly axiomatized, it can be interpreted within its axiomatic constraints, that is, the domain of interpretation is restrained to the situations in which the axioms are true. This observation leads to the *formalization* of a theory: for such an interpretation to be considered precise, one must replace its language with a formal (artificial) syntax.

As for partial interpretation, it is noteworthy to reiterate that this concept carries *no commitment* to a specific set of correspondence rules, thus allegedly distancing from the radical syntactic approach. Moreover, the criticism to which this notion was subjected seems to significantly weakens when the dichotomy between V_T and V_O is properly understood.

And so, this leaves us with the other contentious problem. Without going into much detail, it is well-known that the theoretical/observational distinction was subject to a great number of critical reactions in the philosophical literature (Popper, Putnam, Quine — to only mention a few — argued for the untenability of the distinction) and, as the usual textbook history goes, gave rise to one of the fatal blows responsible for the demise of the logical positivist program.

It is however important to note that the bulk of these criticisms was not directed to the tenability of drawing such a distinction, but to the stronger idea that it could play the role of a counterpart in "reality". What Carnap had in mind was somewhat different. According to him, in the analysis of the language of science, there is no question about the "fair" or "correct" use of the term "observable". "Observable" and "unobservable" represent the two poles of a methodological *continuum* and should not be seen as giving rise to a fixed and irrevocable demarcation. To be able to draw the distinction in this precise manner, we would first need to have a definitive description of our perceptual apparatus but, given the actual development of science, we have not yet [if ever] achieved this point. On the other hand, Carnap points out that, even if the choice of an exact dividing line has to be arbitrary, at least for practical purposes it can be regarded as clear enough.[6] More specifically, Carnap adopted the general strategy of first defending the practical utility of a distinction between terms and then recognizing that it is not irrevocable and subject to variations. As any stipulation motivated by the convenience of structuring the language of science, this separation is flexible and subject to the (often practical) interests of the person who carries it out.[7] Carnap, therefore, refrains from justifying — or trying to ground — the distinction as if it directly represented a fundamental property of terms or their referents, i.e, he opts out of drawing a distinction and instead characterizes it as a merely methodological tool, without the need of "reflecting" or "cutting nature at its joints". This agnostic methodological approach was later adopted by Maxwell (1962), as well as by Lewis (1970) in "How to Define Theoretical Terms": observational terms for Carnap [*O-terms*] are assumed by Lewis to represent "*original* terms", or "*old* terms", i.e., terms that are understood prior to the appearance of a new theory T, without any restriction as to whether these terms belong to an observational language in contradistinction to a theoretical language.[8]

This line of thought renders the famous criticisms — viz. Putnam (1966, pp. 244–248) and Achinstein (1968, pp. 85–91) — regarding the Received View's *partial interpretation* inaccurate when the *moderate* syntactic approach is at stake. The notion of *partial interpretation* is used here in the sense of attributing partial meaning to sentences,[9] e.g., physical meaning. Moreover, as argued by Suppe (1989), this particular feature of the moderate syntactical approach makes sense *only* when attached to semantic concepts such as metalanguage:

> Since it would appear that little more can be said syntactically in the way of characterizing partial interpretation, if we are to find an adequate analysis of the concept, we must turn to semantic considerations. (Suppe, 1989, p. 43)

In this sense, a physical theory allegedly assigns physical meaning (through semantics) to a purely mathematical language that is not necessarily connected to anything but mathematics. The plot thickens right here since this position corroborates our thesis enunciated previously: moderate versions of syntactic and semantic approaches converge, rather than compete with each other.[10]

Let's take this hook to move on to the analysis of the semantic approach.

3 Interpreting scientific theories II: semantic approach

The most basic aspect of the semantic approach is the identification of a scientific theory as a class of models.[11] Here, we adhere to the Suppesian set-theoretical development of models, called by Krause and Arenhart (2016, p. 11) the "hegemonic version of the semantic approach". This choice is made mainly due to the fact, as put by da Costa and French (2000), "models" are said to be structures (of one kind or another) in all these accounts, and as such this notion may configure a better path to the philosophical purposes of this study due to its generality.

Therefore, mentioning the concept of a "model", it should be assumed the acceptance of a set-theoretical entity, typically built in a set theory. In this context the usual Zermelo-Fraenkel set theory. The following sections present the *radical* and the *moderate* version of this view of semantic approaches, beginning with a more *radical* stance.

3.1 The radical semantic approach

Following Krause and Arenhart (2016, p. 11), the radical syntactic approach would assume the following three theses that scientific theories should comply: a scientific theory *is* a class of its models; it is *language-independent*; models are understood as set theoretical structures. According to radical semanticists, "scientific theories are classes of models". Traditionally, mostly due to Tarski (1956), a "model" $\mathcal{M}$ is defined as an ordered pair such as $\mathcal{M} = \langle D, \mathcal{I} \rangle$ where the domain D is a non-empty set and $\mathcal{I}$ is the *interpretation function*. Intuitively, the interpretation function $\mathcal{I}$ involves the interpretation *of a language* $\mathcal{L}$ (the object-language), which means assigning the function truth-values *in a metalanguage* trough mapping of the non-logical elements of a formal language, relating each symbol to an *element* in D, the domain of interpretation: individual *constants*, *functions* and predicate *symbols*. In this sense, to interpret a theory is to correlate a language with set-theoretical elements of a structure, in a purely formal manner — the point is that this notion of interpretation is does not go anywhere outside set theory.

More specifically, the standard textbook approach of this matter (Chang and Keisler, 1990, pp. 18–36) goes as follows: order-1 models are defined as structures in the form of the ordered pair $\langle D, \mathcal{I} \rangle$ (with the domain $D \neq \varnothing$), where the interpretation function $\mathcal{I}$ maps each one of the constant symbols c to a constant $x \in D$, each m-place function symbols F to an m-place function $G : D^m \longrightarrow D$ on D, and each n-ary relation symbols S to an n-ary relation

$R \subseteq D^n$ on D. Constants and relations on D are *extensional* concepts, and thus two relations $\{R, R'\}$ are identical if they have the same extension, that is, if $\forall x\big[(x \in R) \leftrightarrow (x \in R')\big]$, then $R = R'$.[12] However, the existence of two different relation symbols $\{S, S'\}$ with the same extension of their interpretation is possible. If a structure $\langle D, \mathcal{I}\rangle$ contains an exclusive interpretation for multiple relation symbols $\{S_i\}_{i\in I}$ that maps every S to the same R, there would be only one relation, so its image would lead to $\langle D, \{R\}\rangle$. As remarked by Hodges (1993, pp. 1–4) and Lutz (2015, p. 11), this extensional account of relations is precisely the involvement with language. These kinds of structures, formalized with a vocabulary, are called 'labeled structures'.

As famously stated by van Fraassen (1989, p. 366), however, the independence of a specific language (i.e., a syntax) should be a *fundamental* trait of the semantic view: without it, its motivation is lost when "[...] models are defined, as in many standard logic texts, to be partially linguistic entities, each yoked to a particular syntax." Thus, the requirement of *language independence* seems to seriously constrain this traditional Tarskian-like account of models, since it is precisely "yoked to a particular syntax". In fact, Halvorson (2012) argues that any appeal of language, e.g. the labeled-structure account of models, could turn the semantic approach into a syntactic approach.

Moreover, the traditional characterization of "models" via labeled structures is very restrictive. Since it quantifies over *elements* of D, it only functions when considering order-1 structures, which are not able to comprise most of the best contemporary scientific theories. Thus, when adopting models for the characterization of scientific theories, this definition presents itself as problematic. As Bueno and Krause (2007) argued, the literature on models is elusive in this aspect, and any semantic approach which attempts to characterize scientific theories needs some sort of re-conceptualization when intending to characterize contemporary physical theories such as QM. The major drawback presented by this stance is that a proper *model theory* needs to be comprised exclusively in order-1 structures: *strictly speaking, there is no model theory for order-n structures*,[13] since fundamental theorems of standard model theory (such as the Löwenheim-Skølem theorem) only holds for systems of order-1 logic.

The set-theoretical characterization of structures poses as the most common alternative for the traditional, language-dependent, account of models as labeled structures. Following this characterization, $\mathcal{M}$ is defined as a n-tuple such a $\mathcal{M} = \langle D, R_i\rangle_{i\in I}$ where D is a non-empty set and R_i stands for a *family of relations* on the elements of D. Note that R_i does not relate *only* elements of D, therefore it could give rise to order-n structures (with $n > 1$). As for the language-independence criterion, this type of structure is defined independently from a specific vocabulary when written as n-tuples containing a domain and a family of relations (comprised of functions and constants). This can be seen clearly in order-1 structures, through the definition of the algebraic structure of "group". A group G may be written as $\langle G, \circ, e, -\rangle$, where: "$G$" is a non-empty set; "$\circ$" is a binary operation (the composition function on G); "e" is the neutral (identity) element; and "$-$" is the inverse (opposite) operation.

As an example of an order-n structure there is the case of "topological space", written as the structure $\langle D, \tau\rangle$, with $\tau \in \mathfrak{P}(\mathfrak{P}(D))$, since τ is a family of subsets of D. In cases such as this, the family of relations R_i is interpreted as an *indexed structure*, so the structure could be written as $\langle D, R_0, \ldots, R_n\rangle$ and the position of the relations of D in the structure takes the role of the index, so that it could be read as $\langle D, \{R_i\}_{i\in\{1,\ldots,n\}}\rangle$. It should be noted that indexed structures contain more information than the image of an interpretation of labeled structures.

3.2 Moderate semantic approach

In the light of the foregoing, it seems reasonable to drop the "language independence" requirement from the basic traits of the semantic approach to scientific theories. Call, then, the semantic approach following this decision a *moderate* semantic approach. Is it, however, enough to fix an answer for the question of "what is a scientific theory?" in indexed structures? As it will be argued, not quite so. Consider the discussion of group theory previously presented, written as the ordered quadruple $\langle G, \circ, e, -\rangle$: is this an exhaustive answer to the inquiry on what *is* a class of groups? No. Alternatively, one can argue that a group can be represented by the triple $\langle G, \circ, -\rangle$, the triple $\langle G, e, \circ\rangle$, or, yet, the pair $\langle G, \circ\rangle$.[14] None of these structures are the same, and therefore language seems to be important here.

So there seems to be a dilemma: on the one hand, models comprise a vocabulary when identified as labeled structures which yields a not so different result from the syntactic approach (cf. Halvorson, 2012); moreover, if such a language is of order-n, then it is not covered by any model theory at all. On the other hand, models identified as indexed structures do not result in much semantics at stake, insofar as the models do not make anything true. Furthermore, considering a language-free ideal scenario, there shall be no language according to which the structures are interpreted: the semantic approach to scientific theories, then, in agreement with this fairly general characterization, would refer solely to purely formal set-theoretical structures — which are unsound point, since scientific theories must relate to empirical data in some manner (cf. Lutz, 2015, p. 14).

4 Interlude: Towards a unified approach

A fairly well-accepted settlement to this so-called "dilemma" has been presented by Craver (2008), Krause et al. (2011), Lutz (2015) and Halvorson (2016). That is precisely the recognition that issues concerning the "labeled structure" and the "indexed structure" views on the semantic approach arise only when one accepts the ideal of *identifying* scientific theories with something (e.g., these formulations). As soon as this ideal is abandoned, it becomes clear that both views are attempts to apprehend aspects of scientific theorizing, in the sense that they are distinct *representations* of scientific theories —

and thus do not attempt to provide the essence of scientific theorizing itself. In this sense, these views must not be considered as competitors, which is perhaps the major upshot of recent debate.

Following the adaptation made by Lutz (2015, sec. 5.3) of the argument presented by Halvorson (2016, sec. 2), it is possible to interpret an indexed structure (semantic) as a labeled structure (syntactic). Consider the structure A as follows: $A = \langle D, R_1 \rangle$, where $D = \{a, \langle a, a \rangle\}$ and $R_1 = \{\langle a, a \rangle\}$. Presented as such, it is not clear in this indexed structure whether R_1 describes a relation over *a set of elements* or over *a set of tuples of elements of the domain.* As a result, it is also unclear what this structure *is.* If one understands a mathematical structure solely as a set of mathematical entities endowed with mathematical relations, it is not possible to advance from this point. To push the argument a little further, consider now the structure A': $A' = \langle D', R'_1 \rangle$, where $D' = \{a, b, \langle a, b \rangle\}$ and $R'_1 = \{\langle a, b \rangle\}$. A question arises: are A and A' isomorphic structures? This information is not provided by the description. The answer lies in the *arity* of relations R_1 and R'_1, that is, in their classification as unary or binary relations. Thus, if a bijective function $f : D \to D$ and if $\langle l, m \rangle_{\in R_1}$, then $\langle f(l), f(m) \rangle_{\in R_1}$. This crucial information is not explicitly given: if the relations have the same arity, yes; otherwise, no. However, to explicitly provide information regarding non-logical terms of the structure (e.g., a mapping from the index set of the indexed structures to the elements or types of elements of the structure) *is to transform the indexed structure into a labeled structure*, since the result carries the indexed set's index taken as a vocabulary (Chang and Keisler, 1990, p. 19).[15] In other words, the *indexed structure* may be converted to a Tarskian-like structure when the index set I is defined as the non-logical vocabulary of a language, thus transforming the indexed set into a labeled set with concern to the indexation status of interpretation. Therefore, as Krause and Arenhart (2016, p. 16) argue, "[...] an isomorphism may be defined both in the presence as well in the absence of a language in which the structures are interpreted", rendering both approaches to models meaningful.

As Lutz (2015, sec. 7) states, "[...] the syntax-semantics debate was about a distinction that marks no difference", to the extent that one approach can be freely translated into another. Moreover, Krause and Arenhart (2016, p. 17) remarked that, specifically within the semantic approach, the discussion related to whether or not to include the requirement for language independence is unimportant considering that both structures (language-dependent and "independent" are convertible into one another). In fact, that is precisely what Suppes (1967) means when distinguishing between the complementary *intrinsic* and *extrinsic* approaches to theories: the former consists in the axiomatization via linguistic resources (e.g., via *labeled structures*, or *syntactically*); when no such axiomatization is viable, one can proceed with the latter by characterizing the class of models of the theory directly in set theory.

Ultimately, it seems that philosophers of science have not provided a satisfying answer to the question of "what is a scientific theory?", as its most plausible answer would be, after all, "I don't know". Although an answer concerning what a scientific theory *is in its essence* could not be provided, we may say in which ways it could be *represented for explication purposes*. In that sense, both syntactic and semantic *tools* can be employed in philosophical inquiry.

Based on the discussion presented so far, it is relevant to notice that both the syntactic and the semantic approaches are *live options* in the current practice of philosophy of science. Even if both approaches are viable, they are not considered to be equally adequate to treat scientific theories philosophically. For instance, notice that models are models of *something*: when they are models of some axiomatic, it would only make sense to call it "model" as if there is a set of axioms modeled by the structure. Then, it would be necessary to provide the axioms of the theory before modeling it in a set-theoretical structure. Is not that, however, a major shortcoming of the radical syntactic approach mentioned earlier? Recall the syntactic approach: the goal is to axiomatize a theory, by describing it from its underlying logic to its specific axioms. Therefore, this seems to be inconvenient for the syntactic approaches. Naturally, this is a simple matter when the theory in question is simple as well, such as the theory concerning the projective plane. Suppose, however, that the theory to be modeled requires several "sub-theories", which must all be axiomatized. Consider QM, which encompasses several underlying theories such as the tensor calculus, theory of differential equations, vector spaces, and so on. This would render the axiomatization theories that are more complex than those of order-1 such as topology, as Suppes (1957, pp. 248–249) states, an "unduly laborious" task, and even "utterly impractical" in the case of more complex theories such as QM.

Some authors seem to pursue a *syntactic* kind of axiomatization, relating the use of set theory to axiomatization — either focusing on its vocabulary (Worrall, 2007) or in a set-theoretical axiomatization of parts of theories (Zahar, 2004). According to Suppes (1967), this categorizes an "extrinsic approach". Krause and Arenhart (2016, p. 79) argue that this kind of approach either (*i*) cannot provide its models, in the sense that if a set-theory is supposedly *consistent*, it cannot provide its own models in the object language, but only in a metalanguage in which it is constructed. This leads to the objection that (*ii*) this fact would render axiomatization impracticable due to the need for axiomatizing its models in their entirety, which involves huge structures. It seems that not everyone would accept this kind of axiomatization from scratch, but is not entirely correct, as there are philosophers that would *not* decline this kind of axiomatization. In fact, this is precisely how da Costa and Chuaqui (1988) lead their axiomatization process. It should be remarked that both Suppes (2002) and da Costa and Chuaqui (1988) consider their approaches to be part of the so-called "Suppes' predicate", but their approach clearly differs. Krause and Arenhart (2016, chap. 5, chap. 6) present a detailed discussion on these differences, surpasses kinds of axiomatization. However, for the purpose of this paper, a brief account of their differences based on different methodological approaches to axiomatization suffices. This kind of detailed axiomatization is arguably distant from the working scientific practice; by contrast, it is fundamentally the logician's approach, which proves to be more useful to the philosopher of science than to the working scientist. Call this approach "rigid axiomatization" — which is also a *formalization* of scientific theories.

At the same time, Suppes (2002) argues that, ideally, all "step-theories" (axiomatized in the *rigid* approach) can be *presupposed* as obtainable within a set theory. This yields that only the specific axioms of the theory must be presented. The important point, then, is its set-theoretical structures, which model the theory's axioms, i.e., the *models of the theory.* Therefore, this is a *less rigid* axiomatization, yet properly semantic in the sense that the elements of the theory are defined in their satisfiability with the axioms.

Regarding the equivalence, Lutz reduces the syntactic approach to the semantic approach applying set theory in order to demonstrate a certain equivalence. Note, however, that such equivalence is made in a set theory! Thus, it is not a neutral mode of assessing an equivalence between theories. In a way, this begs the question regarding the equivalence of the two approaches. These are tacit assumptions that commonly go unnoticed but can be questioned. Something similar could be said of Halvorson, who presents this equivalence in a stronger theory, namely in a category theory. *Mutatis mutandis*, category theory faces the same problem stressed when addressing set theory: the semantic approach is already bought in advance by the authors who claim to establish an equivalence between the syntactic and semantic approaches. However, it is still possible to question whether it is conceivable to establish a theoretical equivalence from a "cosmic exile". We believe it is not.

5 Interpreting scientific theories III: physics beyond logic

So far, we have climbed a hill to see the debate in a panoramic way. Now it's time to go down to the mean sea level, to analyze how well the syntactic and semantic characterizations about the interpretation of scientific theories deal with fundamental physics. From what has been discussed so far, we have seen what it means to interpret a scientific theory according to two major trends in contemporary philosophy of science. The question now is to check whether this debate helps us to understand the current state of the art in a fundamental physical theory, such as QM. Let us take a first look at the present situation in fundamental physics. The measurement problem is, in QM, the reason why the theory needs to be interpreted.[16] Thanks to Maudlin (1995), the measurement problem can be simply put as a checklist of three simple questions:

1. Is the quantum description complete?
2. Is the quantum description linear?
3. Are there measurement outcomes?

One must say "no" to at least one of those questions in order to solve the measurement problem (which is then defined by the problematic conjunction of those three items). For now, let us take that claim at face value: QM needs to be interpreted, and to do so is to solve the measurement problem. What then? No correspondence rules, structures, or models. Just physics. Seems odd, ain't it?

It is fair to state that almost no one engaged in work on the "interpretation" of quantum theory thinks they are trying to settle the sorts of questions addressed in the other debate, or vice versa. However, there are several approaches to quantum theory that conflates theory and interpretation. Let us see two cases: textbook QM and recent approaches to the axiomatization of QM via Suppes predicates.

5.1 Textbook approaches

Here is a standard textbook approach to the axiomatization of QM, mainly due to the work on von Neumann (1955) in this field. The following is based on Cohen-Tannoudji et al. (1991, chap. 3). QM is obtained with five basic axioms (the notions of "state", "system", and "observable" are taken as primitive). We should acknowledge that the presentation of the axioms is given in a very informal way, as it serves mainly to illustrate the point that we are trying to make, viz., that textbook approaches to the axiomatization of QM axiomatizes an *already-interpreted* quantum theory. Often we find a presentation of the axioms that goes like this.

1. Physical systems are defined by a vector in a Hilbert space $\mathcal{H}$;
2. Observables are measurable physical quantities described by a self-adjoint operator $\hat{A}$, and such operators are the eigenvalues;
3. A probability is assigned to measurement outcomes in the form of the Born rule: $\text{Prob}(a_k)_{\hat{A}} = |a_k|^2 = |\langle a_k|\psi_k\rangle|^2$;
4. A *dynamics* to unmeasured systems (usually in the form of the Schrödinger equation, in which its result may be a vector sum called 'superposition'):
$$i\hbar \frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle, \tag{1}$$
5. The *projection postulate*, in which the measurement outcomes are described as reduction superposed states in the form of
$$\sum_{i=1}^{n} a_i\, |\psi_i\rangle \longrightarrow |\psi_k\rangle_{k\in i}\,.$$

The two first axioms associate primitive concepts with mathematical entities; the third one defines the statistics for measurement outcomes; the fourth defines the dynamics for undisturbed systems (*i.e.*, for systems that are not subject of measurement); the fifth defines the dynamics for disturbed systems (*i.e.*, for systems that are subject of measurement). Let us quote a presentation of this axiom by Cohen-Tannoudji et al. (1991, p. 221):

> If the measurement of the physical quantity $\mathcal{A}$ on the system in the state $|\psi\rangle$ gives the result a_n, the state of the system immediately after the measurement is the normalized projection $\frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$, of $|\psi\rangle$ onto the eigenstate with a_n. (Cohen-Tannoudji et al., 1991, p. 221)

We find also something similar, albeit with a more detailed projection algebra, in Messiah (1961, pp. 260–263). In d'Espagnat (2003, pp. 46–50) we can read that the collapse is a *theorem* of the presentation of QM. In fact, d'Espagnat (2003, p. 50) acknowledges that "[t]his [collapse] theorem is often stated as an independent postulate. We see here that it can be proved on the basis of the other quantum rules".

As Auyang (1995, p. 21) stresses, in order to make empirical statements, the concept of "measurement" represents a "phenomenological statement" about actual (laboratorial) measurement outcomes, rather than a statement buildable within the theoretical apparatus constructed so far. Goldstein (2009, p. 501) emphasizes that the *projection postulate* (or "collapse" or "measurement") plays this phenomenological role in standard quantum mechanics:

> In standard quantum theory, the projection postulate plays a crucial but controversial role: crucial, because standard quantum theory makes contact with physics and the results of experiments via the measurement axioms of quantum theory, the most important of which is the projection postulate; and controversial, because the projection postulate appears to conflict with Schrödinger's equation. (Goldstein, 2009, p. 501)

We can see an interesting parallel between the projection rules and the positivists' rules of correspondence: it is only via this phenomenological statement that QM makes reference to physical concepts. But notice that the projection rule is not general enough to encompass the multiplicity of interpretations of QM (more on that later). For now, it suffices to say that, unlike the first four axioms, there is little consensus about the relationship between them and the fifth one, for its conjunction is a problematic issue known as the 'measurement problem'. This problem lies at the heart of QM, and it is usually seen as a dividing mark between *interpretations* of QM. As remarked by Lewis (2016, p. 50), without an answer to this, QM is *trivialized*; Ruetsche (2018, p. 296) goes even further, calling that problem an "empirical contradiction". We *do not* think that a formal contradiction is involved, even in standard QM. A closer look at the measurement problem is needed to explain such a view since many assumptions were made, and it is not clear whether there is a formal manner to state this so-called contradiction. Regarding *informal* proof, Esfeld (2019, p. 223), claims that Maudlin's taxonomy became the standard way of stating the measurement problem in QM:[17]

> If the entire system is completely described by the wave function [*1*], and if the wave function always evolves according to the Schrödinger equation [*2*], then, due to the linearity of this wave equation, superpositions and entangled states will, in general, be preserved. Consequently,

a measurement of the cat will, in general, not have a determinate outcome [...]. (Esfeld, 2019, p. 223).

It is essential, however, to (at least apparently) determine measurement outcomes [*3*], hence the informal inconsistency. In order to state the measurement problem more precisely, consider the following case. Suppose that one wants to measure a position observable of a physical system, denoted as '$\hat{A}$', by means of a macroscopic apparatus denoted as '$\hat{M}$'. This will be done, in principle, through the interaction of these two physical systems. Suppose, further, that the initial state of $\hat{A}$ in t_0 is $|\psi_0\rangle = \sum_i c_i|\alpha_i\rangle$ and that the initial state of $\hat{M}$ is $|\varphi_0\rangle$, meaning the apparatus presents no reading, i.e., it is in the *reset button*. For $\hat{M}$ to fulfill its purposes as a measuring device, it must be prepared in a certain way in which it is susceptible to measure some quantities of the system of interest $\hat{A}$, to yield an eigenvector of $\hat{A}$. However, by means of $\hat{U}$ only, the state of the composite system $\mathscr{H}_{\hat{A}} \otimes \mathscr{H}_{\hat{M}}$, represented by

$$|\psi\rangle \otimes |\varphi_0\rangle = \left(\sum_{i=1}^{n} c_i|\alpha_i\rangle\right) \otimes |\varphi_0\rangle \tag{2}$$

evolves to

$$\hat{U}(t)\left[\left(\sum_{i=1}^{n} c_i|\alpha_i\rangle\right) \otimes |\varphi_0\rangle\right] \to \hat{U}(t)\sum_{i=1}^{n} c_i\Big(|\alpha_i\rangle \otimes |\varphi_i\rangle\Big) \tag{3}$$

for any $t \neq 0$. Remarkably, this result is not an eigenstate of either $\hat{A}$ or $\hat{M}$, meaning that the measurement process must be *something else* other than the application of $\hat{U}$. Following the notation adopted so far, the measurement process is described in the form

$$\sum_{i=1}^{n} c_i\Big(|\alpha_i\rangle \otimes |\varphi_i\rangle\Big) \to \Big(|\alpha_w\rangle \otimes |\varphi_w\rangle\Big), \tag{4}$$

with the probability given by the statistical algorithm stated in $\mathcal{P}_3$.

To explain *why* and *how* this change in the dynamics occurs is one of the central issues of approaches to the measurement problem, while the attempts to overcome these changes are subject to the so-called *interpretations* of QM. The central point of the matter is, then: how can one reconcile what the theory predicts with what is observed? The answer is: "by interpreting it!"

However, what does it mean to interpret QM, exactly? Notice that the projection postulate is implicit in this textbook formulation of the measurement problem. We wish to stress that the point of departure for interpreting QM is already being taken from a specific *interpretation* of QM! We will take a closer look at this in the next section 6.

5.2 Suppes predicates

Now let us look at Krause and Arenhart's (2016, sec. 5.8.1) presentation of an "axiomatization of non-relativistic quantum mechanics" via Suppes predicates. According to Krause and Arenhart (2016, pp. 105–107), a quantum-mechanical structure is a tuple of the form:

$$\mathrm{QM} = \left\langle S, \{\mathscr{H}_i\}, \{\hat{A}_{ij}\}, \{\hat{U}_{ik}\}\mathscr{B}(\mathbb{R}) \right\rangle \tag{5}$$

where S is the set of physical systems, $\{\mathscr{H}_i\}$ the set of Hilbert subspaces, $\{\hat{A}_{ij}\}$ the set of self-adjoint operators, $\{\hat{U}_{ik}\}$ the set of unitary operators, and $\mathscr{B}(\mathbb{R})$ is the collection of Borel sets over the set of real numbers. And then they present seven postulates (or axioms) of such structure. The first one relates the physical systems with Hilbert (sub-)spaces. The second one deals with superpositions. The third defines self-adjoint operators and the fourth deals with eigenvalues. The 5th axiom gives the Born Rule, the 6th presents the Schödinger equation through the unitary operators, and the 7th postulate presents collapse.

But notice: by including the collapse between the axioms, they are implicitly rejecting Maudlin's item 2. Thus, they commit themselves to an *interpretation* of QM. So this cannot be the quantum theory that is interpreted, in the sense of solving the measurement problem; the above-mentioned structure is not the quantum theory that gives rise to the measurement problem at all. This axiomatization is not an axiomatization of quantum theory *per se*, but an axiomatization of an *interpretation* of QM. So, the models are not standing for an interpretation in this case, but they are *representing* an *already interpreted* structure.[18] But the term "interpretation" here gains another sense, which differs from the uses in the syntactic and semantic approaches. Thus it is clear that the axiomatization presented by Krause and Arenhart (2016) have its own models and an *interpretation function*, and so on, but this is *not* related with the solution of the measurement problem. That is the *sui generis* sense of the word "interpretation" used in the context of QM.

5.3 Towards a new approach

Since some crucial aspects of the previous debate are needed to discuss the efforts of interpreting QM, an assessment of the matter shall be provided, even if in passing. The syntax is needed to better specify some concepts, as well as semantics, in order to interpret scientific theories — recall that a purely syntactical approach is rigid enough to preclude this possibility.

As discussed above, interpreting a scientific theory (according to the syntactic and semantic views) is an issue mainly concerned with concepts in logic. In the syntactic approach, to interpret is to connect an axiomatic system with empirical data. Take as a rough example the following: suppose a structure $\mathscr{S} = \langle \mathcal{F}, \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{F}$ is the set of formulas, $\mathcal{A}$ are the axioms, and $\mathcal{R}$ are the rules of inference. To interpret $\mathscr{S}$ is to add a "physical counterpart". In order to do so, consider the language $\mathcal{L}_{\mathscr{S}}$ of the system $\mathscr{S}$; consider also the addition of new symbols to its primitive alphabet (which may be denominated after 'theoretical terms'), as well as the addition to $\mathscr{S}$ of a set (not necessarily finite) of "specific axioms" of the new system $\mathscr{S}^*$. The rules in $\mathcal{R}$ remain the same. Evidently, there is now an extended axiomatic system, which may be denominated as a 'theory'. In this manner, $\mathscr{S}^* = \langle \mathcal{F}^*, \mathcal{A}^*, \mathcal{R} \rangle$, where $\mathcal{F}^*$ represents the new set of formulas obtained by the application of the grammatical rules of $\mathcal{L}_{\mathscr{S}}$ to the finite sequence of the extended alphabet, and $\mathcal{A}^*$ is the union of $\mathcal{A}$ with the new specific axioms. Thereby, one may go from mathematics to physics.

As stated by Wallace (2012, p. 17), to do so is to surpass the "bare formalism" to an empirically adequate theory, which, in the specific case of QM, is the introduction of the concept of measurement: "[…] if we are to extract empirical content from the mathematics, we seem to have to introduce the notion of measurement as a fundamental concept". Such introduction leads to the *measurement problem* — we'll come back to that later.

As for the semantic approach, all that one is required to do in order to *interpret* a physical theory such as $\mathcal{A}^*$ is to find the models in which its axioms $\mathcal{A}^*$ are true. But is that the case with regard to QM? *In case* there was a single method to axiomatize QM, then to interpret QM would mean finding the models in which its axioms were true. However, were it not the case, both the syntactic and the semantic approach present a relevant issue, and an *interpretation* of QM does not have the same meaning as an interpretation in logic.

Unfortunately, this seems to be precisely the case. Different interpretations of QM have different axioms. For the sake of argument, take as an example only two "families" of interpretations of QM: the collapse interpretations and the no-collapse interpretations. They have different axioms (i.e., the collapse). How to proceed, then? From now on, we discuss our guess as to the correct way to proceed in this matter. What is usually called "QM" has several interpretations. There is a wide philosophical debate about interpretations of axiom theories. The concept of interpretation cannot be used in the same way when dealing with QM. The concept of "interpretation of QM" is not yet properly defined, neither being a correspondence between theoretical and empirical aspects of the theory, (as it is in the syntactic approach), nor a function that crosses domains of a structure (as it happens in the semantic approach). The case of QM is, as we already mentioned, *sui generis* in this sense, which does not conform to traditional characterizations about scientific theories. Therefore, it is clear that it should not follow the parameters of the concept of 'interpretation' based on such approaches. In other words, logic in no way an *endgame* to the discussion about the interpretation of QM — not, at least, in the present state of the debate. From a philosophical point of view, the word 'interpretation', as commonly used in QM, is an unfortunate choice.[19]

It is a settled fact that QM can be formulated in several ways (cf. Styer et al., 2002). Since the seminal work of von Neumann (1955) on the axiomatization of QM, developments and debates on QM employ, predominantly, the Hilbert space formulation.[20] However, when adhering to such formulation, one can still be bothered with the problematic question: *what is QM?*

It is a widespread belief that *the formalism* of QM can be *interpreted* in numerous ways (Jammer, 1974; Lewis, 2016), as if a single theory, *The* QM — with a capital "T" — exists. There are several axiomatic approaches to this so-called Quantum Theory (cf. Muller, 2003; Krause and Arenhart, 2016), and from this Theory, various interpretations emerge as solutions to the measurement problem (Friederich, 2014, chap. 2). This was recently criticized by Acuña (2021) and Wallace (2020b,a) as a categorical mistake that most approaches to the "interpretation of QM" leads to (see Ney and (Eds.) for a contemporary example of this). As Ćirković (2005) stresses, different "interpretations" of QM, such as collapse and no-collapse interpretations, yield *different experimental outcomes*, and, therefore, be considered *different full-blooded quantum theories*. Indeed, as Ćirković (2005) stated, there are many thought experiments that, in principle, yield different results depending on whether one accepts or abandons the axiom of collapse. These results cannot, in fact, be currently tested in practice, contributing to the unfeasibility of opting for a theory.

The so-called 'interpretations of QM', then, are claimed not to consider the same set of axioms (regarding the collapse axiom) neither the same set of equations (regarding the Schrödinger equation), so that interpretations do not depart from the same point in order to "interpret" a single theory. Along these lines, Maudlin (1995, p. 7) stresses that "[a]ny real solution [to the measurement problem] demands new physics". This explains the followed statement made by Sklar (2003, p. 281): "I doubt that one can draw any principled line between replacing a theory and 'merely interpreting' it". Ćirković (2005, pp. 821–822) argues as follows: theory T and T' are different theories if at least one of the three following criteria is fulfilled:

1. T predicts new phenomena, nonexistent in T', subject to empirical verification (even if only in principle);
2. The formal parts of T and T' are different;
3. T and T' differ in the description of *observed* phenomena.

Undisputed cases, such as spontaneous-collapse theories (Ghirardi et al., 1986) are set aside from this discussion.[21] The most difficult cases, represented by collapse and no-collapse versions of QM, constitute the cases of interest here. It is safe to state that these cases do not satisfy item 1. Item 2 may be disputed, as collapse and no-collapse versions of QM can be placed in "external" descriptions as different structures, with different axioms (e.g., one structure *with* the collapse axiom, and another structure *without* it). Since other mathematical aspects of both approaches (e.g., the equations) remain the same, it becomes easy to see how item 2 is traditionally considered unfulfilled in this case. Although we disagree with such an assessment, we will not put it in dispute at this moment, in accordance with the standard practice. Therefore, the debate shall move onto item 3.

Thus far, collapse and no-collapse approaches to QM are empirically indiscernible, meaning that both lead to the same set of laboratorial (i.e. empirical) consequences. However, what to say of *conceivable* experiments, even those not forthcoming in the near future? As Ćirković (2005, sec. 3) emphasizes, there are several thought experiments available in the literature that should not go unnoticed.[22] Considering that *thought experiments* demonstrate experimental differences according to the adoption of collapse or no-collapse approaches to quantum phenomena, item 3 could be considered the epistemologically weakest of the three items. Recently, Dürr and Lazarovici (2020, p. viii) stressed, albeit with other reasons in mind, that the word "interpretation" is a weak one when referring to QM.

> A poem is interpreted if you want to elicit some deeper meaning from the allegorical language. However, physical theories are not formulated in allegories, but with precise mathematical laws, and these are not interpreted, but analyzed. So the goal of physics must be to formulate theories that are so clear and precise that any form of interpretation—what was the author trying to say there?—is superfluous. (Dürr and Lazarovici, 2020, p. viii)

The situation seems to amount to one's definition of 'quantum theory', arising the following dilemma. First off, if the accepted definition is excessively *narrow*, one is unable to comprise several theoretical programs for investigating the phenomena on a quantum level, commonly referred to as QM. This appears to pose a pragmatical drawback for the narrow definition of 'theory', as numerous working physicists inclined to different solutions of the measurement problem could work in various subjects without ever disagreeing, even without realizing that they are working with distinct physical theories.

On the other hand, if one's definition is too *wide*, one may substantially conflate different nuances of several theoretical approaches to quantum phenomena, such as different predictions of experimental outcomes, or different ways of calculating the motion of a quantum object. In this sense, QM presents a unique case in which the theory's very axioms depend on a choice of interpretation.

This paper proposes a characterization of a "quantum theory" considering a modification of the semantic approach, that is, by stating basic requirements in order to obtain the theory's specific axioms, offering a basic formulation of QM that can serve as a common ground for several theoretical programs on the study of quantum phenomena. Collapse (von Neumann, 1955) and no-collapse (Everett, 1957) theories are the examples that serve as focuses of this discussion. We should mention that Wallace (2020b) recently made a similar suggestion, regarding the Hilbert-space formulation of QM as a theoretical framework within which concrete quantum theories can be expressed. Our proposal goes in the same direction, differing in the ways that such framework is taken to be, and how it is expressed by so-called "interpretations".

The offering of a basic scheme, considering convenient set-theoretical tools, allows for further definition of the differences among several approaches to quantum phenomena. Furthermore, the additional assumptions traditionally made upon such a basic scheme clarify the modifications resulting from each response to a foundational problem concerning the basic structure in order to obtain the axioms of the theory at stake.

This basic structure is here called "QM^{bas}". With these efforts, this work seeks to advance towards a more accurate account of what QM could be and what it means to interpret it. In our terms, QM is formed by a basic mathematical structure QM^{bas}. To interpret QM, then, means to instantiate the General Principles of QM^{bas} (purely formal) into specific (physical) axioms, interpretations of QM^{bas} are quantum theories — in contrast, QM^{bas} is a purely formal system, which is what we call the "isolation problem". By doing that, one constraint that the specific axioms of interpreted quantum theories should obey is the measurement problem. In this sense, the notion of "interpretation" of QM is introduced here as the very axiomatic structure of each subsequent "quantum theory" that solves the measurement problem.

The idea that interpretations are indeed different quantum theories has been proposed before (Ćirković, 2005), and the description of the measurement problem and its potential solutions are also not new (Maudlin, 1995). The novelty of this paper is to show precisely what is interpreted, and how it is done. Our guess, as we argue, is that this is done with specific modifications to a basic structure of general postulates. Moreover, we argue that uninterpreted QM is not yet a physical theory. This was proposed before by de Ronde (2016), who calls QM a "proto-theory" and by Wallace (2020b), whose proposal is close to ours. Just as Wallace (2020b), we maintain that an "uninterpreted QM" (here understood as "QM^{bas}") is *not* a physical theory yet — it is a purely formal scheme that furnishes the grounds on which physical theories (quantum theories) are constructed.

6 Presenting QM^{bas}

We present a semantic characterization of QM, briefly explained as follows.[23] Assuming that a semantic axiomatization can be done *at least in principle*, the adequate manner to axiomatize parts of present-day physics, such as the Standard Model of particle physics, is still unknown. Moreover, it should be noted that this study works exclusively at the broader, informal level, for simplicity of presentation. For the sake of precision, if the reader thinks necessary, the Zermelo–Fraenkel set theory with the axiom of Choice can be assumed. "Axiomatize", then, simply means that non-trivial assumptions are at stake. Therefore, this paper does not, of course, present a *full* axiomatization of QM from scratch. As we mentioned in section 4, this would be a (Herculean) task.[24]

A major problem of presenting QM according to the semantic approach is the fact that a quantum theory depends on its axioms. Simultaneously, the theory's axioms largely depend on the chosen interpretation, since QM can be presented with different axioms motivated by a given choice of interpretation. Consequently, as previously stated, the frontiers between replacing a theory and interpreting it are blurred, and this seems to be a *sui generis* case of QM: if the focus were to lie only on the axiomatic structure of QM, it could be presented with *different* axioms, resulting in *different QMs*, without a common starting point.

For instance, von Neumann (1955) presents QM with the so-called "collapse axiom", whereas Everett (1957) drops this axiom in his approach. As Ćirković (2005) argues, however, adopting collapse axiom entails *in principle* that a particular set of experimental predictions divergent from those in which such axiom is dropped.

Thus, to present an axiomatic structure for each quantum theory does not seem to result in a path towards a unified view on QM. The discussion here presents, then, precisely this common starting point. For instance, a recent effort to present an axiomatization of QM conducted by Krause and Arenhart (2016, sec. 5.8.1) is also committed with collapse as an axiom of QM, thus appearing to be an axiomatization of an *interpretation* of QM, and not of QM *per se*. In order to encompass a wider variety of approaches to QM, we propose a different definition of QM with an emphasis on a purely formal system. So, instead of presenting its axioms, something similar to the role played by axiom schema in systems of logic is attempted. In essence, *axiom schemata* generalize the notion of axiom by stating the rules by which axioms are generated. Here, the set of "axiom schema", in QM^{bas} is labeled as "General Principles", denoted as $\mathcal{P}$. These General Principles may establish a common ground that can be instantiated in specific axioms of each quantum theory. As stated above, we conceive each formalized interpretation of QM as a physical theory (Maudlin, 1995; Ćirković, 2005), in order to allow for the examination of theories as different extensions or formulations derived from the same fundamental General Principles: hence, a basic scheme for QM.

In our proposal, modifying the semantic approach, to present QM is to present its (specific) *General Principles*. In this way, our definition of QM is a *basic scheme* of common ground to several independent research programs towards quantum phenomena, known as 'QM'. Each General Principle, in its turn, can be instantiated as a theory's specific axiom.

With these instruments in mind, the basic scheme of QM labeled "QM^{bas}", is now presented. It should be clear that there is no claim that this structure is adequate for all cases; rather, limited cases of the standard Hilbert-space formulation of QM are being considered, hoping to extract some philosophical lessons from it. For instance, only pure states and observables with discrete, non-degenerate spectra are being considered. Following the previous discussion, it is possible, then, to elaborate a structure that furnishes the tools for representing a QM^{bas} as a scheme for the construction of quantum theories, based on the standard (orthodox) formulation of QM. So, QM^{bas} is a structure presented as a triple of the form:

$$\mathrm{QM}^{bas} = \langle \mathcal{F}, \mathcal{P}, \mathcal{R} \rangle \tag{6}$$

where:

1. $\mathcal{F}$ is the set of formulas of the language of QM^{bas}. $\mathcal{F}$ is obtained from a basic language $\mathcal{L}$ consisting of a primitive vocabulary and rules of formation[25] of well-formed formulas or expressions — which in turn are finite sequences of such symbols. So, in $\mathcal{F}$ there is a set of purely mathematical, uninterpreted symbols, which composes the formal (both logical and non-logical) *vocabulary* of the theory. Many of these symbols acquire their meaning in standard mathematics (such as standard functional analysis (Farenick, 2016)), which is where they are defined; nevertheless, some symbols are specific of QM, such as the *bra-ket* notation (known as Dirac's notation), in which the position of the *bras* and *kets* are meaningful. For instance, $|\cdot\rangle$ is a vector; $\langle\cdot|\cdot\rangle$ is a scalar product, and so on.
2. $\mathcal{P}$ is the set of General Principles of QM^{bas}. The General Principles of $\mathcal{P}$ are stated in the language of $\mathcal{F}$. Since we are dealing solely with the Hilbert-space formulation of QM, the mathematical axioms of QM^{bas} are the axioms of standard functional analysis, while the logical axioms and rules of inference of QM^{bas} are the axioms and the rules of classical logic. There is an ongoing debate raised from the seminal work of Birkhoff and von Neumann (1936), to whom the logic of QM *is not* the classical logic. This discussion will not be covered here. Rather, following Dalla Chiara (1977, 1981), we accept that the role of the so-called "quantum logic" is *not* played within the domain of rules of inference. As a consequence, Dalla Chiara (1981, p. 337) argues, the general logic of QM is not quantum logic, but classical logic; quantum logic is to be introduced as "[…] a particular physical sub-language of [QM]". The logic is assumed in the background. Thus, listing rules of inference is not necessary there is the desire to introduce some rule as one of the principles of QM^{bas} — which is not the case. Therefore, the essential matter to be stated here relates to the specific General Principles of the theory, which will be *informally* presented. Again, dealing with the standard Hilbert-space formulation of QM implies a commitment to a specific set of the theory's General Principles. The General Principles of $\mathcal{P}$ are:

   $\mathcal{P}_1$ [*Hilbert Space*]: A Hilbert space $\mathscr{H}$ is a linear space with inner product, complete in relation to the norm introduced by the inner product comprising a set of *vectors* denoted as $\{|\psi\rangle, |\varphi\rangle, |\psi_1\rangle, \ldots\}$. The field is usually taken to be that of complex numbers, where elements are termed "scalars" and denoted by Latin lower-case letters, occasionally with indexes. When $\mathscr{H}$ is infinite-dimensional, it is assumed to be separable (i.e., it has an enumerable orthogonal basis).

   The states of quantum-mechanical systems S are represented by vectors $|\psi\rangle$ in a complex, infinite-dimensional, separable Hilbert space $\mathscr{H}$. A pure quantum state $|\psi\rangle$ is a summary of the physical characteristics of S in a specific instant of time t. The description of S employing $|\psi\rangle$ consists of constant characteristics (such as mass, charge, spin, etc., of the system) and variable characteristics changing over time. A state of a quantum system can be represented by a unitary vector $|\psi\rangle$ (also called 'state vector'), which norm is unity, up to a phase factor. If $|\psi\rangle$

represents a state, then $e^{i\theta}|\psi\rangle$ also represents the same state, where θ is the phase factor (an arbitrary number).

The set of all states permissible for a quantum system to assume is theoretically represented by the concept of "state space", a complex $\mathscr{H}$. $|\psi_i\rangle$ and $|\psi_j\rangle$ are orthogonal if $\langle\psi_i|\psi_j\rangle = 0$ and orthonormal if $\langle\psi_i|\psi_j\rangle = \delta_{ij}$, where $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$.

Vectors can be represented as a linear combination (sum) of other vectors. In the same sense, a state can be represented as a linear combination of other states. A set of vectors $|\alpha_i\rangle$ forms a basis of $\mathscr{H}$ if every vector in $\mathscr{H}$ can be written as a linear combination of its members and this set of vectors is linearly independent. So $|\psi\rangle$ can be written as a set of basis states $\{|\alpha\rangle\}$ in the form:

$$|\psi\rangle = \sum_i \langle\alpha_i|\psi\rangle|\alpha_i\rangle = \sum_i c_i|\alpha_i\rangle, \tag{7}$$

where $c_i \in \mathbb{C}$ are the *Fourier coefficients* $c_i = \langle\alpha_i|\psi\rangle$, where the basis is composed by orthonormal vectors. According to the theorem of Gram–Schmidt, every vector space with an inner product has an orthonormal basis. This is the *superposition principle*. Intuitively, the *sum* of quantities *of the same type* is also a quantity of that same type. Thus, as the sum of two lengths is a length, the superposition principle asserts that the sum of states of a quantum system is a state of such a quantum system.

$\mathcal{P}_2$ [*Quantization Algorithm*]: The elements of set A, termed "observables", are represented by self-adjoint operators in $\mathscr{H}$. The quantization algorithm introduces a set of basis states in which the states of observables can be revealed upon measurement.

A self-adjoint operator is a linear transformation of a Hilbert space $\mathscr{H}$ into itself, and its spectrum consists only of real numbers. The self-adjoint operator maps one state into another. Thus, a state $|\alpha_i\rangle$ for an observable $\hat{A}$ is written as $\hat{A}|\alpha_i\rangle = a_i|\alpha_i\rangle$. The states $|\alpha_i\rangle$ are called the *eigenstates* of $\hat{A}$; they are invariant under the operation of $\hat{A}$, as $\hat{A}$ multiplies the state $|\alpha_i\rangle$ by a numerical factor a_i. From the set of eigenstates $\{|\alpha_i\rangle\}$, in the non-degenerate case, it is possible to obtain a basis of $\mathscr{H}$, such that any state $|\psi\rangle$ can be expressed by a linear combination called 'superposition of states'. Since the interest here lies in observables with *non-degenerate spectra*, each eigenvalue is associated with a single eigenstate.

$\mathcal{P}_3$ [*Statistical Algorithm*]: The statistical algorithm does not mention the probability of a state *to have* a specific eigenvalue; contrarily, probability and eigenvalue are concepts related to measurement outcomes. The concept of probability is used since the observed result of a single pure state is at stake.

The squared norm of the Fourier coefficients $c_i = \langle\alpha_i|\psi\rangle$ is a numerical factor $|c_i|^2$ which gives the probability for a measurement made upon an observable S to yield the eigenvalue a_i when the system is in the eigenstate $|\psi\rangle$. Therefore, for the discrete and non-degenerate state:

$$\mathrm{Prob}_S^{|\psi\rangle}(a_i) = |\langle\alpha_i|\psi\rangle|^2 = |c_i|^2 \tag{8}$$

The statistical algorithm states that the measurement values are probably found in an interval of $\mathbb{R}$. This is frequently called the "Born rule".

$\mathcal{P}_4$ [*Dynamics*]: The motion of quantum states through time is governed by the unitary operator $\hat{U}$, which maps the states from $|\psi(t_0)\rangle$ to $|\psi(t_{x\neq 0})\rangle$ in the form of $\hat{U}(t)|\psi_0\rangle = |\psi(t)\rangle$. Such temporal evolution is represented by linear, differential equations of motion. The linearity feature of $\hat{U}$ implies that $|\psi_1\rangle$ evolves to $|\psi_1'\rangle$, and $|\psi_2\rangle$ evolves to $|\psi_2'\rangle$, then $a|\psi_1\rangle + b|\psi_2\rangle$ evolves to $a|\psi_1'\rangle + b|\psi_2'\rangle$.

3. $\mathcal{R}$ is the set of the inference rules of QM^{bas}, that is, a collection of relations between finite sets of formulas and formulas. Each relation has an arity $n > 0$, and are inference rules of QM^{bas}. Following common practice, we assume that the rules of inference in $\mathcal{R}$ are the standard rules of inference of classical logic.

6.1 The problems of QM^{bas}

There is, however, a crucial problem with QM^{bas}, which prevents even the most instrumentalism-inclined theoretician from leaving it at that. We call it the "isolation problem".

Isolation problem: QM^{bas} is not a physical theory, inasmuch as it does not mention physical systems or physical observables.

The name of the problem is after Carnap (1966, p. 237), who stated that "[a] postulate system in physics cannot have, as mathematical theories have, a splendid isolation from the world". Here's the thing about QM^{bas}: it is a kind of "view from nowhere"[26] in the sense that it is not committed to a particular quantum theory such as its contenders presented in section 5. But the thing is that the view also *to* nowhere! This is the price of isolation.

So we need, as Auyang (1995, p. 21) stated, a "[...] phenomenological statement that can be empirically verified". This, in turn, leads to a very known consistency constraint that all interpretations of QM^{bas} must obey. Traditionally, the introduction of such a "phenomenological statement" leads to the measurement problem, to which interpretations of QM are essentially *responses* to (Friederich, 2014, chap. 2). Recall Maudlin's taxonomy briefly introduced in section 5, now defined as the conjunction of assumptions added to the General Principles ($\mathcal{P}$) and the phenomenological statement:

1. Completeness: The (pure) state vector $|\psi\rangle$ gives a *complete description* of S ($\mathcal{P}_1$)
2. Linearity: The state vector $|\psi\rangle$ is *always* governed by a linear dynamics ($\mathcal{P}_4$).

3. Unicity: Measurement results always have single state vectors as outcomes (phenomenological statement).

While General Principle $\mathcal{P}_4$ states that the description of S is governed by $\hat{U}$, the phenomenological statement determines that the description of S *is not* governed by $\hat{U}$ (but, at best, by a statistical algorithm). Moreover, $\mathcal{P}_1$ states that the description is complete. Therefore, both General Principles, when considered jointly, *seem* to contradict each other. In this sense, the non-trivial role played by interpretation is precisely that of accounting for it: to save the theory's very consistency.

So things cannot be left as it is. QM^{bas} is hardly what QM is, not even to the most shut-up-and-calculate kind of Copenhaguist (cf. Mermin, 2004)! Its value is purely heuristic as a faithful point of departure from which new quantum theories are built upon. Moreover, the phenomenological statement should be addressed by each interpretation of QM^{bas}, e.g., it can be represented by a projection postulate or collapse (von Neumann, 1955), by a branch-recognition process (Everett, 1957), and so on.

Let us see how this unfolds.

7 Interpreting scientific theories IV: building scientific theories

Empirical theories must have an empirical subject matter, this is fairly undisputed. In the case of physical theories, such as QM, physical systems must be taken into account. This is a reason why QM^{bas} is not a physical theory. But even if one grant "physicality" to its postulates, the consistency problem should be addressed. When one does that, one ends up with the measurement problem — which is frequently cashed out in terms of the problem of collapse, as we saw in section 5. In order to move away from such misunderstandings, the notion of "interpretation" put forth in this section is presented as follows.

According to the reasons offered so far, we can conclude that an interpretation of QM must provide: *i*) a solution to the isolation problem, which is to transform QM^{bas} in a physical theory; *ii*) a solution to the measurement problem, which a solution requires the refusal of at least one of the three assumptions mentioned earlier, made in addition to the General Principles and the phenomenological statement. In the language employed thus far, to solve the measurement problem is, therefore, to *instantiate* the General Principles $\mathcal{P}$ of the structure QM^{bas} in equation 6 in specific axioms $\mathcal{A}$. A solution requires the *modification* of the elements of $\mathcal{P}$ or the modification of the phenomenological statement, and the examples in literature are numerous (Jammer, 1974). In doing so, the specific axioms $\mathcal{A}$ are physical axioms. An interpretation, in this sense, is what transforms a purely formal scheme (QM^{bas}) into *full-blooded physical theories*. To cope with Maudlin's taxonomy, three examples are analyzed:

i) The *pilot-wave* interpretations (Bohm, 1952; Bohm and Hiley, 2006) rejects assumption 1 according to the General Principle $\mathcal{P}_1$, and instantiates the

General Principle $\mathcal{P}_4$ in the axiom of dynamics with hidden variables in the differential equations of motion, thus originating the pilot-wave quantum theory QM_{pil}.

ii) The standard *collapse* interpretation (von Neumann, 1955), that rejects assumption 2 in the General Principle $\mathcal{P}_4$ and instantiates the phenomenological statement in the axiom of the state vector collapse, thus originating the collapse quantum theory QM_{col}.

iii) The *branching* interpretations (Everett, 1957), which rejects assumption 3 in the phenomenological statement and instantiates it in the axiom of the branching process, thus originating the branching quantum theory QM_{bra}.

7.1 Pilot-wave quantum theory

Bohm's (1952) pilot-wave solution to the measurement problem is presented as follows. It modifies (or *interprets*) QM^{bas}'s structure, instantiating some of its General Principles in what are called specific axioms of *pilot-wave quantum theories* by denying assumption 1. Therefore, a pilot-wave quantum theory QM_{pil} is a triple:

$$\text{QM}_{pil} = \langle \mathcal{F}, \mathcal{A}_{pil}, \mathcal{R} \rangle \tag{9}$$

where:

1. $\mathcal{F}$ is the language of QM_{pil}, similar to QM^{bas}'s language, with the introduction of the quantum potential Q in $\mathcal{F}$;
2. $\mathcal{A}_{pil}$ are the specific axioms of QM_{pil} (i.e. the instance of the set $\mathcal{P}$ of QM_{pil}). The list of $\mathcal{A}_{pil}$ is:

$\mathcal{A}_{pil1}$ [*Configuration space*]: The same as $\mathcal{P}_1$, except for the following remarks. The set S represents physical systems. Quantum systems are described in a $3N$-dimensional configuration space $\mathscr{Q}$ (not to be confused with the quantum potential Q below), not in a Hilbert space $\mathscr{H}$, where N corresponds to the number of particles in the system; the standard $|\psi\rangle$ wave-function/quantum-mechanical description is not complete, but is supplemented with extra parameters (see $\mathcal{A}_{pil4}$ below).

$\mathcal{A}_{pil2}$ [*Quantization Algorithm*]: The same as $\mathcal{P}_2$, except with the following remarks. The elements of set A are *physical* observables. Observables embody quantum-dynamical variables (position, *momentum*, non-relativistic spin, and so on), and can be *incompatible* in the sense of the theoretical impossibility of simultaneously obtaining the values of two incompatible observables (such as position and momentum). In addition to describing the state, an observable also yields the possible outcomes of measurements.

An observable associated with a quantum system is represented by a self-adjoint operator $\hat{A}$ on its Hilbert space. The spectrum of the operator $\hat{A}$ indicates the possible values that can be found when the observable in question is measured. There are many operators in QM, but only operators in the class of $\hat{A}$ represent observables. For an observable $\hat{A}$, the spectrum $\Lambda(\hat{A})$ of its representing self-adjoint operator stipulates all possible values that the measurements of the physical observables represented by $\hat{A}$ may obtain. As we focus only on observables within the discrete spectrum, the spectrum of an observable $\hat{A}$ is $\Lambda(\hat{A}) = \{a_i\}$, where $\{a_i\}_{\in \mathbb{R}}$ are real numbers named *eigenvalues*, which represent the possible results of experiments, or measurement outcomes.

$\mathcal{A}_{pil3}$ [*Statistical Algorithm*]: The same as $\mathcal{P}_3$.

$\mathcal{A}_{pil4}$ [*Hidden-Variable Dynamics*]: The same as $\mathcal{P}_4$, except for the following remarks. The usual Schrödinger equation (see Equation 1 in $\mathcal{A}_{col4}$) is *supplemented* with additional terms and *hidden variables* that transform its *probabilistic* nature in a *deterministic* equation of motion Bohm (1952). This additional term Q is called 'quantum potential'

$$Q = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}, \tag{10}$$

which supplements the differential equations of motion with additional parameters. In order to do so, the Schrödinger equation is written in function of the *potential* operator V — instead of the Hamiltonian operator H, to yield the total energy of the system, as in Equation 1, as:

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \left(-\frac{\hbar}{2m}\nabla^2 + V\right)|\psi\rangle, \tag{11}$$

where $|\psi\rangle$ is also re-written in its polar form, decomposed between its amplitude and its phase Bohm (1952). It is noteworthy to mention that $|\psi\rangle$ is an abbreviation of $|\psi(\bar{\mathbf{x}}, t)\rangle$, where $\bar{\mathbf{x}}$ stands for spatial coordinates (x, y, z); these are *position coordinates*, taken to be essentially *unknown*. These are the so-called *hidden* variables, thus denying assumption 1. If this is so, the real part of the Schrödinger equation can be rewritten in a *modified Hamilton-Jacobi* equation (also known as its "quantum version" Bohm and Hiley (2006)) as follows:

$$\frac{\partial S}{\partial t} = -\left[\frac{|\nabla S|^2}{2m} + V + \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{12}$$

Equation 12 is commonly referred to as the "guiding equation" of motion.

$\mathcal{A}_{pil5}$ [*Measurement*]: A position measurement, for example, simply yields the *hidden* states that were guided by Equation 12 in a purely deterministic (albeit statistical via Born rule) manner.

3. $\mathcal{R}$ are the rules of inference of QM_{pil}, which are similar to those stated in QM^{bas}.

7.2 Collapse quantum theory

The (standard) collapse solution to the measurement problem, as stated by von Neumann (1955), modifies (or *interprets*) the structure of QM^{bas}, instantiating some of its General Principles in specific axioms of *collapse quantum theories* by denying assumption 2. It is worth mentioning that the so-called "orthodox interpretation" (or the "Copenhagen interpretation") or QM, as well as all the approaches presented in section 5 are instances of what we call a "collapse quantum theory". Therefore, in an axiomatic structure, a collapse quantum theory QM_{col} is a triple:

$$\mathrm{QM}_{col} = \langle \mathcal{F}, \mathcal{A}_{col}, \mathcal{R} \rangle \tag{13}$$

where:

1. $\mathcal{F}$ is the language of QM_{col}, the same as QM^{bas};
2. $\mathcal{A}_{col}$ are the specific axioms of QM_{col} (i.e. instance of the set $\mathcal{P}$ of QM^{bas}). The list of $\mathcal{A}_{col}$ is:

$\mathcal{A}_{col1}$ [*Hilbert space*]: The same as $\mathcal{P}_1$, except for the following remark. The set S represents physical systems.

$\mathcal{A}_{col2}$ [*Quantization Algorithm*]: The same as $\mathcal{A}_{pil2}$.

$\mathcal{A}_{col3}$ [*Statistical Algorithm*]: The same as $\mathcal{P}_3$.

$\mathcal{A}_{col4}$ [*Undisturbed Dynamics*]: Slightly modifies the General Principle $\mathcal{P}_4$; $\mathcal{A}_{col4}$ states that the temporal dynamics of the set $\{A\}$ of observable obeys the linear evolution of $\hat{U}$ *only when* A is not subject to a measurement process, thus denying assumption 2. Moreover, $\mathcal{A}_{col4}$ instantiates the differential equation of motion of $\mathcal{P}_4$ in the Schrödinger equation, as presented in equation 1 (where $i = \sqrt{-1}$, $\hbar$ is the Planck constant divided by 2π, and H is the Hamiltonian, which gives the energy of the system) in the form of $i\hbar\,\partial|\psi(t)\rangle/\partial t = H|\psi(t)\rangle$, where $\hbar$ is the reduced Planck constant and H is the Halmiltonian of the system.

$\mathcal{A}_{col5}$ [*Collapse*]: When the state of a physical system S described as the superposed state

$$|\psi\rangle = \sum_i c_i|\alpha_i\rangle, \tag{14}$$

is subject to a measurement process, the system S ceases to be described by $\hat{U}$ and stats being described by the corresponding eigenstate in the form of

$$|\psi\rangle = \sum_i c_i|\alpha_i\rangle \to |\alpha_w\rangle, \tag{15}$$

where $|\alpha_w\rangle$ is one of the elements of the expansion, with a probability given by the statistic algorithm in $\mathcal{P}_3$, which is $|c_i|^2 = |\langle\alpha_i|\psi\rangle|^2$. It is worth remembering the following: it is agreed that the values of eigenstates are set according to measurement results. An eigenstate, however, is *not* the result of measurement.[27] $\mathcal{A}_{col5}$ states that measurement takes place whenever a quantum system interacts with nonquantum-mechanical systems which collapse a superposed state in a single eigenstate of $\hat{A}$. As $\mathcal{A}_{col4}$ declares the *limited* validity of $\hat{U}$, $\mathcal{A}_{col5}$ states that $\hat{A}$ is found in a single, determined state by virtue of its interaction with a other systems.

3. $\mathcal{R}$ are the rules of inference of QM_{col}, similar to those stated in QM^{bas}.

7.3 Branching quantum theory

Proceeding the branching solution to the measurement problem, according to Everett (1957), modifies (or *interpret*) the structure of QM^{bas}, instantiating some of its General Principles in specific axioms of *branching quantum theories*, by denying assumption 3. Therefore, in an axiomatic structure, a branching quantum theory QM_{bra} is a triple:

$$\mathrm{QM}_{bra} = \langle \mathcal{F}, \mathcal{A}_{bra}, \mathcal{R} \rangle \tag{16}$$

where:

1. $\mathcal{F}$ is the language of QM_{bra}, likewise QM^{bas};
2. $\mathcal{A}_{bra}$ are the specific axioms of QM_{bra} (i.e. instance of the set $\mathcal{P}$ of QM^{bas}). The list of $\mathcal{A}_{bra}$ is:

$\mathcal{A}_{bra1}$ [*Hilbert space*]: The same as $\mathcal{P}_1$, except for the following remark. The set S represents physical systems.

$\mathcal{A}_{bra2}$ [*Quantization Algorithm*]: The same as $\mathcal{A}_{pil2}$.

$\mathcal{A}_{bra3}$ [*Statistical Algorithm*]: The same as $\mathcal{P}_3$.

$\mathcal{A}_{bra4}$ [*Branching*]: Instantiates, also, the differential equation of motion of $\mathcal{P}_4$ in the Schrödinger equation, similarly to $\mathcal{A}_{col4}$, however maintaining its *universal validity*. By maintaining the universal validity of $\hat{U}$, every time A is described by a superposition, $\mathcal{A}_{bra4}$ says that *all terms* of such superposition exist in different *branches*.[28]

$\mathcal{A}_{bra5}$ [*Branch Recognition*]: Instantiates the measurement as the recognition of a relative branch, considering a single eigenstate of A. It is worth noting that, by virtue of $\mathcal{A}_{bra4}$, *all* other states of S are equally real in different branches. Thus, $\mathcal{A}_{bra5}$ implies the denial of assumption 3: S is found in a single, determined state by virtue of a recognition of a particular branch of the universe. Such determinate outcome, however, is relative to a branch, and not absolute. For all practical purposes, $\mathcal{A}_{bra5}$ resembles the concept of collapse, as stated in QM_{col}, but no collapse occurs — just the branching process.

3. $\mathcal{R}$ are the rules of inference of QM_{bra}, similar to those stated in QM^{bas}.

Take a QM_{bra} as an example. It is an axiomatic theory (à la Suppes), whose axioms instantiate (or "interpret") QM^{bas}'s General Principles and the phenomenological statement. In this sense, QM_{bra} is a *physical* theory (whereas QM^{bas} is not yet) capable of making physical predictions. QM_{col} does the same thing as QM_{bra} does, but with different axioms. It is also a physical theory, but different from QM_{bra}. The same goes to QM_{pil}. The axioms of quantum theories are not purely formal, they are theory-specific physical axioms. QM^{bas}'s General Principles are not axioms of a physical theory, but very general conditions (of something that is not yet a theory) for generating physical theories. Moreover, the physical theories developed upon QM^{bas} must meet the consistency constraint and solve the measurement problem. The specific axioms of QM_{pil}, QM_{col} and QM_{bra} do not create the measurement problem: they are physical theories insofar as they introduce physical concepts (viz., the phenomenological statement) to the axiomatic scheme; and they do so in each in a specific way to avoid (or solve) the measurement problem.

8 Final remarks

This paper presented new horizons to old questions. In particular, we addressed the following question: *what is QM, and what does it mean to interpret it?* We argued that QM could not be grasped in its complexity with purely syntactic nor semantic approaches. Because of its interpretation issues, problems such as theory individuation begin to appear as soon as one starts identifying QM with a single set of rules. On the other hand, it seems that all the so-called "interpretations" of QM were departing from a shared point. To pinpoint this point of departure, we needed a radical move: to postulate a basic framework without reference to physical concepts (QM^{bas}) in a "splendid isolation" from physics.

To be sure, the General Principles of QM^{bas} are very close to those mentioned in the textbook approaches to quantum mechanics (section 5). However, even those books present an axiomatic formulation of quantum mechanics that includes in their axioms the collapse or the postulate of projection. This, in our view, is an interpreted version of quantum mechanics, not "The" quantum mechanics that is later interpreted. This is a very common feature in many aspects of our lives. Until getting in touch with other cultures, it is natural to think that one is not being part of a specific culture. Likewise, the presentations of quantum mechanics in textbooks do not feel like presenting quantum mechanics already interpreted; people don't think they are within an interpretation. One reason is that interpretation is the only way to talk about the world. However, this obscures the question: what do the interpretations interpret? This tension was presented in section 5.

Realizing this, we substantially changed the formulation of our QM^{bas} in section 6, in the light of the literature on the syntactic and semantic views of scientific theories. Our solution was to say that what is interpreted is QM^{bas}, which in turn consists of a purely mathematical formalism, that is, something that is not even a physical theory, since it remains, in Carnapian terms, in the "splendid isolation" of the world. Thus, we removed the mention of QM^{bas} systems and physical observables, leaving this to the interpretations. The interpretations, therefore, solve a double problem which we introduce, previously called the "measurement problem": the first one what we called the "isolation problem", which is the placement of QM^{bas} in a purely mathematical framework — after all, non-basic quantum mechanics (the "interpreted" ones) must be physical theories. Doing so consists, among other things, in the introduction of what we called, after Auyang (1995), a "phenomenological statement" which leads to the measurement problem (here presented in the standard version of Maudlin concerning the conjunction of the assumptions of completeness, linearity, and uniqueness of the results).

In this way, our formulation of the QM^{bas} and the subsequent QM_{col}, QM_{pil}, and QM_{bra} are presented originally. The originality does not lie in the content of the axioms of the theories presented, but in the philosophical presentation of the understanding of what is an interpretation of quantum mechanics and what is interpreted — as well as the new meaning that the word "interpretation" urges in quantum mechanics, i.e., a sui generis sense that escapes the tradition of the debate between syntactic and semantic views and urges another use

In order to close this paper indicating further developments, we should mention very briefly the following. One can reify the theoretical mechanisms of each interpretation or not. If they are ontologically reified, then interpretations can have an ontological aspect. But someone can be an empiricist and still be on the QM_{col} team.[29] To take this as an example: one can reify the *cause* of the collapse with an ontological primitive, e.g., one can state that it is "human consciousness" that causes the collapse. (cf. Wigner, 1983; de Barros and Oas, 2017; Arroyo and Arenhart, 2019). But that is not necessary. The gain of our proposal is to remain neutral with the ontological rectification of the theoretical mechanisms of each quantum theory, e.g. collapse and consciousness, branching and many worlds, hidden variables, and pilot waves. This can be done at another, more profound moment of interpretation, which can enrich our understanding of the relationship between theory and the world, and will certainly involve debates concerning scientific realism and anti-realism. Such questions, while interesting, are beyond the scope of this paper.

Notes

[1]We will try to clear up confusion related to the terms "language" and "meaning". Terms are the linguistic representations for certain quantities — for example, the term "mass" designates the physical quantity known as mass. Measuring, thus, is related to quantities, not their linguistic representations. In this sense, *terms*, of course, cannot be measured.

[2]Probably the first author to point out this problem was Maxwell (1962).

[3]It's not our point here to discuss, in a profound way, how the traditional distinction can be if necessary, safeguarded [a more detailed discussion can be found at Silva (2013, p. 144–151)]. Lewis (1970) and Maxwell (1962) suggested how this can be done from a realist perspective and the recent developments in the structural realist view of theories depend/draw heavily on an adequate distinction between observational and theoretical terms.

[4]From this point forward, we will adopt the notation presented by Krause and Arenhart (2016), who call first-order structures as "order-1", and high-order structures (e.g., with $n > 1$) as "order-n".

[5]Something that, again, Carnap was aware (cf. Carnap, 1966, p. 193)

[6](Cf. Carnap, 1959, p. 158) and (Carnap, 1966, p. 228).

[7]In this sense, Carnap is in partial disagreement with Quine's holism: i.e., recognizing the *continuum* between theoretical and observational terms — and so only partially agreeing with Quine's holism — but allowing a distinction to be justified (only) by practical purposes, suspending the judgment about its "nature", that he would regard as a (suspicious) metaphysical commitment.

[8]For a detailed discussion see Silva (2020, pp. 87–95).

[9]It is noteworthy to mention that the terminology of "partial interpretation" is employed here in a non-standard way. It shall not to confused with its standard meaning: if every elementary statement in a theory T has a correspondent in its models, then it is considered to be a "full interpretation"; otherwise, it is a "partial interpretation" (see Haskell (1963, pp. 48)).

[10]We'll say more about this in section 4.

[11]It is not our goal to discuss the implications of this view for scientific realism; the reader interested in this subject may find a helpful discussion in Chakravartty (2001) and references therein.

[12]Although Chang and Keisler (1990) work only with order-1 structures, the idea can be generalized.

[13]This is also the case for non-standard semantics, such as *Henkin semantics*, see Henkin (1950) and Enderton (2015, sec. 3). Nevertheless, we will stick with the standard case.

[14]For simplicity, from this point forward the description of these structures' components shall be fixed as defined in the previous paragraph.

[15]See also Lutz (2015, sec. 5).

[16]This issue is discussed in more detail in section 6.

[17]Another way of stating such problem is relating "open" and "closed" systems — see Pessoa Junior (1997). This work, however, sticks to Maudlin's taxonomy as it better relates to the literature here considered.

[18]In the sense of the word 'interpretation', as usually employed in QM.

[19]In his "Against 'measurement"' (1990), Bell put forth his famous critique of how the notion of "measurement" is ill-defined and even inadequate to the context of the foundations of QM. Our goal here is essentially to do the same, but with the term "interpretation".

[20]For a critical summary of various formulations of QM, see Wightman (1976), Gudder (1979), Styer et al. (2002), and references therein.

[21]For those cases, see the references cited in Ćirković (2005, p. 821).

[22]For a brief analysis of *seven* thought experiments that show the differences in experimental results between collapse and no-collapse approaches versions of QM, see Ćirković (2005, pp. 823–834).

[23]An earlier version of QM^{bas} was presented in Arroyo (2020, chap. 2).

[24]It is assumed, though, that this *can* be done — even if not here.

[25]See $\mathcal{R}$ ahead.

[26]Not to be confused with the "cosmic exile" mentioned in section 4.

[27]There is some consensus concerning this mode of presentation of measurement results. For the sake of precision, it is essential to emphasize that this applies to *measurements of the first kind*. There are, however, measurements where this does not occur: where the eigenstate does not correspond to the eigenvalue. The position measurement satisfies the postulate presented, but, strictly, this does not apply to the measurement of energy.

[28]Recall that such a specific axiom is neutral regarding the question whether the *states* or the *systems* are prone to the branching process.

[29]Basically, this is the lead done by Niels Bohr (cf. Faye, 2012, and references therein).

Acknowledgements We want to thank all the attendants of the *VI International Workshop on Quantum Mechanics and Quantum Information: Identity and Individuality*, held in Florianópolis, Brazil, 2019, in which part of the content of this paper was presented. In special, we also thank Adonai Sant'Anna, Christian de Ronde, David Wallace, Décio Krause, Eduardo Quirino, Jerzy Brzozowski, Jonas Arenhart, Kherian Gracher, Lauro Nunes Filho, Matheus Valente, Osvaldo Pessoa Jr., and Otávio Bueno for many fruitful discussions on related subjects, and for comments made on previous versions of this paper. We would also like to thank the anonymous reviewer for having raised several questions that allowed us to improve our article. Part of the content of this paper was published in chapters 1 and 2 of one of the authors’ doctoral thesis (RWA), which was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

Conflict of interest

The authors declare that they have no conflict of interest.

References

Achinstein P (1968) Concepts of Science: A Philosophical Analysis. Johns Hopkins University Press, Baltimore

Acuña P (2021) Charting the landscape of interpretation, theory rivalry, and underdetermination in quantum mechanics. Synthese 198(2):1711–1740

Arroyo RW (2020) Discussions on physics, metaphysics and metametaphysics: Interpreting quantum mechanics. PhD thesis, Federal University of Santa Catarina (UFSC), Florianópolis, Available via https://tede.ufsc.br/teses/PFIL0381-T.pdf Cited 29 Jun 2021

Arroyo RW, Arenhart JRB (2019) Between physics and metaphysics: A discussion of the status of mind in quantum mechanics. In: de Barros JA, Montemayor C (eds) Quanta and Mind: Essays on the Connection between Quantum Mechanics and the Consciousness, Synthese Library, Springer International Publishing, Cham, chap 3, pp 31–42

Auyang SY (1995) How is Quantum Field Theory Possible? Oxford University Press, Oxford

Bell J (1990) Against ‘measurement’. Physics World 3(8):33–41

Birkhoff G, von Neumann J (1936) The logic of quantum mechanics. Annals of mathematics pp 823–843

Bohm D (1952) A suggested interpretation of the quantum theory in terms of ‘hidden’ variables, i. Physical Review 85(2):166

Bohm D, Hiley BJ (2006) The Undivided Universe: An Ontological Interpretation of Quantum Theory. Routledge, London

Bueno O, Krause D (2007) Scientific theories, models, and the semantic approach. Principia 11(2):187–201

Carnap R (1959) Theoretical concepts in science. Studies in History and Philosophy of Science Part A 31(1):158–172

Carnap R (1966) Philosophical Foundations of Physics: An Introduction to the Philosophy of Science. Basic Books, Editor Martin Gardner

Chakravartty A (2001) The semantic or model-theoretic view of theories and scientific realism. Synthese 127:325–345

Chang CC, Keisler HJ (1990) Model theory, vol 3. Elsevier, Amsterdam

Ćirković MM (2005) Physics versus semantics: A puzzling case of the missing quantum theory. Foundations of Physics 35(5):817–838

Cohen-Tannoudji C, Diu B, Laloe F (1991) Quantum Mechanics, Vol. 1. Wiley, Weinheim

Contessa G (2006) Scientific models, partial structures and the new received view of theories. Studies in History and Philosophy of Science 2(37):370–377

Craver CF (2008) Structures of scientific theories. In: Machamer P, Silbertstein M (eds) The Blackwell Guide to the Philosophy of Science, John Wiley & Sons, Oxford, pp 55–79

da Costa NCA, Chuaqui R (1988) On Suppes' set theoretical predicates. Erkenntnis 29(1):95–112

da Costa NCA, French S (2000) Models, theories, and structures: Thirty years on. Philosophy of Science 67:S116–S127

Dalla Chiara ML (1977) Logical self reference, set theoretical paradoxes and the measurement problem in quantum mechanics. Journal of Philosophical Logic 6(1):331–347

Dalla Chiara ML (1981) Logical foundations of quantum mechanics. In: Agazzi E (ed) Modern Logic – A Survey: Historical, Philosophical and Mathematical Aspects of Modern Logic and its Applications, Springer Netherlands, Dordrecht, pp 331–351

Dalla Chiara ML, di Francia TG (1979) Formal analysis of physical theories. In: di Francia TG (ed) Problems in the Foundations of Physics, Worldcat, Amsterdam, pp 134–201

de Barros JA, Oas G (2017) Can We Falsify the Consciousness-Causes-Collapse Hypothesis in Quantum Mechanics? Foundations of Physics 47(10):1294–1308

de Ronde C (2016) Representational realism, closed theories and the quantum to classical limit, World Scientific, Singapore, pp 105–135

d'Espagnat B (2003) Veiled Reality: An Analysis of Present-Day Quantum Mechanical Concepts. Westview, Boulder

Dürr D, Lazarovici D (2020) Understanding Quantum Mechanics: The World According to Modern Quantum Foundations. Springer, Cham

Enderton HB (2015) Second-order and higher-order logic. In: Zalta EN (ed) The Stanford Encyclopedia of Philosophy, fall 2015 edn, Metaphysics Research Lab, Stanford University, Stanford

Esfeld M (2019) Individuality and the account of nonlocality: The case for the particle ontology in quantum physics. In: Lombardi O, Fortin S, López C, Holik F (eds) Quantum Worlds: Perspectives on the Ontology of Quantum Mechanics, Cambridge University Press, Cambridge, pp 222–244

Everett H (1957) 'Relative state' formulation of quantum mechanics. Reviews of modern physics 29(3):454–462

Farenick D (2016) Fundamentals of Functional Analysis, 1st edn. Universitext, Springer, Switzerland

Faye J (2012) Niels Bohr: His heritage and legacy: An anti-realist view of quantum mechanics, vol 6. Springer Science & Business Media

Friederich S (2014) Interpreting Quantum Theory: A Therapeutic Approach. Palgrave Macmillan, New York

Ghirardi GC, Rimini A, Weber T (1986) Unified dynamics for microscopic and macroscopic systems. Physical Review D 34(2):470
Goldstein S (2009) Projection postulate. In: Greenberger D, Hentschel K, Weinert F (eds) Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy, Springer, Dordrecht, pp 499–501
Gudder SP (1979) A survey of axiomatic quantum mechanics. In: Hooker C (ed) The Logico-Algebraic Approach to Quantum Mechanics – Volume II: Contemporary Consolidation, Springer, Netherlands, pp 323–363
Halvorson H (2012) What scientific theories could not be. Philosophy of Science 79(2):183–206
Halvorson H (2016) Scientific theories. In: Humphreys P (ed) The Oxford Handbook of Philosophy of Science, Oxford University Press, Oxford, chap 27
Haskell CB (1963) Foundations of mathematical logic. McCraw-Hill, New York
Henkin L (1950) Completeness in the theory of types. The Journal of Symbolic Logic 15(02):81–91
Hodges W (1993) Model theory, Encyclopedia of Mathematics and its Applications, vol 42. Cambridge University Press, Cambridge
Jammer M (1974) The Philosophy Of Quantum Mechanics: The Interpretations Of Quantum Mechanics In Historical Perspective. Wiley and Sons, New York
Krause D, Arenhart JRB (2016) The Logical Foundations of Scientific Theories: Languages, Structures, and Models. Routledge, Abingdon
Krause D, Arenhart JRB, Moraes FTF (2011) Axiomatization and models of scientific theories. Foundations of Science 16(4):363–382
Lewis D (1970) How to define theoretical terms. The Journal of Philosophy 67(13):427
Lewis P (2016) Quantum Ontology: A Guide to the Metaphysics of Quantum Mechanics. Oxford University Press, New York
Lutz S (2012) On a straw man in the philosophy of science: A defense of the received view. HOPOS: The Journal of the International Society for the History of Philosophy of Science 2(1):77–120
Lutz S (2015) What was the syntax-semantics debate in the philosophy of science about? Philosophy and Phenomenological Research
Maudlin T (1995) Three measurement problems. Topoi 14(1):7–15
Maxwell G (1962) The ontological status of theoretical entities. Minnesota studies in the philosophy of science 3:1–27
Mermin ND (2004) Could Feynman have said this? Physics Today 57(5):10
Messiah A (1961) Quantum Mechanics, Volume I. North-Holland, Amsterdam, translated by G. M. Temmer
Muller FA (2003) Refutability revamped: How quantum mechanics saves the phenomena. Erkenntnis 56:189–211
Ney A, (Eds) DZA (2013) The wave function: Essays on the metaphysics of quantum mechanics. Oxford University Press, Oxford
Pessoa Junior OF (1997) Can the decoherence approach help to solve the measurement problem? Synthese 113(3):323–346

Putnam H (1966) What theories are not. Studies in Logic and the Foundations of Mathematics 44:240–251

Ruetsche L (2018) Getting real about quantum mechanics. In: Saatsi J (ed) The Routledge Handbook of Scientific Realism, Routledge, New York, pp 291–303

Silva GOd (2013) Carnap and Kuhn on linguistic frameworks and scientific revolutions. Manuscrito 36(1):139–190

Silva GOd (2020) Os empiristas vão à missa: compromissos ontológicos e frameworks linguísticos. PhD thesis, University of Campinas (UNICAMP), Campinas, Available via http://repositorio.unicamp.br/jspui/handle/REPOSIP/342469 Cited 29 Jun 2021

Sklar L (2003) Interpreting theories: The case of statistical mechanics. In: Clark P, Hawley K (eds) Philosophy of science today, Oxford University Press, Oxford, pp 276–284

Styer DF, Balkin MS, Becker KM, Burns MR, Dudley CE, Forth ST, Gaumer JS, Kramer MA, Oertel DC, Park LH, Rinkoski MT, Smith CT, Wotherspoon TD (2002) Nine formulations of quantum mechanics. American Journal of Physics 70(3):288–297

Suppe F (1977) The structure of scientific theories. University of Illinois Press, Illinois

Suppe F (1989) The semantic conception of theories and scientific realism. University of Illinois Press, Illinois

Suppe F (2000) Understanding scientific theories: An assessment of developments, 1969-1998. Philosophy of Science 67:S102–S115

Suppes P (1957) Introduction to Logic. Van Nostrand, New York

Suppes P (1967) What is a scientific theory? In: Morgenbesser S (ed) Philosophy of Science Today, Basic Books, New York, pp 55–67

Suppes P (2002) Representation and invariance of scientific structures. CSLI publications, Stanford

Tarski A (1956) The concept of truth in formalized languages. Logic, semantics, metamathematics 2:152–278

van Fraassen BC (1989) Laws And Symmetry. Clarendon paperbacks, Oxford University Press, Oxford

von Neumann J (1955) Mathematical Foundations of Quantum Mechanics. Princeton University Press, Princeton

Wallace D (2012) The emergent multiverse: Quantum theory according to the Everett interpretation. Oxford University Press, Oxford

Wallace D (2020a) Lessons from realistic physics for the metaphysics of quantum theory. Synthese 197(10):4303–4318

Wallace D (2020b) On the plurality of quantum theories: Quantum theory as a framework, and its implications for the quantum measurement problem. In: Scientific Realism and the Quantum, Oxford University Press, Oxford, chap 5, pp 78–102

Wightman AS (1976) Hilbert's sixth problem: Mathematical treatment of the axioms of physics. In: Browder FE (ed) Mathematical Developments Arising

From Hilbert Problems, Part 1, Proceedings of Symposia in Pure Mathematics, vol XXVIII, American Mathematical Society, Rhode Island, pp 241–268

Wigner E (1983) Remarks on the mind-body question. In: Wheeler J, Zurek W (eds) Quantum Theory and Measurement, Princeton University Press, Princeton, pp 168–181

Worrall J (2007) Miracles and models: Why reports of the death of structural realism may be exaggerated. Royal Institute of Philosophy Supplement 61:125–154

Zahar EG (2004) Ramseyfication and structural realism. Theoria 19(1):5–30