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The Classification of Fuzzy Subgroups of some Finite Non-cyclic Abelian $p-$ Groups of rank 3, with emphasis on the number of distinct fuzzy subgroups

by

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DEDICATION

To Nana Kwadwo-Anyim Appiah and siblings and Lucy A.B Blay, may you be inspired to pursue excellence.

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ABSTRACT

In [6] and [7] we classified fuzzy subgroups of some rank-3 abelian groups of the form $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ for any fixed prime integer p and any positive integer n, using the natural equivalence relation defined in [40]. In this thesis, we extend our classification of fuzzy subgroups in [6] to the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ for any fixed prime integer p; $m = 2$ and any positive integer n using the same natural equivalence relation studied in [40]. We present and prove explicit polynomial formulae for the number of (i) subgroups, (ii) maximal chains of subgroups of G for any $n, m \geq 2$ and (iii) distinct fuzzy subgroups for $m = 2$ and $n \geq 2$. We have also developed user-friendly polynomial formulae for the number of (iv) subgroups, (v) maximal chains for the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ for any $n, m \geq 2$; any fixed prime positive integer p and (vi) distinct fuzzy subgroups of $\mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ for m equal to 2 and 3, and $n \geq 2$ and provided their proofs.

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DECLARATION

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Chapter 1

INTRODUCTION AND PRELIMINARIES

1.1 Introduction

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Fuzzy set theory and fuzzy logic, a mathematical way to represent vagueness in everyday life, were introduced by Zadeh in 1965 as a generalization of classical, Aristotelian and bivalent logic embroiled in scientific and technological investigation. Thus fuzzy set theory in general and fuzzy logic specifically are natural ways to model ambiguous events that occur in human-like reasoning. Ever since its discovery in [68], considerable facets of fuzzy subsets were studied. In 1971, Rosenfeld [56] brilliantly imported the notion of fuzzy sets to the domain of group theory and coined the notion of fuzzy subgroups of a group. Indeed, he showed how many of fundamental properties in group theory should be extended in a basic way to establish the theory of fuzzy groups. Though Anthony and Sherwood [5], redefined fuzzy groups, Rosenfeld's definition appears to be the most essential and accepted one. Based on the Rosenfeld's definition of fuzzy group, several fuzzy algebraic concepts were investigated and vigorous attempts have been made to "fuzzify" a number of important classical mathematical structures such as topological spaces, algebras, categories and groups and also to consider fuzzy automata, fuzzy programmes, fuzzy graphs, fuzzy probability, and so on. P. Das in[18] characterized some fuzzy subgroups of finite cyclic groups using what he termed "level subgroups" of a fuzzy subgroup which is based on the concept of a "level subset" introduced earlier by Zadeh [68]. Das's definition of level subgroup was transformed by Ajmal [1] by restricting $t \in \text{Im } \mu$. This modified definition of level subgroups has been used by Jain and Ajmal [33] to define a new category of fuzzy subgroups. Alkhamees [4] continued then with the investigation of fuzzy cyclic subgroups based on fuzzy cyclic p-groups. In a series of papers, Mukherjee and Bhattacharya [8], [9], [10], [11] have developed fuzzy analogs of a number of notions in classical group theory, and proved fuzzy generalizations of a couple of valuable theorems such as Lagrange's and Cayley's theorems, thereby enriching the theory of fuzzy groups. Bhattacharya [11] in 1987 showed that two fuzzy subgroups of a finite group with identical level subgroups are equal if and only if their image sets are equal. Further, Bhattacharya characterized all fuzzy subgroups of a finite group, thereby generalizing the earlier results by Rosenfeld [56] and P. Das[18]. In [56], Rosenfeld established that a homomorphic image of a fuzzy subgroup is a fuzzy subgroup if and only if the fuzzy subgroup has a sup-property. A homomorphic pre-image of a fuzzy subgroup is always a fuzzy subgroup. Subsequently, Anthony and Sherwood [5] showed that the homomorphic image of a fuzzy subgroup is a fuzzy subgroup irrespective of the sup-property. Similar work have been considered by other authors, see [59], [34], [3] and [57]. M.K. Chakraborty and M. Das [15], [16], [17] considered fuzzy relations in connection with fuzzy functions and equivalence relations. Murali and Makamba in a series of papers [40], [41],[42], [43] studied fuzzy relations and a natural equivalence relation on the class of all fuzzy subsets of a set. This equivalence relation was used by Murali and Makamba in [43] to determine the number of distinct fuzzy subgroups of Abelian groups of the order $p^n q^m$, where p and q are different primes. Zhang and Zou [71] considered a similar problem and obtained the number of fuzzy subgroups of cyclic groups of order p^n, where p is a prime number. There are other versions of equivalence that have been studied by other researchers, see for example Volf [65], Branimir and Tepavcevic [12], Degang et al [20], Tarnauceanu and Bentea [63], Ghafur and Sulaiman [60], Ajmal [2], Mordeson [37], Dixit et al [21], [22], Zhang and Zou [71], Jain [32], Tarnauceanu [62], Mashinchi and Mukaidonon [36] and Iranmanesh and Naraghi [31]. In this study, however it is not our intention to present all such equivalence and make any comparison, for more on comparison, see [46].

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Recent years however, have witnessed a substantial new movement in fuzzy research, with the focus shifting away from fuzzy group to classification of fuzzy subgroups of a finite group. These classifications, however have mostly centered around finite cyclic groups, see for example [41], [45], [64], [61],[30]. In [49], Ngcibi used a suitable equivalence relation defined in [40] to classify the distinct fuzzy subgroups of finite Abelian group of the form $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p$ for any fixed prime p and $n \in N$. Further, the same authors in [49], developed the general formulae for the number of maximal chains and the number of crisp subgroups. S. Ngcibi

[50], later used the same equivalence relation given in [40], to investigate and classify the number of distinct fuzzy subgroups of a finite abelian p- group G of rank two of the form $\mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ for any fixed prime p, positive integers $n \geq 1$ and $1 \leq m \leq 5$. In [51] J. M Oh attempted to perfect Ngcibis work by computing distinct fuzzy subgroups of $\mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ for any positive integers n and m, albeit using a different equivalence defined in [63].
Appiah and Makamba in [7], classified fuzzy subgroups of a rank-3 abelian group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ for any fixed prime p and any positive integer n, using a natural equivalence relation given in [40]. In [7], the authors further presented and proved explicit polynomial formulae for the number of (i) subgroups, (ii) maximal chains of subgroups, (iii) distinct fuzzy subgroups, (iv) non-isomorphic maximal chains of subgroups and (v) classes of isomorphic fuzzy subgroups of G. In fact, the classification of fuzzy subgroups of finite non-cyclic abelian p-groups has until now remained largely unexplored. The present thesis is another step towards filling this gap. In this thesis, we used the equivalence relations defined in [40] to review and enrich the studies conducted in [50] and then extend our previous work in [7] to the form $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$, where p is any fixed prime and any arbitrary positive integers n and $m = 2$.

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1.2 Preliminaries

In this section, a handful of key concepts of fuzzy set theory and fuzzy group theory is given with the view toward our classification of fuzzy subgroups of a finite non-cyclic abelian p-group.

1.2.1 On Fuzzy Sets

Suppose X is a set in an ordinary set theory, then the degree of membership of elements x in X can be defined by a function $f : X \to \{0, 1\}$ such that x is an element of X if and only if $f(x) = 1$. So equivalently and contrapositively, x is not an element of X if and only $f(x) = 0$. Thus the study of set X becomes the study of the membership (or characteristic) function of X. So we may call f a set. Now replace the co-domain $\{0, 1\}$ with the interval $[0, 1]$ and define a subset A of X by $f : X \to [0, 1]$ such that $f(x) \in \{0, 1\}$, for some x. Since $f(x) \neq 0$ and $\neq 1$, we cannot authoritatively say x is in A or x is not in A. In this case we say x is partially in A. The set A is an example of a fuzzy set, since some elements are partially (not fully) in A. We shall give an explicit definition of universe of discourse, fuzzy set, fuzzy subset, review the standard operations on fuzzy set with few examples and discuss some basic concepts in fuzzy set theory under this section. We shall also characterize fuzzy subgroups and give some basic concepts in fuzzy subgroups.

1.2.2 Operations on Fuzzy Sets

Since crisp sets and their associated characteristic functions may respectively be considered as unique cases of fuzzy sets and membership functions, we extend the set theoretic operations from elementary set theory to fuzzy sets. We observe that all those operations which are extensions of non-fuzzy (crisp) notions reduce to their usual meanings whenever the fuzzy sets have membership grades that are drawn from $\{0,1\}$. For this reason, when extending the operations to fuzzy sets, the same notation as in crisp set theory is used. Let β, η and κ be fuzzy sets on the universal set X.

The union: $\mu_\beta(a) \cup \mu_\eta(a) = max(\mu(a), \mu(a))$ for all $a \in X$

The intersection: $\mu_\beta(a) \cap \mu_\eta(a) = min(\mu(a), \mu(a))$ for all $a \in X$

Containment: The fuzzy set β is said to be contained in fuzzy set η if and only if for every $a \in X$ we have $\beta(a) \leq \eta(a)$.

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Theorem 1.2.1. *Let β and κ be two fuzzy subsets of X. If $\mu(x) = (\beta \cup \kappa)(x)$ and $\nu(x) = (\beta \cap \kappa)(x)$ for all $x \in X$, then:*

(a) $\nu \subset \mu$

(b) $\beta \subset \mu$ *and* $\kappa \subset \mu$.

(c) $\nu \subset \beta$ *and* $\nu \subset \kappa$.

Proof: See [67]□

The complement: $\mu_{-c}(a) = 1 - \mu(a)$ for all $a \in X$.

Equality: Two fuzzy sets β and η on the same universe of discourse X are said to be equal if and only if for all $a \in X$, we have $\mu_\beta(a) = \mu_\eta(a)$, see Zadeh [68].

Apart from the above expression, union and intersection of two fuzzy sets can be defined through T-conorm (or S-norm) and T-norm operators respectively. These two operations are functions S,T : $[0,1] \times [0,1] \to [0,1]$ satisfying some convenient boundary, monotonicity, commutativity and associativity properties. As introduced by Zadeh [68], a more intuitive but equivalent definition of union of fuzzy set is the "smallest" fuzzy set containing both η

and κ. Analogously, the intersection of η and κ is the "largest" fuzzy set which is contained in both η and κ.

Note 1.2.2.1. More generally, the intersection of a fuzzy set and its complement is not the empty fuzzy set (whose membership set contains only the number 0) as it is in elementary set theory unless the fuzzy set is a crisp subset, see [47].

Note 1.2.2.2. The union of a fuzzy set and its complement is not the universal set X.

Note 1.2.2.3. With the exception of the axiom of the excluded middle and the axiom of contradiction which are in variance with fuzzy sets, the remaining properties such as associativity, distributivity, idempotency, identity as well as the De Morgans's laws hold on fuzzy sets.

1.2.3 Some Basic Concepts in Fuzzy Set Theory

Here we present some basic concepts in fuzzy set theory that are desirable:

Definition 1.2.1. A fuzzy set $\emptyset$ of X is said to be an empty fuzzy set if for each $x \in X$, we have $\emptyset(x) = 0$.

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Definition 1.2.2. A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except for one and only one element of X, say $x \in X$.

Definition 1.2.3. If $\beta(x) = 0.5$, for a certain point $x \in X$, then such a point is called a crossover point of a fuzzy set β.

Definition 1.2.4. The core of a fuzzy set β, denoted by core(β), is the set of all points $x \in X$ such that $\beta(x) = 1$.

Definition 1.2.5. Let β be a fuzzy set of X. The support of β, denoted supp(β), is the crisp subset of X whose elements all have nonzero membership grades in β. Thus supp$(\beta) = \{x \in X : \beta(x) > 0\}$.

Definition 1.2.6. The co-support of β denoted by co-supp(β), is a non-fuzzy set which consists of all elements that are completely found outside a given fuzzy set and is given by co-supp$(\beta) = \{x \in X : \beta(x) = 0\}$

Definition 1.2.7. A fuzzy set Ψ whose support is a single point in X with $\Psi(x) = 1$, is referred to as a fuzzy singleton.

Definition 1.2.8. A weak α-cut set or an α level-set of S denoted by S_α is a non-fuzzy set which is made up of members whose membership grades are not less than α and is given by $S_\alpha = \{x \in X : \mu_S(x) \geq \alpha\}$, where $\alpha \in (0, 1]$.

Definition 1.2.9. If $S^\alpha = \{x \in X : \mu_S(x) > \alpha\}$, for $\alpha \in [0, 1]$, then S^α is referred to as a strong α-cut.

Proposition 1.2.2. *Let $\beta, \eta \in I^X$, then*

1 . $\beta = \eta$ if and only if $\beta^\alpha = \eta^\alpha$ for all $\alpha \in I$.

2 . $\beta = \eta$ if and only if $\beta_\alpha = \eta_\alpha$ for all $0 < \alpha < 1$.

Proof:. See [35]□

Corollary 1.2.3. *Let S be a fuzzy subset of X. Then*

1 . $S_\alpha = X$ whenever $\alpha = 0$

2 . $S^\alpha = \emptyset$ whenever $\alpha = 1$

Definition 1.2.10. Let β be a fuzzy subset of the set X. Then β is said to be normal if $\sup\beta(x) = 1$, for $x \in X$.

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1.2.4 Characterization of Images and Pre-images of Fuzzy Sets

Let X and Y be two non-empty sets and f a mapping from X into Y. For fuzzy subsets $\lambda, \lambda_k \subseteq X$ and $\mu, \mu_k \subseteq Y$, define the image $f(\lambda)$ of λ under f for $y \in Y$ as

$$f(\lambda)(y) = \begin{cases} sup\{\lambda(x) : x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{, Otherwise} \end{cases}$$

The pre-image denoted by $f^{-1}(\mu)$ of μ under f is the fuzzy subset of X such that, for $x \in X$, $f^{-1}(\mu)(x) = \mu(f(x))$. Thus $f^{-1}(\mu)$ consists of precisely all the elements of λ that are mapped to elements of μ by f.

Definition 1.2.11. [56]. A fuzzy set β in X is said to be f-invariant if $\beta(a_1) = \beta(a_2)$ whenever $f(a_1) = f(a_2)$ for all $a_1, a_2 \in X$.

Definition 1.2.12. Let X be any given set. A fuzzy subset $\beta : X \to [0, 1]$ of X has the sup property if for any subset S of the set X there exists $x_0 \in S$ such that $\beta(x_0) = sup\{\beta(x) : x \in S\}$.

1.2.5 On Fuzzy Subgroups

First of all, we present some basic concepts and results on fuzzy subgroups.

Definition 1.2.13. [56]. Let υ be a fuzzy set on a group G. Then υ is said to be a fuzzy subgroup of G if for all $a, b \in G$, we have

1 . $\upsilon(ab) \geq \min\{\upsilon(a), \upsilon(b)\}$

2 . $\upsilon(a^{-1}) = \upsilon(a)$.

For the identity element $e \in G$, we have $\mu(a) \leq \mu(e)$, for all $a \in G$.

Definition 1.2.14. [9]. If μ is a fuzzy subgroup on a group G and ϑ is a map from G onto itself, we define a map $\mu^{\vartheta} : G \to [0, 1]$ by $\mu^{\vartheta}(g) = \mu(g^{\vartheta})$, for all $g \in G$ where g^{ϑ} is the image of g under ϑ.

Definition 1.2.15. [39]. We define $\mu \circ \mu(g) = \sup_{g=g_1g_2} (\mu(g_1) \wedge (\mu(g_2))$.

Proposition 1.2.4. *A fuzzy subset μ of G is a fuzzy subgroup of G if and only if*

(a) $\mu \circ \mu \leq \mu$ and

(b) $\mu^{-1} = \mu$ where μ^{-1} is defined as $\mu^{-1} : G \to I$, for all $g \in G, \mu^{-1}(g) = \mu(g^{-1})$.

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Proof. See [45]□

1.2.6 On Level Subgroups

As an enrichment of the notion of level subsets studied by Zadeh's[68], Das in [18] characterized level subgroups of a fuzzy group. Since then, several properties of fuzzy groups have been defined by using Das' level subgroups, hence it has become one of the essential tools used in the study of fuzzy groups.

We first define level subset, state a theorem and then proceed with the Das's definition of level subgroup, state theorem, corollary and provide their proofs. We also give the definition of level subgroup revised by Ajmal[1] and Jain [32].

Recall from definitions1.2.8 and 1.2.9 : Let μ be a fuzzy subset of S and $t \in [0, 1]$. Then $\mu_t = \{s \in S : \mu(s) \geq t\}$ is called the level subset of μ at t.

Theorem 1.2.5. *[18]. Let G be a group and μ be a fuzzy subgroup of G, then the level subset μ_t, for $t \in [0, 1]$, $t \leq \mu(e)$, is a subgroup of G, where e is the identity of G.*

Proof. See [18]□

Definition 1.2.16. [18]. Let μ be a fuzzy subgroup of the group G and $0 \leq t \leq \mu(e)$, then μ_t is referred to as the level subgroup of μ at t.

Theorem 1.2.6. *[18] Theorem 3.1*
Let G be a group and μ be a fuzzy subgroup of G. Two level subgroups μ_{t_1}, μ_{t_2}(with $t_1 < t_2$) of μ are equal if and only if there is no $x \in G$ such that $t_1 < \mu(x) < t_2$.

Note 1.2.6.1. Only one implication of the above theorem is true, its converse is false as the next example shows.

Example 1.2.1. *Let $G = S_3 = \{e, a, a^2, b, ab, a^2b\}$ such that $a^3 = e = b^2$, be the symmetric group on 3 symbols. Define a fuzzy subset of S_3 as follows:* $\mu(x) = \begin{cases} 1 & \text{if} \quad x = e \\ \frac{1}{5} & \text{if} \quad x = b \\ \frac{1}{7} & \text{otherwise} \end{cases}$

Now no x in G exists such that $\frac{1}{7} < \mu(x) < \frac{1}{5}$, but $\mu_{\frac{1}{5}} = \{e, b\} \neq \mu_{\frac{1}{7}} = S_3$. This is contrary to the above theorem. Hence the theorem should read

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Theorem 1.2.7. *Let G be a group and μ be a fuzzy subgroup of G. If two level subgroups μ_{t_1}, μ_{t_2}(with $t_1 < t_2$) of μ are equal, then there is no $x \in G$ such that $t_1 < \mu(x) < t_2$.*

Corollary 1.2.8. *[18]Corollary 3.1*
Let G be a finite group of order n and μ be a fuzzy subgroup of G.
Let $Im(\mu) = \{t_i | \mu(x) = t_i$ for some $x \in G\}$. Then $\{\mu_{t_i}\}$ are the only level subgroups of μ.

Proof: see [18]

Proposition 1.2.9. *[57] Prop 2.1. Let μ be a fuzzy subgroup of G with $Im(\mu) = \{t_j : j \in J\}$ and $f = \left\{\mu_{t_j} : j \in J\right\}$ where J is an arbitrary index set. Then:*

(a) there exists a unique $j_0 \in J$ such that $t_{j_0} \geq t_j$, for every $j \in J$,

(b) $\mu_{t_{j_0}} = \bigcap_{j \in J} \mu_{t_j}$,

(c) $G = \bigcup_{j \in J} \mu_{t_j}$,

(d) the members of f form a chain.

Proof. See [57].□

Ajmal in [1] (see also Jain [32]) revised this definition of level subgroup by restricting $\alpha \in \text{Im}$ μ and gave the following,

Definition 1.2.17. [1]. The level subgroup of G, $\mu_t^> = \{x \in G : \mu(x) > t, t \in \text{Im } \mu\}$

1.2.7 On Homomorphic Images and Pre-Images of Fuzzy Subgroups

We present some propositions on homomorphic images and pre-images of fuzzy subgroups from the work studied earlier by Rosenfeld and that of S. Sebastian and S. B. Sundar

Proposition 1.2.10. *[56] Prop 5.8*
A homomorphic image or pre-image of a fuzzy subgroup is a fuzzy subgroup.

Proof. See [56]□

Theorem 1.2.11. *. Let $f : G \to G$ be a homomorphism of G into G. If μ is a fuzzy subgroup of G, then the image $f(\mu)$ of μ under f is a fuzzy subgroup of G.*

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Proof. See [57]□

Proposition 1.2.12. *. Let $f : G \to G'$ be a homomorphism and μ a fuzzy subgroup of a group G'. Then the pre-image $f^{-1}(\mu)$ of μ under f is a fuzzy subgroup of G.*

Proof. See [57]□

1.2.8 On Normal Fuzzy Subgroups

Definition 1.2.18. [57]. Let μ be a fuzzy subgroup of a group G. Then a fuzzy subgroup μ of the group G is said to be a fuzzy normal subgroup if for all $x, y \in G$, we have $\mu(xy) = \mu(yx)$.

Equivalently μ is fuzzy normal if and only if $\mu(xyx^{-1}) = \mu(y)$, for all $x, y \in G$.

Proposition 1.2.13. *[57]. Let μ be a fuzzy normal subgroup of G. Let $f : G \to G'$ be a homomorphism where G' is a group. Then the image $f(\mu)$ of μ under f is fuzzy normal in $f(G)$.*

Proof. See [57]□

1.2.9 On Fuzzy Cosets

Definition 1.2.19. [38]. If μ is a fuzzy subgroup of a group G, then for any $x \in G$, we define a left fuzzy coset of μ, denoted $x\mu$, as the fuzzy subset G defined by $(x\mu)(y) = \mu(x^{-1}y)$ for all $y \in G$. A right fuzzy coset of μ is also defined by $(\mu x)(y) = \mu(yx^{-1})$.

If μ is fuzzy normal, then the set $G_\mu = \{x\mu : x \in G\}$ is a group under the binary operation defined by $(x\mu)(y\mu) = (xy)\mu, \forall\ x, y \in G$. Besides, we also have $x\mu = \mu x$ for all $x \in G$.. (See [38], Proposition 4.3 and Theorem 4.5).

Proposition 1.2.14. *[38]. Let μ be a fuzzy subgroup of G. Then μ is fuzzy normal if and only if $x\mu = \mu x$ for all $x \in G$.*

Proof. See [38]□

1.2.10 On Fuzzy Conjugate

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Definition 1.2.20. [9]. Let G be a group. If μ is a fuzzy subgroup of G and $x \in G$, then the fuzzy subset β of G defined by $\beta(g) = \mu(x^{-1}gx)$, for all $g \in G$ is called the fuzzy conjugate of μ determined by x.

Theorem 1.2.15. *[9]. A fuzzy subgroup μ of a group G is a fuzzy normal subgroup if and only if μ is constant on the conjugate classes of G.*

Proof. See [9]□

1.2.11 On Fuzzy Abelian

Definition 1.2.21. [10]. Let G be a group and let μ be a fuzzy subgroup of G, then μ is said to be fuzzy abelian if G_μ is an abelian subgroup of G.

Proposition 1.2.16. *[10]. A non-empty subset A of G is an abelian subgroup of G if and only if X_A is a fuzzy abelian subgroup of G.*

Proof. See [10]□

1.2.12 Fuzzy Equivalence and Fuzzy Isomorphism

On General Equivalence Relation

One of the elementary facts in science, which is not hard to verify, is that huge systems can best be analysed by arranging them into cells such that each cell consists of elements displaying similar prperties. This leads to the notion of equivalence relation and isomorphism in mathematics, where things with similar properties are normally put together in a class so that one can then critically examine and study a simpler representative of the class.

Definition 1.2.22. [23]. Let $\mathcal{S}$ be a collection of nonempty subsets of an elementary set A. Then $\mathcal{S}$ is said to be a *partition* of A if

(1) $S \cap S^{'} = \emptyset$, for any distinct S and $S^{'}$ in $\mathcal{S}$, and

(2) $A = \bigcup \{S|S \in \mathcal{S}\}$.

Definition 1.2.23. . Given two nonempty sets X and Y, a relation between X and Y is a subset $\Re \subseteq X \times Y$. For a relation $\Re \subseteq X \times Y$ and $x \in X$, $y \in Y$, if $(x, y) \in \Re$, we write $x\Re y$ (we say x is $\Re$-related to y).

Definition 1.2.24. . A binary relation on a nonempty set X is a relation $\Re$ between X and X, that is, a subset $\Re \subseteq X \times X$.

Definition 1.2.25. [23]. A relation $\Re$ on a nonempty set A is an equivalence relation if and only if $\Re$ is

(i) reflexive, i.e $(a, a) \in \Re$,for all $a \in A$

(ii) symmetric, $(a, b) \in \Re \Rightarrow (b, a) \in \Re$ and

(iii) transitive, $(a, b) \in \Re$ and $(b, c) \in \Re \Rightarrow (a, c) \in \Re$.

Definition 1.2.26. [26].

(i) If $\Re$ is an equivalence relation on a nonempty set A, then for an element $a \in A$, the set $[a] = \{x|a\Re x\}$ is called the equivalence class of a.

(ii) The element a in the bracket above is called a representative of the equivalence class.

On Fuzzy Relation

In [68], Zadeh introduced the concepts of fuzzy relation and fuzzy similarity relation and further followed it with the notion of fuzzy equivalence class in [69] and [70], as a natural characterization of the concept of a crisp equivalence class. Murali in [39] defined a fuzzy equivalence relation on a set and observed that there is a correspondence between fuzzy equivalence relations and certain classes of fuzzy sets. De Baets et al in [19] and Ovchinnikov et al in [52] studied fuzzy equivalence relations in terms of fuzzy partitions. We give the following definition by Murali in [39]

Definition 1.2.27. Let X and Y be two universal sets. A fuzzy relation on $X \times Y$ denoted by $\Re(a, b)$ or $\Re$ is defined as the set $\Re$ characterized by the membership function $\mu_{\Re}(a, b)$ where $\Re = \{((a, b), \mu_{\Re}(a, b))|(a, b) \in X \times Y,$
$\mu_{\Re}(a, b) \in [0, 1]\}$.

Note 1.2.12.1. (1) Since the relation $\Re$ defined above is a binary relation, it is said to be:

(i) Reflexive if $\mu_{\Re}(a, a) = 1,$ for all $a \in X$

(ii) Symmetric if $\mu_{\Re}(a, b) = \mu_{\Re}(b, a),$ for all $a, b \in X$

(iii) Transitive if $\mu_{\Re} \circ \mu_{\Re} \leq \mu_{\Re}$ where $\mu_{\Re} \circ \mu_{\Re}$ is defined by $\mu_{\Re} \circ \mu_{\Re}(a, b) = sup_{c \in X}(\mu_{\Re}(a, c) \wedge \mu_{\Re}(c, b))$

If the conditions (i), (ii) and (iii) hold in a fuzzy relation on a set X, then such a relation is called a fuzzy equivalence relation on X.

(2) Because fuzzy relations are fuzzy sets in product space, set theoretic operations such as union, intersection and complement can be defined for them.

The Fuzzy Partition Of A Fuzzy Subset

Definition 1.2.28. . Let X be a nonempty set and let μ be a fuzzy subset of a set X. Then $\sum = \{\kappa : \kappa$ is a fuzzy subset of a set X and $\kappa \subseteq \mu\}$ is said to be a fuzzy partition of μ if

(a) . $\bigcup_{\kappa \in \sum} \kappa = \mu$ and

(b) . any two members of $\sum$ are either identical or disjoint

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Equivalence Relation on Fuzzy Subgroups

Definition 1.2.29. . A fuzzy relation μ on a group G is said to be a fuzzy equivalence relation on G if:

(i) $\mu(x,x) = 1, \forall x \in G$

(ii) $\mu(x,y) = \mu(y,x)$, $\forall x, y \in G$

(iii) $\mu \circ \mu \leq \mu$

Definition 1.2.30. [40]. Let μ and ν be any fuzzy sets in I^X where $I = [0,1]$ and a nonempty set X. We define an equivalence relation $\sim$ on I^X as follows: $\mu \sim \nu$ if and only if for all $a, b \in X$, $\mu(a) > \mu(b)$ if and only if $\nu(a) > \nu(b)$ and $\mu(a) = 0$ if and only if $\nu(a) = 0$.

Note 1.2.12.2. The condition $\mu(a) = 0$ if and only if $\nu(a) = 0$ simply means that the supports of μ and ν are equal.
In the condition of equivalence relation $\mu \sim \nu$, the strict inequality can be replaced by $\geq$. Thus $\mu \sim \nu$ if and only if $\forall\ a, b \in X$, $\mu(a) \geq \mu(b)$ if and only if $\nu(a) \geq \nu(b)$ and $\mu(a) = 0$ if and only if $\nu(a) = 0$. It is easily seen that either of the inequalities in the definition determines the same equivalence class of fuzzy sets.

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The condition $\mu(a) = 0$ if and only if $\nu(a) = 0$ is an important part of the equivalence relation as the following example illustrates.

Example 1.2.2. *Consider* $D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ *where* $a^4 = b^2 = (ab)^2 = e$ *for* e *an identity element of the group. Define the fuzzy sets* μ *and* ν *on* D_4 *as follows:*

$$\mu(x) = \begin{cases} 1 & if \quad x = e \\ \frac{1}{7} & if \quad x = a, a^2, a^3 \\ \frac{1}{9} & otherwise \end{cases}$$

and

$$\nu(x) = \begin{cases} 1 & if \quad x = e \\ \frac{1}{7} & if \quad x = a, a^2, a^3 \\ 0 & otherwise \end{cases}$$

It can be seen that the
$\text{Supp}(\mu) \neq \text{Supp}(\nu)$ even though $\mu(x) > \mu(y)$ if and only if $\nu(x) > \nu(y)\ \forall\ x, y \in D_4$.

Note 1.2.12.3. If $\mu \sim \nu$, then $|\text{Im}(\mu)| = |\text{Im}(\nu)|$.

The converse is not true, thus if $|\text{Im}(\mu)| = |\text{Im}(\nu)|$ or even if $\text{Im}(\mu) = \text{Im}(\nu)$ and $\text{Supp}(\mu) = \text{Supp}(\nu)$, it is not necessary to have $\mu \sim \nu$, as verified by the example below.

Example 1.2.3. *Let $S_3 = \{e, a, a^2, b, ab, a^2b\}$ generated by a and b where $a^3 = e = b^2$ and e the identity element.*

Define fuzzy sets μ and ν as follows:

$$\mu(x) = \begin{cases} 1 \quad if & x = e \\ \\ \frac{1}{7} \quad if & x = b \\ \\ \frac{1}{9} \quad otherwise \end{cases} .$$

and

$$\nu(x) = \begin{cases} 1 \quad if & x = e \\ \\ \frac{1}{7} \quad if & x = ab \\ \\ \frac{1}{9} \quad otherwise \end{cases} .$$

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Observe that in the above example,

$\text{Im}(\mu) = \text{Im}(\nu)$ and $\text{Supp}(\mu) = \text{Supp}(\nu) = S_3$

However, $\mu(b) > \mu(ab)$ but $\nu(b) \not> \nu(ab)$. Therefore μ is not equivalent to ν.

Proposition 1.2.17. . *Let μ and ν be two fuzzy subsets of X. Suppose for each $t > 0$ there exists an $s > 0$ such that $\mu^t = \nu^s$. Then $\mu \sim \nu$.*

Proof. See [40]□

On Fuzzy Isomorphism

An isomorphism of fuzzy groups was first defined by Ray in [55]. There after, Murali and Makamba [40] enriched Ray's definition with another version of fuzzy isomorphism. In this thesis, we used the fuzzy isomorphism defined in [40]. The same authors in [44] studied the relationships among various notions of isomorphism. They compared this notion of isomorphism to that of an equivalence relation of fuzzy subgroups of finite groups. They discovered that this notion of equivalence is finer than the notion of isomorphism. Therefore this definition of fuzzy isomorphism is a generalisation of the definition in [40] of fuzzy equivalence. For an overview on isomorphic fuzzy groups and isomorphism between finite chains of subgroups, see Pruszyriska and Dudzicz in [54] and Ray [55], Murali and Makamba [44]. For completeness, we begin by defining a group homomorphism.

Definition 1.2.31. [27]. Let (G, θ) and $(G^{'}, \circ)$ be groups. A homomorphism is a mapping $f : G \to G'$ such that $f(a\theta b) = f(a) \circ f(b)$, $\forall a, b \in G$.

Note 1.2.12.4. If $f : G \to G^{'}$ is a homomorphism, by $f(\mu)$ we mean the image of a fuzzy subset μ of G and is a fuzzy subset of $G^{'}$ defined by $(f(\mu))(g^{'})$=$\sup\{\mu(g) : g \in G, f(g) = g^{'}\}$ if $f^{-1}(g') \neq \emptyset$ and $f(\mu)(g^{'}) = 0$ if $f^{-1}(g^{'}) = \emptyset$ for $g^{'} \in G^{'}$. Similarly if ν is a fuzzy subset of $G^{'}$, the pre-image of ν, $f^{-1}(\nu)$ is a fuzzy subset of G defined by $(f^{-1}(\nu))(g) = \nu(f(g))$.

Employing the above definition, we now define a mapping that preserves both structure and group operation.

Definition 1.2.32. [45]. An isomorphism is a homomorphism that is bijective.

Murali and Makamba in [40] gave the following definition of isomorphic fuzzy subgroups:

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Note 1.2.12.5. If H is a group, then $F(G)$ is the family of all fuzzy subgroups of G.

Definition 1.2.33. . Let $\mu \in F(G)$ and $\nu \in F(G^{'})$. We say μ is fuzzy isomorphic to ν, denoted by $\mu \cong \nu$, if and only if there exists an isomorphism $f : G \to G^{'}$ such that $\mu(x) > \mu(y) \Leftrightarrow \nu(f(x)) > \nu(f(y))$ and $\mu(x) = 0 \Leftrightarrow \nu(f(x)) = 0$.

Proposition 1.2.18. *For f as in definition1.2.33, if $\mu \cong \nu$ then $f(\mu) \cong f(\nu)$.*

Proposition 1.2.19. *For f as in definition1.2.33, if $\mu \cong \nu$ in $G^{'}$ then $f^{-1}(\mu) \cong f^{-1}(\nu)$ in G.*

Proof. Straightforward.□

1.2.13 On Keychains

Here, we reproduce some basic definitions, and a handful of pertinent known results associated with the concept of keychain. This concept, inherently resulted in the study of fuzzy subsets of a finite set X, with membership grades of elements of X considered within the unit interval.

Definition 1.2.34. By [41]. A collection of real numbers on $[0, 1]$ of the form

$1 > \lambda_1 > \lambda_2 \cdots > \lambda_{n-1} > \lambda_n$, where the last entry may or may not be zero is called a finite n-chain. This is usually expressed in the descending order as $1\lambda_1\lambda_2 \cdots \lambda_n$.

Definition 1.2.35. [41]. The numbers $1, \lambda_1, \cdots, \lambda_{n-1}, \lambda_n$ are referred to as pins. Observe that 1 occupies the first position whilst λ_i occupies the (i + 1)th position for $i = 1, 2, 3 \cdots, n$. Thus the length of an n-chain is $n + 1$, and hence the n-chain contains $n + 1$ available positions.

Definition 1.2.36. . An n-chain is called a keychain if $1 \geq \lambda_1 \geq \lambda_2 \geq \lambda_3 \cdots$

$\geq \lambda_{n-1} \geq \lambda_n$.

Note 1.2.13.1. The pins in a keychain may either be distinct or not. The keychain with only pins 1 and 0 can be considered as a crisp set.

Definition 1.2.37. [42]. A flag on a finite group G is an increasing maximal chain of $n + 1$ subgroups of G starting with the trivial subgroup $\{0\}$.

Definition 1.2.38. [42]. Let ξ be a flag on G and ι be a keychain. The pair $\{\xi, \iota\}$ is called a pinned-flag and we represent it as follows: $0^1 \subset G_1^{\lambda_1} \subset G_2^{\lambda_2} \subset G_3^{\lambda_3} \subset \cdots \subset G_n^{\lambda_n}$.

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Chapter 2

ON CRISP SUBGROUPS, MAXIMAL CHAINS AND DISTINCT FUZZY SUBGROUPS OF FINITE ABELIAN p-GROUPS $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

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In Chapter1, we presented two important sections viz: an introduction section1.1, containing a handful of earlier researches conducted in the domain of fuzzy set theory and a preliminary section1.2, where we discussed some basic concepts of fuzzy set theory, fuzzy relation, fuzzy isomorphism and keychain. In this chapter, we present a concise review of the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ studied in [7]. Understanding how to classify fuzzy subgroups of this particular type of group, is a key ingredient to developing a principled approach in classifying fuzzy subgroups of the group. Before proceeding to classify the distinct fuzzy subgroup of the group, it is convenient to start such discussion with how to find the number of crisp subgroups and maximal chains of group G. Additionally, we shall also give the number of non-isomorphic classes of G to conclude this chapter.

2.1 On Crisp Subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

Theorem 2.1.1. *The number of subgroups of the group* $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ *for* $n \geq 1$ *is* $p(2pn + n + 1) + 3 + n$ *for any prime number* $p \geq 2$.

Proof: See [7]□

2.2 On Maximal Chains of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

Next, we discuss the concept of maximal chain. This old concept has proved to be one of the most accurate and powerful tools to determine the total number of fuzzy subgroups in a group, especially finite non-cyclic ones. After giving the precise definition of maximal chain, we look at some examples. Also we give a handful of results in the form of lemmas.

Definition 2.2.1. . A chain of subgroups of the group G is said to be maximal if it cannot be refined any more or no more subgroups can be inserted in the chain.

Definition 2.2.2. [45]. In a maximal chain δ on G, a subgroup that distinguishes the maximal chain from others is called a distinguishing factor. If there is more than one such subgroups, they are called distinguishing factors.

Examples of Maximal Chains for $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$
Here we construct and count the maximal chains of subgroups of the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ manually using the information of the previous sections. We start with specific cases and then explain our computation in the proofs of the proposition and the theorem that follow. Let us construct the maximal chains for the group $G = \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ for $p = 2, 3, 5, 7$ in that order.
For $p = 2$, the group $\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2$ has the following 21 maximal chains:

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Example 2.2.1. $\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \mathbb{Z}_2 + \{0\} \supseteq \mathbb{Z}_2 + \{0\} + \{0\} \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \mathbb{Z}_2 + \{0\} \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \mathbb{Z}_2 + \{0\} \supseteq \{0\} + \mathbb{Z}_2 + \{0\} \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \{0\} + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \{0\} + \{0\} \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \{0\} + \mathbb{Z}_2 \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \mathbb{Z}_2 + \{0\} + \mathbb{Z}_2 \supseteq \{0\} + \{0\} + \mathbb{Z}_2 \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \{0\} + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \{0\} + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \{0\} + \mathbb{Z}_2 + \{0\} \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \{0\} + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq \{0\} + \{0\} + \mathbb{Z}_2 \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$
$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2 \supseteq < (1,1,0), (1,0,1) > \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,0),(1,1,1) > \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,0),(1,1,1) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,0),(1,1,1) > \supseteq \{0\}+\{0\}+\mathbb{Z}_2 \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(0,1,1) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(0,1,1) > \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(0,1,1) > \supseteq \mathbb{Z}_2+\{0\}+\{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_2+\mathbb{Z}_2+\mathbb{Z}_2 \supseteq < (1,1,1),(1,0,1) > \supseteq \{0\}+\mathbb{Z}_2+\{0\} \supseteq \{(0,0,0)\}$

For $p=3$, the group $G=\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3$ has the following 52 maximal chains:

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\mathbb{Z}_3+\{0\} \supseteq \mathbb{Z}_3+\{0\}+\{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\mathbb{Z}_3+\{0\} \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\mathbb{Z}_3+\{0\} \supseteq < (1,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\mathbb{Z}_3+\{0\} \supseteq \{0\}+\mathbb{Z}_3+\{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\{0\}+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\{0\}+\{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\{0\}+\mathbb{Z}_3 \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\{0\}+\mathbb{Z}_3 \supseteq < (1,0,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \mathbb{Z}_3+\{0\}+\mathbb{Z}_3 \supseteq \{0\}+\{0\}+\mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\{0\}+\mathbb{Z}_3 \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\mathbb{Z}_3+\{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq \{0\}+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (0,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,1,1) > \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,1,1) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,1,1) > \supseteq < (1,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,1,1) > \supseteq \{0\}+\{0\}+\mathbb{Z}_3 \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,1) > \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,1) > \supseteq < (1,2,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,1) > \supseteq < (1,0,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,1) > \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,2) > \supseteq < (1,1,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,2) > \supseteq < (1,2,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3+\mathbb{Z}_3+\mathbb{Z}_3 \supseteq < (1,1,0),(1,2,2) > \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,0), (1,2,2) > \supseteq < (0,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,0) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,0) > \supseteq < (1,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,0) > \supseteq < (1,0,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,0) > \supseteq < (0,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,1) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,1) > \supseteq < (1,2,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,1) > \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,1) > \supseteq \{0\} + \mathbb{Z}_3 + \{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,2) > \supseteq < (1,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,2) > \supseteq < (1,2,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,2) > \supseteq \mathbb{Z}_3 + \{0\} + \{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,1), (1,2,2) > \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,0) > \supseteq < (1,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,0) > \supseteq < (1,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,0) > \supseteq < (1,0,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,0) > \supseteq < (0,1,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,1) > \supseteq < (1,1,2) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,1) > \supseteq < (1,2,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,1) > \supseteq \mathbb{Z}_3 + \{0\} + \{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,1) > \supseteq < (0,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,2) > \supseteq < (1,1,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,2) > \supseteq < (1,2,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,2) > \supseteq < (1,0,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,1,2), (1,2,2) > \supseteq \{0\} + \mathbb{Z}_3 + \{0\} \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,2,0), (1,2,1) > \supseteq < (1,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,2,0), (1,2,1) > \supseteq < (1,2,1) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,2,0), (1,2,1) > \supseteq < (1,2,2) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_3 + \mathbb{Z}_3 + \mathbb{Z}_3 \supseteq < (1,2,0), (1,2,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

Next we consider the maximal chains of subgroups of $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p$.

For $p = 3$, the group $\mathbb{Z}_{3^2} + \mathbb{Z}_3 + \mathbb{Z}_3$ has 136 maximal chains. We do not list the chains here.

For $p = 5$, the group $\mathbb{Z}_{5^2} + \mathbb{Z}_5 + \mathbb{Z}_5$ has 516 maximal chains.

For $p = 7$ and $n = 2$ the group $G = \mathbb{Z}_{7^2} + \mathbb{Z}_7 + \mathbb{Z}_7$ has 1296 maximal chains. The above observations lead to

Proposition 2.2.1. *The group $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p$ has $(p+1) + (3p+2)(p^2+p)$ maximal chains for any given prime number p.*

Proof: See [7]□

We also considered maximal chains of the group $G = \mathbb{Z}_{p^k} + \mathbb{Z}_p + \mathbb{Z}_p$ for $k = 3, 5$ and 7. The observed patterns suggest

Theorem 2.2.2. *For any positive integer n and any prime p, the number of maximal chains of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ is $(p+1) + (p^2+p)[\frac{n(n+1)}{2}p + n]$.*

Proof: See [7]□

2.3 On Distinct Fuzzy Subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

Proposition 2.3.1. *The number of distinct fuzzy subgroups of $G = \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ is equal to $2^4 - 1 + 2^3(2p^2 + 2p) + 2^2p^3$.*

Proof: See [7] □

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Proposition 2.3.2. *The number of distinct fuzzy subgroups of $G = \mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p$ is equal to $2^5 - 1 + 2^4(4p^2 + 3p) + 2^3(3p^3 + p^2)$.*

Proof: See [7]□

Theorem 2.3.3. *The number of distinct fuzzy subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ is equal to $2^{n+3} - 1 + 2^{n+2}[p(2pn + n + 1)] + 2^{n+1}[p + 1 + (p^2+p)(\frac{n(n+1)}{2}p + n) - p(2pn + n + 1) - 1]$.*

Proof: See [7]□

2.4 On Isomorphic and Non-isomorphic Fuzzy Subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

Recall: 1. Two fuzzy subgroups μ and ν of a group G are isomorphic if there is an isomorphism $f : G \to G$ such that for $x, y \in G$, $\mu(x) > \mu(y)$ if and only if $\nu(f(x)) > \nu(f(y))$ and and $\mu(x) = \mu(y)$ if and only if $\nu(f(x)) = \nu(f(y))$.

If the two fuzzy subgroups are not isomorphic, then they are non-isomorphic.

2. Two maximal chains of a group G are isomorphic if they have the same length and corresponding subgroups in the chains are isomorphic.

For example if $G = \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$, then all the maximal chains of G are isomorphic. When computing the number of non-isomorphic classes of fuzzy subgroups, it suffices to collapse all isomorphic maximal chains into one chain and then use the techniques of distinct fuzzy subgroups to calculate the number of non-isomorphic classes of fuzzy subgroups. Thus in $G = \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$, the number of non-isomorphic classes of fuzzy subgroups is $2^4 - 1 = 15$.

For more on isomorphism, see [44], [54], [55].

The Fundamental Theorem of finitely generated abelian groups, see for example Fraleigh [23], is useful in deciding which maximal chains of subgroups are isomorphic.

For an illustrative example on how to compute isomorphic classes of fuzzy subgroups of a group G, see [6].

2.4.1 On Non-isomorphic Maximal Chains of Subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

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Theorem 2.4.1. *The number of non-isomorphic maximal chains of subgroups of the group* $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ *is equal to* $\frac{n(n+1)}{2}$.

Proof: see [7]□

2.4.2 On Non-isomorphic Fuzzy Subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$

Theorem 2.4.2. *The number of non-isomorphic fuzzy subgroups of the group* $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ *is equal to* $2^{n+3} - 1 + 2(n-1)2^{n+2} + \frac{(n-1)(n-2)}{2}2^{n+1}$.

Chapter 3

ON CRISP SUBGROUPS, MAXIMAL CHAINS AND DISTINCT FUZZY SUBGROUPS OF FINITE ABELIAN p-GROUPS $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$

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After a brief discussion of some basic concepts of fuzzy set theory, fuzzy relation and fuzzy isomorphism in the preliminary section1.2, we now turn our attention towards the classification of fuzzy subgroup of a finite non-cyclic abelian p-group of the form $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$. This topic, even though similar to the study conducted in [50], the approach used in this study is different from the one used in [50], thus making the materials contained in this chapter original and has not been published elsewhere. Our intention in this chapter is three-fold: to count the crisp subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ in section 3.1; discuss with specific examples on how to construct maximal chains of G in section 3.2 and study the distinct fuzzy subgroups of the group G in section 3.3.

3.1 ON CRISP SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$

The classification of all finite groups has been one of the primary problems in group theory. Thus the fundamental theorem of finitely generated abelian groups has extensively been used by several authors to compute the number of subgroups, particularly in a direct product of finite cyclic groups. In [28], Edouard Goursat established a theorem that illustrates the subgroup structure of a direct product $G_1 \times G_2$ in terms of quotients in G_1 and G_2. Other researchers such as [13], [14],[24] and [66] have calculated subgroups in specific classes of finite groups. Hao and Jin [29] more recently, studied all finite non-cyclic p-groups which contain a self-centralizing cyclic normal subgroup. J. Petrillo in [53] used Goursat theorem in [28] to compute the number of subgroups, $G = \mathbb{Z}_m \times \mathbb{Z}_n$ for all positive integers m and n and further examined the cases where m and n are relatively prime and powers of the same prime. They then extended their studies to the direct product of arbitrary cyclic and non-cyclic groups. Considerably, it is more tedious to determine the number of the non-cyclic subgroups of any group G than to find a formula which gives the number of the cyclic subgroups contained therein. In this thesis, part of our main focus is to compute the total number of subgroups of a direct product of finite non-cyclic abelian groups, viz (i) $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ where p is a fixed prime and $m, n \geq 1$ and (ii) $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_{\mathbb{P}}$ where p is a fixed prime and $m, n \geq 1$, which shall be treated later in Chapter 4. The study in Case (i), although similar to the one conducted in [14] and [53], a slightly different approach yields user-friendly polynomial formulae. To start with, we count the number of crisp subgroups of a finite non-cyclic abelian p-group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ where p is a fixed prime, $m, n \in N$, and $m, n \geq 1$. We achieve this by first considering all the maximal subgroups in G and then compute the number of new (not counted in the preceding ones) subgroups therein.

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3.1.1 ON SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$

In [58], V.N. Shokeuv developed the following well-known formula $1 + p + p^2 + p^3 + \cdots + p^{d_G - 1}$ to determine the number of maximal subgroups of a p-group G where d_G is the rank of G. The rank of a group G is the minimal number of generators of G. Thus to determine the total number of subgroups in G, it is only necessary to determine the number of maximal subgroups of G since G is a p-group, non-maximal subgroups (subgroups generated by two elements other than the maximal ones) and cyclic subgroups contained therein. As a first step, we use the examples that follow to demonstrate our classification of subgroups of G, by fixing $m = 2$, varying $n \geq 2$ and with specific prime p, leading to a general prime number p. We then characterise our results in the form of propositions and theorems with their proofs.

Note 3.1.1.1. The group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ has rank-2, thus $d_G - 1$ is equal to $2 - 1 = 1$. So the number of maximal subgroups of G is equal to $1 + p$.

3.1.2 On Subgroups Of $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$

Here we use p = 3; 5 and $n = 2 = m$ to show how to compute the subgroups of G, and then proceed with proposition and give its proof.

Example 3.1.1. *Consider the group* $G = \mathbb{Z}_{3^2} + \mathbb{Z}_{3^2}$.

The group G has 23 subgroups, and since G is of rank-2, it has $1 + p$ maximal subgroups and these are $<(1,0),(0,3)>$, $<(1,1),(0,3)>$, $<(1,2),(0,3)>$ and $<(3,0),(0,1)>$. The remaining subgroups of G can be classified as non-maximal subgroups (those generated by two elements) and cyclic subgroups. Non-maximal subgroup of G, i.e. generated by two elements is $<(3,0),(0,3)>$ and cyclic subgroups of G: $<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(1,8)>$, $<(3,0)>$, $<(3,1)>$, $<(3,2)>$, $<(3,3)>$, $<(3,6)>$, $<(0,1)>$, $<(0,3)>$, $<(0,0)>$ and G itself.

Example 3.1.2. *Consider the group* $G = \mathbb{Z}_{5^2} + \mathbb{Z}_{5^2}$.

Group G has 45 subgroups, and $1 + p$ maximal subgroups, these are $<(1,0),(0,5)>$, $<(1,1),(0,5)>$, $<(1,2),(0,5)>$, $<(1,3),(0,5)>$, $<(1,4),(0,5)>$ and $<(5,0),(0,1)>$. The remaining subgroups of G are Non-maximal subgroups of G: $<(5,0),(0,5)>$ and cyclic subgroups of G: $<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(1,8)>$, $<(1,9)>$, $<(1,10)>$, $<(1,11)>$, $<(1,12)>$, $<(1,13)>$, $<(1,14)>$, $<(1,15)>$, $<(1,16)>$, $<(1,17)>$, $<(1,18)>$, $<(1,19)>$, $<(1,20)>$, $<(1,21)>$, $<(1,22)>$, $<(1,23)>$, $<(1,24)>$, $<(5,0)>$, $<(5,1)>$, $<(5,2)>$, $<(5,3)>$, $<(5,4)>$, $<(5,5)>$, $<(5,10)>$, $<(5,15)>$, $<(5,20)>$, $<(0,1)>$, $<(0,5)>$, $<(0,0)>$ and G itself.

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Proposition 3.1.1. *Let* $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$. *Then the number of subgroups of* G *is* $2(1 + 1 + p) + 1 + p + p^2$.

Proof: G has $1 + p$ maximal subgroups, since it has rank 2. By previous Theorem 4.2.9 [6], the number of subgroups of $\mathbb{Z}_{p^2} + \mathbb{Z}_p$ is $n(p + 1) + 2$. One of the maximal subgroups of G is $\mathbb{Z}_{p^2} + \mathbb{Z}_p =<(1,0),(0,p)>$. So, with $n = 2$, this subgroup has $2(p + 1) + 2$ subgroups, including the trivial subgroup. All the other maximal subgroups of G are isomorphic. One of them is $\mathbb{Z}_p + \mathbb{Z}_{p^2} =<(p,0),(0,1)>$. We note that some subgroups of this one have already appeared in $\mathbb{Z}_{p^2} + \mathbb{Z}_p$, for example $<(p,0)>$; $\mathbb{Z}_p + \mathbb{Z}_p$. So we simply identify those subgroups that have not appeared in $\mathbb{Z}_{p^2} + \mathbb{Z}_p$. These are $\mathbb{Z}_p + \mathbb{Z}_{p^2}$; $<(0,1)>$; $<(p,1)>$; $<(p,2)>$; $\cdots$; $<(p,p-1)>$. Hence $\mathbb{Z}_p + \mathbb{Z}_{p^2}$ contributes $p + 1$ to the subgroups of G. There are $p - 1$

such maximal subgroups which are all isomorphic to $\mathbb{Z}_p + \mathbb{Z}_{p^2}$. So these maximal subgroups, including $\mathbb{Z}_p + \mathbb{Z}_{p^2}$, contribute $p(p+1)$ to the subgroups of G. Hence the total number of subgroups of G is $2(p+1) + 2 + p(p+1) + 1$ (including G itself). Rearranging, we get $1 + p + p^2 + 2(1 + 1 + p)$. This completes the proof. $\square$

3.1.3 On Subgroups Of $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$

We use the following two examples to illustrate how to count the subgroups of G and then generalise our result in the form of proposition with proof.

Example 3.1.3. *Consider the group* $G = \mathbb{Z}_{3^3} + \mathbb{Z}_{3^2}$.

The group G has 36 subgroups, and since G is of rank-2, it has $1 + p$ maximal subgroups and these are $<(1,0),(0,3)>$, $<(1,1),(0,3)>$, $<(1,2),(0,3)>$ and $<(3,0),(0,1)>$. The remaining subgroups of G can be classified as non-maximal subgroups(those generated by two elements) and cyclic subgroups. Non-maximal subgroups of G, i.e. generated by two elements are $<(3,0),(0,3)>$, $<(3,1),(0,3)>$, $<(3,2),(0,3)>$, $<(9,0),(0,1)>$, $<(9,0),(0,3)>$ and cyclic subgroups of G: $<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(1,8)>$, $<(3,0)>$, $<(3,1)>$, $<(3,2)>$, $<(3,3)>$, $<(3,4)>$, $<(3,5)>$, $<(3,6)>$, $<(3,7)>$, $<(3,8)>$, $<(9,0)>$, $<(9,1)>$, $<(9,2)>$, $<(9,3)>$, $<(9,6)>$, $<(0,1)>$, $<(0,3)>$, $<(0,0)>$ and G itself.

Example 3.1.4. *Consider the group* $G = \mathbb{Z}_{5^3} + \mathbb{Z}_{5^2}$.

Group G has 76 subgroups, and $1 + p$ maximal subgroups, these are $<(1,0),(0,5)>$, $<(1,1),(0,5)>$, $<(1,2),(0,5)>$, $<(1,3),(0,5)>$, $<(1,4),(0,5)>$ and $<(5,0),(0,1)>$. The remaining subgroups of G are Non-maximal subgroups of G: $<(5,0),(0,5)>$, $<(5,1),(0,5)>$, $<(5,2),(0,5)>$, $<(5,3),(0,5)>$, $<(5,4),(0,5)>$, $<(25,0),(0,1)>$, $<(25,0),(0,5)>$ and cyclic subgroups of G: $<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(1,8)>$, $<(1,9)>$, $<(1,10)>$, $<(1,11)>$, $<(1,12)>$, $<(1,13)>$, $<(1,14)>$, $<(1,15)>$, $<(1,16)>$, $<(1,17)>$, $<(1,18)>$, $<(1,19)>$, $<(1,20)>$, $<(1,21)>$, $<(1,22)>$, $<(1,23)>$, $<(1,24)>$, $<(5,0)>$, $<(5,1)>$, $<(5,2)>$, $<(5,3)>$, $<(5,4)>$, $<(5,5)>$, $<(5,6)>$, $<(5,7)>$, $<(5,8)>$, $<(5,9)>$, $<(5,10)>$, $<(5,11)>$, $<(5,12)>$, $<(5,13)>$, $<(5,14)>$, $<(5,15)>$, $<(5,16)>$, $<(5,17)>$, $<(5,18)>$, $<(5,19)>$, $<(5,20)>$, $<(5,21)>$, $<(5,22)>$, $<(5,23)>$, $<(5,24)>$, $<(25,0)>$, $<(25,1)>$, $<(25,2)>$, $<(25,3)>$, $<(25,4)>$, $<(25,5)>$, $<(25,10)>$, $<(25,15)>$, $<(25,20)>$, $<(0,1)>$, $<(0,5)>$, $<(0,0)>$ and G itself.

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Proposition 3.1.2. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$. Then the number of subgroups of G is* $2[1 + (p + 1)] + 2(1 + p + p^2)$.

Proof: G has $p+1$ maximal subgroups and one of them is $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} =< (p, 0), (0, 1) >$ which yields $2[1 + (p + 1)] + (1 + p + p^2)$ subgroups of G, by the above Proposition 3.1.1. Another maximal subgroup is $H_2 = \mathbb{Z}_{p^4} + \mathbb{Z}_p =< (1, 0), (0, p) >$. We consider subgroups of H_2 not appearing in H_1. These are $< (1, 0) >$; $< (1, p) >$; $< (2, p) >$; $\cdots$; $< (p - 1, p) >$; H_2 giving $p + 1$ subgroups of G not appearing in H_1. Similarly, all the other $p - 1$ maximal subgroups will contribute $p + 1$ subgroups each. By counting, the total number of subgroups of G (including G) is $2[1+(p+1)]+(1+p+p^2)+p(p+1)+1 = 2[1+(p+1)]+2(1+p+p^2)$. This completes the proof.□

Continuing in a similar fashion as above, for $n = 4$ and $m = 2$, we obtain the following proposition.

Proposition 3.1.3. *Let $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2}$. Then the number of subgroups of G is* $2[1 + (p + 1)] + 3(1 + p + p^2)$.

Proof: G has $p+1$ maximal subgroups and one of them is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} =< (p, 0), (0, 1) >$ which yields $2[1 + (p + 1)] + 2(1 + p + p^2)$ subgroups of G, by the above Proposition 3.1.2. Another maximal subgroup is $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (1, 0), (0, p) >$. We consider subgroups of H_2 not appearing in H_1. These are $< (1, 0) >$; $< (1, p) >$; $< (2, p) >$; $\cdots$; $< (p - 1, p) >$; H_2 giving $p + 1$ subgroups of G not appearing in H_1. Similarly, all the other $p - 1$ maximal subgroups will contribute $p + 1$ subgroups each. By counting, the total number of subgroups of G (including G) is $2[1+(p+1)]+2(1+p+p^2)+p(p+1)+1 = 2[1+(p+1)]+3(1+p+p^2)$. This completes the proof. □

Observe that $3 = 4 - 2 + 1 = n - 2 + 1$. Thus the theorem that follows.

Theorem 3.1.4. *Let $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$. Then the number of subgroups of G is* $2[1 + (p + 1)] + (n - 2 + 1)[1 + p + p^2], n \geq 2$.

Proof: By induction on $n \geq 2$. By previous Propositions 3.1.1, 3.1.2 and 3.1.3, the Theorem is true for $n = 2, 3, 4$. Now assume the theorem is true for some $k \geq 2$, i.e. $G = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^2}$ has $2[1 + (p + 1)] + (k - 2 + 1)[1 + p + p^2]$ subgroups. Now consider $G = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2}$. This has $1 + p$ maximal subgroups and one of them is $H_1 = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^2}$ which has $2[1 + (p + 1)] + (k - 2 + 1)[1 + p + p^2]$ subgroups by the induction hypothesis. Another maximal subgroup is $H_2 = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p =< (1, 0), (0, p) >$. This yields the following subgroups not counted in H_1: H_2; $< (1, 0) >$; $< (1, p) >$; $< (2, p) >$; $\cdots$; $< (p - 1, p) >$; $(p + 1)$ of them.

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Similarly, each of the remaining $p-1$ maximal subgroups yields $p+1$ subgroups not counted in other maximal subgroups. Thus the number of subgroups of $G = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2}$ is equal to $2[1+(p+1)] + (k-2+1)[1+p+p^2] + p(p+1) + 1 = 2[1+(p+1)] + (k+1-2+1)[1+p+p^2]$. This completes the proof.$\square$

3.1.4 ON SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$

Here, we give some examples of how to compute the subgroups of G, and then proceed with propositions and theorems with their proofs.

Example 3.1.5. *Consider the group* $G = \mathbb{Z}_{2^3} + \mathbb{Z}_{2^3}$.

This group has 37 subgroups, and since G is of rank-2, G has $1+p$ maximal subgroups and these are $<(1,0),(0,2)>$, $<(1,1),(0,2)>$ and $<(2,0),(0,1)>$. The remaining subgroups of G can be classified as non-maximal subgroups (those generated by two elements) and cyclic subgroups. The following are non-maximal subgroups of G, generated by two elements: $<(1,0),(0,4)>$, $<(1,1),(0,4)>$, $<(1,2),(0,4)>$, $<(1,3),(0,4)>$, $<(2,0),(0,2)>$, $<(2,1),(0,2)>$, $<(4,0),(0,1)>$, $<(2,0),(0,4)>$, $<(2,2),(0,4)>$, $<(4,0),(0,2)>$ and $<(4,0),(0,4)>$.
Cyclic subgroups of G: $<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(2,0)>$, $<(2,1)>$, $<(2,2)>$, $<(2,3)>$, $<(2,4)>$, $<(2,6)>$, $<(4,0)>$, $<(4,1)>$, $<(4,2)>$, $<(4,4)>$, $<(0,1)>$, $<(0,2)>$, $<(0,4)>$, $<(0,0)>$ and G itself.

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Example 3.1.6. *Consider the group* $G = \mathbb{Z}_{3^3} + \mathbb{Z}_{3^3}$.

This group has 76 subgroups, and since G is of rank-2, G has $1+p$ maximal subgroups and these are $<(1,0),(0,3)>$, $<(1,1),(0,3)>$, $<(1,2),(0,3)>$ and $<(3,0),(0,1)>$. The remaining subgroups of G can be classified as non-maximal subgroups (those generated by two elements) and cyclic subgroups.

The following are non-maximal subgroups of G, generated by two elements:

$<(1,0),(0,9)>$, $<(1,1),(0,9)>$, $<(1,2),(0,9)>$, $<(1,3),(0,9)>$, $<(1,4),(0,9)>$, $<(1,5),(0,9)>$, $<(1,6),(0,9)>$, $<(1,7),(0,9)>$, $<(1,8),(0,9)>$, $<(3,0),(0,3)>$, $<(3,1),(0,3)>$, $<(3,2),(0,3)>$, $<(9,0),(0,1)>$, $<(3,0),(0,9)>$, $<(3,3),(0,9)>$, $<(3,6),(0,9)>$,$<(9,0),(0,3)>$ and $<(9,0),(0,9)>$.

Cyclic subgroups of G:

$<(1,0)>, <(1,1)>, <(1,2)>, <(1,3)>, <(1,4)>, <(1,5)>, <(1,6)>, <(1,7)>, <(1,8)>, <(1,9)>, <(1,10)>, <(1,11)>, <(1,12)>, <(1,13)>, <(1,14)>, <(1,15)>, <(1,16)>, <(1,17)>, <(1,18)>, <(1,19)>, <(1,20)>, <(1,21)>, <(1,22)>, <(1,23)>, <(1,24)>, <(1,25)>, <(1,26)>, <(3,0)>, <(3,1)>, <(3,2)>, <(3,3)>, <(3,4)>, <(3,5)>, <(3,6)>, <(3,7)>, <(3,8)>, <(3,9)>, <(3,12)>, <(3,15)>, <(3,18)>, <(3,21)>, <(3,24)>, <(9,0)>, <(9,1)>, <(9,2)>, <(9,3)>, <(9,6)>, <(9,9)>, <(9,18)>, <(0,1)>, <(0,3)>, <(0,9)>, <(0,0)>$ and G itself.

For $p = 5$, the group $G = \mathbb{Z}_{5^3} + \mathbb{Z}_{5^3}$ has 232 subgroups, we omit them here:

For $p = 7$, the group $G = \mathbb{Z}_{7^3} + \mathbb{Z}_{7^3}$ has 532 crisp subgroups, we omit the list:

Proposition 3.1.5. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$. Then the number of subgroups of G is $2[1 + (p+1) + (1 + p + p^2)] + (1 + p + p^2 + p^3)$.*

Proof: G has $1 + p$ maximal subgroups, since it is of rank 2. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$ which yields $2[1 + (1 + p)] + 2[1 + p + p^2]$ subgroups. Another maximal subgroup is $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^3} = <(p,0),(0,1)>$. We list its subgroups that are not appearing in H_1: H_2, $<(0,1)>$, $<(p,1)>$, $<(p,2)>, \cdots, <(p,p-1)>$; $<(p^2,1)>$, $<(p^2,2)>, \cdots, <p^2, p^2-1>$ and $<(p^2,0),(0,1)>$. The number of these subgroups is $1+p+p^2$. All the other $p-1$ maximal subgroups are isomorphic to H_2 and thus each yields $1 + p + p^2$ subgroups not appearing elsewhere. Thus, by counting, G has the following number of subgroups: $2[1+(1+p)]+2[1+p+p^2]+p(1+p+p^2)+1 = 2[1+(1+p)+(1+p+p^2)]+1+p+p^2+p^3$.
□

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Example 3.1.7. *Consider the group $G = \mathbb{Z}_{3^4} + \mathbb{Z}_{3^3}$.*

This group has 116 subgroups, and since G is of rank-2, G has $1 + p$ maximal subgroups and these are $<(1,0),(0,3)>$, $<(1,1),(0,3)>$, $<(1,2),(0,3)>$ and $<(3,0),(0,1)>$. The remaining subgroups of G can be classified as non-maximal subgroups (those generated by two elements) and cyclic subgroups.

The following are non-maximal subgroups of G, generated by two elements:

$<(1,0),(0,9)>, <(1,1),(0,9)>, <(1,2),(0,9)>, <(1,3),(0,9)>, <(1,4),(0,9)>, <(1,5),(0,9)>, <(1,6),(0,9)>, <(1,7),(0,9)>, <(1,8),(0,9)>, <(3,0),(0,3)>, <(3,1),(0,3)>, <(3,2),(0,3)>, <(9,0),(0,1)>, <(3,0),(0,9)>, <(3,1),(0,9)>, <(3,2),(0,9)>, <(3,3),(0,9)>, <(3,4),(0,9)>, <(3,5),(0,9)>, <(3,6),(0,9)>,$

$<(3,7),(0,9)>$, $<(3,8),(0,9)>$, $<(9,0),(0,3)>$, $<(9,1),(0,3)>$, $<(9,2),(0,3)>$, $<(9,3),(0,9)>$, $<(9,6),(0,9)>$, $<(9,0),(0,9)>$, $<(27,0),(0,1)>$, $<(27,0),(0,3)>$ and $<(27,0),(0,9)>$

Cyclic subgroups of G:

$<(1,0)>$, $<(1,1)>$, $<(1,2)>$, $<(1,3)>$, $<(1,4)>$, $<(1,5)>$, $<(1,6)>$, $<(1,7)>$, $<(1,8)>$, $<(1,9)>$, $<(1,10)>$, $<(1,11)>$, $<(1,12)>$, $<(1,13)>$, $<(1,14)>$, $<(1,15)>$, $<(1,16)>$, $<(1,17)>$, $<(1,18)>$, $<(1,19)>$, $<(1,20)>$, $<(1,21)>$, $<(1,22)>$, $<(1,23)>$, $<(1,24)>$, $<(1,25)>$, $<(1,26)>$, $<(3,0)>$, $<(3,1)>$, $<(3,2)>$, $<(3,3)>$, $<(3,4)>$, $<(3,5)>$, $<(3,6)>$, $<(3,7)>$, $<(3,8)>$, $<(3,9)>$, $<(3,10)>$, $<(3,11)>$, $<(3,12)>$, $<(3,13)>$, $<(3,14)>$, $<(3,15)>$, $<(3,16)>$, $<(3,17)>$, $<(3,18)>$, $<(3,19)>$, $<(3,20)>$, $<(3,21)>$, $<(3,22)>$, $<(3,23)>$, $<(3,24)>$, $<(3,25)>$, $<(3,26)>$, $<(9,0)>$, $<(9,1)>$, $<(9,2)>$, $<(9,3)>$, $<(9,4)>$, $<(9,5)>$, $<(9,6)>$, $<(9,7)>$, $<(9,8)>$, $<(9,9)>$, $<(9,12)>$, $<(9,15)>$, $<(9,18)>$, $<(9,21)>$, $<(9,24)>$, $<(27,0)>$, $<(27,1)>$, $<(27,2)>$, $<(27,3)>$, $<(27,6)>$, $<(27,9)>$, $<(27,18)>$, $<(0,1)>$, $<(0,3)>$, $<(0,9)>$, $<(0,0)>$ and G itself.

For $p = 5$, the group $G = \mathbb{Z}_{5^4} + \mathbb{Z}_{5^3}$ has 388 subgroups, we omit them here:

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For $p = 7$, the group $G = \mathbb{Z}_{7^4} + \mathbb{Z}_{7^3}$ has 932 crisp subgroups, we omit the list:

Proposition 3.1.6. *Let $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3}$. Then the number of subgroups of G is equal to* $2[1 + (p+1) + (1 + p + p^2)] + 2[(1 + p + p^2 + p^3)]$.

Proof: G has $1 + p$ maximal subgroups, and one of such subgroups is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$ which has $2[1 + (p+1) + (1 + p + p^2)] + (1 + p + p^2 + p^3)$ subgroups. Another maximal subgroup is $H_2 = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} = <(1,0),(0,p)>$. We list its subgroups that are not appearing in H_1: H_2, $<(1,0)>$, $<(1,p)>$, $<(2,p)>$, $\cdots$, $<(p-1,p)>$; $<(1,p^2)>$, $<(2,p^2)>$, $\cdots$, $<(p^2-1,p^2)>$; $<(1,0),(0,p^2)>$. Thus H_2 yields $1 + p + p^2$ subgroups. Each of the remaining subgroups also yields $1 + p + p^2$ subgroups. Therefore G has the following number of subgroups: $2[1 + (p+1) + (1 + p + p^2)] + (1 + p + p^2 + p^3) + p(1 + p + p^2) + 1 = 2[1 + (p+1) + (1 + p + p^2)] + 2(1 + p + p^2 + p^3)$.□

Note 3.1.4.1. The 2nd 2 in the above formula is $4 - 3 + 1 = n - 3 + 1$. Thus

Theorem 3.1.7. *Let $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$. Then the number of subgroups of G is equal to* $2[1 + (p+1) + (1 + p + p^2)] + (n - 3 + 1)[(1 + p + p^2 + p^3)]$, $n \geq 3$.

Proof: By induction on n. Then use a procedure similar to the above Propositions 3.1.5 and 3.1.6.$\square$

The above arguments can be extended to $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$, $n \geq m$. Hence we have the theorem below.

Theorem 3.1.8. *Let $m, n \in \mathbb{N}$; $n \geq m$. Then for any fixed prime p, the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ has* $2[(1+(p+1)+(1+p+p^2)+\cdots+(1+p+p^2+\cdots+p^{m-1}))]+(n-m+1)(1+p+p^2+\cdots+p^m)$ $= 2\sum_{k=1}^{m-1}(m-k+1)p^k + (n-m+1)\sum_{k=0}^{m} p^k$ *subgroups. If $m > n$, interchange m and n.*

3.2 ON MAXIMAL CHAINS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$

This section consists of maximal chains of G. It is a known fact that to successfully compute the number of distinct fuzzy subgroup of a group, one needs to understand how subgroups are arranged to form maximal chains. This makes the identification of distinguishing factors in the chains easier and convenient. Thus we give an illustrative example below, leading to a generalised formula.

Note 3.2.0.1. By Proposition 4.2.15[49], the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p$ has $(n-1)(p-1)+p+1+n-1$ maximal chains.

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In what follows, is an illustrative example of maximal chain in the above group G.

For $p = 3$, $n = 3$ and $m = 2$ we obtained the following 58 maximal chains:

Example 3.2.1. $\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(3,0)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(3,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(3,6)> \supseteq <(9,0)> \supseteq \{(0,0)\}$ $\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(9,0),(0,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(9,0),(0,3)> \supseteq <(9,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(9,0),(0,3)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(3,0),(0,3)> \supseteq <(9,0),(0,3)> \supseteq <(0,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(1,1)> \supseteq <(3,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(1,4)> \supseteq <(3,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,1),(1,4)> \supseteq <(1,7)> \supseteq <(3,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(1,2),(1,5)> \supseteq <(3,0),(0,3)> \supseteq <(3,0)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(3,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,6)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(0,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(1,2)>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(1,5)>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,2),(1,5)>\supseteq<(1,8)>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(3,0)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(3,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(9,6)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(3,0),(0,3)>\supseteq<(9,0),(0,3)>\supseteq<(0,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(1,3)>\supseteq<(3,0)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\supseteq<(1,6)>\supseteq<(3,0)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(1,3),(0,3)>\mathbb{Z}_{3^3}+0\supseteq<\supseteq<(3,0)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(3,0)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(3,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(3,6)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,6)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,0)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(0,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,0)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(9,6)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(9,0),(0,3)>\supseteq<(0,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(3,1)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(3,4)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}\supseteq<(3,0),(3,1)>\supseteq<(3,1)\,(0,3),>\supseteq<(3,7)>\supseteq<(9,3)>\supseteq\{(0,0)\}$$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(9,0),(0,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(9,0),(0,3)> \supseteq <(9,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(9,0),(0,3)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(9,0),(0,3)> \supseteq <(0,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(3,2)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(3,5)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(3,2)(0,3),> \supseteq <(3,8)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,0),(0,3)> \supseteq <(9,0)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,0),(0,3)> \supseteq <(9,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,0),(0,3)> \supseteq <(9,6)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,0),(0,3)> \supseteq <(0,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,1)> \supseteq <(0,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq <(9,2)> \supseteq <(0,3)> \supseteq \{(0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} \supseteq <(3,0),(3,1)> \supseteq <(9,0)(9,1),> \supseteq 0 + \mathbb{Z}_{3^2} \supseteq <(0,3)> \supseteq \{(0,0)\}$

Proposition 3.2.1. *Let $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$, then G has $1 + 3p + 2p^2$ maximal chains.*

Proof: G has $1 + p$ maximal subgroups. One maximal subgroup of G is $\mathbb{Z}_{p^2} + \mathbb{Z}_p$ which is isomorphic to all the maximal subgroups and has $p - 1 + p + 1 + 1 = 2p + 1$ maximal chains by Note 3.2.0.1. Thus $\mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$ has $(p+1)(2p+1) = 2p^2 + 3p + 1$ maximal chains.□

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Proposition 3.2.2. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$. Then G has $1 + 4p + 5p^2$ maximal chains.*

Proof: G has $1 + p$ maximal subgroups. One maximal subgroup of G is $\mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$ which has $1 + 3p + 2p^2$ maximal chains by Proposition 3.2.1. Another maximal subgroup is $\mathbb{Z}_{p^3} + \mathbb{Z}_p$ which has $2(p-1) + p + 1 + 2 = 1 + 3p$ maximal chains by Note 3.2.0.1 . Thus $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$ has $1 + 3p + 2p^2 + p(1 + 3p) = 1 + 4p + 5p^2$ maximal chains.□

Proposition 3.2.3. *Let $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2}$, then G has $1 + 5p + 9p^2$ maximal chains.*

Proof: G has $1 + p$ maximal subgroups. One of the maximal subgroups of G is $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$ and has $1 + 4p + 5p^2$ maximal chains by Proposition 3.2.2. Another one is $\mathbb{Z}_{p^4} + \mathbb{Z}_p$ which has $3(p-1) + p + 1 + 2 = 1 + 4p$ maximal chains by Note 3.2.0.1. Thus $\mathbb{Z}_{p^4} + \mathbb{Z}_{p^2}$ has $1 + 4p + 5p^2 + p(1 + 4p) = 1 + 5p + 9p^2$ maximal chains.□

Theorem 3.2.4. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$ has $1 + (n+1)p + \frac{(n-1)(n+2)}{2}p^2$, $n \geq 2$ maximal chains.*

Proof: For $n = 2$, the number of maximal chains is $1 + 3p + 2p^2$ from 3.2.1. This agrees with replacing n with 2 in the formula of the group G. Now suppose $\mathbb{Z}_{p^k} + \mathbb{Z}_{p^2}$ has $1 + (k+1)p +$

$\frac{(k-1)(k+2)}{2}p^2$ maximal chains. Let $G = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2}$. We use the maximal subgroups of G viz. $\mathbb{Z}_{p^k} + \mathbb{Z}_{p^2}$ and $\mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p$ to compute the number of maximal chains of G. The subgroup $\mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p$ has $(k+1-1)(p-1)+p+1+k+1-1 = k(p-1)+p+1+k = kp+p+1$ maximal chains by Note 3.2.0.1. Thus, since G has p maximal subgroups that are isomorphic to $\mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p$, then G has $1+(k+1)p+\frac{(k-1)(k+2)}{2}p^2+p(kp+p+1) = 1+kp+p+kp^2+p^2+p+\frac{(k-1)(k+2)}{2}p^2 = 1+(k+1+1)p+\frac{2(k+1)+k^2+k-2}{2}p^2 = 1+(k+1+1)p+\frac{(k^2+3k)}{2}p^2 = 1+(k+1+1)p+\frac{(k+1-1)(k+1+2)}{2}p^2$ maximal chains. Hence the theorem is true for $n = k+1$.□

Proposition 3.2.5. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$. Then the number of maximal chains of G is equal to $1 + 5p + 9p^2 + 5p^3$.*

Proof: For $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$, the number of maximal chains is equal $1 + 4p + 5p^2$ by Proposition 3.2.2. All maximal subgroups of G are isomorphic to $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$. Thus the number of maximal chains of G is equal to $1 + 4p + 5p^2 + p(1 + 4p + 5p^2) = 1 + 5p + 9p^2 + 5p^3$.□

Proposition 3.2.6. *Let $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3}$. Then the number of maximal chains of G is equal to $1 + 6p + 14p^2 + 14p^3$.*

Proof: The maximal subgroup $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$ has $1+5p+9p^2+5p^3$ maximal chains by Proposition 3.2.6. The rest are isomorphic to $\mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} = 1 + 5p + 9p^2$. Thus G has $1 + 5p + 9p^2 + 5p^3 + p(1+5p+9p^2) = 1+5p+9p^2+5p^3+p+5p^2+9p^3 = 1+6p+14p^2+14p^3$ maximal chains.□

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Theorem 3.2.7. $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$ *has* $1 + (n + m - 1)p + \frac{(n+m-3)(n+m)}{2}p^2 + \alpha_3 p^3$ *where* $\alpha_3 = \frac{(n-m+1)(n+2)(n+3)}{m!}$, $n \geq 3$ *maximal chains.*

Proof: By induction on n. For $n = 3, 4$, the result is true by the above Propositions 3.2.5 and 3.2.6.

Now assume that the result is true for any integer $k \geq 3$, that is $G = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^3}$ has $1 + (k + 3 - 1)p + \frac{(k+3-3)(k+3)}{2}p^2 + \frac{(k-3+1)(k+2)(k+3)}{3!}p^3$ maximal chains.

Next, let $G = \mathbb{Z}_{p[k+1]} + \mathbb{Z}_{p^3}$. One of the maximal subgroups of G is $H = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^3}$ which has $1+(k+3-1)p+\frac{(k+3-3)(k+3)}{2}p^2+\frac{(k-3+1)(k+2)(k+3)}{3!}p^3 = 1+(k+2)p+\frac{k(k+3)}{2}p^2+\frac{(k-2)(k+2)(k+3)}{3!}p^3$ maximal chains. (1)

Another maximal subgroup of G is $H_1 = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2}$ which has $1+(k+1+1)p+\frac{(k+1-1)(k+1+2)}{2}p^2 = 1 + (k+2)p + \frac{k(k+3)}{2}p^2$ maximal chains by Theorem3.2.4. So the p maximal subgroups isomorphic to H_1 contribute $p[1+(k+2)p+\frac{k(k+3)}{2}p^2] = p+(k+2)p^2+\frac{k(k+3)}{2}p^3$ maximal chains. (2)

Hence the total number of maximal chains of G is given by summing (1) and (2). This sum

is $1+(k+2)p+\frac{k(k+3)}{2}p^2+\frac{(k-2)(k+2)(k+3)}{3!}p^3+p+(k+2)p^2+\frac{k(k+3)}{2}p^3 = 1+(k+1+3-1)p+\frac{(k+1+3-3)(k+1+3)}{2}p^2+\frac{(k+1-3+1)(k+1+2)(k+1+3)}{3!}p^3 = 1+(k+3)p+\frac{(k+1)(k+4)}{2}p^2+\frac{(k-1)(k+3)(k+4)}{3!}p^3$.
This completes the proof.□

The above Theorem3.2.7, can be extended to the general case where 3 can be replaced by any positive integer $m \geq 3$. Thus we have

Theorem 3.2.8. *Let $m, n \in \mathbb{Z}^+$; $n \geq m$. For any fixed prime p, the number of maximal chains of the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ is $1+(m+n-1)p+\frac{(n+m-3)(n+m)}{2}p^2+\alpha_3 p^3+\alpha_4 p^4+\cdots+\alpha_{m-1}p^{m-1}+\alpha_m p^m$, where $\alpha_m = \frac{(n-m+1)(n+2)(n+3)\cdots(n+m-1)(n+m)}{m!}$ and for $2 \leq k < m, \alpha_k = \frac{(n+m-2k+1)(n+m-k+2)(n+m-k+3)\cdots(n+m-2)(n+m-1)(n+m)}{k!}$*

Proof: Induction on m and with a similar procedure as done in theorems 3.2.4 and 3.2.7.□

3.3 ON DISTINCT FUZZY SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$

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Note 3.3.0.1. Let $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p$. From Theorem 4.5.4[6], the number of distinct fuzzy subgroups of G is $2^{n+2} - 1 + np2^{n+1}$. The coefficient of 2^{n+1}, viz. np, is the number of single distinguishing factors in the maximal chains of G. This shows that there are no pairs (or higher k-tuples) of distinguishing factors. This suggests that for $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$, there will be pairs of distinguishing factors but no triples of distinguishing factors. Indeed, if $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$, then G has m-tuples of distinguishing factors but no higher than m tuples of distinguishing factors in the maximal chains of G. The formula for the number of distinct fuzzy subgroups of G will have $m + 1$ powers of 2.
In Theorem 6.1.4[6], we also observed that $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ has no higher than pairs of distinguishing factors in its maximal chains. So the formula for the number of distinct fuzzy subgroups has $1+1+1 = 3$ powers of 2. These observations will be useful in the next sections and chapters.

3.3.1 Counting Distinct Fuzzy Subgroups Of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$

Proposition 3.3.1. *$G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$ has $[(1 + p) + (1 + p + p^2) + (p - 2)] = p^2 + 3p$ single distinguishing factors in its maximal chains of subgroups.*

Proof: There are $1 + p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_p + \mathbb{Z}_{p^2} =< (p, 0), (0, 1) >$. By the formula 2.3 in [6], the number of single distinguishing factors of H_1 is equal to $2p$ by Note 3.3.0.1. (1).

Note 3.3.1.1. We note that H_1 has no pairs of distinguishing factors.

Another maximal subgroup of G is $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1,0),(0,p) >$. We identify the single distinguishing factors of H_2. These distinguishing factors appear in the 3rd column below:

$$\textbf{BlockH1} = \begin{cases} H_2 \geq< (1,0),(0,p) >\geq< (p,0),(0,p) >\geq< (0,p) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,0) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,p) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,2p) >\geq \cdots \\ \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad \vdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,(p-1)p) >\geq \cdots \end{cases}$$

So H_2 has $p+1$ single distinguishing factors. H_2 represents all maximal subgroups of G other than H_1.
Thus these maximal subgroups yield $p(p+1) = p^2 + p$ single distinguishing factors. (2)
Therefore the total number of single distinguishing factors of G is given by (1) plus (2) which is equal to $2p + p^2 + p = p^2 + 3p$. This completes the proof.□

Proposition 3.3.2. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$ has $[(1+p) + 2(1+p+p^2) + (p-3)] = 2p^2 + 4p$ single distinguishing factors in its maximal chains of subgroups.*

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Proof: There are $1+p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} =< (p,0),(0,1) >$. By the previous Proposition 3.3.1, H_1 has $p^2 + 3p$ single distinguishing factors. (1)
Let $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (1,0),(0,p) >$ be another maximal subgroup of G. We identify the single distinguishing factors of H_2. These distinguishing factors appear in the 3rd column below:

$$\textbf{BlockH2} = \begin{cases} H_2 \geq< (1,0),(0,p) >\geq< (p,0),(0,p) >\geq< (p^2,0,) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,0) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,p) >\geq \cdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,2p) >\geq \cdots \\ \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad \vdots \\ H_2 \geq< (1,0),(0,p) >\geq< (1,(p-1)p) >\geq \cdots \end{cases}$$

So H_2 has $p+1$ single distinguishing factors. H_2 represents all maximal subgroups of G other than H_1.

Thus these maximal subgroups yield $p(p+1) = p^2 + p$ single distinguishing factors. (2)

Therefore the total number of single distinguishing of G is given by (1) plus (2) which is equal to $2p + p^2 + p = 2p^2 + 4p$. This completes the proof.□

A similar argument leads to the Theorem that follows for single distinguishing factors of G.

Theorem 3.3.3. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$ has $[(1+p)+(n-1)(1+p+p^2)+(p-n)]$ single distinguishing factors in its maximal chains of subgroups, where $n \geq 2$.*

Proof: We use arguments similar to Propositions 3.3.1 and 3.3.2. Then use induction on $n > 3$.□

Proposition 3.3.4. *$G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2}$ has $p^2[\frac{n(n-1)}{2}] = p^2$ pairs of distinguishing factors in its maximal chains of subgroups, where $n = 2$.*

Proof: There are $1 + p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_p + \mathbb{Z}_{p^2} =< (p,0),(0,1) >$ which does not yield pairs of distinguishing factors.

Let $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1,0),(0,p) >$ be another maximal subgroup of G. We identify pairs of distinguishing factors yielded by H_2. These pairs of distinguishing factors come from pairing H_2 and 3rd column below:

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$$\textbf{BlockH3} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (0,p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,2p) > \geq \cdots \\ \quad\vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p) > \geq \cdots \end{cases}$$

So the maximal subgroups of G other than H_1 yield $p.p = p^2$ pairs of distinguishing factors. This completes the proof.□

Proposition 3.3.5. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2}$ has $p^2[\frac{n(n-1)}{2}] = 3p^2$ pairs of distinguishing factors in its maximal chains of subgroups, where $n = 3$.*

Proof: There are $1 + p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} =< (p,0),(0,1) >$ which has p^2 pairs of distinguishing factors by the previous Proposition 3.3.4. (1)

Let $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (1,0),(0,p) >$ be another maximal subgroup of G. We identify pairs of distinguishing factors of H_2. These pairs of distinguishing factors come from pairing H_2

and 3rd column of $BlockH_4$ and pairing H_2 and 4th column of $BlockH_5$ below:

$$\textbf{BlockH4} = \begin{cases} H_2 \geq< (p,0),(0,p) >\geq< (0,p) >\geq \cdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p,p) >\geq \cdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p,2p) >\geq \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p,(p-1)p) >\geq \cdots \end{cases}$$

$$\textbf{BlockH5} = \begin{cases} H_2 \geq< (p,0),(0,p) >\geq< (p^2,0),(0,p) >\geq< (p^2,0) >\geq \cdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p^2,0),(0,p) >\geq< (p^2,p) >\geq \cdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p^2,0),(0,p) >\geq< (p^2,2p) >\geq \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad\qquad \vdots \\ H_2 \geq< (p,0),(0,p) >\geq< (p^2,0),(0,p) >\geq< (p^2,(p-1)p) >\geq \cdots \end{cases}$$

H_2 contributes $p + p = 2p$ pairs of distinguishing factors. Thus the maximal subgroups of G other than H_1 yield $p(2p) = 2p^2$ pairs of distinguishing factors. (2)

Hence the total number of pairs of distinguishing factors of G is $p^2 + 2p^2 = 3p^2$. This completes the proof.□

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Theorem 3.3.6. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$ has $p^2[\frac{n(n-1)}{2}]$ pairs of distinguishing factors in its maximal chains of subgroups, where $n \geq 2$.*

Proof: For $n = 2$ and $n = 3$, the result follows from Propositions 3.3.4 and 3.3.5. Thereafter, we use induction on $n > 3$.□

Combining Theorems 3.3.3 and 3.3.6 and using the relevant powers of 2, we obtain the following polynomial formula for $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$:

Theorem 3.3.7. *For $n \geq 2$ and for any fixed prime integer p, the number of distinct fuzzy subgroup for the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$ is given by $2^{n+3} - 1 + [(1+p) + (n-1)(1+p+p^2) + (p-n)]2^{n+2} + \frac{p^2 n(n-1)}{2} 2^{n+1}$.*

3.3.2 Counting Distinct Fuzzy Subgroups Of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$

In the above section, we counted the distinct fuzzy subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2}$. By a similar argument, we compute the distinct fuzzy subgroups for $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$. We begin this section with propositions and theorems to generalise our counting.

Proposition 3.3.8. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$ has $[(1+p)+2(1+p+p^2)+(1+p+p^2+p^3)+(p-3-1)] = p^3+3p^2+5p$ single distinguishing factors in its maximal chains of subgroups.*

Proof: G has $1+p$ maximal subgroups, since G is of rank-2. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} =< (1,0),(0,p) >$. By the previous Proposition 3.3.2, H_1 has $2p^2+4p$ single distinguishing factors. (1)

Let $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^3} =< (p,0),(0,1) >$ be another maximal subgroup of G. We identify the single distinguishing factors of H_2:

$$\textbf{BlockHH1} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,0,) > \geq < (p^2,0,) > \geq \cdots \end{cases}$$

has the single distinguishing factor H_2.

$$\textbf{BlockHH2} = \begin{cases} H_2 \geq < (p,1),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,0,) > \geq < (p^2,0,) > \geq \cdots \\ H_2 \geq < (p,1),(0,p) > \geq < (p,1) > \geq \cdots \\ H_2 \geq < (p,1),(0,p) > \geq < (p,p+1) > \geq \cdots \\ H_2 \geq < (p,1),(0,p) > \geq < (p,2p+1) > \geq \cdots \\ \quad \vdots \qquad\qquad \vdots \quad \vdots \\ H_2 \geq < (p,1),(0,p) > \geq < (p,(p-1)p+1) > \geq \cdots \end{cases}$$

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This **BlockHH2** has $1+p$ single distinguishing factors which appear in the 3rd column, expect the first maximal chain which has the distinguishing factor $< (p,1),(0,p) >$.

$$\textbf{BlockHH3} = \begin{cases} H_2 \geq < (p^2,0),(0,1) > \geq < (p,0),(0,p^2) > \geq < (p,0,) > \geq < (p^2,0,) > \geq \cdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,1) > \geq \cdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,2) > \geq \cdots \\ \quad \vdots \qquad\qquad \vdots \quad \vdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,p-1) > \geq \cdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (0,1) > \geq \cdots \end{cases}$$

This **BlockHH3** has $1+p$ single distinguishing factors which appear in the 3rd column, except the first maximal chain which has the distinguishing factor $< (p^2,0),(0,1) >$.

There are other blocks of maximal chains similar to **BlockHH2**. These are obtained by replacing the 1 in $< (p,1),(0,p) >$ with the integers $2,3,\cdots,p-1$ respectively. Each of these blocks has $1+p$ single distinguishing factors. So H_2 has $(p-1)(p+1)+(p+1)+1 = p^2+p+1$ single distinguishing factors after including the distinguishing factors of **BlockHH1** and

BlockHH3. H_2 represents all maximal subgroups of G other than H_1.
Thus these maximal subgroups yield $p(p^2+p+1) = p^3+p^2+p$ single distinguishing factors.(2)
Therefore the total number of single distinguishing factors of G is given by (1) plus (2) which is equal to $2p^2 + 4p + p^3 + p^2 + p = p^3 + 3p^2 + 5p$. This completes the proof.□

Proposition 3.3.9. *$G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3}$ has $[(1+p)+2(1+p+p^2)+2(1+p+p^2+p^3)+(p-4-1)] = 2p^3 + 4p^2 + 6p$ single distinguishing factors in its maximal chains of subgroups.*

Proof: G has $1 + p$ maximal subgroups, since G is of rank-2. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} =< (p, 0), (0, 1) >$. By the previous Proposition 3.3.8, H_1 has $p^3 + 3p^2 + 5p$ single distinguishing factors. (1)
Let $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^3} =< (1, 0), (0, p) >$ be another maximal subgroup of G. By a similar argument as in Proposition 3.3.8, H_2 and other maximal subgroups (other than H_1) contribute $p(p^2 + p + 1) = p^3 + p^2 + p$ single distinguishing factors. (2)
Thus the total number of single distinguishing factors of G is given by (1) plus (2) which is equal to $p^3 + 3p^2 + 5p + p^3 + p^2 + p = 2p^3 + 4p^2 + 6p$ as required.□

Theorem 3.3.10. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$ has $[(1+p)+2(1+p+p^2)+(n-2)(1+p+p^2+p^3)+(p-n-1)]$ single distinguishing factors in its maximal chains of subgroups, where $n \geq 3$.*

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Proof: For $n = 3$ and $n = 4$, the result follows from Propositions 3.3.8 and 3.3.9. Then use induction on $n > 3$.□

Proposition 3.3.11. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3}$ has $p^2[3 + (p + 2) + (2p + 1)] = p^2[6 + 3p] = 3p^3 + 6p^2$ pairs of distinguishing factors in its maximal chains of subgroups.*

Proof: There are $1 + p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} =< (1, 0), (0, p) >$ which has $3p^2$ pairs of distinguishing factors by the previous Proposition 3.3.5. (1)
Let $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^3} =< (p, 0), (0, 1) >$ be another maximal subgroup of G. We listed in blocks of chains below, the maximal chains containing pairs of distinguishing factors. However, we observe that the first chain in **BlockH4** viz $H_2 \geq< (p, 0), (0, p) >\geq< (p, 0), (0, p^2) >\geq< (p^2, 0), (0, p^2) >\geq< (0, p^2) >\geq \cdots$ has a single distinguishing factor H_2, (not a pair), hence we do not add it to pairs of distinguishing factors. All the next chains have pairs of distinguishing factors. We identify these pairs of distinguishing factors of H_2. These pairs of distinguishing factors come from pairing H_2 and 5th column of *BlockH*4 and pairing H_2 and 4th column

of $BlockH5$ below.

$$\textbf{BlockH4} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\textbf{BlockH5} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots \end{cases}$$

BlockH4 and **BlockH5** contribute $p + p = 2p$ pairs of distinguishing factors. (2)

Next we identify pairs of distinguishing factors in **BlockH6**, **BlockH7**, **BlockH8** and **BlockH9**. These pairs of distinguishing factors come from pairing H_2 in each of the first chain of **BlockH6**, **BlockH7**, **BlockH8** and **BlockH9** with the 3rd column and pairing H_2 and 4th column of the remaining chains in each of the following blocks.

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$$\textbf{BlockH6} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p,p),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,p),(0,p^2) > \geq < (p,p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,p),(0,p^2) > \geq < (p,p+p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,p),(0,p^2) > \geq < (p,p+2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p,p),(0,p^2) > \geq < (p,p+(p-1)p^2) > \geq \cdots \end{cases}$$

$$
\textbf{BlockH7} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p,2p),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,2p),(0,p^2) > \geq < (p,2p) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,2p),(0,p^2) > \geq < (p,2p+p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,2p),(0,p^2) > \geq < (p,2p+2p^2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,2p),(0,p^2) > \geq < (p,2p+(p-1)p^2) > \geq \cdots
\end{cases}
$$

$$
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
$$

$$
\textbf{BlockH8} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p),(0,p^2) > \geq < (p,(p-1)p) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p),(0,p^2) > \geq < (p,(p-1)p+p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p),(0,p^2) > \geq < (p,(p-1)p+2p^2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,(p-1)p),(0,p^2) > \geq < (p,(p-1)p+(p-1)p^2) > \geq \cdots
\end{cases}
$$

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$$
\textbf{BlockH9} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,p) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,2p) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,(p-1)p) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (0,p) > \geq \cdots
\end{cases}
$$

From **BlockH6** to **BlockH9**, we observe that there are p maximal subgroups of the subgroup $< (p,0),(0,p) >$ and each contributes $p+1$ pairs of distinguishing factors, giving rise to $p(p+1) = p^2 + p$ pairs of distinguishing factors. (3)

Next we vary the maximal subgroups of H_2, e.g. replace $< (p,0),(0,p) >$ with $< (p,1),(0,p) >$. We observe that the first chain in $BlockH10$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 5th column of $BlockH10$ and pairing 2nd column and 4th column of $BlockH11$ below:

$$\textbf{BlockH10} = \begin{cases} H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,0)> \geq \cdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,p^2)> \geq \cdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,2p^2)> \geq \cdots \\ \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,(p-1)p^2)> \geq \cdots \end{cases}$$

$$\textbf{BlockH11} = \begin{cases} H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,p)> \geq \cdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,2p)> \geq \cdots \\ \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad\quad \vdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,(p-1)p)> \geq \cdots \\ H_2 \geq <(p,1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(0,p)> \geq \cdots \end{cases}$$

Next we replace $<(p,0),(0,p)>$ with $<(p,2),(0,p)>$. We observe that the first chain in $BlockH12$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 5th column of $BlockH12$ and pairing 2nd column and 4th column of $BlockH13$ below:

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$$\textbf{BlockH12} = \begin{cases} H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,0)> \geq \cdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,p^2)> \geq \cdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,2p^2)> \geq \cdots \\ \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,(p-1)p^2)> \geq \cdots \end{cases}$$

$$\textbf{BlockH13} = \begin{cases} H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,p)> \geq \cdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,2p)> \geq \cdots \\ \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad\quad \vdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,(p-1)p)> \geq \cdots \\ H_2 \geq <(p,2),(0,p)> \geq <(p^2,0),(0,p)> \geq <(0,p)> \geq \cdots \end{cases}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

Similarly, we replace $<(p,0),(0,p)>$ with $<(p,p-1),(0,p)>$. We observe that the first chain in $BlockH14$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 5th column of $BlockH14$ and pairing 2nd column and 4th column of $BlockH15$ below:

$$\textbf{BlockH14} = \begin{cases} H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,0)> \geq \cdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,p^2)> \geq \cdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,2p^2)> \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,(p-1)p^2)> \geq \cdots \end{cases}$$

$$\textbf{BlockH15} = \begin{cases} H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,p)> \geq \cdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,2p)> \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(p^2,(p-1)p)> \geq \cdots \\ H_2 \geq <(p,p-1),(0,p)> \geq <(p^2,0),(0,p)> \geq <(0,p)> \geq \cdots \end{cases}$$

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Next we replace $<(p,0),(0,p)>$ with $<(p^2,0),(0,1)>$. We observe that the first chain in $BlockH16$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 5th column of $BlockH16$ and pairing 2nd column and 4th column of $BlockH17$ below:

$$\textbf{BlockH16} = \begin{cases} H_2 \geq <(p^2,0),(0,1)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p^2,0),(0,1)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,0)> \geq \cdots \\ H_2 \geq <(p^2,0),(0,1)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,p^2)> \geq \cdots \\ H_2 \geq <(p^2,0),(0,1)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,2p^2)> \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq <(p^2,0),(0,1)> \geq <(p^2,0),(0,p)> \geq <(p^2,0),(0,p^2)> \geq <(p^2,(p-1)p^2)> \geq \cdots \end{cases}$$

$$
\textbf{BlockH17} = \begin{cases} H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,0),(0,p) > \geq < (p^2,p) > \geq \cdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,0),(0,p) > \geq < (p^2,2p) > \geq \cdots \\ \quad \vdots \qquad \vdots \quad \vdots \qquad \vdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,0),(0,p) > \geq < (p^2,(p-1)p) > \geq \cdots \\ H_2 \geq < (p^2,0),(0,1) > \geq < (p^2,0),(0,p) > \geq < (0,p) > \geq \cdots \end{cases}
$$

In **BlockH10** and **BlockH11**, we observe that when the maximal subgroup $< (p,0),(0,p) >$ of H_2 is replaced with the maximal subgroup $< (p,1),(0,p) >$, each contributes p pairs of distinguishing factors, this gives rise to $p + p = 2p$ pairs of distinguishing factors. Similarly, other maximal subgroups of H_2 between **BlockH12** to **BlockH17**, including **BlockH12** and **BlockH17** yield $2p$ pairs of distinguishing factors. There are p such maximal subgroups from $< (p,1),(0,p) >, < (p,2),(0,p) >, \cdots, < (p,p-1),(0,p) >$, including $< (p^2,0),(0,1) >$, thus the number of pairs of distinguishing factors yielded by such subgroups is equal to $p(2p)$. (4)

H_2 contributes $p(p+1) + p(2p) + 2p = 3p^2 + 3p$ pairs of distinguishing factors. Thus the maximal subgroups of G other than H_1 yield $p(3p^2 + 3p) = 3p^3 + 3p^2$ pairs of distinguishing factors. (5)

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Hence the total number of pairs of distinguishing factors of G is $3p^3 + 3p^2 + 3p^2 = 3p^3 + 6p^2$. This completes the proof.□

Proposition 3.3.12. *$G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3}$ has $p^2[3+2(p+2)+3(2p+1)] = p^2[10+8p] = 8p^3+10p^2$ pairs of distinguishing factors in its maximal chains of subgroups.*

Proof: There are $1 + p$ maximal subgroups of G. One such maximal subgroup is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} =< (p,0),(0,1) >$ which has $3p^3 + 6p^2$ pairs of distinguishing factors by the previous Proposition 3.3.11. (1)

Let $H_2 = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} =< (1,0),(0,p) >$ be another maximal subgroup of G. We want maximal chains of H_2 contributing pairs of distinguishing factors. Now consider the first chain $H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots$ in **BlockHH4** below, thus H_2 has a single distinguishing factor H_2, (not a pair), hence we do not add it to pairs of distinguishing factor. The next chains have pairs of distinguishing factors. We identify these pairs of distinguishing factors of H_2. These pairs of distinguishing factors come from pairing H_2 and 6th column of $BlockHH4$, pairing H_2 and 5th column of

$BlockHH5$ and pairing H_2 and 4th column of $BlockHH6$ below:

$$
\textbf{BlockHH4} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\
\geq < (0,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\
\geq < (p^3,0) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\
\geq < (p^3,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\
\geq < (p^3,2p^2) > \geq \cdots \\
\quad \vdots \qquad \vdots \quad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\
\geq < (p^3,(p-1)p^2) > \geq \cdots
\end{cases}
$$

$$
\textbf{BlockHH5} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\
\quad \vdots \qquad \vdots \quad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \geq \cdots
\end{cases}
$$

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$$
\textbf{BlockHH6} = \begin{cases}
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\
\quad \vdots \qquad \vdots \quad \vdots \\
H_2 \geq < (p,0),(0,p) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots
\end{cases}
$$

BlockHH4, **BlockHH5** and **BlockHH6** contribute $p+p+p=3p$ pairs of distinguishing factors. (2)

Next we identify pairs of distinguishing factors in **BlockHH7**, **BlockHH8**, **BlockHH9**, **BlockHH10** and **BlockHH11**. These pairs of distinguishing factors come from pairing H_2 in each of the first chain of **BlockHH7**, **BlockHH8**, **BlockHH9**, **BlockHH10** and **BlockHH11** with the 3rd column and pairing H_2 and 4th column of the remaining chains in each of the following blocks.

$$\textbf{BlockHH7} = \begin{cases} H_2 \geq <(p,0),(0,p)> \geq <(p,p),(0,p^2)> \geq <(p^2,0),(0,p^2)> \geq <(p^3,0),(0,p^2)> \\ \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,p),(0,p^2)> \geq <(p,p)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,p),(0,p^2)> \geq <(p,p+p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,p),(0,p^2)> \geq <(p,p+2p^2)> \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,p),(0,p^2)> \geq <(p,p+(p-1)p^2)> \geq \cdots \end{cases}$$

$$\textbf{BlockHH8} = \begin{cases} H_2 \geq <(p,0),(0,p)> \geq <(p,2p),(0,p^2)> \geq <(p^2,0),(0,p^2)> \geq <(p^3,0),(0,p^2)> \\ \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,2p),(0,p^2)> \geq <(p,2p)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,2p),(0,p^2)> \geq <(p,2p+p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,2p),(0,p^2)> \geq <(p,2p+2p^2)> \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,2p),(0,p^2)> \geq <(p,2p+(p-1)p^2)> \geq \cdots \end{cases}$$

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$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\textbf{BlockHH9} = \begin{cases} H_2 \geq <(p,0),(0,p)> \geq <(p,(p-1)p),(0,p^2)> \geq <(p^2,0),(0,p^2)> \geq <(p^3,0),(0,p^2)> \\ \geq <(0,p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,(p-1)p),(0,p^2)> \geq <(p,(p-1)p)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,(p-1)p),(0,p^2)> \geq <(p,(p-1)p+p^2)> \geq \cdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,(p-1)p),(0,p^2)> \geq <(p,(p-1)p+2p^2)> \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq <(p,0),(0,p)> \geq <(p,(p-1)p),(0,p^2)> \geq <(p,(p-1)p+(p-1)p^2)> \geq \cdots \end{cases}$$

$$\textbf{BlockHH10} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,2p) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad\qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^2,(p-1)p) > \geq \cdots \end{cases}$$

$$\textbf{BlockHH11} = \begin{cases} H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^3,0),(0,p) > \geq < (p^3,0),(0,p^2) > \\ \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^3,p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^3,2p) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad\qquad \vdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (p^3,(p-1)p) > \geq \cdots \\ H_2 \geq < (p,0),(0,p) > \geq < (p^2,0),(0,p) > \geq < (0,p) > \geq \cdots \end{cases}$$

From **BlockHH7** to **BlockHH11**, we observe that there are p maximal subgroups of the subgroup $< (p,0),(0,p) >$ and each contributes $p+1$ pairs of distinguishing factors, giving rise to $p(p+1) = p^2 + p$ pairs of distinguishing factors. (3)

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Next we vary the maximal subgroups of H_2, e.g. replace $< (p,0),(0,p) >$ with $< (1,0),(0,p^2) >$. We observe that the first chain in $BlockHH12$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 6th column of $BlockHH12$, pairing 2nd column and 5th column of $BlockHH13$

and pairing 2nd column and 4th column of $BlockHH14$ below:

$$\textbf{BlockHH12} = \begin{cases} H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,0) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,p^2) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\textbf{BlockHH13} = \begin{cases} H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \geq \cdots \end{cases}$$

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$$\textbf{BlockHH14} = \begin{cases} H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,0),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots \end{cases}$$

Next we replace $< (p,0),(0,p) >$ with $< (1,p),(0,p^2) >$. We observe that the first chain in $BlockHH15$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 6th column of $BlockHH15$, pairing 2nd column and 5th column of $BlockHH16$ and pairing 2nd column and 4th column of $BlockHH17$ below:

$$\textbf{BlockHH15} = \begin{cases} H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,0) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,p^2) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,2p^2) > \geq \cdots \\ \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^3,0),(0,p^2) > \\ \geq < (p^3,(p-1)p^2) > \geq \cdots \end{cases}$$

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$$\textbf{BlockHH16} = \begin{cases} H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\ \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\textbf{BlockHH17} = \begin{cases} H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\ \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ H_2 \geq < (1,p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots \end{cases}$$

Next we replace $< (p,0),(0,p) >$ with $< (1,2p),(0,p^2) >$. We observe that the first chain in $BlockHH18$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 6th column of $BlockHH18$, pairing 2nd column and 5th column of $BlockHH19$ and pairing 2nd column and 4th column of $BlockHH20$ below:

$$\textbf{BlockHH18} = \begin{cases} H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,0) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,p^2) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,(p-1)p^2) > \geq \cdots \end{cases}$$

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$$\textbf{BlockHH19} = \begin{cases} H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\textbf{BlockHH20} = \begin{cases} H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq < (1,2p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

Similarly, we replace $< (p,0),(0,p) >$ with $< (1,(p-1)p),(0,p^2) >$. We observe that the first chain in $BlockHH21$ has a single distinguishing factor and thus does not add to pairs of distinguishing factors. We identify pairs of distinguishing factors of H_2 in the remaining chains. These pairs of distinguishing factors come from pairing 2nd column and 6th column of $BlockHH21$, pairing 2nd column and 5th column of $BlockHH22$ and pairing 2nd column and 4th column of $BlockHH23$ below:

$$\textbf{BlockHH21} = \begin{cases} H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (0,p^2) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,0) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,p^2) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \\ \geq < (p^3,0),(0,p^2) > \geq < (p^3,(p-1)p^2) > \geq \cdots \end{cases}$$

$$\textbf{BlockHH22} = \begin{cases} H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,0) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,p^2) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p^2,0),(0,p^2) > \geq < (p^2,(p-1)p^2) > \\ \geq \cdots \end{cases}$$

$$\textbf{BlockHH23} = \begin{cases} H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,0) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,p^2) > \geq \cdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,2p^2) > \geq \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H_2 \geq < (1,(p-1)p),(0,p^2) > \geq < (p,0),(0,p^2) > \geq < (p,(p-1)p^2) > \geq \cdots \end{cases}$$

In **BlockHH12**, **BlockHH13** and **BlockHH14**, we observe that when the maximal subgroup $< (p,0),(0,p) >$ of H_2 is replaced with the maximal subgroup $< (1,0),(0,p^2) >$, each contributes p pairs of distinguishing factors, this gives rise to $p+p+p+p = 4p$ pairs of distinguishing factors. Similarly, other maximal subgroups of H_2 between **BlockHH12**

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to **BlockHH23**, including **BlockHH12** and **BlockHH23** yield $4p$ pairs of distinguishing factors. There are p such maximal subgroups from $< (1,0),(0,p^2) >, < (1,p),(0,p^2) >, < (1,2p),(0,p^2) >, \cdots, < (1,(p-1)p),(0,p^2) >$, thus the number of pairs of distinguishing factors yielded by such subgroups is equal to $p(3p)$. (4)

H_2 contributes $p(p+1) + p(4p) + 3p = 5p^2 + 4p$ pairs of distinguishing factors. Thus the maximal subgroups of G other than H_1 yield $p(5p^2+4p) = 5p^3 + 4p^2$ pairs of distinguishing factors. (5)

Hence the total number of pairs of distinguishing factors of G is $5p^3 + 4p^2 + 3p^3 + 6p^2 = 8p^3 + 10p^2$. This completes the proof.□

Theorem 3.3.13. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$ has $p^2[3+(p+2)(n-2)+\frac{(2p+1)(n-2)(n-1)}{2}]$ pair distinguishing factors in its maximal chains of subgroups, where $n \geq 3$.*

Proof: For $n = 3$ and $n = 4$, result follows from Propositions 3.3.11 and 3.3.12. Then use induction on $n > 3$.□

Theorem 3.3.14. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$ has $\frac{p^3}{3!}[n(n-1)\cdots(n-m+1)(m-2)]$ triples of distinguishing factors in its maximal chains of subgroups, $\forall n \geq 2$ and $m = 3$.*

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Proof: G has $1+(n+2)p+\frac{n(n+3)}{2}p^2+\frac{(n-2)(n+2)(n+3)}{3!}p^3$ maximal chains by a previous Theorem 3.2.7. Out of these, $[(1+p)+2(1+p+p^2)+(n-2)(1+p+p^2+p^3)+(p-n-1)]$ are single distinguishing factors by previous Theorem 3.3.10 and $p^2[3+(p+2)(n-2)+\frac{(2p+1)(n-2)(n-1)}{2}]$ are pairs of distinguishing factors by previous Theorem 3.3.13. Then the number of triples of distinguishing factors is equal to $[(n+2)p+\frac{n(n+3)}{2}p^2+\frac{(n-2)(n+2)(n+3)}{3!}p^3]-[(1+p)+2(1+p+p^2)+(n-2)(1+p+p^2+p^3)+(p-n-1)]-p^2[3+(p+2)(n-2)+\frac{(2p+1)(n-2)(n-1)}{2}] = \frac{p^3}{3!}[n(n-1)(n-2)]$. This completes the proof.□

Note 3.3.2.1. In the above proof, the 1 in $1+(n+2)p+\frac{n(n+3)}{2}p^2+\frac{(n-2)(n+2)(n+3)}{3!}p^3$ counts the first maximal chain one considers and is not included in counting distinguishing factors.

From the above theorems, 3.3.10, 3.3.13 and 3.3.14 , we observe

Theorem 3.3.15. *For $n \geq 2$ and for any fixed prime integer p, the number of distinct fuzzy subgroup for the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^3}$ is equal to $2^{n+4} - 1 + p[(p^2+p+1)n - (p^2-1)(m-1)]2^{n+3} + p^2[3+(p+2)(n-2)+(2p+1)\frac{(n-2)(n-1)}{2}]2^{n+2} + [\frac{p^3}{6}n(n-1)(n-2)]2^{n+1}$.*

Combining theorems 3.3.7 and 3.3.15 , we obtain the following theorem for $n \in \mathbb{Z}^+$; $n \geq 2$; m = 2 and 3, and for any fixed prime p, in the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$:

Theorem 3.3.16. *For $n \in \mathbb{Z}^+$; $n \geq 2$; $m = 2$ and 3, and for any fixed prime p, the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ has $2^{n+m+1} - 1 + [(1+p) + (m-2)(m-1)(1+p+p^2) + (n+1-m)(1+p+p^2+\cdots+p^m) + (p-n+2-m)]2^{n+m} + p^2[m(m-2) + (p+2)(n-2)(m-2) + \frac{(2p+1)(n-2)(n-1)(m-2)}{(m-1)!} + \frac{n(n-1)\cdots(n-m+1)(3-m)}{m!}]2^{n+m-1} + \frac{p^3}{3!}[n(n-1)\cdots(n-m+1)(m-2)]2^{n+m-2}$ number of distinct fuzzy subgroups.*

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Chapter 4

ON CRISP SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$

4.1 Introduction

In [6], we made used of the following well-known formulae by [58]

1 $1+p+p^2+p^3+\cdots+p^{d_G-1}$ for the number of maximal subgroups of a p-group G where d_G is the rank of G. The rank of a group G is the minimal number of generators of G.

2 $\prod_{k=0}^{n-1} \frac{p^{m-k}-1}{p^{n-k}-1}$ for the number of subgroups of order p^n in a p-group G of order p^m,

to derive and prove a user-friendly polynomial formula that computes the number of subgroups of some finite rank-3 abelian group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$. In this thesis and chapter, we will employ that same long established fact in [58] to extend our study in [6] to give and prove an explicit formula that determines the number of subgroups of the group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$.

4.1.1 On Subgroups Of $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$

Here, we use p = 3; 5 and $n = 2 = m$ to show how to compute the subgroups of G, and then proceed with proposition and give its proof.

Example 4.1.1. *Consider the group* $G = \mathbb{Z}_{3^2} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$.

The group G has 126 subgroups, and since G is of rank-3, it has $1 + p + p^3$ maximal subgroups and these are $<(1,0,0),(0,3,0),(0,0,1)>$, $<(3,0,0),(0,1,0),(0,0,1)>$, $<(3,0,0),(1,1,0),(0,0,1)>$, $<(3,0,0),(1,2,0),(0,0,1)>$, $<(1,0,0),(0,1,0)>$, $<(1,0,0),(0,1,1)>$, $<(1,0,0),(0,1,2)>$, $<(1,0,1),(0,1,0)>$, $<(1,0,1),(0,1,1)>$, $<(1,0,1),(0,1,2)>$,

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$<(1,0,2),(0,1,0)>$, $<(1,0,2),(0,1,1)>$ and $<(1,0,2),(0,1,2)>$.

The remaining subgroups of G can be classified as non-maximal subgroups and cyclic subgroups.

Non-maximal Non-cyclic Subgroups of G are:

$<(3,0,0),(0,3,0),(0,0,1)>$, $<(3,0,0),(0,1,0)>$, $<(3,0,0),(0,1,1)>$, $<(3,0,0),(0,1,2)>$, $<(1,0,0),(0,0,1)>$, $<(1,0,0),(0,3,0)>$, $<(1,0,0),(0,3,1)>$, $<(1,0,0),(0,3,2)>$, $<(1,0,1),(0,3,0)>$, $<(1,0,1),(0,3,1)>$, $<(1,0,1),(0,3,2)>$, $<(1,0,2),(0,3,0)>$, $<(1,0,2),(0,3,1)>$, $<(1,0,2),(0,3,2)>$, $<(1,3,0),(0,0,1)>$, $<(1,6,0),(0,0,1)>$, $<(1,1,0),(0,0,1)>$, $<(1,1,0),(0,3,0)>$, $<(1,1,0),(0,3,1)>$, $<(1,1,0),(0,3,2)>$, $<(1,1,1),(0,3,0)>$, $<(1,1,1),(0,3,1)>$, $<(1,1,1),(0,3,2)>$, $<(1,1,2),(0,3,0)>$, $<(1,1,2),(0,3,1)>$, $<(1,1,2),(0,3,2)>$, $<(1,4,0),(0,0,1)>$, $<(1,7,0),(0,0,1)>$, $<(1,2,0),(0,0,1)>$, $<(1,2,0),(0,3,0)>$, $<(1,2,0),(0,3,1)>$, $<(1,2,0),(0,3,2)>$, $<(1,2,1),(0,3,0)>$, $<(1,2,1),(0,3,1)>$, $<(1,2,1),(0,3,2)>$, $<(1,2,2),(0,3,0)>$, $<(1,2,2),(0,3,1)>$, $<(1,2,2),(0,3,2)>$, $<(1,5,0),(0,0,1)>$, $<(1,8,0),(0,0,1)>$, $<(1,0,1),(0,3,0)>$, $<(1,0,2),(0,3,0)>$, $<(0,1,0),(0,0,1)>$, $<(3,0,0),(0,3,0)>$, $<(3,0,0),(0,0,1)>$, $<(3,0,0),(0,3,1)>$, $<(3,0,0),(0,3,2)>$, $<(3,0,1),(0,1,0)>$, $<(3,0,1),(0,1,1)>$, $<(3,0,1),(0,1,2)>$, $<(3,0,2),(0,1,0)>$, $<(3,0,2),(0,1,1)>$, $<(3,0,2),(0,1,2)>$, $<(3,0,1),(0,3,0)>$, $<(3,0,1),(0,3,1)>$, $<(3,0,1),(0,3,2)>$, $<(3,0,2),(0,3,0)>$, $<(3,0,2),(0,3,1)>$, $<(3,0,2),(0,3,2)>$, $<(3,1,0),(0,0,1)>$, $<(3,2,0),(0,0,1)>$, $<(3,3,0),(0,0,1)>$, $<(3,6,0),(0,0,1)>$, $<(0,3,0),(0,0,1)>$

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Cyclic subgroups of G:

$<(1,0,0)>$, $<(1,0,1)>$, $<(1,0,2)>$, $<(1,1,0)>$, $<(1,1,1)>$, $<(1,1,2)>$, $<(1,2,0)>$, $<(1,2,1)>$, $<(1,2,2)>$, $<(1,3,0)>$, $<(1,3,1)>$, $<(1,3,2)>$, $<(1,4,0)>$, $<(1,4,1)>$, $<(1,4,2)>$ $<(1,5,0)>$, $<(1,5,1)>$, $<(1,5,2)>$, $<(1,6,0)>$, $<(1,6,1)>$, $<(1,6,2)>$, $<(1,7,0)>$, $<(1,7,1)>$, $<(1,7,2)>$, $<(1,8,0)>$, $<(1,8,1)>$, $<(1,8,2)>$, $<(3,0,0)>$, $<(3,0,1)>$, $<(3,0,2)>$, $<(3,1,0)>$, $<(3,1,1)>$, $<(3,1,2)>$, $<(3,2,0)>$, $<(3,2,1)>$, $<(3,2,2)>$, $<(3,3,0)>$, $<(3,3,1)>$, $<(3,3,2)>$, $<(3,6,0)>$, $<(3,6,1)>$, $<(3,6,2)>$, $<(0,1,0)>$, $<(0,1,1)>$, $<(0,1,2)>$, $<(0,3,0)>$, $<(0,3,1)>$, $<(0,3,2)>$, $<(0,0,1)>$, $<(0,0,0)>$ and G itself.

Note 4.1.1.1. Number of cyclic subgroups of $G = \mathbb{Z}_{3^2} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$ in Example 4.1.1 is 50. A typical generator of such cyclic subgroup is $<(a,b,c)>$ where $a = 1,3,0$. We observed the following cases:

Case $a = 1$ implies $b = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $c = \{0, 1, 2\}$. So this gives 9×3 cyclic subgroups (1)

Case $a = 3$ implies $b = \{0, 1, 2, 3, 6\}$ and $c = \{0, 1, 2\}$, giving 5×3 cyclic subgroups (2)

Case $a = 0$, $b = \{0, 1, 2\}$ and $c = \{0, 1, 2\}$, but $< (0, 0, 1) >=< (0, 0, 2) >$ implies we do not have 3×3 subgroups but $2 \times 3 + 2 \times 1$ cyclic subgroups, where 2 counts the groups G and $< (0, 0, 0) >$. (3)

Using (1), (2) and (3), G has $3^3 + 5 \times 3 + 2 \times 3 + 2 = 3^3 + (6-1)3 + 2 \times 3 + 2 = 3^3 + 2 \times 3^2 + 3 + 2$ cyclic subgroups. This suggests the formula $p^3 + 2p^2 + p + 2$ for the number of cyclic subgroups for $\mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. (4)

Note 4.1.1.2. Number of non-cyclic non-maximal subgroups of $G = \mathbb{Z}_{3^2} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$ in Example 4.1.1 is 62. There is only 1 rank-3 subgroup here viz $< (p, 0, 0), (0, p, 0), (0, 0, 1) >$ where $p = 3$.

Rank-2 subgroups here are of the form $< (a, b, c), (d, e, f) >$ where $a = 1, 3, 0$:

When $a = 1$, $b = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $c = \{0, 1, 2\}$ implies $d = 0$, $e = \{0, 1, 3\}$ and $f = \{0, 1, 2\}$ So with $a = 1$, we have 36 non-cyclic non-maximal subgroups (1)

When $a = 3$, $b = \{0, 1, 2, 3, 6\}$ and $c = \{0, 1, 2\}$ implies $d = 0$, $e = \{0, 1, 3\}$ and $f = \{0, 1, 2\}$ giving 23 non-cyclic non-maximal subgroups (2)

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When $a = 0$, $b = \{1, 3\}$ and $c = 0$ implies $d = 0$, $e = 0$ and $f = 1$ giving 2 non-cyclic non-maximal subgroups (3)

Using (1), (2) and 1(rank-3), G has $60 = 3^3 + 27 + 2 \times 3 = 3^3 + 3 \times 3^2 + 2 \times 3 = p^3 + 3p^2 + 2p$ non-cyclic non-maximal subgroups (4)

. Thus, the number of non-cyclic non-maximal subgroups for $\mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$, is equal to (3) + (4) $= p^3 + 3p^2 + 2p + 2$, for $p = 3$. (5)

Example 4.1.2. *Consider the group* $G = \mathbb{Z}_{5^2} + \mathbb{Z}_{5^2} + \mathbb{Z}_5$.

The group G has 426 subgroups, and since G is of rank-3, it has $1 + p + p^3$ maximal subgroups and these are $< (1, 0, 0), (0, 5, 0), (0, 0, 1) >$, $< (5, 0, 0), (0, 1, 0), (0, 0, 1) >$, $< (5, 0, 0), (1, 1, 0), (0, 0, 1) >$, $< (5, 0, 0), (1, 2, 0), (0, 0, 1) >$, $< (5, 0, 0), (1, 3, 0), (0, 0, 1) >$, $< (5, 0, 0), (1, 4, 0), (0, 0, 1) >$, $< (1, 0, 0), (0, 1, 0) >$, $< (1, 0, 0), (0, 1, 1) >$, $< (1, 0, 0), (0, 1, 2) >$, $< (1, 0, 0), (0, 1, 3) >$, $< (1, 0, 0), (0, 1, 4) >$, $< (1, 0, 1), (0, 1, 0) >$, $< (1, 0, 1), (0, 1, 1) >$, $< (1, 0, 1), (0, 1, 2) >$, $< (1, 0, 1), (0, 1, 3) >$, $< (1, 0, 1), (0, 1, 4) >$, $< (1, 0, 2), (0, 1, 0) >$, $< (1, 0, 2), (0, 1, 1) >$, $< (1, 0, 2), (0, 1, 2) >$, $< (1, 0, 2), (0, 1, 3) >$, $< (1, 0, 2), (0, 1, 4) >$,

$<(1,0,3),(0,1,0)>$, $<(1,0,3),(0,1,1)>$, $<(1,0,3),(0,1,2)>$, $<(1,0,3),(0,1,3)>$, $<(1,0,3),(0,1,4)>$, $<(1,0,4),(0,1,0)>$, $<(1,0,4),(0,1,1)>$, $<(1,0,4),(0,1,2)>$, $<(1,0,4),(0,1,3)>$, $<(1,0,4),(0,1,4)>$.

The remaining subgroups of G can be classified as non-maximal subgroups and cyclic subgroups.

Non-maximal Non-cyclic Subgroups of G are:

$<(5,0,0),(0,5,0),(0,0,1)>$, $<(5,0,0),(0,1,0)>$, $<(5,0,0),(0,1,1)>$, $<(5,0,0),(0,1,2)>$, $<(5,0,0),(0,1,3)>$, $<(5,0,0),(0,1,4)>$, $<(1,0,0),(0,0,1)>$, $<(1,0,0),(0,5,0)>$, $<(1,0,0),(0,5,1)>$, $<(1,0,0),(0,5,2)>$, $<(1,0,0),(0,5,3)>$, $<(1,0,0),(0,5,4)>$, $<(1,0,1),(0,5,0)>$, $<(1,0,1),(0,5,1)>$, $<(1,0,1),(0,5,2)>$, $<(1,0,1),(0,5,3)>$, $<(1,0,1),(0,5,4)>$, $<(1,0,2),(0,5,0)>$, $<(1,0,2),(0,5,1)>$, $<(1,0,2),(0,5,2)>$, $<(1,0,2),(0,5,3)>$, $<(1,0,2),(0,5,4)>$, $<(1,0,3),(0,5,0)>$, $<(1,0,3),(0,5,1)>$, $<(1,0,3),(0,5,2)>$, $<(1,0,3),(0,5,3)>$, $<(1,0,3),(0,5,4)>$, $<(1,0,4),(0,5,0)>$, $<(1,0,4),(0,5,1)>$, $<(1,0,4),(0,5,2)>$, $<(1,0,4),(0,5,3)>$, $<(1,0,4),(0,5,4)>$, $<(1,5,0),(0,0,1)>$, $<(1,10,0),(0,0,1)>$, $<(1,15,0),(0,0,1)>$, $<(1,20,0),(0,0,1)>$, $<(1,1,0),(0,0,1)>$, $<(1,1,0),(0,5,0)>$, $<(1,1,0),(0,5,1)>$, $<(1,1,0),(0,5,2)>$, $<(1,1,0),(0,5,3)>$, $<(1,1,0),(0,5,4)>$, $<(1,1,1),(0,5,0)>$, $<(1,1,1),(0,5,1)>$, $<(1,1,1),(0,5,2)>$, $<(1,1,1),(0,5,3)>$, $<(1,1,1),(0,5,4)>$, $<(1,1,2),(0,5,0)>$, $<(1,1,2),(0,5,1)>$, $<(1,1,2),(0,5,2)>$, $<(1,1,2),(0,5,3)>$, $<(1,1,2),(0,5,4)>$, $<(1,1,3),(0,5,0)>$, $<(1,1,3),(0,5,1)>$, $<(1,1,3),(0,5,2)>$, $<(1,1,3),(0,5,3)>$, $<(1,1,3),(0,5,4)>$, $<(1,1,4),(0,5,0)>$, $<(1,1,4),(0,5,1)>$, $<(1,1,4),(0,5,2)>$, $<(1,1,4),(0,5,3)>$, $<(1,1,4),(0,5,4)>$, $<(1,2,0),(0,0,1)>$, $<(1,2,0),(0,5,0)>$, $<(1,2,0),(0,5,1)>$, $<(1,2,0),(0,5,2)>$, $<(1,2,0),(0,5,3)>$, $<(1,2,0),(0,5,4)>$, $<(1,2,1),(0,5,0)>$, $<(1,2,1),(0,5,1)>$, $<(1,2,1),(0,5,2)>$, $<(1,2,1),(0,5,3)>$, $<(1,2,1),(0,5,4)>$, $<(1,2,2),(0,5,0)>$, $<(1,2,2),(0,5,1)>$, $<(1,2,2),(0,5,2)>$, $<(1,2,2),(0,5,3)>$, $<(1,2,2),(0,5,4)>$, $<(1,2,3),(0,5,0)>$, $<(1,2,3),(0,5,1)>$, $<(1,2,3),(0,5,2)>$, $<(1,2,3),(0,5,3)>$, $<(1,2,3),(0,5,4)>$, $<(1,2,4),(0,5,0)>$, $<(1,2,4),(0,5,1)>$, $<(1,2,4),(0,5,2)>$, $<(1,2,4),(0,5,3)>$, $<(1,2,4),(0,5,4)>$, $<(1,3,0),(0,0,1)>$, $<(1,3,0),(0,5,0)>$, $<(1,3,0),(0,5,1)>$, $<(1,3,0),(0,5,2)>$, $<(1,3,0),(0,5,3)>$, $<(1,3,0),(0,5,4)>$, $<(1,3,1),(0,5,0)>$, $<(1,3,1),(0,5,1)>$, $<(1,3,1),(0,5,2)>$, $<(1,3,1),(0,5,3)>$, $<(1,3,1),(0,5,4)>$, $<(1,3,2),(0,5,0)>$, $<(1,3,2),(0,5,1)>$, $<(1,3,2),(0,5,2)>$, $<(1,3,2),(0,5,3)>$, $<(1,3,2),(0,5,4)>$, $<(1,3,3),(0,5,0)>$, $<(1,3,3),(0,5,1)>$, $<(1,3,3),(0,5,2)>$, $<(1,3,3),(0,5,3)>$,

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$< (1,3,3),(0,5,4) >, < (1,3,4),(0,5,0) >, < (1,3,4),(0,5,1) >, < (1,3,4),(0,5,2) >,$
$< (1,3,4),(0,5,3) >, < (1,3,4),(0,5,4) >, < (1,4,0),(0,0,1) >, < (1,4,0),(0,5,0) >,$
$< (1,4,0),(0,5,1) >, < (1,4,0),(0,5,2) >, < (1,4,0),(0,5,3) >, < (1,4,0),(0,5,4) >,$
$< (1,4,1),(0,5,0) >, < (1,4,1),(0,5,1) >, < (1,4,1),(0,5,2) >, < (1,4,1),(0,5,3) >,$
$< (1,4,1),(0,5,4) >, < (1,4,2),(0,5,0) >, < (1,4,2),(0,5,1) >, < (1,4,2),(0,5,2) >,$
$< (1,4,2),(0,5,3) >, < (1,4,2),(0,5,4) >, < (1,4,3),(0,5,0) >, < (1,4,3),(0,5,1) >,$
$< (1,4,3),(0,5,2) >, < (1,4,3),(0,5,3) >, < (1,4,3),(0,5,4) >, < (1,4,4),(0,5,0) >,$
$< (1,4,4),(0,5,1) >, < (1,4,4),(0,5,2) >, < (1,4,4),(0,5,3) >, < (1,4,4),(0,5,4) >,$
$< (1,6,0),(0,0,1) >, < (1,7,0),(0,0,1) >, < (1,8,0),(0,0,1) >, < (1,9,0),(0,0,1) >,$
$< (1,11,0),(0,0,1) >, < (1,12,0),(0,0,1) >, < (1,13,0),(0,0,1) >, < (1,14,0),(0,0,1) >,$
$< (1,16,0),(0,0,1) >, < (1,17,0),(0,0,1) >, < (1,18,0),(0,0,1) >, < (1,19,0),(0,0,1) >,$
$< (1,21,0),(0,0,1) >, < (1,22,0),(0,0,1) >, < (1,23,0),(0,0,1) >, < (1,24,0),(0,0,1) >,$
$< (0,1,0),(0,0,1) >, < (5,0,0),(0,5,0) >, < (5,0,0),(0,0,1) >, < (5,0,0),(0,5,1) >,$
$< (5,0,0),(0,5,2) >, < (5,0,0),(0,5,3) >, < (5,0,0),(0,5,4) >, < (5,0,1),(0,1,0) >,$
$< (5,0,1),(0,1,1) >, < (5,0,1),(0,1,2) >, < (5,0,1),(0,1,3) >, < (5,0,1),(0,1,4) >,$
$< (5,0,2),(0,1,0) >, < (5,0,2),(0,1,1) >, < (5,0,2),(0,1,2) >, < (5,0,2),(0,1,3) >,$
$< (5,0,2),(0,1,4) >, < (5,0,3),(0,1,0) >, < (5,0,3),(0,1,1) >, < (5,0,3),(0,1,2) >,$
$< (5,0,3),(0,1,3) >, < (5,0,3),(0,1,4) >, < (5,0,4),(0,1,0) >, < (5,0,4),(0,1,1) >,$
$< (5,0,4),(0,1,2) >, < (5,0,4),(0,1,3) >, < (5,0,4),(0,1,4) >, < (5,0,1),(0,5,0) >,$
$< (5,0,1),(0,5,1) >, < (5,0,1),(0,5,2) >, < (5,0,1),(0,5,3) >, < (5,0,1),(0,5,4) >,$
$< (5,0,2),(0,5,0) >, < (5,0,2),(0,5,1) >, < (5,0,2),(0,5,2) >, < (5,0,2),(0,5,3) >,$
$< (5,0,2),(0,5,4) >, < (5,0,3),(0,5,0) >, < (5,0,3),(0,5,1) >, < (5,0,3),(0,5,2) >,$
$< (5,0,3),(0,5,3) >, < (5,0,3),(0,5,4) >, < (5,0,4),(0,5,0) >, < (5,0,4),(0,5,1) >,$
$< (5,0,4),(0,5,2) >, < (5,0,4),(0,5,3) >, < (5,0,4),(0,5,4) >, < (5,1,0),(0,0,1) >,$
$< (5,2,0),(0,0,1) >, < (5,3,0),(0,0,1) >, < (5,4,0),(0,0,1) >, < (5,5,0),(0,0,1) >,$
$< (5,10,0),(0,0,1) >, < (5,15,0),(0,0,1) >, < (5,20,0),(0,0,1) >, < (0,5,0),(0,0,1) >$

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Cyclic subgroups of G:

$< (1,0,0) >, < (1,0,1) >, < (1,0,2) >, < (1,0,3) >, < (1,0,4) >, < (1,1,0) >,$
$< (1,1,1) >, < (1,1,2) >, < (1,1,3) >, < (1,1,4) >, < (1,2,0) >, < (1,2,1) >,$
$< (1,2,2) >, < (1,2,3) >, < (1,2,4) >, < (1,3,0) >, < (1,3,1) >, < (1,3,2) >,$
$< (1,3,3) >, < (1,3,4) >, < (1,4,0) >, < (1,4,1) >, < (1,4,2) >, < (1,4,3) >,$
$< (1,4,4) >, < (1,5,0) >, < (1,5,1) >, < (1,5,2) >, < (1,5,3) >, < (1,5,4) >,$
$< (1,6,0) >, < (1,6,1) >, < (1,6,2) >, < (1,6,3) >, < (1,6,4) >, < (1,7,0) >,$

$<(1,7,1)>$, $<(1,7,2)>$, $<(1,7,3)>$, $<(1,7,4)>$, $<(1,8,0)>$, $<(1,8,1)>$, $<(1,8,2)>$, $<(1,8,3)>$, $<(1,8,4)>$, $<(1,9,0)>$, $<(1,9,1)>$, $<(1,9,2)>$, $<(1,9,3)>$, $<(1,9,4)>$, $<(1,10,0)>$, $<(1,10,1)>$, $<(1,10,2)>$, $<(1,10,3)>$, $<(1,10,4)>$, $<(1,11,0)>$, $<(1,11,1)>$, $<(1,11,2)>$, $<(1,11,3)>$, $<(1,11,4)>$, $<(1,12,0)>$, $<(1,12,1)>$, $<(1,12,2)>$, $<(1,12,3)>$, $<(1,12,4)>$, $<(1,13,0)>$, $<(1,13,1)>$, $<(1,13,2)>$, $<(1,13,3)>$, $<(1,13,4)>$, $<(1,14,0)>$, $<(1,14,1)>$, $<(1,14,2)>$, $<(1,14,3)>$, $<(1,14,4)>$, $<(1,15,0)>$, $<(1,15,1)>$, $<(1,15,2)>$, $<(1,15,3)>$, $<(1,15,4)>$, $<(1,16,0)>$, $<(1,16,1)>$, $<(1,16,2)>$, $<(1,16,3)>$, $<(1,16,4)>$, $<(1,17,0)>$, $<(1,17,1)>$, $<(1,17,2)>$, $<(1,17,3)>$, $<(1,17,4)>$, $<(1,18,0)>$, $<(1,18,1)>$, $<(1,18,2)>$, $<(1,18,3)>$, $<(1,18,4)>$, $<(1,19,0)>$, $<(1,19,1)>$, $<(1,19,2)>$, $<(1,19,3)>$, $<(1,19,4)>$, $<(1,20,0)>$, $<(1,20,1)>$, $<(1,20,2)>$, $<(1,20,3)>$, $<(1,20,4)>$, $<(1,21,0)>$, $<(1,21,1)>$, $<(1,21,2)>$, $<(1,21,3)>$, $<(1,21,4)>$, $<(1,22,0)>$, $<(1,22,1)>$, $<(1,22,2)>$, $<(1,22,3)>$, $<(1,22,4)>$, $<(1,23,0)>$, $<(1,23,1)>$, $<(1,23,2)>$, $<(1,23,3)>$, $<(1,23,4)>$, $<(1,24,0)>$, $<(1,24,1)>$, $<(1,24,2)>$, $<(1,24,3)>$, $<(1,24,4)>$, $<(5,0,0)>$, $<(5,0,1)>$, $<(5,0,2)>$, $<(5,0,3)>$, $<(5,0,4)>$, $<(5,1,0)>$, $<(5,1,1)>$, $<(5,1,2)>$, $<(5,1,3)>$, $<(5,1,4)>$, $<(5,2,0)>$, $<(5,2,1)>$, $<(5,2,2)>$, $<(5,2,3)>$, $<(5,2,4)>$, $<(5,3,0)>$, $<(5,3,1)>$, $<(5,3,2)>$, $<(5,3,3)>$, $<(5,3,4)>$, $<(5,4,0)>$, $<(5,4,1)>$, $<(5,4,2)>$, $<(5,4,3)>$, $<(5,4,4)>$, $<(5,5,0)>$, $<(5,5,1)>$, $<(5,5,2)>$, $<(5,5,3)>$, $<(5,5,4)>$, $<(5,10,0)>$, $<(5,10,1)>$, $<(5,10,2)>$, $<(5,10,3)>$, $<(5,10,4)>$, $<(5,15,0)>$, $<(5,15,1)>$, $<(5,15,2)>$, $<(5,15,3)>$, $<(5,15,4)>$, $<(5,20,0)>$, $<(5,20,1)>$, $<(5,20,2)>$, $<(5,20,3)>$, $<(5,20,4)>$, $<(0,1,0)>$, $<(0,1,1)>$, $<(0,1,2)>$, $<(0,1,3)>$, $<(0,1,4)>$, $<(0,5,0)>$, $<(0,5,1)>$, $<(0,5,2)>$, $<(0,5,3)>$, $<(0,5,4)>$, $<(0,0,1)>$, $<(0,0,0)>$ and G itself.

We calculated manually, the subgroups of $\mathbb{Z}_{5^2}+\mathbb{Z}_{5^2}+\mathbb{Z}_5$ and obtained 426. Using (4) in Note4.1.1.1 and (5) in Note4.1.1.2, we also obtain 426 upon including G and its maximal subgroups.

The following lemmas are clear.

Lemma 4.1.1. *For any prime p, $G=\mathbb{Z}_{p^2}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$ has p^3+2p^2+p+2 cyclic subgroups.*

Lemma 4.1.2. *For any prime p, $G=\mathbb{Z}_{p^2}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$ has p^3+3p^2+2p+2 non-cyclic non-maximal subgroups.*

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Proposition 4.1.3. *Let* $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. *Then the number of subgroups of* G *is equal to* $2p^3 + 6p^2 + 4p + 6$.

Proof: The result is obtained from Lemmas 4.1.1 and 4.1.2 and the inclusion of G and its $1+p+p^2$ maximal subgroups. Thus we have $p^3+2p^2+p+2+p^3+3p^2+2p+2+1+p+p^2+1 = 2p^3 + 6p^2 + 4p + 6$ as required.□

Example 4.1.3. *Next, consider the group* $G = \mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$.

The group G has 202 subgroups, and since G is of rank-3, it has $1 + p + p^2$ maximal subgroups and these are $<(1,0,0),(0,3,0),(0,0,1)>$, $<(3,0,0),(0,1,0),(0,0,1)>$, $<(3,0,0),(1,1,0),(0,0,1)>$, $<(3,0,0),(1,2,0),(0,0,1)>$, $<(1,0,0),(0,1,0)>$, $<(1,0,0),(0,1,1)>$, $<(1,0,0),(0,1,2)>$, $<(1,0,1),(0,1,0)>$, $<(1,0,1),(0,1,1)>$, $<(1,0,1),(0,1,2)>$, $<(1,0,2),(0,1,0)>$, $<(1,0,2),(0,1,1)>$, $<(1,0,2),(0,1,2)>$.
The remaining subgroups of G can be classified as non-maximal subgroups and cyclic subgroups.

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Non-maximal Non-cyclic Subgroups of G are:

$<(3,0,0),(0,3,0),(0,0,1)>$, $<(3,0,0),(0,1,0)>$, $<(3,0,0),(0,1,1)>$, $<(3,0,0),(0,1,2)>$,
$<(1,0,0),(0,0,1)>$, $<(1,0,0),(0,3,0)>$, $<(1,0,0),(0,3,1)>$, $<(1,0,0),(0,3,2)>$,
$<(1,0,1),(0,3,0)>$, $<(1,0,1),(0,3,1)>$, $<(1,0,1),(0,3,2)>$, $<(1,0,2),(0,3,0)>$,
$<(1,0,2),(0,3,1)>$, $<(1,0,2),(0,3,2)>$, $<(1,3,0),(0,0,1)>$, $<(1,6,0),(0,0,1)>$,
$<(1,1,0),(0,0,1)>$, $<(1,1,0),(0,3,0)>$, $<(1,1,0),(0,3,1)>$, $<(1,1,0),(0,3,2)>$,
$<(1,1,1),(0,3,0)>$, $<(1,1,1),(0,3,1)>$, $<(1,1,1),(0,3,2)>$, $<(1,1,2),(0,3,0)>$,
$<(1,1,2),(0,3,1)>$, $<(1,1,2),(0,3,2)>$, $<(1,4,0),(0,0,1)>$, $<(1,7,0),(0,0,1)>$,
$<(1,2,0),(0,0,1)>$, $<(1,2,0),(0,3,0)>$, $<(1,2,0),(0,3,1)>$, $<(1,2,0),(0,3,2)>$,
$<(1,2,1),(0,3,0)>$, $<(1,2,1),(0,3,1)>$, $<(1,2,1),(0,3,2)>$, $<(1,2,2),(0,3,0)>$,
$<(1,2,2),(0,3,1)>$, $<(1,2,2),(0,3,2)>$, $<(1,5,0),(0,0,1)>$, $<(1,8,0),(0,0,1)>$,
$<(1,0,1),(0,3,0)>$, $<(1,0,2),(0,3,0)>$, $<(0,1,0),(0,0,1)>$, $<(3,0,0),(0,3,0)>$,
$<(3,0,0),(0,0,1)>$, $<(3,0,0),(0,3,1)>$, $<(3,0,0),(0,3,2)>$, $<(3,0,1),(0,1,0)>$,
$<(3,0,1),(0,1,1)>$, $<(3,0,1),(0,1,2)>$, $<(3,0,2),(0,1,0)>$, $<(3,0,2),(0,1,1)>$,
$<(3,0,2),(0,1,2)>$, $<(3,0,1),(0,3,0)>$, $<(3,0,1),(0,3,1)>$, $<(3,0,1),(0,3,2)>$,
$<(3,0,2),(0,3,0)>$, $<(3,0,2),(0,3,1)>$, $<(3,0,2),(0,3,2)>$, $<(3,1,0),(0,3,0)>$,
$<(3,1,0),(0,3,1)>$, $<(3,1,0),(0,3,2)>$, $<(3,1,1),(0,3,0)>$, $<(3,1,1),(0,3,1)>$,
$<(3,1,1),(0,3,2)>$, $<(3,1,2),(0,3,0)>$, $<(3,1,2),(0,3,1)>$, $<(3,1,2),(0,3,2)>$,
$<(3,2,0),(0,3,0)>$, $<(3,2,0),(0,3,1)>$, $<(3,2,0),(0,3,2)>$, $<(3,2,1),(0,3,0)>$,

$<(3,2,1),(0,3,1)>$, $<(3,2,1),(0,3,2)>$, $<(3,2,2),(0,3,0)>$, $<(3,2,2),(0,3,1)>$,
$<(3,2,2),(0,3,2)>$, $<(3,1,0),(0,0,1)>$, $<(3,2,0),(0,0,1)>$, $<(3,3,0),(0,0,1)>$,
$<(3,4,0),(0,0,1)>$, $<(3,5,0),(0,0,1)>$, $<(3,6,0),(0,0,1)>$, $<(3,7,0),(0,0,1)>$,
$<(3,8,0),(0,0,1)>$, $<(9,0,0),(0,1,0),(0,0,1)>$, $<(9,0,0),(3,1,0),(0,0,1)>$,
$<(9,0,0),(3,2,0),(0,0,1)>$, $<(9,0,0),(0,3,0),(0,0,1)>$, $<(9,0,0),(0,1,0)>$, $<(9,0,0),(0,1,1)>$,
$<(9,0,0),(0,1,2)>$, $<(9,0,0),(0,0,1)>$, $<(9,0,0),(0,3,0)>$, $<(9,0,0),(0,3,1)>$,
$<(9,0,0),(0,3,2)>$, $<(9,0,1),(0,1,0)>$, $<(9,0,1),(0,1,1)>$, $<(9,0,1),(0,1,2)>$,
$<(9,0,2),(0,1,0)>$, $<(9,0,2),(0,1,1)>$, $<(9,0,2),(0,1,2)>$, $<(9,0,1),(0,3,0)>$,
$<(9,0,1),(0,3,1)>$, $<(9,0,1),(0,3,2)>$, $<(9,0,2),(0,3,0)>$, $<(9,0,2),(0,3,1)>$,
$<(9,0,2),(0,3,2)>$, $<(9,1,0),(0,0,1)>$, $<(9,2,0),(0,0,1)>$, $<(9,3,0),(0,0,1)>$,
$<(9,6,0),(0,0,1)>$, $<(0,3,0),(0,0,1)>$

Cyclic subgroups of G:

$<(1,0,0)>$, $<(1,0,1)>$, $<(1,0,2)>$, $<(1,1,0)>$, $<(1,1,1)>$, $<(1,1,2)>$,
$<(1,2,0)>$, $<(1,2,1)>$, $<(1,2,2)>$, $<(1,3,0)>$, $<(1,3,1)>$, $<(1,3,2)>$,
$<(1,4,0)>$, $<(1,4,1)>$, $<(1,4,2)>$, $<(1,5,0)>$, $<(1,5,1)>$, $<(1,5,2)>$, $<(1,6,0)>$,
$<(1,6,1)>$, $<(1,6,2)>$, $<(1,7,0)>$, $<(1,7,1)>$, $<(1,7,2)>$, $<(1,8,0)>$,
$<(1,8,1)>$, $<(1,8,2)>$, $<(3,0,0)>$, $<(3,0,1)>$, $<(3,0,2)>$, $<(3,1,0)>$,
$<(3,1,1)>$, $<(3,1,2)>$, $<(3,2,0)>$, $<(3,2,1)>$, $<(3,2,2)>$, $<(3,3,0)>$,
$<(3,3,1)>$, $<(3,3,2)>$, $<(3,4,0)>$, $<(3,4,1)>$, $<(3,4,2)>$, $<(3,5,0)>$,
$<(3,5,1)>$, $<(3,5,2)>$, $<(3,6,0)>$, $<(3,6,1)>$, $<(3,6,2)>$, $<(3,7,0)>$,
$<(3,7,1)>$, $<(3,7,2)>$, $<(3,8,0)>$, $<(3,8,1)>$, $<(3,8,2)>$, $<(9,0,0)>$,
$<(9,0,1)>$, $<(9,0,2)>$, $<(9,1,0)>$, $<(9,1,1)>$, $<(9,1,2)>$, $<(9,2,0)>$,
$<(9,2,1)>$, $<(9,2,2)>$, $<(9,3,0)>$, $<(9,3,1)>$, $<(9,3,2)>$, $<(9,6,0)>$,
$<(9,6,1)>$, $<(9,6,2)>$, $<(0,1,0)>$, $<(0,1,1)>$, $<(0,1,2)>$, $<(0,3,0)>$,
$<(0,3,1)>$, $<(0,3,2)>$, $<(0,0,1)>$, $<(0,0,0)>$ and G itself.

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Note 4.1.1.3. The number of cyclic subgroups of $G = \mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$ is given by $2\times 3^3 + 2\times 3^2 + 3 + 2$ and the number of non-cyclic non-maximal subgroups is given by $2\times 3^3 + 5\times 3^2 + 3\times 3 + 3$, using methods similar to the above Examples 4.1.1 and 4.1.2. Adding, we see that the total number of subgroups of G is equal to $(2\times 3^3 + 2\times 3^2 + 3 + 2) + (2\times 3^3 + 5\times 3^2 + 3\times 3 + 3) + (1 + 3 + 3^2) + 1 = 4\times 3^3 + 8\times 3^2 + 5\times 3 + 7$.

This suggests the following Proposition:

Proposition 4.1.4. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. Then the number of subgroups of G is equal to $4p^3 + 8p^2 + 5p + 7$.*

Proof: G has $1 + p + p^2$ maximal subgroups, since G is of rank-3 and of these, $1 + p$ of them are of rank 3, and so p^2 of them are of rank 2. We pick a rank 3 maximal subgroup, say $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (p, 0, 0), (0, 1, 0), (0, 0, 1) >$ and use the previous Proposition 4.1.3 to find its number of subgroups. So H_1 has $2p^3 + 6p^2 + 4p + 6$ subgroups. Let us pick another rank 3 maximal subgroup, say $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_p + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ and compute the number of subgroups here not counted in H_1:

$$\textbf{Block1} = \begin{cases} < (1,0,0), (0,0,1) > \\ < (1,0,0), (0,p,0) > \\ < (1,0,0), (0,p,1) > \\ < (1,0,0), (0,p,2) > \\ \vdots \quad \vdots \quad \vdots \\ < (1,0,0), (0,p,p-1) > \end{cases}$$

$$\textbf{Block2} = \begin{cases} < (1,p,0), (0,0,1) > \\ < (1,p,0), (0,p,1) > \\ < (1,p,0), (0,p,2) > \\ \vdots \quad \vdots \quad \vdots \\ < (1,p,0), (0,p,p-1) > \end{cases}$$

$$\textbf{Block3} = \begin{cases} < (1,p,1), (0,p,0) > \\ < (1,p,1), (0,p,2) > \\ \dots \quad \dots \quad \dots \\ < (1,p,1), (0,p,p-1) > \end{cases}$$

$$\textbf{Block4} = \begin{cases} < (1,p,2), (0,p,0) > \\ < (1,p,2), (0,p,1) > \\ \dots \quad \dots \quad \dots \\ < (1,p,2), (0,p,p-1) > \end{cases}$$

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Observe that the subgroup $<(1,p,2),(0,p,2)>$ is not part of the above **Block4**.

$$\textbf{Block5} \quad = \quad \begin{cases} <(1,p,3),(0,p,0)> \\ <(1,p,3),(0,p,1)> \\ \quad \cdots \quad \cdots \quad \cdots \\ <(1,p,3),(0,p,p-1)> \end{cases}$$

Observe that the subgroup $<(1,p,3),(0,p,3)>$ is not part of the above **Block5**.

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$$\textbf{Block6} \quad = \quad \begin{cases} <(1,p,p-1),(0,p,0)> \\ <(1,p,p-1),(0,p,1)> \\ <(1,p,p-1),(0,p,2)> \\ \quad \cdots \quad \cdots \quad \cdots \\ <(1,p,p-1),(0,p,p-2)> \end{cases}$$

Blocks 1 and 2 contribute $p+1$ and p subgroups respectively. The remaining $p-1$ blocks yield $(p-1)$ rank 2 subgroups each. Thus, the total number for rank 2 subgroups is $(p-1)^2+2p+1 = p^2-2p+1+2p+1 = p^2+2$. There are other rank 2 subgroups of the form $<(1,kp,0),(0,0,1)>$ for $k=2,3,\cdots,p-1$ not counted in H_1. Their number is $p-2$. Adding this to p^2+2 gives $p+p^2$ rank 2 subgroups of H_2. There are also p^2 cyclic subgroups not counted in H_1. There are still p^2 rank-2 maximal subgroups left. None yields a new subgroup except the group itself. Thus the total number of subgroups is equal to $(2p^3+6p^2+4p+6)+p(1+p+2p^2)+p^2+1 = 4p^3+8p^2+5p+7$. We multiplied $(1+p+2p^2)$ by p because there are p maximal subgroups of G isomorphic to H_2. This completes the proof.□

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Proposition 4.1.5. *Let $G=\mathbb{Z}_{p^4}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$. Then the number of subgroups of G is equal to $6p^3+10p^2+6p+8$.*

Proof: G has $1+p+p^2$ maximal subgroups, since G is of rank-3 and of these, $1+p$ of them are of rank 3, and so p^2 of them are of rank 2. We pick a rank 3 maximal subgroup, say $H_1=\mathbb{Z}_{p^3}+\mathbb{Z}_{p^2}+\mathbb{Z}_p =<(p,0,0),(0,1,0),(0,0,1)>$ and use the previous Proposition 4.1.4 to find its number of subgroups. So H_1 has $4p^3+8p^2+5p+7$ subgroups including H_1. Another rank 3 maximal subgroup is $H_2=\mathbb{Z}_{p^4}+\mathbb{Z}_p+\mathbb{Z}_p =<(1,0,0),(0,p,0),(0,0,1)>$. We look at subgroups of H_2 not appearing in H_1:

$$\textbf{Block1} \quad = \quad \begin{cases} <(1,0,0),(0,0,1)> \\ <(1,0,0),(0,p,0)> \\ <(1,0,0),(0,p,1)> \\ <(1,0,0),(0,p,2)> \\ \quad \ldots \quad \ldots \quad \ldots \\ <(1,0,0),(0,p,p-1)> \end{cases}$$

$$\textbf{Block2} \quad = \quad \begin{cases} <(1,p,0),(0,0,1)> \\ <(1,p,0),(0,p,1)> \\ \quad \ldots \quad \ldots \quad \ldots \\ <(1,p,0),(0,p,p-1)> \end{cases}$$

$$\textbf{Block3} \quad = \quad \begin{cases} <(1,p,1),(0,p,0)> \\ <(1,p,1),(0,p,2)> \\ \quad \ldots \quad \ldots \quad \ldots \\ <(1,p,1),(0,p,p-1)> \end{cases}$$

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$$\textbf{Block4} \quad = \quad \begin{cases} <(1,p,2),(0,p,0)> \\ <(1,p,2),(0,p,1)> \\ \quad \ldots \quad \ldots \quad \ldots \\ <(1,p,2),(0,p,p-1)> \end{cases}$$

Observe that the subgroup $<(1,p,2),(0,p,2)>$ is not part of the above **Block4**.

$$\textbf{Block5} \quad = \quad \begin{cases} <(1,p,3),(0,p,0)> \\ <(1,p,3),(0,p,1)> \\ \quad \ldots \quad \ldots \quad \ldots \\ <(1,kp,0)(0,0,1)> \end{cases}$$

Observe that the subgroup $<(1,p,3),(0,p,3)>$ is not part of the above **Block5**.

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\textbf{Block6} = \begin{cases} <(1,p,p-1),(0,p,0)> \\ <(1,p,p-1),(0,p,1)> \\ <(1,p,p-1),(0,p,2)> \\ \quad \dots \quad \dots \quad \dots \\ <(1,p,p-1),(0,p,p-2)> \end{cases}$$

Also

$$\textbf{Block7} = \begin{cases} <(1,(p-1)p,0),(0,0,1)> \\ <(1,(p-2)p,0),(0,0,1)> \\ \quad \dots \quad \dots \quad \dots \\ <(1,2p,0),(0,0,1)> \end{cases}$$

Observe that there are $p-2$ subgroups in **Block7**. Counting as in the previous Proposition 4.1.4, H_2 contributes $2p^2+p+1$ subgroups not counted in H_1. Each rank-3 maximal subgroup is isomorphic to H_2. So these yield $p(1+p+2p^2) = p+p^2+2p^3$. The other rank-2 maximal subgroups yield themselves only as new subgroup for a p^2 contribution. Counting G as well, the number of subgroups of G contributed is $(4p^3+8p^2+5p+7)+p+p^2+2p^3+p^2+1$. Thus the total number of subgroups is equal to $6p^3+10p^2+6p+8$ as required.□

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This leads to the Theorem

Theorem 4.1.6. *Let $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$, $n \geq 2$. The number of subgroups of G is $(2n-2)p^3 + (2n+2)p^2 + (n+2)p + n + 4$.*

Proof: This is done similarly to the above Propositions 4.1.3, 4.1.4 and 4.1.5, using induction on n.□

4.1.2 On Subgroups Of $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} + \mathbb{Z}_p$

Proposition 4.1.7. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} + \mathbb{Z}_p$. Then the number of subgroups of G is equal to $2p^4 + 6p^3 + 10p^2 + 6p + 8$.*

Proof: Let $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (p,0,0),(0,1,0),(0,0,1) >$. G has $1+p+p^2$ maximal subgroups of which $1+p$ are rank-3 and are isomorphic. By a previous Proposition 4.1.4, H_1 has $4p^3+8p^2+5p+7$ subgroups (with H_1 included). (1)

Let $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1,0,0),(0,p,0),(0,0,1) >$ be a maximal subgroup of G different from H_1. We identify all subgroups of H_2 not appearing in H_1.

Rank-3:

$$\textbf{Block0} \quad = \quad \begin{cases} <(p,0,0),(1,p,0),(0,0,1)> \\ <(1,0,0),(0,p^2,0),(0,0,1)> \end{cases}$$

Rank-2:

$$\textbf{Block1} \quad = \quad \begin{cases} <(1,0,0),(0,0,1)> \\ <(1,0,0),(0,p,0)> \\ <(1,0,0),(0,p,1)> \\ \quad \vdots \quad \vdots \quad \vdots \\ <(1,0,0),(0,p,p-1)> \end{cases}$$

$$\textbf{Block2} \quad = \quad \begin{cases} <(1,p,0),(0,p,1)> \\ <(1,p,0),(0,p,2)> \\ \quad \vdots \quad \vdots \quad \vdots \\ <(1,p,0),(0,p,p-1)> \end{cases}$$

$$\textbf{Block3} \quad = \quad \begin{cases} <(1,p,1),(0,p,0)> \\ <(1,p,1),(0,p,2)> \\ \quad \vdots \quad \vdots \quad \vdots \\ <(1,p,1),(0,p,p-1)> \end{cases}$$

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$$\vdots \qquad \vdots \qquad \vdots$$

$$\textbf{Block4} \quad = \quad \begin{cases} <(1,p,p-1),(0,p,0)> \\ <(1,p,p-1),(0,p,1)> \\ \quad \vdots \quad \vdots \quad \vdots \\ <(1,p,p-1),(0,p,p-2)> \end{cases}$$

$$\textbf{Block5} \quad = \quad \begin{cases} <(1,p,0),(0,0,1)> \\ <(1,2p,0),(0,0,1)> \\ <(1,3p,0),(0,0,1)> \\ \quad \vdots \quad \vdots \quad \vdots \\ <(1,(p-1)p,0),(0,0,1)> \end{cases}$$

The total number of the above subgroups is $p + 1 + (p-1)p + p - 1 = p^2 + p$ (2)

Now we involve p^2 in rank-2: (Combined with p)

$$\textbf{Block6} = \begin{cases} < (1,0,0),(0,p^2,0) > \\ < (1,0,0),(0,p^2,1) > \\ \quad \vdots \quad \vdots \qquad \vdots \\ < (1,0,0),(0,p^2,p-1) > \end{cases}$$

$$\textbf{Block7} = \begin{cases} < (1,p^2,0),(0,p,1) > \\ < (1,p^2,0),(0,p,2) > \\ \quad \vdots \quad \vdots \qquad \vdots \\ < (1,p^2,0),(0,p,p-1) > \end{cases}$$

$$\textbf{Block8} = \begin{cases} < (1,p^2,1),(0,p,0) > \\ < (1,p^2,1),(0,p,1) > \\ \quad \vdots \quad \vdots \qquad \vdots \\ < (1,p^2,1),(0,p,p-1) > \end{cases}$$

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$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\textbf{Block9} = \begin{cases} < (1,p^2,p-1),(0,p,0) > \\ < (1,p^2,p-1),(0,p,1) > \\ \quad \vdots \quad \vdots \qquad \vdots \\ < (1,p^2,p-1),(0,p,p-1) > \end{cases}$$

$$\textbf{Block10} = \begin{cases} < (1,p^2,0),(0,0,1) > \\ < (1,2p^2,0),(0,0,1) > \\ < (1,3p^2,0),(0,0,1) > \\ \quad \vdots \quad \vdots \qquad \vdots \\ < (1,(p-1)p^2,0),(0,0,1) > \end{cases}$$

This yields $p + p - 1 + (p-1)p + p - 1 = p^2 + 2p - 2.$ (3)

Combine p^2 with p^2:

$$\textbf{Block11} \quad = \quad \begin{cases} < (1,p^2,0),(0,p^2,1) > \\ < (1,p^2,0),(0,p^2,2) > \\ \quad \vdots \quad \vdots \quad \quad \vdots \\ < (1,p^2,0),(0,p^2,p-1) > \end{cases}$$

$$\textbf{Block12} \quad = \quad \begin{cases} < (1,p^2,1),(0,p^2,0) > \\ < (1,p^2,1),(0,p^2,2) > \\ \quad \vdots \quad \vdots \quad \quad \vdots \\ < (1,p^2,1),(0,p^2,p-1) > \end{cases}$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$\textbf{Block13} \quad = \quad \begin{cases} < (1,p^2,p-1),(0,p^2,0) > \\ < (1,p^2,p-1),(0,p^2,1) > \\ \quad \vdots \quad \vdots \quad \quad \vdots \\ < (1,p^2,p-1),(0,p^2,p-2) > \end{cases}$$

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Here we get $(p-1)p$ subgroups. (4)

Combine p with p^2:

$$\textbf{Block14} \quad = \quad \begin{cases} < (1,p,0),(0,p^2,0) > \\ < (1,p,0),(0,p^2,1) > \\ \quad \vdots \quad \vdots \quad \quad \vdots \\ < (1,p,0),(0,p^2,p-1) > \end{cases}$$

$$\textbf{Block15} \quad = \quad \begin{cases} < (1,p,1),(0,p^2,0) > \\ < (1,p,1),(0,p^2,1) > \\ \quad \vdots \quad \vdots \quad \quad \vdots \\ < (1,p,1),(0,p^2,p-1) > \end{cases}$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$\textbf{Block16} \quad = \quad \begin{cases} <(1,p,p-1),(0,p^2,0)> \\ <(1,p,p-1),(0,p^2,1)> \\ \quad\vdots \quad \vdots \qquad \vdots \\ <(1,p,p-1),(0,p^2,p-1)> \end{cases}$$

There are $p \times p = p^2$ subgroups in this pack of blocks. Now replace the middle p with kp, for $k = 2, 3, \cdots, p-1$ to obtain packs of blocks with p subgroups. There are $p-1$ packs of blocks. Each pack of blocks has p^2, thus we have $(p-1)p^2$ subgroups. (5)

Combine p^2 with $kp; k = 2, \cdots, p-1$:

$$\textbf{Block17} \quad = \quad \begin{cases} <(1,p^2,0),(0,2p,0)> \\ <(1,p^2,0),(0,2p,1)> \\ \quad\vdots \quad \vdots \qquad \vdots \\ <(1,p^2,0),(0,2p,p-1)> \end{cases}$$

$$\textbf{Block18} \quad = \quad \begin{cases} <(1,p^2,0),(0,3p,0)> \\ <(1,p^2,0),(0,3p,1)> \\ \quad\vdots \quad \vdots \qquad \vdots \\ <(1,p^2,0),(0,3p,p-1)> \end{cases}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\textbf{Block19} \quad = \quad \begin{cases} <(1,p^2,0),(0,(p-1)p,0)> \\ <(1,p^2,0),(0,(p-1)p,1)> \\ \quad\vdots \quad \vdots \qquad \vdots \\ <(1,p^2,0),(0,(p-1)p,p-1)> \end{cases}$$

This yields $(p-2)p$ subgroups. Now repeat the above with 0 in $(1, p^2, 0)$ replaced with 1, then $2, \cdots$, then p.

Thus we have p packs like the above, giving a total of $(p-2)p^2$ subgroups. (6)

Cyclic Subgroups of H_2:

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$$\textbf{Block20} = \begin{cases} <(1,0,0)> \\ <(1,0,1)> \\ \vdots \quad \vdots \\ <(1,0,p-1)> \end{cases}$$

$$\textbf{Block21} = \begin{cases} <(1,p,0)> \\ <(1,p,1)> \\ \vdots \quad \vdots \\ <(1,p,p-1)> \end{cases}$$

$$\textbf{Block22} = \begin{cases} <(1,2p,0)> \\ <(1,2p,1)> \\ \vdots \quad \vdots \\ <(1,2p,p-1)> \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots$$

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$$\textbf{Block23} = \begin{cases} <(1,(p-1)p,0)> \\ <(1,(p-1)p,1)> \\ \vdots \quad \vdots \\ <(1,(p-1)p,p-1)> \end{cases}$$

This yields p^2 subgroups.

$$\textbf{Block24} = \begin{cases} <(1,p^2,0)> \\ <(1,p^2,1)> \\ \vdots \quad \vdots \\ <(1,p^2,p-1)> \end{cases}$$

$$\textbf{Block25} = \begin{cases} <(1,2p^2,0)> \\ <(1,2p^2,1)> \\ \vdots \quad \vdots \\ <(1,2p^2,p-1)> \end{cases}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\textbf{Block26} \quad = \quad \begin{cases} <(1,(p-1)p^2,0)> \\ <(1,(p-1)p^2,1)> \\ \;\vdots\;\; \vdots \\ <(1,(p-1)p^2,p-1)> \end{cases}$$

This yields $(p-1)p = p^2-p$. Therefore, the above yield $p^2+p^2-p = 2p^2-p$ cyclic subgroups for H_2. (7)

The total number for H_2: Using the 2 rank-3's, H_2 and (2) to (7), we have $1+2+p^2+p+p^2+2p-2+p^2-p+p^3-p^2+p^3-2p^2+2p^2-p = 1+p+2p^2+2p^3$ (8)

There are p maximal subgroups other than H_2 and all are isomorphic. Thus these yield $p(1+p+2p^2+2p^3) = p+p^2+2p^3+2p^4$ subgroups of G not counted in H_1. (9)

There are other p^2 maximal subgroups of G that are of rank-2 and yield only themselves as new subgroups of G. Adding this number and G, the total number of subgroups of G, including those in H_1, is $(4p^3+8p^2+5p+7)+1+p+2p^2+2p^3+2p^4 = 2p^4+6p^3+10p^2+6p+8$. This completes the proof.□

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Proposition 4.1.8. *Let $G = \mathbb{Z}_{p^4}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$. Then the number of subgroups of G is equal to $4p^4+8p^3+12p^2+7p+9$.*

Proof: Let $H_1 = \mathbb{Z}_{p^3}+\mathbb{Z}_{p^3}+\mathbb{Z}_p =<(p,0,0),(0,1,0),(0,0,1)>$. G has $1+p+p^2$ maximal subgroups of which $1+p$ are rank-3 and are isomorphic. By a previous Proposition 4.1.7, this yields $2p^4+6p^3+10p^2+6p+8$ subgroups of G (with H_1 included). (1)

Let $H_2 = \mathbb{Z}_{p^4}+\mathbb{Z}_{p^2}+\mathbb{Z}_p =<(1,0,0),(0,p,0),(0,0,1)>$. As in the previous Proposition4.1.5, H_2 yields $1+p+2p^2+2p^3$ subgroups not counted in H_1. There are other p maximal subgroups isomorphic to H_2, whose subgroups are not included in H_1, and their contribution is $p+2p^2+2p^3+2p^4$ (2)

Summing (1), (2) and G, the total number of subgroups of G is equal to $(2p^4+6p^3+10p^2+6p+8)+(1+p+2p^2+2p^3+2p^4) = 4p^4+8p^3+12p^2+7p+9$. This completes the proof.□

In what follows, we consider the subgroups of the group $G = \mathbb{Z}_{p^5}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$ for primes $p = 2,3,5,7,11$.

For $p=2$, the group $G = \mathbb{Z}_{2^5}+\mathbb{Z}_{2^3}+\mathbb{Z}_2$ has $258 = (3+5)(1+2)+2(5+3-(2*2-3))2^2+$

$2(5+3-(2*3-3))2^3+2(5+3-(2*4-3))2^4+2$ subgroups.

For $p=3$, the group $G=\mathbb{Z}_{3^5}+\mathbb{Z}_{3^3}+\mathbb{Z}_3$ has $916=(3+5)(1+3)+2(5+3-(2*2-3))3^2+2(5+3-(2*3-3))3^3+2(5+3-(2*4-3))3^4+2$ subgroups.

For $p=5$, the group $G=\mathbb{Z}_{5^5}+\mathbb{Z}_{5^3}+\mathbb{Z}_5$ contains $5400=(3+5)(1+5)+2(5+3-(2*2-3))5^2+2(5+3-(2*3-3))5^3+2(5+3-(2*4-3))5^4+2$ subgroups.

For $p=7$, the group $G=\mathbb{Z}_{7^5}+\mathbb{Z}_{7^3}+\mathbb{Z}_7$ contains $18588=(3+5)(1+7)+2(5+3-(2*2-3))7^2+2(5+3-(2*3-3))7^3+2(5+3-(2*4-3))7^4+2$ subgroups.

For $p=11$, the group $G=\mathbb{Z}_{11^5}+\mathbb{Z}_{11^3}+\mathbb{Z}_{11}$ contains $102948=(3+5)(1+11)+2(5+3-(2*2-3))11^2+2(5+3-(2*3-3))11^3+2(5+3-(2*4-3))11^4+2$ subgroups.

Hence the theorem:

Theorem 4.1.9. *Let $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$. Then the number of subgroups of G is equal to $(2n-4)p^4+2np^3+(2n+4)p^2+(n+3)p+n+5$, $\forall n\geq 3$.*

Proof: By induction on n.□

This result may be extended to cases $\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p, \forall m\leq n$. Thus we have

Theorem 4.1.10. *For any positive integers $n,m\geq 1$ and any fixed prime number p, the group $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p$ has $(n+m)(1+p)+2\sum_{k=2}^{m+1}[n+m-(2k-3)]p^k+2$ crisp subgroups.*

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Proof: It can be proved by induction on m and by similar arguments as in Theorem 4.1.6 and Theorem 4.1.9.□

Chapter 5

ON MAXIMAL CHAINS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$

5.1 Introduction

In this chapter, we discuss maximal chains, which are of fundamental importance and will form the basis for our study of distinct fuzzy subgroups in the next chapter. Thereafter, we generalise our results as propositions and theorems with their detailed proofs. For an illustrative example on maximal chains of G, see Appendix A7.

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Proposition 5.1.1. *Let $G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. Then the number of maximal chains of G is equal to $1 + 4p + 9p^2 + 11p^3 + 5p^4$.*

Proof: G has $1 + p + p^2$ maximal subgroups. One of the maximal subgroups of G is $H = \mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p =< (1,0,0),(0,p,0),(0,0,1) >$, thus it has $(1+p) + (p+p^2)[\frac{n(n+1)}{2}p + n] = (p+1)+(3p+2)(p^2+p)$ maximal chains by Theorem 2.2.2. All the rank-3 maximal subgroups are isomorphic, hence these contribute $(1+p)\{(1+p)+(p+p^2)[\frac{n(n+1)}{2}p+n]\} = (1+p)[(p+1)+(3p+2)(p^2+p)] = 3p^4+8p^3+8p^2+4p+1$ maximal chains. (1)
The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} =< (1,0,0),(0,1,0) >$ which has $(p+1)(2p+1) = 2p^2+3p+1$ maximal chains by Theorem3.2.4.
Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(2p^2+3p+1) = 2p^4+3p^3+p^2$ (2)
Therefore (1) + (2): $3p^4+8p^3+8p^2+4p+1+2p^4+3p^3+p^2 = 1+4p+9p^2+11p^3+5p^4$.
This completes the proof. □

Proposition 5.1.2. *Let* $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$*. Then the number of maximal chains of* G *is equal to* $1 + 5p + 14p^2 + 24p^3 + 16p^4$*.*

Proof: G has $1 + p + p^2$ maximal subgroups. One of the maximal subgroups of G is $H = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (p, 0, 0), (0, 1, 0), (0, 0, 1) >$, thus it has $1 + 4p + 9p^2 + 11p^3 + 5p^4$ maximal chains, by the previous Proposition 5.1.1. (1)

Another one is $H_1 = \mathbb{Z}_{p^3} + \mathbb{Z}_p + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ has $(1 + p) + (p + p^2)[\frac{n(n+1)}{2}p + n] = 1 + 4p + 9p^2 + 6p^3$ maximal chains by Theorem 2.2.2. The remaining (1 + p - 1) rank-3 maximal subgroups are isomorphic to H_1.

Thus they contribute $p\{(1 + p) + (p + p^2)[\frac{n(n+1)}{2}p + n]\} = p[(p + 1) + (6p + 3)(p^2 + p)] = 6p^4 + 9p^3 + 4p^2 + p$ maximal chains. (2)

The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} =< (1, 0, 0), (0, 1, 0) >$ which has $5p^2 + 4p + 1$ maximal chains by Proposition 3.2.2. Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(5p^2 + 4p + 1) = 5p^4 + 4p^3 + p^2$ (3)

Therefore (1) + (2) + (3): $1 + 4p + 9p^2 + 11p^3 + 5p^4 + 6p^4 + 9p^3 + 4p^2 + p + 5p^4 + 4p^3 + p^2 = 16p^4 + 24p^3 + 14p^2 + 5p + 1$. This completes the proof.□

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Proposition 5.1.3. *Let* $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$*. Then the number of maximal chains of* G *is equal to* $1 + 6p + 20p^2 + 43p^3 + 35p^4$*.*

Proof: G has $1 + p + p^2$ maximal subgroups. One of the maximal subgroups of G is $H = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (p, 0, 0), (0, 1, 0), (0, 0, 1) >$, thus it has $16p^4 + 24p^3 + 14p^2 + 5p + 1$ maximal chains, by the previous Proposition 5.1.2. (1)

Another one is $H_1 = \mathbb{Z}_{p^4} + \mathbb{Z}_p + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ which has $(1 + p) + (p + p^2)[\frac{n(n+1)}{2}p + n] = 1 + 5p + 14p^2 + 10p^3$ maximal chains by Theorem 2.2.2. The remaining (1 + p - 1) rank-3 maximal subgroups are isomorphic to H_1.

Thus they contribute $p\{(1 + p) + (p + p^2)[\frac{n(n+1)}{2}p + n]\} = p[(p + 1) + (10p + 4)(p^2 + p)] = 10p^4 + 14p^3 + 5p^2 + p$ maximal chains. (2)

The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} =< (1, 0, 0), (0, 1, 0) >$ which has $9p^2 + 5p + 1$ maximal chains by Proposition 3.2.3. Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(5p^2 + 4p + 1) = 9p^4 + 5p^3 + p^2$ (3)

Therefore (1) + (2) + (3): $16p^4 + 24p^3 + 14p^2 + 5p + 1 + 10p^4 + 14p^3 + 5p^2 + p + 9p^4 + 5p^3 + p^2 = 35p^4 + 43p^3 + 20p^2 + 6p + 1$. This completes the proof.□

Theorem 5.1.4. $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ *has* $1 + (n+2)p + \frac{(n+2)^2 + (n+2) - 2}{2}p^2 + \frac{(n+2)^3 + 3(n+2)^2 - 10(n+2) - 6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4$ *maximal chains, for* $n \geq 2$*.*

Proof: By induction on n. For $n = 2, 3, 4$, the result is true by the above Propositions5.1.1, 5.1.2 and 5.1.3.

Now assume that the result is true for any integer $k \geq 2$, that is $G = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $1 + (k+2)p + \frac{(k+2)^2+(k+2)-2}{2}p^2 + \frac{(k+2)^3+3(k+2)^2-10(k+2)-6}{3!}p^3 + \frac{2(k-1)(k+1)(k+3)}{3!}p^4$ maximal chains.

Next, let $G = \mathbb{Z}_{p[k+1]} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. One of the maximal subgroups of G is $H = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ which has $1 + (k+2)p + \frac{(k+2)^2+(k+2)-2}{2}p^2 + \frac{(k+2)^3+3(k+2)^2-10(k+2)-6}{3!}p^3 + \frac{2(k-1)(k+1)(k+3)}{3!}p^4$ maximal chains. (1)

Another maximal subgroup of G is $H_1 = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p + \mathbb{Z}_p$ which has $(1+p)+(p+p^2)[\frac{(k+1)(k+2)}{2}p + k+1]$ maximal chains by Theorem 2.2.2. So the p maximal subgroups isomorphic to H_1 contribute $p[(1+p)+(p+p^2)[\frac{(k+1)(k+2)}{2}p+k+1]] = p+(k+2)p^2 + \frac{(k+1)(k+2)+2(k+1)}{2}p^3 + \frac{(k+1)(k+2)}{2}p^4$ maximal chains. (2)

Another maximal subgroup of G is $H_2 = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2}$ which has $1 + (k+2)p + \frac{k(k+3)}{2}p^2$ maximal chains. Thus the p^2 rank 2 maximal subgroups contribute $p^2[1 + (k+2)p + \frac{k(k+3)}{2}p^2] = p^2 + (k+2)p^3 + \frac{k^2+3k}{2}p^4$ maximal chains of G. (3)

Hence the total number of maximal chains of G is given by summing (1), (2) and (3). This sum is $1 + (k+2)p + \frac{(k+2)^2+(k+2)-2}{2}p^2 + \frac{(k+2)^3+3(k+2)^2-10(k+2)-6}{3!}p^3 + \frac{2(k-1)(k+1)(k+3)}{3!}p^4 + p + (k+2)p^2 + \frac{(k+1)(k+2)+2(k+1)}{2}p^3 + \frac{(k+1)(k+2)}{2}p^4 + p^2 + (k+2)p^3 + \frac{k^2+3k}{2}p^4 = 1 + (k+3)p + \frac{(k+1)(k+4)+2(k+3)}{2}p^2 + \frac{k(k+2)(k+7)+3(k+1)(k+2)+12(k+1)}{3!}p^3 + \frac{2k^3+12k^2+16k}{3!}p^4 = 1 + (k+3)p + \frac{(k+3)^2+k+3)-2}{2}p^2 + \frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3 + \frac{2k(k+2)(k+4)}{3!}p^4$. This completes the proof.□

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Proposition 5.1.5. *Let $G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} + \mathbb{Z}_p$. Then the number of maximal chains of G is equal to $1 + 6p + 20p^2 + 43p^3 + 49p^4 + 21p^5$.*

Proof: G has $1 + p + p^2$ maximal subgroups. Out of these, $1 + p$ are of rank-3 and p^2 are of rank-2. One of the rank-3 maximal subgroups of G is $H = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1,0,0), (0,p,0), (0,0,1) >$, thus it has $1+(n+2)p + \frac{(n+2)^2+(n+2)-2}{2}p^2 + \frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4 = 1 + 5p + 14p^2 + 24p^3 + 16p^4$ maximal chains by Theorem 5.1.4. All the rank-3 maximal subgroups are isomorphic, hence these contribute $(1+p)\{1 + (n+2)p + \frac{(n+2)^2+(n+2)-2}{2}p^2 + \frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4\} = (1+p)(1 + 5p + 14p^2 + 24p^3 + 16p^4) = 1 + 5p + 14p^2 + 24p^3 + 16p^4 + p + 5p^2 + 14p^3 + 24p^4 + 16p^5 = 1 + 6p + 19p^2 + 38p^3 + 40p^4 + 16p^5$ maximal chains. (1)

The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} =< (1,0,0), (0,1,0) >$ which has $1 + 5p + 9p^2 + 5p^3$ maximal chains by Proposition3.2.5.

Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(1+$

$5p + 9p^2 + 5p^3) = 5p^5 + 9p^4 + 5p^3 + p^2$ (2)
Therefore (1) + (2): $1 + 6p + 19p^2 + 38p^3 + 40p^4 + 16p^5 + 5p^5 + 9p^4 + 5p^3 + p^2 = 1 + 6p + 20p^2 + 43p^3 + 49p^4 + 21p^5$. This completes the proof.□

Proposition 5.1.6. *Let $G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3} + \mathbb{Z}_p$. Then the number of maximal chains of G is equal to $1 + 7p + 27p^2 + 69p^3 + 106p^4 + 70p^5$.*

Proof: G has $1 + p + p^2$ maximal subgroups. Out of these, $1 + p$ are of rank-3 and p^2 are of rank-2. One of the rank-3 maximal subgroups of G is $H = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$, thus it has $1 + 6p + 20p^2 + 43p^3 + 49p^4 + 21p^5$ maximal chains by the previous Proposition5.1.5. (1)
Another one is $H_1 = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ which has $1 + (n + 2)p + \frac{(n+2)^2+(n+2)-2}{2}p^2 + \frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4 = 1 + 6p + 20p^2 + 43p^3 + 35p^4$ maximal chains by Theorem 5.1.4. All the remaining (1 + p - 1) rank-3 maximal subgroups are isomorphic to H_1, hence they contribute $p[1 + (n + 2)p + \frac{(n+2)^2+(n+2)-2}{2}p^2 + \frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4] = p(1 + 6p + 20p^2 + 43p^3 + 35p^4) = p + 6p^2 + 20p^3 + 43p^4 + 35p^5$ maximal chains. (2)
The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^4} + \mathbb{Z}_{p^3} =< (1, 0, 0), (0, 1, 0) >$ which has $1 + 6p + 14p^2 + 14p^3$ maximal chains by Proposition3.2.6.
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Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(1 + 6p + 14p^2 + 14p^3) = 14p^5 + 14p^4 + 6p^3 + p^2$ (3)
Therefore (1) + (2) + (3): $1 + 6p + 20p^2 + 43p^3 + 49p^4 + 21p^5 + p + 6p^2 + 20p^3 + 43p^4 + 35p^5 + 14p^5 + 14p^4 + 6p^3 + p^2 = 1 + 7p + 27p^2 + 69p^3 + 106p^4 + 70p^5$. This completes the proof.□

Proposition 5.1.7. *Let $G = \mathbb{Z}_{p^5} + \mathbb{Z}_{p^3} + \mathbb{Z}_p$. Then the number of maximal chains of G is equal to $1 + 8p + 35p^2 + 103p^3 + 195p^4 + 204p^5$.*

Proof: G has $1 + p + p^2$ maximal subgroups. Out of these, $1 + p$ are of rank-3 and p^2 are of rank-2. One of the rank-3 maximal subgroups of G is $H = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^3} + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$, thus it has $1 + 7p + 27p^2 + 69p^3 + 106p^4 + 70p^5$ maximal chains by the previous Proposition5.1.6. (1)
Another one is $H_1 = \mathbb{Z}_{p^5} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ which has $1 + (n + 2)p + \frac{(n+2)^2(n+2)-2}{2}p^2 + \frac{(n+2)^3 3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4 = 1 + 7p + 27p^2 + 69p^3 + 106p^4$ maximal chains by Theorem 5.1.4. All the remaining (1 + p - 1) rank-3 maximal subgroups are isomorphic to H_1, hence they contribute $p[1 + (n + 2)p + \frac{(n+2)^2(n+2)-2}{2}p^2 + \frac{(n+2)^3 3(n+2)^2-10(n+2)-6}{3!}p^3 + \frac{2(n-1)(n+1)(n+3)}{3!}p^4] = p(1 + 7p + 27p^2 + 69p^3 + 106p^4) = p + 7p^2 +$

$27p^3+69p^4+106p^5$ maximal chains. (2)

The rank-2 maximal subgroups are isomorphic to $\mathbb{Z}_{p^5}+\mathbb{Z}_{p^3} =< (1,0,0),(0,1,0) >$ which has $1+7p+20p^2+28p^3$ maximal chains by Proposition3.2.6.

Thus, the total number of maximal chains coming from rank-2 maximal subgroups is $p^2(1+7p+20p^2+28p^3) = 28p^5+20p^4+7p^3+p^2$ (3)

Therefore (1) + (2) + (3): $1+7p+27p^2+69p^3+106p^4+70p^5+p+7p^2+27p^3+69p^4+106p^5+28p^5+20p^4+7p^3+p^2 = 1+8p+35p^2+103p^3+195p^4+204p^5$. This completes the proof.□

Theorem 5.1.8. *$G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$ has $1+(n+3)p+\frac{(n+3)^2+(n+3)-2}{2}p^2+\frac{(n+3)^3+3(n+3)^2-10(n+3)-6}{3!}p^3+\frac{(n+4)[(n+3)^3+5(n+3)^2-42(n+3)+24]}{4!}p^4+\frac{3(n-2)(n+1)(n+3)(n+4)}{4!}p^5$ maximal chains, for $n \geq 3$.*

Proof: By induction on n. For $n=3,4,5$, the result is true by the above Propositions5.1.5, 5.1.6 and 5.1.7.

Now assume that the result is true for any integer $k \geq 3$, that is $G=\mathbb{Z}_{p^k}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$ has $1+(k+3)p+\frac{(k+3)^2+(k+3)-2}{2}p^2+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3+\frac{(k+4)[(k+3)^3+5(k+3)^2-42(k+3)+24]}{4!}p^4+\frac{3(k-2)(k+1)(k+3)(k+4)}{4!}p^5$ maximal chains.

Next, let $G=\mathbb{Z}_{p^{[k+1]}}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$. One of the maximal subgroups of G is $H=\mathbb{Z}_{p^k}+\mathbb{Z}_{p^3}+\mathbb{Z}_p$ which has $1+(k+3)p+\frac{(k+3)^2+(k+3)-2}{2}p^2+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3+\frac{(k+4)[(k+3)^3+5(k+3)^2-42(k+3)+24]}{4!}p^4+\frac{3(k-2)(k+1)(k+3)(k+4)}{4!}p^5$ maximal chains. (1)

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Another maximal subgroup of G is $H_1=\mathbb{Z}_{p^{k+1}}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$ which has $1+(k+1+2)p+\frac{(k+1+2)^2+(k+1+2)-2}{2}p^2+\frac{(k+1+2)^3+3(k+1+2)^2-10(k+1+2)-6}{3!}p^3+\frac{2(k+1-1)(k+1+1)(k+1+3)}{3!}p^4 = 1+(k+3)p+\frac{(k+3)^2+(k+3)-2}{2}p^2+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3+\frac{2k(k+2)(k+4)}{3!}p^4$ maximal chains by Theorem5.1.4. So the p maximal subgroups isomorphic to H_1 contribute $p[1+(k+3)p+\frac{(k+3)^2+(k+3)-2}{2}p^2+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3+\frac{2k(k+2)(k+4)}{3!}p^4] = p+(k+3)p^2+\frac{(k+3)^2+(k+3)-2}{2}p^3+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^4+\frac{2k(k+2)(k+4)}{3!}p^5$ maximal chains. (2)

Another maximal subgroup of G is $H_2=\mathbb{Z}_{p^{k+1}}+\mathbb{Z}_{p^3}$ which has $1+(k+1+3-1)p+\frac{(k+1+3-3)(k+1+3)}{2}p^2+\frac{(k+1-3+1)(k+1+2)(k+1+3)}{3!}p^3 = 1+(k+3)p+\frac{(k+1)(k+4)}{2}p^2+\frac{(k-1)(k+3)(k+4)}{3!}p^3$ maximal chains. Thus the p^2 rank 2 maximal subgroups contribute $p^2[1+(k+3)p+\frac{(k+1)(k+4)}{2}p^2+\frac{(k-1)(k+3)(k+4)}{3!}p^3] = p^2+(k+3)p^3+\frac{(k+1)(k+4)}{2}p^4+\frac{(k-1)(k+3)(k+4)}{3!}p^5$ maximal chains of G. (3)

Hence the total number of maximal chains of G is given by summing (1), (2) and (3). This sum is $1+(k+3)p+\frac{(k+3)^2+(k+3)-2}{2}p^2+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^3+\frac{(k+4)[(k+3)^3+5(k+3)^2-42(k+3)+24]}{4!}p^4+\frac{3(k-2)(k+1)(k+3)(k+4)}{4!}p^5+p+(k+3)p^2+\frac{(k+3)^2+(k+3)-2}{2}p^3+\frac{(k+3)^3+3(k+3)^2-10(k+3)-6}{3!}p^4+\frac{2k(k+2)(k+4)}{3!}p^5+p^2+(k+3)p^3+\frac{(k+1)(k+4)}{2}p^4+\frac{(k-1)(k+3)(k+4)}{3!}p^5 = 1+(k+4)p+\frac{(k+4)^2+(k+4)-2}{2}p^2+\frac{(k+4)^3+3(k+4)^2-10(k+4)-6}{3!}p^3+$

$\frac{(k+5)[(k+4)^3+5(k+4)^2-42(k+4)+24]}{4!}p^4 + \frac{3(k-1)(k+2)(k+4)(k+5)}{4!}p^5$. This completes the proof.□

In the above theorem 5.1.8, we may have $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ for $m \geq 3$ and obtain formulae for the number of maximal chains. Hence the following theorem.

Theorem 5.1.9. *Let $x = n + m$; $k \geq 4$ and any fixed prime p, where $m, n \in \mathbb{Z}^+$; $\alpha_1 = k^2 - 5k + 9$; $\alpha_2 = 3k^3 - 16k^2 + 33k - 26$ and $\alpha_3 = 2k^4 - 15k^3 + 41k^2 - 52k + 24$. The number of maximal chains of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ is given by* $1 + xp + \frac{x^2+x-2}{2}p^2 + [\frac{x^3+3x^2-10x-6}{3!}]p^3 + \sum_{s=4}^{k+1} \frac{(x-s+5)(x-s+6)\cdots x(x-1)(x+1)}{s!}[x^3 + \alpha_1 x^2 - \alpha_2 x + \alpha_3]p^s + \frac{m(x-2m+1)(x-m+1)(x-m+3)(x-m+4)\cdots x(x+1)}{m+1!}p^{m+2}$.

Proof: Induction on m and by similar arguments to the proofs in Theorems 5.1.4 and 5.1.8.

□

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Chapter 6

ON DISTINCT FUZZY SUBGROUPS OF $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$

6.1 Introduction

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In this chapter, we use the criss-cut technique used in Section 3.3 to classify the distinct fuzzy subgroups of some finite non-cyclic abelian p-groups of rank three of the form $\mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ for any fixed prime integer p, any positive integers $n \geq 1$ and fixed integer $m = 2$. For a natural number $m = 1$, see Theorem 2.3.

Example 6.1.1. *Let* $G = \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2$.

We construct all the 213 maximal chains of G and then use them to discuss how to compute the distinct fuzzy subgroups of G, using the criss-cut technique.

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,2,0), (0,2,0) > \supseteq < (2,0,0), (0,2,0) > \supseteq$ $< (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,2,0), (0,2,0) > \supseteq < (2,0,0), (0,2,0) > \supseteq < (2,0,0) > \supseteq$ $\{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,2,0), (0,2,0) > \supseteq < (2,0,0), (0,2,0) > \supseteq < (2,2,0) > \supseteq$ $\{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,2,0), (0,2,0) > \supseteq < (1,2,0) > \supseteq < (2,0,0) > \supseteq$ $\{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,2,0),(0,2,0) > \supseteq \mathbb{Z}_{2^2} + \{0\} + \{0\} \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,1,0),(0,2,0) > \supseteq < (1,1,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (1,1,0),(0,2,0) > \supseteq < (1,3,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (2,0,0),(0,1,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (2,0,0),(0,1,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (2,0,0),(0,1,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (2,0,0),(0,1,0) > \supseteq < (2,1,0) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (2,0,0),(0,1,0) > \supseteq \{0\} + \mathbb{Z}_{2^2} + \{0\} \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,2,0) > \supseteq < (1,1,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,2,0) > \supseteq < (1,3,0) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,0,1),(0,2,0) > \supseteq < (2,0,0),(0,2,0) > \supseteq <$

$(0,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(1,0,1)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(1,2,1)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(2,0,0),(0,1,1)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(0,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(2,0,0),(0,1,1)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(2,0,0),(0,1,1)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(2,0,0),(0,1,1)>\supseteq<(2,1,1)>\supseteq<(0,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,0),(1,0,1)>\supseteq<(2,0,0),(0,1,1)>\supseteq<(0,1,1)>\supseteq<(0,2,0)>\supseteq\{(0,0,0)\}$

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$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(0,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(1,0,1)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,0,1),(0,2,0)>\supseteq<(1,2,1)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,1,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(0,2,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,1,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<(2,0,0)>\supseteq\{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2\supseteq<(1,1,1),(1,0,1)>\supseteq<(1,1,1),(0,2,0)>\supseteq<(2,0,0),(0,2,0)>\supseteq<$

$(2,2,0) > \supseteq \{(0,0,0)\}$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,1,1),(0,2,0)> \supseteq <(1,1,1)> \supseteq <(2,2,0)> \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,1,1),(0,2,0)> \supseteq <(1,3,1)> \supseteq <(2,2,0)> \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,1,1),(1,0,1)> \supseteq <(2,0,0),(0,1,0)> \supseteq <(2,0,0),(0,2,0)> \supseteq <(0,2,0)> \supseteq \{(0,0,0)\}$$

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$$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (1,2,0),(0,0,1) > \supseteq < (2,0,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_2 \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (1,2,0),(0,0,1) > \supseteq < (1,2,0) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2} + \mathbb{Z}_{2^2} + \mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) \supseteq < (1,2,0),(0,0,1) > \supseteq < (1,2,1) > \supseteq < (2,0,0) > \supseteq$$

$\{(0,0,0)\}$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq \mathbb{Z}_{2^2}+\{0\}+\mathbb{Z}_2 \supseteq < (2,0,0),(0,0,1) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq \mathbb{Z}_{2^2}+\{0\}+\mathbb{Z}_2 \supseteq < (2,0,0),(0,0,1) > \supseteq < (2,0,1) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq \mathbb{Z}_{2^2}+\{0\}+\mathbb{Z}_2 \supseteq < (2,0,0),(0,0,1) > \supseteq \{0\}+\{0\}+\mathbb{Z}_2 \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq \mathbb{Z}_{2^2}+\{0\}+\mathbb{Z}_2 \supseteq < (1,0,1) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq \mathbb{Z}_{2^2}+\{0\}+\mathbb{Z}_2 \supseteq \mathbb{Z}_{2^2}+\{0\}+\{0\} \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

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$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,0,1) > \supseteq < (0,2,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,0,1) > \supseteq < (2,0,1) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,0,1) > \supseteq \{0\}+\{0\}+\mathbb{Z}_2 \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,1) > \supseteq < (2,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,1) > \supseteq < (2,2,1) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,1) > \supseteq < (0,2,1) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,1),(0,2,1) > \supseteq < (2,2,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq < (1,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,0),(0,2,0),(0,0,1) > \supseteq < (2,0,1),(0,2,1) > \supseteq < (2,0,1) > \supseteq \{(0,0,0)\}$$

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$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,1),(0,2,1)> \supseteq <(0,2,1)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,2,0),(0,0,1)> \supseteq <(2,2,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,2,0),(0,0,1)> \supseteq <(2,2,1)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,2,0),(0,0,1)> \supseteq \{0\}+\{0\}+\mathbb{Z}_2 \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,1),(0,2,0)> \supseteq <(0,2,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,1),(0,2,0)> \supseteq <(2,0,1)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,1),(0,2,0)> \supseteq <(2,2,1)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(0,2,0),(0,0,1)> \supseteq <(0,2,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(0,2,0),(0,0,1)> \supseteq <(0,2,1)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{2^2}+\mathbb{Z}_{2^2}+\mathbb{Z}_2 \supseteq <(1,0,0),(0,2,0),(0,0,1)> \supseteq <(2,0,0),(0,2,0),(0,0,1)> \supseteq <(0,2,0),(0,0,1)> \supseteq \{0\}+\{0\}+\mathbb{Z}_2 \supseteq \{(0,0,0)\}$

The chains are of length 6, thus the first chain contributes $2^6 - 1$ distinct fuzzy subgroups. The second chain contributes a further $\frac{2^6}{2} = 2^5$ distinct fuzzy subgroups since it has at least one subgroup not appearing in the first chain. There are 48 maximal chains like the 2nd chain. Thus each of these has a single distinguishing factor, (not used as a distinguishing factor elsewhere). Thus these chains give 48×2^5 distinct fuzzy subgroups, while 132 out of the remaining chains, each has a unique pair of subgroups, thus they contribute 132×2^4 distinct fuzzy subgroups whereas the remaining 32 chains each has a unique triple of subgroups, thus they contribute 32×2^3 distinct fuzzy subgroups. Thus our group G has $2^6 - 1 + 48 \times 2^5 + 132 \times 2^4 + 32 \times 2^3 = 3967$ distinct fuzzy subgroups.

By a similar argument, we obtained the following results for the indicated variables.

For $p = 2$, $n = 3$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups of $G = \mathbb{Z}_{2^3} + \mathbb{Z}_{2^2} + \mathbb{Z}_2$ to be $16767 = 2^7 - 1 + 74 \times 2^6 + 304 \times 2^5 + 136 \times 2^4$.

For $p = 2$, $n = 4$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $59135 = 2^8 - 1 + 100 \times 2^7 + 544 \times 2^6 + 352 \times 2^5$.

For $p = 2$, $n = 5$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $187903 = 2^9 - 1 + 123 \times 2^8 + 852 \times 2^7 + 720 \times 2^6$.

For $p = 3$, $n = 2$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $13407 = 2^6 - 1 + 120 \times 2^5 + 513 \times 2^4 + 162 \times 2^3$.

For $p = 3$, $n = 3$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $62287 = 2^7 - 1 + 195 \times 2^6 + 1215 \times 2^5 + 675 \times 2^4$.

For $p = 3$, $n = 4$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $230655 = 2^8 - 1 + 270 \times 2^7 + 2196 \times 2^6 + 1728 \times 2^5$.

For $p = 3$, $n = 5$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $755839 = 2^9 - 1 + 345 \times 2^8 + 3456 \times 2^7 + 3510 \times 2^6$.

For $p = 5$, $n = 2$ and $m = 2$, we manually computed the number of distinct fuzzy subgroups to be $72703 = 2^6 - 1 + 420 \times 2^5 + 3075 \times 2^4 + 1250 \times 2^3$.

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Proposition 6.1.1. *$G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $p[1+p+(2n-1)(1+p+p^2) - p(p-2) - (n-2)] = p[2p^2 + 6p + 4]$ single distinguishing factors in its maximal chains of subgroups, where $n = 2$.*

Proof: G has $1+p+p^2$ maximal subgroups since G is of rank-3 and out of these, $1+p$ are of rank-3 while p^2 are of rank-2. One such rank-3 maximal subgroup is $H_1 = \mathbb{Z}_p + \mathbb{Z}_{p^2} + \mathbb{Z}_p = < (p, 0, 0), (0, 1, 0), (0, 0, 1) >$. By the formula 2.3 in [6], the number of single distinguishing factors of H_1 is equal to $p(2pn + n + 1)$. (1)

Another rank-3 maximal subgroup of G is $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p = < (1, 0, 0), (0, p, 0), (0, 0, 1) >$. H_2 contributes cyclic and rank-2 single distinguishing factors. There are no rank-3 single distinguishing factors other than H_2 itself. We list the cyclic distinguishing factors:

$$\textbf{BlockH1} = \begin{cases} 1, p, 0 \\ 1, p, 1 \\ 1, p, 2 \\ \vdots \\ 1, p, p-1 \end{cases}$$

$$\textbf{BlockH2} = \begin{cases} 1, 0, 0 \\ 1, 0, 1 \\ 1, 0, 2 \\ \vdots \\ 1, 0, p-1 \end{cases}$$

$$\textbf{BlockH3} = \begin{cases} 1, 2p, 0 \\ 1, 2p, 1 \\ 1, 2p, 2 \\ \vdots \\ 1, 2p, p-1 \end{cases}$$

$$\textbf{BlockH4} = \begin{cases} 1, 3p, 0 \\ 1, 3p, 1 \\ 1, 3p, 2 \\ \vdots \\ 1, 3p, p-1 \end{cases}$$

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$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\textbf{BlockH5} = \begin{cases} 1, (p-1)p, 0 \\ 1, (p-1)p, 1 \\ 1, (p-1)p, 2 \\ \vdots \\ 1, (p-1)p, p-1 \end{cases}$$

for a total of $p \times p = p^2$. (2)

Next we list the rank-2 single distinguishing factors of H_2:

$$\textbf{BlockH6} = \begin{cases} <(1,0,0),(0,0,1)> \\ <(1,0,0),(0,p,0)> \\ <(1,0,0),(0,p,1)> \\ <(1,0,0),(0,p,2)> \\ \qquad\vdots \\ <(1,0,0),(0,p,p-1)> \end{cases}$$

$$\textbf{BlockH7} = \begin{cases} <(1,0,1),(0,p,0)> \\ <(1,0,1),(0,p,1)> \\ <(1,0,1),(0,p,2)> \\ <(1,0,1),(0,p,3)> \\ \qquad\vdots \\ <(1,0,1),(0,p,p-1)> \end{cases}$$

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$$\textbf{BlockH8} = \begin{cases} <(1,0,2),(0,p,0)> \\ <(1,0,2),(0,p,1)> \\ <(1,0,2),(0,p,2)> \\ <(1,0,2),(0,p,3)> \\ \qquad\vdots \\ <(1,0,2),(0,p,p-1)> \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\textbf{BlockH9} = \begin{cases} <(1,0,p-1),(0,p,0)> \\ <(1,0,p-1),(0,p,1)> \\ <(1,0,p-1),(0,p,2)> \\ <(1,0,p-1),(0,p,3)> \\ \qquad\vdots \\ <(1,0,p-1),(0,p,p-1)> \end{cases}$$

$$\mathbf{BlockH10} = \begin{cases} < (1,p,0),(0,0,1) > \\ < (1,2p,0),(0,0,1) > \\ < (1,3p,0),(0,0,1) > \\ < (1,4p,0),(0,0,1) > \\ \qquad\qquad \vdots \\ < (1,(p-1)p,0),(0,0,1) > . \end{cases}$$

By continuing, the number of such rank-2 subgroups is equal to $p \times p + p = p^2 + p$ single distinguishing factors. (3)
So the contribution from H_2 (including H_2) of single distinguishing factors is equal to $p^2 + p^2 + p + 1 = 2p^2 + p + 1$ (4).

All the rank-3 maximal subgroups are isomorphic, thus there are p maximal subgroups (excluding H_1) yielding $p(2p^2 + p + 1)$ factors. (5)
The rank-2 maximal subgroups contribute only themselves to the single distinguishing factors. Thus their contribution is p^2. (6)
Hence the total number of single distinguishing factors is equal to $p(2pn + n + 1) + p(2p^2 + p+1)+p^2$ from (1), (5),and (6) for $n = 2$. Simplifying, we get $p(4p+3)+p(2p^2+p+1+p) = p[4p + 3 + 2p^2 + p + 1 + p] = p[2p^2 + 6p + 4]$ (7)
Substituting $n = 2$ in the formula of the Proposition6.1.1, we get $p[1 + p + 3 + 3p + 3p^2 - p^2 + 2p] = p[2p^2 + 6p + 4]$ (8). Thus (7) and (8) show the proof is complete.□

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Proposition 6.1.2. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $p[1+p+(2n-1)(1+p+p^2) - p(p-2) - (n-2)]$ single distinguishing factors in its maximal chains of subgroups, where $n = 3$.*

Proof: G has $1 + p + p^2$ maximal subgroups since G is of rank-3 and out of these, $1 + p$ are of rank-3 while p^2 are of rank-2. One such rank-3 maximal subgroup is $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (p, 0, 0), (0, 1, 0), (0, 0, 1) >$. By the previous Proposition 6.1.1, the number of single distinguishing factors of H_1 is equal to $p[2p^2 + 6p + 4]$. (1)
Another rank-3 maximal subgroup of G is $H_2 = \mathbb{Z}_{p^3}+\mathbb{Z}_p+\mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$.
As in the previous Proposition6.1.1, there are p^2 cyclic distinguishing factors from H_2. (2)
There are $(p^2 + p)$ rank-2 single distinguishing factors (3)
and p rank-3 single distinguishing factors (4).
No rank-3's in H_2 except H_2. Thus the number of rank-3 maximal subgroups in G is p (excluding H_1) and the rank-2 maximal subgroups is p^2.

Thus the total number of single distinguishing factors is $p[2p^2+6p+4]+p(2p^2+p+1)+p^2 = 2p^3+6p^2+4p+2p^3+p^2+p+p^2 = 4p^3+8p^2+5p \quad (5)$. This complete the proof.□

The following Theorem is now clear

Theorem 6.1.3. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$, has $p[1+p+(2n-1)(1+p+p^2)-p(p-2)-(n-2)]$ single distinguishing factors in its maximal chains of subgroups, $\forall n \geq 2$.*

Proof: By induction on n. □

Proposition 6.1.4. *$G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $\frac{p^2}{2}[(2p^2+4p+1)n^2+n-2p(p-1)]$ pairs of distinguishing factors in its maximal chains of subgroups, where $n = 2$.*

Proof: G has $1+p+p^2$ maximal subgroups since G is of rank-3 and $1+p$ of them are of rank-3 while p^2 are of rank-2. One such rank-3 maximal subgroup is $H_1 = \mathbb{Z}_p + \mathbb{Z}_{p^2} + \mathbb{Z}_p =< (p,0,0),(0,1,0),(0,0,1) >$. By the formula 2.3 in [6], the number of pairs of distinguishing factors of H_1 is equal to $1+p+(p+p^2)((\frac{n^2+n}{2})p+n)-p(2pn+n+1)-1 = p+(p+p^2)((\frac{n^2+n}{2})p+n)-(2p^2n+np+p)$. Letting $n=2$, this number is $p+(p+p^2)(3p+2)-(4p^2+2p+p) = p+3p^3+2p^2+3p^2+2p-4p^2-3p = 3p^3+p^2 \qquad (1)$

Another rank-3 maximal subgroup is $H_2 = \mathbb{Z}_{p^2} + \mathbb{Z}_p + \mathbb{Z}_p =< (1,0,0),(0,p,0),(0,0,1) >$.

We list the maximal chains of H_2 contributing pairs of distinguishing factors:

$H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \{0\} + \mathbb{Z}_p + \{0\}$. This chain has a single distinguishing factor viz. H_2, (not a pair). The next chains have pairs of distinguishing factors. These pairs of distinguishing factors come from pairing H_2 and 4th column of **BlockH1** below:

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$$\textbf{BlockH1} = \begin{cases} H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \{0\} + \{0\} \cdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,p,0) > \cdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,2p,0) > \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,(p-1)p,0) > \cdots \end{cases}$$

In the next block of maximal chains, the first chain pairs H_2 with the 3rd column while the rest of the chains pair H_2 with the 4th column of **BlockH2** below:

$$\textbf{BlockH2} = \begin{cases} H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \{0\} \cdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \mathbb{Z}_p \geq < (p,0,1) > \cdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \mathbb{Z}_p \geq < (p,0,2) > \cdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \mathbb{Z}_p \geq < (p,0,3) > \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H_2 \geq \mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p \geq \mathbb{Z}_p + \{0\} + \mathbb{Z}_p \geq < (p,0,p-1) > \cdots \end{cases}$$

Replacing $\mathbb{Z}_p + \{0\} + \mathbb{Z}_p$ with other maximal subgroups of $\mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ yields other $p+1$ pairs of distinguishing factors since all the maximal subgroups of $\mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ are isomorphic. Thus the chains of H_2 having $\mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ yield $p + (p+1)(p+p^2) = p^3 + 2p^2 + 2p$ (2) pairs of distinguishing factors since the number of maximal subgroups of $\mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ equals $1 + p + p^2$.

Next we vary the maximal subgroups of H_2, e.g. replace $\mathbb{Z}_p + \mathbb{Z}_p + \mathbb{Z}_p$ with $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$. The chain $H_2 \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \{0\} + \{0\}$ has the single distinguishing factor $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$ and thus does not add to pairs of distinguishing factors.

The next chains add to pairs of distinguishing factors and these pairs of distinguishing factors come from pairing 2nd and 4th columns of **BlockH3** below:

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$$\textbf{BlockH3} = \begin{cases} H_2 \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \{0\} + \mathbb{Z}_p + \{0\} \cdots \\ H_2 \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,p,0) > \cdots \\ H_2 \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,2p,0) > \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H_2 \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,(p-1)p,0) > \cdots \end{cases}$$

The variation of $\mathbb{Z}_p + \mathbb{Z}_p + \{0\}$ does not yield a new pair of distinguishing factors. E.g. if $\mathbb{Z}_p + \mathbb{Z}_p + \{0\}$ is replaced with $\mathbb{Z}_{p^2} + \{0\} + \{0\}$, that chain yields a new single distinguishing factor $\mathbb{Z}_{p^2} + \{0\} + \{0\}$. Similarly, if $\mathbb{Z}_p + \mathbb{Z}_p + \{0\}$ is replaced with $< (1, kp, 0) >$, $k = 1, 2, \cdots, p-1$. Thus in H_2, $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$ gives rise to only p pairs of distinguishing factors. Next we vary the maximal subgroup of H_2, e.g. replace $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$ with $< (1,0,1), (0,p,0) >$. Then the chain $H_2 \geq < (1,0,1), (0,p,0) > \geq < (p,0,0), (0,p,0) > \geq < (0,p,0) >$ has a single distinguishing factor. The following associated chains yield pairs of distinguishing factors. These pairs of distinguishing factors come from pairing 2nd and 4th columns of **BlockH4** below:

$$\textbf{BlockH4} = \begin{cases} H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,0,0) > \cdots \\ H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,p,0) > \cdots \\ H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,2p,0) > \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,(p-1)p,0) > \cdots \end{cases}$$

Similarly to the case of $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$, the maximal subgroup $< (1,0,1),(0,p,0) >$ yields only p pairs of distinguishing factors. Running through all $p + p^2$ maximal subgroups of G of rank-2, we obtain $p(p+p^2) = p^3 + p^2$ pairs of distinguishing factors for H_2. (3)

So the total number of pairs of distinguishing factors for H_2 is obtained by adding (2) and (3), giving $p^3 + 2p^2 + 2p + p^3 + p^2 = 2p^3 + 3p^2 + 2p$ (4)

There are p maximal subgroups of G like H_2 (i.e. of rank-3), thus these yield $p(2p^3 + 3p^2 + 2p) = 2p^4 + 3p^3 + 2p^2$ pairs of distinguishing factors. (5)

Next we consider rank-2 maximal subgroups of G, e.g. $H = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \{0\}$. Then the chain $H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \{0\} + \{0\}$ has the single distinguishing factor H. The next chains have pairs of distinguishing factors. These pairs of distinguishing factors come from pairing 2nd and 4th columns of **BlockHH1** below:

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$$\textbf{BlockHH1} = \begin{cases} H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq \{0\} + \mathbb{Z}_p + \{0\} \cdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,p,0) > \cdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,2p,0) > \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \geq < (p,(p-1)p,0) > \cdots \end{cases}$$

We continue the process of blocks of maximal chains as above.

$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$

In **BlockHH2**, the pairs of distinguishing factors come from pairing 2nd and 3rd columns below:

$$\textbf{BlockHH2} = \begin{cases} H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq \mathbb{Z}_{p^2} + \{0\} + \{0\} \cdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq < (1,p,0) > \cdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq < (1,2p,0) > \cdots \\ \quad \vdots \qquad\qquad \vdots \qquad \vdots \\ H \geq \mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\} \geq < (1,(p-1)p,0) > \cdots \end{cases}$$

Next, let us replace $\mathbb{Z}_{p^2} + \mathbb{Z}_p + \{0\}$ with $\mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} =< (p, 0, 0), (0, 1, 0) >$

In **BlockHH3**, the pairs of distinguishing factors come from pairing 2nd and 3rd columns below:

$$\textbf{BlockHH3} = \begin{cases} H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq \mathbb{Z}_p + \mathbb{Z}_p + \{0\} \cdots \\ H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq < (p, 1, 0) > \cdots \\ H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq < (p, 2, 0) > \cdots \\ H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq < (p, 3, 0) > \cdots \\ \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq < (p, (p-1), 0) > \cdots \\ H \geq \mathbb{Z}_p + \mathbb{Z}_{p^2} + \{0\} \geq < (0, 1, 0) > \cdots \end{cases}$$

This gives $p+1$ pairs of distinguishing factors. Similarly, other maximal subgroups of H yield $p+1$ pairs of distinguishing factors. There are p maximal subgroups of H, thus H yields $2p + p(p+1) = p^2 + 3p$ pairs of distinguishing factors. (6)

(Note: Since $H = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \{0\}$ is a rank-2 subgroup, it has 1 + p maximal subgroups. Hence 1 out of these, contributes 2p and the remaining p yield p + 1)

There are p^2 rank-2 maximal subgroups of G, thus the number of pairs of distinguishing factors yielded by such subgroups is equal to $p^2(p^2 + 3p) = p^4 + 3p^3$ (7)

Hence the total number of pairs of distinguishing factors of G is equal to (5) + (7) + (1) = $(2p^4 + 3p^3 + 2p^2) + (p^4 + 3p^3) + 3p^3 + p^2 = 3p^4 + 9p^3 + 3p^2 = \frac{p^2}{2}[6p^2 + 18p + 6]$ and this equals $\frac{p^2}{2}[(2p^2 + 4p + 1)n^2 + n - 2p(p-1)]$ for $n = 2$.□

Proposition 6.1.5. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $\frac{p^2}{2}[(2p^2 + 4p + 1)n^2 + n - 2p(p-1)]$ pairs of distinguishing factors in its maximal chains of subgroups, where $n = 3$.*

Proof: For $n = 3$, the above formula becomes $\frac{p^2}{2}[(16p^2 + 38p + 12)]2^5 = p^2[8p^2 + 19p + 6]$ (1)

Let $H_1 = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ be maximal subgroup of G. By the previous Proposition 6.1.4, H_1 has $p^2[3p^2 + 9p + 3]$ pairs of distinguishing factors. (2)

Let $H_2 = \mathbb{Z}_{p^3} + \mathbb{Z}_p + \mathbb{Z}_p =< (1, 0, 0), (0, p, 0), (0, 0, 1) >$ be another maximal subgroup of G.

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We identify the pairs of distinguishing factors not counted in H_1

$$
\textbf{BlockH1} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq \\
< (p^2,0,0) > \geq \{0,0,0\} \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq \\
< (p^2,p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \\
\geq < (p^2,2p,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq \\
< (p^2,(p-1)p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p,2p,0) > \geq < (p^2,0,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,0) > \geq < (p,(p-1)p,0) > \geq < (p^2,0,0) > \\
\geq \cdots
\end{cases}
$$

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In **BlockH1**, we pair H_2 with column 5 and H_2 with column 4 respectively for a total of $2p$

pairs of distinguishing factors.

$$
\textbf{BlockH2} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,0,1) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,0,2) > \geq < (p^2,0,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,0,p-1) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq < (p^2,0,0) > \\
\geq \{0,0,0\} \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq < (p^2,0,1) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq < (p^2,0,2) > \\
\geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \\
\geq < (p^2,0,p-1) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq \\
< (0,0,1) > \\
\geq \cdots
\end{cases}
$$

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In **BlockH2**, we pair H_2 with column 4 and H_2 with column 5 respectively for a total of $2p+1$ pairs of distinguishing factors. Similarly, **BlockH3**, **BlockH4** and the other $p-3$

blocks between **BlockH3** and **BlockH4** yield $2p+1$ pairs of distinguishing factors each.

$$
\textbf{BlockH3} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p,p,1) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p,2p,2) > \geq < (p^2,0,0) > \geq \cdots \\
\quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,(p-1)p,p-1) > \geq \\
< (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,p,1) > \geq \\
< (p^2,0,0) > \geq \{0,0,0\} \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,1) > \geq \\
< (p^2,p,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,1) > \geq \\
< (p^2,2p,2) > \geq \cdots \\
\quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,1) > \geq \\
< (p^2,(p-1)p,p-1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,1) > \geq \\
< (0,p,1) > \geq \cdots
\end{cases}
$$

$$
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
$$

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$$
\textbf{BlockH4} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,p-1) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,p-1) > \geq < (p,p,p-1) > \geq < (p^2,0,0) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,p-1) > \geq < (p,2p,p-2) > \geq < (p^2,0,0) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,p-1) > \geq < (p,3p,p-3) > \geq < (p^2,0,0) > \\
\geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p,(p-1)p,1) > \geq \\
< (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,p,p-1) > \\
\geq < (p^2,0,0) > \geq \{0,0,0\} \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,p-1) > \\
\geq < (p^2,p,p-1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,p-1) > \\
\geq < (p^2,2p,p-2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,p-1) > \\
\geq < (p^2,(p-1)p,p-1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,p-1) > \\
\geq < (0,p,p-1) > \geq \cdots
\end{cases}
$$

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$$
\textbf{BlockH5} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,1),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \\
\geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,1),(0,p,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,1),(0,p,2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,1),(0,p,p-1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,2),(0,p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,2),(0,p,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,2),(0,p,2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,2),(0,p,p-1) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,p-1),(0,p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,p-1),(0,p,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,p-1),(0,p,2) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,0,p-1),(0,p,p-1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,p,0),(0,0,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,2p,0),(0,0,1) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p,(p-1)p,0),(0,0,1) > \geq \cdots
\end{cases}
$$

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In **BlockH5**, we pair H_2 with column 3 for a total of $p(p-1)+p-1=(p-1)(p+1)$ pairs of distinguishing factors.

$$
\textbf{BlockH6} = \begin{cases}
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0) > \\
\geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,1),(0,p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,1),(0,p,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,1),(0,p,2) > \geq \cdots \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,1),(0,p,p-1) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,2),(0,p,0) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,2),(0,p,1) > \geq \cdots \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,2),(0,p,p-1) > \\
\geq \cdots \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,p-1),(0,p,0) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,p-1),(0,p,1) > \\
\geq \cdots \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,p-1),(0,p,p-1) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,p,0),(0,0,1) > \\
\geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq < (p^2,2p,0),(0,0,1) > \\
\geq \cdots \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq \\
< (p^2,(p-1)p,0),(0,0,1) > \geq \cdots \\
H_2 \geq < (p,0,0),(0,p,0),(0,0,1) > \geq < (p^2,0,0),(0,p,0),(0,0,1) > \geq \\
< (0,p,0),(0,0,1) > \geq \cdots
\end{cases}
$$

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In **BlockH6**, we pair H_2 with column 4 for a total of $p + 1 + (p-1)p = p^2 + 1$ pairs of distinguishing factors.

$$
\textbf{BlockH7} = \begin{cases}
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,p,0) > \geq \cdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,2p,0) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,(p-1)p,0) > \\
\geq \cdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,2p,0) > \geq < (p^2,0,0) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,(p-1)p,0) > \geq < (p^2,0,0) > \geq \cdots
\end{cases}
$$

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$$
\textbf{BlockH8} = \begin{cases}
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,p,0) > \geq \cdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,2p,0) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,(p-1)p,0) > \\
\geq \cdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,2p,0) > \geq < (p^2,0,0) > \geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq < (1,0,1),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,(p-1)p,0) > \geq < (p^2,0,0) > \geq \cdots
\end{cases}
$$

$$
\textbf{BlockH9} = \begin{cases}
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,p,0) >\geq \cdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,2p,0) >\geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,(p-1)p,0) > \\
\geq \cdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,0,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,p,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,2p,0) >\geq< (p^2,0,0) >\geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq< (1,0,2),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,(p-1)p,0) >\geq< (p^2,0,0) >\geq \cdots
\end{cases}
$$

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$$
\textbf{BlockH10} = \begin{cases}
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,p,0) >\geq \cdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,2p,0) >\geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,(p-1)p,0) > \\
\geq \cdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,0,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,p,0) >\geq< (p^2,0,0) >\geq \cdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,2p,0) >\geq< (p^2,0,0) >\geq \cdots \\
\qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
H_2 \geq< (1,0,3),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,(p-1)p,0) >\geq< (p^2,0,0) >\geq \cdots
\end{cases}
$$

$$
\vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots
$$

$$\textbf{BlockH11} = \begin{cases} H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,0,0) >\geq \cdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,p,0) >\geq \cdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq< (p^2,2p,0) > \\ \geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p^2,0,0),(0,p,0) >\geq \\ < (p^2,(p-1)p,0) >\geq \cdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,0,0) >\geq< (p^2,0,0) >\geq \cdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,p,0) >\geq< (p^2,0,0) >\geq \cdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,2p,0) >\geq< (p^2,0,0) >\geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ H_2 \geq< (1,0,p-1),(0,p,0) >\geq< (p,0,0),(0,p,0) >\geq< (p,(p-1)p,0) >\geq< (p^2,0,0) >\geq \cdots \end{cases}$$

In **BlockH7**, we pair the 2nd column with column 4 and the 2nd column with column 5 respectively for a total of $2p$ pairs of distinguishing factors. Similarly, **BlockH8**, **BlockH9**, **BlockH10**, **BlockH11** and the other $p-5$ blocks between **BlockH10** and **BlockH11** yield $2p$ pairs of distinguishing factors each. There are p blocks above, thus their contribution is

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$p \times 2p = 2p^2$ pairs of distinguishing factors.

$$\textbf{BlockH12} = \begin{cases} H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq< (0,p,1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq< (p^2,p,1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq< (p^2,2p,2) >\geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq \\ < (p^2,(p-1)p,p-1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p,0,0) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p,p,1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p,(p-1)p,p-1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (1,0,1) >\geq< (p,0,0) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (1,p,2) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (1,2p,3) >\geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (1,(p-2)p,p-1) >\geq \cdots \\ H_2 \geq< (1,0,1),(0,p,1) >\geq< (1,(p-1)p,0) >\geq \cdots \end{cases}$$

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$$\mathbf{BlockH13} = \begin{cases} H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p^2,0,0),(0,p,2) > \geq < (0,p,2) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p^2,0,0),(0,p,2) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p^2,0,0),(0,p,2) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p^2,0,0),(0,p,2) > \geq \\ < (p^2,(p-1)p,p-1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p,(p-1)p,p-1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (1,0,1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (1,p,3) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (1,2p,5) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,2) > \geq < (1,(p-2)p,2p-1) > \geq \cdots \end{cases}$$

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$$\textbf{BlockH14} = \begin{cases} H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq < (p^2,0,0),(0,p,3) > \geq < (0,p,3) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq < (p^2,0,0),(0,p,3) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq < (p^2,0,0),(0,p,3) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq < (p^2,0,0),(0,p,3) > \geq \\ < (p^2,(p-1)p,p-1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (p,0,0),(0,p,3) > \geq < (p,(p-1)p,p-1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (1,0,1) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (1,p,4) > \geq \cdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (1,2p,7) > \geq \cdots \\ \quad \vdots \quad \vdots \quad \vdots \\ H_2 \geq < (1,0,1),(0,p,3) > \geq < (1,(p-2)p,2p-1) > \geq \cdots \end{cases}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\textbf{BlockH15} = \begin{cases} H_2 \geq < (1,0,1),(0,p,p-1) > \geq < (p,0,0),(0,p,p-1) > \geq < (p^2,0,0),(0,p,p-1) > \geq \\ < (0,p,p-1) > \geq \cdots \end{cases}$$

In **BlockH12**, we pair the 2nd column with column 5, column 4 and column 3 respectively for a total of $3p$ pairs of distinguishing factors. Similarly, **BlockH13**, **BlockH14**, **BlockH15** and the other $p-5$ blocks between **BlockH14** and **BlockH15** yield $3p$ pairs of distinguishing factors each. There are $p-1$ blocks from **BlockH12** to **BlockH15**, thus their contribution is $3p \times (p-1)$ pairs of distinguishing factors.

$$\textbf{BlockH16} = \begin{cases} H_2 \geq < (1,0,2),(0,p,1) > \geq < (p,0,0),(0,p,1) > \geq < (p^2,0,0),(0,p,1) > \geq < (0,p,1) > \geq \cdots \end{cases}$$

$$\textbf{BlockH17} = \begin{cases} H_2 \geq < (1,0,2),(0,p,2) > \geq < (p,0,0),(0,p,2) > \geq < (p^2,0,0),(0,p,2) > \geq < (0,p,2) > \geq \cdots \end{cases}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

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$$\textbf{BlockH18} = \begin{cases} H_2 \geq< (1,0,2),(0,p,p-1) >\geq< (p,0,0),(0,p,p-1) >\geq< (p^2,0,0),(0,p,p-1) >\geq \\ < (0,p,p-1) >\geq \cdots \end{cases}$$

In **BlockH16**, we pair the 2nd column with column 5, column 4 and column 3 respectively for a total of $3p$ pairs of distinguishing factors. Similarly, **BlockH17**, **BlockH18** and the other $p-4$ blocks between **BlockH17** and **BlockH18** yield $3p$ pairs of distinguishing factors each. There are $p-1$ blocks from **BlockH16** to **BlockH18**, thus their contribution is $3p \times (p-1)$ pairs of distinguishing factors.

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$

There are $p-4$ clusters of blocks, like **BlockH16** to **BlockH18**, between **BlockH18** and **BlockH19**, each contributing $3p(p-1)$ pairs of distinguishing factors, for a total of $3p(p-1)(p-4)$ pairs of distinguishing factors.

$$\textbf{BlockH19} = \begin{cases} H_2 \geq< (1,0,p-1),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq< (0,p,1) >\geq \cdots \end{cases}$$

$$\textbf{BlockH20} = \begin{cases} H_2 \geq< (1,0,p-1),(0,p,2) >\geq< (p,0,0),(0,p,2) >\geq< (p^2,0,0),(0,p,2) >\geq< (0,p,2) >\geq \cdots \end{cases}$$

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$

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$$\textbf{BlockH21} = \begin{cases} H_2 \geq< (1,0,p-1),(0,p,p-1) >\geq< (p,0,0),(0,p,p-1) >\geq< (p^2,0,0),(0,p,p-1) >\geq \\ < (0,p,p-1) >\geq \cdots \end{cases}$$

In **BlockH19**, we pair the 2nd column with column 5, column 4 and column 3 respectively for a total of $3p$ pairs of distinguishing factors. Similarly, **BlockH20**, **BlockH21** and the other $p-4$ blocks between **BlockH20** and **BlockH21** yield $3p$ pairs of distinguishing factors each for a total of $3p \times (p-1)$ pairs of distinguishing factors.

$$\textbf{BlockH22} = \begin{cases} H_2 \geq< (1,0,0),(0,0,1) >\geq< (p,0,0),(0,0,1) >\geq< (p^2,0,0),(0,0,1) >\geq< (0,0,1) >\geq \cdots \end{cases}$$

$$\textbf{BlockH23} = \begin{cases} H_2 \geq< (1,0,0),(0,p,1) >\geq< (p,0,0),(0,p,1) >\geq< (p^2,0,0),(0,p,1) >\geq< (0,p,1) >\geq \cdots \end{cases}$$

$$\textbf{BlockH24} = \begin{cases} H_2 \geq< (1,0,0),(0,p,2) >\geq< (p,0,0),(0,p,2) >\geq< (p^2,0,0),(0,p,2) >\geq< (0,p,2) >\geq \cdots \end{cases}$$

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$

$$\textbf{BlockH25} = \begin{cases} H_2 \geq< (1,0,0),(0,p,p-1) >\geq< (p,0,0),(0,p,p-1) >\geq< (p^2,0,0),(0,p,p-1) >\geq \\ < (0,p,p-1) >\geq \cdots \end{cases}$$

In **BlockH22**, we pair the 2nd column with column 5, column 4 and column 3 respectively for a total of $3p$ pairs of distinguishing factors. Similarly, **BlockH23**, **BlockH24**, **BlockH25** and the other $p-4$ blocks between **BlockH24** and **BlockH25** yield $3p$ pairs of distinguishing factors each for a total of $3p \times p = 3p^2$ pairs of distinguishing factors.

$$\textbf{BlockH26} = \begin{cases} H_2 \geq < (1,p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq < (0,0,1) > \geq \cdots \end{cases}$$

$$\textbf{BlockH27} = \begin{cases} H_2 \geq < (1,2p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq < (0,0,1) > \geq \cdots \end{cases}$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$\textbf{BlockH28} = \begin{cases} H_2 \geq < (1,(p-1)p,0),(0,0,1) > \geq < (p,0,0),(0,0,1) > \geq < (p^2,0,0),(0,0,1) > \geq \\ < (0,0,1) > \geq \cdots \end{cases}$$

In **BlockH26**, we pair the 2nd column with column 5, column 4 and column 3 respectively for a total of $3p$ pairs of distinguishing factors. Similarly, **BlockH27**, **BlockH28** and the other $p-4$ blocks between **BlockH27** and **BlockH28** yield $3p$ pairs of distinguishing factors each for a total of $3p \times (p-1)$ pairs of distinguishing factors.

Contribution from rank-3 maximal subgroups of H_2 come from **BlockH1** to **BlockH6** as follows: $2p + p(1+2p) + (p-1)(p+1) + p^2 + 1 = 4p^2 + 3p$

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Contribution from rank-2 maximal subgroups of H_2 come from **BlockH7** to **BlockH29** as follows:

$2p^2 + 3p(p-1)(p-1) + 3p^2 + 3p(p-1) = 2p^2 + 3p(p^2 - 2p + 1) + 3p^2 + 3p^2 - 3p = 2p^2 + 3p^3 - 6p^2 + 3p + 3p^2 + 3p^2 - 3p = 2p^2 + 3p^3$. Thus total for H_2: $4p^2 + 3p + 2p^2 + 3p^3 = 3p^3 + 6p^2 + 3p$
So rank-3's maximal subgroups other than H_1 yield $p(3p^3 + 6p^2 + 3p) = 3p^4 + 6p^3 + 3p^2$

Let $H_7 =< (1,0,0),(0,1,0) >= \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \{0\}$ be an example of a rank-2 maximal subgroup.

$$
\textbf{BlockH1} = \begin{cases}
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,p,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,2p,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (p^2,(p-1)p,0) > \\
\geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,0,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,p,0) > \geq < (p^2,0,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,2p,0) > \geq < (p^2,0,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,0,0),(0,p,0) > \geq < (p,(p-1)p,0) > \geq < (p^2,0,0) > \geq \cdots
\end{cases}
$$

In **BlockH1**, we pair H_7 with column 5 and H_7 with column 4 respectively for a total of $2p$.

$$
\textbf{BlockH2} = \begin{cases}
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p,1,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p,p+1,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p,2p+1,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p,3p+1,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,1,0),(0,p,0) > \geq < (p,(p-1)p+1,0) > \geq < (p^2,0,0) > \geq \cdots
\end{cases}
$$

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$$
\textbf{BlockH3} = \begin{cases}
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p,2,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p,p+2,0) > \geq \cdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p,2p+2,0) > \geq \cdots \\
\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p,(p-1)p+2,0) > \geq < (p^2,0,0) > \geq \cdots
\end{cases}
$$

$$
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
$$

$$\textbf{BlockH4} = \begin{cases} H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,p-1,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,p-1,0),(0,p,0) > \geq < (p,p-1,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,p-,0),(0,p,0) > \geq < (p,2p-1,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,p-1,0),(0,p,0) > \geq < (p,3p-1,0) > \geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p,2,0),(0,p,0) > \geq < (p,p^2-1,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

$$\textbf{BlockH5} = \begin{cases} H_7 \geq < (p,0,0),(0,1,0) > \geq < (p^2,0,0),(0,1,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p^2,0,0),(0,1,0) > \geq < (p^2,1,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p^2,0,0),(0,1,0) > \geq < (p^2,2,0) > \geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p^2,0,0),(0,1,0) > \geq < (p^2,p-1,0) > \geq < (p^2,0,0) > \geq \cdots \\ H_7 \geq < (p,0,0),(0,1,0) > \geq < (p^2,0,0),(0,1,0) > \geq < (0,1,0) > \geq \cdots \end{cases}$$

In **BlockH2**, **BlockH3**, **BlockH4**, **BlockH5** and other $p-4$ blocks between **BlockH4** and **BlockH5** we pair H_7 with column 3 and H_7 with column 4 respectively for a total of $(p+1)(p-4)$.

$$\textbf{BlockH6} = \begin{cases} H_7 \geq < (1,0,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (1,0,0),(0,p,0) > \geq < (1,0,0) > \geq \cdots \\ H_7 \geq < (1,0,0),(0,p,0) > \geq < (1,p,0) > \geq \cdots \\ H_7 \geq < (1,0,0),(0,p,0) > \geq < (1,2p,0) > \geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (1,0,0),(0,p,0) > \geq < (1,(p-1)p,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

$$\textbf{BlockH7} = \begin{cases} H_7 \geq < (1,1,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (1,1,0),(0,p,0) > \geq < (1,1,0) > \geq \cdots \\ H_7 \geq < (1,1,0),(0,p,0) > \geq < (1,p+1,0) > \geq \cdots \\ H_7 \geq < (1,1,0),(0,p,0) > \geq < (1,2p+1,0) > \geq \cdots \\ \quad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (1,1,0),(0,p,0) > \geq < (1,(p-1)p+1,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

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$$\textbf{BlockH8} = \begin{cases} H_7 \geq < (1,2,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (1,2,0),(0,p,0) > \geq < (1,2,0) > \geq \cdots \\ H_7 \geq < (1,2,0),(0,p,0) > \geq < (1,p+2,0) > \geq \cdots \\ H_7 \geq < (1,2,0),(0,p,0) > \geq < (1,2p+2,0) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (1,2,0),(0,p,0) > \geq < (1,(p-1)p+2,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

$$\textbf{BlockH9} = \begin{cases} H_7 \geq < (1,3,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (1,3,0),(0,p,0) > \geq < (1,3,0) > \geq \cdots \\ H_7 \geq < (1,3,0),(0,p,0) > \geq < (1,p+3,0) > \geq \cdots \\ H_7 \geq < (1,3,0),(0,p,0) > \geq < (1,2p+3,0) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (1,3,0),(0,p,0) > \geq < (1,(p-1)p+3,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

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$$\textbf{BlockH10} = \begin{cases} H_7 \geq < (1,p-1,0),(0,p,0) > \geq < (p,0,0),(0,p,0) > \geq < (p^2,0,0),(0,p,0) > \geq < (0,p,0) > \geq \cdots \\ H_7 \geq < (1,p-1,0),(0,p,0) > \geq < (1,p-1,0) > \geq \cdots \\ H_7 \geq < (1,p-1,0),(0,p,0) > \geq < (1,2p-1,0) > \geq \cdots \\ H_7 \geq < (1,p-1,0),(0,p,0) > \geq < (1,3p-1,0) > \geq \cdots \\ \qquad \vdots \qquad \vdots \qquad \vdots \\ H_7 \geq < (1,p-1,0),(0,p,0) > \geq < (1,p^2-1,0) > \geq < (p^2,0,0) > \geq \cdots \end{cases}$$

In **BlockH6**, **BlockH7**, **BlockH8**, **BlockH9**, **BlockH10** and other $p-5$ blocks between **BlockH9** and **BlockH10** we pair H_7 with column 2 and H_7 with column 3 respectively for a total of $(p+1)(p-5)$.

Total contribution by H_7: $2p + (p+1)p + p(p+1) = 2p + 2p + 2p^2 = 4p + 2p^2$

H_7 represents rank-2 maximal subgroups of G, and there are p^2 rank-2 maximal subgroups of G, thus the rank-2 maximal subgroups of G yield $p^2(4p + 2p^2) = 4p^3 + 2p^4$ pairs of distinguishing factors.

Hence the total number of pairs of distinguishing factors of G is equal to $(3p^4 + 9p^3 + 3p^2) + (3p^4 + 6p^3 + 3p^2) + (4p^3 + 2p^4) = 8p^4 + 19p^3 + 6p^2$. This completes the proof. □

Theorem 6.1.6. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[\frac{p^2}{2}(2p^2 + 4p + 1)n^2 + n - 2p(p - 1)]$ pairs of distinguishing factors in its maximal chains of subgroups, $\forall n \geq 2$.*

Proof: By induction on n. For $n = 2, 3$ the theorem has been proven in the previous Propositions 6.1.4 and 6.1.5. Assume the theorem is true for $n = k \geq 3$.
i.e $H_1 = \mathbb{Z}_{p^k} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[\frac{p^2}{2}(2p^2 + 4p + 1)k^2 + k - 2p(p - 1)]$ pairs of distinguishing factors. H_1 is virtually a maximal subgroup of $G = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$. So from here, the proof proceeds as in Proposition 6.1.5 on $n = 3$, e.g. let $H_2 = \mathbb{Z}_{p^{k+1}} + \mathbb{Z}_p + \mathbb{Z}_p$ and then compute the contribution by H_2 which leads to the contribution by rank-3 maximal subgroups of G. Thereafter, the rank-2 maximal subgroups contribute as in Proposition 3.3.5 on $n = 3$. Summing all the contributions, including H_1, yields the required result. □

Proposition 6.1.7. *$G = \mathbb{Z}_{p^2} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[2p^4]$ triples of distinguishing factors in its maximal chains of subgroups.*

Proof: G has $1 + 4p + 9p^2 + 11p^3 + 5p^4$ maximal chains, by a previous Proposition 5.1.1. Out of these, $2p^3 + 6p^2 + 4p$ are single distinguishing factors by previous Proposition 6.1.1 and $3p^4 + 9p^3 + 3p^2$ are pairs of distinguishing factors by previous Proposition 6.1.4. Then the number of triples of distinguishing factors is equal to $[4p + 9p^2 + 11p^3 + 5p^4 - 2p^3 - 6p^2 - 4p - 3p^4 - 9p^3 - 3p^2] = [2p^4]$. This completes the proof.□

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Proposition 6.1.8. *$G = \mathbb{Z}_{p^3} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[8p^4 + p^3]$ triples of distinguishing factors in its maximal chains of subgroups.*

Proof: G has $1 + 5p + 14p^2 + 24p^3 + 16p^4$ maximal chains, by a previous Proposition 5.1.2. Out of these, $4p^3 + 8p^2 + 5p$ are single distinguishing factors by previous Proposition 6.1.2 and $8p^4 + 19p^3 + 6p^2$ are pairs of distinguishing factors by previous Proposition 6.1.5. Then the number of triples of distinguishing factors is equal to $[5p + 14p^2 + 24p^3 + 16p^4 - 4p^3 - 8p^2 - 5p - 8p^4 - 19p^3 - 6p^2] = [8p^4 + p^3]$. This completes the proof.□

Proposition 6.1.9. *$G = \mathbb{Z}_{p^4} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[20p^4 + 4p^3]$ triples of distinguishing factors in its maximal chains of subgroups.*

Proof: G has $1 + 6p + 20p^2 + 43p^3 + 35p^4$ maximal chains, by a previous Proposition 5.1.3. Out of these, $6p^3 + 10p^2 + 6p$ are single distinguishing factors by previous Theorem 6.1.3 and $15p^4 + 33p^3 + 10p^2$ are pairs of distinguishing factors by previous Theorem 6.1.6. Then the number of triples of distinguishing factors is equal to $[6p + 20p^2 + 43p^3 + 35p^4 - 6p^3 - 10p^2 - 6p - 15p^4 - 33p^3 - 10p^2] = [20p^4 + 4p^3]$. This completes the proof.□

Theorem 6.1.10. *$G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ has $[\frac{p^{m+1}}{(m+1)!}n((2p+1)n + m(p-1))(n-1)]$ triples of distinguishing factors in its maximal chains of subgroups, $\forall n \geq 2$ and $m = 2$.*

Proof: G has $1+(n+2)p+\frac{(n+2)^2+(n+2)-2}{2}p^2+\frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3+\frac{2(n-1)(n+1)(n+3)}{3!}p^4$ maximal chains by a previous Theorem 5.1.4. Out of these, $p[1 + p + (2n-1)(1 + p + p^2) - p(p-2) - (n-2)]$ are single distinguishing factors by previous Theorem 6.1.3 and $[\frac{p^2}{2}(2p^2 + 4p + 1)n^2 + n - 2p(p-1)]$ are pairs of distinguishing factors by previous Theorem6.1.6. Then the number of triples of distinguishing factors is equal to $[1 + (n+2)p + \frac{(n+2)^2+(n+2)-2}{2}p^2+\frac{(n+2)^3+3(n+2)^2-10(n+2)-6}{3!}p^3+\frac{2(n-1)(n+1)(n+3)}{3!}p^4]-p[1+p+(2n-1)(1+p+p^2)-p(p-2)-(n-2)]-[\frac{p^2}{2}(2p^2+4p+1)n^2+n-2p(p-1)] = [\frac{p^3}{(3)!}n((2p+1)n+2(p-1))(n-1)]$. This completes the proof.□

Continuing in this fashion as above, we obtained the following theorem for the number of distinct fuzzy subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ for any fixed prime p and $n \geq 2$.

Theorem 6.1.11. *The number of distinct fuzzy subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^2} + \mathbb{Z}_p$ is given by $2^{n+m+2} - 1 + p[(1+p) + (2n-1)(1+p+p^2) - p(p-2) - (n-2)]2^{n+m+1} + \frac{p^2}{2}[(2p^2 + 4p+1)n^2 + n - 2p(p-1)]2^{n+m} + [\frac{p^{m+1}}{(m+1)!}n((2p+1)n + m(p-1))(n-1)]2^{n+m-1}$ for any fixed prime p; $m = 2$ and $n \geq 2$, where $m, n \in \mathbb{Z}^+$.*

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Proof: We use the previous Theorems 6.1.3, 6.1.6 and 6.1.10 and the relevant powers of 2. □

Chapter 7

CONCLUSION

In this thesis, we have successfully used the criss-cut method introduced in [43] and an equivalence relation introduced by Murali and Makamba in [40] to investigate and classify distinct fuzzy subgroups of a finite abelian p-group of rank 3 of the form $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ for any prime integer p and any positive integer n and $m = 2$, a milestone and a core of this study. We have made no attempt at being encyclopedic, but rather we have delved more deeply into a few areas that readers may achieve a better grasp of how the classification of fuzzy subgroups is unraveled in greater depth. We started by reviewing formulae for the number of subgroups, maximal chains of subgroups and distinct fuzzy subgroups of the rank-3 group $\mathbb{Z}_{p^n} + \mathbb{Z}_p + \mathbb{Z}_p$ studied in [6], see Chapter2. In the same chapter, we reviewed the number of non-isomorphic maximal chains and non-isomorphic classes of fuzzy subgroups.

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Chapter3 deals with the classification of fuzzy subgroups of a rank-2 group $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$. One of our guiding principles has therefore been to develop user-friendly formulae to compute the number of subgroups, maximal chains and distinct fuzzy subgroups of the group G. Thus, our approach of determining all the subgroups of G using these subgroups to construct the maximal chains and counting the distinct fuzzy subgroups of G by identifying all the distinguishing factors contained in these maximal chains, makes the materials presented in chapter3 of this study different from the one studied in [50]. Indeed, the authors in [50] presented some remarkable results in their study of the group G, however, using their approach, one may extremely find it difficult to extend group G from a rank-2 group to a rank-3 group, the foundation of our study. Chapter 3 contains a wealth of examples, propositions and theorems. We have derived the general formulae (i) $2[(1+(p+1)+(1+p+p^2)+\cdots+(1+p+p^2+\cdots+p^{m-1}))]+(n-m+1)(1+p+p^2+\cdots+p^m) = 2\sum_{k=1}^{m-1}(m-k+1)p^k+(n-m+1)\sum_{k=0}^{m}p^k$ for the number of subgroups, (ii) $1+(m+n-1)p+\frac{(n+m-3)(n+m)}{2}p^2+\alpha_3p^3+\alpha_4p^4+\cdots+\alpha_{m-1}p^{m-1}+\alpha_m p^m$, where $\alpha_m = \frac{(n-m+1)(n+2)(n+3)\cdots(n+m-1)(n+m)}{m!}$ and for $2 \le k < m, \alpha_k =$

$\frac{(n+m-2k+1)(n+m-k+2)(n+m-k+3)\cdots(n+m-2)(n+m-1)(n+m)}{k!}$ for the number of maximal chains and (iii) $2^{n+m+1}-1+[(1+p)+(m-2)(m-1)(1+p+p^2)+(n+1-m)(1+p+p^2+\cdots+p^m)+(p-n+2-m)]2^{n+m}+p^2[m(m-2)+(p+2)(n-2)(m-2)+\frac{(2p+1)(n-2)(n-1)(m-2)}{m-1!}+\frac{n(n-1)\cdots(n-m+1)(3-m)}{m!}]2^{n+m-1}+\frac{p^3}{3!}[n(n-1)\cdots(n-m+1)(m-2)]2^{n+m-2}$ for the number of distinct fuzzy subgroups of G.

Chapter4 has prepared the ground for the classification of fuzzy subgroups of the rank-3 group $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p$. We began this study by determining user-friendly formulae for the number of subgroups of this group G. We had the general formula $(n+m)(p^0+p^1)+2\sum_{k=2}^{m+1}[n+m-(2k-3)]p^k+2$ for the number of subgroups of G.

This then enabled us to count the maximal chains of subgroups of G. So we obtained the general formula $1+xp+\frac{x^2+x-2}{2}p^2+[\frac{x^3+3x^2-10x-6}{3!}]p^3+\sum_{s=4}^{k+1}\frac{(x-s+5)(x-s+6)\cdots x(x-1)(x+1)}{s!}[x^3+\alpha_1x^2-\alpha_2x+\alpha_3]p^s+\frac{m(x-2m+1)(x-m+1)(x-m+3)(x-m+4)\cdots x(x+1)}{m+1!}p^{m+2}$ for the number of maximal chains of subgroups of G in Chapter5.

The counting of distinct fuzzy subgroups requires maximal chains and how we distinguish them. We proceeded inductively to count the number of distinct fuzzy subgroups and finally obtained the formula

$2^{n+m+2}-1+p[(1+p)+(2n-1)(1+p+p^2)-p(p-2)-(n-2)]2^{n+m+1}+\frac{p^2}{2}[(2p^2+4p+1)n^2+n-2p(p-1)]2^{n+m}+[\frac{p^{m+1}}{(m+1)!}n((2p+1)n+m(p-1))(n-1)]2^{n+m-1}$ for this number in Chapter6.

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Our research would not have been successful without specific examples involving specific primes and exponents. So we looked laboriously at all subgroups of a specific G, maximal chains of subgroups and how to distinguish them. Painstakingly, we counted all distinct fuzzy subgroups. After many such examples, we started looking for patterns. There seems to be no way of guessing the results without looking first at specific cases.

Further research: The group $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p$ can still be extended to $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_{p^k}$ for any positive integers n,m and k. This should be a further research for a doctoral degree. We have done a lot of good work for a doctoral degree. One could also extend $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p$ to $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^m}+\mathbb{Z}_p+\cdots+\mathbb{Z}_p$, i.e. we can attach any number of the summands $\mathbb{Z}_p$ to $\mathbb{Z}_{p^m}$. There is also the possibility of varying the primes, for example using $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{q^m}+\mathbb{Z}_q$, where p and q are distinct primes. In fact there are many different possible permutations of the primes.

Finally we used the technique of starting with maximal chains of subgroups and then associating each maximal chain with all possible classes of keychains. This technique seems friendlier to handle than other techniques. The equivalence relation used in our classification of fuzzy subgroups seems more complex to handle compared to others available in literature, hence we are confident that our results on subgroups, the maximal chains of subgroups and distinct fuzzy subgroups of $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m}$ and $G = \mathbb{Z}_{p^n} + \mathbb{Z}_{p^m} + \mathbb{Z}_p$ have not been obtained before.

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APPENDIX A

The following example displays all the 2086 maximal chains of the group $G = \mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3$.

Example .0.1. $\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (1,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq < (1,6,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,3,0),(0,3,0) > \supseteq \mathbb{Z}_{3^3}+\{0\}+\{0\} \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,1,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,1,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,1,0),(0,3,0)> \supseteq <(1,1,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,1,0),(0,3,0)> \supseteq <(1,4,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,1,0),(0,3,0)> \supseteq <(1,7,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,0,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(1,2,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(1,5,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(1,2,0),(0,3,0)> \supseteq <(1,8,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <$

$(9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,0,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(3,1,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(3,4,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,1,0),(0,3,0)> \supseteq <(3,7,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,2,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,2,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,2,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(3,0,0),(0,1,0)> \supseteq <(3,2,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,2,0),(0,3,0) > \supseteq < (3,2,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,2,0),(0,3,0) > \supseteq < (3,5,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,2,0),(0,3,0) > \supseteq < (3,8,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,1,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,2,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq \mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq \{0\}+\mathbb{Z}_{3^2}+\{0\} \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,1,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq <$

$(3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (1,1,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (1,4,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (1,7,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,1,0), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (1,0,1) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (1,3,1) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,0,1), (0,3,0) > \supseteq < (1,6,1) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,2,2), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,2,2), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,0), (1,0,1) > \supseteq < (1,2,2), (0,3,0) > \supseteq < (3,0,0), (0,3,0) > \supseteq < (9,0,0), (0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (1,2,2) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (1,5,2) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (1,2,2),(0,3,0) > \supseteq < (1,8,2) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

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$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq <$$

$(9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq < (3,1,2) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,1) > \supseteq < (3,0,0),(0,1,2) > \supseteq < (3,1,2),(0,3,0) > \supseteq < (3,7,2) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,0),(1,0,2)> \supseteq <(1,0,2),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (9,0,0),(0,1,1) > \supseteq < (9,1,1) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,0),(1,0,2) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,1) > \supseteq < (0,1,1) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,0,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,1,1),(0,3,0)> \supseteq <(1,4,1)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,1,1),(0,3,0)> \supseteq <(1,7,1)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,1),(1,0,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <$

$(9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,2,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,2,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,2,1),(0,3,0) > \supseteq < (1,2,1) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,2,1),(0,3,0) > \supseteq < (1,5,1) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (1,2,1),(0,3,0) > \supseteq < (1,8,1) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,1,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (3,1,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,1,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,1,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,1,0),(0,3,0) >\supseteq< (3,4,0) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,1,0),(0,3,0) >\supseteq< (3,7,0) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,2,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,2,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,2,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (3,2,0),(0,3,0) >\supseteq< (3,8,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,1),(1,0,1) >\supseteq< (3,0,0),(0,1,0) >\supseteq< (9,0,0),(0,1,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,1,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,2,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,0,1) > \supseteq < (3,0,0),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq \{0\} + \mathbb{Z}_{3^2} + \{0\} \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1),(1,1,2) > \supseteq < (1,0,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1),(1,1,2) > \supseteq < (1,0,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1),(1,1,2) > \supseteq < (1,0,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1),(1,1,2) > \supseteq < (1,0,1),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1), (1,1,2) > \supseteq < (1,2,0), (0,3,0) > \supseteq < (1,8,0) > \supseteq < (3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$(3,4,1) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,1),(1,1,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq <$
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$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,0,2),(1,1,1) >\supseteq< (3,0,0),(0,1,2) >\supseteq< (3,1,2),(0,3,0) >\supseteq<$
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$(3,1,2) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

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$(3,4,2) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

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$(3,7,2) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,0,2),(1,1,1) >\supseteq< (3,0,0),(0,1,2) >\supseteq< (3,2,1),(0,3,0) >\supseteq<$
$(9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(3,5,1) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

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$(3,8,1) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,0,0),(0,3,0) > \supseteq <$
$(9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,0,0),(0,3,0) > \supseteq <$
$(9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,0,0),(0,3,0) > \supseteq <$
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$(3,6,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq <$
$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (3,1,1) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (3,4,1) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (3,7,1) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,2,2),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (3,2,2),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

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$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (9,0,0),(0,1,1) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (9,0,0),(0,1,1) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$$

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$$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,1),(1,2,2) > \supseteq < (3,0,0),(0,1,1) > \supseteq < (9,0,0),(0,1,1) > \supseteq <$$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,2),(1,2,1) > \supseteq < (1,3,0),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (1,1,2),(1,2,1) > \supseteq < (1,3,0),(0,3,0) > \supseteq \mathbb{Z}_{3^3}+\{0\}+\{0\} \supseteq < (3,0,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,1,2),(1,2,1) > \supseteq < (1,1,2),(0,3,0) > \supseteq < (3,0,0),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,2),(1,2,1)> \supseteq <(1,1,2),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,2),(1,2,1)> \supseteq <(1,1,2),(0,3,0)> \supseteq <(1,7,2)> \supseteq <(3,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,2),(1,2,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(3,0,0),(0,3,0)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,2),(1,2,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(1,5,1)> \supseteq <(3,6,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,1,2),(1,2,1)> \supseteq <(1,2,1),(0,3,0)> \supseteq <(1,8,1)> \supseteq <(3,6,0)> \supseteq$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,1,2),(1,2,1) >\supseteq< (3,0,0),(0,1,2) >\supseteq< (3,1,2),(0,3,0) >\supseteq< (3,7,2) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

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$(3,8,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,0),(0,1,1)> \supseteq <(9,0,0),(0,1,1)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,0),(0,1,1)> \supseteq <(9,0,0),(0,1,1)> \supseteq <(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

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$(9,1,1) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,0),(0,1,1) >\supseteq< (9,0,0),(0,1,1) >\supseteq< (9,2,2) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,0),(0,1,1) >\supseteq< (9,0,0),(0,1,1) >\supseteq< (0,1,1) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,1,1),(0,3,0)> \supseteq <$
$(9,0,0),(0,3,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,1,1),(0,3,0)> \supseteq <$
$(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,1,1),(0,3,0)> \supseteq <$
$(3,1,1)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,1,1),(0,3,0)> \supseteq <$
$(3,4,1)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,1,1),(0,3,0)> \supseteq <$
$(3,7,1)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(3,2,1),(0,3,0)> \supseteq <$

$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (3,2,1) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (3,5,1) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,2,1),(0,3,0) > \supseteq < (3,8,1) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,0,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (3,0,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,1),(0,1,0) > \supseteq < (9,0,0),(0,1,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(9,0,0),(0,1,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(9,0,0),(0,1,0)> \supseteq < (9,1,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(9,0,0),(0,1,0)> \supseteq < (9,2,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,0)> \supseteq <(9,0,0),(0,1,0)> \supseteq \{0\}+\mathbb{Z}_{3^2}+\{0\} \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,1,2),(0,3,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,1,2),(0,3,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,1,2),(0,3,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,1,2),(0,3,0)> \supseteq < (3,1,2)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,1,2),(0,3,0)> \supseteq < (3,7,2)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,2,0),(0,3,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(0,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,2,0),(0,3,0)> \supseteq < (9,0,0),(0,3,0)> \supseteq <(9,0,0)> \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(3,0,1),(0,1,1)> \supseteq <(3,2,0),(0,3,0)> \supseteq <$

$(3,5,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,1),(0,1,1) >\supseteq< (3,2,0),(0,3,0) >\supseteq<$
$(3,8,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,1),(0,1,1) >\supseteq< (3,0,1),(0,3,0) >\supseteq<$
$(9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

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$(3,0,1) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$(3,6,1) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,1),(0,1,1) >\supseteq< (9,0,0),(0,1,1) >\supseteq<$
$(9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) >\supseteq< (9,3,0) >\supseteq \{(0,0,0)\}$

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$(9,1,1) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,2,2) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(0,1,0),(0,0,1) >\supseteq< (3,0,1),(0,1,1) >\supseteq< (9,0,0),(0,1,1) >\supseteq<$
$(0,1,1) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(3,4,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$(3,7,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,2),(0,1,1) > \supseteq < (3,2,1),(0,3,0) > \supseteq <$
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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (3,0,2),(0,1,1) > \supseteq < (3,2,1),(0,3,0) > \supseteq <$
$(9,0,0),(0,3,0) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

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$(3,5,1) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

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$(3,8,1) > \supseteq < (9,6,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0)\,,(0,1,0) >\supseteq \{0\}+\mathbb{Z}_{3^2}+\{0\} \supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$(9,2,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(9,0,0),(0,1,0),(0,0,1)> \supseteq \{0\} + \mathbb{Z}_{3^2} +$
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$(9,0,1),(0,3,0) > \supseteq < (9,6,1) > \supseteq \{(0,0,0)\}$

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$(9,0,1),(0,3,1) > \supseteq < (9,3,2) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,3,0)> \supseteq <(9,6,0)> \supseteq \{(0,0,0)\}$

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$(3,1,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(9,0,0),(3,1,0),(0,0,1)> \supseteq <(3,1,0),(0,3,0)> \supseteq <$

$$(3,4,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (9,0,0),(3,1,0),(0,0,1) > \supseteq < (3,1,0),(0,3,0) > \supseteq < (3,7,0) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$$

$$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (9,0,0),(3,1,0),(0,0,1) > \supseteq < (3,1,1),(0,3,0) > \supseteq < (9,0,0),(0,3,0) > \supseteq < (0,3,0) > \supseteq \{(0,0,0)\}$$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(3,0,0),(0,1,0),(0,0,1)> \supseteq <(9,0,0),(3,1,0),(0,0,1)> \supseteq <(3,1,0),(0,0,1)> \supseteq <(3,1,0)> \supseteq <(9,3,0)> \supseteq \{(0,0,0)\}$

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$(9,3,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(0,1,0),(0,0,1) > \supseteq < (9,0,0),(3,1,0),(0,0,1) > \supseteq < (3,4,0),(0,0,1) > \supseteq <$

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$(9,3,0),(0,0,1) > \supseteq < (9,3,2) > \supseteq \{(0,0,0)\}$

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$(3,7,2) > \supseteq < (9,3,0) > \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq <(1,0,0),(0,3,0),(0,0,1) >\supseteq <(3,0,0),(0,3,0),(0,0,1) >\supseteq <(9,0,0),(0,3,0),(0,0,1) >\supseteq <$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,0,0),(0,3,0),(0,0,1) >\supseteq< (1,3,0),(0,0,1) >\supseteq< (1,3,0) >\supseteq< (3,0,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,0,0),(0,3,0),(0,0,1) >\supseteq< (1,6,0),(0,0,1) >\supseteq< (3,0,0),(0,0,1) >\supseteq< (9,0,0),(0,0,1) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (1,0,0),(0,3,0),(0,0,1) >\supseteq< (1,6,0),(0,0,1) >\supseteq< (3,0,0),(0,0,1) >\supseteq< (9,0,0),(0,0,1) >\supseteq< (9,0,1) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (1,0,0), (0,3,0), (0,0,1) > \supseteq \mathbb{Z}_{3^3} + \{0\} + \mathbb{Z}_3 \supseteq < (3,0,0), (0,0,1) > \supseteq < (9,0,0), (0,0,1) > \supseteq < (9,0,1) > \supseteq \{(0,0,0)\}$

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$(3,0,2) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$(9,3,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$(3,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,2),(0,3,0) >\supseteq< (3,0,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (0,3,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,2),(0,3,0) >\supseteq< (3,0,0),(0,3,0) >\supseteq< (9,0,0),(0,3,0) >\supseteq< (9,6,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,2),(0,3,0) >\supseteq< (3,0,0),(0,3,0) >\supseteq< (3,0,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,2),(0,3,0) >\supseteq< (1,4,2) >\supseteq< (3,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,2),(0,3,0) >\supseteq< (1,7,2) >\supseteq< (3,3,0) >\supseteq< (9,0,0) >\supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) >\supseteq< (9,0,2) >\supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq< (3,0,0),(1,1,0),(0,0,1) >\supseteq< (1,1,0),(0,0,1) >\supseteq< (3,3,0),(0,0,1) >\supseteq< (9,0,0),(0,0,1) >\supseteq \{0\}+\{0\}+\mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0), (1,1,0), (0,0,1) > \supseteq < (1,4,0), (0,0,1) > \supseteq < (3,3,0), (0,0,1) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3} + \mathbb{Z}_{3^2} + \mathbb{Z}_3 \supseteq < (3,0,0), (1,1,0), (0,0,1) > \supseteq < (1,4,0), (0,0,1) > \supseteq < (3,3,0), (0,0,1) > \supseteq <$

$(3,3,1) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(1,1,0),(0,0,1) > \supseteq < (1,4,0),(0,0,1) > \supseteq < (3,3,0),(0,0,1) > \supseteq < (3,3,2) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

$\mathbb{Z}_{3^3}+\mathbb{Z}_{3^2}+\mathbb{Z}_3 \supseteq < (3,0,0),(1,1,0),(0,0,1) > \supseteq < (1,4,0),(0,0,1) > \supseteq < (1,4,0) > \supseteq < (3,3,0) > \supseteq < (9,0,0) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) > \supseteq < (9,0,2) > \supseteq \{(0,0,0)\}$

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$(9,0,0),(0,0,1) > \supseteq \{0\} + \{0\} + \mathbb{Z}_3 \supseteq \{(0,0,0)\}$

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Lemma for Cyclic Subgroups of $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$

Lemma .0.1. *For any prime p, $G=\mathbb{Z}_{p^n}+\mathbb{Z}_{p^2}+\mathbb{Z}_p$ has $(n-1)p^3+2p^2+p+2$ cyclic subgroups.*