An Optimisation Model for Designing Social Distancing Enhanced Physical Spaces

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***Abstract*—In the wake of the COVID-19 pandemic, social distancing has become an essential element of our daily lives. As a result, the development of technological solutions for the design and re-design of physical spaces with the necessary physical distancing measures is an important problem that must be addressed. In this paper, we show how automatic design optimisation can be used to simulate the layout of physical spaces subject to a given social distancing requirement. We use a well known mathematical technique based on the circle packing to address this challenge. Thus, given the dimensions and the necessary constraints on the physical space, we formulate the design as a solution to a constrained nonlinear optimisation problem. We then solve the optimisation problem to arrive at a number of feasible design solutions from which the user can pick the most desirable option. By way of examples, in this paper, we show how the proposed model can be practically applied.**

Keywords: Automatic Optimal Design, Social Distancing Measures, Computer Aided Design of Physical Spaces, COVID-19 Pandemic

I. Introduction

As the COVID-19 pandemic unfolds manually enhanced, adhoc strategies have helped to deal with the complex nature of space re-rearrangements towards the mandatory social distancing requirements in offices, shops, restaurants, cafes and other public places. Due to the uncertain nature of the pandemic, standard operating protocols (SOPs) are dynamic. For example, the physical distance criteria that must be adhered between people in social spaces that tend to be dynamic are based on our understanding of the COVID-19 disease [1]. The architecture of the physical spaces may be inferred as reflecting a specific physical environment by taking into account the different specifications or variables associated with the design layout. The number of people to whom it should be available may include the arrangement of different pieces of furniture, structural elements such as beams and partitions, doors and windows, airflow pattern and the position of lights. Physical environments may vary from schools, workplaces, shopping malls and factories. The optimum designs, with social distancing requirements, of these physical spaces require the identification of potential configurations that take the appropriate requirements into account [2, 3, 4].

Here, we present a simple but flexible method for estimating the potential of public spaces and possible rearrangements of people aligned with social distancing requirements. Here we propose to model the arrangement of a public area as a solution to the packing of circles/disk and this approach can be extended to any private and public area [5]. The aim here is to automate the process of generating optimum arrangements to safely accommodate individuals within a given space. The circle packing method adopted here appears to help us achieve this. It also enables us to account for the existence of containment barriers as steps to regulate crowded occupancy, considering them as a depletion zone [6] along the perimeter of public space. Naturally, this depletion zone occurs because of the external barriers and possible internal obstacles on the discs imposed by the excluded region.

II. Our Proposed Approach

Consider the centre of a circle i as denoted by $\mathbf{x} = (x_i, y_i)$ and the radius of the circle i as denoted by $r_i,\ i = 1, 2, .., n$, where n is the total number of circle needs to be placed within the given domain. Here, assuming that, we are considering uniform sized circle with no overlapping, then radius of the circle can be simply written as $r_i = r,\ \forall\ i$. The pairwise between the centres of the circles i and j with $i < j$, measured using the Euclidean norm denoted by,

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2},\ i, j = 1, 2, ..., n.$$

A. *The Circle Packing*

A circle packing [7] is an optimised arrangement of N arbitrary sized circles within a container (e.g. a rectangular, triangular and/or circular) so that no two circles coincide. The consistency of the packing is usually calculated by (1) the container size, (2) the pairwise weighted average distance between the circle centres, or (3) a linear combination of parameters [8, 9]. This process is composed of three components. The first component minimises the total number of people present in a given physical space at a time; the second component maximises the radius between people to be 2m (it is converted into a minimisation component by a multiplication factor (-1); the last component include constraints for the layout specifications and other constraints like airflow, the present lifts, doors and windows etc.

B. Uniform Sized Rearrangement using the Circle Packing

There are few main variants of the uniform sized rearrangement problem, They include,
1. Finding the maximum number of people N and the maximum radius r_0 of the within the domain considered so that it can hold N people in the form of circles with no overlapping. Assuming that the domain is centred at the origin, this problem is formulated as follows,

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & r_0, \\ \text{subject to} \quad & \sqrt{x_i^2 + y_i^2} \leqslant r_0, \\ & 2r \leqslant d_{ij}, \\ & 1 \leqslant i < j \leqslant n \leqslant N. \end{aligned} \tag{1}$$

2. Finding the maximum radius r of n uniform sized and non-overlapping circles belonging to the rectangular domain. This problem is formulated as follows,

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & r, \\ \text{subject to} \quad & 2r \leqslant d_{ij}, \\ & r \leqslant x_i \leqslant L - r, \\ & r \leqslant y_i \leqslant W - r, \\ & 1 \leqslant i < j \leqslant n < N. \end{aligned} \tag{2}$$

3. Finding the position of n points inside the rectangular domain in such a way that their minimum pairwise distance d is maximised. This problem is written as,

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & d, \\ \text{subject to} \quad & d \leqslant d_{ij}, \\ & -L \leqslant x_i \leqslant L, \\ & -W \leqslant y_i \leqslant W, \\ & 1 \leqslant i < j \leqslant n < N. \end{aligned} \tag{3}$$

Let us point out that there are two main categories of studies dealing with uniform sized circle packings. One of these approaches is to prove the optimality of suggested packings, either (purely) theoretically, or with the help of a computational algorithm, see [5, 10] for detailed discussions and further references. The other approach is to develop efficient numerical solution strategies that can be readily applied, but (in general) without proven optimality of the results obtained. This paper is focused on the latter approach, noting that it can be readily applied to 'arbitrary' packings without substantial modifications.

Result: $R_{\text{b}} \leftarrow 0$, iter $\leftarrow 0$, $n > 0$
Initialization: random/initial positions;
the radius associated with each object;
domain bounds and obstruction positions;
Iterate:
solve constraints;
solve optimisation using solver fmincon using domain specifications and constraints;
correct the radius and update the iteration counter;
Repeat: (solver → correct radius → update counter);

Algorithm 1: Algorithm for the proposed optimisation model.

Note that the description of the Algorithm 1 is such that it is based on the domain being a rectangle and objects in it being circles. The domain types can conveniently be changed using the following considerations.

- The initial solution (x_i, y_i), $i = 1, ..., n$ is generated by randomly placing n circles in the rectangular domain.
- Then, the correction procedure $correct(X, Y)$ ensures that the centres of the circles (x_i, y_i) from the optimiser are inside the domain. Essentially, this will correct the maximum radius of the circle that can be placed in the domain without any overlapping.
- Thus, one of the advantages of this heuristic based algorithm is that it is easy to adapt to encompass a variety of different domains.

III. Rearrangements with Constraints

An important parameter to be addressed in the re-design of physical spaces is the flow of air around a closed area, where people have to interact socially. The air circulation around the specific area can, where possible, be reduced in order to reduce the risk of disease transmission, for example, turning off the air conditioners where possible. Here, we demonstrate how airflow parameters can be taken into account when the optimal design problem is formulated in a closed environment.

The airflow pattern, while under consideration in a three-dimensional physical setting, can be projected into a two-dimensional plane. The flow pattern can be circular or rectangular within an area. For example, the direction of the source from which the air flows, the speed of airflow and the orientation of all windows and doors should also be taken into consideration.

If the airflow source is a fan, for example, the flow pattern will vary depending on where the fan is located which can be written as a problem of optimisation,

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & -r, \\ \text{subject to} \quad & 2r \leqslant ||p_i - p_j||, \\ & p_i \in [r, L - r], \quad p_j \in [r, W - r], \\ & x_i - x_j < a, \quad y_i - y_j < b \quad a, b \in \mathbb{R}, \\ & 1 \leqslant i < j \leqslant n, \end{aligned} \tag{4}$$

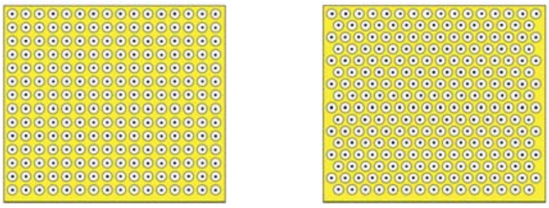

Fig. 1. An example physical arrangement where of people in the form of a circle can be placed in a physicals space of dimension ($90 \times 80m$) in rectangular (left) and triangular (right) pattern.

where, L is the length, W is the width of the room/domain, a and b are the parameters to be modified by form of flow. The limitations of the issue in the Equation (4) can be added or omitted according to the user's particular needs.

Thus, one must note that this case is highly dependent on the user requirements and the setup. However, it is easy to detect the pattern and consider the constraint from the problems defined above and get space rearranged. This kind of systems can be used for designing meeting rooms, common rooms, waiting areas, as well as similar such public areas.

IV. Determining the Maximum Feasible Number of People in a Public Space

To figure out how many people can sit in a classroom and a hallway is to put non-overlapping circles into floor plans if each person must be six feet away from everybody else. This issue raises a very important question of how to manage crowds in areas like supermarkets, particularly open markets, hospitals [10, 11].

Assuming a rectangular domain of dimensions $150m \times 100m$, where we need to find maximum number of people can be placed. For this, we are considering people as a circle of radius $2m$. This implies the area of rectangle (A_r) is $15000m^2$ and the area of one circle (A_c) $12.6m^2$. Then, the maximum number of circles can be placed with a rectangular pattern is 925 and the same with triangular pattern is 1079 shown in Figure 1.

For Rectangular pattern:
Maximum number of circles can be fitted = $\dfrac{l}{d} * \dfrac{w}{d}$,
Circles to Rectangle Area Ratio = $\dfrac{\sum A_c}{A_r} = 77.5\%$
where, l =length of the rectangle, w = width of the rectangle, d = diameter of circle.

For Triangular pattern:
Maximum number of circles can be fitted = $\dfrac{\pi A_r}{2A_c\sqrt{3}}$,
Circles to Rectangle Area Ratio = $\dfrac{\sum A_c}{A_r} = 90.6\%$.

V. Rearrangements within Multi-Purpose Physical Spaces

Given a set of physical spaces, the number of sections/rooms, and a set of walking pathways(corridors), the multi-section model consists of assigning facilities maximum number of people they can allow in the premises with an optimal arrangement while optimising the objective function in equation (1) [12, 13]. Figure 2 provides an illustration example of design with 5 rooms in total [14, 15, 16, 17].
The adaptive layout for social distancing problem in hospitals/clinics seeks to determine the 'most efficient' layout placement of a set of healthcare operating facilities which include corridors, elevators and doors in a designated area subject to a set of constraints healthcare related physical constraints. Existing layouts arrangements are mostly generated from designs based on ad-hoc solution and best practice judgment - which may not be optimal. Due to the lack of scientific rigour and the huge impact of rearrangements on the efficiency and effectiveness of a clinic, the paper proposes nonlinear programming models to find optimal layouts under the design variant. The layout example discussed here consist of 3 doctors' rooms and a lab room with a maximum capacity of 4 people, a waiting area and a reception with a maximum capacity of 55 people. The general objective consists of objectives: the first sub-objective is to minimise the total number of people whereas the second sub-objective is to maximise the distance among people. The maximum capacity of people at a given time in the clinic layout is shown in the left side of Figure 3 and the solution with the 2m social distance among people is shown in the right side of Figure 3 which is one of the optimal situations obtain after solving the nonlinear optimisation problem. Note, each person is represented by a black circular disc.

A. Constraints Within Multi-Section Physical Spaces

To ensure that the corridors are located within the boundary of the section to which they belong, the following four equations are introduced using the left-bottom (x_l, y_b) and the right-top points (x_r, y_t) on the x-axis and y-axis defining a section or a floor with a corridor centre (x_c, y_c). The corridors must have common boundaries between two different sections/rooms.

$$\begin{aligned} x_c + \frac{\partial_x}{2} &\leqslant (x_r + x_{max})\delta, \\ y_c + \frac{\partial_y}{2} &\leqslant (y_t + y_{max})\delta, \\ x_c - \frac{\partial_x}{2} &\leqslant (x_l - x_{max})\delta, \\ y_c - \frac{\partial_y}{2} &\leqslant (y_b - y_{max})\delta, \end{aligned} \tag{5}$$

where, ∂_x and ∂_y is length and width of the corridor and δ is the Dirac delta function which gives one if facility is assigned to the section, otherwise it is zero.

B. Experimental Results

To test our model, by way of a practical example, we approached the University of Bradford, UK, who later shared one of their common areas with us, where they were interested in finding the physical rearrangements of the atrium area shown in the left side of Figure 4. As we can see, the atrium

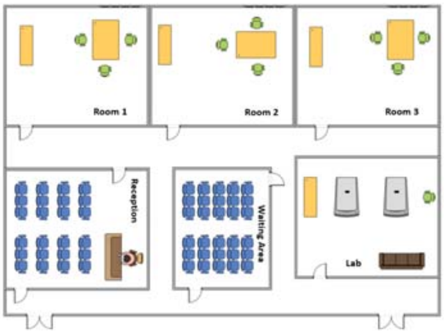

Fig. 2. Arrangement in a medical clinic setting - three doctors' rooms, one lab, waiting area and a reception.

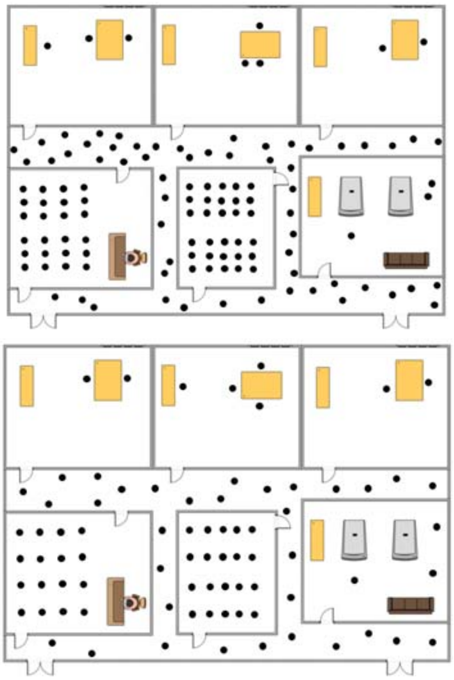

Fig. 3. Arrangement in a medical clinic setting left: before, right: after (MATLAB simulation).

area marked in red is odd-shaped and hence needs to convert into a mathematical shape shown on the right side of Figure 4. For simplicity, we have divided the atrium area into two rough trapeziums. The top trapezium is having a constraint of a lift, where 1m distance is left from all the sides of the lift. To solve this problem, we need to discretise our domain into rectangles shown in Figure 5. One needs to be careful here while discretising the domain, because of the lift constraint in the top trapezium. Discretisation should not cross through the constraints. Discretisation can be done either manually or using a mesh grid depending on the complexity of the domain. Finally, the optimised solution is generated for each region and then combined together. There are few assumptions we need to make before finding a solution as follows,

- $1m$ the distance on both the sides for the common edges in

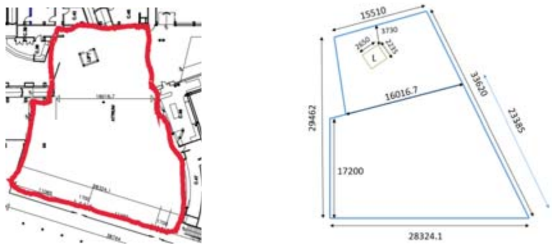

Fig. 4. Atrium area of the University of Bradford, UK. (all dimensions are in mm).

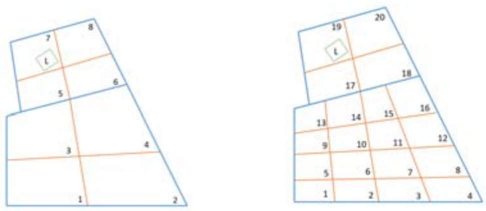

Fig. 5. Domain discretisation (left: First refinement, right: second refinement only in the lower trapezium).

the corresponding element.
- $2m$ the distance from the main wall of the domain.
- $2m$ the distance among people.
- $1m$ the distance from the walls of the lift.

The mathematical optimisation model for Atrium problem is defined below,

$$\begin{aligned} \text{minimize} \quad & -r, \\ \text{subject to} \quad & 2r_i \leqslant ||p_i - p_j||, \\ & r \leqslant \frac{(x_1 - x_i)^2}{a^2} + \frac{(y_1 - y_i)^2}{b^2} - D^2, \\ & 1 \leqslant i < j \leqslant n, \quad a \leqslant b \\ & x_i \in [r, L - r], \ y_i \in [r, W - r]. \end{aligned} \tag{6}$$

Under normal circumstances, the atrium has a maximum capacity of 300 people with no social distancing and 160 with the social distance of 2m shown in left side of Figure 6. The Matlab simulation of the atrium with people rearrangements with 2m distance is shown in the right side of Figure 6. Different possible rearrangements of are also shown in Figure 7.

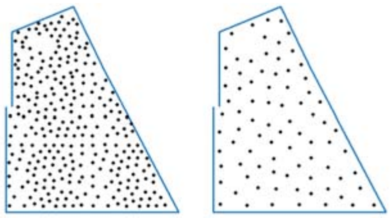

Fig. 6. Proposed arrangement of people in the atrium area. Left: without social distancing, Right: with 2m social distancing.

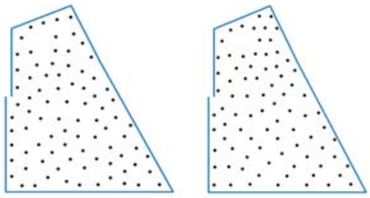

Fig. 7. Different possible solution with 2m social distancing.

VI. Conclusion

In this paper, we have proposed a method of design optimisation for managing the rearrangement of physical spaces with social distancing constraints in the wake of the COVID-19 pandemic. To do this, we have utilised the well-known circle packing technique whereby the design problem is defined as a constrained optimisation problem. Thus, given the dimensions of physical space and other essential requirements such as the social distancing requirements and the position of non-movable physical objects as well as the airflow pattern, the solution resulting from the automated optimisation algorithm can suggest an optimal set of design alternatives from which a user can pick the most feasible option. As a result, we demonstrate that the proposed model outlined here is feasible and can be applied practically.

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