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EFFECTS OF THE USE OF MANIPULATIVE MATERIALS ON GRADE NINE LEARNERS' PERFORMANCE IN FRACTIONS IN PUBLIC HIGH SCHOOLS IN CHRIS HANI WEST EDUCATION DISTRICT, SOUTH AFRICA

by

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DECLARATION

I, George Adom, do hereby declare that this thesis is the outcome of my investigation and research and that this has not been submitted in part or full for any degree at this or any other university.

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DEDICATION

I dedicate this work to my lovely family, my late grandparents and all my students.

ABSTRACT

The purpose of this study was to investigate the effects of the use of manipulative concrete materials on grade 9 learners' performance in fractions in public high schools in Chris Hani west education district, in the Eastern Cape Province of the Republic of South Africa. Two hundred and fifty (250) grade nine (9) learners, whose ages ranged between 13-16 years, and ten (10) educators teaching grade nine mathematics, were selected from 40 public high schools with the use of stratified, systematic random sampling, convenience and purposive sampling methods. One hundred and twenty-five (125) learners were put into the experimental group, and another one hundred and twenty-five (125) learners were put into a control group through systematic random sampling method. Pre-test, Post-test, and Control group quasi-experimental design were used as research designs to collect data. Two research instruments were developed. These included: A Fractions Achievement Test (FAT), and Students Questionnaire on Manipulative Concrete Materials (SQMCM). The experimental group were taught with the Manipulative Concrete Materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative), whilst the control group were taught through the lecture method. Four (4) null hypotheses were generated and tested at 0.05 level of significance. The data collected were analysed using Analysis of Covariance (ANCOVA) to find the Mean, Standard Deviation and t-test. The mean and standard deviation were used to compare the pre-test and post-test results between the Experimental group and the Control group. The analysed results of the mean, standard deviations and t-tests were used to reject the null hypotheses. The analysed results were illustrated as followed: Cuisenaire rods pre-test (mean $\bar{x}$= 8.372, SD=1.770), post-test (mean $\bar{x}$ = 12,428, SD=4.732), t=13,024 p< 0.05. Hypothesis (H_{01}) was rejected : Fraction tiles/fraction bars pre-test (mean $\bar{x}$ = 8.372, SD=1.770), post-test (mean $\bar{x}$ = 11,42, SD=3.67), t=12,10 p< 0.05. Hypothesis (H_{02}) was rejected : Paper folding pre-test (mean $\bar{x}$ = 8.372, SD=1.770), post-test (mean $\bar{x}$ = 11,792, SD=4.256), t=12,024 p< 0.05. Hypothesis (H_{03}) was rejected : Computer assisted manipulative pre-test (mean $\bar{x}$ = 8.372, SD=1.770), post-test (mean $\bar{x}$ = 12,212, SD=4.569), t=12,801 p< 0.05. Hypothesis (H_{04}) was also rejected. The comparison of the mean scores and standard deviation between the Experimental groups and Control groups indicated that there was no significant difference in the Pre-test in all cases. On the other hand, the mean scores and standard deviations between the Experimental group and Control group in the Post-test showed a vast difference in all cases. There were improvement in the mean scores, and slightly difference in the standard deviations in the Experimental groups, whilst there were drops in the mean scores and standard deviations of the Control groups in all cases. From the studies, there was an indication that manipulative concrete materials have significant effects on grade nine learner's performance in fractions. It was therefore suggested that manipulative concrete materials should be incorporated into the instructions of fractions in mathematics.

Key words: Educators, Learners, Manipulative Concrete Material, Fractions, Grade 9.

LIST OF ABBREVIATIONS AND ACRONYMS

AMS	American Mathematics Society
ANA	Annual National Assessment
CAPS	Curriculum Assessment Policy Statements
DBE	Department of Basic Education
DoE	Department of Education
EAUMP	East Africa University Mathematics Program
ECDoE	Eastern Cape Department of Education
ICMI	International Commission on Mathematics Instruction
ICTMMA	International Community for the Teaching of Mathematical Modelling and Application
IMU	International Mathematics Union
JPBM	Joint Policy Board for Mathematics
LTSM	Learning and Teaching Support Material
MCM	Manipulative Concrete Material
MST	Mathematics, Science and Technology
NAEP	National Assessment of Educational Progress
NCII	National Centre on Intensive Intervention
NCSM	National Council of Supervisors of Mathematics
NCTM	National Council of Teachers of Mathematics
NECTA	National Examination Council of Tanzania
NMAP	National Mathematics Advisory Panel
NSCE	National Senior Certificate Examination
OECD	Organisation for Economic Cooperation and Development
PISA	Program for International Student Assessment
SAARMSTE	Southern African Association for Research in Mathematics, Science and Technology Education

SADBE	South Africa Department of Basic Education
SAMSA	Southern Africa Mathematical Sciences Association
SANSMS	South African National Study in Mathematics and Science
SASA	South African Schools Act
SEAMS	Southeast Asian Mathematics Society
TIMSS	Trends in International Mathematics and Science Studies
UNESCO	United Nations Educational, Scientific and Cultural Organization
WAEC	West African Examinations Council
WASSCE	West Africa Secondary School Certificate Examination
WEF	World Economic Forum

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CHAPTER ONE

INTRODUCTION AND BACKGROUND

1.1. INTRODUCTION

Governments and stakeholders all over the world, have channelled a considerable amount of their resources towards the training of educators and learners to improve the quality of teaching and learning of mathematics. Within this context, partnerships were established with the international organizations to achieve this goal (United Nations Educational, Scientific and Cultural Organization, UNESCO, 2015). Research showed that learners are confronted with the task of understanding the concept of the subject, especially fractions (Charalambous & Pitta-Pantazi, 2010). Siegler and Fazio (2010) observed that learners all over the world are faced with the challenge of learning fractions, and it is evident that an average learner never gained an abstract knowledge and understanding of fractions.

Fractions are an essential aspect of mathematics that form the bedrock of every learner's success in the subject, as stipulated by the National Mathematics Advisory Panel (NMAP, 2008). Lortie-Forgues, Tian and Siegler (2015) argued that, the prominence of fractions and decimal calculation for academic accomplishment is not restricted to mathematics courses only. Rational number arithmetic is also ever-present in physics, chemistry, engineering, psychology, sociology, biology, economics, and other spheres of studies.

Gould, Outhred, and Mitchelmore (2006), asserted that educators, learners and academics have typically described fractions learning as a difficult aspect of a mathematics syllabus. Researchers underscored the fact that learners found it problematic to comprehend the idea of "a part as a whole" relationship in mathematics. In 2013, the National Council of Supervisors of Mathematics (NCSM) issued a position statement on the use of manipulative concrete materials in classroom teaching to develop learners' accomplishments in mathematics. In order to develop every learner's mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulative materials into classroom instruction at all grade levels (NCSM, 2013). In a similar vein, the West African Examinations Council (WAEC, 2007) Chief Examiners report, recommended the use of hands-on and physical illustration in teaching abstract ideas to enhance understanding, and to awaken and facilitate learners' interest in mathematics. In this study, the researcher investigated on the effect of use of manipulative concrete materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative) in the instruction of fractions in grade nine.

1.2. BACKGROUND TO THE STUDY

Globally, there are major concerns about the performance of leaners in mathematics, especially in fractions. Through international cooperation and partnership, the United Nations Educational, Scientific and Cultural Organisation (UNESCO) spearheaded the promotion and capacity building for research and innovative teaching in mathematics and mathematics education, to enhance public understanding and appreciation of mathematics in our daily lives (UNESCO, 2015). The American Mathematics Society (AMS), in collaboration with other mathematical organizations, promoted mathematics, science, and research through funding to create consciousness of mathematics education, and to project the mathematics profession (AMS, 2019). The Joint Policy Board for Mathematics (JPBM, 2014) initiated Mathematics Awareness Month every April annually to create public awareness and the importance of mathematics in our daily lives, and also to remunerate and appreciate reporters and other correspondents who, on a regular basis, carried accurate mathematical evidence to a non-mathematical society.

Hanushek and Woessmann (2009), argued that there was a practical motive for directing our attention to mathematics. This subject was predominantly well suited to arduous comparison across countries, and their philosophy in mathematics education. There was a strong global agreement on the mathematical ideas and methods that needed to be learned so that these concepts would be introduced into the mathematics syllabus. The information to be acquired remained the same, irrespective of the principal language spoken in a society.

The Program for International Student Assessment (PISA, 2012) conducted a test with about 510,000 students across the globe in 65 countries on mathematics, science and reading. The average mark for mathematics was 494, while the average mark in reading and science was 496, and 501 respectively. These results showed different levels of performers in mathematics. The East Asian countries of Shanghai and China scored the best result of 613. The United Kingdom, Ireland, Australia and New Zealand had an average score of 494, whilst the USA trailed the group with 481. Analysed results showed that boys did better than girls in mathematics, while girls performed better than boys in reading, and both boys and girls had similar results in science (PISA, 2012). Martin and Mullis (2013) highlighted the proportion of learners attaining the TIMSS/PIRLS 'high' international benchmarks in reading, science and mathematics. Table 1, stipulated the analysed results of grade 4 mathematics, reading and science.

Table 1: Proportion of grade 4 cohort reaching the high international benchmark in Mathematics, Reading and Science.

Proportion of the Grade 4 cohort reaching the high international benchmark				
Country	Mathematics	Reading		Science
Hong Kong SAR	82%	67%		46%
Singapore	78%	62%		68%
Chinese Taipei	74%	55%		54%
Finland	50%	63%		65%
Hungary	37%	48%		46%
Czech Republic	30%	50%		45%
Italy	28%	46%		37%
Austria	26%	39%		42%
Sweden	25%	47%		44%
Croatia	19%	54%		30%
Poland	17%	39%		29%
Spain	17%	30%		28%

Source: TIMSS and PIRLS (2011).

Research showed that mathematics education in East Asia strongly revolved around their cultural setting. This seemed to explain why other systems in Europe, Africa and

North America, lagged behind in mathematics. The high performance of East Asian learners in mathematics could be attributed to the value East Asian countries attached to education by parents, government and non-governmental organizations (Marginson, 2014). The fundamental concern amongst educators in East Asia was how learners gained their understanding of the mathematical concepts, especially fractions, and how to limit the fear people attached to fractions as being a difficult topic (Marginson, 2014). Studies showed that families in East Asia put high importance on their children's education, and are willing to devote much time, effort, and money into the education of their children (Marginson, 2014). Jerrim (2014) avowed that Australian children, with East Asian parents, performed better than their native Australian peers by an average of more than 100 PISA test points.

Researchers conducted an assessment of U.S. Grade 8 learners on fraction addition, to find out the closest whole number to $^{12}/_{13}$ + $^{7}/_{8}$. (The answer choices were 1; 2; 19; 21 and "I don't know"). The outcome of the test indicated that only 27 per cent got the correct answer to be (2) (Lortie-Forgues, Tian, & Siegler, 2015). In a similar vein, the National Assessment of Educational Progress (NAEP), conducted a test to a sample of U.S grade 8 learners. At the end of the test, it was observed that only 50 per cent of the participants could correctly order $^{2}/_{7}$, $^{5}/_{9}$, and $^{1}/_{12}$ from the smallest to the largest (Martin, Strutchens, & Ellintt, 2007).

Siegler and Pyke (2013), observed that in addition and subtraction problems in fractions, unequal denominator problems elicited more errors than equal denominator ones. The results showed that 41per cent of learners in grade 6 correctly answered the problem, and 57 per cent of grade 8 learners also correctly answered the problem. Siegler and Pyke (2013) concurred that sixth and eighth grade learners, representing 68 per cent, correctly answered decimal arithmetic problems. Performance was high on addition and subtraction of fractions, representing 90 per cent and 93 per cent respectively, while the performance of learners was low in multiplication and division of fractions, representing 54 per cent and 35 per cent respectively.

Eurydice (2011) affirmed that most European countries have reviewed their mathematics syllabi, embracing an outcome-based method which was aimed at developing learners' competencies and skills, rather than on a theoretical approach. This integral approach was focused on an all-inclusive and flexible approach in meeting the needs of different levels of learners, as well as to their ability to comprehend the tenacity of mathematical applications, especially fractions, in their daily lives. Studies showed that a section of Mathematics, Science and Technology (MST) learners across Europe, when compared to other subjects, have suffered a great decline. Many European countries have observed that this decline among learners in Mathematics, Science and Technology was a major cause of concern (Eurydice, 2011). Research showed that the use of technology in mathematics has been recommended in most schools in the western world. Computers were rarely used during mathematics instruction. However, the policy makers largely lacked ideas on providing appropriate support to assist educators to implement the reviewed syllabus (Eurydice, 2011).

Other related studies observed that East Asian learners performed better than their colleagues in other continents in mathematics (Son, 2011, Charalambous & Pitta-Pantazi,2010, Watanabe, 2012). Aside the cultural differences and parental involvement among East Asian learners in mathematics, the nature of instruction of mathematics is also a major contributing factor. Fractions, as a mathematical topic, are introduced to Korean learners in grade three. Japan in grade four to the elementary level, while in Taiwan, fractions are introduced to learners at grade three with emphasis on composition and decomposition of fractions (Son, 2011, Charalambous & Pitta-Pantazi, 2010, Watanabe, 2012). In addition, East Asian countries used an amalgamation of carefully selected mathematical materials that have a prolonged existence in terms of their application in demonstrating fractions, and also replicated the idea of fractions as a quantity. The focus was on the linear model in connection with the bar model, which was mostly used in Japanese fractions instruction, as well as Korean and Taiwanese textbooks. This approach, adopted by the East Asian countries, was in variance to the North American approach who are preoccupied with the 'pizza model', or other circular area models (Son, 2011, Charalambous & Pitta-Pantazi, 2010, Watanabe, 2012).

The Southeast Asian Mathematics Society (SEAMS) played an important role in promoting and developing mathematics educators in Southeast Asia. SEAMS served as a tool for attracting sponsors from the larger mathematical community such as the International Mathematics Union (IMU), Australia, and France (Eurydice, 2011).

The Trends in International Mathematics and Science Studies (TIMSS, 2015) edition, affirmed that the five countries that performed lower than Indonesia, with an average score of 388 in the fourth graders, were Jordan (388), Saudi Arabia (383), Morocco (377), South Africa (376) and Kuwait (353). Analysis of TIMSS (2015) results indicated that, only 24.45 per cent of Indonesian grade four learners could correctly answer questions on fractions. This percentage was below the average score of other countries, which had lower TIMSS scores such as Saudi Arabia (29.42%) and Kuwait (25.18%). A similar result was found for TIMSS numeracy in which the average percentage of correct answers of Indonesian students (42.67%), was lower than that of students from lower performing countries such as Jordan (46.76%), and South Africa (48.72%) (Wijaya, 2017).

Studies showed that mathematics development is very low in Africa due to the low numbers of secondary school educators teaching mathematics, and also the low number of graduates and post graduates pursuing mathematics as a course at masters and PhD levels, and, above all, the absence of the use of concrete manipulative materials in most of our schools. The shortfall of professors deemed to produce the subsequent crop of leaders in the field of mathematics cannot be over emphasized. Most African countries lag behind the fast rate of technological development of mathematics, compared to the current technological world (ICMI, 2009).

Research showed that one critical definition of mathematical strength was attributed to the number of PhD graduates a country was able to produce in the field of mathematics. However, it was estimated that the number of PhD holders in mathematics was less than 2,000, and, may be closer to 1,000 in the whole of sub-Saharan Africa, according to 2009 reports (Mathematics in Africa, 2014).

Mathematical strength was also measured based on the research output in terms of mathematical publications. Statistics showed that North Africa has about 0.87 per cent of the world output, while for the rest of Africa, the share was extremely low: for Southern Africa 0.39 per cent ; for West Africa 0.08 per cent ; for Central Africa, 0.03 per cent ; and for East Africa, 0.01 per cent of world output. Within the African continent, there are vast disparities in terms of research publications among countries, with a sampling from MathSciNet indicating that 370 publications are from Egypt, 334 from South Africa, 3 from Benin, while Ghana and the rest recorded none (Mathematics in Africa, 2014).

Study showed that mathematics education is very weak in most African countries at the primary and secondary school level, resulting in the reduction of potential learners choosing mathematics at university. The poor state of mathematics in Africa could be attributed to the inadequate teacher recruitment, and the lack of laws guiding the universal elementary education which have caused an increase in overcrowding in most classrooms, and have reduced the value of instruction and learning (Mathematics in Africa, 2014). Another cause of the poor state of mathematics in Africa has been the persistent lack of experienced educators' in the field of mathematics at all stages of education. Research showed that few learners who made it to university, hopeful of a vocation in mathematics, were overwhelmed by the persistent difficulties involved (Mathematics in Africa, 2014). Some of these difficulties encountered, included: an outdated lecture-style instruction which does not involve learners' participation, an outdated curriculum which does not prepare learners for the job market, and overcrowded lecture theatres (Mathematics in Africa, 2014).

North Africa is comparatively progressive region in mathematics, as a result of governments' extensive involvement in supporting research and promoting quality education at all educational stages, and consistent funding from donors. Among the African countries, Egypt had a uniquely strong mathematical community (International Commission on Mathematics Instruction ICMI, 2009).

The West African Secondary School Certificate Examination (WASSCE) estimated that over ninety thousand WASSCE candidates, translated to 46.6 per cent of candidates, failed mathematics (WASSCE, 2015). This caused overwhelming anxiety from government and stake-holders over the poor performance of students' in mathematics and science. These results were released by the West African Examination Council (WAEC, 2015). The current report, released by the Organisation for Economic Cooperation and Development (OECD) rated Ghana last among 76 other countries regarding the state of students' performance in mathematics and science (OECD, 2015).

Ghana had comparatively robust courses in teacher training, mathematical physics, and robust primary and secondary education courses (Mathematics in Africa, 2014).

However, the concepts of measurement and fractions remained a challenge among most Ghanaian pupils due to their culture setting. Davis (2016), concurred that measurement in the Ghanaian culture involved the use of local physical properties such as “Olonka”, “margarine cup”, which were different in-school and out-of-school contexts. In similar vein, the use of fractions in everyday language in comparing two objects, may not be the same as in the home context in the Ghanaian culture (Davis, 2016). This posed a great challenge for learners to grasp the concept of fractions and measurement in the school environment.

The East African region initiated an independent program called Uwezo, which was part of “Twaweza” to promote access to educational information, civic organisation and improved service delivery outcomes across the region (Uwezo, 2014). This program evaluated what learners were conversant with, and were able to perform in line with the aim of the national Grade 2 curriculum, in reading and performing basic computation in mathematics across Kenya, Uganda, and Tanzania. This program assessed learners at home, and its assessment tools were precise with numeracy tasks measuring counting, recognition of numbers, comparison of numbers and basic computational operations which involved addition, multiplication, division and subtraction of numbers. The test outcome showed that in Uganda, only 44 per cent of learners between the ages of 10 and 16 years passed the numeracy test, whilst the corresponding pass rate in Kenya and Tanzania was 68 per cent (Uwezo, 2014).

Ethiopia made frantic efforts to advance primary and second cycle education in mathematics. All educators teaching first-cycle and second-cycle primary school grades 1- 8 were deemed to have a diploma qualification, whilst high school educators who taught grades 9-10 were required to have a bachelor’s degree. The Ministry of Education also trained educators for master’s degree to handle students in the universities. Added to that, the Ministry recently revised the mathematics curriculum for teaching (Mathematics in Africa, 2014).

The Southern Africa Mathematical Sciences Association (SAMSA), and the Southern Africa Association for Research in Mathematics Science and Technology Education (SAARMSTE), collaborated and promoted exchange programs among other colleagues in the sub-region in the field of mathematical sciences about the most effective way to promote research in an innovative ways to enhance and facilitate communication between mathematical researchers' in the region, and among educators and learners (Mathematics in Africa, 2014).

Studies showed that in 2012, the grade 10 pass rate for mathematics in Zimbabwe in the O-level examination was only 13.9 per cent, which represented the number of learners who could proceed to study mathematics at an advanced level, or senior secondary school (Mathematics in Africa, 2014). The Lusaka Voice, (2013), confirmed that over 6 per cent of 103,000 students who sat for the 2012 exams in Zambia, recorded zero (sic) in each of the two papers written by the 12th graders in mathematics.

The performance of learners in mathematics in South Africa was hardly different from learners in other countries. Generally, the state of mathematics among learners in South Africa was quite low at the primary and secondary school levels, but highly improved at the university and advanced levels (Mathematics in Africa, 2014).

The World Economic Forum (WEF, 2014) rated mathematics and science education in South Africa as second-last compared to other countries in the world. Researchers have reached an agreement that the state of understanding mathematical content among learners in South Africa is uncertain. They further argued that learners in South Africa faced challenges relating to their limited technical vocabulary in mathematics (Van der Walt, Maree & Ellis, 2008:490). Reddy, Visser, Winnaar, Arends, Juan, Prinsloo, and Isdale, (2016), observed that in Trends in International Mathematics and Science Studies (TIMSS, 2015), out of the 39 countries that participated in grade nine mathematics test with a TIMSS achievement scale of centre point 500, and standard deviation of 100, the top five (5) countries were all from East Asia, with Singapore (621), the Republic of Korea (606), Chinese Taipei (599), Hong Kong SAR (594) and Japan (587). The five lowest performing countries were Botswana (391), Jordan (386), Morocco (384), South Africa (372) and Saudi Arabia (368). Among these five (5) countries, with a statistically different score to that of South Africa, was Botswana. Another study conducted by TIMSS (2011) observed that out of the twenty-one middle – income countries that participated in the study, South African learners came last, recording the lowest score. In similar vein, South Africa came last in two major cross-national comparisons of primary school student achievement: the Southern Africa

Consortium for Monitoring Educational Quality (SACMEQ, 2000, 2007) (Spaul & Kotze, 2015).

Table 2 illustrates a comparison of performance in key subjects areas in the National Senior Certificate Examination from 2014 – 2018;

Table 2. Comparison of performance in key subjects (NSCE, 2018)

Subject Description	Exam Date														
	2014			2015			2016			2017			2018		
	Total Wrote	Achieved30-100%	% Achieved at 30% and Above	Total Wrote	Achieved30-100%	% Achieved at 30% and Above	Total Wrote	Achieved30-100%	% Achieved at 30% and Above	Total Wrote	Achieved30-100%	% Achieved at 30% and Above	Total Wrote	Achieved30-100%	% Achieved at 30% and Above
Accounting	125,987	85,681	68.0	140,474	83,747	59.6	128,853	89,507	69.5	103,427	68,318	66.1	90,278	65,481	72.5
Agricultural Sciences	78,063	64,486	82.6	104,251	80,125	76.9	106,386	80,184	75.4	98,522	69,360	70.4	95,291	66,608	69.9
Business Studies	207,659	161,723	77.9	247,822	187,485	75.7	234,894	173,195	73.7	204,849	139,386	68.0	192,139	124,618	64.9
Economics	137,478	94,779	68.9	165,642	112,922	68.2	155,908	101,787	65.3	128,796	91,488	71.0	115,169	84,395	73.3
English First Additional Language	432,933	423,134	97.7	543,941	528,157	97.1	547,292	533,235	97.4	503,151	488,572	97.1	498,959	485,112	97.2
Geography	236,051	191,966	81.3	303,985	234,209	77.0	302,600	231,588	76.5	276,771	212,954	76.9	269,621	200,116	74.2
History	115,686	99,823	86.3	154,398	129,643	84.0	157,594	132,457	84.0	147,668	127,031	86.0	154,536	138,570	89.7
Life Sciences	284,298	209,783	73.8	348,076	245,164	70.4	347,662	245,070	70.5	318,474	236,809	74.4	310,041	236,584	76.3
Mathematical Literacy	312,054	262,495	84.1	388,845	277,594	71.4	361,865	257,881	71.3	313,030	231,230	73.9	294,204	213,225	72.5
Mathematics	225,458	120,523	53.5	263,903	129,481	49.1	265,810	135,958	51.1	245,103	127,197	51.9	233,858	135,638	58.0
Physical Sciences	167,997	103,348	61.5	193,189	113,121	58.6	192,618	119,427	62.0	179,561	116,862	65.1	172,319	127,919	74.2

Source: (NSCE, 2018).

The analysed results of mathematics, from 2014 – 2018 of the (NSCE, 2018) report, clearly showed that learners have a major problem in mathematics. At a media briefing held on 4 January, 2012 in Pretoria, the South African Basic Education Minister Angie Motshekga, declared that the matriculation pass rate for mathematics dropped from 47.4 per cent to 46.3 per cent in 2011 (NSCE, 2012).

The result of the South African National Study in Mathematics and Science (SANSMS) indicated that the performance of learners in all the subjects, especially mathematics, were at variance among the provinces in South Africa in the 2018 results analysis. The Western Cape, Northern Cape and Gauteng outperformed the other provinces, whilst the Eastern Cape and Limpopo performed below expectation (NSCE, 2018). Table 3 highlights the Provincial Performance reported in selected subjects (NSCE, 2018)

Table 3. Provincial performance in selected subjects (NSCE, 2018).

Province Name	Subject Description																					
	Accounting		Agricultural Sciences		Business Studies		Economics		English First Additional Language		Geography		History		Life Sciences		Mathematical Literacy		Mathematics		Physical Sciences	
	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above	Total Wrote	% Achieved at 30% and Above
EASTERN CAPE	11,618	72.8	20,434	74.7	20,976	60.1	14,727	69.6	70,783	96.5	29,046	71.3	21,026	84.6	44,153	73.1	30,031	64.4	36,449	45.5	24,939	66.5
FREE STATE	5,638	82.3	1,951	78.2	10,533	70.7	6,125	78.4	25,147	99.2	11,249	85.1	4,873	91.8	12,714	85.6	15,746	80.9	9,722	74.3	7,876	81.7
GAUTENG	15,491	83.2	854	82.4	41,514	74.3	21,472	83.7	68,212	99.7	47,305	84.5	33,090	95.3	46,340	85.9	60,228	84.1	35,279	74.7	26,763	83.5
KWAZULU-NATAL	24,503	69.1	19,422	73.9	51,588	59.5	27,003	75.4	118,920	94.6	63,897	68.9	38,928	86.7	72,137	76.5	59,387	65.6	61,686	50.6	40,643	73.6
LIMPOPO	14,188	62.3	29,243	63.0	18,814	63.3	22,113	60.2	92,666	96.6	48,008	67.5	15,085	88.0	55,515	71.2	41,393	66.1	39,216	54.9	31,717	71.8
MPUMALANGA	6,608	71.9	15,663	72.0	14,923	62.3	8,095	72.9	50,925	97.9	23,660	72.4	8,306	88.0	30,205	75.4	22,983	66.5	24,207	54.2	20,387	70.2
NORTH WEST	3,357	70.5	5,854	66.8	9,188	69.5	5,102	79.3	30,427	99.6	19,168	76.5	8,719	89.8	16,580	77.2	21,087	72.5	9,083	68.9	7,348	78.6
NORTHERN CAPE	1,185	69.6	1,099	54.2	3,336	50.0	1,575	68.4	10,404	99.5	6,361	69.3	4,350	88.1	6,460	61.9	7,745	69.3	2,798	59.0	2,259	66.9
WESTERN CAPE	7,690	75.0	771	74.1	21,267	64.7	8,957	74.5	31,475	99.4	20,927	82.5	20,159	93.2	25,937	74.7	35,604	79.1	15,418	76.0	10,387	79.5
NATIONAL	90,278	72.5	95,291	69.9	192,139	64.9	115,169	73.3	498,959	97.2	269,621	74.2	154,536	89.7	310,041	76.3	294,204	72.5	233,858	58.0	172,319	74.2

Source: (NSCE, 2018)

Research showed that, "several lecturers who taught first-year mathematics in 2009 reported under-preparedness of student". These under-prepared students, who entered various universities, were already at risk in the sense that they lacked basic mathematical concepts such as fractions, which served as a foundation for mathematics accomplishment (Engelbrecht, Harding & Phiri, 2010:4).

Studies showed that despite a slight improvement in 2018 Matric pass rate in the Eastern Cape, it still remained one of the worst performing provinces (NSCE, 2018). The Province came last in the provincial pass rate in 2015 with 56.8 per cent, in 2016 with 59.3 per cent, and in 2017 with 65 per cent. Eastern Cape, however, did slightly better than Limpopo with a pass rate of 70.6 per cent in the 2018 matric examination (NSC School Performance Report, 2018). Table 4 analysed the provincial pass rate from 2016 – 2018.

Table 4: provincial pass rate from 2016 - 2018 matric examination

Provinces	2016	2017	2018
Eastern Cape	59.3	65.0	70.6
Free State	88.2	86.1	87.5
Gauteng	85.1	85.1	87.9
KwaZulu-Natal	66.4	72.9	76.2
Limpopo	62.4	65.6	69.4
Mpumalanga	77.1	74.8	79.0
North West	82.5	79.4	81.1
Northern Cape	78.7	75.6	73.3
Western Cape	85.9	82.8	81.5

Source: (NSC School Performance Report, 2016, 2017 and 2018).

TIMSS was introduced to monitor trends, and it was administered in a four-year cycle. South Africa participated in five cycles, in 1995, 1999, 2003, 2011 and 2015. The TIMSS (2015) provincial grade nine mathematics result painted a gloomy picture about the Eastern Cape (TIMSS, 2015). The table below illustrated the performance of the various provinces:

Table 5: Difference in provincial performance in mathematics in TIMSS 2015

Province	Score
Gauteng	408
Western Cape	391
Mpumalanga	370
KwaZulu-Natal	369
Free State	367
Northern Cape	364
Limpopo	361
North West	354
Eastern Cape	346

Source: TIMSS 2015 Highlights of Mathematics and Science Achievement of Grade 9 South African Learners.

Research showed that South African learners' poor and varied performances in mathematics could not be linked to one cause only, but were multifaceted. These could be linked to manifold, sophisticated and related sets of issues, including the following: lack of facilities and resources at many schools, large class sizes, inadequate teacher qualification, poor learner commitment, indiscipline among learners, and inadequate parental involvement (CDE, 2014:2). Most of these could be attributed to the country's apartheid history, and the high levels of inequality in society. Mathematics education for blacks had suffered several setbacks in South Africa (Khuzwayo, 2000). Of the numerous factors affecting learners' achievement in mathematics in South Africa, poverty stands tall (Spaull, 2013).

However, as referred to the South African Schools Act, Act 84 of 1996 (SASA), the National Norms and Standards for School Funding (as amended) avowed that the provision of funds to schools was done with the main objective of effectively equipping the previously under resourced schools, and to bring all schools to equal levels in terms of provision of infrastructure and resources.

Schools in quintiles 1 and 2 obtained standard and sufficient Learning and Teaching Support Material (LTSM) for effective curriculum delivery, with the funds allotted to them. Resources are vital component for a school to run effectively. In his inaugural speech of the State of the Nation Address in 2011, the President of the Republic of South Africa, Mr. Jacob Zuma, reiterated that the three Ts (Teachers, Textbooks, and Time) were his major priority areas to improve the quality of education in South Africa (State of Nation Address, 2011).

Ann (2011), mentioned, among other data gathered in 2007, that most of the educators teaching Grade 6 mathematics in South Africa could not answer a question that their learners were supposed to answer in line with the Grade 6 curriculum. A study conducted by TIMSS 2011, concurred that 89 per cent of South African Grade 9 educators felt "very confident" in terms of mathematics teaching. This was in a sharp contrast to the best performing countries in the world, where mathematics educators in Singapore alluded to a 59 per cent confidence level, Finland, a 69 per cent confidence level, and Japan, a 36 per cent confidence level in teaching mathematics.

The confidence level of South Africa educators, however, does not reflect in the performance of Grade 9 learners in the Trends in International Mathematics and Science Studies (Mullis, Martin, Pierre Foy, & Arora, 2012). Research showed that complacency among mathematics educators in South Africa hinders any attempt to introduce innovation to the teaching of mathematics, since mathematics educators are very confident of themselves (Spaull, 2013). The CDE report, however, showed that mathematics instruction in most schools in South Africa was among the worst in the world. A study conducted, ranked learners in South Africa fourth, with a mean score of 48.1 per cent for literacy, and 30 per cent for numeracy (CDE report, (2014).

The National Mean percentage marks for mathematics in 2012, 2013 and 2014, from Grade 1 – 6, and Grade 9, in the Annual National Assessment (DoE, 2014), revealed a gloomy picture about learners' poor performance in mathematics. Table 6 illustrated the average percentage mark of learners in mathematics from grade 1- 6 and grade 9.

Table 6: Mathematics Average percentage mark from Grade 1 – 6 and Grade 9.

Grade	**Mathematics Average Percentage Mark (%)**		
	2012	2013	2014
1	68	60	68
2	57	59	62
3	41	53	56
4	37	37	37
5	30	33	37
6	27	39	43
9	**13**	**14**	**11**

Source: (DoBE, 2014)

Table 7 showed the Grade 9 mean percentage score in mathematics in the Annual National Assessment from 2012, 2013, and 2014 in the nine provinces in South Africa (Department of Basic Education, DoBE 2014):

Table 7: The average percentage mark in grade 9 mathematics 2012, 2013, 2014.

Province	**Average mark (%)**		
	2012	2013	2014
Eastern Cape	14.6	15.8	13.3
Free State	14.0	15.3	12.9
Gauteng	14.7	15.9	12.4
Kwazulu-Natal	12.0	14.4	10.7
Limpopo	8.5	9.0	5.9
Mpumalanga	11.9	13.7	11.3
North West	11.2	13.3	10.6
Northern Cape	13.2	12.6	9.7
Western Cape	16.7	17.0	13.0
National	12.7	13.9	10.8

Source: (DoBE, 2014).

However, the Annual National Assessment program used to assess learners came to a halt in 2014.

Van der Walt et al (2008), and Ndlovu (2011) were of the view that the abysmal performance of learners in mathematics, especially in fractions, in South African could be attributed to the lack of adequate learner support materials, the poor socio-economic background of learners, the medium of instruction, the lack of motivation, the poor quality of educators and inadequate study orientation.

Studies revealed that out of the 16,581 mathematics educators in the Eastern Cape, only 7,090 were really teaching mathematics in their various schools. Of these 7,090 mathematics instructors, 5,032 had no qualification in mathematics to teach the subject (Siyepu, 2013).

Research showed that, understanding fractions is an essential aspect of improving mathematics learning in the classroom. The Curriculum Focus point established in 2006 by The National Council of Teachers of Mathematics (NCTM) indicated that, in Grade 3, learners should have:

a. Improved upon the meanings and understanding of fractions, and demonstrated the representation of various types of fractions, such as fractions as parts of a set, parts of a whole, points, or distances on a number line;
b. Appreciated that the size of a part of fraction was relative to the whole size.
c. Appreciated that fractions could be used to demonstrate less than, greater than and equal to.
d. Demonstrated a high sense of problem-solving involving ordering and comparing fractions by using simulations, standard fractions, common denominators, or numerators;
e. Exhibited and comprehended the use of models, such as the number line to show equivalent fractions (The National Council of Teachers of Mathematics standards for mathematics, NCTM 2000).

However, the mathematics education literature concurred that, understanding fractions was a difficult aspect of the mathematics curriculum for both learners and educators (Hackenberg & Lee, 2012). Research divulged that even adults continued to struggle with the concepts of fractions, whilst fractions played an integral role in our daily activities (Reyna & Brainerd, 2007). The National Council of Teachers of Mathematics (NCTM, 2000), acknowledged that in the American National Test, it was only 50 per cent of Grade 8 learners in America who correctly arranged three fractions from the least to the biggest. Bruce and Ross (2009), argue that fractions involve difficult and complex aspect of mathematics, which make it difficult for educators to clarify the challenges learners face in fractions, and therefore make mathematics a challenging subject. In similar vein, Orpwood, Schollen, Leek, Marinelli-Henriques, and Assiri, (2011), are of the view that these problems with understanding fractions commence early in the basic school level, and proceed through to the elementary school level, then persist through to the secondary and tertiary levels of education.

Understanding fractions enables learners to establish a ground mathematical cognitive process, which includes proportional reasoning and spatial reasoning (Mamolo, Sinclair & Whitely, 2011). Empson and Levi (2011), argue that learning fractions was fundamental to the learning of algebra, since it gives learners the platform to establish basic mathematical connections that form the basis of algebra, as well as arithmetic. There are challenges associated with learning fractions. Some of these challenges include inadequate comprehension of different forms of fractions, and the ability of learners to simplify and deal with the missing elements, which are essential to learning algebra. Learners encountered the problem of understanding mathematical language associated with fractions, and the multidimensional nature of it (Hackenberg & Lee, 2012). Mathematics instructors are confronted with the problem of understanding and making meaning out of some of the concepts of fractions, and how to make the concept realistic to learners.

Yetkiner and Caprano (2009), also outlined the essences of fractions instruction with concrete materials. They reiterated the fact that, “fractional concepts were important building blocks of elementary and middle school mathematics curricula”. Their argument was supported by (Hounsell, 2009) who claimed that “conceptually based instructions of fractions demanded that educators ought to have mastery over the subject matter”. Also, Hounsell, (2009), avowed that educators who made use of concrete manipulative materials in their classroom instructions, ought to display an in-depth conceptual understanding of the subject content which is imparted unto their learners. Yetkiner and Caprano (2009), concurred that the mathematics educator must endeavour to build a connection between the mathematical concept that was to be mastered, using the appropriate concrete materials and the appropriate methodology. When an educator applies the manipulative concrete materials to connect the two types of knowledge, they could then be said to be an important and instructive mathematical material.

1.3 STATEMENT OF THE PROBLEM

Study showed that fractions are challenging topics that educators and learners are confronted with on daily basis (Tobias, 2013). Most educators have little knowledge of fractions necessary for classroom instructions (Harvey, 2012). The Centre for Development and Enterprise (CDE, 2011), indicated that South African learners’ poor performance in national assessments in mathematics could be linked to teacher’s poor content knowledge, and lack of innovative methods of fractions instructions. In support, Davis (2016) concurred that there existed a gap between the way learners experienced fractions in the out-of-school, and in-school settings. The school mathematics curriculum had not made the concepts of fractions relevant to learners in their everyday life activities. Sharing, which formed the basis of the introduction of fractions as division in schools, was also widely practiced in the out of school environment. It was evident that learners lacked the knowledge of linking the concept of fractions at home to the concept of fractions at school due to the physical properties that were used in the instructions (Davis, 2016).

Fractions played an important role in our technological world because our daily lives heavily relied on the ability to compute fractions correctly, competently, and insightfully (Pienaar, 2014). Also, fractions formed the fundamental blocks for future success in mathematics (National Mathematics Advisory Panel (NMAP), 2008).

John Van de Walle, Karp, and Bay (2013), described manipulative concrete material as a mathematical tool or, any item, image, or drawing that embodied an idea, or onto which the connection for that concept could be enacted. In a similar view, manipulative concrete materials are physical objects that could be utilized to demonstrate and undraped mathematical ideas such as fractions.

Research showed that manipulative concrete materials were used by mathematics educators to simplify abstract mathematical theories, made the concept of fractions real to learners, and also filled the gap between out-of-school and in-school settings (Lira & Ezeife, 2008). Lee, (2014) supported the idea that, manipulative concrete materials helped educators in imparting mathematical knowledge to learners, and increased the mathematical understanding of fractions.

However, Maslen, Douglas, Kadosh, Levy and Savulescu, (2014), were of the view that manipulative concrete materials were potentially harmful if used improperly. Improper use of manipulative concrete materials could convince learners that, two mathematical worlds exist: concrete materials and symbolic (Milgram & Wu, 2008). Ormrod, (2014), in his study, argued that manipulative concrete materials used in teaching could make learners uncontrollable, as they could get overly enthusiastic using the physical objects. He further supported his argument by stressing on the fact that, poorly designed lessons, unmotivated learning materials, or unclear expectations, could lead to behavioural problems in classroom which could make class control very difficult.

Studies showed that for every student’s mathematical proficiency in fractions, concrete and virtual manipulative must be systematically integrated into classroom instruction at all grade levels (NCSM, 2013). Adelabu, Adu, and Adjogri (2014), observed that, the use of Information Computer Technology (ICT) to assist in mathematical instruction and learning remained underutilized. They further emphasized that, e-learning was the best representation of the internet to support the provision of skills and knowledge in a universal approach, not limited to a specific subject, and that technologies, or infrastructure, encompassed all media employed in transmitting video, audio, data or multimedia such as cable satellite, fibre optics and wireless. In South Africa, learners were provided with tablets and internet facilities which enabled them to study fractions at home after school hours.

Research revealed that learners' performance in mathematics in South Africa was persistently low (DoBE, 2014). Learners lacked the ingenuity of solving mathematical problems, especially in fractions. NSCE, (2018) highlighted learners who achieved 30 per cent and above pass rate in mathematics, from 2014 to 2018, as follows: 2014 (53.5%); 2015 (49.1%); 2016 (51.1%); 2017 (51.9%) and 2018 (58.0%). Chris Hani West Education District was not immune to learners' poor performance in mathematics. The analysed mathematics matric results in the Chris Hani West Education District revealed that in 2015, out of the 2,028 learners who sat for the exams, only 833 learners passed with 41.1 per cent rate, in 2016, 767 learners passed out of the 1819 learners that wrote the exams, recording 42.2 per cent , 835 learners passed out of 1713 learners who wrote the exams in 2017, giving a 48.7 per cent pass rate, while in 2018, out of the 1707 who wrote the exams, 871 passed, recording a 51.0 per cent pass rate (Eastern Cape Department of Education ECDoE, 2018). The poor performance of learners over the years could be related to the learners' inability to solve fractions. Empson and Levi (2011) revealed that fractions are fundamental to solving algebraic equations. Acquaintance of fractions aided in the proficiency of algebra, and, in effect, prepared learners for the advanced level of education, and also provided vocations in Science, Technology, Engineering, and Mathematics (STEM) field (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014, Siegler et al. 2012).

Grade nine was chosen for this study, due to their abysmal performance in mathematics in the cross-national assessment program. Also, grade nine formed the link between primary and high school, and the problems of learners' mathematics, especially fractions, needed to be investigated and addressed before it proceeded to the higher grades where it would be difficult to rectify. Given the proclamations made by investigators, the researcher was encouraged to investigate the effect of the use of manipulative concrete materials on grade 9 learners' performance in fractions in Chris Hani West Education District of the Republic of South Africa.

1.4 HYPOTHESES

The following null hypotheses were tested at 0.05 level of significance:

i. H_{01}: There is no significant relationship between the use of Cuisenaire rods and grade nine learners' performance in fraction.
ii. H_{02}: There is no significant relationship between the use of Fraction bars/Fraction tiles and grade nine learners' performance in fraction.
iii. H_{03}: There is no significant relationship between the use of Paper folding and grade nine learners' performance in fraction.
iv. H_{04}: There is no significant relationship between the use of computer assisted manipulative and grade nine learners' performance in fraction.

1.5 RESEARCH QUESTIONS

Main research question

What is the effect of the use of manipulative concrete materials on grade nine learners' performance in fractions?

Sub-research questions

i. What is the relative effect of the use of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative on grade nine learners' performance in fractions?
ii. What is the composite effects of the use of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative on grade nine learners' performance in fractions?
iii. What is the predicted effect of the use of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative on grade nine learners' performance in fractions?

1.6 OBJECTIVES OF THE STUDY

This study aimed to:

i. Examine the relative effect of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative on grade nine learners' academic performance in fractions.

ii. Examine the composite effect of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computer assisted manipulative on grade nine learners' academic performance in fractions.

iii. Examine the predictor effect of Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computer assisted manipulative on grade nine learners' academic performance in fractions.

1.7 SIGNIFICANCE OF THE STUDY

This study looked at the effects of the use of manipulative materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding, and Computer assisted manipulative) on grade nine learners performance in fractions. The study highlighted the effects and appreciation of the use of manipulative concrete materials in the instructions of fractions in mathematics among learners in South Africa.

The researcher anticipated that the study would contribute to the body of knowledge in mathematics education in assisting educational planners, policy makers, and governments in the instructions of fractions for both educators and learners.

The researcher could use the findings from the study to develop seminars and workshop training for provincial education directors, district directors, principals, government agencies, NGOs, parents and learners in other provinces, in South Africa and the world at large.

1.8 SCOPE OF THE STUDY

The scope of this study was restricted to Grade 9 learners and educators teaching grade nine mathematics in public High Schools in Chris Hani West Education District in the Eastern Cape of the Republic of South Africa.

1.9 DEFINITION OF TERMS

The following terminologies were used in the study.

Educator:

Somebody, such as an instructor, or a school administrator, who worked at an educational institution (DoE, 2015). An educator is a person who facilitated and imparted knowledge into a learner in an educational environment.

Learner:

A person who has undergone formal education at a school, or a recognized institution (DoE, 2015). A learner is a person who has received tutelage from an educator in an educational environment.

Manipulative concrete material:

A manipulative concrete material is a physical object that can be touched, felt, moved around by learners, appealed to the faculties of the senses, and also conveyed a mathematical knowledge (Swan & Marshall, 2011). A manipulative concrete material is an object that can be felt, touched and used in teaching and learning. They are in different shapes, sizes and colours.

Fractions:

Fractions are an aspect of rational numbers which are expressed in the form $\frac{a}{b}$ where "a" and "b" are both integers, and "b" is not equal to 0 (Olanoff, Lo and Tobias, 2014).

A fraction is a part of a whole, and it includes a proper fraction, an improper fraction, and a mixed fraction.

Grade 9:

Grade 9 is the last stage of GET, which laid the foundation for FET that starts from grade 10-12 (Department of Basic Education, (DoBE, 2013). It is the grade after the completion of grade 8.

1.10 THEORETICAL FRAMEWORK

Theoretical framework is the 'blueprint' or guide for a research work (Osanloo & Grant, 2014). The researcher based his study on the cognitive development theory and constructivist theories for the study. Cognitive development theory is the process of receiving information through the faculties of the mind, and the construal of the information (Donald, Lazarus, Lolwana, 2010:58; Robinson & Lomofsky, 2010:34). Cognitive theory enables learners to make intellectual decisions by involving all the mental faculties to learn, and also to make meaningful discernment. Cognitive development is employed in this study to help the researcher access the thinking process of learners to use concrete materials in solving fractional problems in mathematics. Another theory used in the study was constructivist theory. In constructivist theory, meanings and interpretation of information were best explained through the individual's own experience and clarifications associated with certain elements for better understanding. These elements are diverse and multifaceted in nature, resulting in the multiplicity of views rather than restricting the understanding of ideas (Creswell, 2013:8). The important aspect of this approach is the disintegration of each mathematical idea into a progressive aspect, in line with Piagetian theory of cognitive development, which is based on observation and interaction with learners as they acquire new knowledge of understanding (Mathforum, 2015). Paily (2013) explained that, "in a constructivist learning environment, the role of the educator is to facilitate and guide the knowledge construction process by engaging learners in meaningful learning". In constructivism, learners are persistently assessed with assignments that challenged them to acquire new knowledge and understanding, different from what they already know. As the constructivist philosophy concurred, learners are central to their own learning, and how they perceived their learning environment. Learners understand their role in class and their interaction with the educator, other learners, and the content (Stefl-Mabry, Radlick, & Doane, 2010).

1.11 LITERATURE REVIEW

The literature review outlined the basis for establishing the study, as well as the standard for equating results with other findings (Creswell, 2015). In this study, the researcher reviewed his literature in line with what scholars had already written on "the effect of the use of manipulative concrete materials on learners' performance in fractions".

1.12 RESEARCH METHODOLOGY

Research methodology is the method adopted by the researcher to carry out the research (Jonker & Pennink, 2010). It involved the practical considerations to which the researcher structured his research, given the questions he wanted to answer. Creswell, (2013:16), argued that research methodology involved the procedure of data collection, analysis and interpretation that researchers anticipated for their research work. The research methodology for this study comprised a research paradigm, a research approach, a research design, population and sample, sampling technique and data analysis. The researcher employed a quantitative research method.

1.13 POPULATION AND SAMPLING TECHNIQUE

The target population for this study consisted of all grade nine (9) learners in Chris Hani West Education District. Forty (40) public high schools were selected out of eighty-nine (89) high schools, which were combined schools (i.e. high school and primary school which had grade nine) for the study through multiple sampling techniques of stratified, and convenience methods. A sample of two hundred and fifty (250) grade nine (9) learners, and ten (10) educators teaching Grade 9 mathematics were also selected through multiple sampling techniques of stratified, systematic sampling, purposive and convenience sampling method.

1.14 DATA ANALYSIS

Numerical data collected were coded, sorted and categorized to find Percentages, Mean, and Standard Deviation. The t-test was used to test the hypotheses raised in the study to find the significant effects between the manipulative concrete materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative) and grade nine learners' performance in fractions. The hypotheses were tested at 0.05 level of significance.

1.15 ETHICAL CONSIDERATION

Voluntary Participation: the researcher ensured that participation in the research was completely voluntary. Thus, nobody was forced to participate.

Consent of the participant: the researcher sought the consent of respondents before administering the research instrument. This was done by providing respondents with consent forms which were filled before administering the questionnaires.

Anonymity of the participant: the researcher ensured that the identities of the respondents were not identified on the forms. Names of the respondents were not provided.

Confidentiality of respondents: confidentiality of the respondents was ensured by the researcher. Thus, the researcher held respondent's information as confidential as possible, and the information provided was used only for the intended purpose.

Permission: permission was sought from the Department of Education and the principals of High schools from which the research was carried out. This was done by taking an introductory letter from the researcher's supervisor introducing the researcher as a student of University of Fort Hare to the District director of Education, and to the principals of the various high schools.

1.16 CHAPTER ORGANIZATION

Chapter one: consisted of the introductory section of the study. It encompassed the background of the study, statement of the problem, objectives of the study, significance of the study, scope of the study and organization of the study.

Chapter two: was made up of the literature review. The chapter began with an introduction, followed by the literature of the study.

Chapter three: was made up of the research methodology; it defined the research design, the study area, study population, sample size, sampling techniques, sources of data, data collection instruments and technique of data analysis.

Chapter four: encompassed of the presentation of analysed data of the findings of the study.

Chapter five: dealt with the summary of the analysed results after which conclusions were drawn. Commendations were made to the government, department of education, Non-governmental Organizations, principals, educators, and learners based on the findings of the researcher.

1.17 CHAPTER SUMMARY

The chapter presented the background of the study, statement of the problem, research hypotheses, research questions, objectives of the study, significance of the study, scope of the study, and organization of the study. Terms were defined according to how they were used in the study. Chapter one provided the blueprint of the study by looking at the state of mathematics among learners in various continents such as Europe, America, Asia and Africa. The chapter briefly discussed the effect of the use of manipulative concrete materials in fractions. Also, the chapter made a forensic analysis into learners' performance in mathematics in South Africa as a whole, and narrowed it down to the Eastern Cape Province, and to the Chris Hani West Education District where the researcher carried out his studies. Chapter two delves into detailing the theoretical frame work and the literature relevant to the study.

CHAPTER TWO

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 INTRODUCTION

The literature review summarized and analysed previous studies related to the current study (Creswell, 2015). In this research, the researcher reviewed literature on what scholars had written about the effects of the use of manipulative concrete materials on fractions instructions. The literature was grouped under two main categories; the theoretical framework, and the empirical framework. It also dealt into detailing the conceptualization of fractions and the conceptualization of manipulative concrete materials on fractions. Research showed that the knowledge acquired from the literature helped the researcher to compare the outcome of the results of the study to previous knowledge.

2.2 THEORETICAL FRAMEWORK

Grant and Osanloo, (2014) concurred that theoretical framework is the 'blueprint', or a framework, for a research study. It is the basis on which an existing theory in a field of study is established, which correlated to the hypotheses being investigated. In similar vein, Fulton and Krainovich-Miller (2010) related the function of the theoretical framework to a road map. The theoretical framework guided the researcher to meet the international standard of the theories which made his final contribution scholarly. Brondizio, Leemans, and Solecki (2014) asserted that the theoretical framework comprised precise theories that dealt with human endeavour which could be useful in the study of events. Grant and Osanloo, (2014) highlighted that the theoretical framework encompassed: ideologies, paradigms, concepts, and theories. All aspects of the research work are interlinked to the theoretical framework. Imenda (2014) submitted that a research without the theoretical framework rendered the study inconclusive, because it would lack the precedence on which appropriate literature and academic debates are established for the current study. In this study, the researcher reviewed various theories of scholars and adopted Cognitive Development Theory and Constructivism Theory, which best fits the study.

2.2.1 The essence of theoretical framework in research

The essence of theoretical framework in a research work are underpinned on the following:

- ❖ It helped the researcher define his study theoretically, epistemologically, procedurally and logically (Grant & Osanloo, 2014).
- ❖ It guided academics in conducting and formulating theories into their research work which made their work scholarly (Ravitch & Carl, 2016).
- ❖ It made academic investigation important and generalizable (Akintoye, 2015).
- ❖ It provided a common view about a problem to be investigated, carried out a research and analysed data to provide sufficient evidence for other scholars in the field (Grant & Osanloo, 2014).

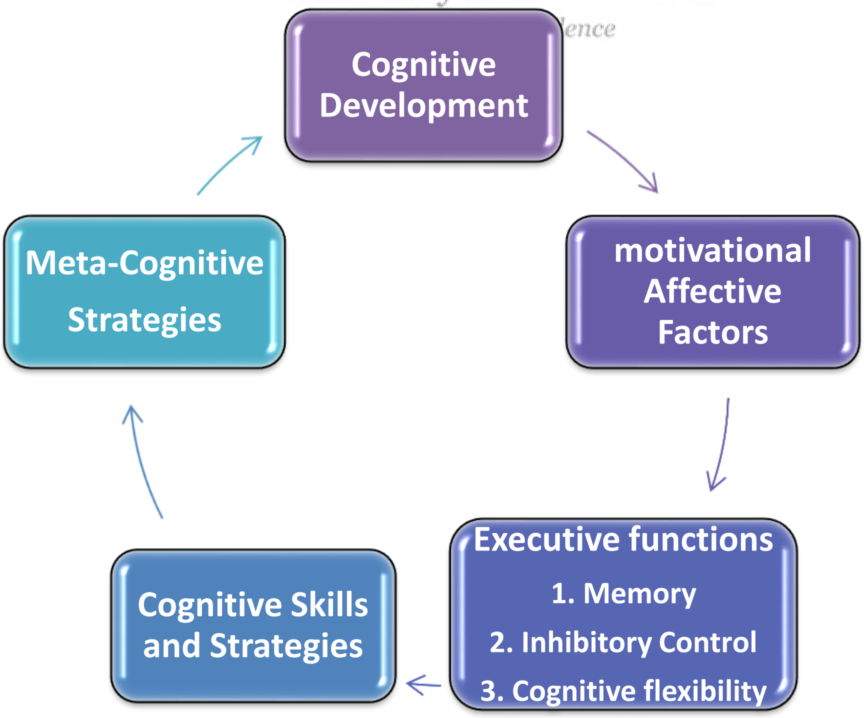
Simon and Goes (2011) outlined some important key points that underpinned the theory for an informed research. These included:

- ❖ Identifying the main elements that goes into the research work.
- ❖ Itemizing the main elements and variables that might be important for carrying out the research work.
- ❖ Reviewing the literature by including the 'theory' and the theorists that have contributed to knowledge in line with what the researcher is reaching for.
- ❖ Ascertaining the key variables in the research.
- ❖ Acknowledging other theories that challenged the viewpoints of the researcher.
- ❖ Bearing in mind how the variables are related to the theory.
- ❖ Reading and revising contemporary literature that is linked to the topic under discussion by using specific search words.
- ❖ Making provisions for limitations that come with the selected theory in line with the problem to be investigated, and provided logical explanations to ameliorate the problem.

2.2.2 Conceptualization of Cognitive Development Theory

Cognitive development theory is the process of receiving information through the senses, as well as the clarification of the information (Donald et. al., 2010:58; Robinson & Lomofsky, 2010:34). De Witt (2011), defined Cognitive development as the ability to make intellectual judgement through the process of involving all the mental faculties to learn, pay attention, recall, verbalize, make meaningful discernment, innovation and ingenuity. In similar vein, researchers argued that cognitive development is the ability of modifying mental capabilities or skills, such as; language, learning, thinking, attention, creativity, and reasoning (Lerner & Johns, 2009:153; Papalia, Wendkos Olds, Duskin Feldman, 2008:10). Rapid mental development among children takes place between the ages of 0 to 9. During this age, period children are able to address problems associated to cognitive development (Lerner & Johns, 2009; Rademeyer, 2007, Lerner, 2006). The development of intellectual abilities and skills are important in solving problems, making effective decisions and transforming passive, dependent learners into dynamic enthusiastic learners who could apply their cognitive ability into an extensive range of real life situations (Donald et al., 2010:58; Eggen & Kauchak, 2010:30; Benjamin, 2009; Lerner & Johns, 2009:164). Cognitive and metacognitive skills and approaches, as well as the dynamics to deal with motivational-affective elements, are rooted in the attainment of reasoning ability. This is illustrated in the figure 1;

Figure 1: A Conceptualization of cognitive development

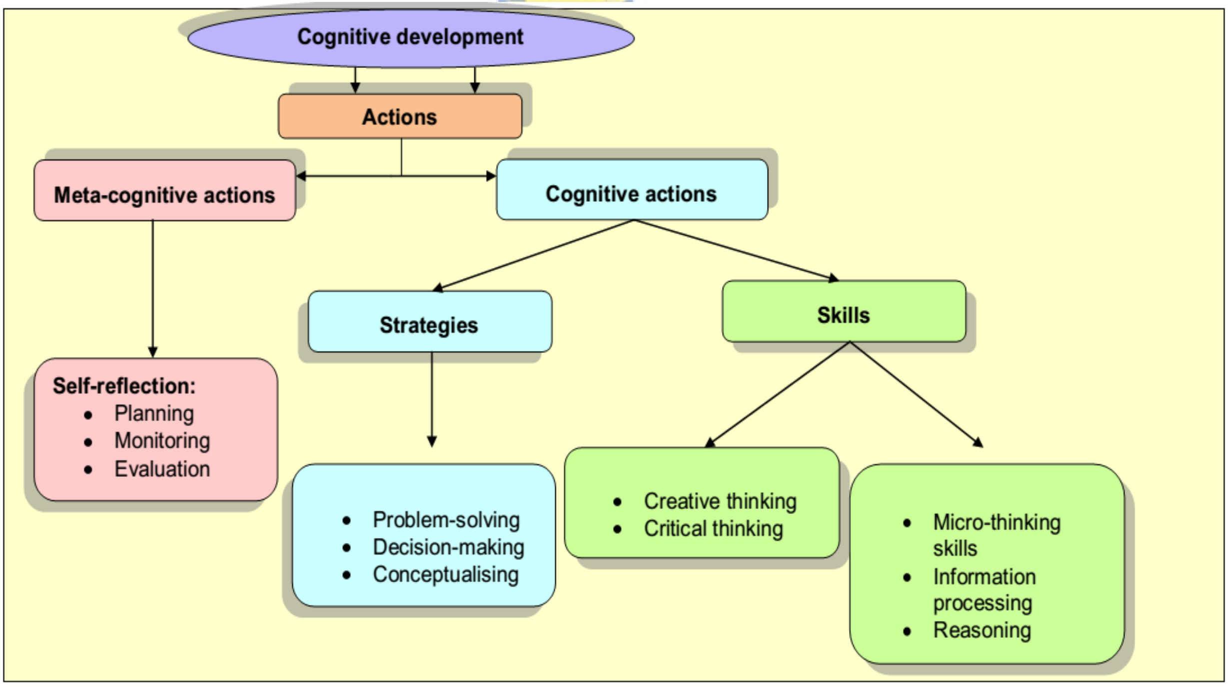


Source: Esterhuizen, (2012).

Mental ability could be classified under lower order (ability for memorizing information) and higher order (ability to order, classify, combine, examine, and think logically and innovatively and to assess information) (Brewer, 2007; Wegerif, 2006). On the other hand, Metacognition entails an individual's awareness of, and control of, their cognitive process when faced with cognitive challenges. The Metacognitive approach assisted learners to become well informed learners, who could strategize, regulate and assess their cognitive ability (Fisher, 2005). In a similar vein, Fitzpatrick, (2012) reiterated that the importance of Metacognition is to cognitively self-regulate practices to withstand obligation, engagement and perseverance during learning, remaining fixated on tasks and being flexible to adjust to diverse task demands.

Research showed that cognitive development is a pattern of change in mental abilities or skills, such as learning, thinking, memory, attention, creative and reasoning (Lerner & Johns, 2009:153; Papalia et al., 2008:10). This pattern of change of metal ability is illustrated in figure 2;

Figure 2: A chart of cognitive development.



Source: Esterhuizen, (2012).

Cognitive ability and developments are needed in problem solving, to make effective decisions in transforming passive learners into active self-motivated learners who could apply their intellectual ability into a wide range of daily problems (Donald et al., 2010:58; Eggen & Kauchak, 2010:30; Benjamin, 2009; Lerner & Johns, 2009:164). The researcher adopted this theory in his study because he wanted to assess how learners apply their cognitive abilities in the use of manipulative concrete materials in solving mathematical problems involving fractions.

Different developmental cognitive psychologists have given their views on the cognitive development of learners. Beneath are the following:

2.2.3 Jean Piaget's perspective of cognitive development

Cognitive development theory was first propounded by the Swiss developmental psychologist Jean Piaget (1896 – 1980). Piaget asserted that cognitive development is an incessant process through which children exhibited and comprehended certain concepts at certain developmental stages of their lives. These stages included the Sensory-motor stage (birth - 2 years), the Pre-operational stage (2years – 7years), the Concrete operational stage (7years – 12years) and the Formal-operational stage (adolescence – adulthood) (Awudetsey, Grosser, Karstens, Lombard, Meyer, 2010: 63-68; Byram & Dube, 2008: 14-19, Berger, Kathleen & Stassen, 2008).

2.2.3.1 The Sensorimotor stage

This is the early phase of cognitive development which extends from birth to the acquisition of language (Tuckman, Bruce, & David 2010). In this phase, children increasingly create knowledge and understanding of their environment by coordinating experiences such as vision and hearing, coupled with physical communications with items such as grasping, sucking, and stepping (Bernstein, Penner, Clarke-Stewart, & Roy 2012). Infants acquire intellectual knowledge of their environment through the physical activities they perform within it (McLeod, 2015). They advance from reflexive, intuitive action at birth to iconic representation of ideals toward the end of this phase of development (McLeod, 2015).

2.2.3.2 Pre – Operational stage

The pre – operational stage starts from age 2 – 7. At this phase of development, toddlers begin to learn how to communicate. During this second phase of cognitive development, infants cannot make tangible and logical reasoning, and cannot intellectually operate information reasonably (McLeod, 2015). Children are pre-occupied with playing and fantasising during these stages. Children's play at this stage is characterized symbolically. Coupled with operation of objects. Children play is demonstrated by using objects to represent the real world, such as using leaves of plants as plates, using toys as babies and acting like father and mother. Children's world of symbols exemplifies the idea of play with the absence of the real materials (Loftus & Geoff, 2009). Santrock and John (2004) are of the view that the pre-operational phase is categorized into two sub – stages: the iconic function sub-stage, and the intuitive thought sub-stage. The iconic function sub-stage is the stage at which children are able to comprehend, symbolise, recall, and represent images in their cognitive without having the real object in front of them. On the other hand, the intuitive thought sub-stage is the enquiry stage where children ask questions "why"? And "how come"? This stage is the inquisitive stage where children want to make meaning of everything (Santrock & John, 2004).

2.2.3.3 Concrete operational stage

The concrete operational stage is the third phase of Piaget's philosophy of cognitive development. This stage is between the ages of seven and eleven years. In this phase of development, children make use of suitable and logical reasoning (Ginsburg & Opper, 1979). An individual's thinking level becomes more advanced and "adult like". They begin to solve challenges in a more intellectual manner. However, children lack abstract or imaginary thinking, and can only apply their minds to challenges that require the manipulation of concrete objects. At this phase of development, children go through different transitional periods where they learn rules of preservation (Concrete Operation, 1993). Piaget (1972) observed that children at this stage are able to integrate Inductive thinking into whatever they do. Inductive thinking encompasses the representation of inferences from observations to make a conclusion. Children at this stage of development are faced with challenges of figuring out intellectual reasoning. Piaget (1972) concurred that children in the concrete operational stage are able to integrate inductive reasoning to whatever they do. Children at the concrete operational phase of development are usually challenged with the use of logical reasoning, which encompasses the use of universal opinions to forecast the result of a particular event. However, children lack the ability of thinking abstractly, which is more prevalent among adults. Santrock (2008) observed that teenagers are able to verbally solve problems, without laying hands on concrete objects. In this instance, teenagers begin to reason like a scientist by developing strategies to solve challenging and scientifically testing opinions. Adolescents often use hypothetical-deductive thinking, which employs assumptions and systematically draw a conclusion, which is the best approach in solving mathematical problems (Santrock, 2008).

2.2.3.4 Testing for concrete operations

Piagetian tests are universally known and accomplished testing for concrete operations. The most predominant tests are those for conservation. Researchers argued that the most important aspects for testing for conservation is the water level task (Tran & Formann, 2005). This could be demonstrated by using two glasses that were of the same size filled with water to the same level which was acknowledged by the learners that the water levels are the same. The experimenter then emptied one of the small glasses into a tall, thin glass. The experimenter then enquired from the learners if the taller glass has more water, less water, or the same amount of water.

- **Justification:** the response the learners gave indicated to the experimenter what the learner concluded about the experiment. Karplus and Lavatelli (2010) asserted that the response of the learner is important, because it enables the examiner to evaluate the learners’ cognitive level.
- **Number of times asked:** researchers claimed that when a learner agreed that the water in the first set of glasses was equal in the first instance, and the same amount of water was emptied into the taller glass, the learner tends to doubt that the amount of water in the taller glass was the same. McLeod (2015) observed that the learner then doubts that the original levels were not equal,

which influenced their thought of the same amount of water being equal in the taller glass.

Word choice: researchers argued that the choice of words that the experimenter used may influence the response learners provided. If, in the liquid and glass demonstration, the experimenter asked, “which of these glasses had more liquid?’’, the learners may assume that their thought of them being equal was erroneous, because the examiner was alluding that one must be more. On the other hand, if the examiner asked, “Are these equal?”, then the learner presumably is going to say that they were, because the examiner was alluding that they were equal.

2.2.4 Formal operational stage

According to Piaget (1972), the formal operational stage is the concluding phase of the cognitive development. It ranged from adolescence to adulthood. In this phase of development, intelligence is established through the intellectual use of concrete associated material to abstract ideas. This form of thought involves “assumptions that are not necessary to reality”. During this stage, an individual could reason hypothetically and deductively. It also involved the ability to think abstractly. Piaget argued that “hypothetic deductive reasoning” is significant through the formal operational phase. This form of reasoning includes hypothetical thinking: “what-if” conditions are not practical. It is often applicable in science and mathematics.

2.2.4.1 Abstract thought: occurred during the formal operational stage. At this stage, learners reasoned concretely and precisely in their earlier stages, and begin to study likely results and consequences of actions.

2.2.4.2 Metacognition: is the ability of “thinking about thinking”, which enabled adults and adolescents to think about their mental processes and monitor them (Arnett, Jeffrey, & Jensen, 2013). Metacognition is also defined as an individual responsiveness of and regulation over his intellectual abilities, meta-attention and attentiveness (Donald et al., 2010:82; Eggen & Kauchak, 2010:217; De witt, 2009:14, 55; Learner & Johns, 2009:172-175). Meta-cognition starts in children between the ages of four and six years (Robson, 2006:84). Meta-cognition could be established in learners by making them conscious of **what** they think, and **how** they think. Nursery learners are expected to be taught the Developmentally Appropriate Practices (DAP) (information of learner's requirements and abilities at various phases of the developmental levels of learning) through which they developed knowledge about how they learn to advance their meta-cognition skills and tactics (Eggen & Kauchak, 2010:219; Brewer, 2007:4).

Problem-solving is observed when learners use a trial-and-error method in solving mathematical problems. However, mathematics deals with the manipulation of mathematical problems in a reasonable and procedural manner. Researchers are of the view that most learners applied the inductive reasoning during their primary school years. In their adolescent years, learners reasoned deductively to draw precise deductions from abstract ideas. This ability results from their ability to reason theoretically (Berger, 2014). Different studies showed that there was an absence of meta-cognition consciousness among mature learners (Eggen & Kauchak, 2010:219). Bolani, Pissarra, Hendricks, Swanepoel, Opie-Jacobs, (2007:2-4) asserted that South African learners lacked the ability to think analytically, even at the high school level. It is therefore imperative for educational policy makers to focus on the developmental of meta-cognition skills among learners at the pre-school level. Research showed that children who knew how to study, performed better academically than others (Eggen & Kauchak, 2010:217; Papalia et al., 2008:365-366). Meta-cognition encompasses: Cognitive strategies, Cognitive actions and Cognitive skills.

2.2.4.2.1 Cognitive strategies

Cognitive strategies are made up of advanced level of thinking skills which involve sophisticated methods of problem solving, forming judgement and information conceptualization (Epstein, 2008:40; Meltzer et. al., 2007:165; Learner, 2006:103-188). Researchers are of the view that cognitive strategies are at variance to **ability,** in the sense that ability is a subject of capacity, whereas **cognitive strategy** deals with **habit**. Cognitive strategies and styles showed an individual's ability of systematized or assimilated information needed to do different tasks (Epstein, 2008:40; Learner, 2006:103). Research showed that children begin to use different approaches in solving mathematical problems at the age of four. This development proceeds to adulthood with the guide of their cognitive schemes. (Eggen & Kauchak, 2010:219; Brewer, 2007:4).

2.2.4.2.2 Cognitive actions

Cognitive actions are reasoning activities that are considered as intellectual actions that include thinking, making judgement and solving problems (Brewer, 2007: 29; Robson, 2006:9). Cognitive actions are actions that are significant for tutors to appreciate the cognitive actions of students to cultivate proper learning skills.

2.2.4.2.3 Cognitive skills

Cognitive skills are defined as the metal activities of an individual that include cataloguing, organising, evaluating and inferential reasoning (Brewer, 2007:29; Robson, 2006:9). Young learners need to acquire knowledge and understanding that interconnects to the acquisition of the basic intellectual skills needed to master the basic concepts required to interpret and understand, in order to learn successfully (De Witt, 2011; Hansen, 2009:11). The National Curriculum Statement of South Africa, and the Department of Education, stipulated in the policy document, that the cognitive and meta-cognitive abilities and approaches of pre-school learners should be directly linked to abilities that were talked about in the Children's Inferential Thinking Modifiability Test (CITM) (Department of Education, 2002:1; Tzuriel, 1990:2-11). When these abilities and approaches are well harnessed at the pre-school level, it goes a long way to prepare learners mentally, physically, and emotionally for high school mathematics. The Table 8 showed the skills and strategies list of pre-school learners.

Table 8: A Cognitive and meta-cognitive skills and strategies of pre-school learners.

National Curriculum Statement	CITM
☺ Paying attention	☺ Concentration
☺ Remembering	☺ Storing of information
☺ Interpreting	☺ Inferential thinking
☺ Classifying and categorising	☺ Considering different aspects of the data, rules of elimination, negation, search for objects
☺ Comparison	☺ Eliminating clues, comparative ability, simultaneous consideration, negation
☺ Analysing	☺ Gathering information systematically
☺ Problem-solving	☺ Solving inferential problems
☺ Evaluating	☺ Reflection, coping with complex presentation of information
☺ Inferring principles and deducing rules	☺ Inferential thinking, transfer
☺ Imagining possibilities	☺ Transfer of strategies and rules
☺ Generating strategies	☺ Rules of elimination and negation, transfer of strategies and rules
☺ Critical evaluation and reflection	☺ Improving general efficiency of performance

Source: DoE, 2002.

2.3 Lev Vygotsky perspective of cognitive development

Lev Vygotsky, a Russian psychologist, although approved of Piaget's assertion that children construct their own understanding of the world by actively participating in the learning process. He disagreed, however, with Piaget's cognitive developmental changes in children. Vygotsky argued that language, the communal and traditional atmosphere in which a child grows up, plays a vital role in their cognitive development (Lerner & Johns, 2009:179; Patterson, 2008:25; Brewer, 2007:9; Louw et al., 2004:90-91; Vygotsky, 1986:13). According to Vygotsky, the socio-cultural development of children depended mainly on well-educated, successful and knowledgeable members in the society. Children develop cognitively through the assistance of well-educated people in the community. Their attitude, and way of life in the society, have great influence on the young generations. They become role models to the young learners. In effect, good attitude replicated good cognitive behaviour in the young learners, and bad attitude is also replicated in their lives. (De Witt, 2009:55; Patterson, 2008: 24; Meintjes, 2007:126; Donald et al., 2006:59; Lerner, 2006:189; Louw et al., 2004:90-91; Vygotsky, 1986: 13-18). This is in a sharp contrast to Piaget's assertion of children as being self-regulating, individual learners. Vygotsky argued that mediation is the process through which effective learning occurred. He reiterated that knowledge is established through social communications between the learner and the facilitator (a parent, an educator, a well-informed colleague), that the learner internalised the information resulting in the development of more sophisticated cognitive processes. This mediation occurred in the "Zone of Proximal Development" (ZPD), which he described as the "distance" between the already established mental buildings of the student (the "actual developmental level"), and the "potential developmental level" (Vygotsky, 1986: 13-18). Vygotsky is of the view that a critical space of possible development, or the Zone of Proximal Development (ZPD), is a stage where children could not do anything meaningful on their own without the guidance of an experienced educator (Patterson, 2008:301; Papalia, et. al., 2008:363, Meintjes, 2007:126; Donald et al., 2006:59; Lerner, 2006:189; Vygotsky, 1986: 13-18). Researchers are of the opinion that the dichotomy between Piaget and Vygotsky's cognitive development is that children are actively adapted to their environment "from the inside out", while Vygotsky asserted that children learn from the tutelage of an experienced adult, thus learning "from the outside in". (Patterson, 2008:300; Donald et. al., 2006:83; Robson, 2006:17).

2.4 Jerome Bruner perspective of cognitive development

Although Jerome Bruner approved with certain developmental stages in Piaget's cognitive developmental studies. Which begins from embryonic stage to adulthood such as enactive, iconic and symbolic stages, (Byram & Dube, 2008: 59-64), study revealed that the cognitive developmental stage proceeds from the symbolic learning phase to developing intellectual images, and finally, abstract knowledge through language. By contrast, Bruner endorsed discovery learning as the ultimate for intellectual development in learners (Byram & Dube, 2008: 59-64). Discovering learning involved, *inter alia,* formulating and testing theories rather than merely acknowledging the tutor's presentation. Discovery learning encourages and motivates learners to discover for themselves the underlying rules and principles regarding a concept. One unique aspect of discovery learning is the classroom atmosphere where mistakes are regarded as learning opportunities for learners and learners are free to express themselves without intimidation from their peers and educators (Byram & Dube, 2008: 59-64). Bruner is of the view that learners should constantly interact with their environment to discover for themselves. Discovery learning theory placed much emphasis on learning by participating and innate reasoning, where learning is a vigorous and unmethodical process, while reasoning is unstructured and unplanned. Byram and Dube, (2008) and Meintjes, (2007), argued that discovery learning increased learners' confidence and self-dependence for cognitive development. In this study, learners discovered for themselves, the effective use of concrete manipulative materials on their performance in fractions through the use of their cognitive abilities.

2.5 Importance of Cognitive Development

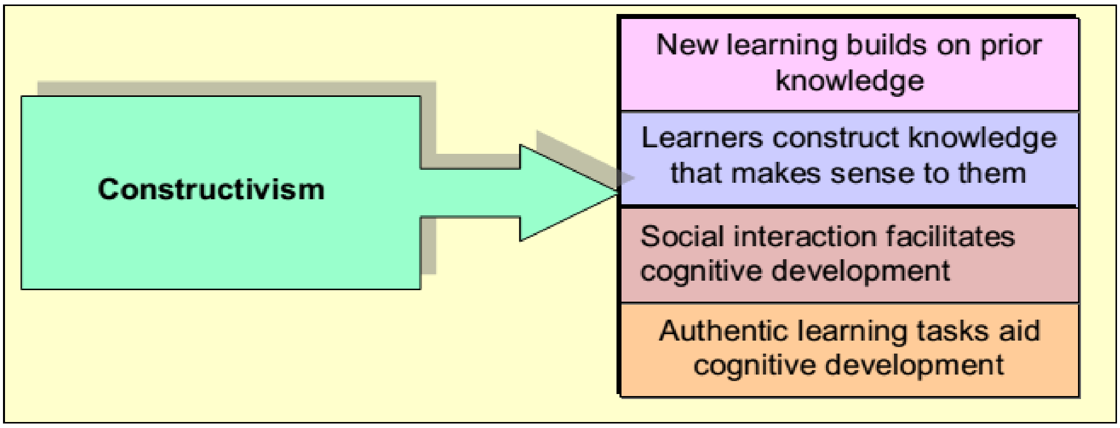
The curriculum, instructional and assessment developers need to understand the children's world. This pre-supposed that, educational and curriculum developers should put into consideration the cognitive level and needs of the learners, and not presume that what is good for them, is necessarily good for the child in developing the educational curriculum. It is therefore imperative on curriculum developers to design educational policies based on the learners need and readiness. Understanding the cognitive development of a child enables educators to present to learners what is most necessary to teach at a particular time. Educators sometimes use the diagnosis approach to decide a learner's level of development, and then designed a personalised instruction to provide the maximum amount of stimulus and assistance. In this process, children learned the laws for specific situation, which resulted in the attainment of a new organization of working materials (real learning) from the equilibration process (Piaget, 1994).

2.6 CONSTRUCTIVIST THEORY

Constructivist theory deals with knowledge acquisition through which individuals gained information and comprehended the information from their personal experience. In constructivism, entities developed independent meanings of their experience and meanings directed towards specific items. These items are diverse and multifaceted, leading the investigator to look for varied views rather than limiting understanding into a few categories or thoughts (Creswell, 2013:8). The significant aspect of this theory is the decomposition of each mathematical concept into the developmental phase, in line with the Piagetian theory of intellectual development based on observation and interviews with students, as they tried to learn a concept (Mathforum, 2015). Robson, (2006:13-14); Fraser, (2006:6); Troutman and Lichtenberg, (2003), asserted that the constructivist technique of tuition identified the significance of the learner in the learning process, and allows the learners to discover their own knowledge through the self-discovery method. Learners are enthusiastically involved in the learning and teaching process. In a similar view, constructivism is a learning process where learners are the cardinal point of the learning process. In this view educators structured their lesson plan, which actively involved learners during lesson delivery to achieve a common academic goal. Learners introduced to this approach of learning seemed to be highly motivated and learned tactical skills that propelled them to think scientifically and solve problems intellectually (Zakaria & Iksan, 2007; Johnson, Kimball, Melendez, Myers, Rhea & Travis, 2009; Froyd & Simpson, 2010). Froyd and Simpson, (2010) asserted that in constructivism, knowledge acquired by learners tends to have ripple effects on them in future leading to building robust committed team- work, and positive collaboration among learners which enabled them to retain information learned. Paily, (2013:40) explained that "In a constructivist learning environment, the role of the educator is to facilitate and guide the knowledge construction process by engaging learners in meaningful learning". Atherthon, (2010) contended that in constructivism, learners are more keenly involved in collaboration effort with the educator in constructing new meaning. In constructivism, learners are persistently challenged with tasks that refer to the application of skills and knowledge, further than their present level of mastery. As the constructivist philosophy supports, learners are central to their own learning, and how they perceived their learning environment is an important element in how they formed an understanding of their role in the class, their self-reflection, and their interaction with the educator, other learners, and course content (Stefl-Mabry, Radlick, & Doane, 2010).

Constructivist learning concept showed that constructivists acknowledged that learning should be realistic and feasible in a conducive learning environment. Constructivists argued that assessment of learners needed to be incorporated into the tasks that learners had performed, and have experienced of, since abstract thinking occurred through negotiation of knowledge, as a result of having a prior experience (Fraser, 2006:7). Eggen and Kauchak (2010:230) outlined constructivism as integrated in Figure 3.

Figure 3: A chat showing Characteristics of Constructivism



Source: Esterhuizen, (2012).

In constructivist theory, learners build new knowledge on existing knowledge which enables them to understand comprehensively, and also develop cognitively on the evidence that new knowledge is adequately acquired. Cognitive constructivism, however, equipped learners with mental processes that are needed to make meaning of the universe around them with the help of an experienced educator. An experienced educator comprehends and accommodates learners' differences brought to the classroom, and develops a strategy to meet the different needs of learners. Fraser, (2006:6) contended that cognitive constructivists are of the view that learning is a process of constructing new information, and not only a process of obtaining information. Learners are able to formulate their own idea of what is being thought, by making relations with the learning material through the help of an educator.

2.7 CONCEPTUALISATION OF MANIPULATIVE CONCRETE MATERIALS

Understanding mathematical skills are important in today's technological world (Burns & Hamm, 2011; Carbonneau, Marley, & Selig, 2013). Studies showed that these mathematical skills are not only necessary in mathematics classrooms, but are also relevant in our daily lives. Golafshani (2013), argued that everybody acknowledged that fractions are very significant in our lives, though a lot of learners have poor mathematical skills which attested to the fact that there is the need for an urgent change in the method of presentation of mathematics to learners (Golafshani, 2013:140). Golafshani (2013), further observed that there is a unanimous agreement of the use of manipulative concrete materials in the instruction of mathematics. The ancient Chinese adage which says, "I hear and I forget, I see and I remember, I do and I understand", seemed to suggest that mathematics is best understood by doing. The likelihood of this coming into effect would advance the use of manipulative concrete materials (Burns & Hamm, 2011; Bjorklund, 2014).

Johann Pestalozzi (1746 –1827) influenced educators in the 19th century to use manipulative concrete materials in teaching number sense at the basic level of education, including basic blocks (Saetter, 1990). During the teaching of young children in the first Montessori School in 1907, Maria Montessori discovered that children learned best when they are introduced to manipulative concrete materials (Encyclopaedia of Social Reforms, 2013). In a similar study, Piaget's constructivism viewpoint of the 1970s, observed that theoretical knowledge is established through discovering, while using physical materials rather than through auditory information via person to person (Piaget, 1973). In this modern world, there are a variety of manipulative concrete materials stretching from virtual computer software programmes to teacher-made materials (Gaetano, 2014 p.5). A manipulative concrete material is a physical object that can be touched, felt, moved around by learners, appeals to the faculties of the senses, and also conveys mathematical knowledge (Swan & Marshall, 2011). In addition, Cramer and Henry (2013) asserted that manipulative are physical materials which range in size, shape, and colour. They encompass physical prototypes such as fraction circles, paper folding, pie pieces, Cuisenaire rods, fraction bars, dice, and chips that enable learners to establish cognitive images of fractions.

The Gauteng Department of Education (GDE, 2012) is of the view that Learning and Teaching Support Material (LTSM) are any material that assists in the instructional and learning in our educational institution, which includes materials for learners with special instructional needs. The Gauteng Department of Education (GDE, 2012) further categorised the Learning and Teaching Support Materials as follows:

- **Non-LTSM:** This includes all the materials that are essential for effective curriculum delivery in schools (e.g. photocopiers, cleaning equipment, sporting equipment, telephones, and fax machines). These materials can again be categorised into capital and non-capital items, considering their durability.

- **Consumable items:** These are educational materials which are used over a period of time, and are used to attain intended outcomes, which in effect become consumed. These materials includes textbooks and reference materials.

- **Non-consumable items:** These are resources that have a long lifespan. Non-consumable resources are generally "once-off" purchases that would need schools to make allocation for their upkeep.

- **Other materials:** These materials are wide spectrum of resources that helps learners to attain intended outcomes. These includes photocopying paper, concrete materials and electronic projectors.

- **Library resources:** These are the educational tools that are used in the library by students and instructors. They are used for relaxation and investigation purposes. Library tools includes educators' and learners' resourced pools, books, articles, audio-visual software, young and adult literature, journals, reference books, and government publication.

- **E-learning materials:** E-learning resources are electronic learning support resources such as electronic projectors, electronic Boards, and educational hardware and software. E-learning resources also includes E-books and related resources. (GDE, 2012).

Mathematics educators adopted manipulative concrete materials such as Cuisenaire rods, Fraction bar/Fraction tile, Paper folding, and Computer assisted manipulative in teaching mathematics to make the lesson real and practical to the learners. Manipulative concrete materials have been universally acknowledged as useful mathematical objects that support practical learning through the application of concrete objects (Burns & Hamm, 2011).

Researchers are of the opinion that, manipulative concrete materials are influential objects that are useful in the instruction of mathematics, especially fractions (Council of Chief State School Officers (CCSSO), 2010). In a similar study, researchers have approved that manipulative concrete materials have an adverse positive effect on learners in learning mathematics and mathematical achievement, compared to the orthodox way of teaching mathematics. Educators used manipulative concrete materials to simplify abstract mathematical concepts that may be challenging to learners. Mathematical concepts such as fractions, algebra, addition and subtraction are made simpler to learners by using manipulative concrete materials (Lira & Ezeife, 2008). Research showed that the use of manipulative concrete materials helped educators to create a conducive mathematical classroom environment for learners (Ross, 2008). Merriam and Brockett, (2011) are of the view that manipulative concrete materials aided in motivating and sustaining the interest of learners in learning mathematics. Special Connections, (2009) observed that learners started learning through visual, physical, and kinaesthetic methods to establish basic understanding, and then improved their knowledge through pictorial representations (drawings, diagrams, or sketches). In the advanced stage in life, learners begin to think abstractly using the mathematical symbols to model and solve mathematical problems.

2.8 TYPES OF MANIPULATIVE CONCRETE MATERIALS

There are different types of Manipulative Concrete Materials used in conveying mathematical knowledge. Manipulative Concrete Materials are visual objects that allows leaners to explore mathematical concepts by using hands-on activities. These mathematical tools are made up of blocks, shapes, money, papers, cubes of different colours and size (MathematicsAtube, 2012:1). Cockett and Kilgour (2015) concurred that manipulative concrete materials include blocks, plastic dinosaurs, counters and interactive whiteboards. Table 9 showed some examples of manipulative concrete materials.

Table 9: Showed a variety of manipulative concrete materials.

Manipulative	Common Core Math Standard Covered	Image of Manipulative
Geoboards	*CCSS.Math.Content.3.MD.C.5* Recognize area as an attribute of plane figures and understand concepts of area measurement.	
TI Explorer Plus Calc.	*CCSS.Math.Content.8.EE.A.4* Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Two-sided Counters	*CCSS.Math.Content.6.NS.C.5* Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	

Pattern Blocks	***CCSS.Math.Content.K.G.A.3*** Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	
Tangrams	***CCSS.Math.Content.1.G.A.1*** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	
Color Tiles	***CCSS.Math.Content.2.G.A.2*** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	
Unifix/Snap Cubes	***CCSS.Math.Content.5.MD.C.3*** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. ***CCSS.Math.Content.5.MD.C.3a*** A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. ***CCSS.Math.Content.5.MD.C.3b*** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	

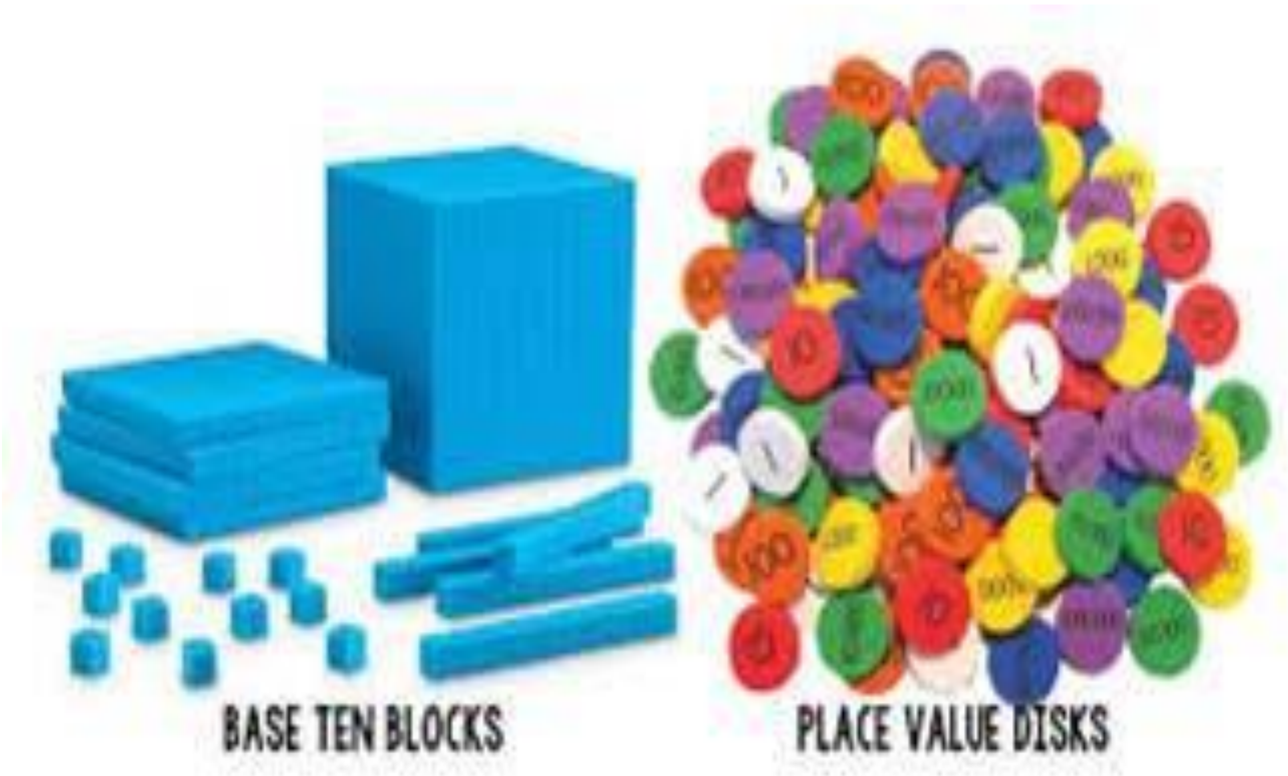
Source: Furner & Worrell (2017).

Visual manipulative also includes technology such as computer games and interactive whiteboards that are two-dimensional representations of three-dimensional space. Manipulative concrete materials are mainly made from wood, paper and plastics.

2.8.1 Wooden manipulative concrete materials

These are mathematical tools made from wood. They are designed into different shapes, sizes and colours. Examples are; Base 10 blocks, Cuisenaire Rods, Pattern blocks, Unfix cubes, Fraction tiles, Snap cubes, Two-sided counters, Geoboards etc. (CCSSO, 2010).

Figure 4: Wooden manipulative concrete material.

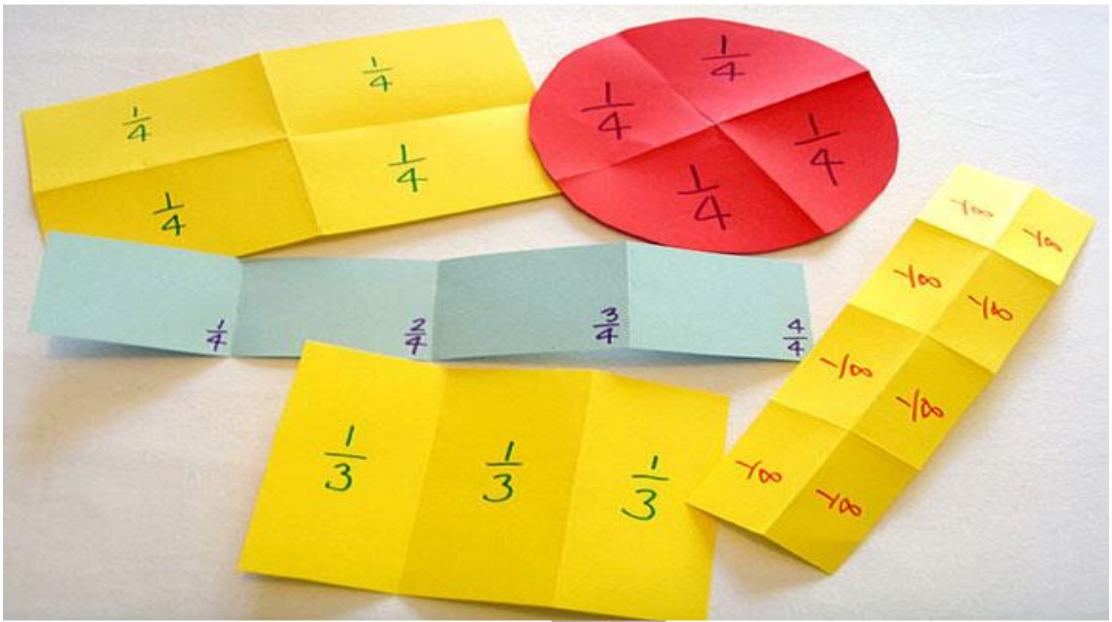


Source: Furner & Worrell (2017).

2.8.2 Paper manipulative concrete materials

Paper manipulative concrete materials are mathematical tools made from papers, mostly hard cards. They are made in different shapes, size and colour. They includes; paper folding, puzzles, hundred charts etc. (Special connection, 2009).

Figure 5: Paper manipulative concrete materials.



Source: Ervin 2017.

2.8.3 Plastic manipulative concrete materials

These are mathematical tools made from hard plastics. They also come in different shapes, sizes, and colours. Examples are; Base 10 block, Cuisenaire rods, pattern blocks, unfix cubes, snap cubes, fraction bars, fraction tiles, two-sided counters, 10 frames, 100 beads, beans and cup etc. (CCSSO, 2010).

Figure 6: Plastic manipulative material.

Source: Furner & Worrell (2017).

2.8.4 Electronic Manipulative Concrete materials

Electronic learning materials are electric learning support materials which includes; Smart Boards, educational software and hardware, data projectors, E-books and related electronic resources (Gauteng Department of Education, 2012). Electronic manipulative concrete materials also includes calculators and computers (Special connection, 2009).

Figure 7: Electronic manipulative concrete material.

Source: Furner & Worrell (2017).

Ochohi and Ukwumunu, (2008) argued that e-learning in mathematics could not be underestimated. E-learning made mathematics more interesting, more enjoyable and important to learner's day-to-day activities. It stimulated a learners' cognitive level and assisted them to make meaning of mathematical ideas by sustaining their interest in the subject, become more innovative, and also gain different mathematical skills when using e-learning tools. Johnson (2012) opined that computers are manipulative concrete materials because they performed simulations just as manipulative concrete materials. Websites have been established to allow educators and learners free access to learn from the internet and download useful materials for their studies (Bouck & Flanaga, 2009). Johnson (2012), observed that the internet can be useful to strengthen instructional practices as well as to extend the horizon of evaluation.

2.9 APPROACHES TO TEACHING FRACTIONS

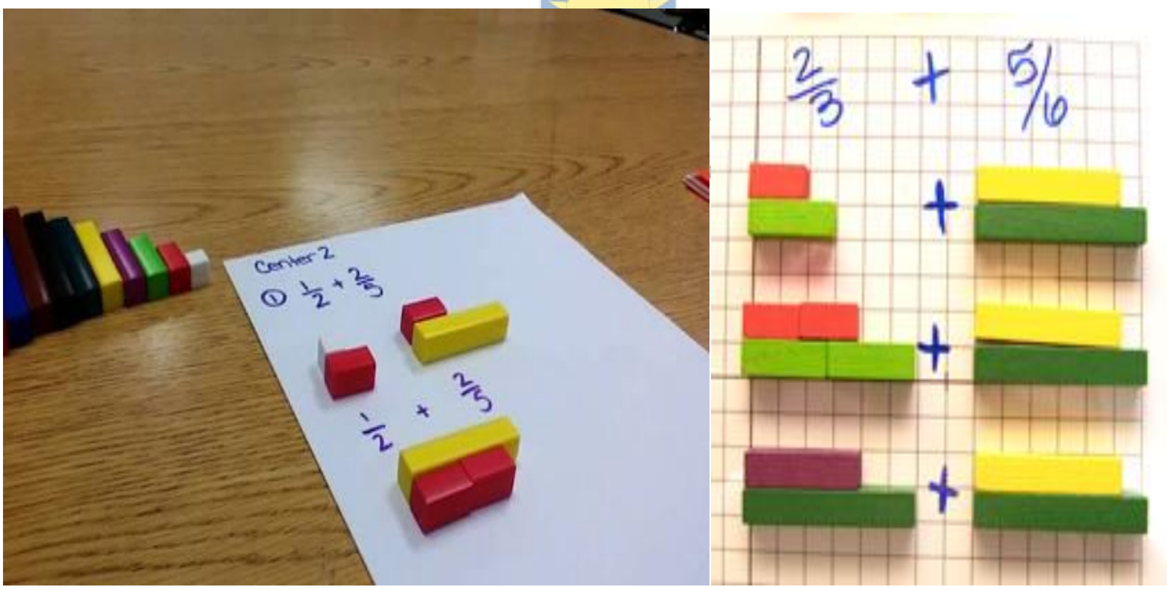
There are different instructional approaches mathematics educators adopt in teaching fractions. These includes: Cuisenaire rod's approach, Paper folding approach, Fraction bar/fraction tile approach and Electronic manipulative approach.

2.9.1 Cuisenaire Rod's Approach to Teaching Fractions

Elia, Gagatsis, and Demetrico (2007), are of the view that Cuisenaire rods are a hands-on and minds-on physical material use for mathematical instruction of abstract concepts. It is a significant mathematical material use for modelling mathematical concepts of what is taught in the mathematics classroom, and what pertained at home relating to classroom experienced to everyday life activities. Cuisenaire Rods are invented over the past nine decades by George Cuisenaire, a Belgian mathematics educator. This distinctive mathematical tool is to help learners understand abstract mathematical concepts by manipulating painted wooden strips of different dimensions called Cuisenaire rods. A package of Cuisenaire rods consisted of 74 rectangular rods in 10cm different dimensions and 10 varied colours. Each colour relates to a particular length. The content of the packet is made up of 22 white rods of 1cm each, 12 red rods of 2cm each, 10 light green rods of 3cm each, 6 purple rods of 4cm each, 4 yellow rods of 5cm each, 4 dark green rods of 6cm each, 4 black rods of 7cm each, 4 brown rods of 8cm each, 4 blue rods of 9cm each and 4 orange rods of 10cm each. These rods are used as physical objects to teach any concept in mathematics (Kurumeh, 2009:20). George Cuisenaire applied these coloured rods of different lengths to basically enable learners to experience where colours created shapes, or become numbers. This harmonious interconnection of the senses acted as a tool to enhance learning and memory. Cuisenaire rods enable learners to discover mathematical problems on their own (Akarcay & Sevilay, 2012).

Figure 8: A picture of a Cuisenaire Rod.

Source: Kurumeh 2009.



Source: Pinterest.com

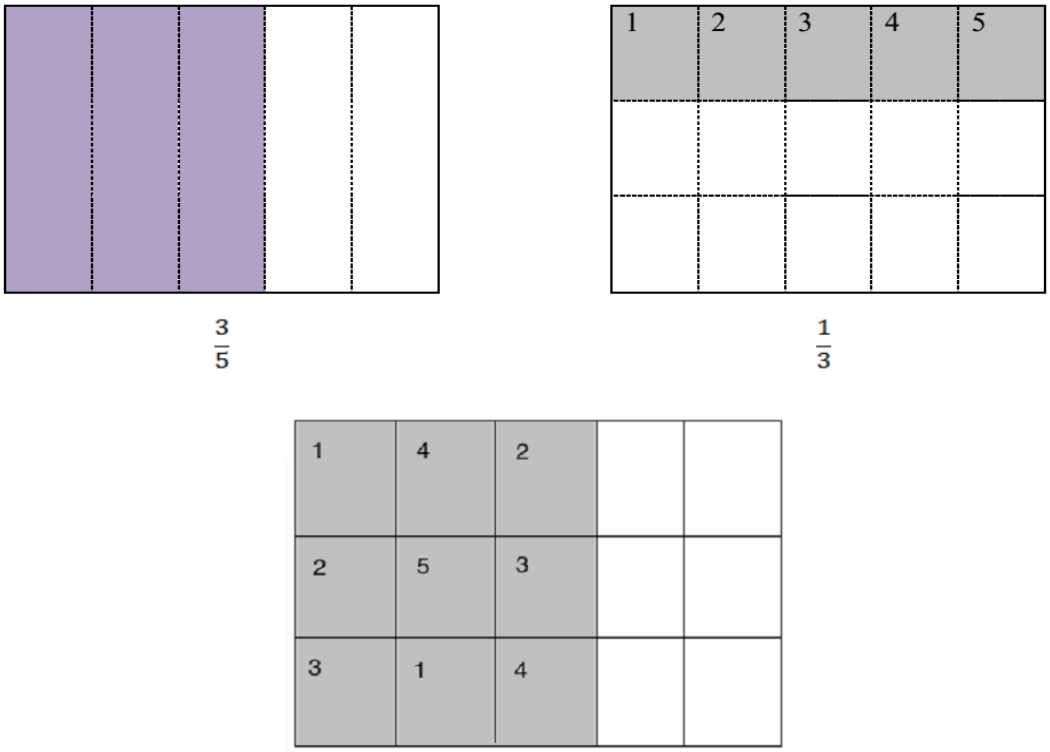
Using Cuisenaire rod teaching approach prepares learners to meet daily standards with daily critical thinking activities. It prepares learners for school success in fractions, and addresses the needs of every age group. Cuisenaire rods enable every learner to work individually, and in groups, on important mathematical contents such as fractions, while the educator offers individual assistance to learners (Akarcay & Sevilay, 2012;

Van de Walle, 2007). In a similar vein, Kurumeh (2009) argued that Cuisenaire rods are ready-made tools, therefore it minimizes preparation and set up time for both the educator and the learners. Cuisenaire rods' approach is an easy to follow activity that helps learners' master estimation and measurement. It promoted successful learning experience that changed mathematics education forever (Elia, et al., 2007). Rule and Hilagan (2006) supported the idea that the Cuisenaire rod approach of teaching assisted in developing fundamental skills in learners such as ordering, analytical thinking, and problem solving in mathematics. Study revealed that as learners manipulated and played with the rods, they merged touch and sight into the learning process, which made the lesson motivating, exciting, relaxed, memorable, and imaginative (Akarcay & Sevilay, 2012). Cuisenaire rods are tremendously flexible as learning tools, such that educators could assess learners in the process of learning, without interfering in the learners learning process. Cuisenaire rods can be employed into any mathematical learning environment to help create a tangible and visual situation (Akarcay & Sevilay, 2012). Kurumeh (2010) concurred that the use of Cuisenaire rods made mathematics real to learners since it is learner friendly, activity oriented, and aroused learners' comprehension of the mathematical concepts, and accelerated higher understanding of mathematical ideas, facts and principles. Cuisenaire rods enable learners to work independently and in a group on meaningful mathematical contents (Kurumeh, 2010). In this study, the researcher used Cuisenaire rods because it was an already prepared manipulative tool, and easy to manipulate to demonstrate the concepts of fractions to the learners. The colours were also fascinating, so it fully attracted the attention of the learners during demonstration.

2.9.2 Paper Folding Approach to Teaching Fractions

This demonstration was generated by folding a piece of paper into equal sizes in relation to the problem under study (Ervin, 2017:265). Paper folding plays a vital role in learners' comprehension of division in fractions (Johanning & Mamer, 2014). Through paper folding modelling, learners are able to visualise problems in figurative form, through a lens that highlighted the scale of the dividend and divisor. This enables learners to make better judgement of their solutions (Ervin, 2017:265). Figure 9 illustrated paper folding approach in solving fraction division problem. For example: solve $^{3}/_{5} \div {^{1}/_{3}}$

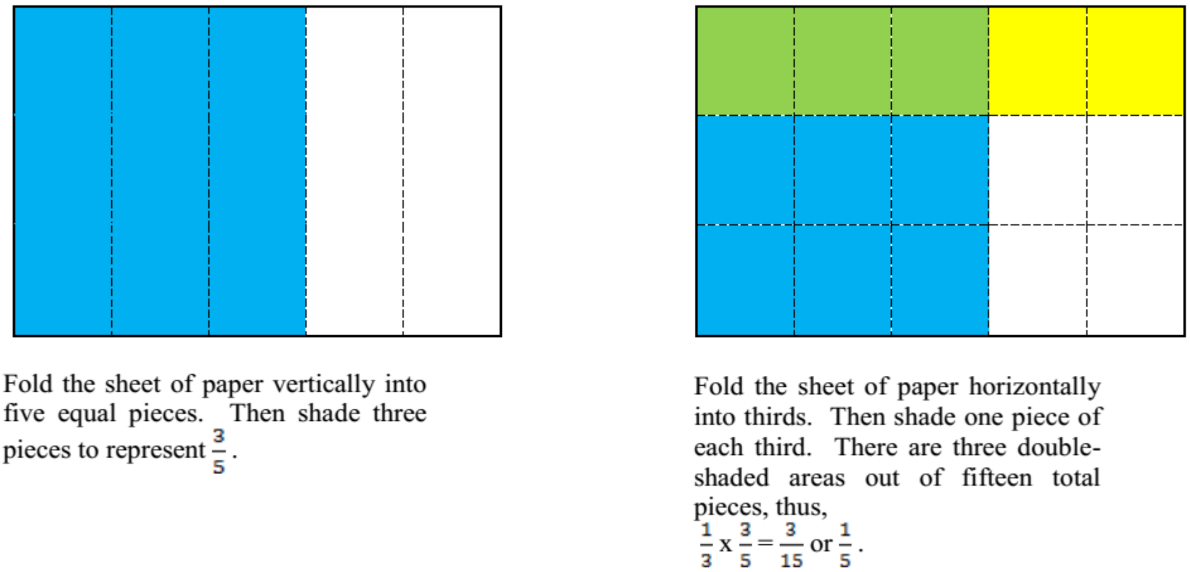
Figure 9: Paper folding showing division of fractions



Source: Ervin 2017

How many $^1/_3$ fits into $^3/_5$? Five grey blocks made up $^1/_3$ of the whole unit. One whole set of these grey blocks, and four out of five of a second set of grey blocks, would fit into the purple region, if we considered the purple region to contain nine grey blocks. Thus $^3/_5 \div {}^1/_3 = 1\frac{4}{5}$ (Ervin, 2017). In a similar scenario, fraction multiplication can also be modelled by means of a piece of paper as a physical object. This illustration was generated by folding a piece of paper into the same size in line with the problem under investigation (Tsankova & Pjanic, 2009; Van de Walle et al., 2008). Equal size fragments of paper are vital in answering fraction multiplication problems so that connections between the two dissimilar fractions could be compared (Ervin, 2017:263-265). Ervin (2017:263-265), emphasised that it is necessary for pre-service educators to be able to sketch and fold area models, since paper folding exposed pupils to physical experience. Papers are folded into horizontal and vertical shapes in solving problems involving multiplication of fractions (Van de Walle et al., 2008). For example: what was $^1/_3 \times {}^3/_5$? Figure 10 illustrated how paper folding was used to solve fraction multiplication problem. Also refer to Figure 5 for the illustration of division of fractions using paper folding.

Figure 10: Paper folding showing multiplication of fractions.



Source: Ervin 2017.

2.9.3 Fraction Bar/Fraction Tile Approach of Teaching Length Model in Fractions

Length models are quite different from area models in terms of measurements or dimensions. These models supported learners in making relations with linear problems (Ervin, 2017:263-265). Fraction bars/Fraction tiles models are frequently used to analyse fractions. One of the key concepts conveyed through the use of fraction bars is the unit, and how learners compare fraction bars with the whole dimension being equal, and illustrated the equal unit. Fraction tiles are a clear demonstration of how models enabled learners to pictorially observe the part in relation to the entire unit (Ervin, 2017:265). Figure 12 showed an illustration of fraction bars used to compare fractions.

Figure 11: A Fraction bars/Fraction tiles model.

Source: Ervin 2017

Fraction tiles and fraction strips conveyed the same mathematical message. Fraction strips are folded strips of pieces of papers in which the whole strip illustrated the whole. Cuisenaire rods, or strips of paper, are usually used as dimension prototypes because dissimilar dimensions could be identified with dissimilar colours, and any dimension signified the entire length (Ervin, 2017). These models are important models in conveying mathematical connections between fractions, compared dimensions, and examined corresponding fractions (Ervin, 2017). Boggan, Harper and Whitmire (2010) argued that it is better to use fraction strips for the instruction of addition and subtraction of fractions to learners, or use to represent equivalent fractions.

2.9.4 Electronic Manipulative Approach

Bouck and Flanagan (2009) defined a virtual manipulative as web-based pictures on a computer screen that permitted learners to operate with graphical images as if they were three-dimensional. This virtual manipulative enabled the design structures to regulate instructional difficulties, scaffolding mathematical concept, and increased the amount of virtual manipulation to intensify mathematical instructions for learners with learning disabilities. Also, the solicitation of virtual manipulative served as personalised adjustments for learners with learning disabilities and mathematical difficulties, especially in fractions (Bryant 2011; Edyburn, 2013). Satsangi and Bouck (2015), supported the idea that virtual manipulative tools enabled learners to actively contribute in class during the instructional period. In a similar vein, countless virtual manipulative websites are available for easy access. Educators and learners made use of these ready-to-use online resources at a less, or no cost, rate. In using virtual manipulative, educators made good use of their time to plan and implement instructions for their learners (Bouck & Flanagan, 2010).

Figure 7. A picture of Electronic manipulative.

Source: Furner & Worrell (2017)

2.10 PERCEPTION OF THE USE OF MANIPULATIVE CONCRETE MATERIALS ON LEARNERS' PERFORMANCE IN FRACTIONS

Study showed that a considerable number of investigations have been conducted on the use of manipulative concrete materials on the effect of learners' mathematical achievement. Many researchers have varied views on the application of manipulative concrete materials in teaching. The use of concrete manipulative materials in mathematical instruction has gone through a lot of evolution. Golfashani (2013) argued that mathematical instruction had metamorphosed from using beans, or counters, to associating cubes, fractions rings and other technologies.

Despite the benefits associated with the application of manipulative physical objects in mathematical instruction, many scholars are of the view that the use of manipulative concrete materials does not necessarily warrant the understanding of mathematical ideas. Researchers are of the notion that virtual manipulative concrete materials, in reality, do not help learners in cultivating mathematical comprehension (Moyer-Packenham & Westenskow, 2013). Virtual manipulative concrete materials have varied learning consequences for learners with diverse learning capabilities (Moyer-Packenham & Suh, 2012). Ross (2008) attested to the fact that educators who are not in tune with the application of concrete manipulative materials, are mostly liable of limiting the success of teaching, classroom organisation, and learners' mathematical attainment. Also, The National Centre on Intensive Intervention (NCII, 2016) was of the view that although students may exhibit the proper use of a manipulative, this does not guarantee that they understood the mathematical concepts behind the use of the manipulative. Explicit instruction and student verbalizations, such as explaining the mathematical concept, or demonstrating the use of the manipulative while they verbally described the mathematical procedure, cannot be over emphasised (NCII, 2016). Other researcher's supported the idea that a lot of educators have not been educated on the selection of the appropriate mathematical tool designed to meet learners' individual needs (McMahon & Walker, 2014). Study revealed that majority of learners' required extra time to accomplish a particular task involving the manipulation of concrete materials, and to develop fractional knowledge (Cramer & Henry, 2013). The least amount of time a learner needed to understand a mathematical concept using a manipulative concrete material, relied extensively on the learners' inherent enthusiasm and intellectual abilities (Gaetano, 2014). In a similar view, Uribe-Florez and Wilkins (2010) argued that educators are of the notion that the application of manipulative concrete materials in teaching is time consuming. A research indicated that educators suggested that time spent on concrete materials could best be used with other teaching methods (Trespalacios, 2008).

Lee and Chen (2010), opined that using manipulative concrete materials enabled learners to gain insight into other mathematical topics to enrich their mathematical proficiencies. Also, manipulative concrete materials helped learners develop intellectual pictures and abstract concepts due to their experience. "Learners who see and manipulate a variety of objects have clearer mental images, and can represent abstract ideas more completely than those whose experiences are meagre" (Gaetano, 2014:4). Studies showed that virtual manipulative are less expensive, and time effective when employed in the classroom. Virtual manipulative are interactive by nature, and also web-based graphic representations allow learners to simulate using manipulative concrete materials (Moyer, 2002). Several websites are created to give educators free access to use with their learners (Bouk & Flanagan, 2010). In support, Belenky and Nokes (2009) asserted that learning with manipulative aids helped learners in building confidence by boosting the level of commitment in using concrete materials in the future. Research conducted by Driscoll, (as cited in Ligget, 2017), asserted that manipulative concrete materials could be used at every grade level to support the teaching of mathematics. He further argued that manipulative concrete materials have a place in the basic grades, which helps in imparting mathematical ideas and abilities into the learners, by providing remedial services to learners who may lag behind in grasping mathematical concepts. In support, Frasher (2013), asserted that learners of all categories of age group could benefit from the use of concrete manipulative materials in the instruction of mathematical concepts. This could be done through efficient lesson preparations that are grounded on the principle of concrete, to pictorial, and to abstract presentations of mathematical ideas in words and symbols. Swan and Marshall (2010), argued that there are possible advantages to be gained in the use of manipulative concrete materials in the instruction of mathematics where sufficient skills are applied in a logical way. A further study, involving students from Grade 3 and 4, by Burns and Hamm, (2011), concluded that manipulative reinforced mathematical concepts, and increased average test scores.

Morris (2013) concurred that learners become enthusiastic and energetic in the use of manipulative concrete materials in the instruction of mathematics, as educators used these manipulative materials as a motivation to learners' good behaviour, rather than use it as a mathematical tool to convey mathematical concepts in a more practical manner. Uribe-Floez and Wilkins (2010) supported the idea that when manipulative concrete materials are applied, learners are able to understand the mathematical concepts in a more physical manner. Manipulative concrete materials assisted learners to make mathematical links that would otherwise not have been realised by the learners. Mathematics scholars have showed that the application of manipulative concrete materials enabled learners to appreciate and comprehend abstract mathematical concepts, and to achieve a better result in their studies (Bjorklund, 2014; Burns & Hamm, 2011; Freer, 2006; Swan & Marshall, 2010). Ligget (2017), observed that the use of manipulative concrete materials in the instruction of mathematics in elementary and middle schools, has improved learners' academic achievement in mathematics. He further stated that, if learners applied mathematical concrete materials, not only would it increase their mathematical success, but it would also improve their intellectual skills in solving mathematical problems.

2.11 CHALLENGES ASSOCIATED WITH MANIPULATIVES AND LEARNING MATHEMATICS

Study showed that the challenges educators face in the use of manipulative concrete materials in mathematics instructions, especially fractions, is the problem of methodology. Mathematics, as a subject, is basically grounded on a well-defined method which helps in getting the correct result. However, most educators and learners battle with the right use of manipulative concrete materials in solving mathematical problems involving fractions (Jansen & Spitzer, 2009). Carney, Brendefur, Thiede, Hughes and Sutton's (2014) argued that educators become more biased towards the informal methods of instruction, rather than the formal methods when manipulative concrete materials are used in mathematical instruction in classrooms. In support, Boggan, Harper and Whitemire (2011), asserted that there is a paradigm shift among educators who used formal teaching methods to more imaginative methods, through the application of manipulative concrete materials in teaching mathematics. Researchers observed that the national efforts by (NCTM,

2000) to increase focus related to solving K-12 mathematical challenges in the classroom, have not yielded the desired results, because mathematics educators have shifted to the conventional methods of instruction (Boggan et al., 2011; Holton, Cheung, Kesianye, de Losada, Leikin, Makrides, Yeap, 2009; McNeil, Weinberg, Hattikudur, Stephens, Asquith, Knuth, Alibali, 2010; Moyer-Packenham, Salkind & Bolyard, 2008). Neubrand, Seago, Agudelo-Valderrama, DeBlois, Leikin, and Wood (2009) argue that virtual recreations of physical manipulative that are often used in mathematics lessons, such as Cuisenaire rods, tangrams are effortlessly found online. The introduction of technology compelled mathematics educators to participate in skilled development courses to improve their knowledge in the use of manipulative concrete materials in teaching mathematics.

2.12 EDUCATORS SELF-EFFICACY IN MATHEMATICS INSTRUCTION

Bruce, Esmonde, Ross, Dookie, and Beatty, (2010), defined educators' self-efficacy as the ability to self-evaluate their capabilities in line with their learners' academic success in mathematics. Educators' intellectual abilities and technical know-how imparted meaningfully to their philosophies about self-efficacy in the use of manipulative concrete materials. Research showed that an individual's mastery and experiences, social influences and mental conditions contributed to educators' intellectual knowledge of self-efficacy (Bruce et al., 2010). Mathematics educators are convinced that when they can teach more challenging topics, it increased their confidence level in contributing to the mathematical achievement of their learners (Tschannen-Moran & Barr, 2004). In South Africa, majority of mathematics educators have a high level of confidence in teaching mathematics (Mullis et al., 2012). Mathematics educators, with high levels of confidence, coupled with different types of instructional strategies, helped to increase the comprehension level of their learners (Bruce et al., 2010; Brown, 2012). Bruce et al., (2010), observed that professional development courses enabled educators to improve self-confidence in the comprehension of the concepts and instruction of the subject, and to overcome fears and anxiety in the instruction of mathematics.

2.13 EDUCATORS PROFESSIONAL DEVELOPMENT

Researchers are of the notion that, continuous refresher courses are vital in training mathematics educators to improve their mathematical concepts and skills, which, in effect, also improved their learners' academic performance (Brown, 2012; Coleman & Goldenberg, 2010; Francis-Poscente & Jacobsen, 2013; Tschannen-Moran & Barr, 2004). Studies showed that collaboration among educators is a major component of improving educators' professional development in teaching mathematics. Brown (2012) asserted that educators should be ready to learn and adopt new strategies for teaching, even if they are laborious and demanding. In a similar view, Zambo and Zambo (2008) argued, that regular participation of educators in courses to improve their professional development cannot be underestimated. They further argued that, when educators participate in the professional development programs, it boosts their self-efficacy and improves their confidence level in their instructional method to impact positively in their learners' performance in mathematics (Zambo & Zambo, 2008).

2.14 CONCEPTUALISATION OF FRACTIONS

Olanoff, Lo and Tobias (2014), opined that fractions are an aspect of rational numbers which are expressed in the form $\frac{a}{b}$ where "a" and "b" are both numerals, and "b" ≠ 0. Fractions are an aspect of study of rational numbers. In similar vein, Lortie-Forgues et. al. (2015 p.206) asserted that a fraction is made up of three components, a numerator, a denominator, and a line separating the two numbers eg. $^{1}/_{2}$. Studies showed that for one to advance in the understanding of the concept of rational numbers in general, one must study the different interpretations of fractions (Lamon, 2007, 2012). Ball (cited in Olanoff et al. 2014 p. 272) asserted that fractions may be referred to as: (a) in part-whole terms, where the whole unit may varied; (b) as a number on the number line; (c) as an operator (or scalar) that could shrink or stretch another quantity; (d) as a quotient of two integers; (e) as a rate; and (f) as a ratio.

Lamon (2007, 2012) emphasised that learners need to be introduced to all these types of fractions, and that learners, whose instructions are centred only on a part-whole fractions, are limited in understanding rational number concepts. In the same vein, Olanoff et al. (2014) reiterated that, for learners to develop a deeper comprehension of fractions, they must not only be taught operations of fractions, but ought to be introduced to fractions number sense, which would enable them to have an in-depth understanding of fractions as numbers in a system. Fractions number sense develops an intuition that helps learners to appreciate and make appropriate connections to determine order, size and equivalence, and make an appropriate judgement (Lamon, 2012). In addition, Lamon (2012) outlined three different approaches for ordering fractions: (a) same number of parts; (b) same-size parts; and (c) compared to a benchmark.

In the same number of parts, or "common numerator" form of fractions, the two fractions will have equal numerators, or number of parts that can be compared by observing the size of the separate parts e.g. $^{2}/_{4}$ **>** $^{2}/_{7}$. In this scenario $^{2}/_{4}$ is greater than $^{2}/_{7}$, because when two oranges are shared among four people, and same number of oranges are shared among seven people, it would be observed that the portion of oranges the four people would get, would be bigger than the portions of oranges the seven people would receive when compared. Also, in the same-size parts form of fraction, which is also known as the "common denominator" approach, it shows that when two fractions have the same denominator, or size of parts, then they can be compared by looking at the numerators. For example, $^{4}/_{7}$ < $^{5}/_{7}$. In this example, $^{5}/_{7}$ is greater than $^{4}/_{7}$ because, 5 is greater than 4 of the same thing. Another form of fractions involved the comparison strategy, which looks at comparing two fractions to another "benchmark" fraction such as $^{1}/_{2}$ and $^{2}/_{3}$ **.** In comparing $^{1}/_{2}$ and $^{2}/_{3}$, it would be observed that, $^{2}/_{3}$ > $^{1}/_{2}$. This is because, 2 is more than half of 3, and 1 is exactly half of 2 (Lamon, 2012).

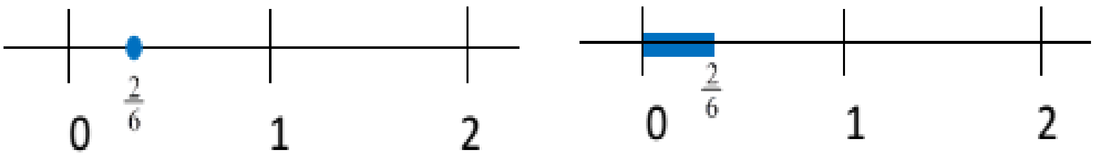
2.14.1 Multiple Interpretations of Fractions

Fractions indicate the connection between two numbers. These two quantities offer information about the parts, the units under analysis, as well as the whole (Catherine, Diana & Tara, 2013 p.8). However, research shows that there is a universal agreement about the different forms of interpretations of fractions (Empson & Levi, 2011; Clark & Roche, 2011; Steffe & Olive, 2010; Petit, Laird & Marsden, 2010). The interpretations of fractions adopted from Math for Teaching: Ways We Use Fractions (Ontario Ministry of Education, in publication) outlined the following as the interpretation of fractions (Bruce, Chang, & Flynn, 2013).

2.14.1.1 A linear interpretation

The linear interpretation of fractions is centred on the fraction's distance from zero, and allowed for the mathematical value of the fraction to be positioned in relation to the unit of 1. The number line interpretation of fractions e.g. $^2/_6$ can is represented in Figure 12:

Figure 12: A Number line showing a linear interpretation of fractions.

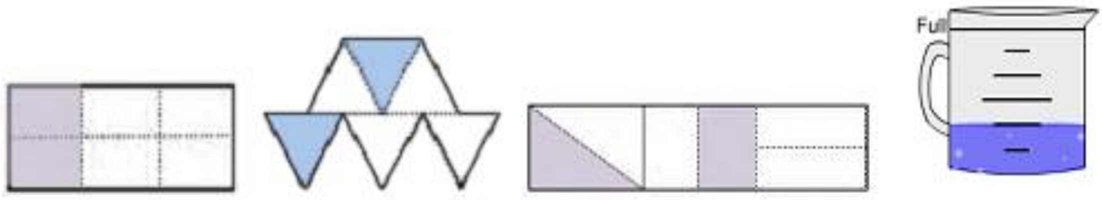


Source: (Bruce, Chang, & Flynn, 2013).

2.14.1.2 The Part – Whole interpretation

The part – whole fractions is established on either a continuous model (such as an area or a volume) or a discrete model (such as a set). For continuous models, entities are parsed into equal countable parts (e.g. an apple cut into equal slices, or a rectangle divided into equal squares), whilst for discrete models, they are entities, or a set of individual objects, that cannot be broken down into natural equal units (e.g. Marbles, balloons, or grapes). (Rapp, Bossok, DeWolf & Holyoak, 2015). Partitioned continuous models of fractions, e.g. $^2/_6$ can be illustrated with the shaded regions as shown in Figure 13:

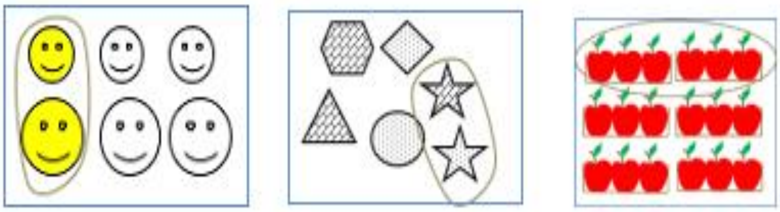
Figure 13: A Continuous model of fractions.



Source: (Bruce, Chang, & Flynn, 2013).

One unique feature of the discrete model is that: the areas can be of equal size, but not essentially the same shape. For the discrete models, features other than size, are more prominent. Figure 14 illustrated $^{2}/_{6}$ of a discrete model.

Figure 14: A Discrete Models of fractions.

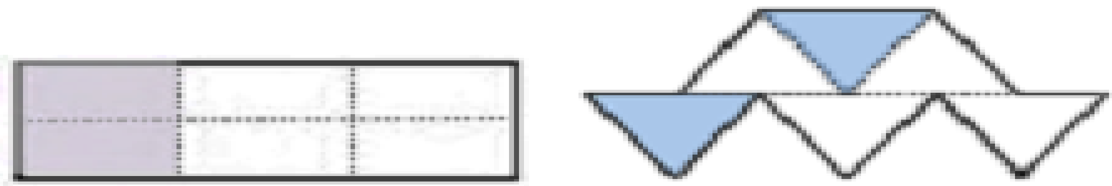


Source: (Bruce, Chang, & Flynn, 2013).

2.14.1.3 Part – Part Relationship

Fractions can be used to represent a part – part relationship. In this instance, fractions are used to compare the size of two measures. In the part-part connection, the entities are the sum of the parts. Part – part connections can be illustrated by using linear, continuous or discrete prototypes. An example of part-part relationship is shown in Figure 15. Two equal areas shaded to four equal unshaded areas. In all there are six equal parts.

Figure 15: Shows a part – part relationship in fractions.

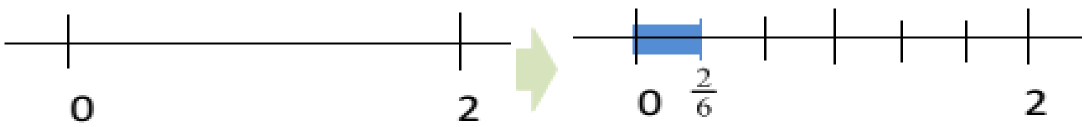


Source: (Bruce, Chang, & Flynn, 2013).

2.14.1.4 Fraction as a quotient

A fraction is also referred to as a quotient, or a division statement. For instance, $^2/_6$ is also expressed as 2÷ 6, or 2 divided into 6 equal parts, Figure 16 is illustrated below:

Figure 16: Shows a representation of fractions as a quotient on the number line.



Source: (Bruce, Chang, & Flynn, 2013).

2.14.1.5 Fractions as an operator

Fractions can also be expressed as an operator. In this case, the fractions act as a transformer by either expanding or decreasing the operand. An example of this can be illustrated as $^2/_6$ of the area of the flat surface, or $^2/_3$ of the recipe. The Mathematical Education of Teachers II, outlined various components of fractions as an important aspect of mathematics to be taught by mathematics educators (Conference Board of the Mathematical Science, 2012).

A study conducted by Moseley and Okamoto (2008) observed that, unlike the highly intellectual learners, the average learners are slow in understanding the various aspects of rational numbers, which resulted in a learner focusing on the similarities of the symbols rather than the arithmetic meaning. In a similar vein, it is observed that, learners who are conversant with both the part – part, and part – whole interpretations have in-depth comprehension of rational numbers (Moseley, 2005). This indication, however, calls for the need to extend learners' fractional knowledge further than the usual meaning of fractions as part – whole relationships. This will enhance learner's comprehension of fractions, and better equip them for a more meaningful and comprehensible switch to operations with fractions (Conference Board of the Mathematical Sciences, 2012).

2.14.2 Importance of Fractions

Fractions form the basis of elementary and middle school mathematics. It is widely acknowledged that fractions are important in learning algebra and more advanced mathematics (National Mathematics Advisory Panel (NMAP, 2008), but also our day-to-day activities such as handling personal investments, understanding bank loan interest rates, cooking, therapeutic dosage, and doing home repairs (Siegler et al. 2012). The significance of fractions is elaborated in the U.S. Common Core State Standards (Council of Chief State School Officers & National Governors Association Centre for Best Practices, 2010). In a similar vein, acquaintance fraction aids in the proficiency of algebra, and, in effect, prepares learners for the advanced level of education, and also provides vocations in the science, technology, engineering, and mathematics (STEM) field (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014, Siegler et al. 2012).

Lortie-Forgues, Tian and Siegler (2015) observed that, the essence of fractional knowledge is not limited to mathematics courses only. Rational number arithmetic is also significant in areas such as biology, physics, chemistry, engineering, economics, sociology, psychology and many other aspects of lives. Knowledge in these fields forms the basis in acquiring jobs in which higher mathematics knowledge is not a pre-requisite, such as professional nurses and druggists for dosage calculation (Lortie-Forgues et al., 2015). Also, fractions and decimal arithmetic concepts are universally significant in intellectual and arithmetical development. Jean Piaget and his researchers are perhaps the first researchers to acknowledge the essence of studying rational numbers, such as ratios and proportions for the appreciation of cognitive development (Lortie-Forgues et al. 2015). More so, Lortie-Forgue et al., (2015 p. 202) reiterated that fraction and decimal arithmetic are common in our everyday activities. For instance, in recipes (eg. If $^3/_4$ of a cup of flour is needed to make a dessert for 4 people, how much flour is needed for 6 people?), and measurement (e.g. Can a piece of logs 36 inches long be cut into 4 pieces each 8.75 inches long?) Fractions and decimal arithmetic are vital to the comprehension of simple statistical and probability reports in mass media, and to understanding business reports, such as compound interest and the unevenness of percentage fluctuations in the stock exchange market (Lortie-Forgues et al. 2015 p.202).

In addition, basic school learners' fractions comprehension foretells their algebraic success in high school, even after going through several levels of tuition grades, intellectual ability, and knowledge of whole number arithmetic (Siegler et al., 2012). Studies show that learners who do not do well in algebra have less chance of graduating from high school, and also have fewer career opportunities in the field of science, technology, engineering, and mathematics, compared to learners of higher achievement. (NMAP, 2008; Sadler & Tai, 2007).

Early, in-depth knowledge of fractions distinctively foretells success in advanced mathematics. Analysis of large data-sets from the U.S. and the U.K., show that fractions knowledge in grade five is an exceptional predictor of general understanding of mathematics in grade ten. This is confirmed to be true after studies confirmed that acquaintance of whole number arithmetic, oral and nonverbal IQ working memory, family education, race, ethnicity, and family income is vital (Siegler et al., 2012). Similar studies have led to the same conclusion. For example, a sample, representing one thousand U.S. educators who had poor knowledge in algebra, including "rational numbers and operations involving fractions and decimals", was one of the two greatest hurdles preventing their learners from learning algebra (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Weak understanding of fractions has serious long-term consequences.

2.14.3 Fractions A Challenging Topic in Mathematics

A study conducted by the Queensland Studies Authority (QSA, 2013), observed that, learners' comprehension of fractions using data from NAPLAN testing, indicated that almost all learners find fractions a difficult topic, and tussle with it throughout their studies. It was observed that fractions are not only a challenging topic to learners, but to their educators as well (QSA, 2013).

Lortie-Forgues et. al. (2015) asserted that fractions arithmetic is a complicated aspect of mathematics for young learners. Fractions arithmetic involved learning a larger number of distinct techniques, perhaps more than for any other mathematical operation taught in elementary school. Researchers are of the view that teaching fractions is complex, and that handling the instruction of this topic can be challenging for classroom practitioners, leading to gaps in learning being overlooked (QSA, 2015). In a similar vein, Watanabe (2012) asserted that the central point of mathematics instruction should be on fractions as quantity, which enables learners to make a strong link between their previous knowledge of whole numbers as quantity. Also fractions models need to be established informally through using open-minded questions, and with the same substantial and consistent use of concrete materials (Brown & Quinn, 2006; Petit et al., 2010; Empson & Levi, 2011).

Lortie-Forgues et. al. (2015 p. 202) argued that, after a series of educational reform, with billions of dollars invested into educational research on mathematics innovation and instruction, only little progress had manifested in learners' comprehension of fractions arithmetic. Several studies and investigations conducted in the past years have recognised learners' weak comprehension of fractions (Perle, Moran, & Lutkus, 2005; Stigler, Givvin, & Thompson, 2010). The difficulties learners face in mathematics is not limited to fractions alone, or to USA learners only. Countries, where mathematics is well established, or learners perform well internationally, also face challenges in multiplication and division of decimal arithmetic (Liu, Ding, Zong, & Zhang, 2014; OECD, 2014). Researchers are of the view that mathematics anxiety of learners can be traced to the challenges learners face in fractions arithmetic (Heitin, 2015).

2.14.4 Factors Contributing to Learners Difficulties in Fractions and Decimal Arithmetic

There are many factors that contribute to learners' problems in fractions and decimal arithmetic. Some of these difficulties emanate from genetics (nature), while others come from the surroundings (nurture) (Papalia, Wendkosold, Duskin & Feldman, 2008:12). These factors include: inherent sources of difficulty in fractions and decimal arithmetic, environmental factors, educators' presentation of mathematical knowledge, reading difficulties, effect of sensory motor skills regarding the learning of mathematics, attention deficit related problems, educators' inability to understand learners' construction of mathematical ideas, the language used in teaching fractions in mathematics, dyscalculia paradigm, lack of school resources, anxiety of learners, and perception etc.

2.14.4.1 Inherent Sources of Difficulty in Fractions and Decimal Arithmetic

Researchers have identified and discussed seven root causes of learners' poor performance in mathematics that are inherent to fractions and decimal arithmetic. Lortie-Forgues et al. (2015: 206) observed that these difficulties includes: (1) fractions and decimal symbolization, (2) approachability of fractions and decimal magnitudes, (3) opacity of standard fractions and decimal arithmetic procedures, (4) complex relations between rational and whole number arithmetic procedures, (5) multifaceted relations of rational number arithmetic techniques to each other, (6) differing direction of effects of multiplying and dividing positive fractions and decimals below and above one, and, (7) sheer number of distinct components of fractions and decimal arithmetic procedures. Lortie-Forgues et al. (2015:206) reiterated that this list of inborn challenges should not be considered exhaustive, but outlined some challenges that contributed to the difficulty that learners usually experienced with fractions and decimal arithmetic.

2.14.4.2 Fractions and decimal symbolizations

Lortie – Forgues et al., (2015:206), submitted that one contributing element that brings about the difficulties among learners in fractions and decimal arithmetic is the symbol used to express fractions and decimals. A fraction is made up of three components, a numerator, a denominator, and a line splitting the two numbers. This arrangement gives rise to the fractional symbol being challenging to understand. For example, learners in their initial phases of learning, repeatedly misconstrued fractions as two separate whole numbers (e.g. ½ as 1 and 2), as opposed to arithmetic operation (e.g. 1 + 2), or as whole number (e.g. 12) (Lortie-Forgues et al., 2015:207). It is acknowledged that, even after going through the rudiment of fractional instructions, learners still battled with fractional concept due to the symbols applied in the process of working. Working with ${}^{336}/_{14} * {}^{234}/_{18}$ through memorization entailed substantially more intellectual resources than solving mathematical problems using two corresponding whole numbers, 24 * 13. The excessive use of memory in representing fractions trimmed down to the cognitive resources available for intellectual reasoning. This is desirable for solving mathematical problem, and also checking improvement. While implementing the method needed for solving the mathematical problem. Applying the rules to decimal arithmetic, and not complicating learners with the rules of whole numbers, required recollecting memory hassles of learning decimal arithmetic (Lortie-Forgues et al., 2015; Fuchs et al., 2013; Hecht & Vagi, 2010; Jordan et al.,2013; Siegler & Pyke, 2013).

2.14.4.3 Approachability of magnitudes of operands and answers

Lortie – Forgues et al., (2015) observed that, in accessing the magnitude of fractions, it called for comprehension of natural number division which was normally thought of as the more problematic of the four arithmetic operations. Researchers are of the opinion that challenges have led certain authors to suggest that learners have to go through elementary restructuring of comprehending figures in advance before being able to perform fraction arithmetic (Vamvakoussi & Vosniadou, 2010; Carey, 2011; Smith, Solomon, & Carey, 2005). In a similar view, Smith et al., (2005) asserted that making sense of numerous ideas connected to rational numbers (e.g. the occurrence of figures between 0 and 1, the point that numbers are substantially divisible) appeared to arise at the same time within an individual. Learners with an in-depth understanding of fractions magnitudes, generally perform better on fractions arithmetic, even after appropriate variables such as awareness of natural numbers, operational retention, and decision-making abilities have been statistically regulated (Hecht & Vagi, 2010; Jordan et al.,2013; Siegler & Pyke, 2013; Siegler et al., 2011).

Studies show that the connection between decimal magnitude knowledge and decimal arithmetic, have not been given much consideration when measured up to the connection with fractions. In contrast, research shows that understanding the significance of separate decimals is definitely connected to the exactness of decimal arithmetic learning (Rittle, Johnson & Koedinger, 2009). Demonstrating the significance of decimals devoid of a “0”, directly to the right of the decimal point, is as precise and exactly as swift as demonstrating natural number significance. Conversely, demonstrating decimals with one or more “0”, directly to the right of the decimal point, is significantly more challenging (DeWolf et al., 2014).

2.14.4.4 Opacity of Rational Number Arithmetic Procedure

Research showed that the theoretical foundation of fractions arithmetic techniques are frequently far from being understandable (Lortie – Forgues et al., (2015:208). Questions such as: why are equal denominators compulsory for adding and subtracting, but not for multiplying and dividing? The following questions therefore linger in the minds of learners':

- ❖ Why should the natural number process be separately applied to the numerator and denominator?
- ❖ Why is the denominator overturned and multiplied when performing division of fractions?

Lortie – Forgues et al., (2015:208), concurred that all these questions have responses. However, the responses are not instantaneously fetched, and an in-depth comprehension of algebra, which is commonly taught after fractions, so that learners who do not have the relevant understanding of the process of learning fractions, and possibly would never have studied how algebra could be applied to explain fraction algorithms, after they acquired an in-depth understanding of algebra.

Decimal arithmetic techniques are in some ways more understandable than fractions arithmetic techniques. This was buttressed with situation to the corresponding of whole number process, which are similar. For instance, in performing the addition of 123 + 456, it involved adding ones, tens, and hundreds. On the other hand, in adding 0.123 + 0.456, one must add the tens, hundreds and the thousands. However, some features of decimal arithmetic, and their reasons, is normally vague to learners Lortie – Forgues et al., (2015:208).

2.14.4.5 Complex relations between rational and whole number arithmetic procedures

The relationship between whole number and fractions arithmetic processes is multifaceted. For instance, in addition and subtraction of fractions, once the least common multiple is found, the numerators are added or subtracted as though they are whole numbers, whereas the denominator is approved to the result with any action being executed. In case of multiplication, the numerators and denominators of the multiplicands are handled as though they are separate multiplication problems with whole numbers, irrespective of the denominators being the same, or not. In the normal division technique, the denominator is overturned, and then the numerator and denominator are handled as if they are separate whole numbers in multiplication problems. These complex relations between whole numbers and fractions procedures probably contribute to the prevalence of independent whole number errors (e.g. adding numerators and denominators separately as in $^{2}/_{3}$ + $^{2}/_{3}$ = $^{4}/_{6}$) (Lortie – Forgues et al.,2015:209),

Studies showed that independent whole number errors accounted for 22 per cent of sixth and eighth graders' answers on fraction addition and subtraction problems (Siegler & Pyke, 2013). Lortie – Forgues et al., (2015:209), argued that the mapping between decimal and whole number arithmetic procedures was also complex. The procedures for adding and subtracting decimals are similar to the corresponding procedures with whole numbers.

2.14.4.6 Multifaceted relations of rational number arithmetic procedures to each other

Lortie-Forgues et al., (2015:209) maintained that complicated connections between techniques of dissimilar fractions arithmetic processes contributed immensely to the challenges learners' face in fractions arithmetic. For instance, addition and subtraction of fractions with an identical denominator, necessitated leaving the denominator unaffected in the result, while multiplication of fractions equal denominator necessitated multiplying the denominators. Lortie – Forgues et al., (2015:209), echoed that unsuitable introduction of addition and subtraction process into multiplication resulted in blunders such as this: $^{2}/_{3}$ X $^{2}/_{3}$ = $^{4}/_{3}$.

Siegler and Pyke (2013) opined that about 55 per cent of response to division of fraction problems, and 46 per cent of response to multiplication of fractional problems, comprised of unsuitable introduction mechanisms from other fractions arithmetic processes.

2.14.4.7. Differing direction of effects of multiplying and dividing positive fractions and decimals below and above one

Siegler and Lortie – Forgues (2015), underscored the fact that acknowledging the impact of performing multiplication and division of proper fractions involving decimals (those between 0 and 1), created peculiar difficulties to learners understanding. In performing multiplication on counting numbers, the product always ended up in an answer bigger than either multiplicand, but in multiplying two proper fractions or decimals, the product is always lesser than the multiplicand. Research further showed that performing fractions on counting numerals never gave an answer bigger than the numerals being divided, whereas performing division on proper fractions, or decimal, constantly resulted in the product being greater than the numbers being divided. Similarly, the impact of multiplying and dividing numbers from 0 to 1 might be complicated owing to the fact that, adding and subtracting numbers from 0 to 1 have similar directional impact as adding and subtracting whole figures, as occurred in all the four-arithmetic processes with fractions and decimals bigger than one (Siegler & Lortie-Forgues, 2015).

2.14.4.8 Sheer number of distinct procedures

Fraction arithmetic entails the learning of huge numbers of varied techniques, more so than any other mathematical processes imparted in high school. It involves abilities in all four whole numeral techniques. These include: equivalent fractions, simplifying fractions, changing fractions to mixed numbers and mixed numbers to fractions. Becoming acquainted with capsizing the numerator or denominator when working on fractions arithmetic, and comprehending when equal denominators are preserved in the answer (addition and subtraction) and when the process in the problem should be used as the denominator and as the numerator (multiplication and division) (Lortie – Forgues et al., 2015 p.210).

2.14.4.9 Reading Difficulties

A study conducted by Baroody and Coslick (2008), suggested that learners facing challenges in reading find it difficult to read numerals such as seven, six and nine. They reversed the numerals and write seven back to front and confuse nine with six. Poor reading causes difficulties in reading fractions, expressing fractions, expressing fractions notation, mathematical combinations and constructed word sums, correctly. To improve reading levels among South African learners, the Department of Education (DoE) introduced the National Reading Strategy in 2008. The objective behind this is to ensure that every South African learner become a fluent reader who reads to understand, reads for enjoyment and enrichment (Department of Education, 2008). This venture is aimed at improving the academic performance of learners in every subject at all grade levels. Researchers are of the view that if learners' reading competence is poor, their writing and comprehension competencies would also be poor. This strategy adopted by the Department of Education is therefore motivated by the results of the systemic evaluation, which is conducted on the Intermediate Phase learners. It is recorded that 14 percent of learners are outstanding in their language competence, and 23 per cent are satisfactory, or partly competent, whilst the clear majority of learners representing, 63 per cent, are below the required competence when compared to their age level (Department of Education, 2008). This attests to the fact that the government ought to equip the various schools with good textbooks and teaching/ learning materials. Learners who cannot read and write tend to be passive in the mathematics classroom, especially when learning fractions.

2.14.4.10 Dyscalculia Paradigm

Dyscalculia paradigm deals with learners who lack the ability to learn mathematics. Research showed that learners do well in other subjects, but underperformed in mathematics. Szucs and Goswami (2013) concurred that dyscalculia paradigm is a selective weakness of mathematics, and about 6 per cent of children and adults suffer from it. Szucs and Goswami (2013) further argued that intelligence, reading and motivation to learn are normal. Access to appropriate educational provision helps people suffering from developmental dyscalculia. Study shows that about 50 – 60 per cent of children with developmental dyscalculia have a persistent condition of learning mathematics. About 95 per cent of children with developmental dyscalculia showed long-term weakness in mathematics. Shalev et. al., (cited in Szucs & Goswami, 2013).

2.14.4.11 Attention Deficit Related Problems

Research shows that short attention span of learners in class is a challenge to learners to keep an eye on all the processes necessary to accomplish a mathematical problem which involves fractions. Learners often leave their work unfinished or skip the steps involve in solving mathematical problems. They often ask for repetition of an explanation in class (Dednam, 2005). A study conducted by Serame (2013) observed that learners' lack of attentiveness in class, and feeling bored during mathematics classes, are the most prevalent misconducts exhibited in South African rural schools, which is a worrying situation to educators.

2.14.4.12 Visual-motor and visual-perception abilities

Researchers are of the view that the establishment of visual-motor and visual-perception skills in teaching should be given major priority at the early stage of education (De Witt, 2009:61; Patterson, 2008: 149; Papalia et. al., 2008:199). Learners with difficulties in counting numbers in series by pointing to each of them before counting, are likely to have problems with visual-motor and visual-perception. Such learners may likewise find it difficult to recognize figures of items correctly. According to research, learners with visual-motor and visual-perception problems are not able to comprehend geometric shapes as a complete object, but rather observed it as lines. These learners later faced challenges in life when working with numbers (De Witt, 2009:61; Patterson, 2008:149; Papalia et al., 2008:199, Lerner, 2006:479).

2.14.4.13 Lack of Perception Skills

Perception is the ability to categorize and ascribe connotation to stimuli (De Witt, 2009:61; Patterson, 2008:149; Papalia et al., 2008:199). Lack of perception skills have an adverse effect on learners' cognitive development. Learners at the pre-school age are deeply and actively involved in learning. At this stage of their academic life, they major in many pre-academic skills and gain an accumulated amount of understanding, information and skills essential for later academic work. Hearing skills, thinking skills, visual skills, memory skills, language skills and comprehension skills are acquired in the pre-school years. Young learners compare objects, sort out objects according to colour, shape and size, due to their experience in manipulating objects at the pre-school. For learners to cope with difficult tasks of high school mathematics, all these skills must be harnessed and developed at the pre-school stage to be able to meet the tasks at the high school level. For example, sorting of objects according to shapes, size and colour when developed in learners at the pre-school stage, would go a long way to help them in solving fractions using Cuisenaire rods, which are made up of different colours of different sizes. Study shows that learners who are not exposed at the pre-school lacked attention in class. They experienced perceptual skills, difficulties in motor development due to inadequate exposure to manipulative activities that guaranteed the comprehension of space, time, size, colour, distance and time, and this affects their mathematical skills in fractions in high school (Papalia et al., 2008:199; Lerner, 2006:477).

2.14.4.14 Effect of Sensory Motor Skills Regarding Fractions

The lack of fine motor coordination, optical motor and psycho-motor skills, especially at pre-school level, unfavourably affects learners' mathematics performance at the advanced stage in their schooling. Study shows that a lack of fine motor coordination, tactual kinaesthetic and optical motor incorporation also affects and impairs learning of mathematics in children (Dednam, 2005). Learners who suffer from auditory problems find it difficult to distinguish between numbers, which echoed almost the same and this adversely affect the learning of mathematics especially in handling of addition and subtraction of operations simultaneously. In a similar study, Dednam (2005), emphasized that visual difficulties that are manageable with fundamental concepts in mathematics connect to position value in numbers such as 15, posed a challenge to learners and these are referred to as "visual perception difficulties". In the same vein, researchers observed that learners are not able to differentiate between mathematical operations such as **+, - , x** and **÷.**

2.14.5 THE ENVIRONMENTAL EFFECT ON LEARNERS' FRACTIONAL ARITHMETIC ABILITY

Akkus (2016) made available useful materials for appreciating the atmosphere in which learners in the U.S. learn rational number arithmetic. Study shows that arithmetic knowledge in mathematics is not inborn and, in effect, is developed from the learners' communication with the surroundings and the attitude of the people in the environment. Mathematical knowledge is unearthed through the type of explanations given by the learner. Thus, learning is the outcome of what pertained in the learning environment of the learner. An educators' involvement is crucial in the sense that it creates a conducive atmosphere for effective learning in mathematics (al-Absi & Nofal, 2010). Study revealed that more than 80 per cent of U.S. states adopted Common Core State Standards Initiative (CCSSI) as a certified course of action in relation to which subject matter should be taught and when. Also, commendations from (CCSSI) have been integrated into the harmonized evaluations document, which stipulates what is be taught and at what time. A study conducted by Davis, Choppin, McDuffie, and Drake (2013), affirmed that the revised evaluations document which is prepared to reflect the (CCSSI) objectives, stimulate mathematical instructions. The (CCSSI) recommended textbook series, such as "Everyday Math", serves as a regulator to U.S. educators and learners as to when to teach rational number arithmetic. In South Africa, the two fold National Curriculum Statements (NCSs) for grades R to 9, and grades 10 to 12 respectively, were combined into a distinct document which is referred to as NCS grades R to 12. The NCS for grades R to 12 provides clearer specification to educators of what is to be instructed and taught on a term – by - term basis across public and private, Junior and Senior High schools, in South Africa (DoE, 2009). In a similar view, the Common Core State Standards Initiative (CCSSI, 2010) recommended that fractions arithmetic is a core topic of instruction in grades 4, 5, and 6 (roughly ages 9 – 12). CCSSI (2010) suggested that fractions arithmetic instruction should be taught, with addition and subtraction of fractions with common denominators, then continue to instructions of operations with uneven denominators, and to multiplication of fractions, and then to division of fractions. CCSSI (2010), asserted that the revised operations and instructions of learners of how to apply problem solving such as ratios, rates, and proportions should be taught in grades 7 and 8. Research has shown that fewer learners in a well-ventilated classroom tend to do better than learners in an overcrowded classroom. It has been observed that associating with high-ability learners in class, helps to boost learners' performance in mathematics, while associating with low ability learners in class, lead to low performance of mathematics (Burke and Sass, 2011). Rajoo (2013) opined that classroom learning environment is an important factor for motivating learners' mathematics achievement, especially in fractions. Formative evaluation is also one of the most essential features relating to success at all educational levels (Creemers & Kyriakides, 2008). Stears and Gopal (2010) argued that informative and collaborative methods are the best way of evaluating learners in mathematics. The environmental effect on learners' fractional arithmetic ability consists of the following: educators' inability to understand learners' construction of mathematical ideas, the language use in teaching fractions in mathematics, lack of school resources, lack of sense of concepts of time and direction, and lack of sense of spatial relationship.

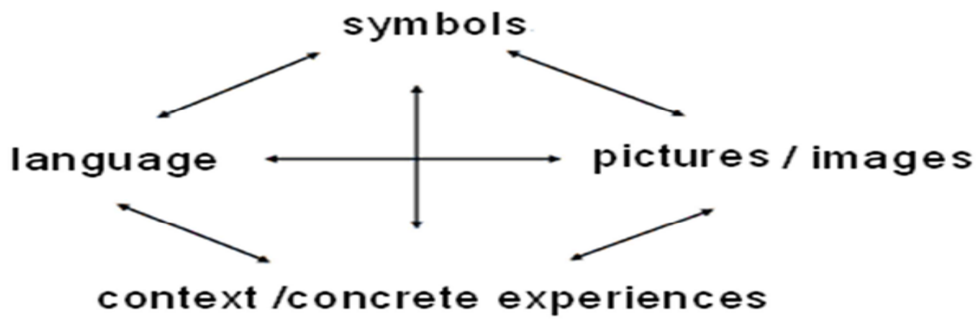
2.14.5.1 Educators' Inability to Understand Learners' Construction of Mathematical Ideas

Studies have shown that most educators do not understand learners' construction of mathematical ideas. Marake, (2013), argued that educators' inability to comprehend with learners' diverse ways of understanding mathematics is one major factor of environmental impediment to learners understanding of fractions arithmetic. For example, a fairy-tale of two learners and their educator who understood a mathematical problem in different ways. The problem involved two pizzas, where each was divided into twelve pieces. The problem enquired which fraction of the two pizzas was consumed by seven learners if each consumed one piece from each pizza. The two learners represented two pizzas as their unit and informed that $^{14}/_{24}$ of the two pizzas was consumed. The educator who represented one pizza as her unit expected the learners to response $^{7}/_{12}$ of a pizza was consumed. The learners' response was not incorrect with respect to the construction of the question, but the educator's presentation of a unit was at variance with the learners' unit, hence she saw the learners' presentation inappropriate. She clarified her stance to the learners but did not allow them to use physical materials to establish an understanding from her clarifications. When posed with the same question the subsequent year, the learners applied their previous understanding that they had established and arrived at their original answer (Ward, 2001). This demonstrated that an established knowledge gained from hands-on activities was retained, and the knowledge gained through direct tuition was forgotten. Study shows that interaction between learners and educators is a crucial element in constructivism. The illustration above seemes to suggest that if the educator would have analysed the learners' explanations carefully, the educator would have realized that the learners' answers were correct (Ward, 2001). This, therefore, brings to light how interaction between educators and learners assisted educators to understand learners' construction of mathematical knowledge. Marake, (2013) stressed that, educators should acknowledge that learners establishe their response from their previous knowledge which may be different from the educators' presentation. Therefore, they must be ready to accept other views from the learners, as far as the response is correct.

2.14.5.2 The Language Use in Teaching Fractions in Mathematics

Language is a vital tool in motivating and establishing learners' mathematical concepts, such as fractions. Mathematics is a language which needs to be understood by both learners and educators. To understand this language, it is imperative that educators resort to the use of apposite mathematical practise in which symbols, pictures, context and language are evocatively connected (Association of Teachers of Mathematics 2010). These collaborative efforts enhances learners' appropriate use of the mathematical language, and this improves their communication in class.

Figure 17: Showed the cognitive model of learning mathematics



Source: Association of Teachers of Mathematics 2010.

The Association of Teachers of Mathematics (2010), asserted that learners are not deeply involved in fractional activities as compared to numbers. Learners therefore become unfamiliar with fractional language and, in effect, do not exhibit assuredly from the context to the symbols and images of the mathematical concept. The Association of Teachers of Mathematics (2010) further argue that learners should be exposed to daily activities such as sharing of items equally, e.g., sharing of pizza among five learners to have a **concrete experience** of fractions. More so, learners should be exposed to **pictorial objects** by drawing a circular pizza cut into five equal pieces on the blackboard. When doing sharing among the five learners, the appropriate mathematical **language,** such as one part out of five should be used. The **symbol** $^1/_5$ should then be written on the blackboard for the learners to see. This demonstration enables learners to have a complete understanding of fractions. Marake, (2013) avowed that if this all-inclusive method to fractions instruction is not demonstrated, learners would continually face teething troubles in the use of fractional language, and disintegrate learning will always occur. In South Africa, mathematics is taught in the second language which serves as an impediment to the learners who cannot read and write English or Afrikaans. This affects the performance of learners because they face challenges to understand and demonstrate their mathematical knowledge in the second language (Education White Paper 6:19). Karslake (as cited in Marake, 2013), observed that the word “cancelling” in mathematics is one of the main causes of the learning development, especially in fractions. The word “cancel”, in general, means to “undo, annul, or remove” which is often used by both educators and learners in working mathematical problems involving fractions. It is obvious that most learners do not appreciate that cancelling in mathematics is applied when dividing. In effect, most learners use it as an inexpedient procedure typically when simplifying fractions. Learners do not associate it correctly to the theory learned. Inappropriate use of the word “cancelling” often results in teaching through memorization (Barmby, Bilsborough, Harries & Higgins, 2009). This attests to the fact that, if educators do not use manipulative concrete materials and the appropriate mathematical terms, learners would cancel inappropriately due to lack of understanding the mathematical concept. Learners are challenged in reading fractions correctly to express the mathematical meaning associated with it. UNESCO (2008) observed that over 50 per cent of students dropped out of school due to the incompetence of speaking the language in which they were being taught. Munkacsy (2007), is of the notion that learners’ problem in learning mathematics is as a result of lack of communication skills and social skills. The development of everyday communication skills is an important part of mathematics learning, especially fractions (Munkacsy, 2007). Celik and Korkmaz (2012:895), contend that when teaching learners about a concept, the educator must begin from familiar to unfamiliar, from indigenous to international, from simple to complex, and from concrete to abstract for learners to achieve meaning out of the study. It is important to acknowledge that language must be an entirety, rather than fragments through meaningful contexts which promotes rich response for language practice (Celik & Korkmaz, 2012:895).

2.14.5.3 Lack of School Resources

The resources and infrastructure of a school helps to enhance learners’ performance in mathematics, especially fractions. Research showed that well-resourced schools and class size are major factors that affects learners’ performance in fractions. Schools with well-equipped mathematical tools tend to do better than ill-equipped and over populated schools (Mohammadpour, 2012). In a similar view, Kyei and Nemaorani (2014), avowed that the location of schools affects learners’ mathematical performance in South Africa. They reiterated that schools situated in towns performed poorly in mathematics, because they are preoccupied by entertainment activities. However, Yusuf and Adigun (2010) are of the view that, there is no correlation between the influence of school location and learners’ mathematical achievement in fractions.

2.14.5.4 Lack of Sense of concepts of time and direction

Learners with difficulties of the basic concept of time and directions encounter difficulties with estimation (De Witt, 2009:61; Patterson, 2008:149; Papalia et al., 2008:199; Lerner, 2006:479). This is because of lack of exposure to manipulative concrete materials at the pre-school stage. Learners, are faced with difficulties to approximate how long it would take them to complete a task, or find it difficult to locate a place, or move in the right direction.

2.14.5.5 Lack of Sense of spatial relationship

The sense of space, sequence and order play a significant part in the manipulative activities of a learner. Researchers assert that learners should be actively engaged in activities that have to do with fixing objects of different shapes, sizes and colours into each other (Patterson, 2008:192; Papalia et al., 2008:190; Lerner, 2006:478). This in effect would help learners to develop a sense of area, categorization and uniformity. Pre-school learners should be exposed to different types of activities involving blocks, models, puzzles etc. Learners who lack the sense of space, find it difficult to compare and order in fractions learning.

2.14.6 Errors and Misconceptions in Fractions

Zwelithini and Kibiringe (2014), observed that errors and misconceptions in fractions are lengthy blunders and deficiencies that learners exhibit in their answers to mathematics tasks. In a similar vein, Nesher (cited in Zwelithini & Kibiringe, 2014), described misconstructions as a demonstration of a previously developed system of ideas and procedures that have been erroneously applied. Luneta and Makonye (2010) outlined two categories of errors in mathematics: Systematic and Unsystematic errors. Systematic errors may be recurring, methodically created, or recreated over a period of time due to comprehension of improper notions of answering a specific problem (Idris, 2011), while unsystematic errors are caused devoid of the purpose of the learners. Learners may not replicate such mistakes and learners could correct them by themselves (Zwelithini & Kibiringe, 2014). Luneta and Makonye (2010), argued that errors and misconceptions are related, but are different. Gabriel, Coche, Szucs, Carette, Rey and Content (2013) pointed out that the wrong misconception created by learners lead to struggling in working with fractions as a result of its complicated nature which involves ratio, operator, quotient and measure. In this context, ratio meant, e.g., constructing fractions as a ratio of boys to girls, 4:5, meaning $^4/_5$; operator $^4/_5$ meant 4 divided by 5 or $^4/_5$; quotient is the outcome of division. Measure defined fractions as numbers and intervals. Research conducted by Zwelithini and Kibiringe (2014) observed that learners' in South Africa, at both the Senior phase (Grades 7-9) and Further Education and Training (FET) phase (Grades 10-12) respectively, commits blunders and misunderstanding of fractions that are drawn back to their early stages in school.

2.14.6.1 Misconception of Multiplication and Division of fractions

Cindy, Jonathan, Edna, Meredith and Mallory (2016:13), observed that learners have misconceptions of multiplication and division of decimals. For example, when learners enter 0.4 x 8 into a calculator, their result is less than 8, and when they enter 8 ÷ 0.4, their answer is greater than 8. Research shows that this answer confuses most learners, who then ask for a new calculator.

2.14.6.2 Misconception of Decimals Arithmetic

Misconception learners are confronted with in decimals arithmetic includes, "The longer the number, the larger the number". Also, learners have misconceptions of whole numbers when they examine numbers to the right of a decimal (Karp, Bush, & Dougherty, 2014:23). For example, a learner might incorrectly reason that 2.255 > 2.5 because 0.255 is greater than 0.5. Learners difficulties are: the lengthier the decimal, the greater the number (Griffin, 2016). Cindy et al. (2016) argue that learners have misconceptions about addition or subtraction of decimals in the sense that, numbers to the right of the decimal cannot be rearranged into one entire number. For example, 2.6 + 3.6 = 5.12. This happens when learners observe the decimal part of a number as distinct from the whole number. Griffin (2016) asserted that, one common misconception prevalent among learners, is how to simplify and compare between decimals and fractions. This misunderstanding leads to learners erroneously altering $^3/_4$ to 0.34 or $^1/_4$ to 0.14.

2.14.6.3 Misconception of Addition of Fractions

Bush and Karp, (2013) avowed that learners tend to focus on numbers in addition of fractions and disregard the conceptual knowledge that involves the combination of two or more concepts. For example, learners perform the addition of fractions in this manner $^1/_5 + ^1/_3 = ^2/_8$. In this illustration, learners consider the figures on the fraction as distinct whole numbers, and then add the numerators and the denominators. This is one misconception learners have in the addition of fractions. Representations are important when using it to relate to mathematical methods, procedures of adding fractions and to assist learners' comprehension of the concepts of adding fractions and their connections (Cramer, Wyberg, & Leavitt, 2008). Research shows that when learners are exposed to proper illustrations of addition of fractions, learners will understand the concept and correctly apply them in solving mathematical problems (Osana & Royea, 2011).

2.14.6.4 Misconception of Part-Whole Fractions

The "part – whole" relationship of fractions that applies in division, is also a misconception learners' exhibit (Gabriel et al. 2013). For example, one litre of water distributed equally amongst five pupils. In this scenario, the one litre of water needed to be shared into five equal parts. The idea of the operator for division may be applied. When 1000ml of water is shared amongst three learners, each learner would get 333.333ml**.** Gabriel et al. (2013) observed that, the theoretical form of understanding fractions calculations posed misunderstanding in learners' choice of detail idea that needs to be used to a particular problem.

2.14.6.5 Misconception of circles in fractions

The use of pies and pizzas to teach fractions posed a challenge to learners. Usually, learners have the misconception of using pizzas or pies. The misconception learners have in using pizza is that pizzas always come cut into pieces which are not automatically equal (Pearn, 2007). Pearn (2007) suggested that learners should be encouraged to use paper folding strips, or pieces of paper streamers, mark fractions on number lines to understanding the density of the number system, and make their own fractions wall. Researchers emphasised that educators should use the right language of fractions by asking learners to write in words expressions such as: $1/_2$ x $1/_2$ (one half of a half: a quarter), and $1/_3 - 1/_6$ (The dissimilarity between one third and one sixth; one sixth).

2.15 CONCEPT OF LEARNING IN PERSPECTIVE

Schunk (2012:4) defined learning as a permanent change in conduct, or the ability to conduct yourself in a certain manner, which results from training, or other methods experienced. Schunk (2012:4) classified learning into three criterions:

One principle of learning is that it involves permanent change in behaviour, or in the ability to change behaviour. Learning occurs when a person can do something differently, and learning is inferential in nature. Learning is determined by the product and outcome of a person. Learning encompassed a change in ability to behave in a particular style, because it is not unusual for a person to acquire skills, knowledge, form opinions, or manners without proving them at the time learning ensued (Schunk, 2012:4). The second criterion of learning, according to (Schunk, 2012:4), is that learning withstands for a longer period of time. This eliminates short-term behavioural change (e,g., slurred speech) which occurs as a result of the influence of drugs, liquor, and/or exhaustion. These changes are momentary because when a person regained consciousness, the behaviour returns to its normal state. Learning, however, may not be everlasting due to forgetfulness. The third criterion of learning come about through practice (e.g., revision, imitation of others). This condition eliminates behavioural changes that are mainly determined by genetics such as maturational variations in children. Individuals may be hereditarily subjected to appear in a manner, but the real development of behaviour is influenced by environmental factors. Learners' phonological achievement, instruction and social communications with families, educators, and colleagues employ a resilient effect on learners' attainments (Mashburn, Justice, Downer, & Pianta, 2009).

2.16 TEACHERS PRESENTATION OF MATHEMATICAL KNOWLEDGE IN FRACTIONS

The responsibility of the educator in the presentation of mathematical understanding cannot be overemphasised. Realistic problem solving is greatly applied to progressively enhance learners' mathematical proficiencies and educators mathematical Pedagogical Content Knowledge (PCK), as well as their Mathematical Content Knowledge (MCK) (Buchholtz & Mesrogli, 2013). Studies have shown that mathematical modelling has been tremendously encouraged in the school mathematics syllabi in most countries all over the world in the last decade, as well as in South Africa (CAPS, 2011), and realistic learning has been established beyond uncertainty (Campbell, Oh, Maughn, Kiriazis & Zuwallack, 2015; Kang & Noh, 2012). Jacobs and Durandt (2017:63) argued that with problem – solving and modelling forming part of lots of mathematics instructional curriculum these days, it is imperative that educators integrate modelling in their classrooms. In contrast, researchers have cautioned against the use of modelling, and under preparedness of educators in respect of a detailed comprehension and teaching of modelling (Karali & Durmus, 2015; Ng, 2013; Ikeda, 2013). These authors reiterated that the open–end of model–prompting activities, as well as the development of classroom values, which contribute towards modelling, are persistently problematic for them. Educators may possibly not be able to acknowledge and appreciate the values and significance of establishing learners' mathematical modelling proficiencies, if they themselves are not sufficiently subjected to such assignments and events (Soon & Cheng, 2013).

2.16.1 Retention and memory model approach in mathematics

Research shows that cognitive psychologists are of the view that there are phases of attaining and processing information in order to establish one's own knowledge in learning. The initial phase of information process in mathematics learning is the “sensory memory phase”, or “sensory register” (Woolfork, 2013:228). While Schunk, (2008) argued that during the first phase of acquiring knowledge, the minds chose specific stimuli. Woolfork, (2013) claimed that the human mind is selective in paying attention, and this is influenced by the environmental factors that go on concomitantly. Therefore, learners’ inattentiveness can be attributed to as one major factors affecting learners’ mathematics performance.

Research indicates that the second phase is the intellectual process stage in learning mathematics. Salvin (2009) acknowledged that the human brain can accumulate information in the short – term memory, whilst (Woolfork, 2013) is of the view that the capacity of the short-term memory has a limited capacity, and can hold up to five to nine items of information at a time for about 15 to 20 seconds. Information that are well processed and assimilated are stored permanently in the long-term memory. Information which is not well processed, is kept in the short – term memory for a temporary period of time, and later discarded to make space for new information to be processed. Study shows that learners’ undesirable behaviour on functioning memory is that it overloads the limited space of the functioning memory, and decreased the span of attention (Schenk, 2011). Researchers are of the view that the last phase in memory processing in mathematics learning, which includes recalling educators’ methods of teachings, formulas, methodological operations and hypotheses. Conversely, Schenck (2011) emphasised that learners’ absentmindedness is a routine manifestation in a mathematics lesson. As a matter of fact, Salvin (2009) concurred that one important reason of forgetfulness among learners is interruption of other information which is mixed up with, or instantly pushed aside, by other irrelevant information. Salvin (2009) suggested that the reason for this forgetfulness is as a result of interference of external factors, which inhibited learners from absorbing the information taught, and practising it intellectually to establish it in their operational memories (Salvin, 2009). In view of this, Lee and Chen (2010) indicated that learners must be exposed to concrete manipulative materials to get a first-hand experience of what is taught. This, in effect, would eliminate the level of high rate of forgetfulness among learners in learning mathematics. In a similar view, Marake (2013:9) outlined the following strategies to enhance learners’ retention memory in learning mathematics:

- **Rehearsal strategies:** this deals with repetition of the same thing repeatedly until the concept is well established. The partitioning method can be applied to different forms of fractions repeatedly with learners to establish the concept of equality of parts in their long-term memory. Marake (2013:9) emphasised that a repetition method of teaching fractions does not only enhance rote learning, but it also promotes an in-depth understanding of the concepts among learners.
- **Elaboration strategies:** in this approach, new mathematical information can be associated with what is already known. Learners associate fractions to sharing of objects in their everyday situation.
- **Organizational strategies:** this approach involves the way information is presented to learners to make meaning of it. E.g., $1/2$ can be presented as a part of a whole.
- **Comprehension and monitoring strategies:** this involves things we do to keep us abreast with our learning. This includes taking notes during lessons, mnemonic, and probing to get answers.
- **Affective strategies:** Marake (2013) asserted that the attitude of learners during mathematics instruction has a great influence on their memory. If learners are fed up during a mathematics lesson, it affects their motivation to study the subject and, in effect, affects their memory retention.

2.17 DIFFERENT CULTURAL APPROACHES TO FRACTIONS INSTRUCTIONS

There are different approaches to fraction instructions across cultures and continents. Beneath, are some of the perspectives of fractions instruction among some best performing countries in mathematics compared to South Africa;

2.17.1 Japanese Perspective

Fractions instruction is formally introduced to learners in grade four in Japan. The educators' guidebook that goes together with Japanese textbooks, stipulates that educators need to establish two main concepts during fractions instructions (Watanabe, 2007).

Firstly, fractions are used to represent numbers less than one, and secondly, fractions are quantities similar to whole numbers. Both of these are fundamental concepts that educators emphasise during the course of fractions instruction in Japan, from grade four to the elementary stage (Watanabe, 2007). Instruction of fractions in Japan is focused on establishing the significance of fractions, and the concept of mixed numbers is made known to learners in grade four (Watanabe, 2007). Even though the Japanese mathematics syllabus also highlights on part-whole relationships, it is acknowledged that when learners are introduced to mixed numbers and improper fractions in the early stages of fractions, learners develop the misconception that all forms of fractions must be parts of one whole. Also, decimal numbers are introduced to learners together with the instruction of fractions in grade four. The relationship between fractions, decimals and whole numbers are introduced to learners in the fifth grade, and finally investigations of fractions arithmetic are taught in the sixth grade (Watanabe, 2007).

The Japanese elementary mathematics curriculum comprises of five components:

(i) Part-whole relationship,

(ii) Unit and non-unit fractions,

(iii) Fractions as operators,

(iv) Fractions as quotients, and

(v) Fractions as ratios.

In the Japanese curriculum, Part-whole relationship, Unit and non-unit fractions and fractions as operators are introduced to learners in the fourth-grade, whilst Fractions as quotients and Fractions as ratios are taught in the fifth and sixth grades, once learners have an in-depth comprehension of fractions (Watanabe, 2007). Watanabe, (2007) observed that, the Japanese textbooks reveal that most challenges learners encounter in fractions are structured within a well-defined framework where linear illustrations of fractions are applied. Table 10, shows an excerpt of the content of a Japanese textbook in fractions instructions.

Table. 10: An excerpt of content taught in fractions in Japan.

Chapter 11: Expressions and Calculations (Numbers and Calculations)
1. Represent the Expressions
2. Rules for Calculations
3. Calculation of Whole Numbers

Chapter 12: Area (Measurement)
1. Area
2. Area of Rectangles and Squares
3. Units for Large Areas

Chapter 13: Decimal Numbers (Numbers and Calculations)
1. How to Represent Decimal Numbers
2. Structure of Decimal Numbers
3. Addition and Subtraction of Decimal Numbers

Chapter 14: Thinking about How to Calculate (Numbers and Calculations)

Chapter 15: Arrangement of Data (Data and Relations)
1. Arrangement of Table
2. Arrangement of Data

Chapter 16: Multiplication and Division of Decimal Numbers (Numbers and Calculations)
1. Calculations of (Decimal Number) x (Whole Number)
2. Calculations of (Decimal Number) ÷ (Whole Number)
3. Division Problems
4. What Kind of Expression?

Review 2

Chapter 17: Fractions (Numbers and Calculations)
1. Fractions Larger than 1
2. Equivalent Fractions
3. Addition and Subtraction of Fractions

Chapter 18: Rectangular Prisms and Cubes (Shapes and Figures)
1. Rectangular Prisms and Cubes
2. Nets
3. Perpendicular and Parallel Faces and Edges
4. How to Represent Positions

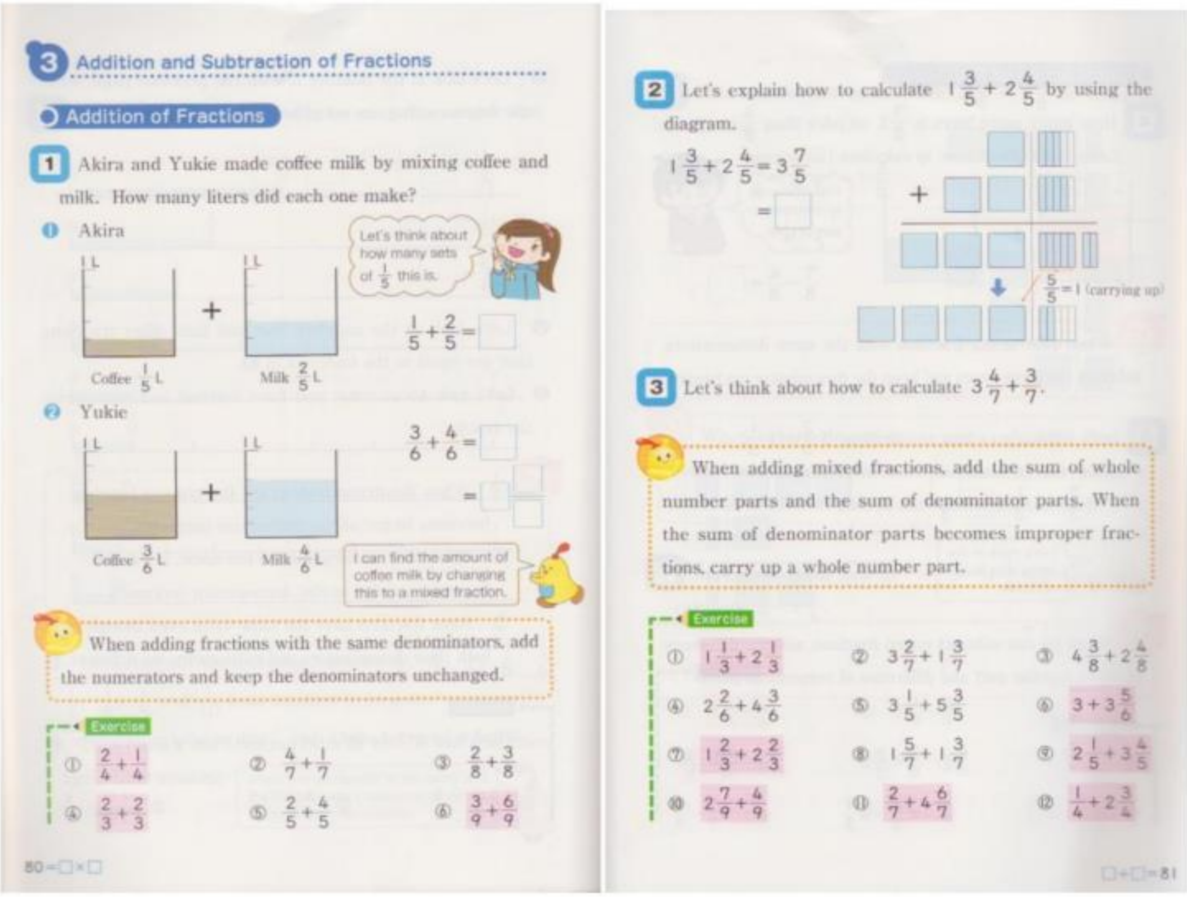
Chapter 19: Quantities Change Together (Data and Relations)
1. Quantities Which Change Together
2. Mathematical sentence using □ and o

Chapter 20: Summary of the Fourth Grade

Math Adventure:
1. How to Win Rock-Paper-Scissors
2. Getting on the Shinkansen Bullet Train
3. Getting on a Train
4. Forestry Industry in Japan

Source: Bruce, Chang and Flynn 2013.

Figure18: Excerpt of addition and subtraction of fractions in Japan



Source: Hitotumatu, (2011)

2.17.2 Korean Perspective

Son (2011), proclaimed that the instruction of fractions among learners in Korea is not different from the Japanese and the North American curriculum. However, some unique features in the Korean curriculum may have an additional contribution to its advancement. Fractions are presented to learners in the third grade, which is a year earlier than the Japanese learners. In Korea, the introduction of fractions to learners is focused on fractions as a demonstration of parts of a whole, points on a line, and parts of a set. This is in sharp contrasts to the Japanese instruction of fractions where decimals are introduced alongside fractions to enhance a strong foundation of fractions to learners (Son, 2011). The Korean syllabus stresses on presenting fractions as parts of a whole, and parts of a set to learners in the third grade (Son, 2011). In a similar vein, the Japanese also introduce unit and non-unit fractions in a measurement context to learners. Finally, Korean learners are taught how to associate fractions with like denominators, whilst learning about the connection between fractions and decimals. In grade four, different categories of fractions, such as mixed numbers and improper fractions are introduced to the learners. Learners are therefore introduced to fundamental arithmetic with fractions and are instructed to understand fractions as quotients, or as ratios. Learners continue to learn arithmetic fractions, in line with how to apply procedures such as multiplication and division with various types of fractions in the fifth and sixth grades (Son, 2011).

The Korean curriculum is comprised of five fractions constructed just like the Japanese curriculum (Son, 2011). These include: (1). Part-whole relationship, (2). Measurement (the treatment of unit and non-unit fractions, (3). Fractions as quotients, (4). Fractions as ratios, (5). Fractions as operators. All of these are introduced in the earlier stages of learners, and then come back to teach it again during the course of the elementary years. Fractions as operators are presented in connection with part-whole relationships in third grade, and then repeated the topic again in the fifth and sixth grades.

Son (2011) maintained that in relation to fractions illustrations, the Korean syllabus presented fractions with area models to highlight fractions as parts of an entity and then proceeds to apply distinct models of fractions as parts of a set. The Korean syllabus emphasises on reasonable application of a few carefully chosen models in diverse problem contexts. Also, the Korean textbooks encompassed more problem solving than Japanese and American textbooks (Son, 2011).

2.17.3Taiwanese Perspective

In comparing the instruction of fractions with other countries, it is observed that the Taiwanese have the most unique comprehensive textbooks and effective fractions instruction program (Charalambous et. al., 2010). The Taiwanese mathematics syllabus presents fractions to learners in the third grade, with much emphasis on the integration and disintegration of fractions (Charalambous et. al., 2010). In grade three, the sense of fractions is presented to learners, whilst the addition and subtraction of fractions are made known to learners in the fourth grade. At this stage, learners are taught how to add and subtract proper and improper fractions, and mixed numbers with like denominators. Later in the elementary stage of schooling, the addition and subtraction of fractions with unlike denominators are then introduced to learners (Charalambous et. al., 2010).

The Taiwanese textbooks emphasize the teaching of unit and non-unit fractions in measurement perspectives through illustrations such as number line, weight, or the volume of liquids in volumetric glasses (Charalambous et. al., 2010). The Taiwanese textbooks also make use of the incorporation of area model, set, and linear illustrations of fractions. The adoption of varieties of fractions illustrations rests on specific fractions construct. More so, Charalambous et. al., (2010) argued that the Taiwanese textbooks displays many graphical images such as cartoon figures in solving mathematical problems involving fractions in a step-by-step procedure.

2.17.4 South Africa Perspective

In South Africa, the CAPS document outlines the topic approached system which is characterised as the “week-by-week” ,and “hour-by-hour” method which is geared towards attention to critical thinking in teaching and learning (Republic of South Africa Department of Basic Education, RSA DBE 2011). CAPS (2011) argued that if educators follow the syllabus and ‘teach’ the topics in the week-by-week and hour-by-hour as prescribed, some of the skills in mathematics, especially fractions, will be acquired.

In a similar vein, CAPS (2011) emphasized that the arrangement and development of the topics are cautiously calculated, so that the theoretical preceding ideas would most probably be taught prior to the more advanced topics. The CAPS document stated that fractions should be introduced to learners at the Intermediate Phase, at the fourth and fifth grades respectively, and only in the sixth grade when it is acknowledged that learners have gripped the concept of fractions, after which the decimal fractions concept should also be taught (CAPS, 2011). The major idea behind the CAPS document in the instructions of fractions, is to guarantee that the instruction of decimal fractions is connected to the previous knowledge of place-value and fractions. In addition, this provides grades four and five prior knowledge of decimal fractions in the sixth grade. It is assumed that, at this stage, common fractions with denominators of multiples of ten are taught, and that the relationship of one tenth ($\frac{1}{10\,m}$) to ten hundred ($\frac{10}{100m}$) is understood (Long & Dunne, 2014 p. 138). The connections with measurement, particularly the length of a decimetre, one tenth of a metre ($\frac{1}{10\,m}$), and ten centimetres, ten hundredths of a metre ($\frac{10}{100m}$) – may offer the context with which learners will improve and exhibit the concept of equivalence of fractions (Long & Dunne, 2014 p. 138). Spaull, (2013a:32), acknowledged that an opinion has been portrayed in the air wave which states that, "Teachers must be taught what the workbooks, structured in the curriculum per week of teaching time, allowing them to ensure that the full curriculum is covered". The third approach of instruction in mathematics is labelled as the theoretical fields' method. A theoretical field is created by precise structural standards through multiplicative conceptual field recognised by common multiplicative constructs. The significant features of the field are comprised of problem situations, applications of thought and symbolic demonstrations (Long & Dunne, 2014 p. 137). The CAPS policy clearly indicates the content and skills that learners are expected to acquire in each year of study. DBE (2011) stipulated the instructional time for fractions in the intermediate phase for term 1, term 2 as 20 hours, and term 3 and 4 as 10 hours. The policy also stipulated the content to cover, methodology and teaching timeframes at all grade levels. The learning content is spread across the grades as illustrated in the table below (Department of Education, 2011). Below is an excerpt of the South African CAPS document for Grade 7 – 9.

Table. 11: A South African CAPS document for grade 7 – 9 Mathematics content.

CONTENT	GRADE 7	GRADE 8	GRADE 9
1.4 Common Fractions	**Percentages** • Revise: - Find percentages of whole numbers • Calculate the percentage of part of a whole • Calculate percentage increase or decrease of whole numbers • Solve problems in contexts involving percentages	**Percentages** • Revise: - finding percentages of whole numbers - calculating the percentage of part of a whole - calculating percentage increase or decrease • Calculate amounts if given percentage increase or decrease • Solve problems in contexts involving percentages	
	Equivalent forms • Revise: - recognize and use equivalent forms of common fractions with 1-digit or 2-digit denominators (fractions where one denominator is a multiple of the other) - recognize equivalence between common fraction and decimal fraction forms of the same number - recognize equivalence between common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** • Revise equivalent forms between: - common fractions (fractions where one denominator is a multiple of the other) - common fraction and decimal fraction forms of the same number - common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** • Revise equivalent forms between: - common fractions where one denominator is a multiple of another - common fraction and decimal fraction forms of the same number - common fraction, decimal fraction and percentage forms of the same number

CONTENT	GRADE 7	GRADE 8	GRADE 9
	Ordering and comparing decimal fractions • Revise: - count forwards and backwards in decimal fractions to at least two decimal places - compare and order decimal fractions to at least two decimal places - place value of digits to at least two decimal places - rounding off decimal fractions to at least 1 decimal place • Extend all of the above to decimal fractions to at least three decimal places and rounding off to at least 2 decimal places	**Ordering and comparing decimal fractions** • Revise: - ordering, comparing and place value of decimal fractions to at least 3 decimal places - rounding off decimal fractions to at least 2 decimal place	**Calculations with decimal fractions** • Multiple operations with decimal fractions, using a calculator where appropriate • Multiple operations, with numbers that involve the squares, cubes, square roots and cube roots of decimal fractions
	Calculations with decimal fractions • Revise: - addition and subtraction of decimal fractions of at least two decimal places - multiplication of decimal fractions by 10 and 100 • Extend addition and subtraction to decimal fractions of at least three decimal places • Multiply decimal fractions to include: - decimal fractions to at least 3 decimal places by whole numbers - decimal fractions to at least 2 decimal places by decimal fractions to at least 1 decimal place • Divide decimal fractions to include: decimal fractions to at least 3 decimal places by whole numbers	**Calculations with decimal fractions** • Revise: - addition, subtraction, multiplication and of decimal fractions to at least 3 decimal places - division of decimal fractions by whole numbers • Extend multiplication to multiplication by decimal fractions not limited to one decimal place • Extend division to division of decimal fractions by decimal fractions • Calculate the squares, cubes, square roots and cube roots of decimal fractions	
	Calculation techniques • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate	**Calculation techniques** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate	**Calculation techniques** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate

Source: (DBE, 2011).

The comparison of instructions of fractions between different cultures clearly shows that every culture has a unique way of imparting fractional knowledge to their learners in a systematic way. This also enables the researcher to ascertain what is being done in countries that are known to perform well in mathematics in the world, and what is also done in South Africa, where mathematics is a major problem to most learners.

2.18 MATHEMATICAL MODELLING

Mathematical modelling is an act of creating mathematical symbols in making an effort to solve real life problems (English, Fox & Watters, 2005; Ikeda, 2013). Kang and Noh, (2012), argued that modelling is a recurring process which encompasses: (1) the creation of a sample model, which originated from (2) a sequence of collaborative activities, which should be (3) repeatedly verified and developed in order to advance or validate it. The modelling process at any level, includes different forms of language, such as computer software packages, sketches, illustrations, tables, spreadsheets and the likes. A mathematical modelling cycle typically consists of four sequential phases, namely: "mathematization" (demonstrating an actual problem mathematically), "working with mathematics" (applying a suitable mathematical method to unravel the problem), "interpretation" (applying the appropriate mathematical knowledge in solving a real life situation) and "reflection" (scrutinizing the expectations and successive restrictions of recommended solution) (Balakrishnan, Ven & Goh, 2010:237-257). These illustrations are then authenticated, applied and unceasingly developed (Ang, 2010).

The International Community for the Teaching of Mathematical Modelling and Applications (ICTMMA), (Stillman, Gailbrath, Brown and Edwards, 2007:689), appropriately differentiated between mathematics applications and mathematics modelling. Mathematics application is an effort to associate mathematics to everyday life situations. Kang and Noh (2012) and (Ng, 2013), observed that there are three distinct stages of model – eliciting activities. These includes: orthodox problem solving, which is referred to as the level 1-problem solving. In this category, problems are systematically identified, and no extra figures are needed to create a model which necessitates a particular mathematical technique. Problems at level 2 have a bit of ambiguity, since an inadequate clue is provided to effectively complete the task. Level

3 – problems are the most reliable and open-ended type. It is accompanied by an unstructured and puzzling level of complication.

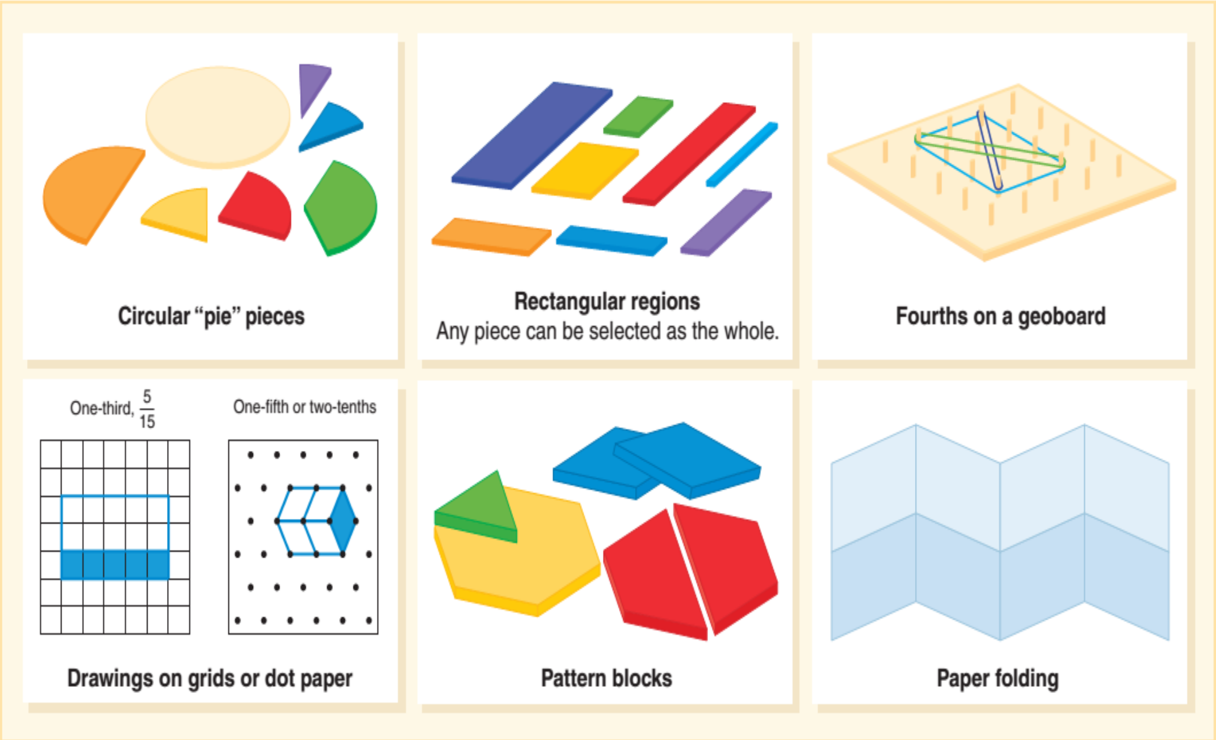
2.18.1 Types of Mathematical Models

Ervin (2017) proclaimed that there are a variety of mathematical models that may influence learning and appreciating fractions multiplication and division. Van de Walle, Karp, and Bay-Williams (2008) supported the notion that models are significant in the learning and comprehension of fractions and fractions operation. Models are applied to elucidate ideas that may be perplexing, when thought only in symbolic form. Van de Wall et al., (2008) supported that models provide learners with prospects to see mathematical problems in diverse ways, and from different viewpoints. Some models lead themselves to more straightforward specific circumstances than others. For instance, an area model can assist learners to distinguish between the parts and the whole, whilst a linear model explains that an additional fractions can be found between any two given fractions. There are three main types of models as stipulated by (Van de Walle et al., 2008). These includes: Area model, Length model, and Set model, in the case of learning and understanding fractions.

2.18.1.1 Area model

The knowledge of fractions being associated with area model is an important theory when learners work on sharing tasks (Van de Walle et al., 2008). These models can be represented in different forms. Circular fractions piece models are familiar in the mathematical community, and enjoys a lot of advantages in the part-whole theory of fractions as highlighted, as well as the meaning of the comparative size of a part to a whole concept. Subsequently, related area models can be created of rectangular regions, on geoboards, drawings on grids or dotted paper, pattern blocks, and by folding paper. (Van de Walle et al., 2008). Figure 19, shows the different forms of area models.

Figure 19: Showed different forms of area models in fractions.



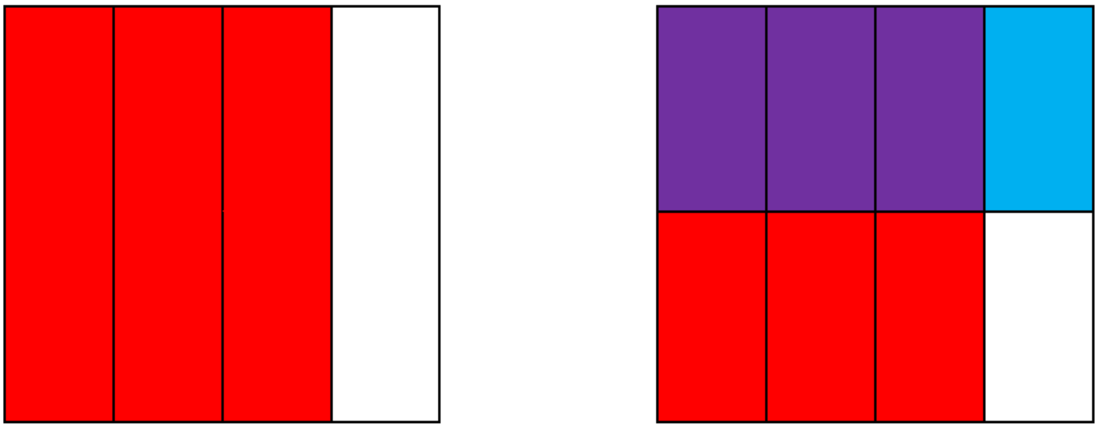
Source: Ervin (2017)

Van de Walle (2008) maintained that the area model can be used to illustrate a variety of mathematical modelling, including: area model of fractions multiplication and area model of fractions division.

2.18.1.2 Area model of fractions multiplication

Research shows that the area model of fractions multiplication appears to be extremely productive for different motives. It enables learners to acknowledge that multiplication of fractions stemmed from a smaller product and helps to understand fractional number sense, number sense associated to fractions as opposed to whole numbers (Krach as cited in Ervin, 2017). Research observed that this model shows a graphic for two fractions being related to one fractions resulting in a product close to one. Finally, the area model of fractions multiplication "was a good model for connecting to the standard algorithm for multiplying fractions" (Van de Walle et al., 2008). Typical area models are illustrated using rectangles and squares, as well as fractions circles (Tabor, 2001). Figure 20, illustrates a rectangle of area model in solving multiplications of fractions (Van de Walle et al., 2008).

Figure 20: Area model for multiplication Rectangle.
For Example; what is the product of ½ and ¾ ?

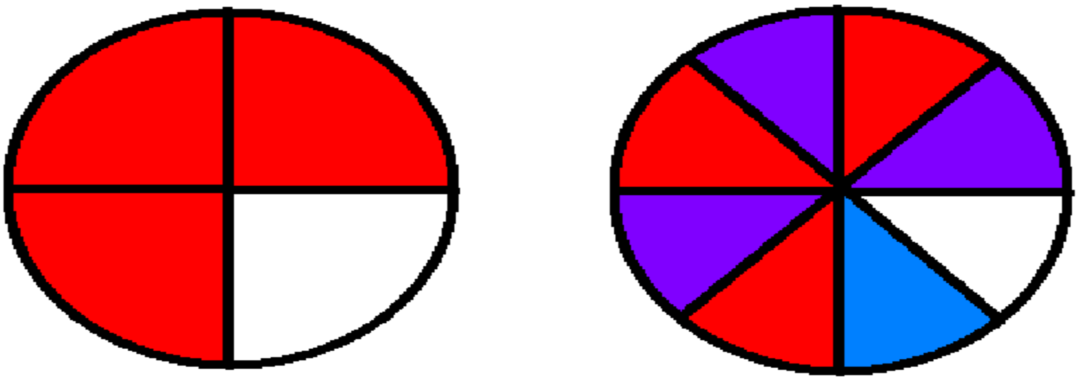


Partition one unit into four equal pieces. Then shade three pieces to represent $\frac{3}{4}$.

Partition each of the four pieces into two equal parts and shade one of each of the two parts. There are three double-shaded areas out of eight total pieces, thus, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

Figure 21: Area model for multiplication Circle.

What is $\frac{1}{2} \times \frac{3}{4}$?



Partition one unit into four equal pieces. Then shade three pieces to represent $\frac{3}{4}$.

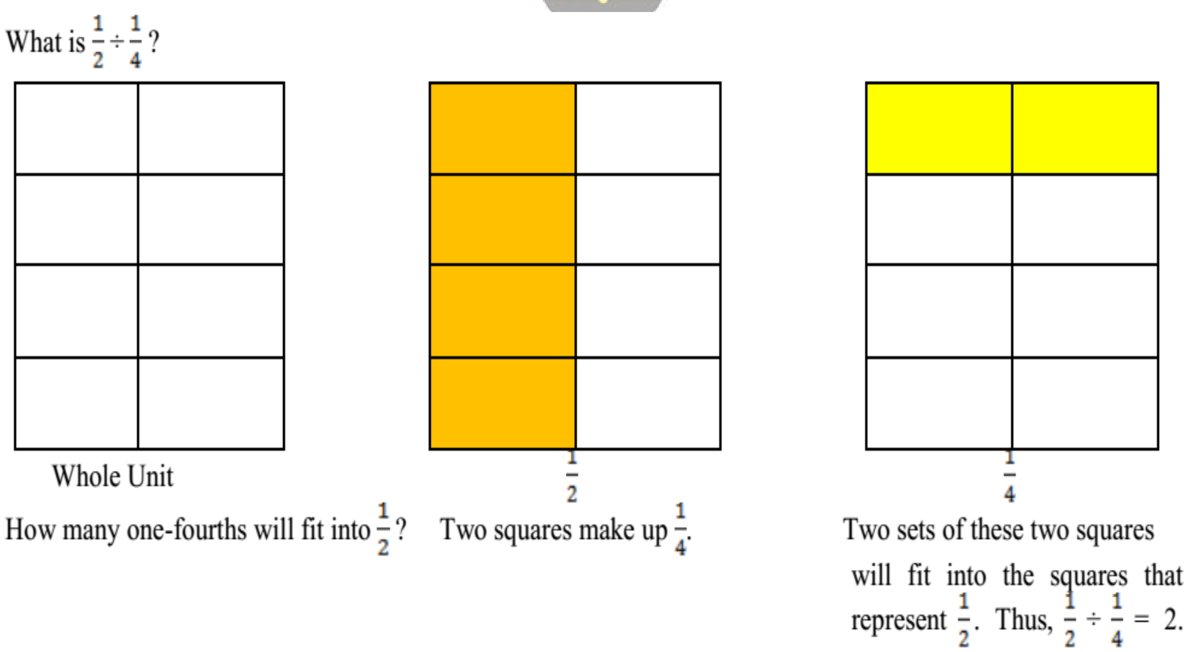
Partition each of the four pieces into two equal parts and shade one of each of the two parts. There are three double-shaded areas out of eight total pieces, thus, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

Source: Ervin (2017).

2.18.1.3 Area model for division

Ervin (2017), argued that the area model for division of fractions is an illustration that enables learners to visualize the process. Area model for division helps learners to establish fractional number sense by acknowledging that the quotient can be larger than the dividend (Ervin, 2017). In illustrating the area model, the entity is divided into horizontal lines to signify one divisor, and vertical lines to signify the other divisor. Study shows that this type of arrangement helps in solving problems where the equal size pieces may be challenging to create, and establish to learners the common-denominator procedure (Van de Walle et al., 2008). Figure 22, shows how area model is applied to resolve a fractions division problem, where horizontal lines and vertical lines illustrates the least common denominator (Van de Walle et al., 2008). Eg. Calculate ${}^{1}/_{2} \div {}^{1}/_{4}$

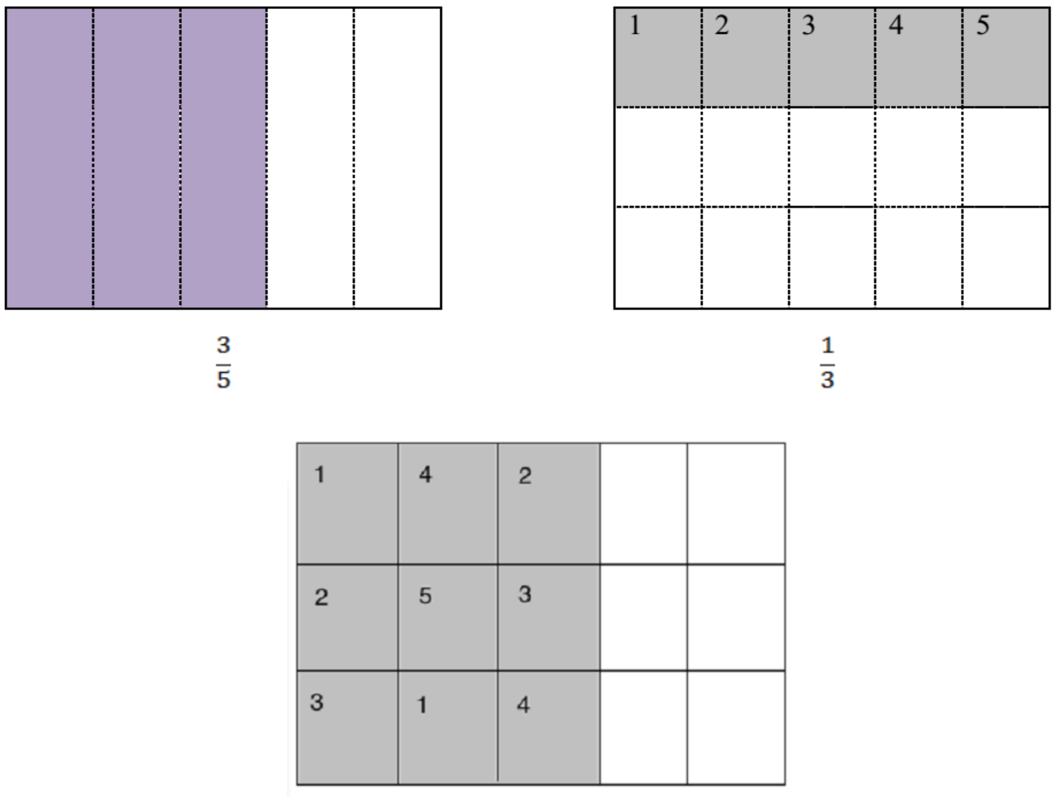
Figure 22: Shows area model of division without least common denominator.



Source: Ervin (2017).

Division of fractions can also be modelled by means of a piece of paper as a form of the area model, precisely as fractions multiplication is modelled with a piece of paper (Ervin, 2017). This model is designed in a similar form to the multiplication sample of folding a piece of paper into identical size pieces, in line with the problem under discussion (Taber, 2001). It is observed that this illustration is the same universal idea as the area model for division, but an alternative method of physically folding a paper is adopted (Ervin, 2017). Whether drawn, or illustrated, in a more physical way such as folding paper, “modelling played an important role in students’ understanding, and visualized what a division problem was enacting” (Johanning & Mamer, 2014). Through modelling, learners’ understanding of solving mathematical problems increases through using symbols by focusing on the magnitude of the dividend and divisor, and to evaluate whether their answers are judicious. Figure 23 illustrates how paper folding is used to solve mathematical problem involving fractions.

Figure 23. Showed paper folding for division.
Example. What is ${}^{3}/_{5} \div {}^{1}/_{3}$?



Source: Ervin (2017).

Find out how one-thirds $({}^{1}/_{3})$ will fit into ${}^{3}/_{5}$ five grey blocks made up ${}^{1}/_{3}$ of the whole unit. One whole set of these grey blocks and four out of five of a second set of grey blocks would fit into the purple area if we consider the purple area to be made up of nine grey blocks. Thus, $3/5 \div 1/3 = 9/5$ or $1\frac{4}{5}$.

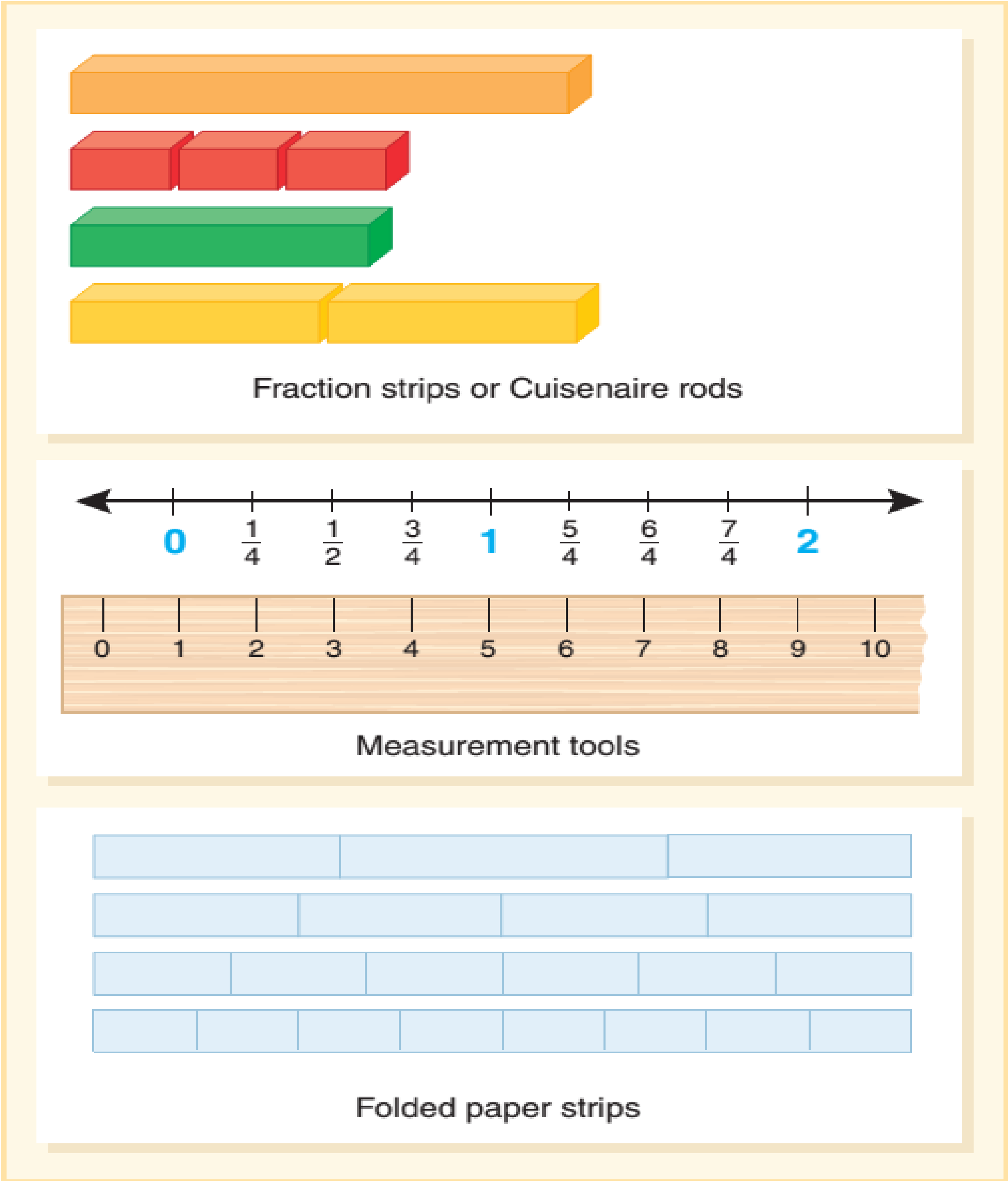
2.18.1.4 Length Model

Research showed that with length models, lengths or measurements are compared instead of areas. Physical quantities are compared based on length, or number lines are subdivided as shown in figure 24. Length models are significant in the development of students understanding of fractions. Siegler et al., (2010) observed that the number line enables students to understand a fraction as a number (rather than one number over another number). This helps students to develop other fractions concepts. Linear models are closely connected to the real-world contexts in which fractions are commonly used, such as measuring.

Cuisenaire rods, or paper strips are usually use as length models. Cuisenaire rods are made up of pieces in lengths of 1 to 10 measured in terms of the smallest strip or rod. Each length is made up of a different colour for ease identification. Strips of paper can be folded to produce student-made fractions strips. Cuisenaire rods, or strips, provides flexibility because any length can represent the whole. For example, if you want students to work with $1/4$ and $1/8$, students select the brown Cuisenaire rod, which is 8 units long. They therefore select the four rods (purple) which become $1/2$, the two rod (red) become $1/4$ and the one rod (white) become $1/8$. To explore twelfths, orange rods and red rods are put together to make a whole that is 12 units long. Refer to Cuisenaire rods in Figure 8.

Models, such as fraction bars are frequently used to compare fractions (Ervin, 2017). One of the key ideas behind the application of fraction bars is that of the unit and how learners can associate fraction bars as a whole length which epitomised the same unit. Fraction bars are mathematical models that enables learners to visibly acknowledge that fragment as a part of the whole (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). Figure 24 illustrates different quantities used to compare fractions in the length model.

Figure 24: Showes how fraction bars are used to compare fractions.



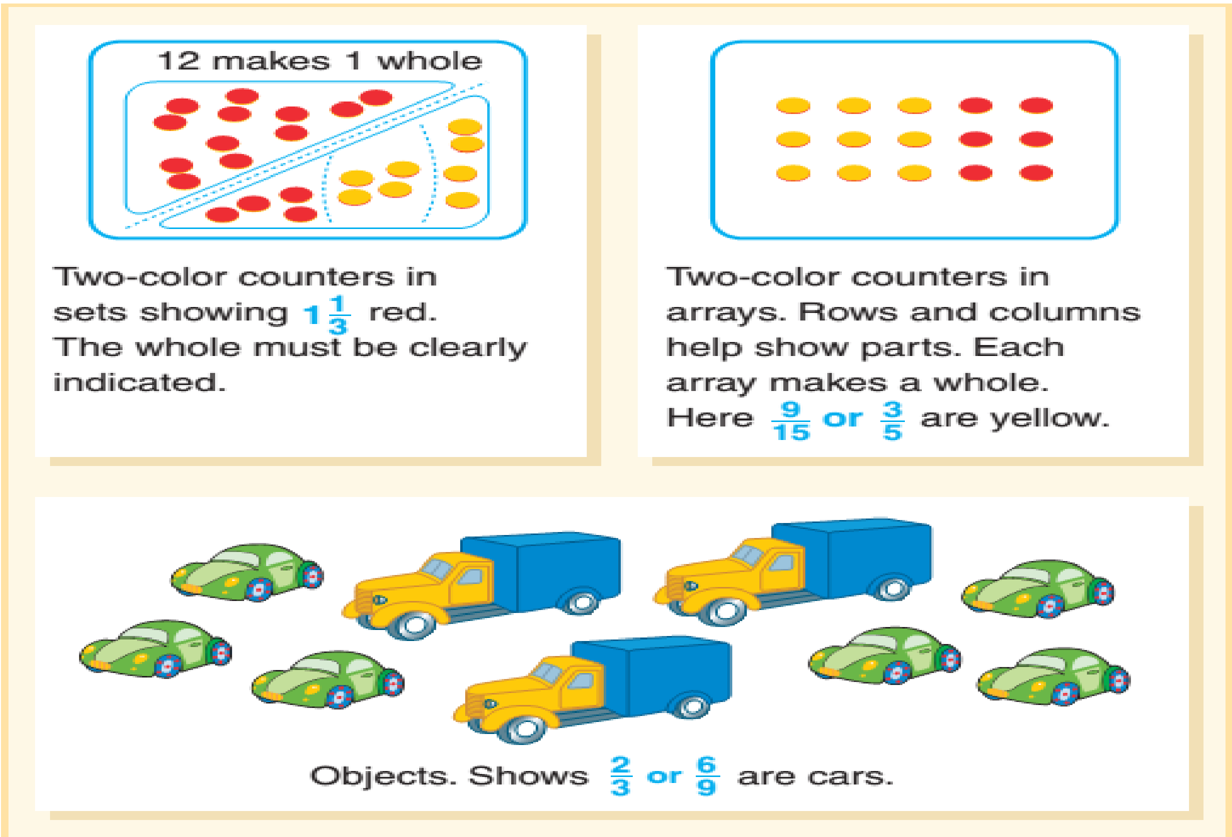
Source: Siegler et al., (2010).

Fraction bars and fraction strips served the same knowledge of length models. Fraction strips of paper are used to symbolise the entire length of paper. These models are used to demonstrate the connections between fractions, equate lengths, and investigate the equivalent of fractions (Ervin, 2017).

2.18.1.5 Set Model

Set models are made up of a group of the same elements where subsections of the entire set represents the parts of the whole (Ervin. 2017). The whole set denoted one entity. Research shows that applying a set of objects or physical materials to represent one entity is a challenging concept to understand. Despite this limitation, set models can be beneficial when making connections with practical presentations of fractions and proportionality concepts. It is also important to represent set models in different colours to illustrate fractional parts (Van de Walle et al., 2008).

Figure 25: Shows set models



Source: Van de Walle et al., (2008).

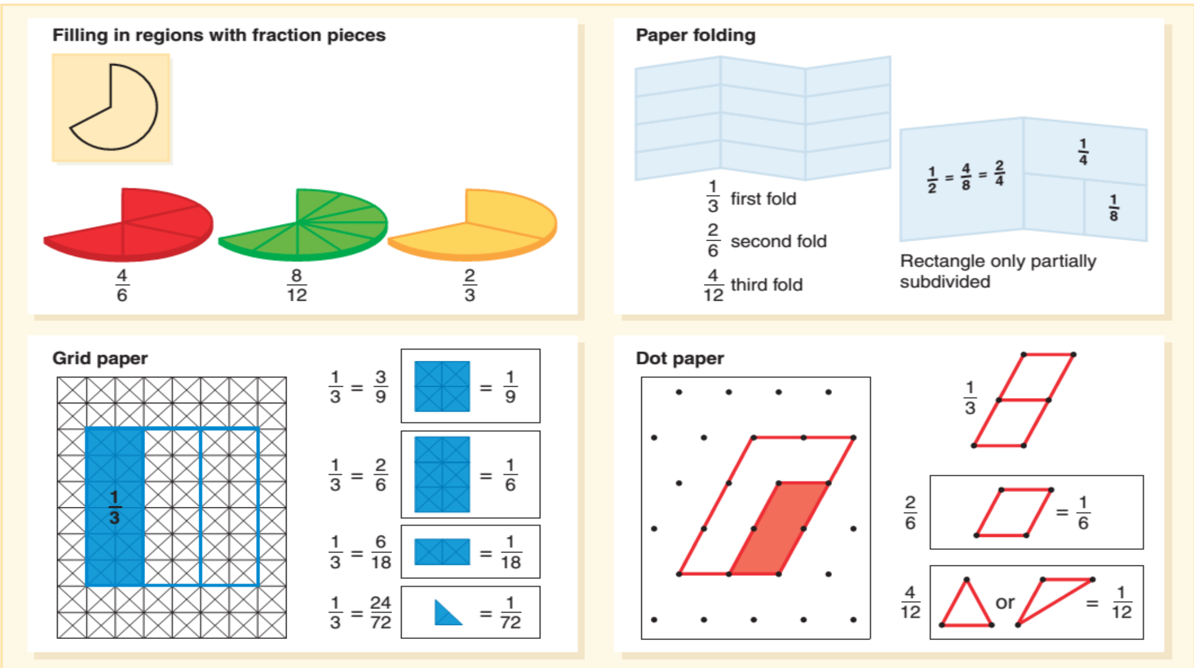
Research shows that counters, which forms a part of a set model, is applied to represent fraction multiplication. These models are useful, principally if learners are exposed to applying counters in solving mathematical problems. However, one difficulty learners' face with the use of counter model is the lack of understanding of what is considered to be the whole (Ervin, 2017).

In contrast, (Van de Walle et al., 2008) asserted that learners should not be dissuaded from applying counter models in solving mathematical problems, but instead educators should aid learners in applying counters to determine the whole.

2.18.1.6 Equivalent Fraction Models

The general approach used to help learners understand equivalent fractions, is to use contexts and models to find different names for fractions. The Dot Paper Equivalences activity involves "unitizing". That is finding different ways of chunking a quantity into parts to name it (Lamon, 2012). Length models are used in activities similar to the Making Stacks task. An example of this is asking learners to locate $^{2}/_{5}$ and $^{4}/_{10}$ on a number line. This example helps learners to see that the two fractions are equivalent (Siegler et al., 2010). Figure 26 illustrates different examples of equivalent fractions.

Figure 26: Shows different forms of equivalent fractions.



Source: (Siegler et al., 2010)

2.18.2 Importance of Models

Models are important mathematical tools that enhances learners understanding of fractions, as well as algebra (Brown & Quinn, 2007; Siegler et al., 2010; Son, 2011). Research shows that many learners do not do well in mathematics, especially in the United States because learners do not have power over the mathematical skills needed to master the subject (Siegler et al., 2010). An in-depth understanding of fractions is improved through the application of symbols, physical objects and other visual illustrations. Models are also used in the instruction of ratio, rate, and proportional problems, which are key elements in learning algebra (Ervin, 2017). Prediger (2011) asserted that knowledge of fractions and fractions operations may help learners surmount challenges associated to word problems, and enhanced their mathematical abilities.

2.18.3 Exposure of Educators to Mathematical Modelling

Mathematical model – eliciting activities have the proven capability to improve educators' intellectual thinking, interaction, elucidating and presentations of problems (Kang & Noh, 2012; Ng, 2013). These tasks subsequently develops an individual in decision making since classroom mathematics situations are related to our everyday life situations. Researchers all over the world, including South Africa, have acknowledged that educators' acquaintance to problem solving, way of thinking and opinions about mathematics, are elements that either enforce or imped their involvement in model – eliciting activities (Ng, 2013; Stillman, et. al. 2012; Julie & Mudaly, 2007; Tan & Ang, 2012). Lack of experience to model – eliciting tasks are identified as one main root cause of educators lack of readiness to apply other innovative methods into the mathematics classroom, since many educators see mathematics as a formula – based subject (Ng, 2013). Soon and Cheng (2013) are of the opinion that if educators acknowledged and appreciated the contributions that mathematical modelling contribute to the instruction of fractions, it would go a long way to improve the instruction of the subject. Especially if educators become more acquainted with the fundamental principles governing the modelling approach.

2.19 TEACHING PERSPECTIVE AND TEACHING METHODS

There are different teaching styles and teaching perspective adopted by educators in teaching fractions. A teaching perspective is an action taken by an educator, and whether the reasons behind his/her actions are laudable and justifiable, whilst teaching style or teaching method, is defined as an undying personal attribute and demeanour that shows how educators comport themselves in a mathematics classroom (Deggs, Machtmes & Johnson, 2008:1; Hunt, Barent, Lex, Grapentine, Liguori & Trivedi, 2008:358; Fung & Chow, 2002:314; Pratt, Collins & Selinger, 2001).

Pratt, Collins and Selinger, (2001) outlined five perspectives concerning fruitful teaching and learning. However, the researcher employed the following in his studies: Nurturing perspective, Developmental perspective, and Apprenticeship perspective.

2.19.1 Nurturing Perspective

Effective change of attitude to achieve individual goals and objectives come from the heart, as well as the mind-set. In the nurturing perspective of teaching, educators encourage, provide support and create a conducive classroom environment to learners to explore and solve daily problems, without fear or failure. Researchers avowed that in the nurturing perspective, educators guide their teaching by respecting the views of their learners, treating learners with dignity and mutual trust, whilst learners do the same to their educators (Deggs et al. 2008:2; Hunt et al. 2008:358; Fung & Chow 2002:314). Learners are encouraged to know that (a) they are capable of succeeding at learning if they put in their best; (b) their accomplishment would come from their own effort and resilience rather than the generosity of an educator; (c) their endeavour to learn receive a boost from their educators and colleagues.

For learners to adapt to the use of manipulative concrete materials in solving mathematical problems, especially fractions, the researcher makes the classroom stress free. Most educators in South Africa do not use manipulative concrete materials in the instruction of fractions. Due to this, most of the learners are scared and nervous to use these concrete materials during the research study. However, the researcher and the research assistants calmed the learners and encouraged them. Participants were guided through the exercises by the researcher and his assistants. Participants were motivated to ask questions and share ideas with their colleagues, after which they were assessed. In nurturing perspective, the evaluation of learning weighed up individual development, or progressed as well as total accomplishment (Courneya, Pratt, & Collins, 2007).

2.19.2 Developmental Perspective

In developmental perspective, comprehensive instructions need to be structured and piloted "from the learner's point of view'. Educators, with developmental teaching perspective, serve as facilitators in the instruction and learning process (Pratt, 1992:210). In view of this, educators should understand learners' construction of knowledge and understanding to maximise and develop their cognitive knowledge. A good educator should understand how leaners comprehend with the content knowledge. Educators' basic goal in this aspect is to assist learners develop increasingly sophisticated and complex cognitive structures for understanding the content. There are two skills needed to understand the cognitive structure. These includes: (a) 'bridging knowledge', which makes available examples that are evocative to the learner; and (b) effective interrogation that compel learners to advance from comparatively modest to more sophisticated form of reasoning.

For the researcher to ensure that his manipulative concrete materials are workable and can achieved his objectives, he first did pilot studies to ascertain the effectiveness of the tools. In this study, the researcher began his demonstrations of the use of manipulative concrete materials in solving fractions from simplest forms of fractions, and then proceeded to the complex fractions after he was convinced that participants can solve the simplest fractions using the concrete materials. In developmental perspective of teaching, educators develop in learners simple to more difficult and challenging topics, which demands a higher level of thinking. A good educator stimulates and motivates learners to apply their knowledge to each learner's level of thinking and understanding the content knowledge. (Courneya, et al., 2007)

2.19.3 Apprenticeship perspective

An apprenticeship perspective encourages a social-constructivist method of instruction, where teaching and learning develops learners to become independent learners, and also cultivate the social norms to work with others (Deggs et al., 2008:2; Hunt et al., 2008:358; Kramer, 2007:100-108). In this approach, educators are regarded as extremely experienced at what they impart to learners. Educators exhibit an in-depth knowledge and technical ingenuity which transforms into manageable language, and a systematic set of activities. Learning activities generally start from known to unknown, making available diverse opinion and understanding depending on the learner's competence. Experienced educators acknowledged that learners can work independently with little supervision and direction, by involving them with their 'zone of development'. As learners develop and become more knowledgeable, the educator's responsibilities change and, with time, educators offer little supervision and engage learners with more tasks as they advance from dependent learners to independent workers (Courneya, et al., 2007). At the later part of the study, the experimental group worked independently using the manipulative concrete materials to solve different types of fractions without the aid of the researcher and the research assistants. The main ideology behind this study is for learners to develop self-confidence and be able to use manipulative concrete materials to solve mathematical problems on their own. It also encourage learners to work in groups, interact with each other, and offer support to the slow learners.

2.20 TEACHING METHOD

Adu, Moyo and Olaoye, (2014), are of the view that instructional technique in classroom teaching is a comprehensive one in which instructional approach is most efficient. This contributes to a paradigm shift in which an educator serves as a presenter of knowledge to learners with the conceptualization of varied theories in the classroom tutoring which includes: tutoring, personalized teaching, structured learning, the school environment, audio visual aids, electronic learning workrooms, computer assisted terminal, and dial-access recovery systems. These methods largely advance educator's choice of accomplishing a specific learning outcomes. Adu et. al., (2014) asserted that instructional approach can be categorized into two groups: the orthodox, and the current or contemporary method. They reiterated that, in the orthodox methods of teaching, educators are loaded with too many responsibilities for classroom instruction to ensure that, what is taught, is well understood by the learners. The contemporary method of instruction involves an agreement between the educator and the learners concerning how each will participate and conduct themselves in the classroom, to establish a learner's anticipation towards independence. With this, learners established a strong bond exists between the educator and the learner which leads to student-teacher friendship. This friendship enables learners to share their mathematical difficulties with the educator devoid of fear or intimidation. In a similar view, Fletcher (2009), argued that there are different teaching methods adopted by educators in mathematical instructional theories to different levels of success. These includes: 'transmission' and 'interactive' methods. Studies shown that, the interactive approach of teaching is more effective to the transmission method of teaching. Fletcher (2009) further explained that the "transmission" method of teaching is referred to as traditional teaching method, or teacher centred approach of instruction. In this method of instruction, the educator is seen to be the repository of knowledge, where he/she feeds learners with the necessary information and superintends over every activity in the instructional process. In this method, learners are only passive in the instructional process. Research shows that this non-participatory method of teaching creates boredom in class and does not encourage learners to participate in class. It also makes learners feel they have nothing to offer in the teaching and learning process (Fletcher, 2009). The use of manipulative concrete materials is encouraged in the mathematics classroom, to enable full participation of learners during mathematics instruction. Mathematics is a subject which involves participation in classroom, and not just passive learners during the instruction process.

2. 21 LEARNING STYLE

Awla, (2014), contended that learning style is the "the complex manner in which, and conditions under which, learners effectively perceive, process, store, and recall what they are attempting to learn". Learning styles are considered as techniques employed by learners to gain knowledge, and are regarded to be less established, while cognitive styles are seen as an individual's instinctive, usual, and ideal way(s) of grasping, processing and recalling and acquiring new strategies (Awla, 2014). Learning style concepts have been named as an applicable way of assisting educators to identify the exceedingly varied needs learners bring into the learning environment (Williamson & Watson, 2007). Research shows that appreciating the ways learners acquire knowledge, helps the educator in selecting the appropriate instructional strategies suitable to meet learners needs (Zapalska & Dabb, 2002). In support, these concepts makes available structures that enables educators to acknowledge advanced and diversified methods of teaching to meet every learners need (Williamson & Watson, 2007).

Even though there is a comprehensive theoretical basis for the existence of learning styles, the prerequisite for advanced research, regarding the relationship between learning styles and learners' academic achievement, remains the same (Romanelli, Bird, & Ryan, 2009). Romanelli et. al., (2009) further indicated that investigators have not systematically delved into the relations between learning styles and accomplished learning effects, thereby hampering the application of learning styles theory in instructional application.

2.21.1 Classification of Learning Styles

Awla (2014) highlighted that learning styles can be categorised into three major groups: **Cognitive**, **Personality** (psychology), and **Sensory**.

Cognitive learning style consists of logical, reliant or self-determining, spontaneous, or contemplative learning styles (Awla, 2014). Personality learning styles are made up of overenthusiastic or introvert, random-intuitive or physical chronological, and closure-oriented or open-oriented learning styles (Awla, 2014). Sensory learning styles are a form of learning style which makes use of the senses. These are subdivided into three categories: Visual, Tactile or Kinaesthetic and Auditory (Awla, 2014; Dornyei, 2005).

2.21.2 Visual versus Verbal

Study showed that visual learners learn better through reasoning with pictures, and acquire knowledge through visual processes such as illustrations and videotapes. In contrast, verbal learners learn better with the aid of spoken instructions. Learners listen attentively in class or lectures to make meaning of what is being taught without the illustration of pictures. Such learners make use of abstract thinking (Awla, 2014). Manipulative concrete materials help learners to have a physical image of what is been taught in the mathematics classroom and by so doing improve their understanding and performance in fractions.

2.21.3 Auditory learners

Research showed that auditory learners learn better through audio channels such as oral debates, and paying attention to speeches and lectures. Auditory learners comprehend what is being discussed by focusing on the pitch, tone and swiftness of oral speech. They acquire knowledge better from reading- aloud activities, rather than by printed material (Awla, 2014). Auditory learners also acquire knowledge during the demonstration of the manipulative concrete materials by an educator, or their colleagues. During demonstration, the experimenter explained the process to the participants, which enhanced their understanding.

2.21.4 Kinaesthetic or tactile learners

Awla (2014) asserted that kinaesthetic, or tactile learners, learn better through touching of objects and feeling objects with their fingers, or any part of the body. Such learners' delights in intermittent pauses to enable them take a leisurely walk around the classroom. The use of manipulative concrete materials enable tactile learners to touch, and manipulate the tools to arrive at their desired results. This also makes the lesson practical to learners. In effect, it improves their understanding of the mathematical knowledge in fractions.

2.21.5 Importance of Learning Style

Awla, (2014) concurred that learning styles are significant in the lives of learners. If learners will be able to identify their learning style, they will be able to incorporate it into their learning process. In effect, the learning process becomes stress-free, more rapidly, and more efficacious. Learners who are able to identify their learning style will be able to cope with their mathematical difficulties, and will better positioned themselves to control their own lives (Biggs, 2001).

Also, learners who are able to identify their learning style, are able to learn on their own. Learners become independent and self-motivated in the learning process. Subsequently, learners' become more confident in learning and educators spend less time in teaching. In this instance, the lesson becomes learner centred, while the educator offers little or no assistance in the instructional process. Learners control their world in the learning process. (Gilakjani & Ahmadi, 2011). Another important benefit of comprehending learning styles, is that it assists educators to structure their lesson plans to equal their learners learning styles. Corresponding is crucial when working with slow learners, as they become bored during the learning process.

In addition, Alaka, (2010) and Lauria, (2010), asserted that a possible advantage of integrating learning styles into classroom instruction, is to assist educators and learners to better appreciate and comprehend the features associated to each personality in any given environment. Research identified three benefits associated with learning styles. These includes: Academic, Personal, and Professional benefits. Academic benefits includes: improve learners capabilities, surmounting all educational levels, recognizing how to learn to obtain an academic excellence, to observe regulated boundaries in the classroom, to ease tension and anxiety, and adapt to new learning approaches. Personal benefits includes: boosting learners' self-image and self-assurance, looking out for the best method to improve learners' understanding, identifying learners' strengths and weaknesses, planning to fashion out learning which will be more pleasurable, boosting enthusiasm for learning, and developing strategies to support learners' inborn talents and expertise (Awla, 2014). Learning styles aid educators to acknowledge learners possible strengths and weaknesses (Hawk & Shah, 2007). Professional benefits include: the awareness of specialised topics, attaining improvement over rivalry, being efficient in group organisation, establishing learners' transactions skills, and having control of learning (Awla, 2014).

Furthermore, researchers support the idea that using a variety of learning styles is more advantageous than adopting only one instructional method for learners' (Alaka, 2011; Martin, 2010). It is therefore significant for educators to concentrate on teaching learners and assisting them with teaching activities that will help them to become successful learners. Educators ought to be encouraged to improve their method of instruction to advance and apply their ability and instinct, and action tactics that will meet the needs of the learners (Martin, 2010). Educators consciousness of adopting different learning style preference, and the readiness to integrate diversity of instructional methods suitable to improve and become acquainted with the different needs of all their learners, is highly acknowledged by researchers (Cox, 2008; Hawk & Shah, 2007; Hsieh, Jang, Hwang, & Chen, 2011; Lauria, 2010).

2.22 CHAPTER SUMMARY

This chapter reviewed literature relevant to this study. This included literature on the theoretical and empirical framework of the study, conceptualization of fractions and manipulative concrete materials. Constructivism and cognitive theories were employed for this study, since the study looked at the effect of the use of manipulative concrete materials on learners' performance in fractions. The study reviewed the different types of manipulative concrete materials, approaches to teaching fractions using concrete materials such as Cuisenaire rods, paper folding, fractions tiles, and computer assisted manipulative. Perception of the use of manipulative concrete materials, challenges associated with the use of manipulative concrete materials, and errors and misconceptions in fractions were also discussed. Related concepts on educators' self-efficacy in mathematics instruction, educators' professional development, the environmental effect on learners' fractional arithmetic ability, the concept of learning in perspective, mathematical modelling, teaching methods and learning styles were reviewed. The chapter also reviewed the cultural approaches to fractions instructions among some of the best performing countries in the world in mathematics, and in South Africa. The next chapter looks at the methodology of the study.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

Research methodology is described as the action plan of the researcher (Jonker & Pennink, 2010). It involves the practical considerations to which the researcher structured his research study, given the questions he wanted to answer. Creswell, (2013:16), argued that research methodology involves the procedures of data collection, analysis, and interpretation that researchers recommended for their studies. This chapter elaborates on the research paradigm, research approach, research design, area of study, population and sample, sampling technique, data collection instruments, pilot study, validity of data, ethical considerations and limitations of the study.

3.2 RESEARCH PARADIGM

The genesis of research paradigm stemmed from Thomas Kuhn's book: "*The structure of scientific revolutions"* initially published in 1962 (Mouton, 1996:203). When Kuhn published the second edition of his book in 1970, the idea of a paradigm had already existed; and this attracted a lot of consideration to the role of paradigms in the history of the natural sciences. Studies showed that investigators and writers such as; (Mouton & Marais, 1990:150; Mouton, 1996:203; Creswell, 2007:19; Collis & Hussey, 2009:55; Babbie, 2010:33; 2011:34; De Vos, Strydom, Fouche, & Delport, 2011:40; Neuman, 2011:94) have previously applied the term to support their philosophy of paradigms, which has had a great impact on philosophy and methodology of the philosophical framework of their studies (Collis & Hussey, 2009:55). A research paradigm consisted of the established theories, models, approaches, frame of reference, traditions, research and methodologies, and it could also be a model or basis for observation and comprehension (Creswell, 2007:19; Babbie, 2010:33; Rubin & Babbie, 2010: 15; Babbie, 2011:34).

Research Paradigm is used to describe a researcher's 'worldview' (Mackenzie & Knipe, 2006). In a similar vein De Vos, Strydom, Schulze & Patel, (2011) concurred that a research paradigm is an all-inclusive system of interconnected procedure and cognitive activities that referred to the nature of enquiry along the line of epistemology, ontology and methodology. The ancestries of the qualitative and quantitative approaches extend into varieties of philosophical research paradigms, namely those of positivism and post-positivism (Wisker, 2008:68; Collis & Hussey, 2009: 55; Creswell, 2009:6,16; Gratton & Jones, 2010:23,26; Rubin & Babbie, 2010:37; Blumberg, Cooper & Schindler , 2011:16; Denzin & Lincoln, 2011a:1; Lincoln et al., 2011:117; Muijs, 2011:3,5). Post-positivism (post-modernism) is made up of two sub-paradigms, namely interpretivism (constructivism) and critical theory (critical post-modernism), while realism served as a bridge between positivism and post-positivism (Blumberg, Cooper & Schindler., 2011:18; 6). In this study the researcher adopted the positivism research paradigm, due to its connection to the study. Positivism is adopted for the study, because it is linked to quantifiable interpretations that are analysed statistically (Collins, 2010).

3.2.1 Positivism

Babbie, (2011:35) argued that the genesis of positivism can be traced down to Auguste Comte, who observed the human being as a phenomenon that needs to be studied scientifically. Positivism, therefore, is an approach to social research that requires the application of the natural science model of research as the point of departure for investigations of social phenomena, and for the justifications of the social world (Denscombe, 2008:14; 2010b:120).

The Positivist defined a worldview to research, which is based on a scientific method of enquiry. Collins (2010), is of the view that positivism hung on quantifiable interpretations that lead themselves to statistical analysis. It has an atomistic, ontological view of the world which includes distinct, noticeable features and events that relates in an observable, determined and unvarying manner. Positivism is often associated with the quantitative research method. For this reason, positivists preferred an analytical interpretation of quantifiable data (Druckman, 2005:5). A positivist is of the view that physical events can be perceived empirically, and clarified with scientific scrutiny. Welman, Kruger, Mitchell and Huysamen (2009:6) argued that positivism is directly linked with scientific model. This model formulated laws that are applicable to the population of study. These laws substantiated the bases of visible and quantifiable behaviour. Positivists are of the notion that an unbiased reality exists outside personal understanding, with its own cause-and-effect relationships (Babbie & Mouton, 2008:23; Saunders, Lewis, & Thornhill, 2009:113; Muijs, 2011:4).

De Vos et al., (2011b:6) concurred that positivists used scientific theories in postulating hypotheses which are subjected to empirical testing. This seeks to suggest that science is empirical, starting from specific propositions and from universal explanations of reality. A hypothesis is therefore established to assist investigators to subject the hypotheses to rigorous experimental examination for accepting, refuting or revising the hypotheses. In evaluating the validity of a scientific theory, the researcher took into consideration whether the information gathered (i.e. theory-based predictions) were reliable with the data obtained, or not. Positivism research encouraged experimentation to determine the realities of the study to eliminate the complexity of the external world. In addition, Welman et. al., (2009) affirmed that positivism is associated with the creation of laws which are universal to all study. Collis and Hussey (2009) contended that the main aim of positivism is to seek generalisations (theories). The theories are, however, affiliated to natural science laws which are not essentially appropriate to social structures. In all, positivism "*equates legitimacy with science and scientific methods*" (Scott & Usher, 2011:13).

In this study, the researcher employed a positivism paradigm because it assisted the researcher to investigate and experiment the effect of using manipulative concrete materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computer assisted manipulative) on grade nine (9) learners performance in fractions. The researcher used treatments on experimental group and control group, as stipulated by the positivism theory of laboratory experiment of study. This enabled the investigator to empirically observe the results in the form of data, and analyse it scientifically through inferential statistics. The researcher also used a Pre-test and Post-test to collect data from the experimental group, and the control group, which was later analysed using statistical method.

The four essential features or postulations of a paradigm for the Positivist includes: its epistemology, which is said to be objectivist, its ontology naïve realism, its methodology experimental, and its axiology beneficence. A critical look at these foundational elements enabled the researcher to understand the paradigm better.

Kivunja and Kuyini, (2017) asserted that the objectivist epistemology understood that human comprehension is acquired through cognitive application on issues. This attested to the fact that through investigation, we can acquire knowledge which enlightens us in comprehending the universe around us. The *naïve realist ontology* acknowledged the following five theories (Putnam, 2012; Searle, 2015):

- The world is made up of physical objects.
- Pronouncement of these physical objects existed through the physical experience of them.
- These physical objects exist whether they are perceived or not. These objects of awareness are presumed to be essentially perception-independent.
- These physical elements are capable of retaining features of what they are presumed to be, even when they are not being perceived. Their features are perception-independent.
- With the help of our senses, we imagined the universe directly, and beautiful as much as it is. Meanwhile our assertions to have a clue about them are reasonable.

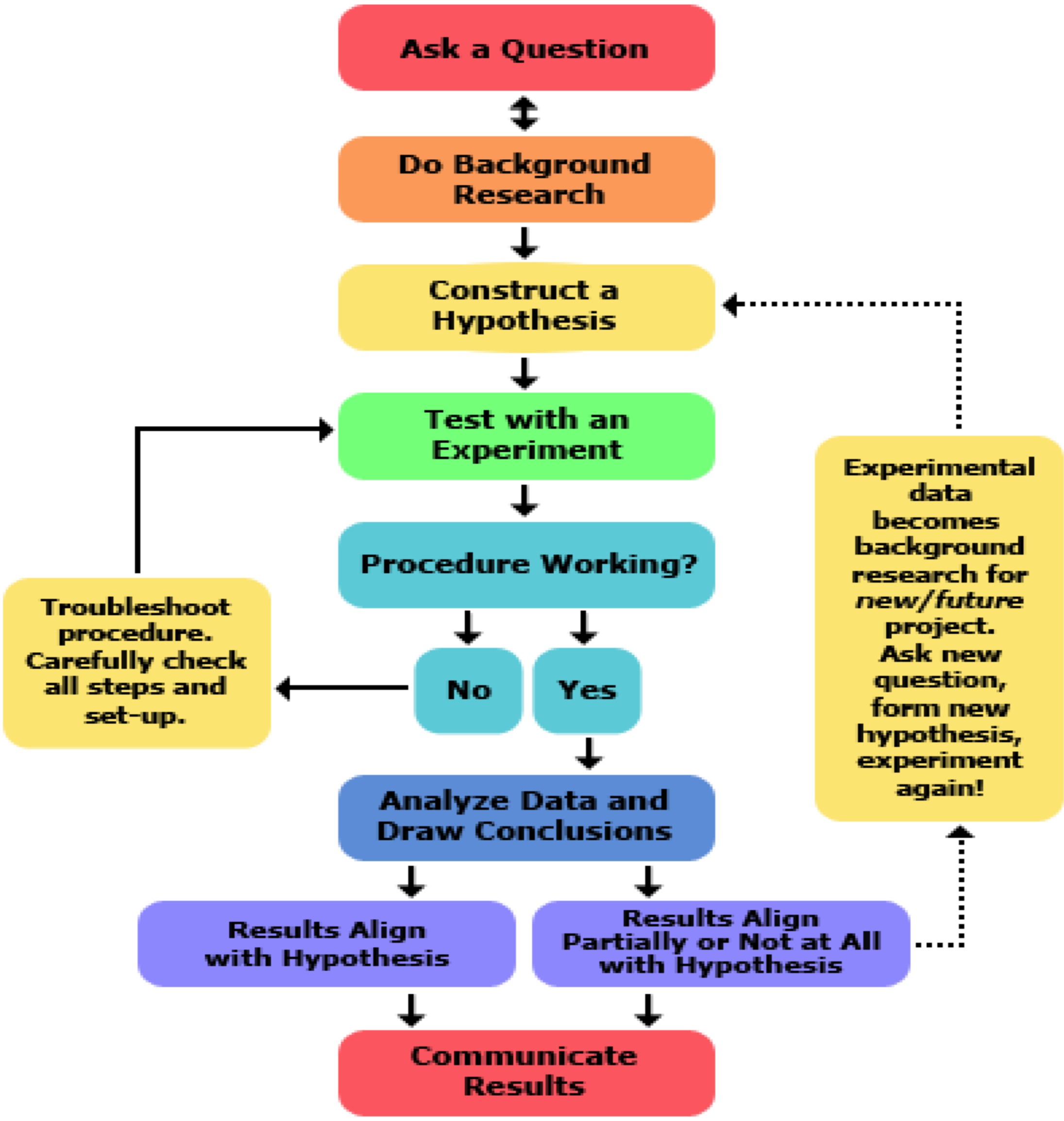
Kivunja and Kuyini, (2017), contended that the experimental methodology aspect involves the manipulation of one variable (experimental variable) to decide whether alterations in that variable can have effects on another variable (control variable), or not. In this study, the researcher grouped participants into an experimental group and a control group. The experimental group went through the treatment of manipulative concrete materials, whilst the control group did not. Pre-test and Post-test were administered to the groups, after which data were collected and analysed. This methodology is applicable if we can manipulate the experimental variables. The control variable enabled the investigator to accept or refute the hypotheses stated in chapter one. Mertens, (2015) argued that *beneficence axiology* referred to the condition that all investigation studied must be geared towards the capitalization of good results for the research study, for the human race in general, and for the research participants.

The basic characteristics associated with Positivist Research Paradigm as stipulated by (Fadhel, 2002) are as follows:

- The assertion that investigation should follow the Scientific Method of enquiry
- The assertion that theory is worldwide and universally accepted across all situations.
- The assertion that context is not paramount
- The assertion that facts are 'out there to be discovered' by studies.
- The assumption that cause and effect are divergent, and systematically distinguishable.
- The conviction that outcomes of investigation can be quantified.
- The belief that theory could be used to predict and to control outcomes.
- A scientific investigation is carried for evidence
- Relied on construction and testing of hypotheses
- Relied on the ability to perceive knowledge
- Employed empirical or analytical approaches
- The researcher's main aim is to formulate an inclusive general theory, to the explanation of human and social behaviour.
- The use of the scientific method.

Figure 27 illustrates the chart of scientific method of Positivist Research Paradigm.

Figure 27: A Scientific method of Positivist Research Paradigm



Source: (Fadhel, 2002)

Even though the positivist paradigm had been in the system for a very long time, and had been used by many educational researchers, in the latter half of the 20th century, it was challenged by interpretivist and post-positivism, because it was not subjective in interpreting social reality (Kumur, 2011). Their arguments were based on the fact that there is the need to replace objectivity with subjectivity in undertaking scientific research. Flick (2015), acknowledged that knowledge can be derived through observation and testing of hypotheses and theories, with the aim of gaining secure knowledge. The limitations of positivism had resulted in the development of other perspectives such as: interpretivism, post-positivism, realism, and the critical approach paradigm. Positivists are of the notion that there is an objective information out there to be studied, captured and understood, whilst post-positivists are of the view that reality can never be fully apprehended, only approximated (De Vos et al., 2011b:7). Upon all the arguments put up by the various theorists, the researcher opted for the positivism theory in this study because it enabled the researcher to experimentally, and scientifically, reach conclusions about the effect of using manipulative concrete materials on grade nine learners performance in fractions.

3.3 RESEARCH APPROACH

Research Approach is the organization, and the method, for an investigation that spanned from general assumptions to the detailed methods of data collection, analysis, and interpretation. The research approach consisted of: (a) Qualitative Research Approach, (b) Quantitative Research Approach, and (c) Mixed Methods Research Approach (Creswell, 2014). In view of the research paradigm adopted for this study, a Quantitative Research Approach was adopted by the researcher to gather data on the effect of the use of manipulative concrete materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computer assisted manipulative) on learners' performance in fractions in grade nine (9) in Chris Hani West Education District. This approach was adopted for the study by the researcher because, a scientific experiment, a questionnaire, and a performance test on fractions was used to collect quantitative data from both the control and experimental groups. Creswell, (2014) asserted that quantitative research is a method for testing objective philosophies by investigating the connection between variables. These variables can be measured, essentially on statistical instruments, so that numbers are generated for analysis using statistical methods.

3.4 RESEARCH DESIGN

Bless, Higson-Smith and Kagee (2006) asserted that research design is “operations to be performed, to test a specific hypothesis under a given condition”. Research design is the logical expectation the investigator brings to the study procedures of inquiry (Creswell, 2013:3). Similarly, Welman et al., (2009:46) concurred that research design is the plan, in which the participants of a research are selected, as well as the means of collecting data whilst Babbie and Mouton (2008:74) argued that research design is a strategy in which research is conducted. Denzin and Lincoln (2011), are of the view that research designs are approaches of inquiry of a study. In support Kumur, (2011) argued that a research design is a procedural plan of answering research questions accurately and precisely. It is the methodical, principled and laid down procedure of answering questions pertaining to research.

Researchers are of the notion that the research design should have varieties of approaches which implied that it will enable researchers to choose from different alternatives that will best suit their study. Zikmund, Babin, Carr, and Griffin, (2010:66) concurred that the research problem defined the procedural techniques to adopt, the type of method, the sampling technique, the data collection and the data analysis used for the research under studies.

For this study, the researcher adopted a Pre-test, Post-test, and Control group quasi-experimental design to determine the effect of the use of manipulative concrete materials in fractions on learners’ performance in grade nine (9). Pre-test are often used in a study to get prior knowledge of the actual situation, before a more rigorous investigation is conducted, whilst a Post-test is conducted after the participants have gone through rigorous training and experimentations (Creswell, 2014). In this study, the researcher used a pre-test and post-test to access participants, and came out with results. If there were differences between the results of the Pre-test and the Post-test, then it will assumed that the training had an effect on the participants. But, if the results remained the same after the post-test, then it will assumed that no learning had taken place, or the treatment have not had any effects on the participants. Quasi-experimental design is a laboratory-based experiment which is used to test practical situations to see if the findings are useful. Laboratory studies revealed whether, under highly controlled conditions, concrete manipulative materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative) would improve learners' performance in fractions, or not. This experimental design was used by the researcher to assess the effect of manipulative concrete materials on learners' performance in fractions before and after an experimental treatment. Pre-test and post-test were used to collect data on grade 9 learners' performance in fractions, and the results gathered were analysed using statistical techniques (SPSS) to determine the t-test, which was used to accept or reject the hypotheses. The researcher grouped the research design as systematically illustrated below:

O_1 X_1 O_3 ---------------------- Experimental group

O_2 X_2 O_4 --------------------- Control group

Where:

O_1 and O_2 were Pre-test observations for both experimental and control group's respectively.

O_3 and O_4 were Post-test observations for both experimental and control group's respectively.

X_1 was the experimental strategy of participatory learning using manipulative concrete materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative).

X_2 was the control condition of a convectional instructional strategy.

Table 12: A 2 x 2 x 2 Factorial Matrixes.

Treatment	Gender	Performance in Mathematics	
		Good	Poor
Experimental	Male		
	Female		
Control	Male		
	Female		

Source: Field study 2019.

3.5 VARIABLES IN THE STUDY

Creswell, (2014) defined a variable as a feature or characteristic of a sample. A population can be measured or observed and it varies among the people or population being investigated. Psychologists often referred to a variable as a *construct*, whilst Social scientists typically use the term *variable*. Variables are used in research to describe: gender, socioeconomic status (SES), age, and attitudes or behaviours such as racism, social control, political power, or leadership. Variables are grouped into two categories: (a) temporal order, and (b) their measurement (or observation) (Creswell, 2014). Punch, (2005) argued that temporal order means that quantitative investigators think about variables in a direction from "left to right", and order the variables in a specific statement, research questions, and visual models are grouped into left-to-right, in a cause-and-effect form of presentation. In a quantitative research study, variables are connected to answer a research question (e.g., "How do concrete materials affect learners' academic performance?") or to make extrapolations about what the researcher presumed the outcome to be. These forecasts are called hypotheses (Creswell, 2014). In addition, Creswell, (2014) enumerated two categories of variables as: *control variables,* and *confounding variables*.

Control variables are actively used in a quantitative research. These are a specific type of independent variable that investigators assess because they theoretically affect the dependent variable. Investigators use statistical tools (e.g., analysis of covariance [ANCOVA]) to assess these variables. They may be demographic, or special variables (e.g., age or gender), that needs to be "controlled", so that the real effect of the independent variables on the dependent can be assessed.

On the other hand, confounding (or spurious) variable, is not essentially assessed or observed in a research. It exists, but its effect cannot be determined directly. Studies show that, researchers' observe the effect of confounding variables after the research had reached a conclusion, because these variables are used to clarify the connection between the independent variable and dependent variable. In this study, the researcher used three categories of variables. These included: (a) Independent Variable, (b) Moderator Variable, and, (c) Dependent Variable.

3.5.1 Independent Variable

Independent variable is the treatment variable. Creswell, (2014) acknowledged that Independent variable can be scientifically measured, such as demographics (e.g., gender or age). Other independent variables may simply be observable variables in which no manipulation occurs (e.g., attitudes, or personal features of participants). In an experimental research as this, a variable such as different teaching methods are manipulated to determine learners' performance in fractions. In other words, manipulating the independent variable is expected to cause a change in the dependent variable. It is also referred to as a treatment variable. In this study, the researcher attempted to find the effect of manipulative concrete materials on learners' performance in fractions. The researcher employed the treatment on independent variables, whilst the control group were not. This involved the Participatory Learning Strategy (PLS), and the Conventional Teaching Method (CTM).

3.5.2 Moderator Variables

Creswell, (2014) argued that moderator variables are independent variables that influence the direction and the strength of the connection between independent and dependent variables, or between predictor and outcome variables. It is the variable that brings about changes between variables. In other words, it modifies the effects of another variable, and consequently cause statistical interaction. The moderator variable in this study were gender and school location.

3.5.3 Dependent Variables

Dependent variable is the reaction, or the standard variable assumed to be produced by, or affected by the independent treatment condition, and any added independent variables (Creswell, 2014). Rosenthal and Rosnow (cited in Creswell, 2014), advanced three prototypic consequence measures: (a) the bearing of experiential change, (b) the expanse of this change, and (c) the effortlessness with which the participant changes. In this study, knowledge was the dependent variable because it depended on the teaching methods adopted by the researcher. These included;

Application of New Knowledge (ANK), and Learning Outcome (LO)

Beneath is the summary from of the variables adopted for the study. These included:

A. Independent variable: These are the mode of instruction manipulated at two levels.

(i) Participatory Learning Strategy (PLS)

(ii) Conventional Teaching Method (CTM)

B. Moderator Variables

Two moderator variables were involved in the study.

(i) Participants' location at two (2) levels: Rural and Urban

(ii) Gender at two (2) levels: Male and Female

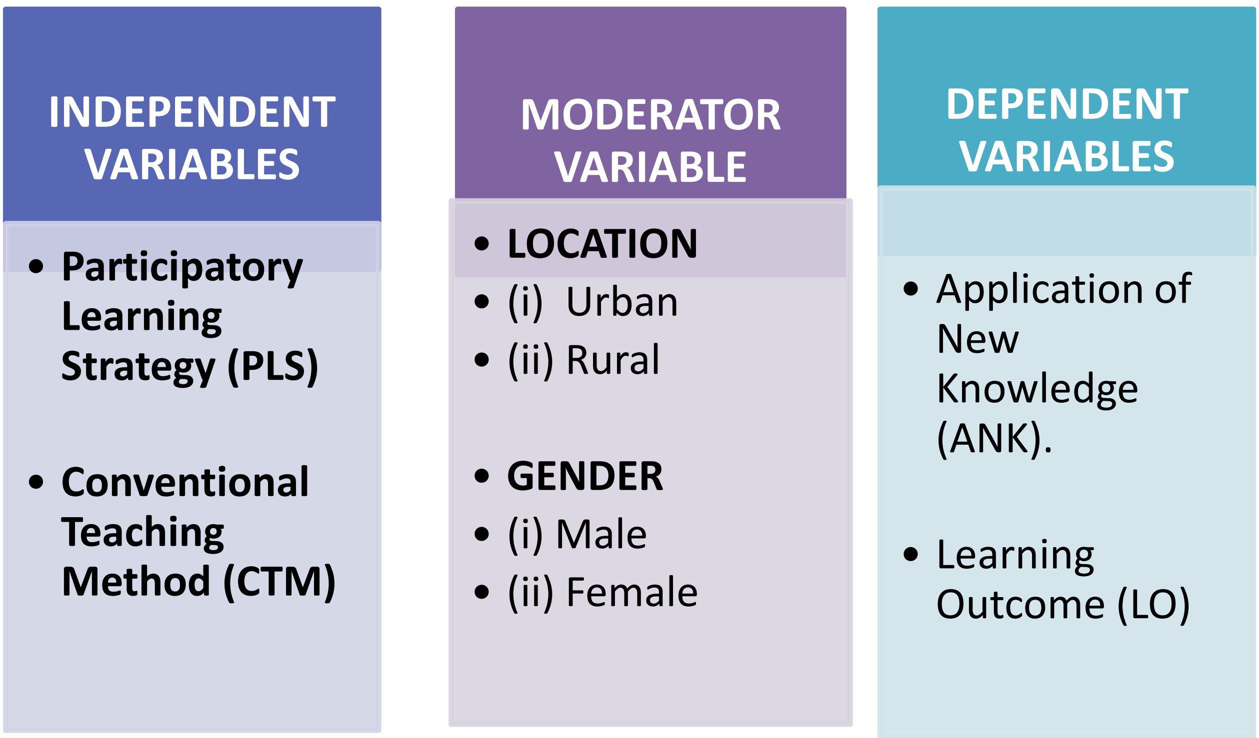
C. Dependent variables

Two dependent variables were involved in the study. These included:

(i) Application of New Knowledge (ANK)

(ii) Learning outcome.

Figure 28: A Framework of the variables in the study.



3.6 AREA OF STUDY

This study was conducted in the Chris Hani West Education District of the Eastern Cape Province of the Republic of South Africa. Komani was founded in early 1853 under the auspices of Sir George Cathcart, and then named it after Queen Victoria. The town was situated on the Komani River, which formed an enclave of the Great Kei system of rivers. Komani is in the middle of the Eastern Cape Province, and in the middle of the smaller towns of Cathcart and Sterkstroom (South African Government, 2016). It has a total land square of 71.3km^2 (27.5 sq mi), and a total population of 68,872. The town is made up of the following racial groups; Black African 81.8%, Coloured 10.0%, Indian/Asian 1.1%, White 6.5% and other 0.6%. The spoken languages in the town are; Xhosa 75.2%, Afrikaans 13.8%, English 7.3% and other 3.7% (Census, 2011). Komani is the commercial, administrative and educational hub of the neighbouring farming district. It has several high schools serving the town and the surrounding areas. Grade 9 learners and educators teaching grade 9 mathematics within Chris Hani West public high schools, were considered for the study.

3.6.1 Population of the study

Ogundipe, Lucas and Sanni (2006:100), opined that population is the entirety of all the members that enjoys a quantified set of one or more common features. Creswell (2014), described a population as a group in which the investigator is interested in gathering data and drawing inferences from it. The target population for this study consisted of all grade nine (9) learners in Chris Hani West Education District. Forty (40) public high schools were selected out of eighty-nine (89) high schools, which were combined schools (i.e. high schools which were combined with primary schools) for the study through multiple sampling techniques of stratified, systematic random sampling, convenience and purposive methods. Two hundred and fifty (250) grade nine (9) learners, and ten (10) educators teaching grade nine 9 mathematics in the schools where the study took place, were also selected through multiple sampling techniques of stratified, systematic random sampling, purposive and convenience sampling methods.

3.6.2 Sample

A sample is a representation of the target population being investigated, and findings from the sample are normally used to draw inferences about the population (Field, 2009). A research sample informs the quality of conclusions made by the investigator that stemmed from the essential outcomes (Burns & Bush, 2010). In this study, a sample of forty (40) public high schools out of eighty-nine (89) combined high schools, within the research locale, were selected for the study. This was a fair representation of combined high schools within the Chris Hani West Education District. The schools were selected, taking into account the proximity of the schools to the researcher, the performance of the schools in the previous matric examinations, and the Annual National Assessment (ANA). The forty (40) public high schools were divided into five (5) strata, taking into account the age, gender, race performance of the schools in national examinations, and resources of the schools. In each stratum, a school was chosen for the study. The researcher considered schools that performed poorly in the previous matric examinations, also in the previous Annual National Assessment (ANA). This was to ascertain the cause of the poor performance, especially in mathematics. Two hundred and fifty (250) learners' out of one thousand two hundred and fifty (1250), representing 20 per cent of the population, were considered for the study, which was a fair representation of the population according to the sampled guide stipulated by (Stoker, 1985 as cited by Adu, 2014). The sample were selected through multiple sampling techniques of stratified, systematic simple random, purposive and convenience sampling method. Ten (10) educators teaching grade nine (9) mathematics were also considered for the study. The educators were selected from the schools where the study took place. Table 13 shows the guideline formulated by Stoker, (1985) as cited by Adu, (2014) to give researchers a clear indication of sample size that was required of a population in a quantitative research.

Table.13: Stoker's sample guideline table.

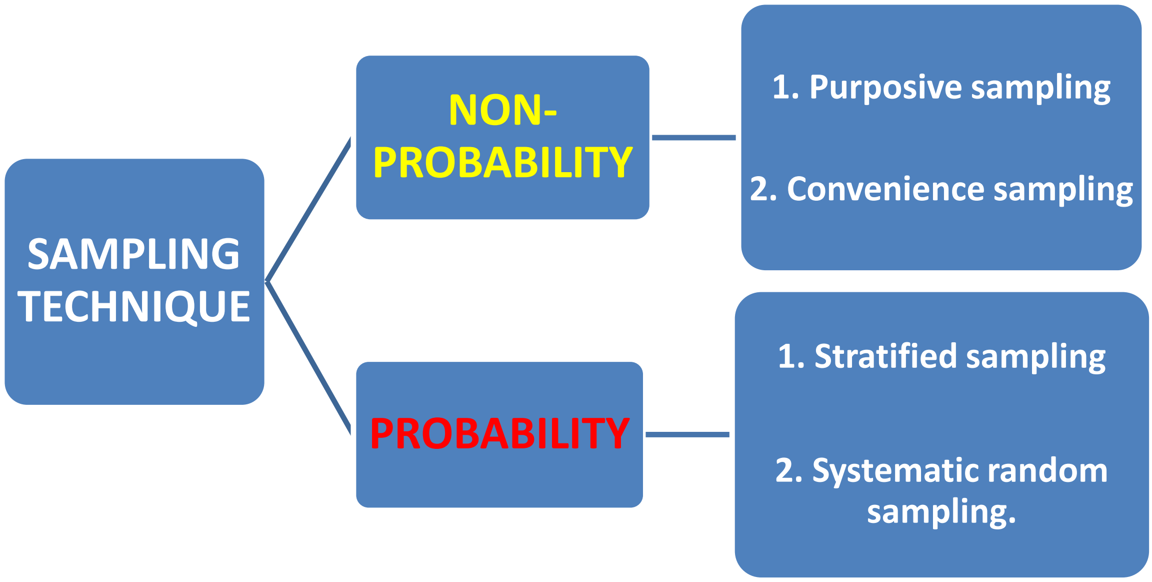
Population	Percentage Required	Number of Sample
20	100%	20
30	80%	24
50	64%	32
100	45%	45
200	32%	64
500	20%	100
1 000	14%	140
10 000	4,5%	450
100 000	2%	2 000
200 000	1%	2 000

Source: (Stoker, 1985 cited by Adu 2014).

3.6.3 Sampling Technique

Saunders, Lewis and Thornhill (2009), argued that sampling techniques are procedures applied in selecting a sample from a population by narrowing it down to a controllable size. This includes Non-Probability Sampling and Probability sampling. Non-Probability Sampling is a sampling technique that does not guarantee the probability that a population in the universe will have a chance to be selected into the study sample. Non-Probability Sampling includes: Quota sampling, Snowball sampling, Purposive sampling, Expert sampling, Accidental sampling, Modal instant sampling and Convenience sampling (Etikan & Bala, 2017). On the other hand, probability sampling, also refers to as random sampling, is a selection process which enables every distinct element from the universe to have an equal opportunity in the sample. Probability sampling is made up of: Multi-stage sampling, Cluster sampling, Systematic random sampling, Stratified sampling, and Simple random sampling, (Etikan & Bala, 2017). However, in this study the researcher employed; purposive sampling, convenience sampling, stratified sampling and systematic random sampling.

Figure 29: shows the components of sampling techniques used in the study.



Source: Etikan & Bala, (2017).

3.6.3.1 Purposive sampling

Rahi (2017) defined a purposive sampling as a technique where researchers use their own judgement to hand-pick a specific group of sample who are aware of the problem under investigation. Purposive sampling, also known as judgemental sampling, contains a specific purpose. In addition, Etikan and Bala, (2017) concurred that purposive sampling, or judgemental sampling, relies on the discretion of the researcher as to who would provide the appropriate information to fulfil the purpose of the study. In this study, the researcher used his own discretion to hand-pick schools from each stratum taking into consideration the proximity of the schools, the performance of the schools in the previous national examinations, and the resources available in the schools to enable him carry out his research successfully. Educators, who also served as the research assistants, were hand-picked from the schools because they taught mathematics in the same school, were familiar with the learners and helped the researcher in teaching. Researchers observed that purposive sampling is convenient, cost effective and less stressful (Rahi, 2017).

3.6.3.2 Convenience sampling

Convenience sampling technique is a process of data gathering from a population that is near to the researcher, and easily reached. Convenience sampling is cost effective, and it enables researchers to conduct interviews, or get responses in their comfort zone, and at their own convenient time. However, this method of sampling is prone to bias due differences in the target population (Rahi, 2017). This sampling method was used in this study because the researcher selected schools from each stratum that were not far from where he resided. This enabled the researcher to get to the various schools at his own convenient time to conduct his research with the learners. The researcher also accessed information from the schools at his own convenient time.

3.6.3.3 Stratified Sampling method

Stratified sampling is a process of apportioning the sample frame into strata to obtain fairly homogenous subgroups (Sanni, 2011). De Vos, Strydom, Fouche and Delport, (2011) observed that this method is appropriate for a heterogeneous population to enable the small subgroups, in terms of percentages, to be included in the sample frame. Specific characteristics of the individuals, such as gender (male or female), are represented in the sample, in relation to the proportion of the population (Fowler, 2009). For example, if a sample size of 200 is to be selected from a sample frame of 4000 that consisted of four strata of 700, 1300, 1100 and 900, this can be done by using the sample size, either proportional stratified, or non-proportional stratified sampling. The proportional stratification involves the following:

Subgroup 1. $^{700}/_{4000}$ X 200 = 35

Subgroup 2. $^{1300}/_{4000}$ X 200 = 65

Subgroup 3. $^{1100}/_{4000}$ X 200 = 55

Subgroup 4. $^{900}/_{4000}$ X 200 = 45

The proportional selection is as follows: 35 + 65 + 55 + 45 = 200 sample size. This is proportional, because all the subgroups are proportionally represented. However, in the non-proportional, or disproportional sample, the number of the elements chosen from each subgroup are disproportionate to its size in the sample frame. Using the sample frame from the first example given, selecting 200 from the four strata of 700, 1300, 1100 and 900, requires that we divide the sample size by the number of strata $200/_4$ = 50. This meant that 50 elements are selected from each stratum, or sub-group, irrespective of the size of each stratum. In this research, the researcher grouped forty (40) public high schools under study into five (5) distinct strata, taking into consideration the gender, age, race, performance in the national examinations, and the resources of the schools. Sanni, (2011) outlined some of the features used in stratifying the population as being: educational level, gender, and race etc.

The stratum are labelled with the letters A, B, C, D, and E. Each stratum consisted of eight (8) schools with equal features. The stratification is illustrated in Table 14.

Table. 14: shows the stratum with number of schools.

STRATUM	NUMBER OF SCHOOLS
A	8
B	8
C	8
D	8
E	8
TOTAL	40

Source: Field study (2019).

In this study, the researcher employed the non-proportional, or the disproportional sample method to select a sample of 250 participants from the five strata. This was done by dividing the sample of 250 by the 5 strata i.e. $250/_5$ = 50. This meant that the researcher selected 50 participates from each stratum or sub-group, irrespective of the size of each stratum. Fifty (50) grade nine (9) learners were selected from each of the schools considered to participant. Two-hundred and fifty (250) grade nine learners, and ten (10) educators, teaching grade nine mathematics, from the five (5) selected schools were considered for the study. The teachers were hand-picked by the researcher from the schools where the study took place to assist in the study. Out of these (250) learners, one hundred and two (102) learners were boys, and one hundred and forty-eight (148) learners were girls. This showed that there were more girls than boys in the grade 9 classes selected for the study. There were six (6) female, and four (4) male educators, selected for the study. Table 15 illustrates the number of learners and the number of educators in each stratum.

Table.15: Shows the number of learners and educators selected from each stratum.

STRATUM	NUMBER OF LEARNERS	NUMBER OF EDUCATORS
A	50	2
B	50	2
C	50	2
D	50	2
E	50	2
TOTAL	250	10

Source: Field study (2019).

The teachers selected were responsible for teaching grade nine mathematics, and also have different qualifications and years of teaching. Table 17 provides the profile of the teachers used for the study.

Table 16: Shows the profiled of Educators.

School	**Teachers**	**Years of Teaching**	**Qualifications**	**Gender**
A	Mrs. Brown	10	Diploma and Degree	Female
	Mr. Azo	8	Diploma	Male
B	Mr. Kiz	12	Diploma, Degree and Honours	Male
	Miss. Love	3	Diploma	Female
C	Mrs. Tsa	12	Diploma and Degree	Female
	Miss. Peace	5	Diploma	Female

D	Mr. Paul	12	Diploma and Degree	Male
	Miss. Jane	6	Degree	Female
E	Miss. Siviwe	5	Diploma	Female
	Mr. John	6	Degree	Male

Source: Field study (2019)

Denscombe, (2007), concurred that the main advantage of stratified sampling method over pure random sampling method, is that the social investigator can have some level of restriction over the selection of the sample, to ensure that some specific elements, or essential people, are catered for, and, in the right proportion as they occur in the broader population. This method made it feasible for the researcher to select the desirable schools for the study, considering the performance of the schools, resources of the schools, and the proximity of the schools to the researcher's residence.

3.6.3.4 Systematic random sampling

In systematic sampling technique, the preliminary sampling point is chosen at random, and then the subsequent elements are selected at a specific interval (Rahi, 2017). In this method each element in the population is allotted a number (Sanni, 2011; Ogundipe, Lucas & Sanni, 2006). For example, in the selection process, the researcher systematically selected the first number that is 5, and the subsequent numbers selected were at regular intervals of 25th, 35th, 45th, 55^{th} positions of elements from the population, until the desired sample size is reached (Rahi, 2017). In this case, the items for inclusion in the sample could be determined by dividing the size of the population by the same sample size (Hammed & Popoola, 2006). This was obtained by dividing the study population (N), by the sample size (n), which gave the item to be selected (K^{th}), $K^{th} = N/n$. Assuming a sample (n) of 500 elements is to be selected from a population (N) of 10000, this can be done by using the formula

$K^{th} = N/n$, the interval for selection will be $K^{th} = 10000/500$ therefore $K^{th} = 20^{th}$. This suggested that there would be a random start between 1 and 20, and thereafter picked every 20^{th} item until we have the 500 elements. In this study the researcher employed this method to select participants into control and experimental groups. Before the commencement of the research, the researcher allotted numbers to each participant. The researcher used the formula $K^{th} = 50/25$, therefore $K^{th} = 2^{nd}$. There was a random start between 1 and 2, and thereafter the researcher picked every 2^{nd} elements i.e., 2^{nd} until the 25^{th} participant. The selected learners were put into experimental groups and the rest into the control group. This was done to avoid any form of bias on the part of the researcher, and it also gave the chance to every learner to be selected. The researcher coded both the experimental group and the control group of each school. Table 17 illustrates the codes assigned to the participants.

Table 17: codes assigned to the experimental and control groups of the various schools.

Schools	Code
A	Control group: 001 – 025
	Experimental group: 026 – 050
B	Control group: 051 – 075
	Experimental group: 076 – 100
C	Control group: 101 – 125
	Experimental group: 126 – 150
D	Control group: 151 – 175
	Experimental group: 176 – 200
E	Control group: 201 – 225
	Experimental group: 226 – 250

Source: Field work (2019).

In all one hundred and twenty-five learners (125) formed the Experimental group, and rest one hundred and twenty-five learners (125) formed the Control group for the study.

3.7 DATA COLLECTION PROCEDURES

The method for gathering data entailed the procedure of gaining access to the schools, creating good rapport with the school authorities and the learners, data collection instruments, the method of data collection, pilot study, administration of the measuring instrument, validity of data, ethical considerations, limitations, and conclusion.

3.7.1 Gaining access

An introductory letter from my supervisor was given to the Chris Hani West Education District director seeking authorisation to conduct an academic research in the district (Appendix A). The Chris Hani West Education District also issued the researcher an introductory letter to the principals of the various public schools where the research was to be carried out (Appendix B). This was done in compliance with the rules and regulations governing public schools with respect to getting access to schools in the Republic of South Africa. Arrangements were put in place by the researcher, the schools and the participants. The time scheduled was also agreed upon by the researcher and the schools. Denzin and Lincoln (2010) asserted that considering the likelihood of the location and the problem under study, two types of research entrance may be obtained;

- “Covert” access without the participant being aware of the researchers’ presence.
- “Overt” access required the awareness of participants of the researcher’s presence, and obtaining permission from the participants, which is often done through ‘gatekeepers’. In this study, gatekeepers were the Department of Education, and the principals of the various schools. In view of this research, the researcher opted for the “Overt” access which is in line with this study, because the participants ought to be aware of their roles and responsibilities, and have the right to participate, or not, as stipulated by (Denzin & Lincoln, 2010).

3.7.2 Creating a rapport

Creating a good rapport between the researcher and the school is an important element in conducting a research. Research shows that the presentation of oneself is important due to the fact that it leaves a great impression on the minds of the participants, and has a great impact on the success or failure of the study. This also creates a relax atmosphere with regards to the participants, since the researcher is a stranger to them. The purpose of the research to the people concerned, and the authorization from the Department of Education, principals, and the participants, was categorically done. The researcher clarified to the participants that participation was voluntary, and that the data collected from them would be treated with the outmost confidentiality. The researcher indicated to the participants that the purpose of the study was to find out the effects of the use of manipulative concrete materials on their performance in fractions. This study provided an insight to educators and stakeholders about the use of manipulative concrete materials in the instruction of fractions in Grade 9.

3.7.3 Data Collection Instruments

Two, self-developed research instruments, were constructed by the researcher, validated by experts in the field and my supervisor, and used to test the hypotheses. These included:

- Students' Questionnaire on Manipulative Concrete Material (SQMCM) and
- Fractions Achievement Test (FAT).

3.7.3.1 Students' Questionnaire on Manipulative Concrete Material (SQMCM)

The Students' Questionnaire on Manipulative Concrete Material (SQMCM) was made up of five (5) sections:

Section A: consisted of respondents' demographic information. This included: gender, age, grade and race.

Section B: consisted of five (5) items structured in line with hypothesis H01, Section C: was made up of five (5) items structured in line with hypothesis H02,

Section D: consisted of five (5) items structured in line with hypothesis H03, and

Section E: was made up of five (5) items structured in line with hypothesis H04.

In all, a total of twenty-four (24) question items were prepared on the Students' Questionnaire on Manipulative Concrete Material (SQMCM) (Appendix E). Babbie (2012) argued that, questionnaires are designed to elicit appropriate information from respondents, and are subjected to scientific analysis.

The researcher applied a 4-point Likert scale to grade learners for this study. These included options such as: 'strongly agree' (SA), 'agree' (A), 'disagree' (D) and 'strongly disagree' (SD). Prayag, (2007) opined that a 4-point Likert scale is better in a research study than a 7-point Likert scale, because it reduces the level of frustration among respondents and it increases the rate and quality of the responses. Saunders, Lewis and Thornhill (2007), argued that the Likert scale is a popular instrument used to elicit information for survey, and also measures attitudes that call for respondents to choose a statement from a collection of statements that ranges from 'strongly agree' (SA) to 'strongly disagree' (SD).

3.7.3.2 Fraction Achievement Test (FAT)

The Fraction Achievement Test (FAT) was made up of multiple-choice objective test of twenty (20) items. Each item had one correct option (key), and three distractors, i.e. options A, B, C, and D. The content area covered different formed of fractions such as:

- Proper fractions.
- Improper fractions.
- Mixed fractions.

Refer to appendix C.

3.7.4 Pilot study

Van Teijlingen and Hundley (2001) asserted that a pilot study is a trial form of a full-scale study (also called a 'feasibility' study), as well as the piloting of a research instrument for the necessary adjustment to be made. The trial study is an important element in a good study design. However, a trial experiment does not assure success in the main research study, but gives an insight into what will happen. Researchers are of the view that pilot testing is planned to access whether the interventions will work, or not. In a similar vein, pilot study helps to determine the effectiveness of the intervention, and ascertains which elements of the prototype may be reviewed (De Vos, 2005: 402). The Students' Questionnaire on Manipulative Concrete Material (SQMCM) and Fraction Achievement Test, were piloted on fifteen Grade 9 learners. In the process, it was observed that there was an ambiguity with one of the questions which the learners were not clear with. The question was not specific, so the researcher had to restructure the question for a clearer understanding.

3.7.5 Research Procedure

Field Work Activities

S/N	WEEK	RESEARCH ACTIVITIES
1	Prior to the study	- Selection of schools
2	1st week	- Selection and training of Participants and Research Assistants - Categorization of participants into experimental and control groups - Arrangement of classrooms - Administration of pre-test to experimental and control groups. - Provision of treatment to experimental and control groups
3	2nd week	- Provision of treatment to experimental and control

groups

- Assessment

4 3rd week - Provision of treatment to experimental and control groups

- Administration of Post-test to the experimental and control groups.

3.7.5.1 Instructional Approach for Experimental Group

The participatory instructional approach was adopted from the British Council and modified by the researcher. The main steps involved were:

- Organizing learners into small groups by the facilitator
- Group activities by members of the individual small groups
- Group presentation to the whole class by group representatives, or the entire members of a group
- Whole class input/discussion by all the participants in the class to freely contribute, critique and clarifie issues on the presentations by the group where necessary
- Summary by facilitator and participants
- Evaluation by facilitator and participants.

3.7.5.2 Instructional Approach for Control Group

The control groups were taught with a well-constructed lecture series on the same selected contents with those in the experimental groups. Lessons were delivered using the modified conventional lecture method as indicated below:

- Educator introduced the concepts
- Educator discussed facts or ideas on the concepts
- Educator gave notes on the concepts
- Educator asked questions
- Educator gave assignment to students.

At the end of the third week, Post-test was administered to both the experimental and control groups (Appendix D). This was done according to the codes assigned to them in the pre-test. This was to ascertain the performance of each learner in the Pre-test and the Post-test. The test lasted for forty-five minutes, after which they were collected and marked. To ascertain the effectiveness of the use of each of the manipulative concrete material for the study, the Experimental group and Control group were made to take a post-test for each of the manipulative concrete material for forty-five minutes, according to their codes in the Pre-test in each school.

This was done to enable the researcher to gain an insight into the effect of the use of each of the manipulative concrete material (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative) on learners performance in fractions. Questionnaires on manipulative concrete materials were also administered to the experimental group, only to ascertain their perception of the manipulative concrete materials used for the study in solving mathematical problems involving fractions (Appendix E). Background information was also collected from the control groups. The data collected were coded, sorted and categorized to find Percentages, Mean, and Standard Deviation using an SPSS. The t-test was used to test the significant difference of the hypotheses. The hypotheses were tested at 0.05 level of significance.

3.7.6 Validity of data

Validity of data is determined by the degree of authenticity of data collected, and it is an uninterrupted process of collecting evidence and devised argument to reinforce score of interpretations (Phye, Robinson & Levin, 2005:128). In a similar vein, Sanni (2011) avowed that validity is the degree to which a test assess what it is presumed to measure. The degree of validity is ascertained by the researcher using certain procedural measures to determine the accuracy of his research findings (Adefioye, 2015). There are different forms of establishing the validity of a research instrument. These includes:

(a) Content validity

(b) Predictive or Concurrent validity

(c) Construct validity

(d) Face validity (Rubin & Bellamy, 2012, Creswell, 2014).

For the validity of the research instruments to be authentic in this study, the researcher gave a draft copy of the instrument to his supervisor and experts in the field of research studies for their perusal and necessary corrections. The content validity was ensured by pilot-testing the research instruments on 15 learners. After validation, the instrument were subjected to pilot-testing to establish its reliability. The researcher adopted all four (4) forms of validity to ensure that the research instruments used to collect data from the respondents were authentic. These included: content validity, predictive or concurrent validity, construct validity and face validity.

3.7.6.1 Content validity

Mosby (2008) argued that content validity consists of how the research instrument, or tool, epitomises the universe, or domain, of subject matter for the concept being measured. Content validity is one of the commonly used validity by researchers to validate teacher-made test. Content validity is employed when the experiment makes available adequate analysis of the subject being studied. The researcher adopted content validity in this study to ensure that the research instrument adequately covered the domain of the subject matter, namely, the use of concrete materials to solve different types of fractions (Proper fractions, improper fractions and mixed fractions). The researcher ensured that the instrument equitably and systematically covered items it deemed to cover. A quantitative approach enables researchers to refer content validity questionnaires to experts working at different environments, whereby distance is not a hindrance (Taherdoost, 2016). The sampling is an added degree of quality assurance. The researcher, in this study, maintained uniformity all through the process of data gathering, because it was an important feature of validity. Changingminds (2016) asserted that content validity is linked closely to good experimental design.

3.7.6.2 Concurrent and Predictive validity

This type of validity are two fundamental type of criterion-related validity. Concurrent validity relates to relationships between two tests (the test and the criterion), which are administered concurrently, and the latter relates to the level at which a language test can foretell the future (academic) performance of participants (Pearson, 2012). In a similar vein Taherdoost, (2016) argued that predictive validity is the ability of one valuation instrument to foretell future performance, either in some activity, or, on another evaluation of the same concept. Predictive validity assesse the operationalization's ability to foretell something it should hypothetically be able to predict. Isangedighi, Joshua and Ekuri, (2004) observed that predictive validity is used to find out how effective the test will work when used to predict success, or failure. In this study, the researcher used the predictive validity to predict the outcome of the use of manipulative concrete materials on learners' performance in fractions. The researcher also used the predictive validity to establish how workable the instruments would be used to determine the learners' performance in fractions. Predictive validity is useful for the ability test. It is obtained by calculating the correlation coefficient between distributions of scores at the pre-test stage against the distribution of scores at the post-test stage of the studies (De Vos, Strydom, Fouche & Delport, 2011).

3.7.6.3 Construct validity

Construct validity is the degree at which a test measured an individual in terms of human traits such as intelligence, self-control, anxiety, honesty, creativity, innovation, motivation, conformity, anger etc. Babbie (2007), contended that construct validity is complex and difficult to predict. Construct validity has two components: convergent and discriminant validity (Taherdoost, 2016). Construct validity ensured that when the hypothetical theories perfectly influenced the real-world circumstances, they were anticipated to model. A good research turned the concept (constructs) into real things that could be quantified. Construct validity was adopted for this study to assess the quality of the research tool for the experiment. It was used to measure the construct it was supposed to measure. Changingminds (2016) is of the view that when construct validity is not used in the study, there is the likelihood that the conclusion drawn from the study will be incorrect.

3.7.6.4 Face validity

De Vos, Strydom, Fouche and Delport (2011), assert that face validity deals with the superficial outlook of a face value of a measurement technique. The aim of the face validity is to show that the test meets the expectation of the researcher. Face validity is typically constructed by a group of experts in the field, similar to content validity (Kraska-Miller, 2014; Jackson, 2016). The experts appraised each of the measuring items to ascertain whether they matched with the given theoretical domain of the concept. In this study, the researcher ensured that the research instrument measured what it was expected to measure. To ensure the face validity of the study, the researcher gave a draft copy of the instrument to his supervisor and experts in the field of research studies for their perusal, and for any necessary corrections. A pilot study was also conducted by the researcher to ensure that the instruments met the face value. Sanni (2011), suggested that the researchers examined the items to confirm that the test was a valid concept, intended to measure just on the face of it.

3.7.7 Reliability of the Research Instrument

Babbie (2012), concurred that a research instrument is reliable when it can consistently produce the numerical results each time it is applied; not subject to variations except when there are changes in the variables being measured. In a similar vein Sanni, (2011), argued that reliability is the level of uniformity between two sets of data, or outcomes gained, with the same instrument, or a similar set of instruments. Adefioye, (2015) is of the view that reliability is the regularity, constancy and repeated results that emanated from the researcher over a period of time, in similar situations, but under different circumstances. Reliability occurs when a research instrument assessed the same thing repeatedly, and the same result is produced (Wellington, 2015; De Vos, Strydom, Fouche & Delport, 2011; Sanni, 2011).

In this study the researcher subjected the research instruments to all forms of reliability tests to ensure that they were devoid of errors on the instruments. Reliability is therefore the absence of error of measurement in a measuring instrument (Isangedighi, Joshua, Asim & Ekuri, 2004). De Vos et. al., (2011) outlined the following measures to increase the reliability of a research instrument:

- ❖ Remove obscurities items
- ❖ Eliminate external influence
- ❖ The number of items must be increased
- ❖ Pilot test your research instrument.
- ❖ use standard conditions for the test
- ❖ Standard instructions must be used
- ❖ Increase the level of instruments or measurement
- ❖ Follow a consistent scoring technique.

There are four basic techniques to improve the reliability of a research instrument. These include:

- ❖ Internal consistency
- ❖ Equivalence form reliability
- ❖ Test-retest reliability and
- ❖ Split-half reliability

3.7.7.1 Internal consistency

Internal consistency is important to ensure the reliability of the research instrument. In this study, the researcher ensured that there was internal consistence between the items and the research instrument. The Cronbach Alpha Coefficient and test-retest was applied to assess the internal consistency and reliability of the Manipulative Concrete Material. This statistic is a complete item correlation where the values ranged between 0 and 1. Values above 0.7 are often considered to be acceptable. The Cronbach's alpha coefficient for this study was 0.75.

The Cronbach co-efficient alpha is a generalized form of Kuder-Richardson (KR 20), except that $\sum si^2$ replaces the value $\sum pq$. The basic assumption is that items requiring responses such as "strongly agree", "agree", "disagree", and "strongly disagree", do not correspond with the usual right or wrong format, resulting in making coding difficult. For example, all positively worded items such as "strongly agree", response may attract 4 points, 3 points for "agree", while 2 points for "disagree", and 1 point for "strongly disagree", respectively. The scoring order is reversed for all negatively worded items (Cohen, Manion & Morrison, 2007). This was what the researcher applied in this study to the Students Questionnaire on Manipulative Concrete Materials

(SQMCM). On the other hand, the Kuder-Richardson formula (KR 20) is used to measure the validity and the reliability of the Fraction Achievement Test of the learners. This method made use of psychometric data obtained from one test administration. It is estimated that items in the instrument are homogeneous, and so possessed inter-item consistency (Isangedighi, Joshua, Asim & Ekuri, 2004). In applying K-R20, the items should be scored dichotomously (right or wrong), followed by the preparation of person-by-item matrix. This matrix indicates how each member of the sample answers each item in the test either correctly or incorrectly.

3.7.7.2 Equivalence form reliability

Equivalence form of reliability is also referred to as parallel form of reliability. Cohen, Manion, and Morrison (2007), approved that equivalence form of reliability involves two equivalent research instruments which are administered to the same group of participants at the same time, but in chronological order. In this study, Pre-test and Post-test were used to measure the cognitive level of learners on the effective use of manipulative concrete materials on learners' performance in fraction between the experimental group and the control group. The reliability is therefore used to demonstrate the equivalent forms of a test if applied simultaneously to match samples. The reliability was measured using a t-test to establish a high correlation coefficient between the experimental group and the control group, and it also established the mean and standard deviations of the study.

3.7.7.3 Test-retest reliability

Cohen, Manion, and Morrison (2007) argued that for the reliability of an instrument to be authentic, the instrument must be administered more than once to the same group of participants at different time frames. The time frame can be a day interval, or within a two weeks interval, depending on the researcher. In this study, the researcher administered the test and retest to the same participants within three weeks. It was assumed that the time interval between the test and retest prevented participants from reproducing the same response. The degree of score from both pre-test and post-test measured reliability of the research instrument.

3.7.7.4 Split-half reliability

The split-half reliability technique involves questionnaires that are split into two halves, and then compared results to see if they are the same on the basis that one subset of the questionnaire contained even number items, and the other subset contained odd number items (Sanni, 2011; Cohen, Manion & Morrison, 2007). In this study the Students' Questionnaire on Manipulative Concrete Material (SQMCM), which consisted of twenty-four (24) question items, was divided into five (5) sections. Each of the sections contained a set of question items which were scored differently.

3.8 ETHICAL CONSIDERATIONS

Ethical issues are one of the critical aspects of conducting research work. Ethical consideration is a moral sensitivity to the right of others in a research study (Fouka & Mantzorou, 2011). Balnaves and Caputi (2001) argued that the aim of ethical consideration is to make researchers responsive to the issues that may possibly arise in their work, and to caution them to conduct themselves ethically. In conducting a research, the rights of the participants are paramount in protecting their integrity and in establishing the outmost confidentiality of participants' information's. In a similar vein, De Vos et.al. (2011) supported the idea that research must be established on a common understanding, respect, trust, cooperation, acceptance, and expectations between the researcher and the participants. In addition, McMillan and Schumacher (2014) asserted that the fundamental human rights and welfare of learners in the study must be protected. Robertson and Dearling (2004:33) are of the view that ethics is about the moral principles embraced by the researcher, and those funding the research work. Each party has the moral obligation to protect participants from any form of harm that is likely to occur from participating in the study.

In this study, the researcher strictly adhered to the following ethical measures:

- Permission
- Voluntary Participation
- Informed consent
- Anonymity

- Confidentiality
- Avoiding harm to participants
- Professionalism and
- Plagiarism

3.8.1 Permission

The researcher applied for ethical clearance from the University of Fort Hare Research Ethics Committee, and obtained permission also from his supervisor, and the Education Department of the University of Fort Hare. The introductory letter from his supervisor was presented to the District Director of Chris Hani West Department of Education to permit him to conduct research in some selected High schools in the district. An introductory letter from the District Director of Chris Hani West Education District was then presented to the principals of the selected High schools where the research was going to be conducted. This was in the accordance with the policy of the Department of Education of the Republic of South Africa. Having gone through all the formalities, a date was scheduled with the schools to begin the field work.

3.8.2 Voluntary Participation

De Vos et al., (2011:117) agreed that researchers are obliged to communicate complete accuracy of information, so that the participants are fully aware of the detail of the study, which will give them an informed decision to take part in the study, or not. In this study, the researcher explained extensively to the participants what the study was about, and all the likely dangers that may be involved in taking part. The researcher explained that participation in the study was voluntary. The participants can choose not to participate. No participant was forced or coerced to participate. Anyone who felt the need to opt out, was free to do so. Babbie (2007) asserted that no participant should be forced to participate in a study, and it should be voluntary. The researcher explained into detail, the rights and responsibilities of the participants in the study.

3.8.3 Informed consent

Informed consent deals with seeking the formal permission from the participants to participate in the research study. Consent was sought from the school, the participants and the parents of the participants. The researcher sent letters to the schools and consent letters were given to the learners and their parents for approval before the commencement of the study. White (2002) argued that the crucial element of informed consent is not necessarily the exhaustiveness of the information provided to participants, but rather its relevance to the participants' decision is important. Grinnell and Unrau (2008) outlined the following as the process for obtaining an informed consent:

- The possible merits and demerits and the ethical hazards the participants are likely to encounter
- The procedures that will be followed during the study
- All the necessary information about the study needs to be known to the participants
- The credibility of the researcher needs to be scrutinized by the potential subjects or their legal representatives
- The duration in which participants will be involved in the study must be known.

In this study, the researcher took into consideration all the above measures before he carried out his research.

3.8.4 Anonymity

Anonymity is an act of keeping participants identities and information from the public domain. Every participant has the right to privacy, anonymity and confidentiality. It is the prerogative of the researcher to ensure that all these are adhered to in carrying out the research study (Burns & Grove, 2005). The information given by the participants should remain confidential. In this study, the researcher did not use the real names of participants, either during or after the data collection. Codes were used to indicate the names of the schools that participated to keep them anonymous. The researcher did not provide any space on the questionnaire for participants to write their names, and no names were written on the questionnaires. De Vos et. al., (2011) asserted that everybody has the right to privacy, and it is the right of the individual to indicate where, to whom, when, and to what extent his or her beliefs, attitudes and behaviours, should be put in the public domain.

3.8.5 Confidentiality

Confidentiality is an act of keeping respondents' information confidential, and not revealing it to any other person. In other words, Polit and Beck, (2006) referred to confidentiality as the protection of participants' identities and information from the public domain. In support, Macmillan and Schumacher (2014) avowed that confidentiality can be guaranteed by ensuring that data gathered from an individual cannot be linked to individual participants by name. McMillan and Schumacher (2014) outlined the following measures to maintain the confidentiality of an individual in data collection:

- Participants should use self-styled names during the data collection.
- Data collected should be handled with the utmost confidentiality and be anonymous.
- The researcher should use a third party to link names to data and should receive results without names.
- The researcher should report group instead of individual results
- The researcher should use an interim system of names that are linked to data, and delete those names later on (McMillan & Schumacher, 2014:134).

In this study, the researcher kept respondents' information as confidential as possible, and the information gathered was only used for the intended purpose. Names were not attached to the questionnaires. Only the researcher and his supervisor had access to the returned questionnaires and the research data. The questionnaires were destroyed after the research work came to an end. Giordano , O'Reilly, Taylor, and Dogra, (2007) conceded that participants must be well educated about the likely risks of non-confidentiality, such as information in the final report that participants may not be aware of, and information that invaded the privacy of others should remained hidden from the public domain, and so forth.

3.8.6 Avoiding harm to participants

Avoiding harm, refers to the circumvention of physical, psychological, mental and emotional trauma to the participants. Babbie (2007) is of the view that, in a research study, it is an ethical principle that no participant should suffer any form of harm, such as physical, psychological, emotional or mental torture. In a similar vein, White (2002) observed that participants in a study must be protected from physical, emotional, psychological, mental torture, harm and danger, and that if there is any possibility for any of the foregoing to occur, the researcher must inform participants of any potential risk. McMillan and Schumacher (2014), and White (2002), asserted that in educational research, the possible harm may be emotional rather than physical. In this study, the researcher ensured that the questionnaires were not degrading questions to the family background, educational background, or religious background of the participants. The paper on which the questionnaires were written was the usual A4 sheet used for educational work at school. The researcher ensured that the papers had not been in contact with anything that could cause harm to the participants. Participants were asked to use their own pens, or pencils, in answering the questionnaires. The responses from the participants were kept in an envelope and sealed to prevent them from a third party. McMillan and Schumacher (2014) are of the view that protection from harm implies not exposing information that may result in humiliation or danger to a family, academic performance, religion, health, and/or relationships. The researcher observed that the rights of every participant was respected, according to the principles governing ethics in research work.

3.8.7 Professionalism

Professionalism, in this context, refers to the procedural and the interpersonal relationship between the researcher and all those involved in the research process. Professionalism involves the process of obtaining the ethical clearance from every sector that matters before going into the field. The respect accorded the participants is of the nature of handling respondents' information to ensure that confidentiality, anonymity and no harm is caused to anybody. In this study, the researcher observed that all the ethical principles of research were applied to the latter. Nobody was forced to participate, and all the information gathered from the participants were kept from the public domain.

3.8.8 Plagiarism

Plagiarism is an act of copying extensive material from others without acknowledging the source of the information. Researchers are expected to acknowledge the work of others, and apply quotations to point out the exact words used. The objective of this is for the researcher to not present the work of other scholars as being their own (American Psychological Association, (APA, 2010). Acknowledgement must be given to the original source, even when the material is rephrased (Creswell, 2014). Plagiarism is an academic injustice, and these dishonest habits are not accepted in professional research communities, and they lead to academic misconduct (Neuman, 2009). Research shows that plagiarism can be avoided by giving credit to the contributions of other scholars and people, including organizations that are used in the study. In this study, the researcher duly acknowledged all the sources from which information was gathered. This was done in the text, and in the references column.

Table 18 shows the summarised form of ethical issues, and how to address the issues in research study.

Table: 18: A table showing a summary of Ethical issues in a research.

Where in the Process of Research the Ethical Issue Occurs	Type of Ethical Issue	How to Address the Issue
Prior to conducting the study	• Examine professional association standards. • Seek college/university approval on campus through an institutional review board (IRB). • Gain local permission from site and participants. • Select a site without a vested interest in outcome of study. • Negotiate authorship for publication.	• Consult the code of ethics for professional association in your area. • Submit proposal for IRB approval. • Identify and go through local approvals; find gatekeepers or key personnel to help. • Select sites that will not raise power issues with researchers. • Give credit for work done on the project; decide on author order in future publication.
Beginning the study	• Identify a research problem that will benefit participants. • Disclose purpose of the study. • Do not pressure participants into signing consent forms. • Respect norms and charters of indigenous societies. • Be sensitive to needs of vulnerable populations (e.g., children).	• Conduct a needs assessment or informal conversation with participants about their needs. • Contact participants, and inform them of the general purpose of the study. • Tell participants that they do not have to sign form. • Find out about cultural, religious, gender, and other differences that need to be respected. • Obtain appropriate consent (e.g., parents, as well as children).
Collecting data	• Respect the site, and disrupt as little as possible. • Make certain that all participants receive the same treatment. • Avoid deceiving participants. • Respect potential power imbalances and exploitation of participants (e.g., interviewing, observing). • Do not "use" participants by gathering data and leaving site. • Avoid collecting harmful information.	• Build trust, and convey extent of anticipated disruption in gaining access. • Put into place wait list provisions for treatment for controls. • Discuss purpose of the study and how data will be used. • Avoid leading questions. Withhold sharing personal impressions. Avoid disclosing sensitive information. Involve participants as collaborators. • Provide rewards for participating. • Stay to questions stated in an interview protocol.
Analyzing data	• Avoid siding with participants (going native). • Avoid disclosing only positive results. • Respect the privacy and anonymity of participants.	• Report multiple perspectives. • Report contrary findings. • Assign fictitious names or aliases; develop composite profiles of participants.
Reporting, sharing, and storing data	• Avoid falsifying authorship, evidence, data, findings, and conclusions. • Do not plagiarize. • Avoid disclosing information that would harm participants. • Communicate in clear, straightforward, appropriate language, • Share data with others. • Keep raw data and other materials (e.g., details of procedures, instruments). • Do not duplicate or piecemeal publications. • Provide complete proof of compliance with ethical issues and lack of conflict of interest, if requested. • State who owns the data from a study.	• Report honestly. • See APA (2010) guidelines for permissions needed to reprint or adapt work of others. • Use composite stories so that individuals cannot be identified. • Use unbiased language appropriate for audiences of the research. • Provide copies of report to participants and stakeholders. Share results with other researchers. Consider website distribution. Consider publishing in different languages. • Store data and materials for 5 years (APA, 2010). • Refrain from using the same material for more than one publication. • Disclose funders for research. Disclose who will profit from the research. • Give credit for ownership to researcher, participants, and advisers.

SOURCES: Adapted from APA (2010); Creswell (2013); Lincoln (2009); Mertens and Ginsberg (2009); and Salmons (2010)

3.9 CHAPTER SUMMARY

The purpose of this chapter was to present the quantitative research methodology that was used in this research work. The chapter illustrated a systematic analysis of the procedure followed in conducting the study. It explored the following key areas that stipulated a clear image of the whole work. These included; research paradigm, research approach, research design, area of study, population, sample and sampling technique, data gathering tools, pilot study, validity and reliability of data instruments that were structured questionnaires on manipulative concrete materials and fraction achievement test, and ended with ethical considerations. Chapter three stipulated the guideline that ensured that this research achieved its purpose and objectives. A reflection on the outcome of the experiment, and the scientific analysis of the results forms the content of the next chapter, and serves as a true reflection of the outcome of the study.

CHAPTER FOUR

DATA ANALYSIS

4.1 INTRODUCTION

This chapter, presents the analysed data gathered, using questionnaires and fraction achievement tests from the field. Data were collected through Pre-test, Post-test and structured questionnaire in line with the hypotheses formulated, and subsequently tested. Data collected were coded, sorted and categorized to find Percentages, Mean, and Standard Deviation between the experimental and control groups. The analyses were done using quantitative analyses, whilst descriptive statistics of mean and standard deviations were used in explaining and comparing the pre-test and post-test scores of the experimental and control groups. The t-test was used to test the hypotheses raised in chapter one.

4.2 TESTING OF NULL HYPOTHESES

Hypothesis1 (H_01):

There is no significant relationship between Cuisenaire rods and grade nine learners' performance in fractions

Table 19: Analysed results of Cuisenaire rods.

Pair	Mean	Std. Deviation	Std. Error Mean
Cuisenaire rods manipulative Post-test	12.428	4.732	.299
Pre-test	8.372	1.770	.112

Source: Field work (February, 2019).

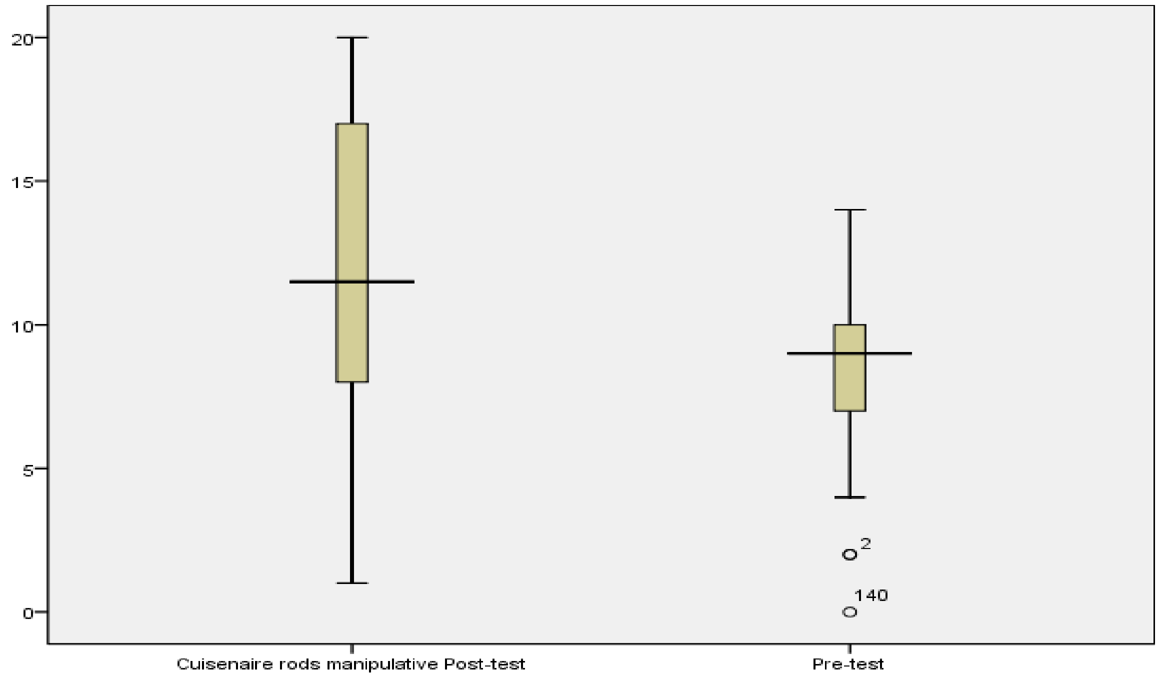
Pair	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Cuisenaire rods manipulative Post-test - Pre-test	4.056	4.924	.311	3.443	4.669	13.024	249	0.000

Source: Field work (February, 2019).

Effect size

Mean	Std. Deviation	Effect size
4.056	4.924	0.82

Figure.30: Showed a Box and Whisker plot of post-test and pre-test of Cuisenaire rods



Source: Field work (February, 2019).

To prove that there is a significant relationship between Cuisenaire rods and grade nine learners' performance in fractions, the pre-test and post-test mean and standard deviations scores of Cuisenaire rods were compared using a sample paired t-test. Table 19 shows the Pre-test and Post-test mean and standard deviation, Pre-test scores (mean $\bar{x}$ =8.372, SD=1.770), and Post-test scores (mean $\bar{x}$ =12,428, SD=4.732), respectively. The scores indicated that there were improvement in the mean scores and standard deviation of the Post-test. The t-test (t=13,024, $p < 0.05$) indicated that there was a significant relationship between Cuisenaire rods and grade nine learners' performance in fractions, therefore the null hypothesis (H_{01}) was rejected. The effect size was (0.82) or 82 per cent, which meant that there was a significant difference in the mean scores of the post-test, and the mean scores of pre-test of learners' performance in fractions.

Hypothesis 2 (H_0 2)

There is no significant relationship between Fraction bars/Fraction tiles and grade nine learners' performance in fractions

Table 20: Analysed results of Fraction bars/Fraction tiles

Pair	Mean	Std. Deviation	Std. Error Mean
Fraction tiles manipulative Post-test	11.4240	3.66633	.23188
Pre-test	8.3720	1.77035	.11197

Source: Field work (February, 2019).

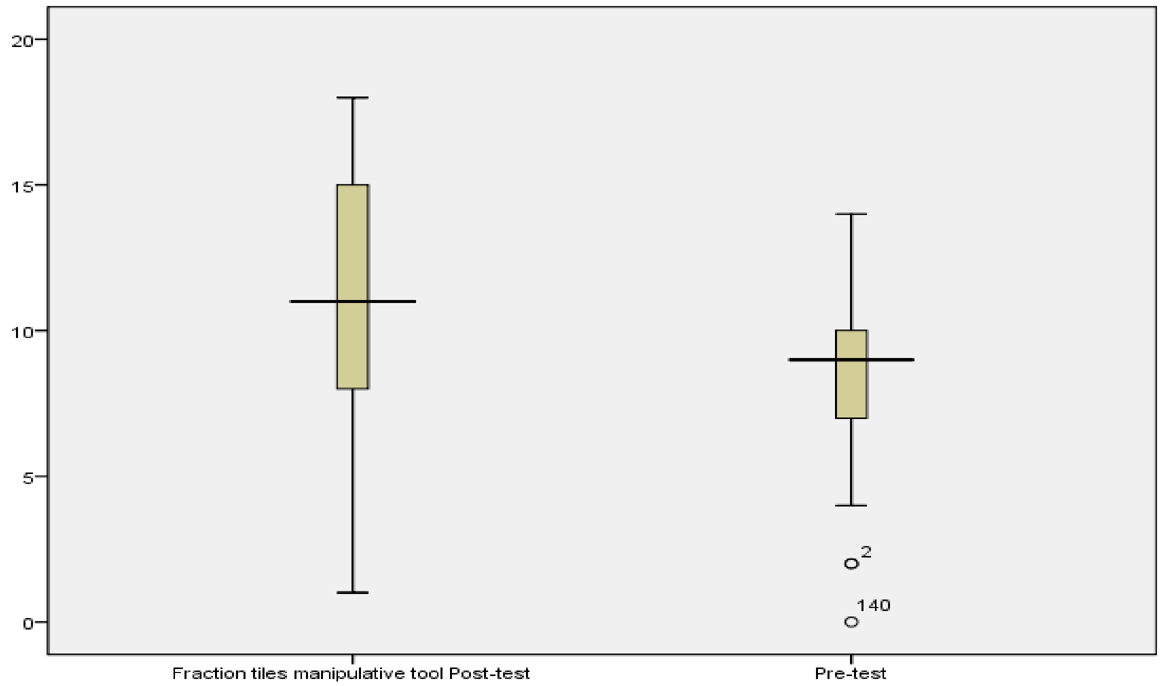
Pair	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Fraction tiles manipulative Post-test - Pre-test	3.052	3.988	.252	2.555	3.549	12.100	249	.000

Source: Field work (February, 2019).

Effect size

Mean	Std. Deviation	Effect size
3.052	3.99	0.77

Figure. 31: Showed a Box and Whisker plot showed the pre-test and post-test of Fraction tiles/fraction bars



Source: Field work (February, 2019).

To prove that there is a significant relationship between fraction tiles/fraction bars and learners' performance in fractions, the pre-test and post-test mean and standard deviations scores were compared, using paired sample t-test. Table 20, shows results of fraction tiles/fraction bars of the pre-test (mean $\bar{x}$ =8.37, SD=1.77), and post-test (mean $\bar{x}$=11.42, SD=3.67), respectively. The scores of the mean and standard deviation show that there was an improvement in the mean score and standard deviation of the Post-test. This indicated that the experimental group gained higher scores in the post-test. The result of the t-test (t=12,10 $p < 0.05$), suggested that there was significant relationship between fraction tiles/fraction bars and grade nine learners performance in fractions. Thus, the null hypothesis (H_{02}) was rejected. The effect size (0.77), or 77 per cent, meant that there was significant difference between the mean score of the post-test and pre-test of grade nine learners' performance in fractions.

Hypothesis 3 (H_03):

There is no significant relationship between Paper folding and grade nine learners' performance in fractions

Table. 21: Analysed results of Paper folding

Pair	Mean	Std. Deviation	Std. Error Mean
Paper folding manipulative Post-test	11.792	4.256	.269
Pre-test	8.372	1.770	.112

Source: Field work (February, 2019).

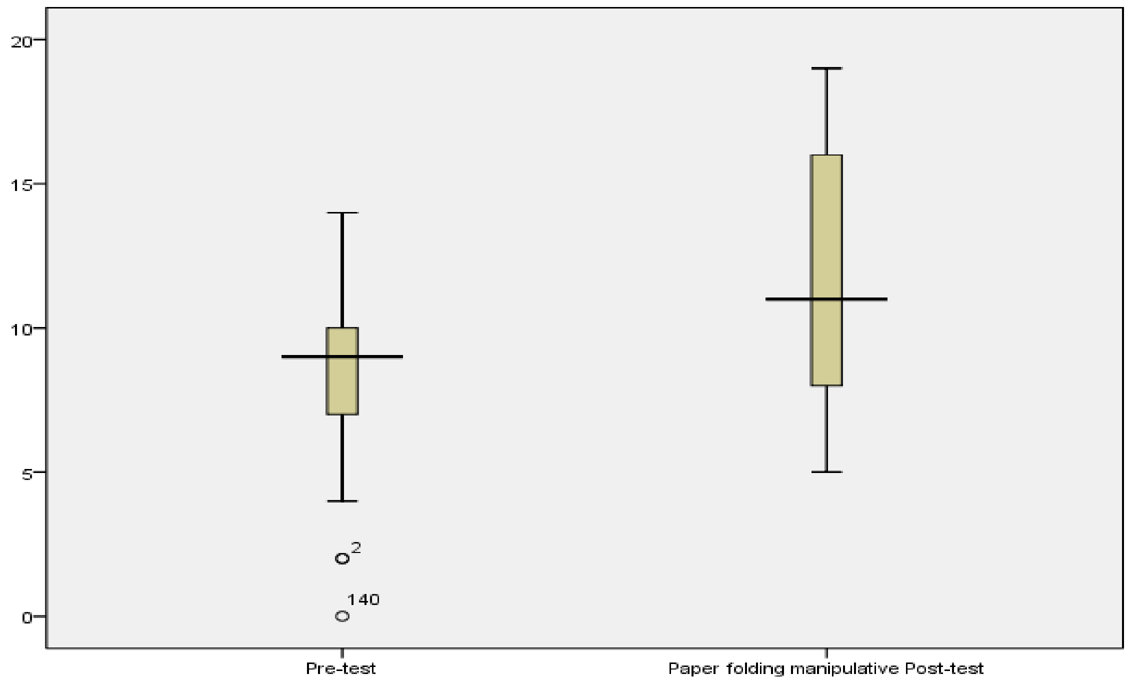
Pair	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Paper folding Post-test - Pre-test	3.420	4.426	.280	2.868	3.971	12.219	249	.000

Source: Field work (February, 2019).

Effect size

Mean	Std. Deviation	Effect size
3.420	4.426	0.77

Figure. 32: Showed a Box-and-Whisker plot of pre-test and post-test of Paper folding.



Source: Field work (February, 2019).

To prove that there is a significant relationship between Paper folding and learners' performance in fractions, the pre-test and post-test mean and standard deviations scores were compared using sample paired t-test. Table 21, shows the scores of mean and standard deviation of Paper folding in the pre-test (mean $\bar{x}$=8.372, SD=1.770) and post-test (mean $\bar{x}$=11.792, SD=4.256), respectively. This indicated that the experimental group gained higher scores in the post-test. The result of the t-test (t=12,219; $p < 0.05$) indicated that there was a significant relationship between Paper folding and grade nine learners' performance in fractions. Thus, the null hypothesis (H_{03}) was rejected. The effect size (0.77), or 77 per cent, meant that there was a significant difference between the mean score of the post-test and pre-test of grade nine learners' performance in fractions.

Hypothesis 4 (H_{04})

There is no significant relationship between computer assisted manipulative and grade nine learners' performance in fraction.

Table 22: Analysed results of Computer assisted manipulative

Pair	Mean	Std. Deviation	Std. Error Mean
Computer assisted manipulative Post-test	12.212	4.569	.289
Pre-test	8.372	1.770	.112

Source: Field study (February, 2019).

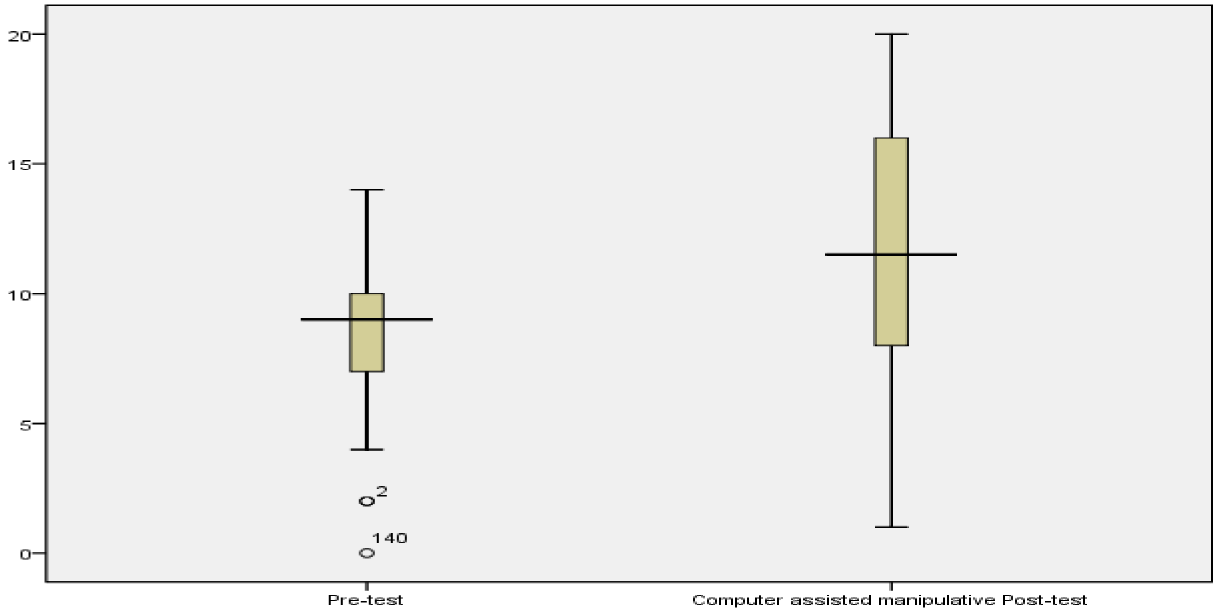
Pair	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Computer assisted manipulative Post-test - Pre-test	3.84000	4.74304	.300	3.249	4.431	12.801	249	.000

Source: Field work (February, 2019).

Effect size

Mean	Std. Deviation	Effect size
3.84	4.743	0.81

Figure 33: Showed a Box and Whisker plot of Pre-test and Post-test of Computer manipulative.



Source: Field work (February, 2019).

To prove that there is a significant relationship between Computer assisted manipulative and grade nine learners' performance in fractions, the pre-test and post-test mean and standard deviation were compared using sample paired t-test. Table 22, shows the scores of mean and standard deviation of the pre-test (mean $\bar{x}$=8.372, SD=1.770), and post-test (mean $\bar{x}$=12.212, SD=4.569), respectively. The results indicated that there were increases in the mean score and standard deviation in the post-test. The sample t-test score (t=12,801; $p < 0.05$) suggested that there are significant relationships between computer assisted manipulative and grade nine learners performance in fractions. Thus, the null hypothesis (H_{04}) was rejected. The effect size (0.81) or 81per cent, meant that there is a significant difference between the mean score of the post-test and pre-test of grade nine learners' performance in fractions.

The sample T-test was used to determine the level of significance between the manipulative tools and grade nine learners' general performance in fraction. Also, Cohen's *d* was used to measure the significance effect in paired sample t-test. Cohen (1988) defined effect size as the difference between two means divided by a standard deviation for the data ($d = \frac{Mean\ Difference}{Standard\ Deviation}$). For the difference between two-group experiments, the suggested measure of effect size (the statistic d) is as follows:

Effect size (d)	**Size of Effect**
0.2 to 0.5	Small
0.5 to 0.8	Medium
>0.8	Large

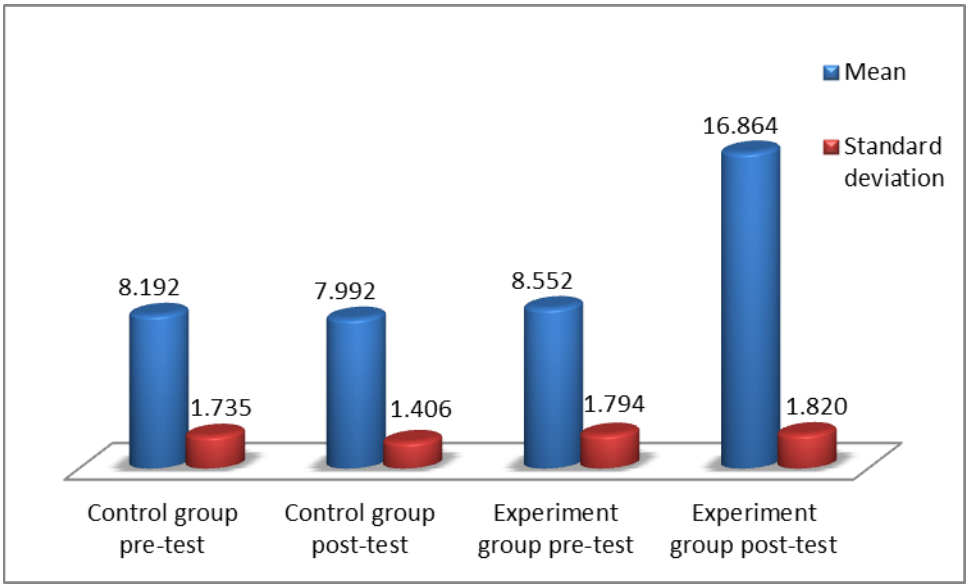
4.3 ANALYSIS OF PRE-TEST AND POST-TEST

4.3.1 Findings of Cuisenaire Rods Manipulative Tool Data Set

Table.23: Shows Findings of Cuisenaire Rods Manipulative Tool Data Set

Group	N	Trial	Mean	Standard deviation	Min	Max
	125	Pre-test	8.192	1.735	0	14
Controls	125	Post-test	7.992	1.406	1	11
	125	Pre-test	8.552	1.794	2	11
Experiment	125	Post-test	16.864	1.820	12	20

Figure 34: Shows a bar chart of findings of Cuisenaire Rods Manipulative Tool



Source: Field work (February, 2019).

Table 23, illustrated the descriptive statistics for Cuisenaire Rods Manipulative Tool. The table showed the mean scores, and standard deviations of the experimental group, and the control group in the Pre-test (mean $\bar{x}$ =8.552, SD=1.794), and (mean $\bar{x}$ =8.192, SD=1.735), respectively. The pre-test scores showed that there was no significant difference in the mean scores and standard deviation between the experimental group and the control group in the Pre-test. This could suggest that the initial competencies of the two groups in fractions were equivalent, prior to the study. The mean scores and standard deviation of the experimental group and control group in the Post-test were as follows, (mean $\bar{x}$ =16.864, SD= 1.820), and (mean $\bar{x}$ = 7.99, SD=1.406), respectively. This vast disparities in the mean scores and slightly difference in the standard deviation in the Post-test between the Experimental Group and Control Group could be attributed to the effects of Cuisenaire Rods on the Experimental Group. However, the drop in the mean scores and Standard deviation of the Control Group in the Post-test (mean $\bar{x}$ = 7.992, SD=1.406), suggested that the learners could not comprehend well with the instructions of fractions without the use of concrete materials. They might have forgotten the process of working fractions since they were taught via the lecture method, and they might have lost concentration during the instructional process. Kurumeh (2010) concurred that the use of Cuisenaire rods made mathematics real to learners, since it was learner friendly, activity oriented, and aroused learners' comprehension of the fractions, and also accelerated a higher understanding of mathematical concepts, facts and principles. Cuisenaire rods enabled learners to work independently and in groups on meaningful mathematics contents (Kurumeh, 2010). This suggested that Cuisenaire rods manipulative tool had a positive influence in improving grade 9 learners' performance in fractions. Elia, Gagatsis, and Demetrico (2007), suggested that Cuisenaire rods are hands-on, and minds-on physical materials used for mathematical instruction of abstract concepts, and made the concepts of fractions real to learners.

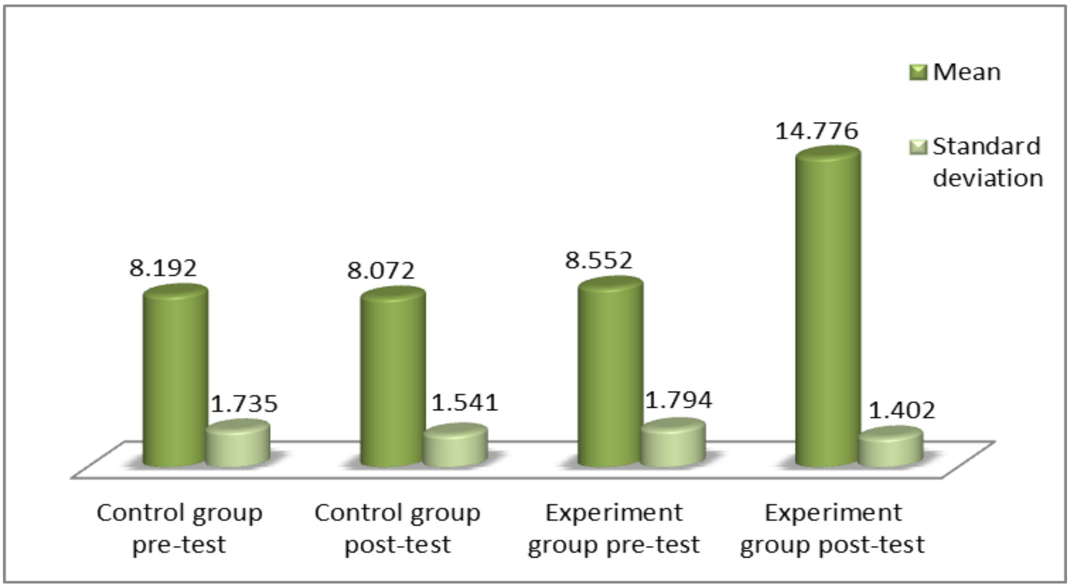
4.3.2 Findings of Fraction Bar/Fraction Tiles Manipulative Tool Data Set

Table.24: Findings of fraction Tiles/fraction bars Manipulative Tool Data Set

Group	N	Trial	Mean	Standard deviation	Min	Max
	125	Pre-test	8.192	1.735	0	14
Control	125	Post-test	8.072	1.541	1	11
	125	Pre-test	8.552	1.794	2	11
Experiment	125	Post-test	14.776	1.402	8	18

Source: Field work (February, 2019).

Figure 35: Showed a bar chart of fraction Tiles/fraction bars Manipulative Tool



Source: Field work (February, 2019).

Table 24, showed the descriptive statistics for Fraction bars/Fraction Tiles Manipulative data set. The Pre-test mean and standard deviation for Experiment group and control group were (mean $\bar{x}$ = 8.552, SD=1.402), and (mean $\bar{x}$= 8.192, SD=1.735), respectively. There was no significant difference in the Pre-test mean and standard deviation of the Experiment group and the Control group. This suggested that the initial competencies of the two groups in fractions were equivalent prior to the study. However, the Post-test mean and standard deviation for the Experiment group and control group were (mean $\bar{x}$ = 14.776, SD=1.402), and (mean $\bar{x}$=8.072, SD=1.541), respectively. The huge difference in the Post-test mean and standard deviation between the Experimental group and the Control group could be attributed to the effect of the fraction tiles/fraction bars on the Experiment group. Learners in the Experimental group had a practical understanding of solving different types of fractional problems using fraction tiles/fraction bars. The Experimental group were fully involved in the instructional process due to the practical nature of instructions. However, there was a drop in the mean score and standard deviation of the control group in the Post-test (mean $\bar{x}$ = 8.072, SD=1.541). This could be attributed to the fact that the learners could not understand the concept of fractions fully, since there was no concrete material involved to explain the concept better to them. Also, the learners might have forgotten the process of solving different types of fractions since they were taught orally, and might have not paid attention in class during the instructional process. Boggan, Harper and Whitmire (2010) agreed that fraction tiles/fraction bars enhanced learners' understanding of addition and subtraction of fractions, and also facilitated learners understanding of the representation of equivalent fractions. The standard deviations from the (table 25) for both the control and experimental group was low, which suggested that the data was close to the mean scores in all instances. Literature suggested that Fraction tiles/Fraction bars are a clear demonstration of how models enabled learners to pictorially see the part in relation to the entire unit (Ervin, 2017:265). Other scholars are of the opinion that Fraction tiles/Fraction bars models are important models in conveying mathematical connections between fractions, compared dimensions, and examined corresponding fractions (Ervin, 2017).

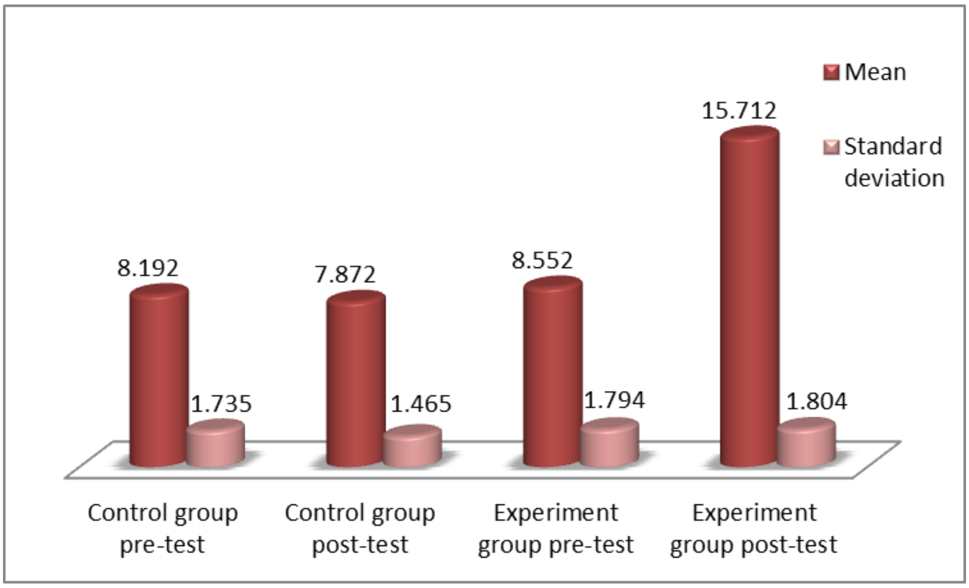
4.3.3 Findings of Paper folding Manipulative Tool Data Set

Table.25: Findings of Paper Folding Manipulative Tool Data Set

Group	N	Trial	Mean	Standard deviation	Min	Max
	125	Pre-test	8.192	1.735	0	14
Control	125	Post-test	7.872	1.465	5	11
	125	Pre-test	8.552	1.794	2	11
Experiment	125	Post-test	15.712	1.804	6	19

Source: Field work (February, 2019).

Figure.36: Shows a bar chart of Paper Folding Manipulative Tool



Source: Field work (February, 2019).

Table 25, illustrated the Mean scores and Standard deviation of the Paper folding manipulative tool in the Pre-test and Post-test of both the Experimental group and Control group. The Pre-test mean scores and standard deviation of Experimental group (mean $\bar{x}$ = 8.552, SD=1.794), and Control group (mean $\bar{x}$ = 8.192, SD=1.735), respectively. There was no significant difference in the mean scores and standard deviation of both the experimental group and the control group in the pre-test. This suggested that the initial competencies of the two groups in fractions were equivalent prior to the study. The Post-test mean scores and standard deviation showed Experimental group (mean $\bar{x}$ = 15.712, SD= 1.804), and Control group (mean $\bar{x}$ =7.872, SD=1.465), respectively. The huge difference between the mean score and slightly difference in the standard deviation of the Experiment group and Control group revealed that the Paper folding manipulative tool had effects on the experimental group. It aided the experimental group to have first-hand information of solving fractions practically. It also helped the experimental group in the process of solving different types of fractional questions in mathematics. Learners in the experiment group were fully involved in the instructional process due to the practical nature of instructions. There was a drop in the mean scores and the Standard deviation of the control group in the Post-test (mean $\bar{x}$ = 7.872, SD=1.465). This could be attributed to the fact that the control group could not get a better understanding of the concept of fractions, since it was taught without a concrete material. It also suggested that the learners had forgotten the process of solving different types of fractions since they were taught orally, and they might not have been concentrating in class during the instructional process. Ervin, (2017) opined that through paper folding modelling, learners are able to visualise problems in figurative form through lens that highlighted the scale of the dividend and divisor, and are able to make a better judgement whether their solutions are viable, or not. In addition, (Johanning & Mamer, 2014) agreed that Paper folding played a vital role in learners' comprehension and in imagining what a division problem was enacting. Table 25, showed that the Standard deviation in the Experiment group and control group, in both Pre-test and Post-test, were low. This suggested that the data was close to the mean.

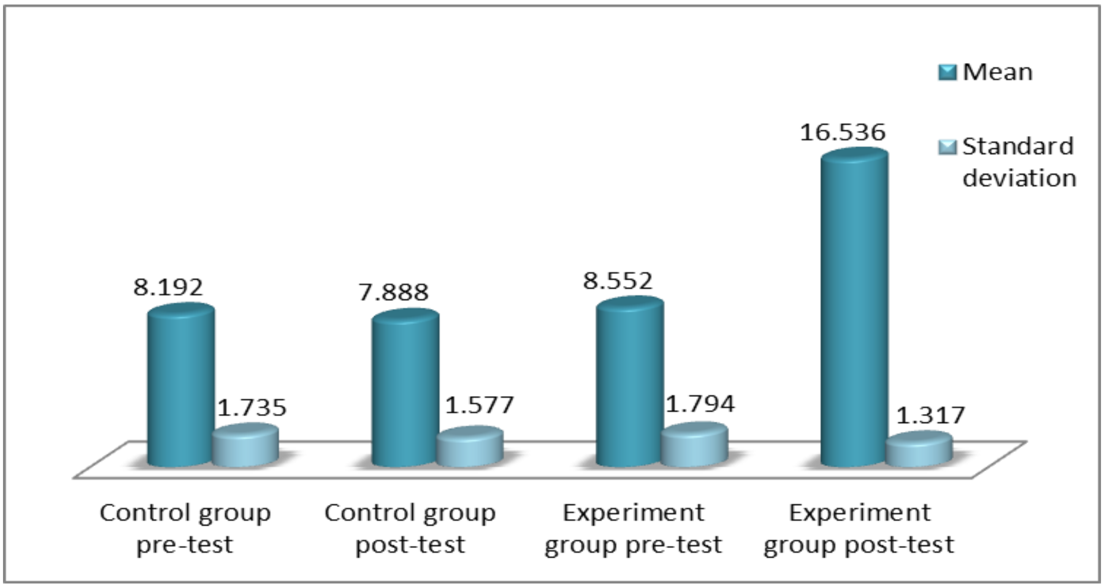
4.3.4 Findings of Computer Assisted Manipulative Tool Data Set

Table.26: Findings of Computer Assisted Manipulative Tool

Group	N	Trial	Mean	Standard deviation	Min	Max
	125	Pre-test	8.192	1.735	2	11
Control	125	Post-test	7.888	1.577	1	11
	125	Pre-test	8.552	1.794	0	14
Experiment	125	Post-test	16.536	1.317	12	20

Source: Field work (February, 2019).

Figure 37: Showed a bar chart of Computer Assisted Manipulative Tool



Source: Field work (February, 2019).

Table 26, showed the descriptive statistics for Computer Assisted Manipulative Tool Data Set. The Pre-test mean and standard deviation of the Experiment group were (mean $\bar{x}$= 8.552, SD=1.794), and control group (mean $\bar{x}$=8.192, SD=1.735), respectively. Pre-test mean and standard deviation of the Experiment group and the Control group showed that there was no significant difference between the two groups. This suggested that the initial competencies of the two groups in fractions were equivalent prior to the study. However, the Post-test mean and standard deviation of the Experiment group and control group were (mean $\bar{x}$=16.536, SD=1.317), and (mean $\bar{x}$=7.888, SD=1.577), respectively. The huge difference in the Post-test mean between the Experimental group and the Control group could be attributed to the effect of the Computer assisted manipulative tool on the Experiment group. Learners in the Experimental group had a practical understanding of solving different types of fractional problems using Computer assisted manipulative tool. The Experimental group were fully involved in the instructional process due to the practical nature of instructions. On the other hand, there was a drop in the mean score and standard deviation of the control group in the Post-test. This could be attributed to the fact that learners could not understand the concept of fractions fully, since there was no concrete material involved to explain the concept better to them. Also, learners might have forgotten the process of solving different types of fractions since they were taught orally, and they might have not paid attention in class in the instructional process. Satsangi and Bouck (2015), supported the idea that virtual manipulative tools enabled learners to actively contribute in class during the instructional period and also facilitated the understanding of fractions. Websites have been established to allow educators and learners free access to learn from the internet and download useful materials for their studies (Bouck & Flanaga, 2009). Johnson (2012), observed that the internet could be helpful to strengthen instructional practices, as well as to extend the horizon of evaluation. The solicitation of virtual manipulative serves as personalised adjustments for learners with learning disabilities, and mathematics difficulties, especially in fractions (Bryant & Bryant 2011; Edyburn, 2013). Ochohi and Ukwumunu, (2008) supported that e-learning in mathematics made it more interesting, more enjoyable and important to learner's day-to-day activities. However, the standard deviations from (Table 26) showed that both the control group and experimental group was low, which implies that the scores of the learners were close to the mean scores in all cases.

4.4 Findings of Learners' perception of Manipulative Data Tool Set

Questionnaire on Manipulative Concrete Material (MCM) were administered to One hundred and twenty-five (125) learners who formed the Experimental group, to ascertain their perception on the effect of the use of Manipulative Concrete Materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding, and Computer assisted manipulative) on their performance in fractions. Questionnaires were also given to the control group to collect their background information. Tables 36 and 37 present information on learners' background in the experimental group and control group respectively.

4.4.1 Socio-demographic information of Experimental group

Table 27: Socio-demography variables on experimental group.

Variable	Frequency	Percentage
Age		
11 – 13	15	12
14 – 16	110	88
Gender		
Male	41	32.8
Female	84	67.2
Race		
Black	119	95.2
White	2	1.6
Coloured	4	3.2
Grade		
9	123	98.4
Not indicated	2	1.6
Total	125	100

Source: Field work (February, 2019).

A total of one hundred and twenty-five (125) learners completed the questionnaire in the experimental group. Table 27 shows that there were 84 (67.2%) female, and 41 (32.8%) male learners. More than half of the learners' were in the age group 14 – 16 years, which translates into 88 per cent, whilst 15 learners, representing (12%) were in the age group of 11-13 years. The black race was made up of 119 learners and were the majority, representing (95.2%), 4 (3.2%) learners were coloured, and 2 (1.6%) were whites. 123 (98.4%) learners indicated that they were in Grade 9, and 2 (1.6%) learners did not indicate their grades.

4.4.2 Socio-demographic information of the control group

Table 28: Socio-demography variables on control group.

Variable	Frequency	Percentage
Age		
11 – 13	10	8
14 – 16	115	92
Gender		
Male	61	48.8
Female	64	51.2
Race		
Black	110	88
White	6	4.8
Coloured	9	7.2
Grade 9		
	124	99.2
Not indicated	1	0.8
Total	125	100

Source: Field work (February, 2019).

Table 28, shows that there were 64 (51.2%) female and 61 (48.8%) male. Most of the learners' were in the age group 14 – 16 years (115) which translated into 92 per cent, whilst 10 learners representing (8%) were in the age group 11-13 years. The black race was made up of 110 learners representing (88%), 9 learners translating to (7.2%) belonged to the coloured race, and 6 learners, translating to (4.8 %) were whites. 124 learners translating to (99.2%) indicated their grade and 1 learner translating to (0.8%) did not indicate the grade. This information enabled the researcher to get an insight of the participants' background, compared to their performance in the study.

Learners' general perception on the manipulative concrete materials were collected from the experimental group. Refer to Appendix E. The items were arranged in a four option Likert scale of 'Strongly Agree' (SA), 'Agree' (A), 'Disagree' (D) and 'Strongly

Disagree' (SD). The scoring for positive items 'Strongly Agree' (SA), 'Agree' (A), 'Disagree' (D) and 'Strongly Disagree' (SD) were coded 4, 3, 2 and 1 respectively, and the reversed for negatively worded items.

4.4.3 Learners' general perception on Cuisenaire Rods

Table 29: Learners' general perception of Cuisenaire rods manipulative tool

Variable	Strongly Disagree n (%)	Disagree n (%)	Agree n (%)	Strongly Agree n (%)	Mean	SD
My teacher uses Cuisenaire rods in solving mathematical problems involving fractions	7 (5.6)	35 (28)	42 (33.6)	41 (32.8)	2.936	0.914
I have never used Cuisenaire rods in solving mathematical problems involving fractions	30 (24)	39 (31.2)	39 (31.2)	17 (13.6)	2.344	0.993
I often use Cuisenaire rods in solving mathematical problems involving fractions	6 (4.8)	34 (27.42)	56 (44.8)	29 (23.2)	2.864	0.826
I am comfortable in using Cuisenaire rods in solving mathematical problems involving fractions	20 (16)	45 (36)	48 (38.4)	12 (9.6)	2.416	0.872
Cuisenaire rods help my academic performance in fractions	15 (12)	66 (52.8)	37 (29.6)	7 (5.6)	2.288	0.749

Source: Field work (February, 2019).

Table 29 shows that 42 learners, translating to (33.6%), agreed that teachers used Cuisenaire rods in solving mathematical problems involving fractions, whilst 7 learners, translating to (5.6%), strongly disagreed on the assertion that their mathematics educators used Cuisenaire rods in instruction of mathematics. Thirtynine learners, translating to (31.2%), and 30 learners translating to (24%), disagreed, and strongly disagreed respectively, on the notion that they have never used Cuisenaire rods in solving mathematical problems involving fractions. This seemed to suggest that most of the learners used Cuisenaire rods in working mathematical problems involving fractions. Fifty-six learners, translating to (44.8%), conceded that they often used Cuisenaire rods in solving mathematical problems involving fractions. The study showed that 48 learners, translating to (38.4%), were comfortable in using Cuisenaire rods. The result showed that most learners were comfortable in using Cuisenaire rods. However, 20 learners, translating to (16%), conceded that they were not comfortable using Cuisenaire rods. This could be attributed to the failure on the part of the educators in using Cuisenaire rods in classroom instruction of fractions. Ross (2008) attested to the fact that educators who were not in tune with the application of concrete manipulative materials were most liable to limit the success of teaching, classroom organisation, and learners' mathematical attainment. Sixty-six learners, translating to (52.8%), agreed that Cuisenaire rods helped their performance in fractions. The results showed that the majority of learners concurred that Cuisenaire rods helped their performance in fractions. This could be attributed to the fact that Manipulative concrete materials have been universally acknowledged as useful mathematical objects that support practical learning through the application of concrete objects, as asserted by (Burns & Hamm, 2011).

4.4.4 Learners' general perception on Fraction tiles/Fraction bars

Table 30: Learners general perception of Fractional tiles/Fraction bars manipulative tool

Variable	Strongly Disagree n (%)	Disagree n (%)	Agree n (%)	Strongly Agree n (%)	Mean	SD
My teacher uses Fractions bar /Fraction tiles in solving mathematical problems involving fractions	7 (5.6)	23 (18.4)	61 (48.8)	34 (27.2)	2.976	0.828
I have never used Fractions bar/Fraction tiles in solving mathematical problems involving fractions	21 (16.8)	66 (52.8)	22 (17.6)	16 (12.8)	2.264	0.890
I often use Fractions bar/Fraction tiles in solving mathematical problems involving fractions	6 (4.8)	18 (14.4)	82 (65.6)	19 (15.2)	2.912	0.696
I am comfortable in using Fraction bars/Fraction tiles in solving mathematical problems involving fractions	18 (14.4)	24 (19.2)	67 (53.6)	16 (12.8)	2.648	0.882
Fraction bars/Fraction tiles help my academic performance in fractions	15 (12)	48 (38.4)	48 (38.4)	14 (11.2)	2.488	0.848

Source: Field work (February, 2019).

Table 30 shows that 34 (27.2%) learners Strongly Agreed that teachers' used Fractions bar/Fraction tiles in solving mathematical problems involving fractions. However, 7 (5.6%) learners Strongly Disagreed with the assertion. The result indicated that most of the learners agreed that Fractions bar/Fraction tiles were used by educators in fraction instructions. Fifty-six (52.8%) learners disagreed, and 21 (16.8%)

learners Strongly Disagreed with the notion that they have never used Fractions bar/Fraction tiles in solving mathematical problems involving fractions. The result showed that the majority of learners have used Fractions bar/Fraction tiles in solving fractional problems. With respect to how often Fractions bar/Fraction tiles were used in solving mathematical problems involving fractions, 82 (65.6%) learners Agreed, and 19 (15.2%) Strongly Agreed that they often use Fraction tiles/Fraction bars. However, 18 (14.4%) learners Disagree and 6 (4.8%) learners Strongly Disagreed to that assertion. The results confirmed that the majority of learners often used Fraction tiles/Fraction bars in solving questions involving fractions. Sixty-seven (53.6%) learners, and 16 (12.8%) learners, respectively Agreed and Strongly Agreed that they were comfortable in using Fraction bars/Fraction tiles in solving mathematical problems involving fractions. However, 24 (19.2%) learners Disagreed, and 18 (14.4%) learners Strongly Disagreed with the assertion. The result showed that most of the learners affirmed that they were comfortable using Fraction bar/Fraction tiles. Also, 48 (38.4%) learners, and 14 (11.2%) learners, respectively, Agreed and Strongly Agreed that Fraction bars/Fraction tiles helped their performance in fractions. On the other hand, 48 (38.4%), and 15(12%) learners, respectively Disagreed and Strongly Disagreed with the assertion. The result indicated that a fair number of learners agreed that Fraction bars/Fraction tiles helped their academic performance. This could be attributed to their improvement in the post-test. Learners had a deep understanding of fractions with the use of fraction tiles/fraction bars.

4.4.5 Learners' general perception on Paper Folding

Table 31: Learners' general perception of Paper folding manipulative tool

Variable	Strongly Disagree n (%)	Disagree n (%)	Agree n (%)	Strongly Agree n (%)	Mean	SD
My teacher uses Paper folding in solving mathematical problems involving fractions	10 (8)	36 (28.8)	51 (40.8)	28 (22.4)	2.776	0.888
I have never used Paper folding in solving mathematical problems involving fractions	25 (20)	43 (34.4)	43 (34.4)	14 (11.2)	2.368	0.929
I often use Paper folding in solving mathematical problems involving fractions	4 (3.2)	36 (28.8)	66 (52.8)	19 (15.2)	2.8	0.730
I am comfortable in using Paper folding in solving mathematical problems involving fractions	16 (12.8)	46 (36.8)	52 (41.6)	11 (8.8)	2.464	0.828
Paper folding helps my academic performance in fractions	10 (8)	61 (48.8)	40 (32)	14 (11.2)	2.464	0.799

Source: Field work (February, 2019).

Table 31 shows that 51 learners, translating to (40.8%), Agreed that their teachers used paper folding in solving mathematical problems involving fractions. However, 36 learners, translating to (28.8%), Disagreed with the assertion that their teachers used paper folding in fractions instructions. The results indicated that the majority of learners acknowledged that paper folding is used by their educators in the instructions of fractions. Forty-three (34.4%) learners Disagreed that they have never used paper folding in solving fractions, whilst 14 (11.2%) learners, Strongly Agreed that they have never used paper folding in solving mathematical problems involving fractions. The result showed that most of the learners Disagreed with the notion that they had never used paper folding in solving mathematical problems involving fractions. With respect to how often paper folding was used in solving mathematical problems involving fractions, 66 (52.8%) learners Agreed, and 19 (15.2%) learners Strongly Agreed to the assertion. However, 36 (28.8%) learners Disagreed, and 4 (3.2%) learners Strongly Disagreed. The results showed that learners often used paper folding in solving problems involving fractions. In response to comfort in using paper folding, 52 (41.6%) learners Agreed, and 11 (8.8%) learners Strongly Agreed to the assertion. However, 46 (36.8%) learners Disagreed, and 16 (12.8%) learners Strongly Disagreed to the assertion. The results showed that most of the learners were comfortable in using paper folding in solving mathematical problems. Also 61 (48.8%) learners Disagreed that paper folding helped their performance in fractions, while 40 (32%) learners Agreed, and 11 (11.2%) learners Strongly Agreed that paper folding helps their academic performance in fraction. However, the result showed that quite a number of learners were in affirmation that paper folding enhanced their academic performance in fractions. This could be attributed to the effects paper folding contributed to their post-test. Their mean scored improved in the post-test due to the use of paper folding.

4.4.6 Learners' general perception on Computer Assisted Manipulative

Table 32: Learners' general perception of Computer assisted manipulative tool

Variable	Strongly Disagree n (%)	Disagree n (%)	Agree n (%)	Strongly Agree n (%)	Mean	SD
My teacher uses Computer assisted manipulative in solving mathematical problems involving fractions	14 (11.2)	48 (38.4)	36 (28.80	27 (21.6)	2.608	0.949
I often use Computer assisted manipulative in solving mathematical problems involving fractions	8 (6.4)	51 (40.8)	44 (35.2)	22 (17.6)	2.64	0.846
I am comfortable in using Computer assisted manipulative in solving mathematical problems involving fractions	18 (14.4)	45 (36)	49 (39.2)	13 (10.4)	2.456	0.866
Computer assisted manipulative helps my academic performance in fractions	18 (14.4)	62 (49.6)	34 (27.2)	11 (8.8)	2.304	0.825

Source: Field work (February, 2019).

Table 32 shows that 48 (38.4%) learners Disagreed, and 14 (11.2%) learners Strongly Disagreed on the assertion that teachers used Computer assisted manipulative in solving mathematical problems involving fractions. On the other hand, 36 (28.80%), and 27 (21.6%), learners respectively Agreed and Strongly Agreed on the assertion. The results showed that a fair number of learners agreed that teachers used Computer assisted manipulative in teaching. The results showed that a considerable number of learners Agreed that teachers used computer assisted manipulative in teaching. Fifty-one (40.8%) learners Disagreed, and 8 (6.4%) learners Strongly Disagreed that they often used Computer assisted manipulative in solving mathematical problems involving fractions. However, 44 (35.2%), and 22 (17.6%), learners Agreed and Strongly Agreed that they often use Computer assisted manipulative in solving mathematical problems involving fractions. The results indicated that a fair number of learners Agreed that they often used Computer assisted manipulative in solving mathematical problems involving fractions. The study also showed that 49 (39.2%) learners Agreed, and 13 (10.4%), Strongly Agreed that they were comfortable using Computer assisted manipulative in solving mathematical problems involving fractions. However, 45 (36%) learners Disagreed and 18 (14.4%) learners Strongly Disagreed that they were comfortable using Computer assisted manipulative in solving mathematical problems involving fractions. The results showed that quite a number of learners were comfortable in using computer assisted manipulative. Sixty-two (49.6%) learners Disagreed that computer assisted manipulative helped their academic performance. However, 34 (27.2%) learners Agreed and 11 (8.8%) learners Strongly Agreed that computer assisted manipulative helped their academic performance. The results showed that a fair number of learners Agreed that computer assisted manipulative helped their academic performance. This could be attributed to their performance in the post-test. The use of computer assisted manipulative helped learners in improving their performance. It also helped learners understand the concept of fractions in mathematics.

4.4.7 Learners’ general perception on manipulative tools

Table 33: Learners' general perception on manipulative tools

Manipulative tools	Mean	Minimum	Maximum	Mean rank
Students' perception of Cuisenaire rods manipulative tool	12.848	5	19	3
Students' perception of Fraction tiles manipulative tool	13.288	7	19	1
Students' perception of paper folding manipulative tool	12.872	8	18	2
Students' perception of computer assisted manipulative tool	12.528	7	20	4

Low= <mean, High= >Mean, n=frequency

Source: Field work (February, 2019).

Table 33 described learners’ perception towards the use of Manipulative Concrete Materials for the study (Cuisenaire rods, Fraction bar/Fraction title, Paper folding, and Computer assisted manipulative). The Mean explained the weight of perception learners have towards the manipulative tools.

From Table 33, the Mean score for the manipulative tools were as follows: Fraction bar/Fraction tiles (M=13.288), Paper folding (M=12.872), Cuisenaire rods (12.848) and Computer assisted manipulative (12.528) respectively. The results indicated that learners’ have a higher level of perception for Fraction bar/Fraction tiles, followed by Paper folding, Cuisenaire rods and Computer Assisted Manipulative respectively. The Mean rank shows the level of perception in hierarchical order.

4.5 CHAPTER SUMMARY

The chapter analysed data on the effect of use of manipulative concrete materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative) on learners' performance in fractions. Twenty-item multiple-choice test of Pre-test and Post-test were administered to the Experimental and Control group. Questionnaires were also administered to the experimental group. The data collected were analysed through coding, sorting and categorization to find Percentages, Mean, and Standard Deviation. The sample t-test was used to test the hypotheses. The analysed results showed that, prior to the application of the manipulative concrete materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative), there was no significant difference between the mean scores and standard deviation of the Experimental group and the Control group in the Pre-test. However, after the treatment of the experimental group with the manipulative concrete materials, the results in the Post-test showed a significant improvement in the mean scores and slightly improved standard deviation of the Experimental group in all cases, whilst there was a drop in the mean scores and standard deviation in the control group. The results indicated that manipulative concrete materials have a significant effect on learners' performance in fractions. The study also showed that learners' have high perception towards the manipulative concrete materials used for the study. The next chapter looks at the summary of the findings, and also, recommendations for further study.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter presents significant findings of the effect of manipulative concrete materials on learners' performance in fractions, conclusions drawn based on the findings, and also recommendations to learners, educators, principals, the Department of Education, government and non-governmental organizations (NGO's). In all, two hundred and fifty (250) grade nine learners', selected from five (5) different high schools, made up of 102 boys and 148 girls participated in the study. Their ages ranged from 13 to 16 years old. Ten educators, who taught grade nine mathematics, were also selected. The data was collected in both the first term and the second term of the 2019 academic year. The duration for the data collection was three months. Approximately three weeks was spent in a school. Pre-test and Post-test was administered to both the control group and experimental group. The pre-test was administered prior to the commencement of the study. The pre-test and post-test were made up of 20-item multiple-choice questions of different types of fractions. The Curriculum Assessment Policy Statement (CAPS) document for grade nine mathematics, Platinum mathematics learner's book grade 9 and "Spot On" Mathematics Learner's book for Grade 9, were used. The experimental group had vigorous instruction on different types of fractions with different manipulative concrete materials (Cuisenaire rods, Fraction bar/Fraction title, Paper folding and Computer assisted manipulative) throughout the learning process, whilst the control group was instructed without manipulative concrete materials.

5.2 SUMMARY OF THE FINDINGS AND DISCUSSIONS

5.2.1 Summary of the Findings of the Use of Manipulative Concrete Materials on the Instructions of Fractions

- Manipulative concrete materials such as: (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative) have significant effects on grade nine learners' performance in fractions.

- The methodology employed in this study, and the prepared manipulative concrete materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative) have significantly increased learners' performance in fractions', and also sustained the interest of learners' to learn more.
- Learners' were able to manipulate paper folding and fraction titles/fraction bars within the stipulated time, but spent considerable time on Cuisenaire rods, and computer assisted materials.
- There were no much difference between the mean score and standard deviations of the Experimental group and Control group in the Pre-test.
- There was a significant difference in the mean score and standard deviations between the Experimental group and Control group in the Post-test in all cases.
- The mean scores and standard deviations in the post-test of the experimental groups for all the manipulative concrete materials were higher than the mean scores of the pre-test in all cases.
- There were drops in the mean scores and standard deviations of the post-test in the control group in all cases.
- Learners' have a high perception level for Fraction tile/Fraction bar, Paper Folding, Cuisenaire rods and Computer assisted manipulative tools respectively.

5.2.2 Discussions of the findings

The findings of the study showed that there were no much difference in the mean scores and standard deviations between the Experimental group and the Control group in the Pre-test. This could be attributed to the fact that the initial competencies of the two groups in fractions were equivalent, prior to the study. However, there were vast differences in the mean scores and slightly difference in the standard deviations of the Post-test between the Experimental group and the Control group in all cases. This suggested that manipulative concrete materials; Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative, have significant effects on grade nine learners' performance in fractions. It also helped learners to be actively involved in the learning process. In effect, the instructional process became learner centred. Learners were fully involved in the manipulation of the concrete materials which improved their understanding of working fractions.

Ligget (2017), observed that the use of manipulative concrete materials in the instructions of mathematics at elementary and middle schools, have improved learners' academic achievement in mathematics. In support, Belenky and Nokes (2009), asserted that learning with manipulative concrete materials helped learners' in building practical confidence by boosting the level of commitment in the use of concrete materials in the future. Other studies showed that manipulative concrete materials aided in motivating and sustaining learners' interest in learning mathematics (Merriam & Brockett, 2011). Satsangi and Bouck (2015) avowed that virtual manipulative tools enabled learners to actively contribute in class during the instructional period.

The study observed that there are drops in the mean scores and standard deviations in the control group in the post-test in all cases. This could be attributed to lack of motivation to sustain the attention and interest of learners' towards the instruction of fractions, due to the orthodox method of instruction. The abstract nature of the topic under discussion could also be a factor. Moreover, it could suggest that the learners had forgotten the process of working fractions since they were taught without a concrete material. Research showed that learners' lacked attentiveness in class when the instruction process was teacher centred. They felt bored during mathematics instructions, which led to misconduct exhibited in most South African schools (Serame, 2013). Rajoo (2013) opined that the classroom learning environment is an important factor for motivating learners' Mathematics achievement. This, therefore, suggested that appropriate teaching methods and learning styles must be employed in the teaching of abstract concepts in mathematics. Studies showed that appreciating the ways learners' acquired knowledge, helps the educator in the selection of the appropriate instructional strategies suitable to meet learners' needs (Zapalska & Dabb, 2002).

Considering the sample sizes, the models' specifications, and the methodology employed for this study, it was observed that the prepared manipulative concrete materials have significantly increased learners' performance in fractions. This could be attributed to the effect of manipulative concrete materials, such as a hands-on material which made mathematics practical to learners'. Uribe-Floez and Wilkins (2010) are of the view that in applying manipulative concrete materials, learners' are able to envisage the mathematical concepts in a physical manner. Learners' who see and manipulate a variety of objects, have clearer mental images and can represent abstract ideas more completely than those whose experiences are meagre (Gaetano, 2014:4). The study observed that considerable time was spent in manipulating some of the concrete materials, such as the Cuisenaire rods, and computer assisted materials, whilst less time was spent on paper folding and fraction titles/fraction bars. This could suggest that some of the learners were not conversant in the use of some of the manipulative concrete materials. Other studies revealed that, the majority of learners' required extra time to accomplish a particular task using manipulative concrete materials, and to develop fractional knowledge among learners' (Cramer & Henry, 2013). Uribe-Florez and Wilkins (2010) argued that educators are of the notion that the application of manipulative concrete materials in teaching is time consuming.

In reference to the comparison of the mean scores and standard deviations of the Post-test and the Pre-test of the experiment group, and the control group of this study, it could be observed that the prepared manipulative concrete materials (Cuisenaire rods, Fraction bars/Fraction tiles, Paper folding and computers assisted manipulative) for the topic "fractions", in Grade nine, contributed significantly to the performance of learners'. Burns and Hamm, (2011) concurred that manipulative concrete material reinforced mathematical ideas and improved the average test scores of learners'. Swan and Marshall (2010), argued that there are possible advantages to be gained in the use of manipulative concrete materials in the instruction of mathematics, where sufficient skills are applied in a logical way. However, researchers are of the view that manipulative concrete materials, in reality, do not help learners' in cultivating mathematical comprehension (Moyer-Packenham & Westenskow, 2013). Educators who are not in tune with the application of concrete manipulative materials are most likely to limit the success of teaching, classroom organisation, and learners' mathematical attainment (Ross, 2008). The study observed that learners' perception towards the use of manipulative concrete materials in the instruction of fractions was high.

5.3 RECOMMENDATIONS

The following recommendations are made by the researcher to the learners', educators, principals, the Department of Education, government and non-governmental organizations (NGO's) based on the outcome of the study, the use of manipulative concrete materials, and how to improve learners' performance in fractions in mathematics:

5.3.1 Recommendations to learners

- Learners' ought to use manipulative concrete materials frequently in their mathematical lessons, so that they will be abreast with it, and increase their understanding in fractions.
- Learners' ought to solve more challenging mathematical problems involving fractions with the use of different manipulative concrete materials.
- Learners' ought to approach experts in the field to help them in the use of manipulative concrete materials.

5.3.2 Recommendations to Educators

- Educators ought to make adequate use of appropriate manipulative concrete materials in mathematics instructions.
- Mathematics educators should ensure that they incorporate manipulative concrete materials in their mathematical instructions.
- Educators ought to motivate and sustain the interest of their learners' during mathematics instruction by using different teaching methods. Mathematics classrooms must be learner centred, and not teacher centred form of instruction.
- Educators ought to have the ability to accommodate every learner during mathematics lesson.
- Educators ought to identify every learners' strength and weakness in mathematics in order to organise remedial classes for learners' with the application of manipulative concrete materials.

5.3.3 Recommendations to school principals

- Principals ought to ensure that there are adequate manipulative concrete materials in their schools to enhance the teaching and learning of mathematics.
- The principals, with the collaboration of the mathematics educators, ought to establish mathematics laboratory where the manipulative concrete materials are kept for generations to use.
- Principals ought to ensure that mathematics educators integrate manipulative concrete materials into the instructions of mathematics in their schools.

5.3.4 Recommendations to the Department of Education

- It is obligatory for the Department of Education to ensure that every school is well resourced with manipulative concrete materials to enhance the mathematical proficiency of the learners'.
- The Department of Education ought to ensure that there are adequate facilities in every school to improve the teaching and learning of mathematics. Schools must be provided with computers, science and mathematics laboratories where practical teaching can take place, and also for safe keeping of the equipment.
- The Department of Education should ensure that the Curriculum Assessment Policy Statement (CAPS) is provided to every school, and is strictly adhered to by educators at every grade level.
- The Department of Education ought to ensure that every mathematics educator incorporates the use of manipulative concrete materials in the instruction of mathematics.

5.3.5 Recommendations to government

- The government ought to enact policies that would strictly involve the use of manipulative concrete materials in the instructions of mathematics at all grade levels.
- The government ought to ensure that qualified educators in mathematics are posted to the various schools to teach the subject. This will help to reduce the bottleneck in the subject.

- The government ought to build schools and ensure that every school is well resourced with manipulative concrete materials to enhance the mathematical proficiency of the learners'.
- The government must set aside funds to motivate schools, educators, and learners' performing well in mathematics. This will encourage the schools, educators and learners' to do more.

5.3.6 Recommendations to Non-governmental Organisations (NGO's)

- Companies and Non-governmental Organisations (NGO's) can supplement the effort of government by contributing resources to the development of mathematics in the country, since education is the tool for development in every nation.
- NGO's should establish mathematics resource centres in every community to promote the learning of mathematics in every community in the country.
- Companies ought to supplement the effort of government by rewarding schools, educators and learners' who excel in mathematics by offering them manipulative concrete materials, scholarships and other incentives, such as mathematical labs.

5.4 CONCLUSION

Based on the findings of this study, it was established that manipulative concrete materials helped greatly in learners' understanding of fractions. Learners' performance improved in the post-test due to the use of manipulative concrete materials. The concrete materials motivated and sustained learners' interest during the instructional period. This made learners take control of the learning process and boosted their understanding of the topic under discussion. The study also observed that the manipulative concrete materials made the lesson real to the learners. The abstract nature of mathematics will be made real to every learner in South Africa if teachers incorporated manipulative concrete materials in their instructional process.

5.5 CONTRIBUTIONS TO KNOWLEDGE

1. This study established that learners' mathematical knowledge in fractions will improve if educators use manipulative concrete materials in the instructional process.
2. This study established that with the use of manipulative concrete materials, the instructional process becomes learner centred, rather than teacher centred. This enabled learners to fully participate in the instructional process.
3. This study inspired educators to see the need to incorporate manipulative concrete materials in the instruction of fractions, so as to bridge the gap between abstractness and a real life situation in fractions. This also enabled learners to see fractions in both the classroom situation and the out-of-school situation.
4. This study will help society at large, as it will enable learners', educators', governments' and non-governmental organizations to place emphasis on the use of manipulative concrete materials in the instructions of mathematics, especially fractions, in all South African schools.

5.6 LIMITATIONS OF THE STUDY

The researcher limited his study to selected schools in the Chris Hani West district of the Department of Education, due to time constriction. The outcome of the study was conclusive. However, it may not be reflective of same situation in all schools in South Africa.

The researcher encountered communication barriers. He therefore employed the services of one of the mathematics educators who could speak English and isiXhosa fluently, to translate some of the terms in the questionnaire to two learners who could not understand some of the terms. This could, however, alter the actual meaning of what the researcher was demanding.

Some of the manipulative concrete materials were improvised by the researcher. The researcher could not get the original manipulative concrete materials from the shops and the schools. This therefore could have altered some of the results generated, especially for learners suffering from colour blindness, since some of the tools were not brightly coloured as the colours learners see in their textbooks.

The data collection took place for only three months, which was practically insufficient due to the number of manipulative concrete materials to study, and the mathematical concepts to be acquired. This, therefore, could affect slow learners.

5.7 SUGGESTIONS FOR FUTURE RESEARCH

This study called for further research into the effects of the use of manipulative concrete materials on learners' performance in fractions. One major question that needed further research, was: which manipulative concrete material was best suited to enhance learners' mathematical proficiency and skills in fractions in Grade nine, and why? There are many manipulative concrete materials on the market use to teach fractions. But, for leaners' to acquire the mathematical proficiency and skills in fractions, the mathematics educator ought to understand which manipulative concrete material will be most appropriate to enhance learners' proficiency when teaching fractions.

Another research question that needs additional research, is the duration of the retention skills. Specifically, how long would the mathematical skills acquired be retained by the learners' after the practical activities? The retention skills could not be tested in this study due to time constricts. The researcher therefore entreated other researchers to investigate the retention skills of learners' in terms of long-term and short-term memory, after the practical activities.

This study could be taken up by other researchers to find out the effect of the use of manipulative concrete materials in all the high schools in South Africa, since the study was limited to only a few schools in the Chris Hani West Department of Education.

5.8 CHAPTER SUMMARY

This chapter presented on the summary of the findings of the study and discussions. Recommendations were made to stakeholders, such as: learners, educators, principals of schools, department of education, government and non-governmental organizations, and ended with a conclusion. The study also presented the contributions to knowledge, limitations of the study and suggestions were made for future research. The purpose of this study was to find out the effect of the use of manipulative materials on grade nine learners' performance in fractions.

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APPENDICES

APPENDIX A
INTRODUCTORY LETTER FROM MY SUPERVISOR

University of Fort Hare
Faculty of Education
School of General & Continuing Education(SGCE)

East London Campus:
Private Bag X9083, 50 Church Street, East London, 5200, RSA
Tel: +27 (0) 43 704 7221 • +27 (0) 43 704 7216 • Fax: +27 (0) 86 628 2153
Email: nsibeko@ufh.ac.za • aboysen@ufh.ac.za

University of Fort Hare
Together in Excellence

10 January 2019

TO WHOM IT MAY CONCERN

Dear Sir/Madam

RE: Mr. **George Adom (201509244)**

This document serves to confirm that:

(a) the above student is registered for PhD at this university;

(b) The Faculty Research & Higher Degrees Committee approved the research proposal for his doctoral thesis.

In order for him to administer his questionnaire, he needs your express permission as a department before he can have access to schools for the collection of data. Please grant him the necessary assistance.

May I also request your cooperation and support to the student.

Thank you

Professor E.O. Adu **PhD**
Email: eadu@ufh.ac.za
Cell: 084 925 1948

www.ufh.ac.za

APPENDIX B

INTRODUCTORY LETTER FROM THE DEPARTMENT OF EDUCATION

CHRIS HANI WEST EDUCATION DISTRICT

Province of the
EASTERN CAPE
EDUCATION

OFFICE OF THE DISTRICT DIRECTOR

Physical Address: The Homestead Building, 02 Limpopo Drive, LAURIE Dashwood Park, Queenstown; **Postal Address:** Private Bag X7053, QUEENSTOWN 5320, REPUBLIC OF SOUTH AFRICA, Contact no: 083 2750706 / 083 418 8856 **e-mail:** philisa.mqoboli@gmail.com

CHRIS HANI WEST DISTRICT

TO	**: MR. GEORGE ADOM**
FROM	**: DISTRICT DIRECTOR**
SUBJECT	**: YOUR REQUEST TO CONDUCT EDUCATIONAL RESEARCH**
DATE	**: 02 FEBRUARY 2019**

The above mentioned and your letter dated 11th January 2019 has reference.
Your request to conduct educational research in schools in the Chris Hani education is hereby approved, provided that you provide my office with a copy of your research outcome.

I wish you all the best in your endeavor.

Thank you

Yours in Education

NRW. DE BRUYN
DISTRICT DIRECTOR

building blocks for growth

PGDP EASTERN CAPE
Ikamva eliqaqambileyo!

APPENDIX C
PRE-TEST QUESTIONS ON FRACTION ACHIEVEMENT TEST

FOR BOTH CONTROL GROUP AND EXPERIMENTAL GROUP.

PRE-TEST CONTROL GROUP

Solve the following questions.

DO NOT write your name on any part of the paper.

Choose the correct ANSWER FROM THE OPTIONS A-D. Each correct answer carries a mark of one (1).

1. Solve $^{20}/_{5}$

   A. 5

   B. 4

   C. 3

   D. 2

2. What is the product of $^{1}/_{2}$ and $^{3}/_{4}$

   A. $^{3}/_{8}$

   B. $^{3}/_{5}$

   C. $^{8}/_{3}$

   D. $^{5}/_{8}$

3. Solve $^{1}/_{2}$ X 2

   A. 2

   B. 4

   C. 1

   D. 3

4. Solve -4 ÷($^{1}/_{8}$ − $^{3}/_{8}$)

   A. 32

   B. 23

   C. 12

   D. 20

5. Solve 5 –(1½ +1 ¾ + $^{5}/_{6}$)

   A. $\frac{12}{11}$

   B. $\frac{11}{12}$

   C. $\frac{13}{12}$

   D. $\frac{10}{12}$

6. Solve 5⅕ X 6⅔

   A. 34⅔

   B. 32⅔

   C. $34\frac{3}{2}$

   D. 34

7. Find the sum of $^{2}/_{3}$ and $^{5}/_{6}$

A. $^{7}/_{9}$

B. $^{9}/_{7}$

C. $^{9}/_{10}$

D. $^{6}/_{9}$

8. Find the sum of $^{1}/_{2}$ and $^{2}/_{5}$

A. $^{10}/_{9}$

B. $^{3}/_{7}$

C. $^{9}/_{10}$

D. $^{7}/_{3}$

9. Calculate $^{3}/_{4} \div {}^{5}/_{8}$

A. 1⅕

B. $^{5}/_{6}$

C. $^{2}/_{3}$

D. 1½

10. Calculate 2 ⅝ ÷ 1 ¾

A. $^{5}/_{6}$

B. 1½

C. $^{2}/_{3}$

D. 1⅕

11. Calculate $^{3}/_{5} \div {}^{1}/_{3}$

A. $^{20}/_{30}$

B. $^{7}/_{12}$

C. $^{9}/_{5}$

D. $^{6}/_{12}$

12. Calculate $^{45}/_{3}$

A. 16

B. 3

C. 10

D. 15

13. Solve $^{1}/_{2} \times 1\frac{2}{4}$

A. $^{1}/_{12}$

B. $^{3}/_{4}$

C. $^{1}/_{8}$

D. $^{1}/_{18}$

14. Solve $1\frac{1}{5} + 3\frac{1}{3} - {}^{13}/_{15}$

A. $^{50}/_{15}$

B. $^{55}/_{15}$

C. $^{15}/_{55}$

D. $^{20}/_{15}$

15. Solve $(^1/_3)^2$

A. $^2/_6$

B. $^1/_9$

C. 18

D. 81

16. Calculate $^6/_{19}$ X $(^2/_4 + {}^3/_9)$

A. $^5/_{19}$

B. $^{19}/_5$

C. $^{20}/_{19}$

D. $^{30}/_{19}$

17. Solve $^5/_{14}$ X $\left(\frac{3}{10} + \frac{2}{5}\right)$

A. $\frac{1}{6}$

B. $\frac{30}{120}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Use the symbol <, > or = to answer the following questions

18. $^3/_4$ ………… $^3/_6$

A. $^3/_4 > ^3/_6$

B. $^3/_4 < ^3/_6$

C. $^3/_4 = ^3/_6$

D. None of the above

19. $^3/_4$ ……………$^1/_4$

A. $^3/_4 < ^1/_4$

B. $^3/_4 = ^1/_4$

C. $^3/_4 > ^1/_4$

D. None of the above.

20. $^2/_4$…………$^2/_7$

A. $^2/_4 = ^2/_7$

B. $^2/_4 < ^2/_7$

C. $^2/_4 > ^2/_7$

D. None of the above.

Thank you.

PRE-TEST EXPERIMENTAL GROUP

DO NOT write your name on any part of the paper.

Solve the following questions

Choose the correct ANSWER FROM THE OPTIONS A-D. Each correct answer carries a mark of one (1).

1. Solve $^{20}/_{5}$
   A. 5
   B. 4
   C. 3
   D. 2
2. What is the product of $^{1}/_{2}$ and $^{3}/_{4}$
   A. $^{3}/_{8}$
   B. $^{3}/_{5}$
   C. $^{8}/_{3}$
   D. $^{5}/_{8}$
3. Solve $^{1}/_{2}$ X 2
   A. 2
   B. 4
   C. 1
   D. 3
4. Solve -4 ÷$(^{1}/_{8} - ^{3}/_{8})$

A. 32

B. 23

C. 12

D. 20

5. Solve 5 –(1½ +1 ¾ + $^5/_6$)

A. $\frac{12}{11}$

B. $\frac{11}{12}$

C. $\frac{13}{12}$

D. $\frac{10}{12}$

6. Solve 5⅕ X 6⅔

A. 34⅔

B. 32⅔

C. $34\frac{3}{2}$

D. 34

7. Find the sum of $^2/_3$ and $^5/_6$

A. $^7/_9$

B. $^9/_7$

C. $^9/_6$

D. $^6/_9$

8. Find the sum of $^1/_2$ and $^2/_5$

A. $^{10}/_{9}$

B. $^{3}/_{7}$

C. $^{9}/_{10}$

D. $^{7}/_{3}$

9. Calculate $^{3}/_{4} \div {}^{5}/_{8}$

A. $1\frac{1}{5}$

B. $^{5}/_{6}$

C. $^{2}/_{3}$

D. $1\frac{1}{2}$

10. Calculate $2\frac{5}{8} \div 1\frac{3}{4}$

A. $^{5}/_{6}$

B. $1\frac{1}{2}$

C. $^{2}/_{3}$

D. $1\frac{1}{5}$

11. Calculate $^{3}/_{5} \div {}^{1}/_{3}$

A. $^{20}/_{30}$

B. $^{7}/_{12}$

C. $^{9}/_{5}$

D. $^{6}/_{12}$

12. Calculate $^{45}/_{3}$

A. 16

B. 3

C. 10

D. 15

13. Solve $^{1}/_{2} \times 1\frac{2}{4}$

A. $^{1}/_{12}$

B. $^{3}/_{4}$

C. $^{1}/_{8}$

D. $^{1}/_{18}$

14. Solve $1\frac{1}{5} + 3\frac{1}{3} - {}^{13}/_{15}$

A. $^{50}/_{15}$

B. $^{55}/_{15}$

C. $^{15}/_{55}$

D. $^{20}/_{15}$

15. Solve $(^{1}/_{3})^{2}$

A. $^{2}/_{6}$

B. $^{1}/_{9}$

C. 18

D. 81

16. Calculate $^{6}/_{19}$ X $(^{2}/_{4} + {^{3}/_{9}})$

A. $^{5}/_{19}$

B. $^{19}/_{5}$

C. $^{20}/_{19}$

D. $^{30}/_{19}$

17. Solve $^{5}/_{14}$ X $\left(\frac{3}{10} + \frac{2}{5}\right)$

A. $\frac{1}{6}$

B. $\frac{30}{120}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Use the symbol <, > or = to answer the following questions

18. $^{3}/_{4}$ ………… $^{3}/_{6}$

A. $^{3}/_{4} > {^{3}/_{6}}$

B. $^{3}/_{4} < {^{3}/_{6}}$

C. $^{3}/_{4} = {^{3}/_{6}}$

D. None of the above

19. $^{3}/_{4}$ ……………$^{1}/_{4}$

A. $^{3}/_{4} < {^{1}/_{4}}$

B. $^{3}/_{4} = {^{1}/_{4}}$

C. $^{3}/_{4}>^{1}/_{4}$

D. None of the above.

20. $^{2}/_{4}$………..$^{2}/_{7}$

A. $^{2}/_{4} = ^{2}/_{7}$

B. $^{2}/_{4}<^{2}/_{7}$

C. $^{2}/_{4}>^{2}/_{7}$

D. None of the above.

Thank you

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APPENDIX D

POST-TEST QUESTIONNAIRES ON FRACTION ACHIEVEMENT TEST

FOR BOTH CONTROL GROUP AND EXPERIMENTAL GROUP.

CONTROL GROUP

Answer the following questions.

DO NOT write your name on any part of the paper.

Choose the correct ANSWER FROM THE OPTIONS A-D. Each correct answer carries a mark of one (1).

1. Solve $^{20}/_{5}$

   A. 5

   B. 4

   C. 3

   D. 2

2. What is the product of $^{1}/_{2}$ and $^{3}/_{4}$

   A. $^{3}/_{8}$

   B. $^{3}/_{5}$

   C. $^{8}/_{3}$

   D. $^{5}/_{8}$

3. Solve $^1/_2$ X 2

   A. 2

   B. 4

   C. 1

   D. 3

4. Solve -4 ÷$(^1/_8 \;\; - \, ^3/_8)$

   A. 32

   B. 23

   C. 12

   D. 20

5. Solve 5 –(1½ +1 ¾ + $^5/_6$)

   A. $\frac{12}{11}$

   B. $\frac{11}{12}$

   C. $\frac{13}{12}$

   D. $\frac{10}{12}$

6. Solve 5⅕ X 6⅔

   A. 34⅔

   B. 32⅔

   C. 34$\frac{3}{2}$

   D. 34

7. Find the sum of $^{2}/_{3}$ and $^{7}/_{9}$

A. $^{7}/_{9}$

B. $^{9}/_{7}$

C. $^{9}/_{6}$

D. $^{6}/_{9}$

8. Find the sum of $^{1}/_{2}$ and $^{2}/_{5}$

A. $^{10}/_{9}$

B. $^{3}/_{7}$

C. $^{9}/_{10}$

D. $^{7}/_{3}$

9. Calculate $^{3}/_{4} \div {}^{5}/_{8}$

A. $1\frac{1}{5}$

B. $^{5}/_{6}$

C. $^{2}/_{3}$

D. $1\frac{1}{2}$

10. Calculate $2\frac{5}{8} \div 1\frac{3}{4}$

A. $^{5}/_{6}$

B. $1\frac{1}{2}$

C. $^{2}/_{3}$

D. $1\frac{1}{5}$

11. Calculate ${}^{3}/_{5} \div {}^{1}/_{3}$

A. ${}^{20}/_{30}$

B. ${}^{7}/_{12}$

C. ${}^{9}/_{5}$

D. ${}^{6}/_{12}$

12. Calculate ${}^{45}/_{3}$

A. 16

B. 3

C. 10

D. 15

13. Solve ${}^{1}/_{2} \times \ 1\frac{2}{4}$

A. ${}^{1}/_{12}$

B. ${}^{3}/_{4}$

C. ${}^{1}/_{8}$

D. ${}^{1}/_{18}$

14. Solve $1\frac{1}{5} + 3\frac{1}{3} - {}^{13}/_{15}$

A. ${}^{50}/_{15}$

B. ${}^{55}/_{15}$

C. ${}^{15}/_{55}$

D. $^{20}/_{15}$

15. Solve $(^{1}/_{3})^{2}$

A. $^{2}/_{6}$

B. $^{1}/_{9}$

C. 18

D. 81

16. Calculate $^{6}/_{19}$ X $(^{2}/_{4} + {}^{3}/_{9})$

A. $^{5}/_{19}$

B. $^{19}/_{5}$

C. $^{20}/_{19}$

D. $^{30}/_{19}$

17. Solve $^{5}/_{14}$ X $\left(\frac{3}{10} + \frac{2}{5}\right)$

A. $\frac{1}{6}$

B. $\frac{30}{120}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Use the symbol <, > or = to answer the following questions

18. $^{3}/_{4}$ ………… $^{3}/_{6}$

A. $^{3}/_{4} > ^{3}/_{6}$

B. $^{3}/_{4} < ^{3}/_{6}$

C. $^{3}/_{4} = ^{3}/_{6}$

D. None of the above

19. $^{3}/_{4}$ ……………$^{1}/_{4}$

A. $^{3}/_{4} < ^{1}/_{4}$

B. $^{3}/_{4} = ^{1}/_{4}$

C. $^{3}/_{4} > ^{1}/_{4}$

D. None of the above.

20. $^{2}/_{4}$…………$^{2}/_{7}$

A. $^{2}/_{4} = ^{2}/_{7}$

B. $^{2}/_{4} < ^{2}/_{7}$

C. $^{2}/_{4} > ^{2}/_{7}$

D. None of the above.

Thank you.

POST-TEST EXPERIMENTAL GROUP

Use any of the following manipulative concrete materials provided to solve the following questions.

DO NOT write your name on any part of the paper.

Choose the correct ANSWER FROM THE OPTIONS A-D. Each correct answer carries a mark of one (1).

1. Solve $^{20}/_{5}$
   A. 5
   B. 4
   C. 3
   D. 2

2. What is the product of $^{1}/_{2}$ and $^{3}/_{4}$
   A. $^{3}/_{8}$
   B. $^{3}/_{5}$
   C. $^{8}/_{3}$
   D. $^{5}/_{8}$

3. Solve $^{1}/_{2}$ X 2
   A. 2
   B. 4

C. 1

D. 3

4. Solve -4 ÷($^{1}/_{8}$ – $^{3}/_{8}$)

A. 32

B. 23

C. 12

D. 20

5. Solve 5 –(1½ +1 ¾ + $^{5}/_{6}$)

A. $\frac{12}{11}$

B. $\frac{11}{12}$

C. $\frac{13}{12}$

D. $\frac{10}{12}$

6. Solve 5⅕ X 6⅔

A. 34⅔

B. 32⅔

C. $34\frac{3}{2}$

D. 34

7. Find the sum of $^{2}/_{3}$ and $^{5}/_{6}$

A. $^{7}/_{9}$

B. $^{9}/_{7}$

C. $^{9}/_{6}$

D. $^{6}/_{9}$

8. Find the sum of $^{1}/_{2}$ and $^{2}/_{5}$

A. $^{10}/_{9}$

B. $^{3}/_{7}$

C. $^{9}/_{10}$

D. $^{7}/_{3}$

9. Calculate $^{3}/_{4} \div {}^{5}/_{8}$

A. $1\frac{1}{5}$

B. $^{5}/_{6}$

C. $^{2}/_{3}$

D. $1\frac{1}{2}$

10. Calculate $2\frac{5}{8} \div 1\frac{3}{4}$

A. $^{5}/_{6}$

B. $1\frac{1}{2}$

C. $^{2}/_{3}$

D. $1\frac{1}{5}$

11. Calculate $^{3}/_{5} \div {}^{1}/_{3}$

A. $^{20}/_{30}$

B. $^{7}/_{12}$

C. $^{9}/_{5}$

D. $^{6}/_{12}$

12. Calculate $^{45}/_{3}$

A. 16

B. 3

C. 10

D. 15

13. Solve $^{1}/_{2} \times \; 1\frac{2}{4}$

A. $^{1}/_{12}$

B. $^{3}/_{4}$

C. $^{1}/_{8}$

D. $^{1}/_{18}$

14. Solve $1\frac{1}{5}$ + $3\frac{1}{3}$ - $^{13}/_{15}$

A. $^{50}/_{15}$

B. $^{55}/_{15}$

C. $^{15}/_{55}$

D. $^{20}/_{15}$

15. Solve $(^{1}/_{3})^{2}$

A. $^{2}/_{6}$

B. $^{1}/_{9}$

C. 18

D. 81

16. Calculate $^{6}/_{19}$ X $(^{2}/_{4} + {}^{3}/_{9})$

A. $^{5}/_{19}$

B. $^{19}/_{5}$

C. $^{20}/_{19}$

D. $^{30}/_{19}$

17. Solve $^{5}/_{14}$ X $\left(\frac{3}{10} + \frac{2}{5}\right)$

A. $\frac{1}{6}$

B. $\frac{30}{120}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Use the symbol <, > or = to answer the following questions

18. $3/4$ ……….. $3/6$

A. $3/4 > 3/6$

B. $3/4 < 3/6$

C. $3/4 = 3/6$

D. None of the above

19. $3/4$ ……………$1/4$

A. $3/4 < 1/4$

B. $3/4 = 1/4$

C. $3/4 > 1/4$

D. None of the above.

20. $2/4$………..$2/7$

A. $2/4 = 2/7$

B. $2/4 < 2/7$

C. $2/4 > 2/7$

D. None of the above.

Thank you.

APPENDIX E

QUETIONNAIRE ON MANIPULATIVE CONCRETE MATERIAL

EXPERIMENTAL GROUP

Students' Questionnaire on Manipulative Concrete Material (SQMCM)

NOTE: Please do not write your name on the questionnaire.

Please use [x] to tick the appropriate answer.

SECTION A

1. Age: 8 – 10 [ ] 11 – 13 [ ] 14 – 16 [ ]
2. Gender: Male [ ] Female [ ]
3. Race: Balck [ ] White [ ] coloured [ ]
4. Grade: 9 [ ] 10 [ ] 11 [ ]

SECTION B*(questions on Cuisenaire rods)*

5. My teacher uses Cuisenaire rods in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

6. I have never used Cuisenaire rods in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

7. I often use Cuisenaire rods in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

8. I am comfortable in using questionnaire rods in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

9. Cuisenaire rods help my academic performance in fractions

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

SECTION C*(questions on Fractions bar /Fraction tiles)*

10. My teacher uses Fractions bar /Fraction tiles in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

11. I have never used Fractions bar/Fraction tiles in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

12. I often use Fractions bar/Fraction tiles in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

13. I am comfortable in using Fraction bars/Fraction tiles in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

14. Fraction bars/Fraction tiles help my academic performance in fractions

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

SECTION D *(questions on Paper folding)*

15. My teacher uses Paper folding in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

16. I have never used Paper folding in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

17. I often use Paper folding in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

18. I am comfortable in using Paper folding in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

19. Paper folding helps my academic performance in fractions

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

SECTION E *(questions on Computer assisted manipulative)*

20. My teacher uses Computer assisted manipulative in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

21. I have never used Computer assisted manipulative in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

22. I often use Computer assisted manipulative in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

23. I am comfortable in using Computer assisted manipulative in solving mathematical problems involving fractions.

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

24. Computer assisted manipulative helps my academic performance in fractions

Strongly Agree [ ] Agree [ ] Disagree [ ] Strongly Disagree [ ]

Thank you.

CONTROL GROUP

Background information

NOTE: Please do not write your name on the questionnaire.

Please use [x] to tick the appropriate answer.

SECTION A

1. Age: 8 – 10 [ ] 11 – 13 [ ] 14 – 16 [ ]
2. Gender: Male [ ] Female [ ]
3. Race: Balck [ ] White [ ] coloured [ ]
4. Grade: 9 [ ] 10 [ ] 11 [ ]

Thank you.

APPENDIX F

Ethical Clearance Certificate

University of Fort Hare

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ETHICAL CLEARANCE CERTIFICATE REC-270710-028-RA Level 01

Certificate Reference Number:	ADU011SADOM01
Project title:	**Effects of the use of manipulative materials on grade nine learners' performance in fractions in Public High Schools in Chris Hani West Education District, South Africa.**
Nature of Project	PhD in Education
Principal Researcher:	George Adom
Supervisor:	Prof E.O Adu
Co-supervisor:	N/A

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above- mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document;
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research.

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

Special conditions: *Research that includes children as per the official regulations of the act must take the following into account:*

Note: The UREC is aware of the provisions of s71 of the National Health Act 61 of 2003 and that matters pertaining to obtaining the Minister's consent are under discussion and remain unresolved. Nonetheless, as was decided at a meeting between the National Health Research Ethics Committee and stakeholders on 6 June 2013, university ethics committees may continue to grant ethical clearance for research involving children without the Minister's consent, provided that the prescripts of the previous rules have been met. This certificate is granted in terms of this agreement.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
  - Any unethical principal or practices are revealed or suspected;
  - Relevant information has been withheld or misrepresented;
  - Regulatory changes of whatsoever nature so require;
  - The conditions contained in the Certificate have not been adhered to.

- Request access to any information or data at any time during the course or after completion of the project.

- In addition to the need to comply with the highest level of ethical conduct principle investigators must report back annually as an evaluation and monitoring mechanism on the progress being made by the research. Such a report must be sent to the Dean of

Research's office.

The Ethics Committee wished you well in your research.

Yours sincerely

Professor Pumla Dineo Gqola

Dean of Research

25 July 2018

University of Fort Hare

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APPENDIX G

Language Editor certificate

BRUCE WESSON - NMMU Language Editor

8 Quay One
Mitchell Street
SOUTH END 6001

Email bruce@wesson.co.za
Mobile 082 555 5204

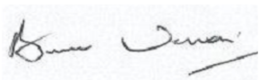
02 July 2020

TO WHOM IT MAY CONCERN

I, Bruce Wesson, hereby declare that I have proof read and copy edited the thesis paper given to me by George Adom, Student Number : 201509244

I declare that the content remains solely that of the student, and that any changes made, were only to language and grammar errors.

Yours faithfully,



BRUCE WESSON
NMMU Language Editor