GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11: A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT

by

ISRAEL YEUKAI MARANGE

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ABSTRACT

The aim of this study was to establish the influence of using GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry at one school in OR Tambo Inland. The study adopted a quantitative approach and utilised the quasi-experimental research design. The sample consisted of 107 Grade 11 mathematics learners. Sixty (60) learners were in the experimental group and 47 in the control group. Pre-test and post-test, and likert-scaled questionnaires were used as instruments. Reliability and validity were ensured through test-retest, as well as member checking and a pilot study. All ethical requirements were followed. Findings revealed that pre-test results did not show much difference in the performance of experimental and control groups. After using GeoGebra, it statistically emerged that control group respondents performed lower than the experimental group. Findings obtained from the questionnaire also showed similar patterns. The study also found that participants who learnt circle geometry using GeoGebra were significantly motivated and that GeoGebra allowed learners to be exceedingly creative and discover skills of solving geometry problems by themselves. The positive impact of using GeoGebra resulted in significant differences on academic performance. Based on gender, more females compared to males agreed that GeoGebra was an effective manipulative tool in learning circle geometry. Interestingly, boys performed better than girls in the post-test. The study concluded that students who used GeoGebra showed higher ability in conceptual knowledge compared to students who used conventional methods. It also concluded that GeoGebra had positive effects on learners' understanding as learners became significantly active and responsible for their own learning process as the software allows a self-learning process. Recommendations included the need for teachers to use the latest technology and to vary their methods of teaching to motivate learners. There is also a need for a workable alternative opposed to the rigid axiomatic approaches to circle geometry to facilitate and enhance learners' ability to make and test conjectures. Curriculum planners and subject specialists need to emphasise on education systems that shift teaching and learning away from the traditional methods and emphasise on learning rules for manipulating geometry problems.

DECLARATION

I, Israel Yeukai Marange, declare that the dissertation "GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11: A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT" is my own work and has not been submitted in its entirety to Walter Sisulu University or any other university. The sources that I have used have been acknowledged by means of complete references and the list is provided.

21/06/2019

Israel Yeukai Marange — Date

PLAGIARISM DECLARATION

I:	Israel Yeukai Marange, Masters candidate in the Faculty of Education at WSU
Student number:	217303404

Declare the following:

1. I understand what plagiarism entails and am aware of the University's policy in this regard.
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3. I did not copy and paste any information directly from an electronic source (e.g., a web page, electronic journal article or CD ROM) into this document.
4. I did not make use of another student's previous work and submitted it as my own. Further professional guidance was provided by my supervisor, Prof. J.K Alex.
5. I did not allow and will not allow anyone to copy my work with the intention of presenting it as his/her own work.

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DEDICATION

To my wife Nyasha and my two daughters, Tyla-lee and Tamara.

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CHAPTER 1

ORIENTATION AND BACKGROUND OF THE STUDY

1.1 Introduction

Geometry is the "study of shapes, their relationships, and their properties" (Bassarear 2012, p.463). According to Akaazua, Bolaji, Kajuru, Mu, Musa and Bala (2017), the importance of geometry in providing learners with a vehicle for enhancing logical reasoning and deductive thinking for modelling abstract problems has made policy makers to consider geometry as an essential branch of Mathematics curriculum at all grade levels in South Africa. Geometry has historically been a topic that many learners struggle with and increasingly dislike as they progress through the grades.

According to Adolphus (2011, p.144), "Euclidian Geometry is the most problematic area to teach and learn". The inclusion of Circle Geometry in the Curriculum and Assessment Policy Statement (CAPS) was a curriculum change of great magnitude to mathematics teachers and students alike. Since my employment as a Mathematics teacher, my experience has not been different from authors with regard to learners' performance in Geometry. Chimuka (2017) claims that teachers are pedagogically ill-equipped to effectively teach circle geometry as most of them were not taught the topic well in school. According to the Mathematics Paper 2 Chief Markers' Report (2018, p.31) suggested that "GeoGebra could be used to provide visual reinforcement of the theorems work and how they are applied in various situation". Ojose and Sexton (2009) pointed out that the use of manipulative materials has become one way of involving learners in fun learning that encourages motivation of students. In the same article, they further claim that manipulatives have also been useful in making abstract ideas concrete for learners and thereby making for conceptual understanding. The study sought to investigate the effectiveness of GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry.

1.2 Background to the study

The National Council of Supervisors of Mathematics (NCSM) urges mathematics teachers and other involved stakeholders on the use of manipulatives when teaching and learning mathematics (NCSM, 2013). Manipulatives are capable of engaging learners and boosting both the attentiveness and enjoyment of mathematics (Moore, 2013). Cain-Caston (1996) and Heuser (2000) claim that when students learn mathematics with manipulatives and then provided an opportunity to give feedback and reflect on their experiences, it is found that mathematics anxiety is significantly lessened and mathematical learning intensified. National Council of Teachers of Mathematics (NCTM) (2000, p.11) declared that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning". Roberts (2012) added that amalgamating technology in circle geometry teaching provides pronounced learning opportunities for students.

In 2006 Euclidean Geometry was made a voluntary section (Paper 3) of mathematics in South Africa. Research show that Euclidean Geometry went out of the mainstream curriculum between 2009 and 2013 because most schools were not teaching it (Van Putten, Howie & Stols, 2010). Many schools that did not have trained and experienced teachers to teach geometry decided to opt out. Schools that chose to do Paper 3, however, had most of their learners choose not to do it because of the extra load towards their chances of passing mathematics (Bowie, 2009). This disadvantaged mathematics learners because geometry is the basis for learners' success with further studies at tertiary level in mathematical, engineering and health sciences (Kearsley, 2009).

The researcher believes that Euclidean Geometry is a topic where learners have to actually get to understand by being actively involved, which is not a common culture in practice in South African schools even though Curriculum and Assessment Policy Statement (CAPS) emphasises on the matter (Van Putten, Stols & Howie, 2014). While CAPS embodies the learner-centred teaching approach, its implementation is questionable. This has been indicated by the studies done by Morar (2000) and Van Putten et al. (2014) who claim that despite South African teachers' beliefs about learner-centered teaching approaches, they use conventional approaches in their classrooms. Circle geometry, in many cases, is taught using conventional methods in numerous secondary schools (Van Putten et al., 2014). Reed (1996) cautioned that this approach of instruction does not readily lend itself to student input and creativity. She further explained that learners are not given a chance to discover and explore theorems or properties on their own. When using this kind of approach, learners are told about the theorems and are asked in many cases to memorise them.

Geometry learning involves the ability to visualise and involves a number of issues that require imaginations. Students are discouraged to learn the subject when it is not presented in a modern and accessible way. According to Majerek (2014), the main obstacles to teaching mathematics are concepts without an adequate illustration and mathematics diagrams or objects that are static. Static objects do not allow for generalisation of concepts. In South Africa, the 2012 presentation by Linda Chisholm to the Minister of Basic Education highlighted inadequate content and pedagogical knowledge (Department of Basic Education (DBE), 2012). Teachers' lack of content knowledge results in teachers not explicitly explaining the concepts to the satisfaction of the learners. Hill, Ball and Schilling (2008) define pedagogic content knowledge as knowledge about the purposes for teaching a given subject matter, knowledge about the order in which subject matter should be presented and knowledge about the instructional strategies useful for teaching content. According to Guzman (2008), many concepts in circle geometry require learners to visually perceive objects and identify their properties by comparing them with their previous experiences involving similar objects. By being able to "touch-see-and-do" and interacting with the objects of their learning, learners can learn circle geometry in a more imaginative and successful way (Tay, 2003, p.1).

1.3 Statement of the problem

Circle geometry poses a great challenge to most learners in OR Tambo Inland District. Most learners encounter challenges in proving and applying the theorems to answer questions on Euclidean geometry. Most questions in the circle geometry section aim at finding the reason why given statements are true, completing the gaps in a seen proof, direct application of known facts or recall of elements of the proof of a theorem that is designated as bookwork. They lack skills to prove and apply theorems from CAPS Grade 11 Guidelines (see Appendix G) which state:

(i) The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

(ii) The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference (on the same side of the chord as the centre).

(iii) The opposite angles of a cyclic quadrilateral are supplementary.

(iv) The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

In the researcher's teaching experience, the researcher observed that questions that demand the application of circle geometry theorems and corollaries when solving riders posed problems to the learners. Most learners were leaving questions unanswered and if they answered, the researcher observed that there were many errors that were being made. These errors included failing to find unknown angles and stating inaccurate reasons for statements they wrote. All these challenges were observed during marking controlled tests, end of term examinations and final examinations for many years since the topic was introduced. The researcher shared his experiences with other mathematics teachers during departmental meetings. Surprisingly, the same problems were common with Grade 11 and Grade 12 mathematics teachers. All these were observed when the researcher was still a teacher.

Currently, the researcher is involved in advisory services on curriculum implementation, monitoring and evaluation of high school mathematics working for Department of Basic Education as Senior Education Specialist commonly known as subject advisor. The researcher has observed that besides learners having challenges in answering questions on circle geometry, teachers themselves have challenges in preparing and teaching the topic. This has resulted in under-teaching of circle geometry concepts. The researcher was then convinced that the instructional approaches that mathematics teachers are using when teaching circle geometry are not helping learners enough to learn circle geometry theorems and their application.

Teaching in a systematic approach is required from teachers in the process of developing knowledge through suitable methodology to induce effective learning in the classroom. For instance, official guidelines for Grade 10 (Terminale in France), published in 2011 (cited in Hausberger, 2015, p.221), make it clear that:

> elements of epistemology and history of mathematics fit in naturally in the implementation of the curriculum. To know the names of a couple of famous mathematicians, the period in which they lived and their contribution to mathematics are an integral part of the cultural baggage of all students taking scientific education. Presentation of historical documents is an aid to the understanding of the genesis and evolution of certain mathematical concepts.

Zawawi (2002) claims that the methodology used by mathematics teachers were still teacher-centred and they are influenced by conventional methods. Learners tend to memorise mathematical theorems and formulae without understanding the concepts that lie behind it. This situation is producing learners who are able to calculate but do not know how to solve common problems that involve concepts and mathematical skills.

Moreover, the idea that present-day learners are enthusiastic about technology motivated the researcher to explore the instruction in incorporating manipulatives that is technology-enabled in the learning of circle geometry. Mathematics teachers are required to gradually develop instructional approaches that make learners to like, experience, apprehend and appreciate circle geometry (NCTM, 2000). Twenty first century learners are modern students and are growing up in a digital world. Teaching methods that involve technology are now viewed as crucial to this generation. The use of digital gadgets by students are viewed as their everyday experience out of a classroom setting. Moreover, the present-day student has a much lower need for traditional resources such as libraries of physical content, as Howie (2012, p.5) suggests that "all teachers need digital pedagogies". According to Efandi et al. (2007), teachers must be able to intelligently improvise teaching when and where necessary. A variety of approaches can be used to increase learners' mathematical skills and their understanding of geometric concepts. Zakaria et al. (2010) cited Oldknow and Taylor (2000) who demanded that an alternative approach which is the use of information and communication technology (ICT) that could help teachers not only in the teaching of geometric concepts but also to lighten their workload and allow teachers to solve learners' problems individually. The use of computers in mathematics education was able to make the teaching and learning methodology of circle geometry more up-to-date and interesting as compared to the conventional method, according Norazah and Effandi (2007).

This has motivated the researcher to research into an instructional approach that may make learners able to understand and recall the theorems and ultimately apply them when solving circle geometry problems. This study, therefore, was designed to explore an alternative technology-enhanced instructional approach to teach so that learners could possibly appreciate learning and apply circle geometry concepts appropriately. Based on the problem statement presented above, the following were the research questions and objectives.

1.4 Main and sub-research questions

1.4.1 Main research question

The main research question of this study was: What is the influence of using GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry at one school in OR Tambo Inland District?

1.4.2 Sub-research questions

In order to carry out the study, the following sub-research questions were considered.

1.4.2.1 What are the effects of conventional methods of teaching in learning the concepts of circle geometry?

1.4.2.2 How does learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation?

1.4.2.3 What is the significance of using GeoGebra on the academic performance of learners learning circle geometry?

1.4.2.4 What are the gender differences in terms of students' achievement after learning circle geometry using GeoGebra?

1.5 Aim and objectives of the study

1.5.1 Aim of the study

The aim of this study was to establish the influence of using GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry at one school in OR Tambo Inland District.

1.5.2 Objectives

The study was guided by the following specific objectives:

1.5.2.1 To assess the effects of conventional teaching methods in learning the concepts of circle geometry.

1.5.2.2 To determine how learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation.

1.5.2.3 To establish the significance of using GeoGebra on the academic performance of learners learning circle geometry.

1.5.2.4 To establish if there are gender differences in terms of students' achievement after learning circle geometry using GeoGebra.

1.6 Rationale of the study

In the researcher's experience of teaching high school mathematics, the researcher found that most Grade 11 learners experienced problems with circle geometry. The researcher sought to encourage learners to understand theorems and solving of riders by exposing them to experiential learning. This type of learning does not promote rote learning instead learners understand through experience. Circle geometry is an area of mathematics where experiential learning can be used and where technology may be utilised.

For many years of teaching mathematics particularly circle geometry, the researcher had been using transmission approaches to teaching commonly known as a conventional methods. The researcher used to ask if learners were following the taught procedures. This instruction encourages passive attitude to learners and makes learners feel they have nothing to contribute. The lesson is however conducted through explicit teacher explanation of lectures and teacher-led demonstrations. The researcher believed that experiential learning would allow learners to learn and understand circle geometry concepts by interacting with the dynamic software called GeoGebra and sharing their experiences.

A research conducted by Ferrance (2000) indicated that "studies conducted by teachers themselves, in a familiar school setting, with their students, would help solve real problems in schools and thus contributes towards improving teaching and learner achievement". In this study, the researcher needed to let Grade 11 mathematics learners be at the centre of learning by using the manipulative tool that fully supports learners' engagement and interest in learning circle geometry.

1.7 Significance of the study

The South African education system is confronted by the under-preparedness of teachers particularly in the teaching of circle geometry (Aldridge, Fraser and Ntuli, 2009). One of the aims as stipulated in the National Curriculum Statement Grades R–

12 is that teachers of mathematics need to produce learners who are able to communicate effectively using visual, symbolic or language skills in various modes (Department of Basic Education, 2011). In order to prepare geometrically literate citizens for the 21st century, classrooms need to be restructured so that mathematics can be learned with understanding (Carpenter & Lehrer, 1999). The significance of this study lies in that it is likely to motivate learners when learning circle geometry. The findings would introduce a learner-centred approach where learners would explore and discover the processes involved when learning circle geometry concepts. It might lessen the cases of learners leaving circle geometry questions unanswered. The errors that learners make when answering circle geometry questions might be lessened because of the experiences learners acquire when learning the concepts. Thus, the study is significant since it meets the needs of students who learn better, when the concept is reinforced through a variety of media.

Information and Communication Technology (ICT) represents a fundamental paradigm shift in mathematics education. ICT allows multiple representations of mathematics and enhances the interaction between learners and the mathematics that they learn (Leung, 2006). It is envisioned that the study will assist mathematics teachers in instructing geometry with technology assisted learning. Employing ICTs, it is planned that learners will have a better understanding of the geometrical concepts than students who have undergone traditional instructions.

It is envisaged that the study might also significantly benefit the body of knowledge in mathematics education. Mathematics teachers, geometry teachers, and curriculum and professional development planners are likely to gain insight into the students' learning and thinking around circle geometry concepts, which are important for learner-centred classrooms. The findings of the study provide relevant background study and literature to other researchers who wish to further research into the teaching and learning of circle geometry using technology.

1.8 Conceptual and Theoretical Frameworks

A framework can be defined as a hypothetical description of a complex entity of process or the underlying structure or a structure supporting or containing something. The study will be guided by the conceptual and theoretical frameworks.

1.8.1 Conceptual Framework

The following concepts will guide this study: manipulatives, technology in teaching and learning of mathematics, and GeoGebra as a virtual manipulative tool.

1.8.2 Theoretical Framework

This study will be framed by three theories namely, Constructivist Theory, Experiential Learning Theory and Concrete Representational Abstract. In a constructivist classroom, the teacher is expected to provide learners with resources and activities that ensure that they are actively involved and participate in, while constructing their own knowledge. Experiential learning theory acknowledges learning as a process where knowledge is created through the transformation of experience as well as thinking about those experiences. In Concrete Representational Abstract theory learners first use concrete materials to solve problems and look for patterns and generalisations.

1.9 Definition of pertinent terms

The following key terms are defined in this study.

- **Geometry**

  Bassarear (2012, p.463) defines geometry as the 'study of shapes, their relationships, and their properties.' In this study, geometry takes the same definition.

- **GeoGebra**

  Hohenwarter (2009) defines GeoGebra as a dynamic mathematics software for all levels of education that join arithmetic, geometry, algebra and calculus. GeoGebra lets students construct dynamic-mathematics objects such as points, segments, lines, circles, angles and other dynamic functions and investigate them interactively. In this study, GeoGebra takes the same definition.

- **Virtual Manipulatives**

  Moyer and Bolyard (2016, p.3) define virtual manipulatives as "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all programmable features that allow it to be manipulated, that presents an opportunity for constructing mathematical knowledge". In this study, virtual manipulatives refer to the use of GeoGebra as a mathematical tool that learners use to demonstrate their understanding of circle geometry concepts.

- **Learning**

  Learning is the lifelong process of transforming information and experience into knowledge, skill, behaviours and attitude (Cobb, 2009). In this study, learning refers to the acquisition of circle geometry concepts by learners either individually, cooperatively and/or through mediating with peers.

- **Processes**

  Geometry processing is concerned with mathematical models and algorithms for analyzing and manipulating geometric data (Botsch, Kobbelt, Pauly, Alliez & Lèvy, 2010). Processes in this study will mean learners' motivation, demonstration of concept understanding and academic performance of Grade 11 circle geometry.

- **Conventional teaching method**

  According to Akaazua, Bolaji, Kajuru, Mu, Musa and Bala (2017, p.89), "conventional teaching method is a teaching procedure in which there is a one-way channel of communication where the teacher makes an oral presentation of the subject matter content and students react by silently listening and taking notes". In this study, conventional teaching method takes the same definition.

1.10 Research outline

Chapter 1 introduces the study, the background of the study, local and international mathematics standards on technology and the use of manipulatives in teaching and learning of mathematics. The author also presented the problem statement, a brief discussion of the main and sub-research questions, the rationale and the significance of the study. The chapter ended with the definitions of pertinent terms.

Chapter 2 focuses on developing the study through a detailed discussion of research questions and a deeper understanding of the literature of GeoGebra as a manipulative tool in providing processes to Grade 11 circle geometry. Theoretical framework underpinning the study in the teaching and learning of circle geometry is discussed in this chapter.

Chapter 3 presents the research methodology for the study. The research design, procedure, instruments of the study, data collection procedure and analysis are presented in this chapter.

Chapter 4 presents the analysis of quantitative data gathered during the study. Discussions of the results will be done in this chapter.

Chapter 5 summarises, concludes and gives recommendations of the study.

1.11 Summary

In chapter 1, the researcher introduced the study by discussing the background and the problem statement of the study. Research questions and their associated objectives were also presented in this chapter. The researcher explained the rationale and the significance of the study. Chapter 1 ended with a definition of the relevant terms used in the study. Chapter 2 focuses on study development of the literature and theoretical and conceptual frameworks underpinning the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Chapter 1 presented an overview of the study. The scope of this study and the questions it answered was premised on the adopted theoretical framework. Eisenhart (1991, p.205) defined a theoretical framework as "a structure that guides research by relying on a formal theory constructed by using an established, coherent explanation of certain phenomena and relationships". "The ideas of constructivism, experiential learning and the Concrete Representation Abstract (CRA) concept of learning using technology were employed as the theoretical framework underpinning the study. The researcher stitched theoretical framework with conceptual framework which, in this study, served as the basis for understanding the causal or correlational patterns of interconnections across events, ideas, observations, concepts, knowledge, interpretations and other components of experience" (Svinicki, 2010, p.5). The importance of manipulatives, technology in the teaching and learning of mathematics and GeoGebra as a manipulative tool in providing processes of circle geometry were employed as the conceptual framework underpinning the study. This chapter focuses on developing the study through a detailed review of the empirical literature pertaining to the research questions, presenting theoretical framework of the study and the conceptual framework.

2.2 Study development

In this section, the researcher developed the study based on the objectives.

2.2.1 The effect of conventional teaching methods in learning the concepts of circle geometry

A conventional teaching method is a collection of teaching methods other than the concrete manipulative approach. According to Akaazua et al. (2017, p.89), "conventional teaching method is a teaching procedure in which there is a one – way channel of communication where the teacher makes an oral presentation of the subject matter content and students react by silently listening and taking notes". Such approach creates boredom in class, encourages passive attitude among learners and make them feel they have nothing to contribute (Fletcher, 2009). According to Larbi and Mavis (2016), the teacher-centred approach contradicts the vision of mathematics instructions as indicated by the standards of National Council of Teachers of Mathematics (NCTM). Tomljenović (2015) cited Jagodzinski (2009) who argued that the conventional approach of transmitting knowledge, where learners are simply passive recipients of information, must be replaced with more effective student-centred teaching strategies such as active, experiential, independent, investigative, cooperative and problem-solving learning. According to Luneta (2013), the main cause of learners' errors and misconceptions are mathematics teachers. The author further identifies teaching habits and ineffective teaching approaches such as persistent teacher-centred approaches as some of the sources of learners' errors. Wrong questioning techniques, such as incomplete, ambiguous, or unnecessarily difficult questions can cause learners to make mistakes.

Romani and Patadia (2012) commented on teacher-learner ratio in normal South African classroom especially townships. They claimed that it is difficult for the teacher to reach out to all the learners, thus allowing for computers and other devices to provide a possible solution to this problem. According to the authors it may be one of the possible reasons of poor performance by learners in mathematics. The larger the class the more difficult it is for teachers to teach learners effectively, maintain discipline in class, immediately mark and give feedback to learners and provide individual attention to learners. However, learners are computer enthusiasts and computers have the ability to show each learner where they will be making errors. Moreover; according to Skemp (1976), the use of computer assisted instructions tends to limit the teacher's involvement in the learning process. This may force learners to create their own solution paths, thus developing a stronger relational understanding of circle geometry concepts, as compared to learners who are exposed to conventional teaching instructions where the teacher is tempted to tell learners what to do.

2.2.2 Impact on learners' motivation of learning circle geometry using GeoGebra as a manipulative tool

Technology has an important role to play in the teaching and learning of Geometry according to the NCTM (2000). The NCTM's (2000) view is also supported by Department of Education (DoE) (2003) when it suggests that learners should be able to use science and technology effectively, critically, and in the development of models. Instructional technology has its impact on teaching and in learning mathematics (Soman & Sivarajan, 2009). According to Majerek (2014) and NCTM (2000), technology is essential in teaching and learning mathematics, it influences the mathematics that is taught and enhances students' learning. New learning opportunities are provided in technological environments, potentially engaging students of different mathematical skills and levels of understanding with mathematical tasks and activities (Hollebrands, 2007). Tools, both representations and virtual manipulatives, are helpful for communicating ideas and thinking that are otherwise difficult to describe, talk about, or write about (Anthony & Walshaw, 2007).

GeoGebra was used to visualize mathematical concepts as well as to create instructional materials. It has the potential to foster active and student-centred learning by allowing for mathematical experiments, interactive explorations, as well as discovery learning (Preiner, 2008 & Bruner, 1961). Pandiscio (2002) as cited by Bhagat and Chang (2015) pointed out that GeoGebra can help learners by enabling them to comprehend the ideas embedded in the theorems and problems more fully than they would have understood without the aid of technology. GeoGebra can encourage students to explore mathematics and offer opportunities for critical thinking, which is vital to constructivism. Reisa (2010) (cited by Bhagat and Chang, 2015), claims that making more use of GeoGebra in circle geometry teaching would be an important factor in an effective mathematics teaching and a permanent learning.

Diamond (2012) argues that an examination of learners' motivation was a stepping stone to improving learner mathematical proficiency and success. A study conducted by Mansukhani (2010) concluded that increasing the levels of learner motivation could be increased by creating an environment in which learners are inventors of their own knowledge through hands on projects and discovery activities. Learners view a mathematics class environment as motivating when teachers present concepts in various ways such as models and diagrams. According to Korenova, (2012), digital environment motivates students in the teaching and learning of Mathematics. It also encourages both teachers and students to engage in learning and teaching (Ozdamli, Mus & Nizamoglu, 2013). Arbain and Shukor (2015) cited Yenilmez (2009) who claims that many scientific studies show that computers have made it easier not only to understand mathematical concepts, but also enhance students' motivation and self-confidence.

Venkataraman (2012) in Singapore carried out a study on innovative activities to develop the geometrical reasoning skill in secondary mathematics with the help of GeoGebra and found that students taught with the software made progress towards mathematical explanations, which provide a foundation for further deductive reasoning in mathematics. He found that the dynamic nature (drag feature) of the software influences the form of the explanation and that the students were able to generalize the solution and respond with an adequate statement. Venkataraman (2012) concluded that GeoGebra makes learning abstract concepts far more meaningful and helps students to visualize related concepts.

Effective teachers draw on a range of representations and tools to support learners' mathematical development. Anthony and Walshaw (2009) cited Blanton and Kaput (2005) who pointed out that teachers have a critical role to play in ensuring that tools are used effectively to support students to organise their mathematical reasoning and support their sense-making. In the same article Anthony and Walshaw (2009) claim that providing students access to multiple representations helps them to develop conceptual and computational flexibility. They further claim that tools are helpful in communicating ideas that are otherwise difficult to talk about or write about.

GeoGebra might be one of the various motivational ways in which circle geometry concepts may be presented that may eventually lead to improved performance in the topic. GeoGebra can help in teaching circle geometry because the students can move, rotate, or stretch the figure, and observe what properties stay the same. Arbain and Shukor (2015) cited Lunar et al. (2010) who stated that the use of technology in teaching and learning of mathematics is not only to improve student performance, but also to motivate them. This means if learners are motivated to learn circle geometry using GeoGebra, they may put more time in trying to understand the topic resulting in improved performance. Keengwe (2013) argues that technology provides learners with greater access to a vast array of information and resources, thus empowering them to become free agent learners, able to create meaningful personalised learning experiences outside the traditional classroom. In Slovakia, Guncuga, Majherova and Jancek (2012) found that GeoGebra can be a motivational tool for teaching and learning while in Malaysia, Noorbaizura and Leong (2013) studied the effect of using it to teach fractions to students. The study showed that the students in the experimental group performed better than those in the control group who were taught using the traditional learning method. The software also enhanced visualization and understanding of the concept of fractions for both the teacher and students.

2.2.3 The relationship between use of GeoGebra in teaching and learning and students' achievement in circle geometry

Academic achievement is commonly shown by students' performance in school. Higher academic achievement depicts that students are leaning toward excellence (Robiah, 1994). Achievement in this study referred to students' success in post-test. Studies done at all different grade levels and in several different countries show that mathematics attainment increases when manipulatives are put to good use (Clements and Battista, 1990; Dienes, 1960; Driscoll, 1981; Sugiyama, 1987 & Suydam, 1984). Various teaching and learning aids are used to encourage students' involvement in class. According to Kiuru, Pakarinen, Vasalampi, Silinskas, Aunola, Poikkeus and Nurmi (2014), students' achievement can be improved by teachers who wholeheartedly support their students by providing interesting teaching methods to attract students' interest.

Moore (2013) cited Parham (1983) and Sowell (1989) who found that achievement in mathematics could be increased by the long-term use of manipulatives. Bhagat and Yen Chang (2015) cited Guven (2012) who reported that using GeoGebra as a teaching tool, the experimental group outperformed the control group not only in academic attainment but also in levels of learning of transformation geometry.

Suydam and Higgins (1976) believe that lessons involving manipulative materials, if employed appropriately, will yield greater mathematical achievement than will lessons in which manipulative materials are not used. Ball (1992) posits that manipulative usage is widely accepted as an effective way to teach mathematics. Long-term interest in doing mathematics elucidate into increased mathematical propensity (Sutton & Krueger, 2002). Subrahmany, Greenfield, Kraut and Gross (2001) concluded in their study of the impact of using technology on children and adolescent development that there was evidence that using technology supported the development of visual mental rotation, spatial visualisation, the ability to deal with two dimension and three dimension space, the ability to keep track of too much of different information at the same time, and the ability to read pictures and diagrams, all of which are skills essential in circle geometry. Subrahmany et al., (2001) hint that more study is needed between the mentioned skills and academic achievement. Trexler (2007) carried out a study on the effect of computer-assisted instruction on mathematics performance and concluded that learners who were taught using computer assisted instruction improved their performance in mathematics as compared to those who were taught using conventional teaching instructions. This was attributed to the computer program that provided varying problem formats, immediate and correct feedback to the learners.

A survey carried out by Jackson (2005) in at risk environment of learners below grade level and those who are economically disadvantaged concluded that technological aligned mathematics activities had a significant effect on learner achievement as compared to conventional teaching approaches. Jackson (2005)'s study has support of Lindsey (2005) and Lin (2008). Lindsey (2005) investigated the effect of computer use on academic achievement as compared to Socio-Economic Status (SES) variables such as family income, and school variables such as class size that affect achievement. The study concluded that a statistically significant difference existed between learners who had relatively easy access to computers as compared to those who had little or no access to computers. On the other hand, Lin (2008) concluded that using technology effectively as a learning tool improved learners' mathematical achievement by providing visual representations of shapes in addition to it being very colourful, thus making mathematics exciting for learners.

2.2.4 Gender differences in terms of learners' achievement after learning circle geometry using GeoGebra as a manipulative tool

Gender issues have become the talk of today's forum. The issue of gender differences in mathematics performance by various researchers has raised a major concern in the teaching and learning of mathematics. Researchers believe that gender differences in mathematics achievement may be influenced by gender differences in mathematics strategies (Carr & Jessup, 1997). Pillow (2008) examined the gender difference among students on their academic performance and revealed that in individuals, background effect on students' cognitive and non-cognitive development is one of the most significant and influential characteristics in academic performance.

According to Sousa (2011), girls are generally scoring higher on standardised tests, especially in language skills and verbal expression. Sousa (2011) claims that the difference in brain structure between girls and boys results in learning differences. Sousa further states that the language areas in girls' brains are denser than in the brains of boys. Kommer (2006) mentioned the different brain chemistry as one of the causes, but also discussed the difference in brain structure. He cited research, which suggested that boys are largely right hemisphere dominant, and therefore, are better in visual-spatial tasks such as math, while girls are more able to use both hemispheres. Academics attribute academic gender difference to the feminist movement, which forced schools to focus on creating girls friendly educational environments and experiences.

Bonomo (2010) points out that girls have more cortical areas. According to her, girls are more able in verbal expression, and are more adapted to sensory memory, sitting still, listening and writing related functions. All these skills are emphasised in schools putting girls at an advantage than boys (Bonomo, 2010). On the other hand, Bonomo (2010) claims that boys' brains are more adapted to symbols, abstractions and pictures. She concluded that boys generally learn mathematics and physics more easily than girls. Gender differences become more ostensible in the higher classes with boys performing better than girls do in the areas involving calculations. Fennema (1990) asserts that recent studies have shown on male superiority in all subjects. On contrary, Wainer and Steinberg (1992) in their research have found that female students receive higher levels (grades) than more male students do because of their ability to work harder and attend class more frequently than male counterparts do.

Bonomo (2010, p.259) argues, "Studies have found significant difference in the way boys and girls hear, see, and smell". She further commented that girls and boys are drawn to different visuals and draw with different focus. Girls focus better on faces and people, while boys focus on objects. She also suggested that boys' teachers must keep instructions brief and engage them in competitive activities which allows for physical movement and lessons should be kinesthetics and experiential, Bonomo (2010, p.163). Furthermore, Maccoby and Jacklin (1974) found that many studies have shown that boys' mathematics achievement is superior to that of girls. On contrary to other to the latter scholars, Betz (1994) found that girls perform much better than boys in many school subjects including mathematics, sciences and engineering.

Other studies have revealed that boys perform better than their female counterparts (Alkateeb, 2001 as cited in Larbi & Okyere, 2014). Larbi and Okyere (2014) cited a study conducted by Beller and Gafni (1996) to explore differential performance of boys and girls in mathematics performance for ages 9 and 13, which sign posts that mathematics performance was in favour of the boys. Eshun (2000) investigated the pattern of mathematical achievement of secondary schools; he found that boys performed relatively better than girls did. A study conducted and cited in Larbi and Okyere (2014) noted that problems involving spatial rapport, in the skills of measurement, geometry, application and statistics and probability, boys scored higher than girls did.

Researchers found out that one gender is more creative than another gender. Onekutu (2002) argues that male learner put up a superior performance as compared to female learner. A study carried out on Spanish students indicates that some difference exists between males and females on aspect of creativity related to academic achievement.

Xixia (1999) also found that the degree of creativity is more famous in males than females. Gender as a predicator of mathematics achievement in Bahurudin and Luster (1998) found the gap between the average scale scores of males and females was small. Zhang and Manon (2000) found that males had a larger variance in mathematics scores than females.

From experience and literature, it is safe to conclude that boys and girls learn differently because of these genetically generated, environmental and cultural differences. As expected, taking these gender differences into consideration will mean revamping curriculum and lesson planning to include choices that match the different skills interest and traits of each gender. In view of the literature that suggests that there are gender differences in mathematics education, it is therefore pertinent to investigate gender differences in the effect of the use of GeoGebra as a manipulative tool on providing processes of grade 11 circle geometry.

2.3 Theoretical Framework

This section discusses the theoretical framework underpinning the study. Eisenhart (1991, p.205) defined a theoretical framework as "a structure that guides research by relying on a formal theory…constructed by using an established, coherent explanation of certain phenomena and relationships". It consists of the selected theory (or theories) that undergirds researchers' thinking with regards to how the researcher understand and plan to research the topic, as well as the concepts and definitions from that theory that are relevant to the topic. It is the focus of the study to address epistemological issues since epistemological beliefs are defined as beliefs about the nature and acquisition of knowledge. It is typically assumed that epistemological beliefs have an influence on how people think and reason, as well as on their motivational processes (Hofer & Pintrich, 1997). As such, the ideas of constructivism, experiential learning and the Concrete Representation Abstract (CRA) concept of learning using technology were employed as the theoretical framework underpinning the study.

2.3.1 Constructivist theory

Constructivism is defined as an idea that emphasises the active role of learners in creating their own knowledge by creating understanding and construction sense of information (Woolfolk, 2010). According to Cobb, Yackel and Wood (1992) constructivist learning is described as an active construction and the representational view of the mind, whereby learners modify their internal mental representations to construct. Cheek (1992) cited in Paulsen (2009) maintains that in constructivism, learners actively take in knowledge, connect it with prior knowledge and make it their own knowledge by constructing their own interpretations. In a constructivist classroom, the teacher provides learners with resources and activities that ensure they are actively involved and participate in, while constructing their own knowledge. Thus, constructivism was used as a theoretical framework underpinning this study.

During the study, the researcher adopted the constructivist approach of teaching and learning. Participants in the experimental group were more actively involved in building their own dynamic mathematical understanding of circle geometry as they interacted with devices installed with GeoGebra and each other. The control group used other approaches besides GeoGebra to solve processes of circle geometry. GeoGebra was expected to be a teaching and learning manipulative tool that lessens the dominance of the teacher in the learning setting while increasing learner participation as compared to conventional teaching approaches. The use of GeoGebra was expected to generate a more learner-centred environment. The researcher aimed to determine if GeoGebra was superior or not in applying constructivism as compared to conventional approaches of teaching. Questionnaires and pre-and-post tests were used to determine the mode of instruction that was more significance in implementing the constructivist approach. To emphasize classroom interaction, experiential learning theory is presented next.

2.3.2 Experiential learning theory

Experiential learning theory defines learning as the process whereby knowledge is created through the transformation of experience (Kolb, 1984). Bibian (2014, p.39) cited Dewey (1997) who defined the theory of experiential learning as a continuity of interaction. He asserts that learning was characterized not as the experience itself but as thinking about the experience. The study needed to provide the tool (GeoGebra) for thinking about such experience. According to Moore (2013) the work of Piaget suggests that learners begin to recognise mathematical ideograms and abstract notions only after having acquired and experienced the ideas on a concrete level. Moore (2013) further explained that the work of Dienes (1960) extended Piaget's work to suggest that learners whose mathematical learning is strongly grounded in manipulative experiences are probably to close the gap between the world in which they live and the abstract world of mathematics. This research focused on the importance of experience in the learning process. The holistic theory is consistent with how people learn, grow and develop. The next theory is presented below.

2.3.3 Concrete Representational Abstract (CRA)

The Concrete Representational Abstract (CRA) approach (presented in Figure. 2.1) is a three-step gradual and systematic instructional approach that has been found to be highly effective in teaching mathematics concepts. In this approach, built on Bruner's (1966) work, learners first use concrete materials to solve problems and look for patterns and generalisations. The CRA sequence of instructions integrates the use of hands-on manipulatives in the concrete stages (learners working with computers), followed by pictorial displays in the representational phase (learners discovering properties and making generalisation of theorem using GeoGebra) and in the next phase facilitates abstract reasoning with numerical symbols (learners solving riders) (Miller & Mercer, 1993).

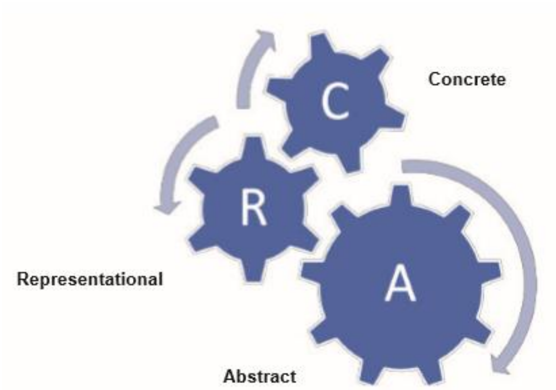

Figure 2.1: The Concrete–Representational–Abstract (CRA) approach (Moore, 2013, p. 2)

Teaching mathematics through a CRA sequence of instructions has abundant support for its effectiveness for learners struggling in mathematics (Kennedy & Tipps, 1998; Van DeWalle, 1994). Research-based studies show that students who use concrete materials like computers, develop more precise and more comprehensive mental representations. Arefreh, Dragoo, Luke and Steedly (2008) cited a research performed by the Access Centre in 2004 which demonstrated that CRA works well in both primary and secondary levels and that it can be used successfully in classroom settings. They further explained that learners showed good on-task behaviour and motivation.

Additionally, Boggan, Harper and Whitmire (2009) revealed a study conducted in 2007 that consisted of two classes in which the geometrical concepts were presented. One class used charts and drawings and the other used manipulatives to present the concept. They found that the class that used the manipulatives method scored significantly higher on the test that both groups sat for at the end of the concept.

2.4 Conceptual Framework

This section discusses the conceptual framework underpinning the study. "A conceptual framework is an interconnected set of ideas (theories) about how a particular phenomenon functions or is related to its parts. The framework serves as the basis for understanding the causal or correlational patterns of interconnections across events, ideas, observations, concepts, knowledge, interpretations and other components of experience" (Svinicki, 2010, p.5). The study gives details of the importance of manipulatives and technology in the teaching and learning of mathematics and GeoGebra as a manipulative tool in providing processes of circle geometry.

2.4.1 Manipulatives

Hynes defines manipulatives as "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (1986, p. 11). Clements further claims that "Good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas" (1999, p.49).

The use of manipulatives in teaching and learning mathematics has a strong and long history. Researchers have found that manipulatives are effective tools in addition to mathematics injunctions. The National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (SSM) highlighted the power of using manipulatives and visual delineation when teaching and learning mathematics (Shaw, 2002). Moore (2013), Suydam and Higgins (1977), Parham (1983) and Sowell (1989) found that achievement in mathematics could be increased by the long-term use of manipulatives.

Manipulatives allow learners to formulate their own understanding for philosophical and mathematical thoughts and approaches. Manipulatives are capable of engaging learners and boosting both the attentiveness and enjoyment of mathematics (Moore, 2013). Students who are given a chance to use manipulatives when doing mathematics report that they are more interested in learning mathematics. Long-term interest in doing mathematics translates into increased mathematical propensity (Sutton & Krueger, 2002).

2.4.2 Technology in teaching and learning of mathematics

Technology allows easy access to information. The world's largest association of mathematics teachers declared technology as one of their six principles for school mathematics (Majerek, 2014). It is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning (NCTM, 2000). Young people are accustomed to pictorial culture through wide access to the internet. Rapid growth of the internet in combination with its increasing accessibility for the public has opened up a completely new digital world. This makes that education of young people is increasingly inclined to accept the content given to them in this way. In circle geometry where a number of issues require a lot of imagination, students are discouraged to learn the subject when it is not provided in a modern and accessible way. According to Lytras, Gasevic, Ordonez de Pablos and Huang (2008, p.189 cited in Bester & Brand, 2013), "present day learners are more used to gripping information from the screen than from the printed page, and they find educators who use technology to be more dependable and well-informed than those who do not."

According to Bester and Brand (2013), computer technology assists students to make meaning of the learning material, and the interactive effects of sound, animation, narration and additional definitions provided by technology (computers) appeal to today's learners, motivating them to concentrate better and to achieve higher average scores. Willougby and Wood (2008) argue that learning takes place on the computer without the learners realizing the amount of attention they are paying to the material. Similarly, Trifonas (2008) shows that achievement can be improved in the classroom with the active involvement of the students making optimal use of technological innovations. Modern learners seem to focus on their work longer when using technology (Bitter & Legacy, 2008). "Many learners have become visual learners, having been brought up with technology, so without visuals in a presentation the learners may not learn effectively" (Smaldino, Lowther & Russell, 2008, p.259).

Results of a study by Stols (2012) to investigate the geometric cognitive development of students in a technology-enriched environment (dynamic geometry software) compared with students in a learning environment without any technological enhancement suggest that the technology-enriched environment helped to improve the conceptual geometric growth of students. The study further reveals that technology can help to create an active learning environment in which students can discover, explore, conjecture and visualise. One possible way to improve students' learning and achievement in circle geometry could be the integration of computer technology with the teaching and learning of the topic. Integrating computer technology software such as GeoGebra with mathematics teaching and learning is supported by many studies. Hohenwarter, Hohenwarter and Lavicza (2008) cited Hollebrands (2007) who asserts that new learning opportunities are provided in technological environments, potentially engaging learners of different mathematical skills and levels of understanding with mathematical tasks and activities. Hohenwarter et al. (2008) further cited Van Voorst (1999, pp. 2) who claimed that technology helps students to "visualize certain math concepts better" and that it adds "a new dimension to the teaching of mathematics".

2.4.3 GeoGebra as a virtual manipulative tool

GeoGebra was created by Markus Hohenwarter in 2001/2002 (Hohenwarter & Lavicza, 2007). Haciomeroglu, Bu, Schoen and Hohenwarter (2009) have proven that the GeoGebra Software facilitates the process of teaching Geometry, Algebra and Calculus. GeoGebra is freely downloadable from the internet and thus it is available both in schools and at home without any limitations (Hohenwarter & Lavicza, 2007). GeoGebra has a very clear and intuitive interface divided into parts corresponding to Geometry and Algebra (see Figure 1.1).

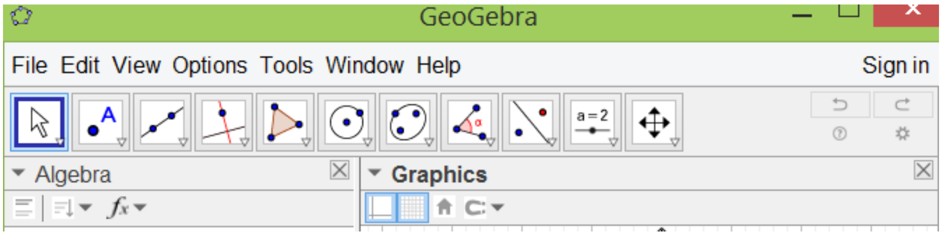

Figure 2.2: GeoGebra menu bar

GeoGebra is the first among other Information and Communication Technology (ICT) tools which are currently being explored in order to achieve integration of ICT into education (Ogwel, 2009). GeoGebra can be applied in Mathematics especially in teaching and learning Geometry, Algebra and Calculus (Antohe, 2009; Haciomeroglu, et al., 2009). GeoGebra allows leaners to construct dynamic-Geometry diagrams that are equivalent to paper-and-pencil drawings. Learners can drag objects around, changing their measurements but maintaining the dependencies that they design into their construction.

The application of GeoGebra software creates a conducive learning environment as it is a very dynamic educational technology with the potential to aid students in their mathematical exploration, for instance through problem solving, calculation, development, modelling and reflection (Bu & Schoen, 2011). According to Bu and Schoen (2011), the GeoGebra software improves the learning environment through its presentation of entities, calculation utilities, documentation tools and user-friendly web characteristics.

GeoGebra was found to be a motivational tool for teaching and learning in Slovakia (Guncuga, Majherova & Jancek, 2012) while in Malaysia, Noorbaizura and Leong (2013) studied the effect of using it to teach fractions to students. Noorbaizura et al. (2013) reveal that the students in the experimental group performed better than those in the control group who were taught using the traditional learning method. The software also enhanced visualization and understanding of the concept of fractions for both the teacher and students.

A study carried out in Singapore on innovative activities to develop the geometrical reasoning skill in secondary mathematics with the help of GeoGebra found that students taught with the software made progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics. The study reveals that the dynamic nature (drag feature) of the software influences the form of the explanation and that the students were able to generalize the solution and respond with an adequate statement. The study concluded that GeoGebra makes learning abstract concepts far more meaningful and helps students to visualize related concepts (Venkataraman, 2012).

In this study, the open-source software GeoGebra was selected from the pool of available software packages for mathematics teaching and learning because GeoGebra is a versatile tool that combines the ease of use of dynamic geometry software with features of computer algebra systems (Hohenwarter, Hohenwarter & Lavicza, 2008). The main idea of using GeoGebra in this study was to provide opportunities for learners of different mathematical skills and levels for better understanding concepts and fostering them to doing circle geometry in a new and attractive way.

2.5 Summary

In chapter 2, the researcher presented and explained the conceptual framework of the study. The section also revealed how the study was developed by addressing each research sub question. The section ended with a discussion of the theoretical framework of the study. Research methods are discussed in chapter 3 of the study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

In this chapter, the researcher gives an account of the strategy that was used to gather the information that was needed to answer research questions. A detailed overview of the research paradigm, approach, design, the data gathering process and data representation would also be given. A discussion concerning weaknesses and strengths of the method chosen is explained in this section. The ethical issues, reliability and validity, delimitation and limitations of the study are also presented.

3.2 Research paradigm

A paradigm is a shared worldview that represents the beliefs and values in a discipline and that guides how problems are solved (Schwandt, 2001). Positivism holds that the scientific method is the only way to establish truth and objective reality (Wagner, Kawulich & Garner, 2012). Positivism is based upon the view that science is the only foundation for true knowledge (Wagner et al., 2012). Collins noted that as a philosophy, positivism is in accordance with the empiricist view that knowledge stems from human experience (Chimuka, 2017).

The researcher was detached from the participants of the study by creating a distance, which is imperative in remaining emotionally impartial to make clear delimitations and boundaries between reason and feeling. Lastly, the researcher used statistical and mathematical methods, as there are essential to positivist research. Therefore, this study was conducted within the positivism research paradigm because of its scientific nature.

3.3 Research approach

The study adopted a quantitative methodology. It solicited quantitative data on the perceptions of learners following the method used to them when learning circle geometry. Quantitative data was accessed through pre-test; post-test and scaled questionnaires to establish the views of learners on their learning experiences in circle geometry. This method was found to have significant benefits, but its major weakness is that of overlooking human factor that is, depth of an issue in research (Zendah, 2017). The quantitative approach attempts to produce findings that are precise and generalisable (Rubin & Babbie, 2016).

3.4 Research design and procedure

Research design refers to the complete approach that the researchers select to assimilate the different components of the study in a coherent and logical manner, thereby ensuring the study effectively addresses the research problem (Mthethwa, 2015). The researcher utilised the quasi-experimental design. A quasi-experiment is an empirical study used to estimate the casual impact of an intervention on its target population (Dinardo, 2010). The quasi-experiment was the best approach due to the availability of the original classes of students (Wiersma, 2000). The researcher purposely and systematically manipulated a natural phenomenon to observe a series of changes experienced by the phenomenon (Sekaran, 1992).

Selected Grade 11 mathematics learners were grouped into experimental and control group. The experimental group was the one where the manipulative tool (GeoGebra) was incorporated in the teaching and learning of circle geometry. The control group was with participants who were taught circle geometry using the conventional methods of teaching. The design is depicted in Figure 3.1.

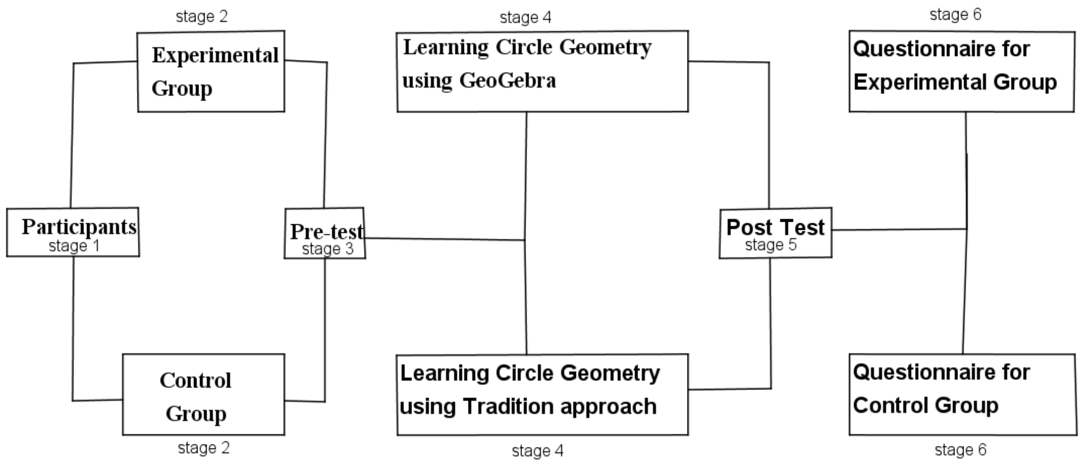

Figure 3.1: Research procedure (Marange, 2018)

3.5 Population

The population is a complete set of elements (persons or objects) that possess some common characteristics defined by the sampling criteria established by the researcher (Ogwel, 2009). The population for this study were all Grade 11 learners in Eastern Cape Province of South Africa. The target population for the study consisted of all Grade 11 mathematics learners from 67 senior secondary schools in OR Tambo Inland District. According to Kadam and Bhalerao (2010), target population is a subject of individuals with specific clinical and demographic characteristics in whom one wants to study one's intervention. The accessible population to which the researcher had reasonable access in this study was the target population.

3.6 Sample and sampling techniques

According to Salaria (2012) and Yates, Moore and Stames (2008), sampling is the process by which a relatively small number of individuals, objects or events is chosen and analysed to find out something about the entire population from which the sample was chosen. A sample is the number of participants who are selected from the target population and from whom data are collected (Mamali, 2015). The sample consisted of 108 randomly sampled Grade 11 mathematics learners who were divided into two classes at one school. Sixty (60) learners were in the experimental group and 48 in the control group. GeoGebra software was installed on all laptops and ipads in the experimental group.

Figure 3.2: Experimental class sitting layout (Marange, 2018).

3.7 Sample of lessons conducted in Experimental class

The topic was taught for a period of 15 days as prescribed by the Annual Teaching Plan (ATP). There are four examinable theories and their converses (see Appendix F). The flow diagram below shows how each theorem was taught. Each theorem was taught for three days following the three-step flow diagram (see Figure 3.3). Each lesson was 1hr long and according to CAPS, mathematics must be taught for four and half hours per week. In Experimental class, each learner had either a laptop or an ipad and the researcher was using an interactive SmartBoard (see figure 3.2).

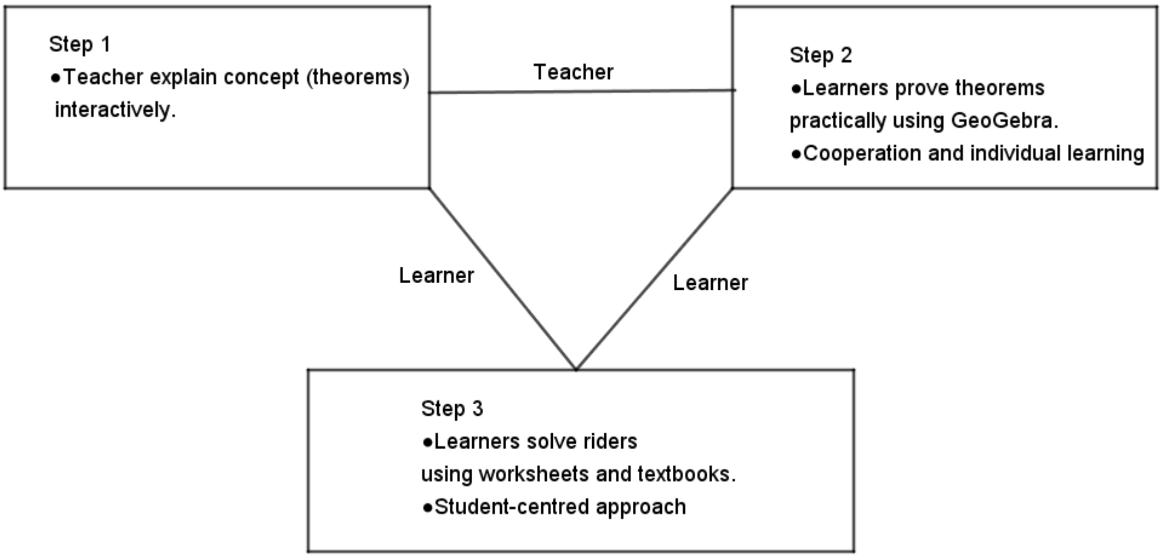

Figure 3.3: Lesson flow diagram (Marange, 2018)

Introduction to GeoGebra and its dynamic tools

During this lesson, participants were trained on how to use different tools of GeoGebra. GeoGebra is an interactive geometry system. Constructions with points, vectors, segments, lines, polygons and conic sections. The researcher showed and explained how lines, angles and circles are constructed on GeoGebra platform (see Figure 3.4)

After starting GeoGebra, the following window appears.

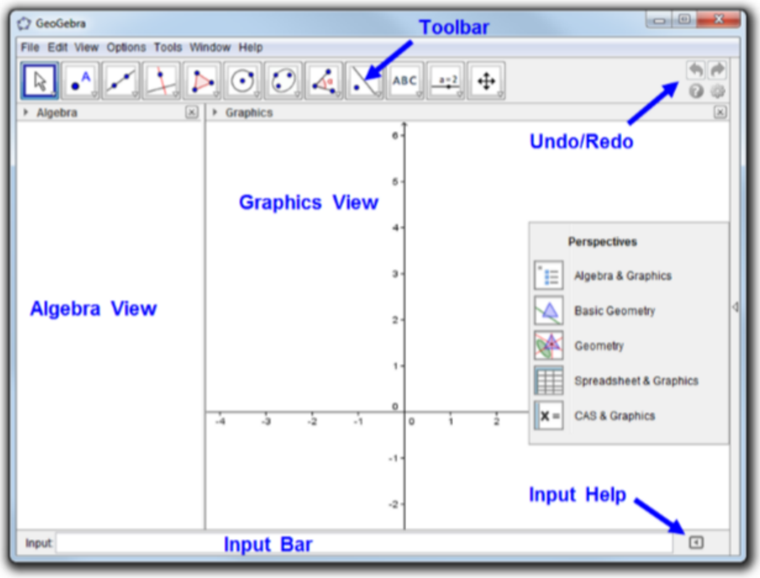

Figure 3.4: A snapshot of GeoGebra's user interface (Hohenwarter & Hohenwarter, 2008, p.8)

Using the provided geometry tools in the Toolbar (see Figure 3.5); students can create geometric constructions on the Graphics View with the mouse. At the same time the corresponding coordinates and equations are displayed in the Algebra View. On the other hand, students can directly enter algebraic input, commands, and functions into the Input Bar by using the keyboard. While the graphical representation of all objects is displayed in the Graphics View, their algebraic numeric representation is shown in the Algebra View. In GeoGebra, geometry and algebra work side by side.

1		Create segment *AB*.
2		Construct a circle with center *A* through *B*. Hint: Drag points *A* and *B* to check if the circle is connected to them.
3		Construct a circle with center *B* through *A*.
4		Intersect both circles to get point *C*.

5		Create the polygon *ABC* in counterclockwise direction.
6		Hide the two circles.
7		Show the interior angles of the triangle by clicking somewhere inside the triangle. Hint: Clockwise creation of the polygon gives you the exterior angles!
8		Save the construction.
9		Apply the drag test to check if the construction is correct.

	Point Hint: Click on the *Graphics View* or an already existing object to create a new point.	**New!**
	Move Hint: Drag a free object with the mouse.	**New!**
	Line Hint: Click on the *Graphics View* twice or on two already existing points.	**New!**
	Segment Hint: Click on the *Graphics View* twice or on two already existing points.	**New!**
	Delete Hint: Click on an object to delete it.	**New!**
	Undo / Redo Hint: Undo / redo a construction step by step (on the right side of the Toolbar).	**New!**
	Move Graphics View Hint: Click and drag the *Graphics View* to change the visible part.	**New!**
	Zoom In / Zoom Out Hint: Click on the *Graphics View* to zoom in / out.	**New!**

Figure 3.5: GeoGebra construction tools and Steps: GeoGebra tools (Hohenwarter & Hohenwarter, 2008, p.14)

Sample Lesson 1: Theorem: The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference (on the same side of the chord as the centre).

During this lesson, participants were now able to draw lines, construct circle and measure angles. The following diagram is a sample of one of the constructions done by participants when trying to understand the above theorem (see Figure 3.6)

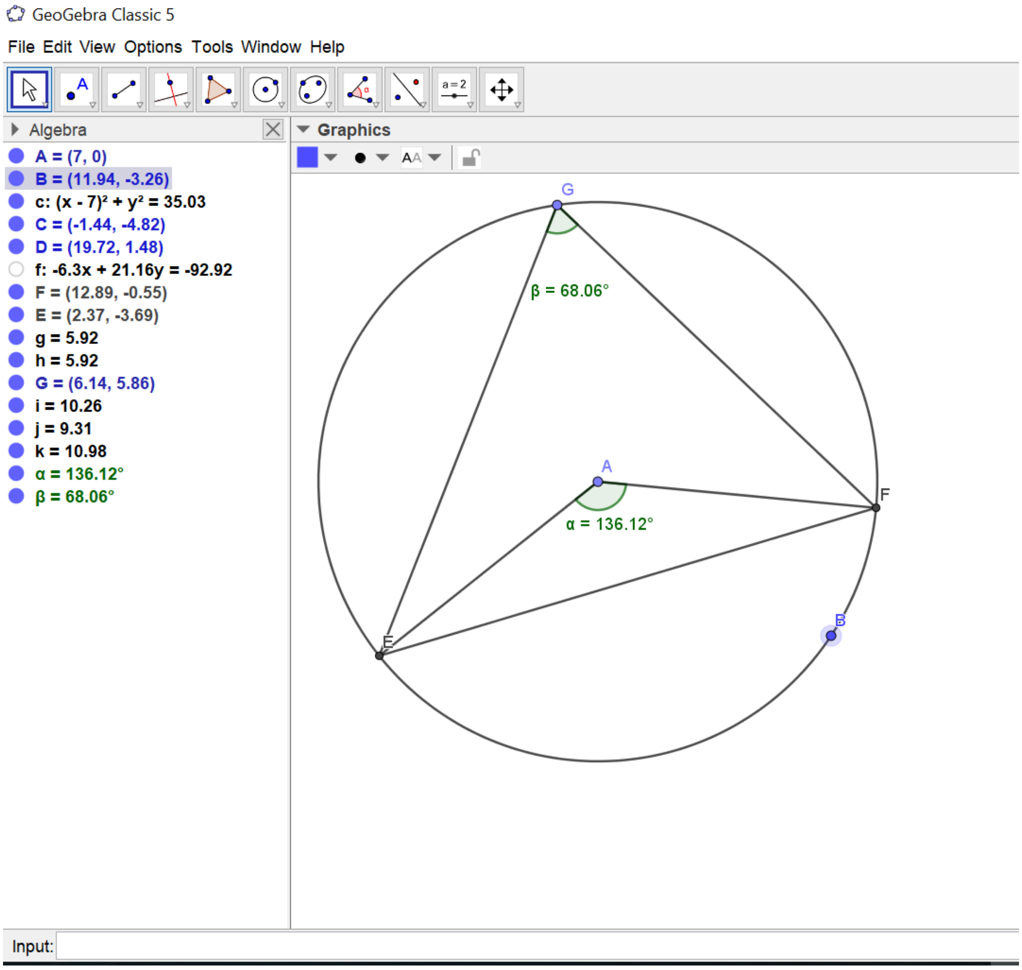

Figure 3. 6: Angle at centre theorem

In this activity participants observed that angle at point A ($136,12^0$) is twice angle at circumference at point G ($68,06^0$). Participants were also able to move and drag point G and they learnt that the angle remained the same because it was still subtended at the same point at A, the centre. They also learnt that if they move or drag point A or B, the angle at the centre changes but remain twice the angle at the circumference, that is point B.

The following activity was given to participants to use GeoGebra and answer the questions (see Figure 3.7)

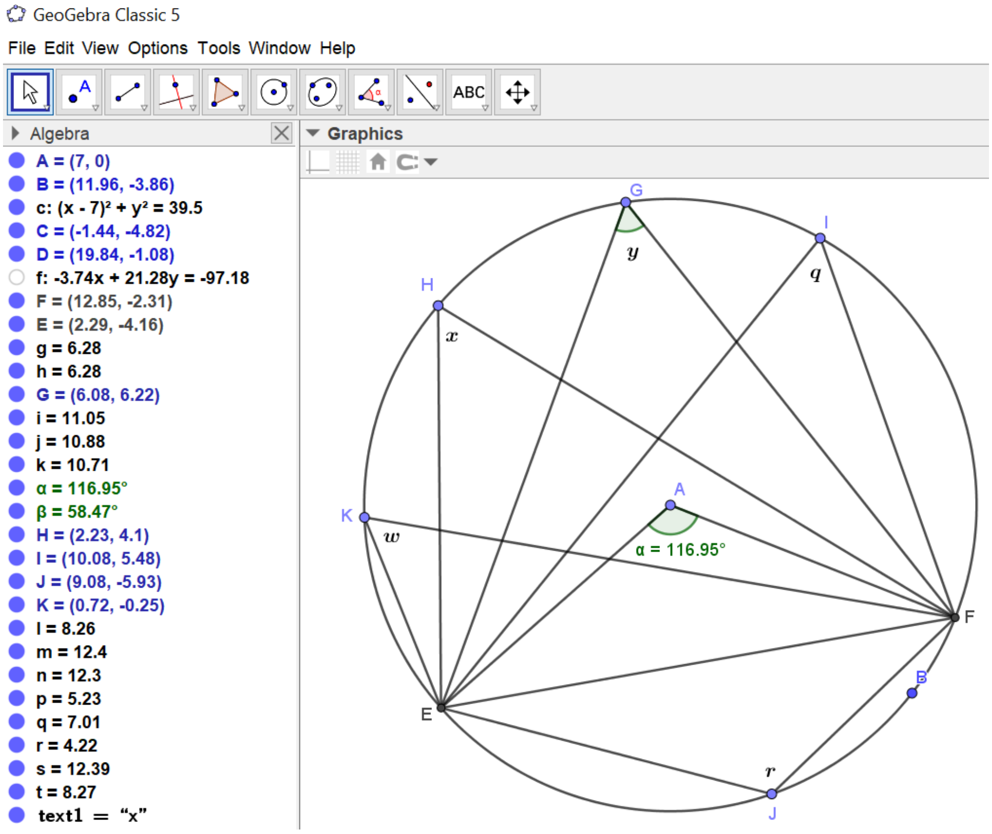

Figure 3.7: Angle at centre theorem: Informal activity

On the above activity learners were asked to calculate the size of ***w, x, y, q*** and ***r***, giving reasons.

Sample Lesson 2: Theorem: The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

During this lesson, participants constructed circle and measure angles. Figure 3.8 is a sample of one of the constructions done by participants when trying to prove and understand the above theorem.

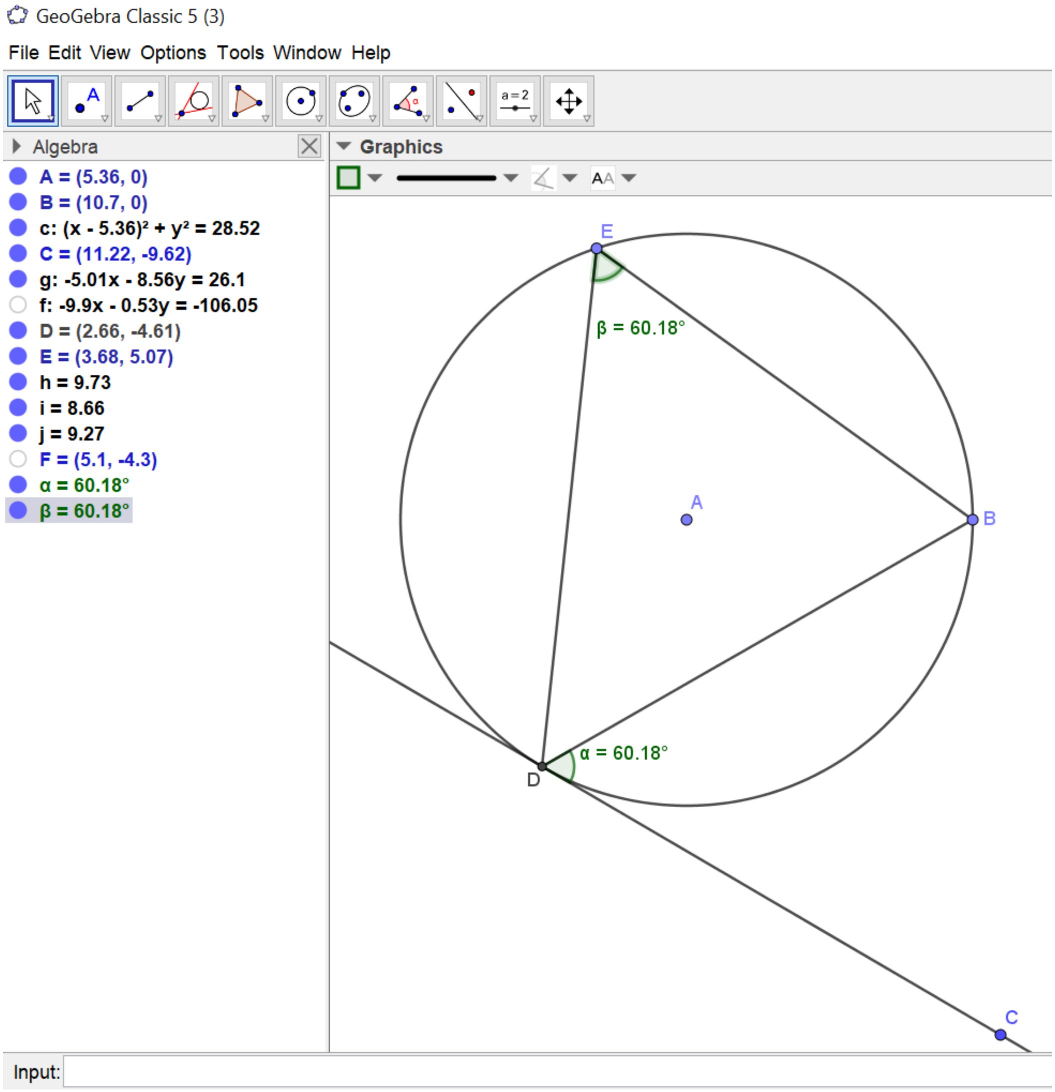

Figure 3.8: Tan-chord theorem

Participants were able to see that the angle between the chord BD and the tangent DC that is angle BDC = $60{,}18^0$ is equal to the angle subtended by same chord BD at point E that is angle DEB = $60{,}18^0$. They also learnt that moving point C in any direction, the two angles would always be the same. Moving point A or point B will make the circle bigger or smaller, but it does not affect the angles. They always remain the same.

The following activity was given to participants to use GeoGebra and answer the questions (see Figure 3.9).

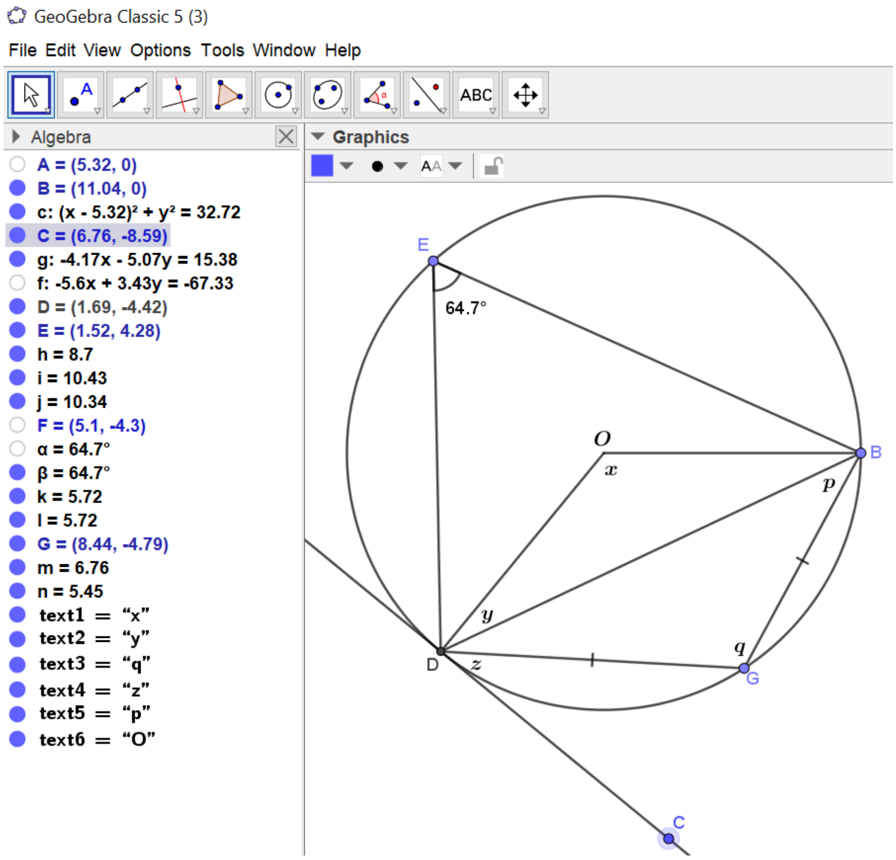

Figure 3.9: Tan-chord theorem: Informal activity

From the above activity (figure 3.9), participants were asked to calculate the size of ***x, y, z, p*** and ***q***, giving reasons. Participants were also expected to integrate two theorems (angle at centre and tan-chord theorem) when answering the questions.

3.8 Data Collection and Instrumentation

Data collection instruments are the fact-finding strategies for obtaining relevant information concerning the problem under study. There are many strategies of data collection and the choice of an instrument or research tool depends mainly on the attributes of the subjects, research topic, problem question, objectives, design, expected data and results (Wandera, 2011). Three instruments were used for the study as follows:

(a) Circle geometry pre-test,

(b) Learners' motivation scaled questionnaires,

(c) Circle geometry post-test.

Learners' questionnaire comprised of two parts; Part A: Demographic data and Part B: Usability of GeoGebra or conventional methods in the learning of Circle Geometry.

3.8.1 Pre- and-Post Circle Geometry Tests

The tests were set according to the requirements of the CAPS policy document and it met all the expectations as stated in the Grade 11 mathematics examination guideline (see Appendix F). The questions were also within the scope of the learners' content knowledge and academic ability. The pre-and-post-test was the same and was out of 50 marks. After the development of the test, it was taken to my supervisor and to a researcher in the field of geometry for suggestions and it was modified. After piloting it on a class of learners, further modifications were done.

Experimental and control groups were asked to sit for pre-test and post-test circle geometry tests (see Appendix H) in different venues. Pre-and-post-tests for both group were recorded for further analysis (see Appendix I and J). In this study, it was the researcher's wish that a post-test be applied to all the students to whom the pre-test had been applied and, in this way, loss of subject would be prevented.

3.8.2 Scaled questionnaires

According to Jupp (2006, p.252), "a questionnaire is a set of carefully designed questions given in the same form to a group of people in order to collect data about some topic in which the researcher is interested". The questionnaire adopted by Bryan (2009) in a study to investigate high school learners' motivation to learn Science was modified in this study to measure learners' motivation towards Circle Geometry. Bryan (2009) proved that the questionnaire was reliable and valid in Science. The questionnaire designed for this study was administered as a post-study tool to check what learners thought about the method of learning approach used when learning circle geometry.

The Likert scale questions were used to collect data. Likert-scales are used to register the extent of agreement or disappointment with a statement of attitude, belief or judgement (Tuckman, 1994). The advantage of Likert-scales is that they provide "greater flexibility since the descriptors on the scale can vary to fit the nature of the question or statement" (McMillan & Schumacher, 1993, p.245). In scoring and measurement of participants' perception questionnaires, all the points were labelled starting with 5 (Strongly Agree), 4 (Agree), 3 (May Agree), 2 (Disagree) and concluded with 1 (Strongly Disagree).

Part B for the questionnaire in both groups was sub-divided into three sections namely:

1. Learners' general views of circle geometry;
2. Learners' views of GeoGebra or conventional methods advantages in learning circle geometry, and

3. Learners’ experiences of learning circle geometry using GeoGebra or conventional methods.

Table 3.1 shows the constructs and numbers of items of part B of the control group questionnaire.

Table 3.1: Construct and Number of items of part B Control group

Construct	**Number of items**
Learners’ views of circle geometry	5 items (1-5)
Learners’ views of the advantages of conventional methods	11 items (6-16)
Learners’ experiences of conventional methods	11 items (17-27)

The questionnaire had 27 items for which 19% (5 items) were focusing on learners’ views of circle geometry, 41% (11 items) focused on learners’ views on the advantages of conventional methods when learning circle geometry and 41% (11 items) assessed on learners’ experiences of conventional methods in the learning of circle geometry concepts.

Table 3.2 shows the constructs and numbers of items of part B of the experimental group questionnaire.

Table 3.2: Construct and Number of items of part B Experimental group

Construct	Number of items
Learners' views of circle geometry	5 items (1-5)
Learners' views of GeoGebra advantages	11 items (6-16)
Learners' experiences of GeoGebra	11 items (17-27)

For consistency the questionnaire also had 27 items for which 19% (5 items) were focusing on learners' views of circle geometry, 41% (11 items) focused on learners' views on the advantages of using GeoGebra when learning circle geometry and 41% (11 items) accessed on learners' experiences of using GeoGebra in the learning of circle geometry concepts.

The administration of the questionnaire after the interventions aimed to determine the effect of using GeoGebra as a manipulative tool in providing process of circle geometry on the motivation levels of learners in the experimental group as compared to learners in the control group. According to Woolfolk (2010), motivation to learn may be described as the way one thinks and feels about academic activities. The constructed items for the questionnaire probed learners on issues that related to their motivation levels towards circle geometry after using GeoGebra and conventional approaches.

3.9 Reliability and validity of the study

There are various ways of magnifying the reliability and validity of the instruments, data and findings of the study.

3.9.1 Reliability

According to Kothari (2004), reliability is defined as the extent to which a questionnaire, test, observation or any measurement procedure produces the same results on repeated trials. De Vos (2002) claims that the method of test-retest reliability addresses the question of consistent responses from multiple occasions of instrument use. The researcher uses the test-retest reliability as a measure of reliability obtained by administering the same test twice over a period of time to the same group of individuals. The scores of the first and second times could then be correlated in order to evaluate the test for stability over time using the Pearson correlation coefficient *(r)*. An *r-value equal* or greater that 0.7 will be considered an acceptable value for an instrument to be viewed as reliable (Burns & Grove, 2007). In this study the researcher used the test-retest reliability as a measure of reliability obtained by administering the same test twice over a period of one month to the same four Grade 11 learners that were drawn from non-participating classes. When the scores of the first and second times were then correlated in order to evaluate the test for stability over time using the Pearson correlation coefficient, the questionnaire for this study was considered reliable since the *r* was 0.74.

3.9.2 Validity

Mugenda (2008, p.256) defined validity as "the degree to which an instrument measures what it purports to measure". The face and content validity were carried out on the following instruments:

(a) Circle geometry pre-test and post-test.

(b) Learners' motivation scaled questionnaires.

Face validity refers to the extent to which an instrument looks as it measures what it is intended to measure (Patton, 2015). According to Drost (2011) content validity refers to qualitative type validity where the domain of the concept is made clear and the analyst judges whether the measures fully represent the domain. In this study the pre-test and post-test were validated by two other Grade 11 Senior Educators while a pilot study was first done to a non-participating class of Grade 11 learners that were one year ahead of the participating learners. Pre-and-post-test marks of pilot study were recorded (see Appendix G) and analysed. The results of pilot study are shown in Table 3.3 below.

Table 3.3: Pilot study results

Pilot Study Group	N	Mean	Standard deviation	Minimum	Maximum
Pre-test	43	2,81	2,45	0	11
Post-test	43	12,65	10,24	0	44

Table 3.3 shows that out of the 43 non-participating Grade 11 learners, in a pre-test out of 50 marks, the average performance was 2,81 with a standard deviation of 2,45. The maximum mark obtained was 11 with a minimum mark of 0. The same group of learners were given the same test after exposing them to learning circle geometry with GeoGebra. The average performance in post-test was 12,65 with a standard deviation of 10, 36. The maximum mark obtained was 44 with a minimum mark of 0. Pilot study results shows that using GeoGebra in teaching and learning of the circle geometry concept had a positive impact over conventional approaches.

3.10 Ethical issues

According to Reddy (2015, p.53), "ethical issues are the precautions, steps and effort that the researchers put into practice to protect the research participant while working with them for data production". The researcher considered the following ethical principles.

3.10.1 Permission

The researcher got permission to undertake the study, firstly, from the ethical clearance committee of the institution (Appendix A) which was followed by the provincial clearance (Appendix B). Thereafter, the researcher applied for an approval to carry out the study at the school (Appendix C). The permission was granted (Appendices D & E).

3.10.2 Informed consent

The researcher issued the informed consent letters to the respondents to give to their parents or guardians for signing and return to the researcher indicating whether they agreed or not to allow their children to participate in the study. Both the parent/guardian and the child had to sign (Appendix F). The researcher only engaged respondents whose parents or guardians had agreed to let them participate.

3.10.3 Autonomy

Autonomy refers to gaining consent from every learner participating in the research. The researcher considered what Christian (2000) declared as necessary conditions when conducting a study which involves human beings. Christian (2000, p.138) asserted that "Proper respect for human freedom generally includes two necessary conditions", being that these conditions include that participants must concur to participate willingly without physical or emotional force and that their agreement to take part in the study must be based on the full and open information.

3.10.4 Non-maleficence

Non-maleficence means that the study does not do any harm (Reddy, 2015). No physical, emotional, and social or any other type of harm was inflicted on the participants. Ethical principles as listed by Carlsen (2008) were not ignored. Thus, there was no invasion of privacy of respondents, lack of informed consent, harm to participants and deception. The researcher did everything in his power to make sure that these principles were not violated.

3.10.5 Anonymity

The names of the participants are not mentioned anywhere in the study. When responding to the pre-test and post-test, the participants used codes instead of their names on their scripts to preserve their anonymity.

3.10.6 Confidentiality

The researcher used the codes instead of names to ensure confidentiality. Results obtained from the study were not used for any other purpose except for this study. However, without mentioning of names, respondents, parents and the school can access the outcomes of the study through consultation with the researcher.

3.10.7 Privacy

There was no violation of respondents' privacy. Respondents participated willingly without physical or emotional force. Individual rights were respected. Respondents were informed that they could withdraw at anytime they felt not to continue participating in the study.

3.11 Delimitation and Limitations of the study

3.11.1 Delimitation

3.11.1.1 Geographical Delimitation

The study was conducted in OR Tambo Inland District in the Eastern Cape Province of South Africa.

3.11.1.2 Population Sample Delimitation

The population of the study was all 67 high schools in OR Tambo Inland and all Grade 11 learners learning mathematics in these schools. The sample for this study was derived from Grade 11 mathematics learners at one school.

3.11.1.3 Conceptual Delimitation

There are many mathematical application programmes, however, the study focused on one dynamic mathematical software called GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry.

3.11.2 Limitations

3.11.2.1 Computer Literacy

The level of computer literacy among participants was one of the limiting factors when conducting this study. Fortunately, the researcher was an educator at the school where the study was conducted, so the researcher conducted extra classes to train respondents on how to use GeoGebra tools (see Figure 3.4) when learning circle geometry.

3.11.2.2 Time

Scarcity of time was another limiting factor. The researcher organised morning classes to overcome the issue of time as a limiting factor.

3.11.2.3 Cost

The cost of carrying out the study cannot be undermined. Each laptop and ipad were to be installed with GeoGebra for which this application had to be downloaded. Fortunately, the application could be downloaded freely but it required internet bundles to download. Unfortunately, the researcher's school WiFi was not working so the researcher had to purchase the bundles using his own money to download and install GeoGebra.

3.11.2.4 Accessibility

Finally, the target population was too large to carry out the study due to lack of resources such as human resources and time. To overcome this, since the objective of this study was to investigate whether there is a significant relationship between learning circle geometry (independent variable) and achievement and motivation (dependent variables), the researcher used the results of the analysis of the sample to generalise information about the population that the sample represented. The findings of the sample were assumed most representative of schools with the same resources in the entire OR Tambo Inland district.

3.12 Data Analysis

This research generated quantitative data from the pre-test, post-test and scaled questionnaires. Data was analysed using SPSS Version 24 and GeoGebra Classic 5 methods. Data analysis in quantitative approach is believed to be relatively less time consuming, for it can be done using statistical software (Zendah, 2017). Descriptive statistics such as frequency distributions and measures of central tendencies were used to describe and compare sets of data from the study. However, inferential statistics was employed in computing the T test on the impact of using GeoGebra on gender difference.

3.13 Summary

In this chapter, the researcher discussed the research methodology, design, procedures and justification of instruments used. Methods used to analyse data were also explained in this section. Issues of data and results validity and reliability were also discussed. The next chapter presents the findings of the study.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 Introduction

This chapter presents and analyses the findings using GeoGebra and conventional methods in providing processes of circle geometry. The aim of the study was to establish the influence of using GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry at one school in OR Tambo Inland. The research approach used in this study was quantitative. Quantitative data was analysed using SPSS Version 24 and GeoGebra Classic 5.

The following sub-research questions guided the study:

- What are the effects of conventional methods of teaching in learning the concepts of circle geometry?
- How does learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation?
- What is the significance of using GeoGebra on the academic performance of learners learning circle geometry?
- What are the gender differences in terms of learners' achievement after learning circle geometry using GeoGebra?

The findings are presented and discussed according to the research questions stated above. The researcher uses tables, figures, descriptive and inferential statistics to organise and present results in this chapter. In this section, the researcher was interchanging the presentation and analysis of the questionnaire findings with those of the pre-test and post-test in an effort to give a clearer picture.

4.2 Demographic data of the respondents

The following section presents the tables and interpretation of the biographical data of the respondents.

4.2.1 Gender

Table 4.1 shows data for the 107 learners who took part in the study. All respondents were black Africans.

Table 4.1: Demographic data of the respondents (N=107)

Variable	Group			**Total**
		Experimental	Control	
Gender	Female	43	25	**68**
	Male	17	22	**39**
Age	16-17	47	37	**84**
	18+	13	10	**23**

Table 4.1 shows that out of 107, 64% (69) participants were female learners and 36% (39) participants were male learners. The study was a quasi-quantitative research for which 56% (60) participants were from the experimental group and 44% (47) participants were learners from the control group. In the experimental group, Table

4.1 shows that 73% (44) were female participants and 27% (16) were male participants. The table also shows that 53% (25) were females in the control group for which 47% (22) were male participants. It can be concluded that more females took part in the study than males.

The table also shows that out of 107 participants, 79% (84) were learners aged between 16 and 17 and 21% (23) were aged 18 and above. Out of 85 participants, aged 16-17, 56% (47) were from the experimental group and 44% (37) were in the control group. Out of 22 participants aged 18 and above, 57% (13) were in the experimental group and 43% (10) were in the control group.

4.3 Data presentation pertaining to sub-research questions

In this section, the researcher presented the results from respondents' responses in the questionnaire as well as the results from the pre-test and post-test. Tables and graphs were used to show the results.

4.3.1 What are the effects of conventional methods of teaching in learning the concepts of circle geometry?

The researcher addressed this research sub-question by assessing and comparing pre-test and post-test performances of participants in control group and experimental group. The sub-headings of the questionnaire were also used to compare and measure the extent of variation of participants' views on selected statements. Discussion was also done.

4.3.1.1 Pre-test and post-test results

Figure 4.1 presents the results for the experimental and control pre-test.

Table 4.2 Results for Experimental and Control pre-test

Group	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
Experiment	60	0,83	1,40	0	0	0	1	8
Control	47	0,36	1,23	0	0	0	0	8

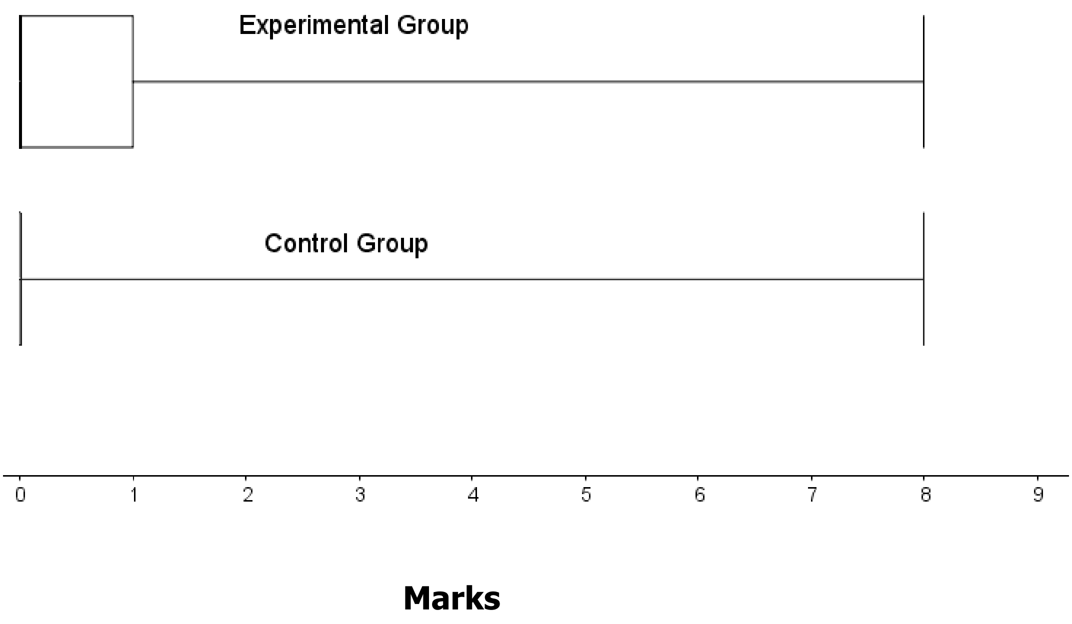

Figure 4.1: Experimental and Control pre-test box and whisker diagram

Table 4.2 and Figure 4.1 shows that out of 60 participants in the experimental group, 25% of participants scored at least 1 out of 50 marks whereas 50% of 47 participants in the control group scored 0 out of 50 marks (see Figure 4.1). The table also shows that the minimum mark in both groups was 0 when the maximum was 8 which gave a range of 8 in both groups. The mean score of the experimental group was 0,83 with a standard deviation of 1,40. Control group obtained the mean score of 0,36 with a standard deviation of 1,23 which shows a difference of 0.47 from the experimental group. This shows that, even though experimental group performed slightly better than their counterparts did. All participants were almost at the same level of thinking before the experimental group were exposed to GeoGebra.

Table 4.3: Results for Experimental and Control post-test

Group	N	Mean	Standard deviation	Minimum	Lower Quartile	Median	Upper quartile	Maximum
Experiment	60	11,33	9,82	0	3,5	7,5	17,5	37
Control	47	3,81	4,76	0	1	3	5	31

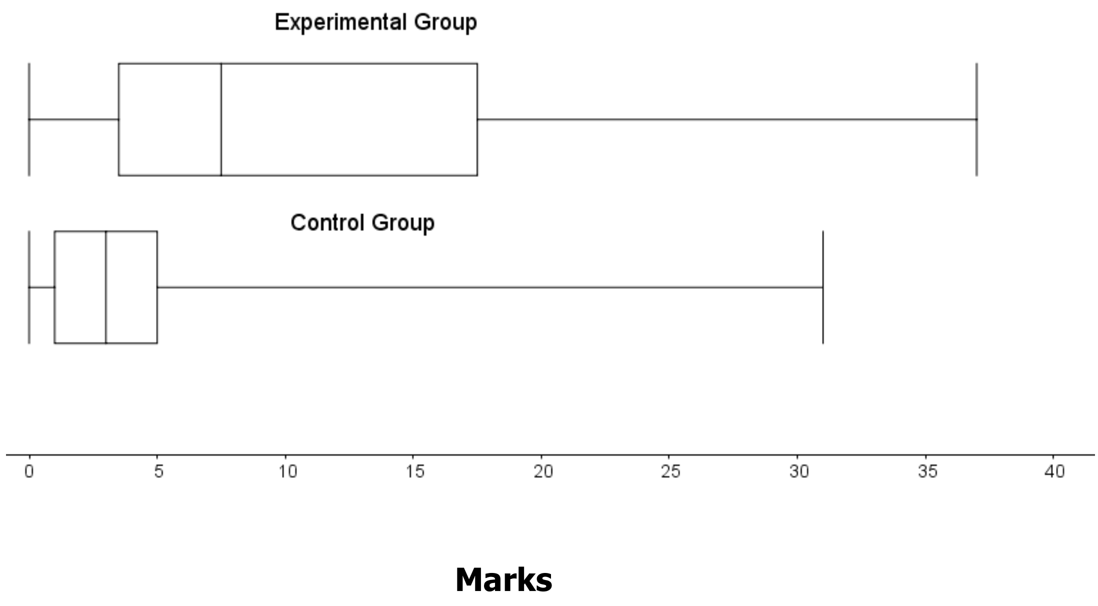

Figure 4.2: Experimental and Control post-test box and whisker diagram

Table 4.3 and Figure 4.2 indicate that out of 60 participants in the experimental group 50% of participants scored above 8 out of 50 marks whereas in the control group 50% of participants scored above 4 out of 50 marks. Moreover, 25% of participants in experimental group scored 18 out of 50 marks whereas 25% of participants in the control group scored above 5 out of 50 marks. The minimum mark obtained in the experimental group was 0 with a maximum of 37. The range recorded in this group was 37. The mean score of 11,33 with a standard deviation of 9,81 was recorded in the experimental group. The minimum mark obtained in the control group was 0 with a maximum of 31. The range recorded in this group was 31. The mean score of 3,81 with a standard deviation of 4,76 was recorded in the control group.

4.3.1.2 Control and Experimental questionnaire results

The researcher also sought data using questionnaires to compare what learners perceived about the approach used to learn circle geometry. Results were tabulated in terms of the headings from the questionnaire: (i) Learners' general views of circle geometry, (ii) Learners' views of advantages of GeoGebra or conventional methods in learning circle geometry, and (iii) Learners' experiences of learning circle geometry using GeoGebra or conventional methods. Data for the 60 experimental group learners and the 47 control group learners is presented. Therefore, the results represented responses for 107 learners.

4.3.2 How does learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation?

Table 4.4 shows that there were no significant differences regarding learners' views towards mathematics as 23% (N=14) experimental compared to 21% (N=10) control group learners strongly agreed that they like mathematics, while 40% (N=24) experimental compared to 53% (N=25) control group learners agreed to liking mathematics.

4.3.2.1 Learners' mathematics attitude, learning styles and preferences

Table 4.4: Comparative learners' attitude towards mathematics and circle geometry (N=107)

Variable	Responses																			
	Strongly agree				Agree				May agree				Disagree				Strongly disagree			
	Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group	
	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%
I like mathematics	14	23	10	21	24	40	25	53	15	25	9	19	5	8	2	4	2	3	1	2
I like the section of Circle Geometry in mathematics	6	10	5	11	19	32	7	15	14	23	13	28	18	30	13	28	3	5	9	19
In Circle Geometry you can be creative and discover things by yourself	24	40	7	15	19	32	10	21	11	18	1	19	5	8	12	26	1	2	9	19
Memorisation helps me understand theorems of Circle Geometry	16	27	7	15	22	37	6	13	16	27	9	19	6	10	16	34	0	0	9	19

Generally, learners liked mathematics. In terms of liking the section on Circle Geometry, it also emerged that only 10% (N=6) compared to 11% (N=5) of the learners from the experimental group and the control group strongly agreed that they liked the Circle Geometry section, while 32% (N=19) experimental group learners compared to only 15% (N=7) control group learners agreed to that notion. While 23% experimental group learners compared to 28% from the control group indicated may agree, 35% experimental group learners compared to 47% control group learners disliked the section of circle geometry in mathematics. 40% experimental group compared to 15% control group learners strongly held the view that in Circle Geometry you can be creative and discover things by yourself. The same trend followed as 32% experimental group learners compared to 21% from the control group agreed to that view, while 18% experimental group learners compared to 19% from the control group indicated may agree.

Generally, the experimental group was more positive than the control group on that notion. Ironically, on the notion of memorisation helping learners to understand theorems of Circle Geometry, 64% of the experimental group agreed or strongly agreed to that in comparison to only 28% from the control group. Ten percent (10%) experimental group learners compared to 53% control group learners actually disagreed that memorisation helped them to understand theorems of Circle Geometry.

4.3.2.2 Preference to learn circle geometry

Figure 4.3a and Figure 4.3b present the learners' views on the preferred approaches to learning Circle Geometry.

Figure 4.3a shows that 37% (N=22) strongly agreed while 25% (N=15) agreed that they preferred to learn circle geometry using GeoGebra, 20% (N=12) indicated may agree while only 19% generally disagreed indicating that they did not favour the use of GeoGebra in learning circle geometry. In the control group, out of 47 respondents in Figure 4.3b, 17% (N=7) and 32% (N=15) respectively strongly agreed and agreed that they preferred the conventional approach when learning circle geometry. Thirty-eight percent (38%) (N=18) were neutral while only 15% did not support learning circle geometry through the traditional approach.

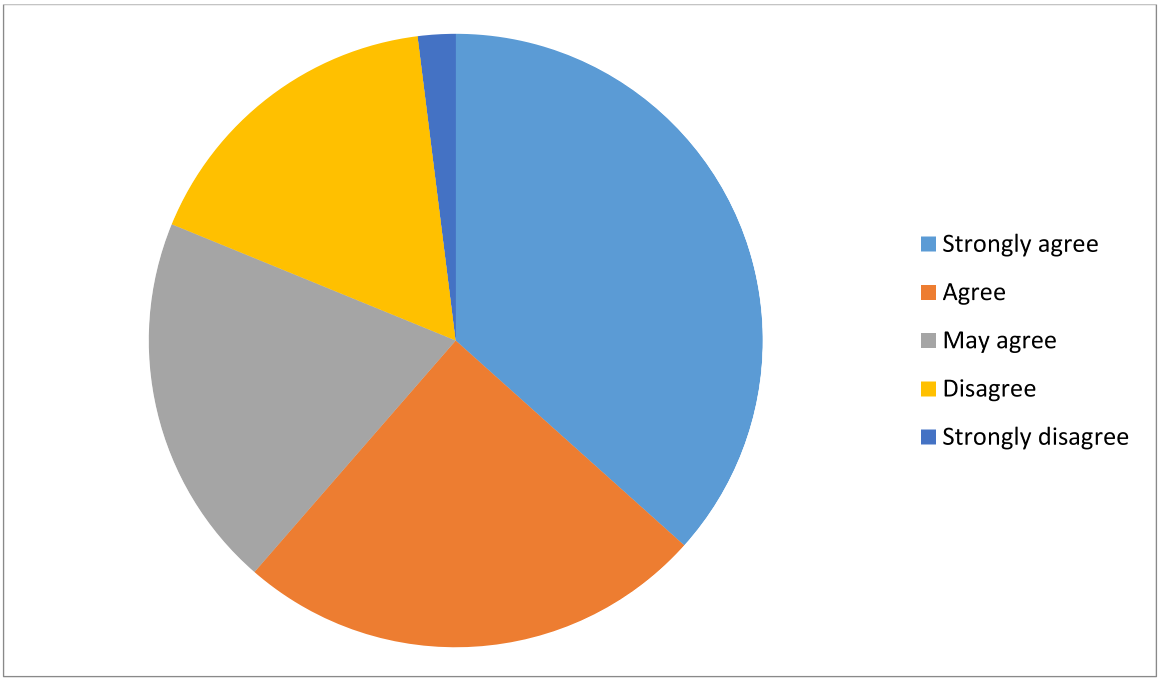

Figure 4.3a: Experimental Group: I prefer to learn Circle Geometry using GeoGebra

Figure 4.3b presents preference to learn circle geometry using conventional approach.

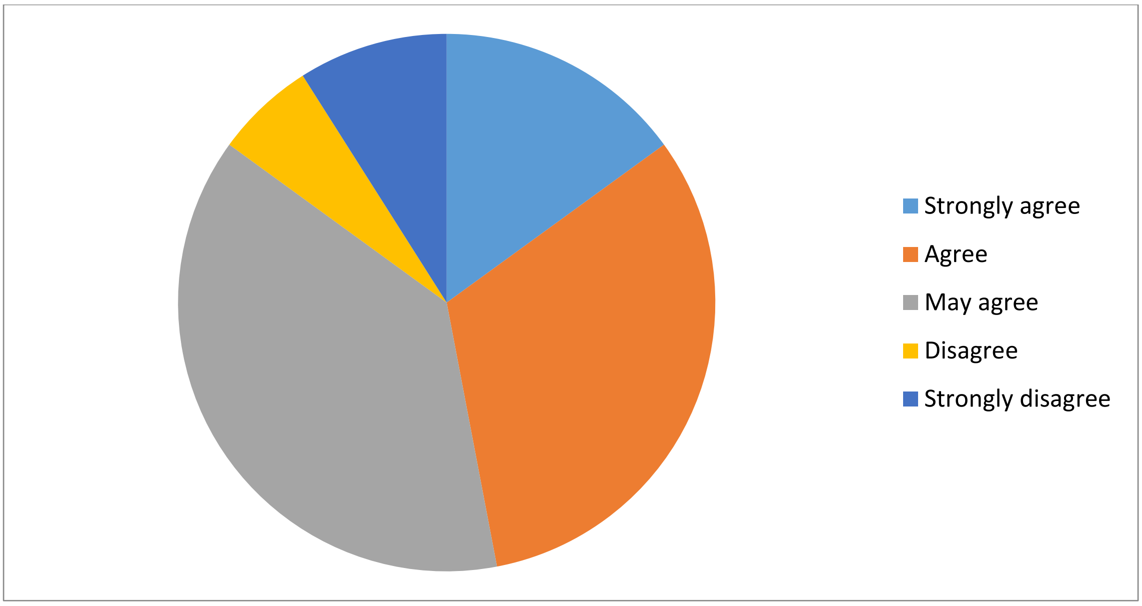

Figure 4.3b Control Group: Presents preference to learn circle geometry using conventional approach

4.3.3 Learners' views on advantages of learning approaches utilised in learning Circle Geometry

4.3.3.1 Learning tools and participation in class

The two groups of learners were to indicate their views about available learning tools and how they participated in class as a result of these tools.

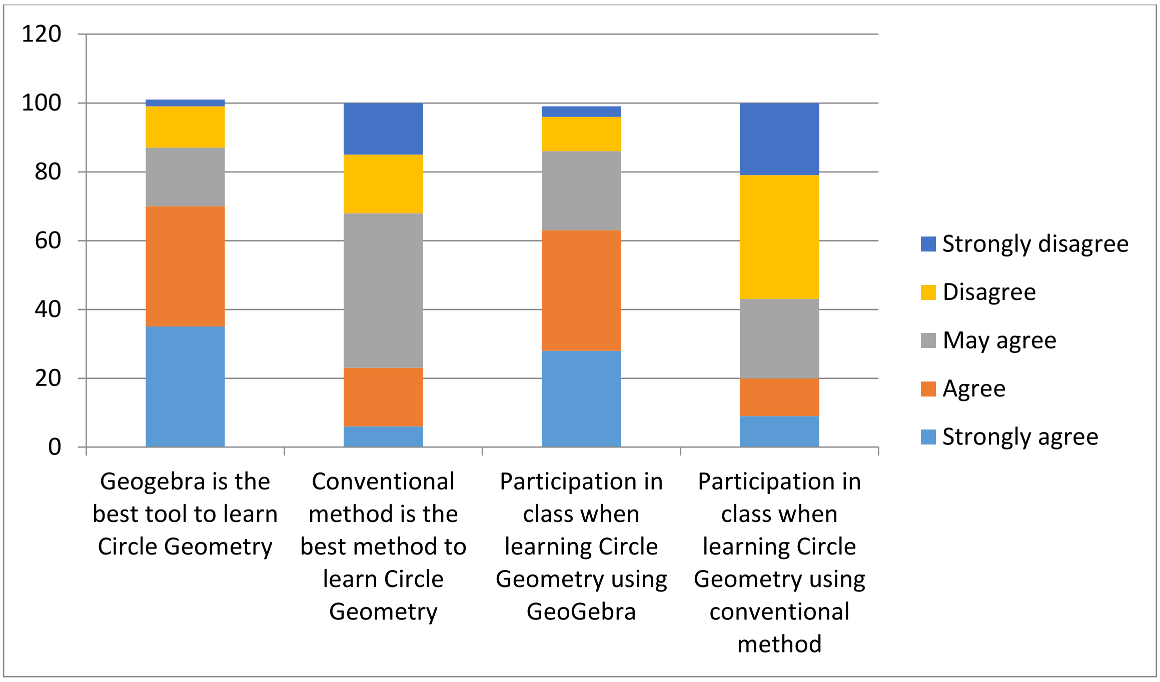

Figure 4.4: Learners' views on learning approaches and participation in class

Figure 4.4 shows that out of a total 60 respondents in the experimental group, an equal number of 21 (35%) respectively strongly agreed and agreed that learning circle geometry using GeoGebra as a manipulative tool was the best method. In that group 10 (17%) indicated may agree, 7 (12%) disagreed and only 1 (2%) strongly disagree. Compared to the control group, out of 47 respondents only 3 (6%) and 8 (17%) respectively strongly agreed and agreed that conventional methods were the best approach to learning circle geometry, with 21 (45%) indicating that they may agree. 8 (17%) disagreed while 7 (15%) strongly disagreed with that notion.

Figure 4.4 also shows that in comparison, 17 (28%) of the experimental group compared to 4 (9%) of the control group strongly agreed that they participated more when learning circle geometry using GeoGebra and the conventional method respectively; while 21 (35%) from experimental group and only 5 (11%) from control group also respectively agreed. An equal percentage of 23% (14 from experimental and 11 from control group) said may agree, whereas only 13% from the experimental group disagreed or strongly disagreed compared to 57% from the control group.

4.3.3.2 Motivation to learn resulting from the approach adopted

The following graph represents respondents' views on their motivation levels due to teaching approaches used.

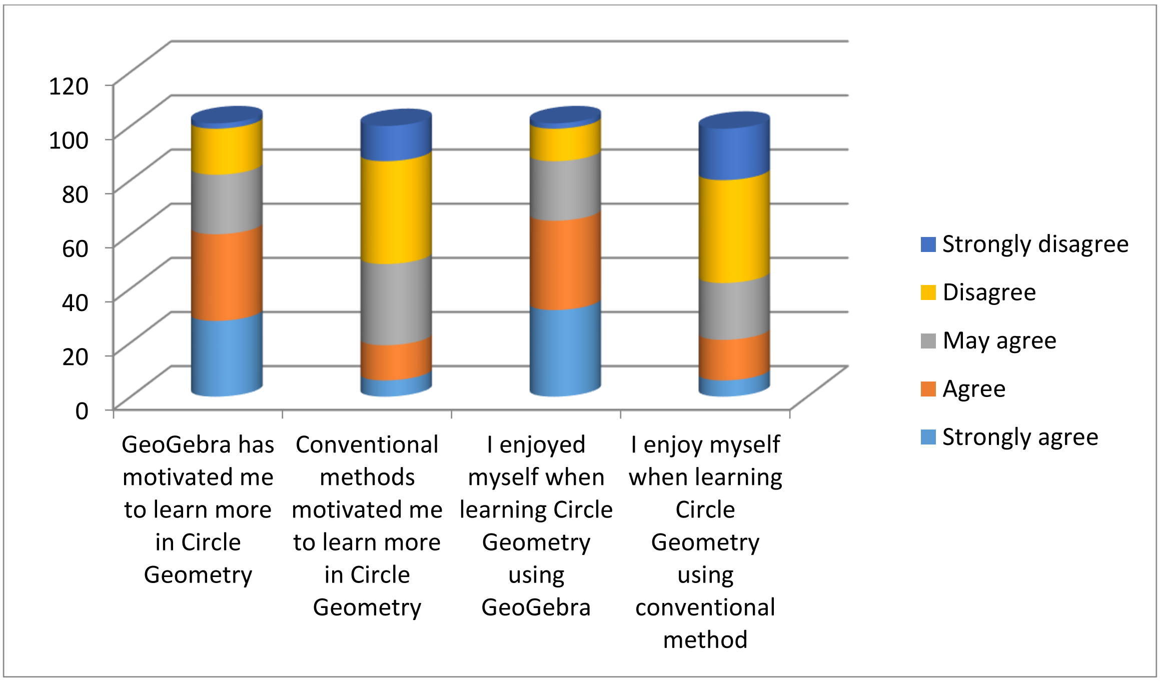

Figure 4.5: Motivation to learn resulting from the approach adopted

It is evident from Figure 4.5 that experimental group learners were more motivated than the control group. For example, in the experimental group 17(28%) strongly agreed that GeoGebra motivated them to learn circle geometry while 3 (6%) from the control group had the same notion. In addition, 19 (32%) from the experimental group agreed to have been motivated while 6 (13%) from the control had a similar view. Only 24 (40%) respondents from the experimental group compared to 38 (81%) from the control group either might agree or disagreed to being motivated by the used teaching approach.

Figure 4.5 also shows that in the experimental group, 19 (32%) and 20 (33%) respectively strongly agreed and agreed that they enjoyed learning circle geometry using GeoGebra compared to only 3 (6%) and 7 (15%) from the control group who held respectively similar views. Only 21 (36%) respondents from the experimental group compared to 37 (78%) from the control group either might agree or disagreed to not enjoying the teaching approach that was being used.

4.3.3.3 Teacher competence

Respondents were asked about how they viewed their teachers' competences to teach.

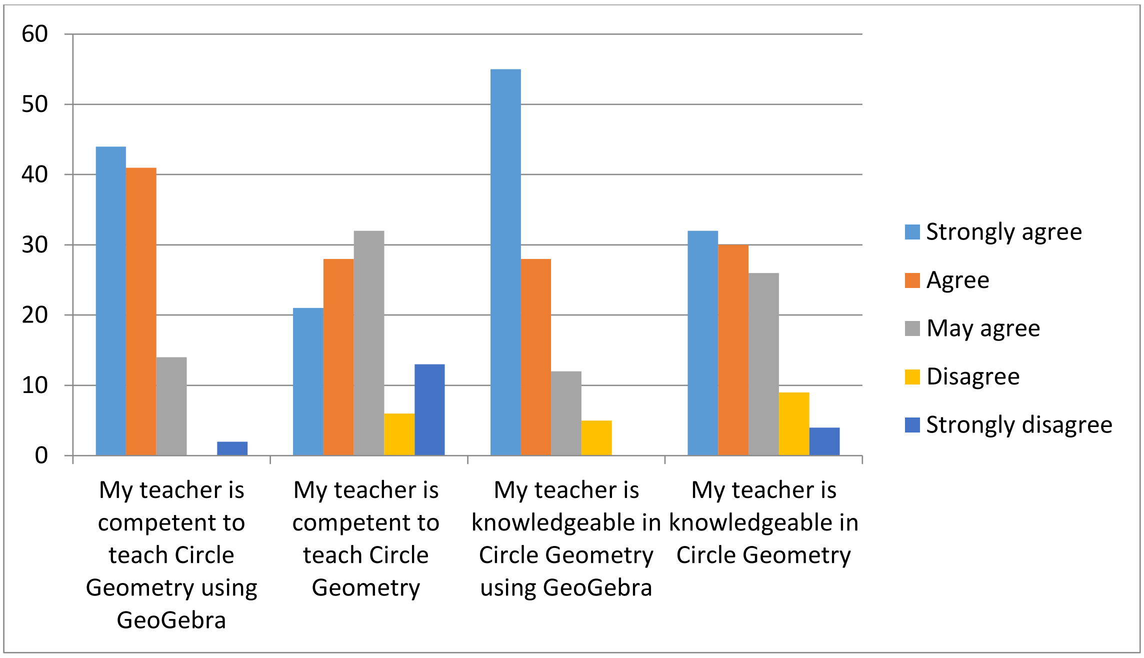

Figure 4.6: Respondents' views on teacher competence

It is shown in Figure 4.6 that out of the 60 respondents in the experimental group, 26 (44%) strongly agreed that their teacher was competent in teaching circle geometry using GeoGebra compared to 10 (21%) in the control group; 24 (41%) agreed in the experimental group compared to 13 (28%) in the control group. and 8 (14%) indicated may agree compared to 15 (32%) in the control group. No participant disagreed and only 1 (2%) strongly disagreed that the teacher was competent in the experimental group compared to respectively 3 (6%) and 6 (13%) in the control group.

Regarding knowledge, Figure 4.6 shows that in the experimental group, 33 (55%) and 17 (28%) respectively strongly agreed and agreed that their teacher was knowledgeable in using GeoGebra compared to 15 (32%) and 14 (30%) from the control group. Respectively in the experimental and control class only 7 (12%)

compared to 12 (26%) might agree, with 5% compared to 13% respectively in the experimental and control class either disagreed or strongly disagreed to that view.

Further findings are presented in Table 4.5 which shows that out of 60 respondents in the experimental group 22 (37%) strongly agreed that GeoGebra helped them improve their performance in circle geometry. In the same group an equal number 14 (23%) agreed and might agree, 7 (12%) disagreed and 3 (5%) strongly disagreed. In the control group out of 47 respondents only 5 (11%) strongly agreed that learning circle geometry using conventional methods improved their performance. 6 (13%) agreed, 12 (26%) indicated may agree, 16 (34%) disagreed and 8 (17%) strongly disagree.

It is also shown that an equal number of 17 (28%) in the experimental group both strongly agreed and agreed that they had problems with circle geometry before they were exposed to GeoGebra and admitted that they will approach the topic with confidence. 18 (30%) indicated may agree with 8 (15%) of the respondents disagreed or strongly disagreed. From the control group, only 7 (15%) and 11 (23%) of the respondents respectively strongly agreed and agreed that they can now approach circle geometry with confidence; 10 (21%) indicated may agree, 13 (28%) disagreed and 6 (13%) strongly disagreed.

Table 4.5: Comparative learners' views on GeoGebra and conventional methods when learning Circle Geometry (N=107)

Variable	Responses																			
	Strongly agree				Agree				May agree				Disagree				Strongly disagree			
	Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group	
	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%
I think learning Circle Geometry using this method improved my marks in Geometry	22	37	5	11	14	23	6	13	14	23	12	26	7	12	16	34	3	5	8	17
I had problems with Circle Geometry before but now I think I will approach the topic with confidence	17	28	7	15	17	28	11	23	18	30	10	21	7	12	13	28	1	2	6	13
I recommend this approach when learning Circle Geometry	19	32	2	4	17	29	7	15	14	24	11	23	9	15	20	43	0	0	7	15

While 23% experimental group learners compared to 28% from the control group indicated that they may agree, 35% experimental group learners compared to 47% control group learners disliked the section on circle geometry in mathematics. Furthermore, 40% experimental group compared to 15% control group learners strongly held the view that in Circle Geometry you can be creative and discover things by yourself. The same trend followed as 32% experimental group learners compared to 21% from the control group agreed to that view, while 18% experimental group learners compared to 19% from the control group might agree. Generally, the experimental group was more positive than the control group on that notion. Ironically, on the notion of memorisation helping learners to understand theorems of Circle Geometry, 64% of the experimental group agreed or strongly agreed to that in comparison to only 28% from the control group. Ten percent (10%) experimental group learners compared to 53% control group learners actually disagreed that memorisation helped them to understand theorems of Circle Geometry.

4.3.3.4 Learners' comparison of learning circle geometry using GeoGebra and conventional methods

The researcher presents the results of learners' experiences of learning circle geometry using either conventional methods or GeoGebra.

4.3.3.4.1 Perception of participants towards circle geometry

Figure 4.7 presents learners experiences relating to changes in learning attitude

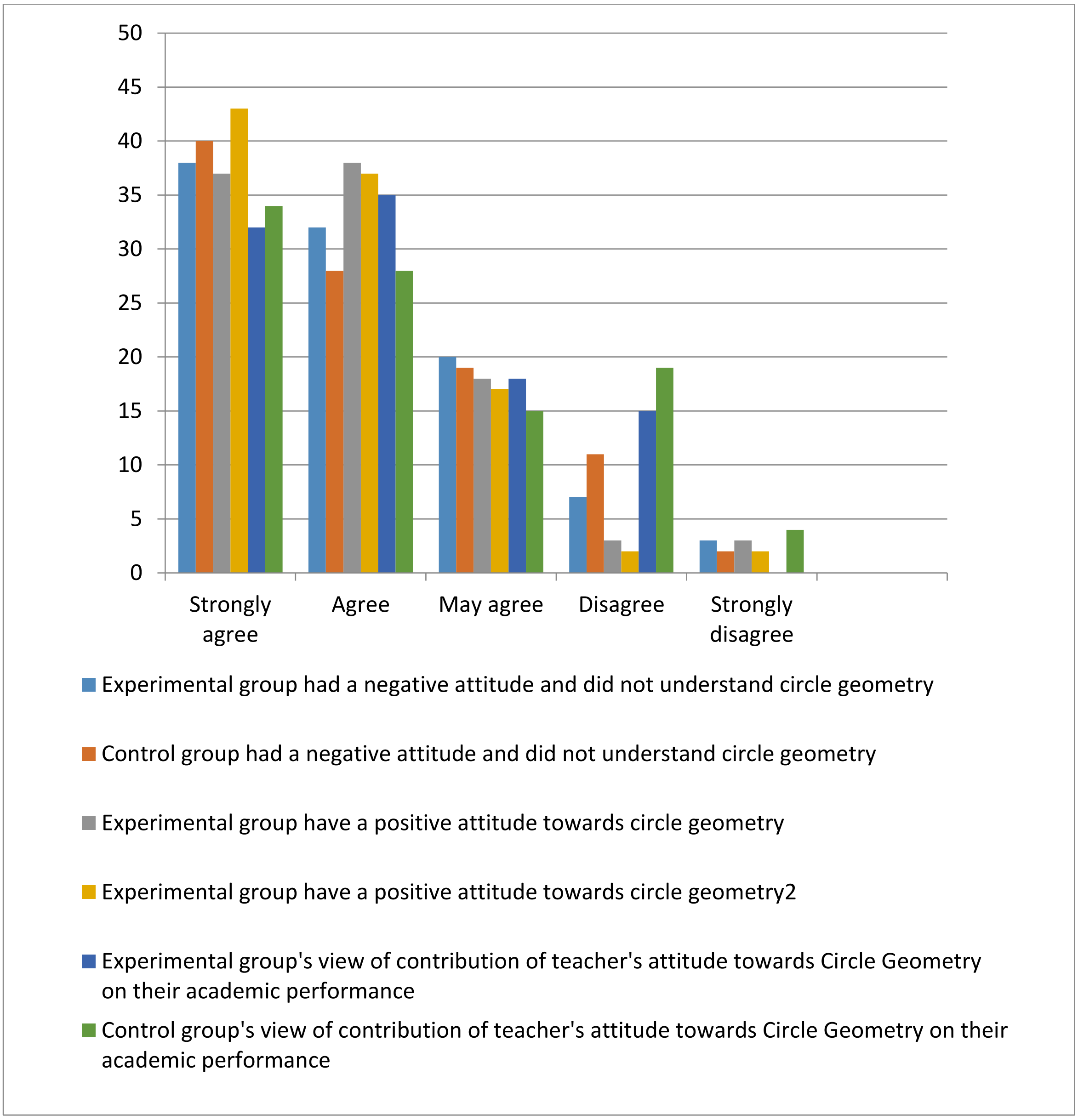

Figure 4.7: Learners experiences relating to changes in learning attitude

Figure 4.7 shows that out of 60 respondents in the experimental group, 23 (38%) strongly agreed that they had negative attitude and did not understand circle geometry compared to 19 (40%) from the control group, while 19 (32%) of the experimental group compared to 13 (28%) from the control group agreed. The learners who indicated may agree on that notion were about 20% for either group, while those who disagreed were slightly above 10% for both groups. In the experimental group, 22 (37%) strongly agreed that they were positive and that they would change their attitudes towards circle geometry compared to 20 (43%) from the control group.

Almost an equal percentage agreed to that notion, and the trend continued in a similar fashion for the learners who indicated that they may agree and learners who disagreed. The figure also reveals that in the experimental group 19 (32%) strongly agreed that their teacher's attitude contributed to their academic performance with 17 (35%) agree compared to respectively 16 (34%) and 13 (28%) from the control group. Learners who might agree were almost equal at 18% and 15% respectively for experimental and control groups. While 15% from the experimental group disagreed, 9 (19%) and 2 (4%) from the control group respectively agreed and strongly disagreed.

4.3.3.4.2 Support from home

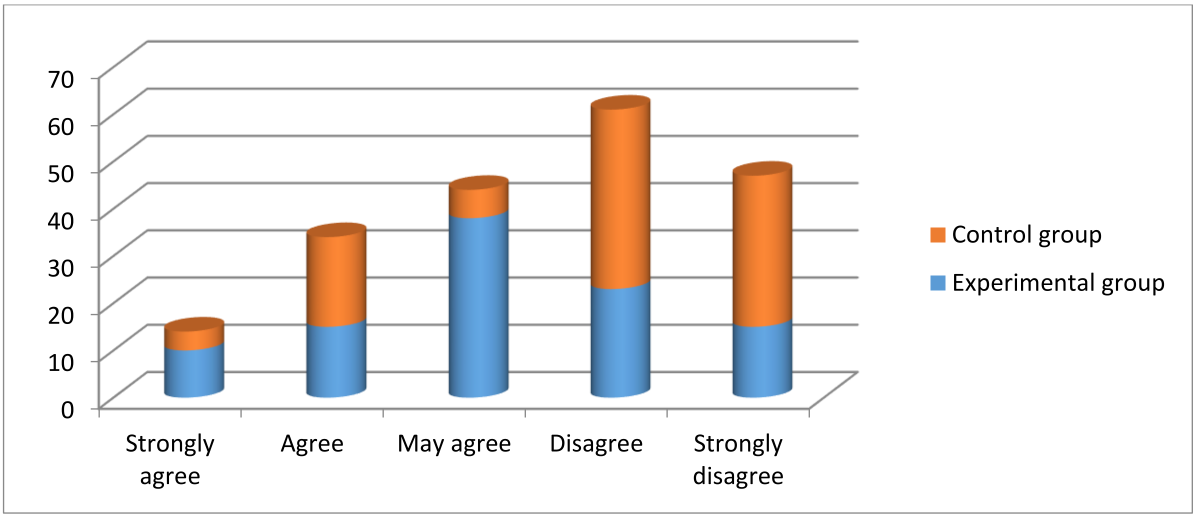

Figure 4.8: Support from home

From Figure 4.8, the experimental group revealed that 10% of the respondents strongly agreed that there were people at home assisting them with problems of circle geometry with 4% from the control group strongly agreeing. Both groups equally agreed by a mean of 18.5%, with 38% experimental indicated may agree, 11 (23%) disagreed and 7 (15%) strongly disagreed. Compared to respondents in the control group, 6% indicated may agree, 38% disagreed and 32% strongly disagreed.

Table 4.6 reveals what the participants in the experimental group indicated on whether extra support from their teacher affected their performance in circle geometry. Out of 60 respondents 21 (35%) strongly agreed, 15 (25%) agreed, 21 (35%) indicated may agree and none strongly disagreed. Responding to the same item, control group responded with only 6 (13%) strongly agreeing, 15 (32%) agreed, 12 (26%) responded may agree, 10 (21%) disagreed and 4 (9%) strongly disagreed.

4.3.2.1 Learners' mathematics attitude, learning styles and preferences

Table 4.4: Comparative learners' attitude towards mathematics and circle geometry (N=107)

Variable	Responses																			
	Strongly agree				Agree				May agree				Disagree				Strongly disagree			
	Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group		Experimental group		Control group	
	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%
I like mathematics	14	23	10	21	24	40	25	53	15	25	9	19	5	8	2	4	2	3	1	2
I like the section of Circle Geometry in mathematics	6	10	5	11	19	32	7	15	14	23	13	28	18	30	13	28	3	5	9	19
In Circle Geometry you can be creative and discover things by yourself	24	40	7	15	19	32	10	21	11	18	1	19	5	8	12	26	1	2	9	19
Memorisation helps me understand theorems of Circle Geometry	16	27	7	15	22	37	6	13	16	27	9	19	6	10	16	34	0	0	9	19

Thus experimental learners were positive about teacher extra support. On willingness to learn on their own when learning circle geometry, the experimental group had cumulatively 53% who strongly agreed or agreed compared to the 26% from the control group. On the other hand, only 24% from the experimental group either cumulatively disagreed or strongly disagreed compared to the 60% from the control group on the same notion.

With reference to small groups, 48% from the experimental group compared to 64% from the control group strongly agreed that small groups helped them understand the concepts of circle geometry. In addition, 30% experimental group compared to 19% control group agreed to that notion. Only at most 10% from either group disagreed or strongly disagreed. Table 4.6 also reveals the results of how hands-on activities in circle geometry helped them understand the topic. In the experimental group 18 (30%) strongly agreed while 25 (42%) agreed compared to 10 (21%) who strongly agreed while 16 (34%) agreed in the control group. 23% indicated may agree from each group. Only 3 (5%) disagreed in the experimental group compared to 5 (11%) in the control group.

When it comes to forming better connections between previous learning and new learning, Figure 4.9 shows that in the experimental group 12 (20%) strongly agreed that they can form better connections between previous learning and new learning while 28 (47%) agreed compared to respectively 8 (17%) and 13 (28%) in the control group. Almost an equal percentage indicated may agree from each group, while only

7 (12%) in the experimental group compared to 11 (23%) in the control group disagreed.

4.3.3.4.4 I'm able to form better connections between previous learning and new learning

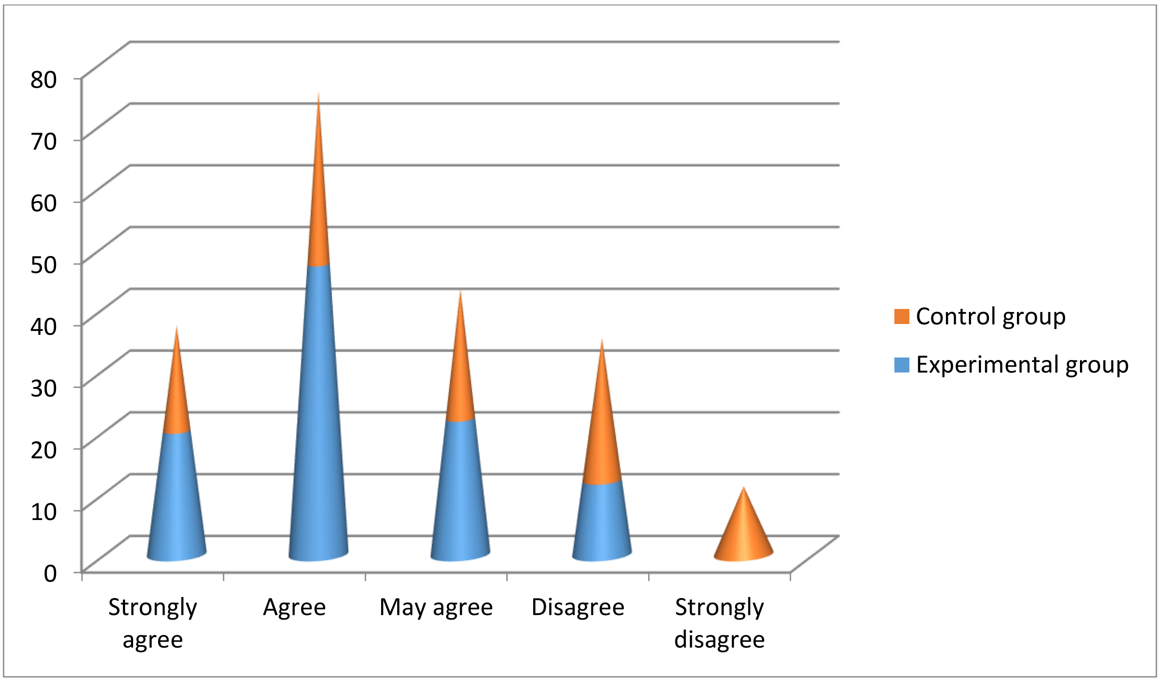

Figure 4.9: I'm able to form better connections between previous learning and new learning

Discussion

The aim of the study was to establish the influence of using GeoGebra as a manipulative tool in providing processes of Grade 11 circle geometry. Findings from pre-test and post-test show that the mean score of the experimental group increased more than that of the control group. This indicates that learners' performance in the experimental group improved more than in the control group. This is consistent with the conclusion made by Romani and Patadia (2012) who claim that conventional methods of teaching are the possible reasons of poor performance by learners in mathematics. Therefore, learners taught using GeoGebra as a manipulative tool performed better than those taught using conventional methods of teaching. It may also be argued according to Couco et al. (1995) that GeoGebra was more appropriate in developing habits of mind, Hiebert and Lefvres' (1986) conceptual knowledge, and Skemp's relational understanding in circle geometry. The results obtained also agree with a study carried out by Skemp (1976). He claims that the use of computer-assisted instructions may force learners to create their own solution paths as compared to learners who are exposed to conventional teaching instructions. Moreover, Jagodzinski (2009) and Larbi and Mavis (2016) found that when using conventional approach learners are passive recipients of information and that conventional approach must be replaced with student-centred teaching strategies such as active, experiential, independent, investigative, cooperative and problem-solving learning. These findings agree with the constructivism approach where learners are provided with resources and activities to ensure active involvement and participation while constructing their own knowledge.

Learners could not apply properties of geometric concepts in correct contexts to simplify geometry problems. When facing a geometry problem, learners usually gave a reason unrelated to a problem. This is consistent with the studies conducted by Cunningham and Roberts (2010) confirmed that learners have difficulties in understanding geometry concepts. This is also alluded to by Siyepu (2005), whose study also reveals that learners display errors because of their inability to understand geometric concepts. According to Oberdorf and Taylor-Cox (1999), lack of exposure to proper instructional approach is one of the reasons for learners' errors in geometry. For a success in learning Geometry the understanding of geometrical concepts is essential.

Questionnaire findings indicated significance differences between the participants in experimental group and those in control group. The findings from the experimental group showed that participants were highly motivated with the use of GeoGebra when learning circle geometry compared with their counterparts in the control group. These findings agree with the experiential learning theory whereby knowledge is created through transformation of experience and continuity of interaction with manipulatives. Furthermore, these discoveries concur with the claims made by Diamond (2012) and Mansukhani (2010) that increasing level of learner motivation results in learner academic performance. Recommendations made by Hollebrands (2007), Anthony and Walshaw (2009), Keengwe (2013), Reisa (2010) and Pandiscio (2002) indicate that technology (GeoGebra) can encourage students to explore mathematics and offer opportunities for critical thinking, which is vital to constructivism. Lunar et al. (2010) stated that the use of computers in teaching and learning is not only to improve learners' performance, but also motivation. This was shown by the students' interest in using GeoGebra software in learning circle geometry. In the study by Noorbaizura and Leong (2013), they found that learning process experienced by the experimental group (using GeoGebra) allowed them to communicate openly with the teachers and students and among the students themselves. It showed that learning with software could also trigger on-task interactions. The interactions resulting from learning increased students' interest in learning circle geometry.

4.3.3 What is the significance of using GeoGebra on the academic performance of learners learning circle geometry?

The researcher used pre-test and post-test to present the findings of the above sub-research question. The results are presented using tables.

Table 4.7: Summary of experimental pre-test and post-test results

Experimental Group	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
Pre-Test	60	0,83	1,40	0	0	0	1	8
Post Test	60	11,33	9,81	0	3,5	7,5	17,5	37

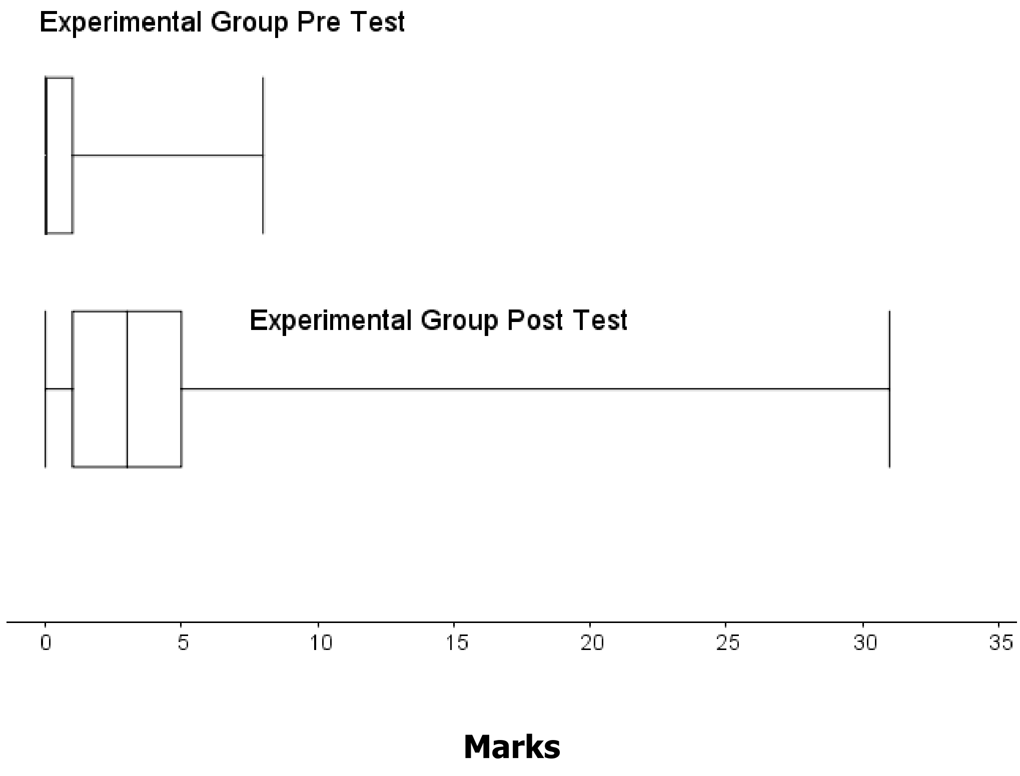

Figure 4.10: Experimental pre-and-post-test box and whisker diagram

Out of 60 participants who sat for the pre-test in the experimental group, Table 4.7 and Figure 4.10 reveal that out of 50 marks the maximum mark obtained was 8 with a minimum mark of 0. On average, the participants obtained 0,83 with a standard deviation of 1,40. The results also revealed that 50% (median) of the participants obtained 0 mark, 25% (lower quartile, Q1) of the participants obtained 0 mark. Furthermore, Figure 4.10 also shows that 75% (upper quartile, Q3) of the participants obtained 1 mark.

However, when GeoGebra was used as a manipulation tool to address the processes of circle geometry, the post-test results reveal to a greater extend the impact of GeoGebra in learners' performance. Out of 60 participants in the experimental group, a test out of 50 marks the maximum mark obtained was 37 as compared to 8 in pre-test with a minimum mark of 0. On average, the participants obtained 11,33 with a standard deviation of 9,82 compared to mean of 0,83 and standard deviation of 1,40 in pre-test. The results also revealed the median (50%) mark obtained by the participants was 8 as compared to 0 mark in the pre-test, 25% (lower quartile, Q1) of the participants obtained 4 marks and 75% (upper quartile, Q3) of the participants obtained 11 marks.

Table 4.8: Results analysis for control pre-test and post-test

Control Group	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
Pre Test	47	0,36	1,23	0	0	0	0	8
Post Test	47	3,81	4,76	0	1	3	5	31

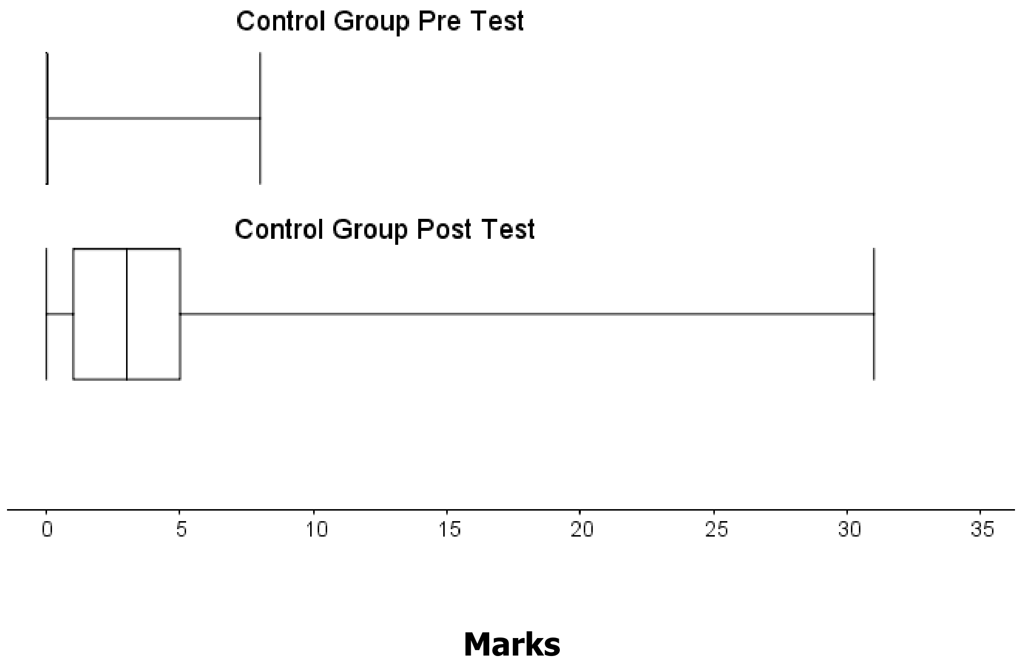

Figure 4.11: Control group pre-and-post-test box and whisker diagram

In the control group (see Table 4.8 and Figure 4.11) 47 participants sat for the pre-test and post-test respectively. Out of a total of 50 marks, the minimum mark obtained in both pre and post-test was 0. The maximum mark obtained in the pre-test was 8 and in post-test was 31. With a standard deviation of 1,23 and 4,76 in pre-test and post-test respectively the average mark obtained was 0,36 and 3,81 respectively. The lower quartile (25%) obtained 0 mark and 1 mark in pre-test and post-test respectively. The table also reveals that the middle mark (median) obtained by participants in pre-test was 0 and 3 in post-test. It also shows that 1 mark was obtained in the pe-test and 5 marks at 75% (upper quartile).

Table 4.9: Comparison results analysis for experimental and control pre-tests

Group	N	Mean	Standard deviation	Minimum	Lower Quartile	Median	Upper quartile	Maximum
Experimental	60	0,83	1,40	0	0	0	1	8
Control	47	0,36	1,23	0	0	0	0	8

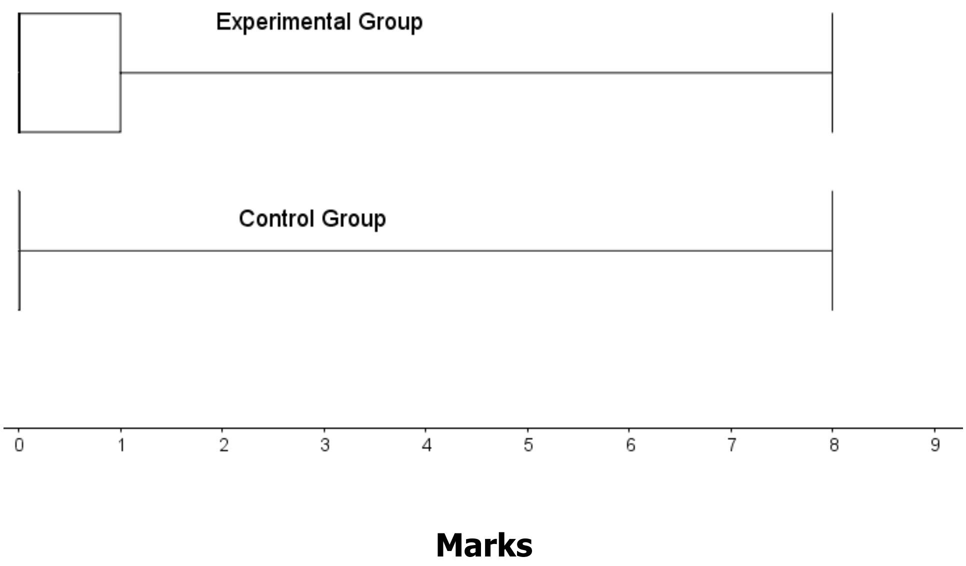

Figure 4.12: Experimental and Control group pre-test box and whisker diagram

Out of 60 participants in the experimental group and 47 participants in the control group, Table 4.9 and Figure 4.12 reveal that in both groups the five-number summary is almost the same for the pre-test. The experimental group recorded a mean of 0,83 with a standard deviation of 1,40 while control group recorded mean of 0,36 with a standard deviation of 0,23. Comparison results for the post-test are presented below.

Table 4.10: Comparison results analysis for experimental and control post-test

Group	N	Mean	Standard deviation	Minimum	Lower Quartile	Median	Upper quartile	Maximum
Experimental	60	11,33	9,82	0	3,5	7,5	17,5	37
Control	47	3,81	4,76	0	1	3	5	31

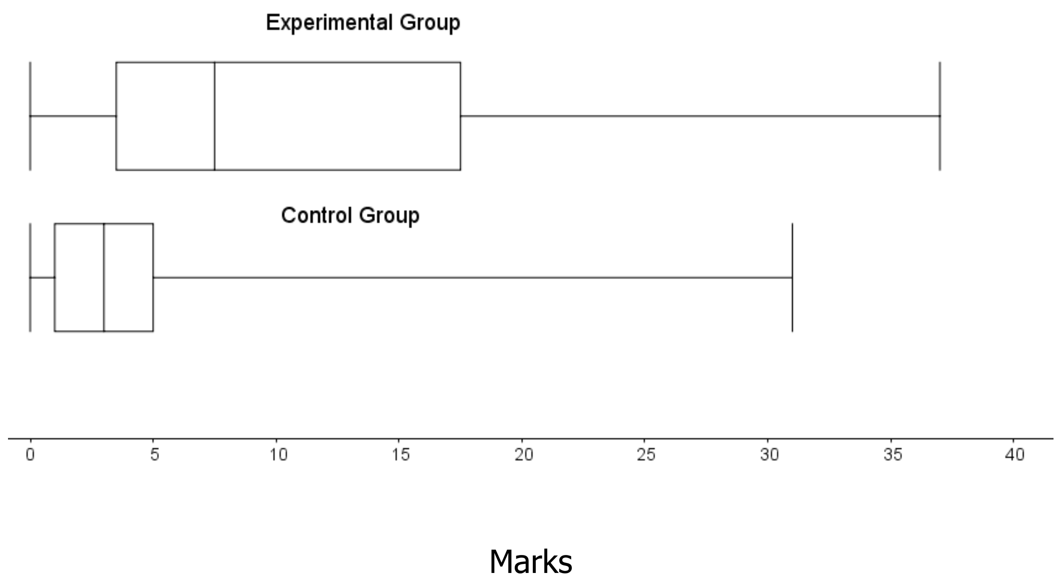

Figure 4.13: Experimental and Control group post-test box and whisker diagram

The average mark obtained in the experimental group was 11,33 with standard deviation of 9,82 and in the control group, the average mark was 3,81 with a standard deviation 4,76. On contrary to the five-number summary of the pre-test, the post-test results reveals a five-number summary for both groups. Except for minimum mark all other items of five number summary in both groups were not the same.

Discussion

Findings presented in Tables 4.7 to 4.10 and Figures 4.10 to 4.13 indicate that there were small statistical differences in marks obtained by learners in the control group in the pre-test and the post-test, as compared to statistical differences in the marks obtained by experimental learners in the pre-test and the post-test. While both control group and experimental group learners appeared to have almost the same characteristics in the pre-test, they showed significant statistical differences in the post-test as experimental learners outperformed control group learners. These results agree with findings of studies done by Clements and Battista (1990); Dienes (1960); Driscoll (1981); Sugiyama (1987) and Suydam (1984) who revealed that mathematics attainment increases when manipulatives are put to good use. The findings of the current study had also support of the findings by Parham (1983) and Sowell (1989) who found that achievement in mathematics could be increased by the long-term use of manipulatives. The use of technology (GeoGebra) as a manipulative as concluded by Lin (2008) when learning circle geometry has made the experimental group outperform the control group. Guven (2012) reported the same findings as cited in Yen Chang (2015). All lessons that the researcher did when teaching circle geometry in the experimental group involved the use of technology (GeoGebra) and the researcher noted significant difference as ultimately proved by learner performance. These findings are also in line with those of Suydam and Higgins (1976), Ball (1992), Sutton and Krueger (2002) and Jackson (2005) who believe that lessons which involve the use of computers, if employed appropriately, will yield greater mathematical achievement than will lessons in which manipulative materials are not used.

4.3.4 What are the gender differences in terms of learners' achievement after learning circle geometry using GeoGebra?

The researcher addressed the above sub-research question by first checking whether gender differences in terms of motivation had an impact in terms of learners' attainment. To address that the researcher used mean individual responses summed up from the research instruments used. For gender differences in terms of learners' achievement after learning circle geometry the researcher used pre-test and post-test. The results are presented below.

Females and males' perceptions on the usability of GeoGebra after learning circle geometry

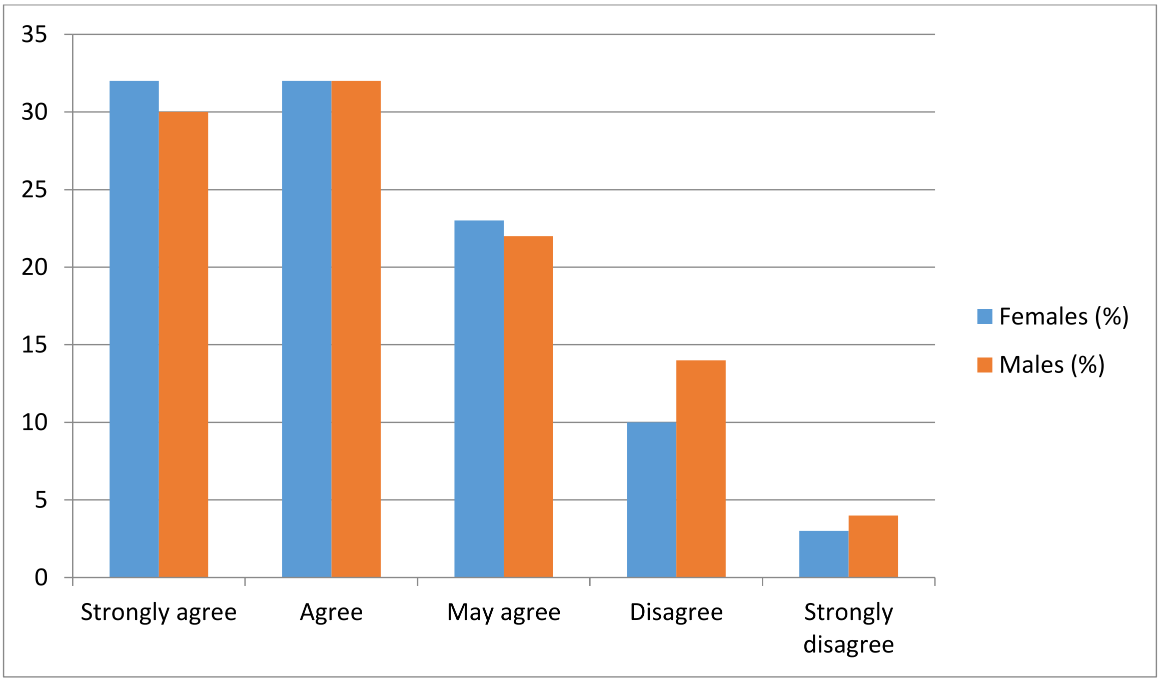

Fig 4.14: Females and males' perceptions on the usability of GeoGebra after learning circle geometry

Fig 4.14 reveals comparison gender perceptions on the usability of GeoGebra after learning circle geometry. Out of a total of 60 learners in the experimental group, 32% of the 43 females compared 30% of the 17 males strongly agreed that GeoGebra was an effective manipulative tool in learning circle geometry. Interesting a similar number (32%) agreed for each gender. 23% of females compared to 22% males responded may agree, with 10% females compared to 14% males disagreeing. Only 3% strongly disagreed for females compared to 2% for males.

Table 4.11 (a): Descriptive analysis of females and males' pre-test results in experimental group.

Experimental Group	N	Mean	Standard deviation	Minimum	Lower Quartile	Median	Upper quartile	Maximum
Females	43	0,93	1,56	0	0	0	1	8
Males	17	0,59	0,84	0	0	0	1	3

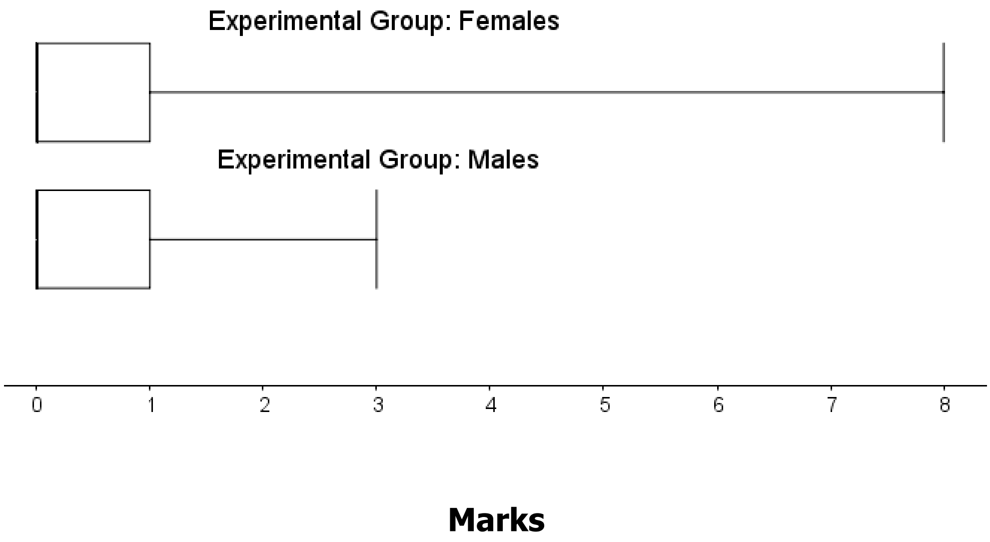

Figure 4.15: Females and Males' experimental pre-test box and whisker diagram

Table 4.11 (b): Inferential analysis of females and males' pre-test results in experimental group

GENDER	N	MEAN	SD	D	DF	t	*p*
MALE	17	0,59	0,84				
				0,34	43	1,02	0,31
FEMALE	43	0,93	1,49				

Descriptively, Table 4.11 (a) and Figure 4.15 show the results of females and males pre-test. Out of 43 females who sat for pre-test in the experimental group the minimum mark was 0 with maximum mark of 8. The median mark was 0 with an average mark of 0,93 and a standard deviation of 1,56. Compared with their male counterpart, out of 17 males who sat for the pre-test the minimum mark and maximum mark recorded was 0 and 3 respectively. Males' average mark was 0,59 with a standard deviation of 0,84. Further analysis, inferentially, Table 4.11 (b) reveals that, although the mean response of females with regard to their performance before using manipulatives was higher than that of the males, the table shows no significant difference in the mean response of both boys with mean = 0,59 and standard deviation (SD) = 0,84 and girls with mean = 0,93 and SD = 1,56. Extract of t-test comparison of males and females before using GeoGebra as a manipulative tool shows that t(43) = 1,02 with a *p*-value of 0.310.

Table 4.12 (a): Descriptive analysis of females and males' post-test results in experimental group

Experimental Group	N	Mean	Standard deviation	Minimum	Lower Quartile	Median	Upper quartile	Maximum
Females	43	10,91	8,53	1	4	7	17	30
Males	17	13,18	12,42	0	1,5	8	26	37

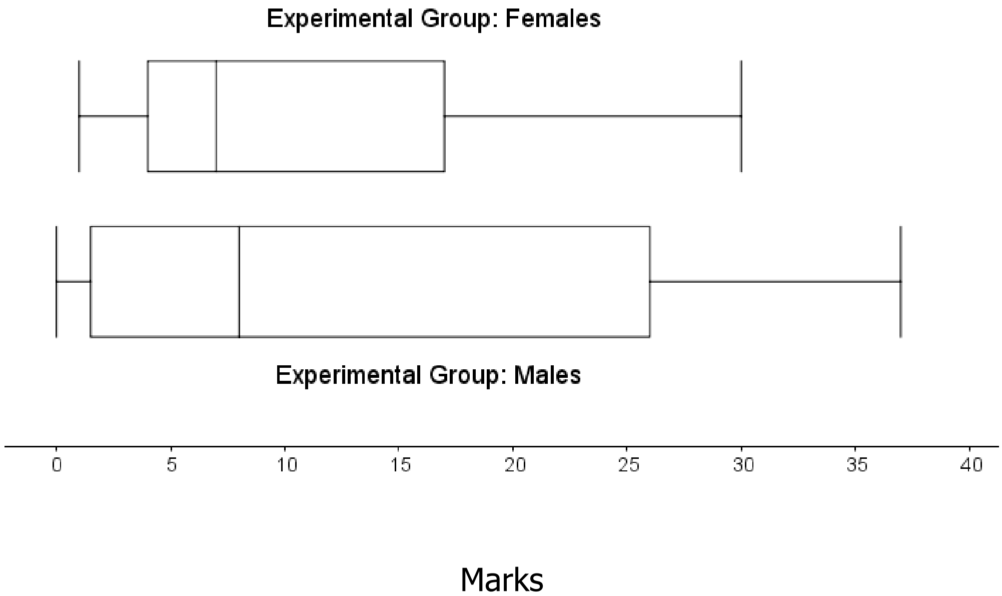

Figure 4.16: Females and Males experimental post-test box and whisker diagram

Table 4.12 (b): Inferential analysis of females and males' post-test results in experimental group

GENDER	N	MEAN	SD	D	DF	t	*p*
MALE	17	13,18	12,42				
				2,27	22	-0,6916	0,4963
FEMALE	43	10,91	8,53				

Table 4.12 (a) and Figure 4.16 shows the results of females and males post-test. Out of 43 females who sat for post-test in the experimental group the minimum mark was 1 with maximum mark of 30. The median mark was 7 with an average mark of 10,91 and a standard deviation of 8,53. Compared with their male counterparts, out of 17 males who sat for the post-test the minimum mark and maximum mark recorded were 0 and 37 respectively. Males' average mark was 13,18 with a standard deviation of 12,42. Inferentially, Table 4.12 (b) shows that, the mean response of females with regard to their performance in using GeoGebra as a manipulative tool was higher than their counterparts. Table 4.12 (b) reveals that there is no significant difference in the mean response of both boys with mean = 13,18 and standard deviation (SD) = 12,42 and girls with mean = 10,91 and SD = 8,53. Extract of t-test comparison of males and females after using GeoGebra as a manipulative tool reveals that t(22) = -0,6916 with a p-value of 0.4963.

Discussion

The results shown in Table 4.11 (a) and Table 4.12 (a) and Figures 4.14 to 4.16 revealed that girls outperformed their counterparts before the introduction of GeoGebra as a manipulative tool. Table 4.12 show different results as boys outperformed girls with a difference of 2,27 in their mean score. The differences in the findings shown agree with findings by Carr and Jessup (1997) that mathematics achievement may be influenced by gender differences in mathematics strategies. This could be so as the CRA sequence of instructions integrates the use of hands-on manipulatives in the concrete stages (learners working with computers), followed by pictorial displays in the representational phase (learners discovering properties and making generalisation of theorem using GeoGebra) and in the next phase facilitates abstract reasoning with numerical symbols (learners solving riders) (Miller & Mercer, 1993).

The findings also agree with findings noted by Sousa (2011) who claim that girls are generally scoring higher on standardised tests, especially in language skills and verbal expression. Bonomo (2010)'s findings further support the results as she claims that girls are more able in verbal expression, and are more adapted to sensory memory, sitting still, listening and writing related functions whilst boys are more adapted to symbols, abstractions and pictures. She concluded that boys generally learn mathematics and physics more easily than girls when lessons are presented with manipulatives (objects).

Inferentially, if all conditions of inference are met and working with significance level of 0.05, the results in table 4.11(b) and 4.12(b) could be explained differently. Since p -value of table 4.11(b) is 0.310 and greater than 0.05 it shows that there is no evidence to suggest that differences existed in the boys and girls performance before the use GeoGebra in solving processes of circle geometry. However, both groups had a lower mean score. Likewise, table 4.12(b) with p -value of 0.4963 shows no significant difference in the mean score of boys who were taught circle geometry using the GeoGebra. Since $p > 0.05$, we fail to reject the null hypothesis of no difference of boys and girls performance after they have been taught geometry with GeoGebra as a manipulative tool.

Results revealed in 4.11(b) and 4.12(b) maintains claim as cited by Larbi & Mavis, (2016) that the uses of manipulatives arouse and sustain students' interest and ensures their active participation in the learning process (Heddens, 1997; Munger, 2007). Learners assimilate knowledge when they are given the opportunity to explore and talk about their discoveries. Although the difference in boys and girls performance to the use of the manipulatives materials was not significant, the higher mean score of the boys seems to suggest boys are having more interest in using manipulatives than the girls. Interestingly, boys and girls who received instruction through this strategy performed at pall (Larbi & Mavis, 2016). This finding supports (Kurumeh, Chiawa & Ibrahim, 2010; al-Absi, & Nofal, 2010) both studies, which found no difference in boys and girls performance after they have been taught with GeoGebra. The use of GeoGebra as a manipulative enables learners to model abstract concepts of circle geometry and gives them the opportunity to communicate ideas among themselves and to the teacher, and enhance memory. According to Bruner (as cited in Resnick, & Ford, 1984), knowledge acquired without sufficient structure to tie it together is knowledge that is likely to be forgotten. Conclusively, since mathematics is abstract and comes from the real world, the use of real materials like GeoGebra in teaching of circle geometry can help learners' formation of concepts and develop positive attitude towards its teaching and learning.

4.4 Summary

The researcher presented and analysed the data according to subsidiary research questions. Data was presented in a manner that involved combining results from the different instruments used. Discussion of the findings was also done in this chapter. The next chapter presents the summary and conclusions of the study.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

The previous chapter presented findings of the study based on the data that was gathered. This chapter summarises and concludes the research project. It ends with recommendations for the study. The purpose of this study was to establish the influence of using GeoGebra as a manipulative tool in providing processes of grade 11 circle geometry at one school in OR Tambo Inland.

5.2 Summary of the research

The summary of this study is presented in line with the following sub-research questions:

- What are the effects of conventional methods of teaching in learning the concepts of circle geometry?
- How does learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation?
- What is the significance of using GeoGebra on the academic performance of learners learning circle geometry?
- What are the gender differences in terms students' achievement after learning circle geometry using GeoGebra?

5.2.1 What are the effects of conventional methods of teaching in learning the concepts of circle geometry?

This question was addressed using data obtained from all the research instruments. Pre-test results did not show many differences though both experimental and control group learners did not show superior performances showing that participants were almost at the same level of thinking before the experimental group were exposed to GeoGebra. After using GeoGebra, the researcher found that participants who learnt circle geometry using conventional methods performed lower than their counterparts. Findings obtained from the questionnaire also show similar patterns as post-test results as learners from the two groups showed that experimental learners were more positive about using GeoGebra to learn circle geometry while control group learners did not show much appreciation for continued use of conventional teaching and learning methods.

5.2.2 How does learning circle geometry using GeoGebra as a manipulative tool impact learners' motivation?

On average experimental group compared to the control group strongly agreed that they participated more when learning circle geometry using GeoGebra and the conventional method respectively. When it comes to learning circle geometry, it emerged that experimental group learners were more motivated than the control group. More so, the experimental group strongly agreed or agreed that they enjoyed learning circle geometry using GeoGebra compared to experience of the control group who were using the conventional methods. The study thus revealed that participants who learnt circle geometry using GeoGebra were more motivated than their counterparts.

The experimental group also strongly held the view that in Circle Geometry you can be creative and discover things by yourself than the control group. That could have been due to the nature of the tool they were using. Surprisingly on the notion of memorisation helping learners to understand theorems of Circle Geometry, more than double the experimental group agreed or strongly agreed with that view in comparison with the control group. Therefore, five times less experimental group learners compared to control group learners actually disagreed that memorisation helped them to understand theorems of Circle Geometry.

5.2.3 What is the significance of using GeoGebra on the academic performance of learners learning circle geometry?

Experimental respondents strongly agreed or agreed that learning circle geometry using GeoGebra as a manipulative tool was the best method whereas the majority of the control group disagreed or strongly disagreed that conventional methods were the best approach to learning circle geometry. Comparing pre-test and post-test results after GeoGebra was used as a manipulation tool to address the processes of circle geometry, the post-test results revealed to a greater extent the positive impact of GeoGebra in learners' performance. That was due to the better performance of learners in the experimental group than the control group. In many ways the researcher found that there is a significance difference on the academic performance of learners who learnt circle geometry using GeoGebra and those learners who learnt without. This study reveals that learners who learnt circle geometry using GeoGebra outperformed their counterparts.

5.2.4 What are the gender differences in terms of students' achievement after learning circle geometry using GeoGebra?

Based on the totality of individual gender responses, 67% of the females compared to 63% of males agreed or strongly agreed that GeoGebra was an effective manipulative tool in learning circle geometry. However, a slightly higher percentage of boys performed better than girls in the post-test despite the notion above. Thus, the findings of the study revealed that boys learn mathematics more easily than girls when lessons are presented with manipulatives as boys outperformed their counterparts.

5.3 Conclusion

- Students who used the GeoGebra software showed higher ability in conceptual knowledge compared to students who learnt using the conventional method. It was found that learners regardless of their level of ability who learnt the circle geometry topic with the help of the GeoGebra software had indeed higher conceptual knowledge than learners taught using the conventional method. Overall, it can be concluded that students who used the GeoGebra software would have higher conceptual knowledge than those who did not.

- The GeoGebra software has positive effects and it does help to enhance learners' understanding in circle geometry. The software was designed for users regardless of their ability, to easily understand abstract geometry concepts. It also assists students in applying the concepts.

- Learners became more active and responsible for their own learning process, as they were personally involved in the use of GeoGebra, which allows a self-learning process. Learners were given the opportunity to use their own ideas and to present their own works. This was different than the conventional learning where participants were passively waiting for the teacher to deliver information as they did not have the chance to present their own ideas.

- Learning circle geometry with GeoGebra software provided possible active interaction between teacher and participants, which is very rare in conventional learning.

5.4 Recommendations

The results of the study showed that learners who learnt the topic using GeoGebra performed better and were motivated more than those who learnt using conventional methods. The study thus makes the following recommendations.

5.4.1 Recommendations for practice

- Teachers should familiarize themselves with the current changes and strive to realise the use of the latest technology in the classroom.

- Teachers should vary their methods of teaching as a way of motivating and accommodating the post-modern learners, within heterogeneous classes. Teachers should first develop their own relational understanding of teaching methods, and knowledge of when and how to use a variety of teaching methods.

- The researcher also recommends a workable alternative to the rigid axiomatic approaches to circle geometry by utilising GeoGebra to facilitate and enhance learners' ability to the making and testing of conjectures.

- Seemingly using GeoGebra to visualise a problem helps learners to have a global picture of the problem to be solved. Text-books give static and limited information about geometric constructions but dynamic geometry can help learners to better visualise, provided it is used in conjunction with measurements.

- Curriculum planners and subject specialist must emphases on education systems that shift teaching and learning away from a traditional and emphasis on learning rules for manipulating geometry problems.

- Effective mathematics teachers must make sure that all learners are provided with opportunities to find ways around mathematics for themselves. Teachers can make everyone feel included by respecting and valuing the mathematics and the cultures that students bring to the classrooms. Ensuring that all students feel safe allows every student to get involved.

- Furthermore, building on students' thinking, teachers are urged to plan mathematics learning experiences that allow learners to build on their existing proficiencies, interest, and experiences.

- Providing students access to multiple representations helps them to develop conceptual and computational flexibility. Tools are helpful in communicating ideas that are otherwise difficult to talk about or write about. Teachers and students can use representations, such as pictures, symbols, concrete objects, and virtual manipulatives, to assist in communicating their thinking to others.

5.4.2 Recommendations for further studies

- Conducting the same study with a larger sample for instance, considering two or more schools in order to generalise the findings.

- Conducting a study on the impact of the use of technology gargets when writing examinations.
- The researcher also recommends the inclusion of technology in other topics such as functions and calculus.

5.5 Conclusion of the study

In concluding the study,it can be stated that the aim and objectives of the study have been achieved. The results of the study have answered the main and sub-research questions beyond reasonable doubt. The use of GeoGebra as a manipulative tool has proved that conventional teaching approach may not be the best to when teaching and learning GeoGebra in Grade 11, and this could apply to other grades. Finally, gender differences have shown significant results which might be worth considered when teaching using technology since boys have shown to have understood circle geometry better when GeoGebra was employed.

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APPENDICES

APPENDIX A: APPROVAL FROM INSTITUTIONAL ETHICS CLEARANCE COMMITTEE FOR PERMISSION TO CONDUCT STUDY

WSU
Walter Sisulu University

FACULTY OF EDUCATIONAL SCIENCES

Postgraduate Research and Higher Degrees

FEDS RESEARCH ETHICS COMMITTEE
CLEARANCE CERTIFICATE

Nelson Mandela Drive, Private Bag X1, Mthatha, WSU, Eastern Cape, South Africa
Tel: 047-5022541 E-mail: mbuka@wsu.ac.za

MEETING	: 26/09/2018
PROTOCOL NUMBER	: EREC22-09-18
TITLE OF PROJECT	: Geogebra as a manipulative tool in providing processes of circle Geometry in grade 11: A case of one school in OR Tambo Inland District
RESEARCHER	: Marange Israel Yeukai
SUPERVISOR	: Dr JK Alex
CO-SUPERVISOR	:
DEPARTMENT/UNIT	: CPTD
DEGREE	: Master of Education
COMMITTEE DECISION	: Approved

DR AM Buka
CHAIRPERSON

27/09/2018
Date

Please Note
Should there be any changes in the research procedures as approved, the researcher(s) must re-submit the protocol to the committee

PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES

Walter Sisulu University

APPENDIX B: APPLICATION TO EASTERN CAPE DEPARTMENT OF EDUCATION FOR PERMISSION TO CONDUCT STUDY

Province of the
EASTERN CAPE
EDUCATION

APPLICATION TO CONDUCT RESEARCH IN THE EASTERN CAPE DEPARTMENT OF EDUCATION

THIS APPLICATION FORM MUST BE COMPLETED AND SUBMITTED IN HARD AND ELECTRONIC COPY TO:

The Director
Strategic Planning Policy and Research
Eastern Cape Department of Education

Private Bag X0032
Bhisho
5605 (Postal address)

OR

Fax to: 040 608 4574/ 086 742 4942

OR
email: babalwa.pamla@ecdoe.gov.za
cc fundiswa.pakade@ecdoe.gov.za
and jabulile.mazibuko@ecdoe.gov.za

OR

Deliver to
Steve Vukile Complex
Zone 6
Zwelitsha
5608 (Physical address)

ENQUIRIES: Babalwa or Jabu
Tel: 040 608 4537/4035/4773

CHECKLIST – Please ensure all documents are attached	
Departmental Application Form (this form)	✓
Proposal as approved by relevant institution	✓
Research Instruments	✓
Ethical Clearance Certificate	EREC22-09-18

EASTERN CAPE DEPARTMENT OF EDUCATION
RESEARCH APPLICATION FORM

SECTION A
TO BE COMPLETED BY RESEARCHER

1. PARTICULARS OF THE RESEARCHER

1.1	Details of Researcher
Surname:	MARANGE
First Name/s:	ISRAEL JEUKAI
Title (Prof / Dr / Mrs / Ms / Mr):	MR
Student/Staff Number (if applicable):	217303404 / 40007842

1.2	Contact Details		
Institution/Home Address		Postal Address (if different)	
17 DDT JABAVU STREET			
SOUTHRIDGE PARK			
MTHATHA			
EASTERN CAPE			
Postal Code:	5100	*Postal Code:*	
Contact No.:	0731466352	Fax No:	047 537 0956
Email address:	marangeisrael@yahoo.com.		

2. DETAILS OF THE PROPOSED RESEARCH

2.1	Level of Study (place an "X" in the appropriate column)	
Honours	*Masters*	*Doctorate*
	X	
Other (specify):		

2.2	**Full title of Thesis / Dissertation / Research Project (attach detailed research proposal)** Application will not be considered if proposal is not attached
GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11 : A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT	

2

2.3	**Student and Postgraduate Enrolment Particulars (if applicable)**
Name of institution where enrolled:	WALTER SISULU UNIVERSITY
Faculty:	EDUCATION
Department:	MATHS AND SCIENCE
Name of Supervisor:	DR. J.K ALEX

3. **RESEARCH INFORMATION**

3.1. District where research will be undertaken:

Institutions where research will be undertaken

Name of Institution	*Type of Institution (primary school, secondary school, technical school, ECD centre, LSEN, FET college)*	*District*
HOLY CROSS HIGH SCHOOL	SECONDARY SCHOOL	OR TAMBO INLANDS

If Head Office/s (Please indicate Chief Directorate/s and Directorates)

3.2. Total number of learners and staff to be involved:

	Learners	Educators	Principals	Support Staff	Administrative Staff	Lecturers	Other:
Number	108	2					

3

3.3. Time of day that you propose to conduct your research. *Please mark with an "X".*

School Hours	*During Break*	*After School Hours*
X		X

3.4. Did you receive a bursary from the ECDOE? Yes/No
NO

3.5. Expected date of commencement of study (DD/MM/YYYY): As soon as possible

3.6. Expected date of completion of study (DD/MM/YYYY): MAY 2019

SECTION B
TO BE COMPLETED BY THE UNIVERSITY/INSTITUTION WHERE THE RESEARCHER IS REGISTERED FOR RESEARCH

University application to access Eastern Cape schools for research purposes

This form is to be completed in any of the following three cases:

A. Student undertaking an M.Ed or PhD within Eastern Cape school/s
B. Academic undertaking research in Eastern Cape school/s
C. Academic applying for group project undertaken by a number of students within a particular programme in Eastern Cape schools (for example for Honours level research project)

This form is not to be completed in the case of research undertaken outside of a University structure.

1) Name of university	WALTER SISULU UNIVERSITY	
2) Type of application (See above and indicate one)	A. Student	✓
	B. Academic	
	C. Group project	
3) Name of student/academic/ group project coordinator	ISRAEL JEUKAI MARANGE	
4) Student number/ Staff number	217303404	
5) Qualification (where applicable, or indicate if not for qualification purposes)	MASTER OF EDUCATION	

6) Title of research: GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11 : A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT.

4

7) Supervisor/s' names (where applicable)	Dr JK Alex
8) Contact email for (A) supervisor, or (B) academic researcher, or (C) programme coordinator (as applicable)	jalex@wsu.ac.za

The completion of this form indicates that the university's processes for proposal approval by the Higher Degrees Committee and Ethical clearance have been followed.

Reference number and documentary proof of Ethical Clearance: Ref number: EREC22-09-18 (proof must be attached)

Reference number and documentary proof of approval by Higher Degrees Committee: Ref no: EREC22-09-18 (proof must be attached)

This entailed ensuring that the proposed research meets the criteria of, inter alia:

- Sensitivity - towards participants and institutions, including issues of informed consent and ethical considerations around beneficence and non-maleficence;
- Significance – that the study has merit and meaning and has a contribution to make;
- Accountability – that the researcher understands the responsibilities associated with research in schools and takes issues of validity, reliability and trustworthiness into account;
- Appropriateness – that the research design is aligned to its intentions and to the context of the study.

Date	20/11/2018
University Research Office stamp / signature	WALTER SISULU UNIVERSITY WSU FACULTY OF EDUCATIONAL SCIENCES DEPARTMENT : CPTD 20 NOV 2018 MTHATHA CAMPUS PRIVATE BAG X 1, NMD, 5117 0475022541

5

5. **COMMITMENT FORM FOR CONDUCTING RESEARCH IN THE EASTERN CAPE DEPARTMENT OF BASIC EDUCATION**

I, (Title, surname and names in full) MR MARANGE ISRAEL JEUKAI
residing at (Full address)

17 DDT JABAVU STREET SOUTHRIDGE PARK MTHATHA

commit myself to the following 9 items regarding my research:

1. To effect no changes with respect to my questionnaire/method of work after having my research application approved by the Department. Any changes I might make shall be submitted to the Department for approval.
2. That I am prepared on request of the department, at my own cost, do a presentation to one preferred audience, once off.
3. That, after having obtained permission to continue with my research project from the Department, I shall negotiate with the relevant areas and/or schools regarding final arrangements for visits.
4. That I will not to use the Department's written letter of consent as a means of making unreasonable demands on an office/institution.
5. To involve persons in my research project on an absolutely voluntary basis – these persons being all those concerned (including pupils) and all others associated with the Department as well as with all offices/institutions under the control of the Department. Parental/community approval shall be obtained should such a measure be prescribed by the Department.
6. Not to remove files/records/documents from the offices and institutions of the Department should information contained in these files/records/documents be needed; to obtain such information under the supervision of a Departmental official assigned by the Department; and to select only information applicable to my research project.
7. To present the Department with a copy of my final paper/report/dissertation/thesis free of charge in hard copy and electronic format.
8. Not to visit (conduct research or any field work) at institutions (schools) during the fourth school term unless permission has been granted.
9. To allow the research to be published on the Departmental website.

SIGNATURE OF APPLICANT: [signature]

PRINT NAME IN FULL: ISRAEL JEUKAI MARANGE

DATE: 24|10|2018

PLACE: MTHATHA

Tel no (h): 0605050830 Tel no (w): 047 537 0956

Cell no: 0731466352 Fax no.: 0475370956

FOR THE DEPARTMENTAL COMMITTEE ONLY

APPROVED/ NOT APPROVED

SIGNATURE (Department of Education): ....

NY KANJANA
DIRECTOR: STRATEGIC PLANNING POLICY RESEARCH AND SECRETARIAT SERVICES

DATE: ....

END OF DOCUMENT

6

APPENDIX C: APPROVAL FROM EASTERN CAPE DEPARTMENT OF EDUCATION FOR PERMISSIONTO CONDUCT STUDY

Province of the
EASTERN CAPE
EDUCATION

STRATEGIC PLANNING POLICY RESEARCH AND SECRETARIAT SERVICES
Steve Vukile Tshwete Complex • Zone 6 • Zwelitsha • Eastern Cape
Private Bag X0032 • Bhisho • 5605 • REPUBLIC OF SOUTH AFRICA
Tel: +27 (0)40 608 4773/4035/4537 • Fax: +27 (0)40 608 4574 • Website: www.ecdoe.gov.za

Enquiries: B Pamla Email: babalwa.pamla@ecdoe.gov.za Date: 07 February 2019

Mr Israel Yeukani Marange

17 DDT Jabavu Street

South Park

Mthatha

5100

Dear Mr Marange

PERMISSION TO UNDERTAKE A MASTERS' STUDY: GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11. A CASE STUDY OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT.

1. Your application to conduct the above mentioned research from one selected Secondary School under the jurisdiction of Sarah Baaartman District of the Eastern Cape Department of Education (ECDoE) is hereby approved based on the following conditions:

   a. there will be no financial implications for the Department;

   b. institutions and respondents must not be identifiable in any way from the results of the investigation;

   c. you seek parents' consent for minors;

   d. it is not going to interrupt educators' time and task;

   e. you present a copy of the written approval letter of the Eastern Cape Department of Education (ECDoE) to the Cluster and District Directors before any research is undertaken at any institutions within that particular district;

   f. you will make all the arrangements concerning your research;

   g. the research may not be conducted during official contact time;

PGDP EASTERN CAPE 2004 2014

h. should you wish to extend the period of research after approval has been granted, an application to do this must be directed to Chief Director: Strategic Management Monitoring and Evaluation;

i. your research will be limited to those institutions for which approval has been granted, should changes be effected written permission must be obtained from the Chief Director: Strategic Management Monitoring and Evaluation;

j. you present the Department with a copy of your final paper/report/dissertation/thesis free of charge in hard copy and electronic format. This must be accompanied by a separate synopsis (maximum 2 – 3 typed pages) of the most important findings and recommendations if it does not already contain a synopsis.

k. you present the findings to the Research Committee and/or Senior Management of the Department when and/or where necessary.

l. you are requested to provide the above to the Chief Director: Strategic Management Monitoring and Evaluation upon completion of your research.

m. you comply with all the requirements as completed in the Terms and Conditions to conduct Research in the ECDoE document duly completed by you.

n. you comply with your ethical undertaking (commitment form).

o. You submit on a six monthly basis, from the date of permission of the research, concise reports to the Chief Director: Strategic Management Monitoring and Evaluation

2. The Department reserves a right to withdraw the permission should there not be compliance to the approval letter and contract signed in the Terms and Conditions to conduct Research in the ECDoE.

3. The Department will publish the completed Research on its website.

4. The Department wishes you well in your undertaking. You can contact the Director, Ms. NY Kanjana on the numbers indicated in the letterhead or email nelisa.kanjana@ecdoe.gov.za should you need any assistance.

ANJANA
CTOR: STRATEGIC PLANNING POLICY AND RESEARCH
SUPERINTENDENT-GENERAL: EDUCATION

APPENDIX D: REQUEST TO CONDUCT RESEARCH AT YOUR SCHOOL

17 DDT Jabavu Street
Southridge Park
Mthatha

2 August 2018

The Principal
Holy Cross High School
101 Chief Jojo Street
Southridge Park
Mthatha

RE: Request to conduct research at your school

Dear Principal

My name is Israel Yeukai Marange. I am registered with Walter Sisilu University (WSU) for a degree of Master of Education (MEd), with a specialisation in Mathematics Education. My supervisor is Dr J.K. Alex. As part of completing the MEd, I am required to conduct a school-based research. The topic of my research is, "GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11: A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT". The purpose of my research is to work with a group of Grade 11 Mathematics learners to determine the effectiveness of mathematical software called GeoGebra when learning Circle Geometry.

Learners will write a pre-test; post-test and they will complete a questionnaire. The collection of data for the research will take place in August 2018 to July 2019. The aim of this research is to contribute in improving the performance of Grade 11 learners in mathematics. In case you decide to allow your school to participate in this research, the researcher will call a meeting in which the objectives of the research will be explained. The participation in this research is voluntary. There shall be no incentives given to those who are chosen to participate in this research. If your school, or a child from your school, decides to withdraw from participation during the course of this research, there will be no penalty incurred.

There are no foreseen risks to those who decide to participate in this research. All names of participants and those of participating school will not be revealed, and pseudonyms will be used instead. Please sign and complete the consent slip below and return it to me if you agree. Your assistance is greatly appreciated.

Yours faithfully

Mr I.Y. Marange

APPENDIX E: SCHOOL PERMISSION TO CONDUCT RESEARCH

HOLY CROSS HIGH SCHOOL

HOLY CROSS HIGH SCHOOL

RUN TO WIN

TEL: 047 537 0956

FAX: 047 537 0956

101 CHIEF JOJO DRIVE

SOUTHRIDGE PARK

Permission to conduct research

Dear Mr Marange, I.Y

I, MADIKIZELA W, the Principal of HOLY CROSS HIGH SCHOOL, agree that I have read and understood the content of the letter that was sent to me. I have read and understood the purpose of the research entitled: GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11: A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT. I therefore **agree/ do not agree** to allow my school to participate in the research.

Signature of the Principal: ____________________________

Date: 10/08/2018

Signature of researcher:

APPENDIX F: INFORMED CONSENT FORM

My name is Israel Yeukai Marange, a Master of Education student at Walter Sisulu University. I am currently a teacher at Holy Cross High School, and I am carrying a research entitled, "GEOGEBRA AS A MANIPULATIVE TOOL IN PROVIDING PROCESSES OF CIRCLE GEOMETRY IN GRADE 11: A CASE OF ONE SCHOOL IN OR TAMBO INLAND DISTRICT". I am gathering data for my study using tests and questionnaires so I would appreciate your participation throughout this process. The whole process is expected to take 16 weeks and will take place at Holy Cross High School.

Data obtained will be confidentially treated and there will be no mention of names. The only people to have access to the data will be me and the supervisor. I will also honestly report your performance and responses. Participation is voluntary and you can withdraw from participating at any stage of the interview process.

For any questions feel free to contact me or the school principal. Thank you.

Parent's signature: ________________________ Date: __________________

Learner's signature: _______________________ Date: __________________

Yours sincerely

I.Y Marange

APPENDIX G: Grade 11 Examination Guidelines

EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of Trigonometry and Euclidean Geometry.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. Candidates must know the formulae for the surface area and volume of the right prisms.
4. If the question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.
5. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)
   - The opposite angles of a cyclic quadrilateral are supplementary
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment
6. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circles
   - Equal chords subtend equal angles at the circumference of a circle
   - Equal chords subtend equal angles at the centre of a circle
   - In equal circles equal chords subtend equal angles at the circumference
   - In equal circles equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic
   - Tangents drawn from a common point outside the circle are equal in length
7. The theory of quadrilaterals will be integrated into questions in the examination.
8. Concurrency theory is excluded.

APPENDIX H: PRE- AND POST-TEST AND MEMO: GRADE 11 CIRCLE GEOMETRY

Pre- and Post-Test: Grade 11 Circle Geometry

Time: 1Hr Marks: 50

Instructions

1. Answer ALL the questions.
2. Use the answer book provided.
3. You may use a calculator.

Question 1

Complete the following statements by filling in the missing word(s) so that the statements are CORRECT.

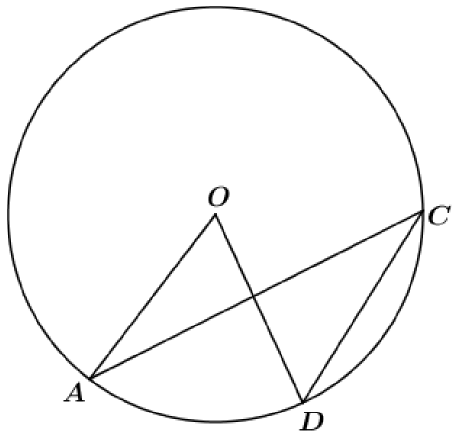
1.1 The angle subtended by a chord at the centre of a circle is ………………………….. (1)

1.2 The angle between the tangent and a chord is ……………………………….. (1)

1.3 The opposite angles of a cyclic quadrilateral are ……………………………… (1)

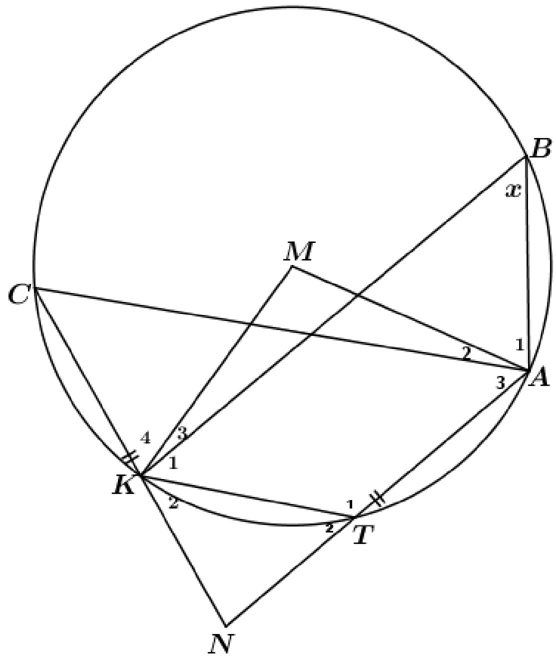
Question 2

2.1 In the diagram, O is the centre of a circle and A, D and C are points on the circle. Use Euclidean Geometry methods to prove the theorem which states that: $A\hat{O}D = 2A\hat{C}D$



(6)

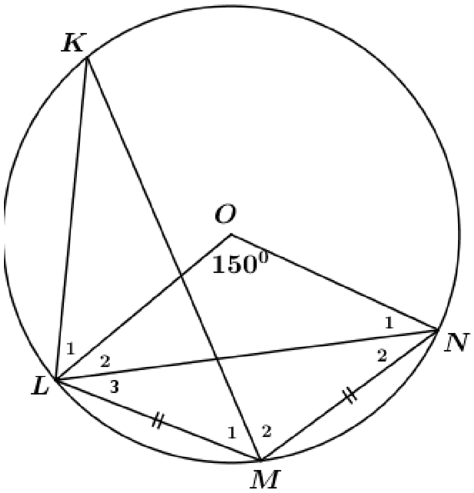
2.2 In the diagram below, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also $NA = NC$ and $\hat{B} = x$



2.2.1 Name, with reasons, three other angles equal to x. (3)

2.2.2 Determine in terms of x the size of the angle $K\hat{M}A$. (2)

In the diagram below, O is the centre of the circle. K, L, M and N are points on the circumference of the circle such that $LM = MN$. $L\hat{O}N = 150^0$



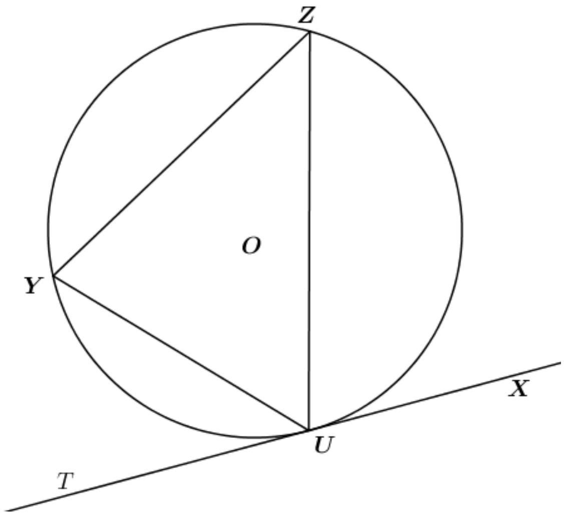
Determine, with reasons, the values of the following;

2.3.1 $L\hat{M}N$ (3)

2.3.2 $L\hat{K}M$ (3)

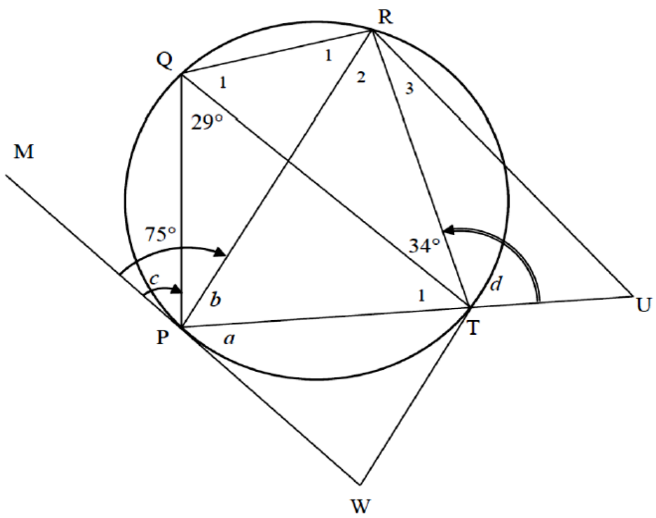
Question 3

3.1 In the diagram below O is the centre of the circle UYZ. XUT is a tangent to the circle at U.



Prove that $T\hat{U}Y = \hat{Z}$

3.2 In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. $M\hat{P}R = 75^0, P\hat{Q}T = 29^0; Q\hat{T}R = 34^0$.

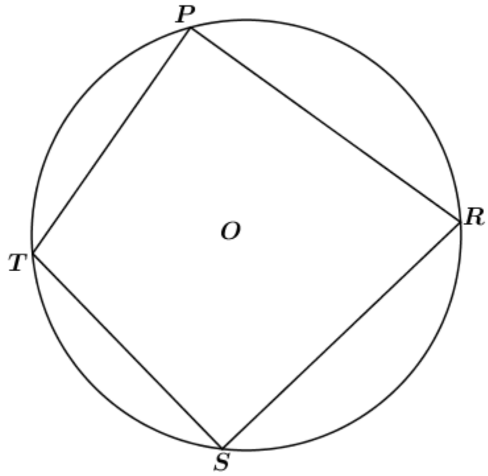


Calculate, with reasons, the values of the angles labelled: a, b, c and d.

Question 4

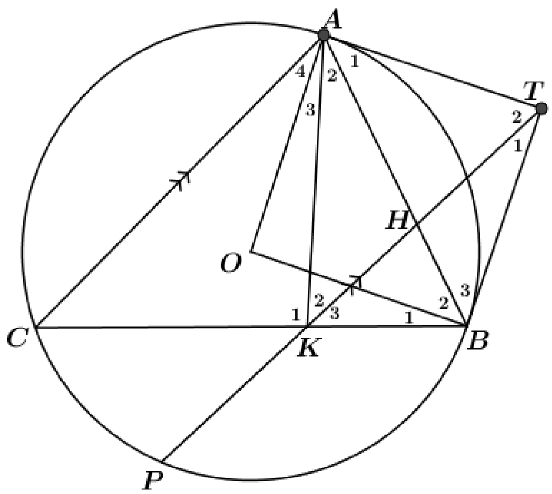
4.1 In the figure below O is the centre of the circle and $PRST$ is a cyclic quadrilateral.

Prove the theorem that states $P\hat{R}S + P\hat{T}S = 180^0$.



(5)

4.2 In the sketch TA and TB are tangents to the circle with the centre O. THP is a secant and chord AC is parallel to PT. PT cuts AB and BC in H and K respectively.



Prove that:

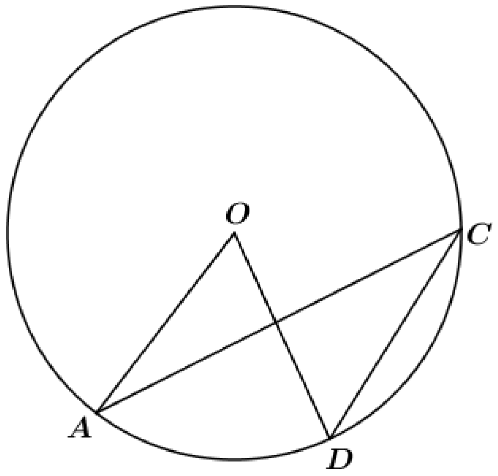
4.2.1 $AOBT$ is a cyclic quadrilateral (5)

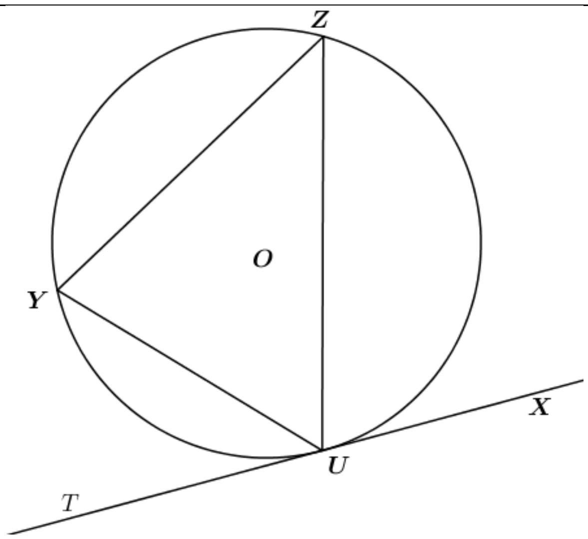
4.2.2 $T\hat{K}B = T\hat{A}B$ (4)

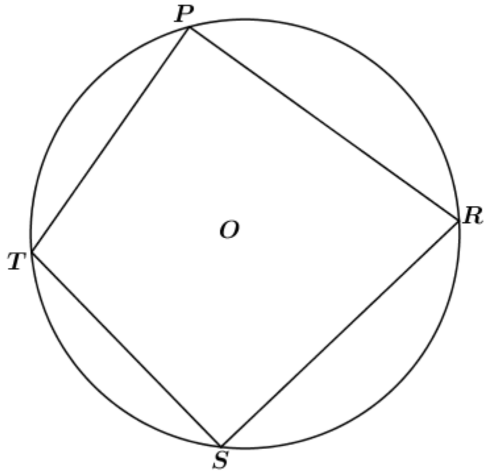
4.2.3 $AKBT$ is a cyclic quadrilateral (2)

[50]

Grade 11 Circle Geometry Mathematics Memorandum (50 Marks)

Question 1		
1.1	Twice the angle subtended by the same chord at the circumference.	✓(1)
1.2	Equal to the angle subtended by the same chord in the alternate segment.	✓(1)
1.3	Supplement.	✓(1)
Question 2		
2.1	 Construct OC and produce it to B Let $\angle AOB = O_1$ and $\angle DOB = O_2$. $O_1 = \angle ACO + \angle CAO \quad (\text{ext.} \angle\Delta = \text{sum int. opp.} \angle\text{s})$ $\text{but } \angle ACO = \angle CAO \quad (\text{equal radii}, OA = OC)$ $O_1 = \angle ACO + \angle ACO$ $O_1 = 2\angle ACO.$ Similarly, we can also show that $O_2 = 2\angle DCO.$ $\angle AOD = O_2 - O_1$ $= 2\angle DCO - 2\angle ACO$ $= 2(\angle DCO - \angle ACO)$ $= 2\angle ACD$	✓OC to B ✓O_1 ✓Reason ✓O_2 ✓Reason ✓ $\angle AOD = O_2 - O_1$ (6)
2.2.1	$T_2 = B = x \quad (Ext \angle \text{ of a cyclic quad})$ $C = B = x \quad (\text{sub. by the same chord})$ $A_3 = C = B = x \ (NA = NC \text{ or angles opp} = \text{sides})$	S/R✓ S/R✓ S/R✓(3)
2.2.2	$\angle KMA = 2x \quad (\text{angle at the centre})$	S✓R✓(2)

2.3.1	$\angle LMN$ $O_1 + \angle LON = 360^0$ (Revolution angles) $O_1 = 360^0 - 150^0$ $O_1 = 210$ but $O_1 = 2\angle LMN$ (angle at the centre) $\therefore \angle LMN = 105^0$	S/R✓ O_1 ✓ answer✓ (3)
2.3.2	$N_1 = L_1 = 37.5^0$ (angles opp. equal sides.) but $N_1 = k = 37.5^0$ (sub. by the same chord ML)	S/R✓ S/R✓✓(3)
Question 3		
3.1	 Construct diameter UW and join W to Y Let $\angle YWU = W_1$ $\angle YUT + \angle YUW = 90^0$ (tangent $\perp$ radius) $\angle \text{UYW} = 90^0$ ($\angle$ in semi circle) $\angle \text{YUW} + \text{W}_1 = 90^0$ ($\angle$ sum of ΔUYW) $\angle \text{YUT} = \text{W}_1$ but $Z_1 = W_1$ ($\angle$s in the same segment) $\text{Z}_1 = \angle YUT$	✓UW produced to W and Y ✓ $\angle YUT + \angle YUW = 90^0$ ✓ Reason ✓ $\angle \text{UYW} = 90^0$ /R ✓ $Z_1 = W_1$ /R ✓ $\text{Z}_1 = \angle YUT$ (6)
3.2	$a = 29^0$ (tan - chord theorem) $b = 76^0$ (ext $\angle$ of a cyclic quad the sub. by chord RT) $c = 41^0$ (75 - 34 = c, sub. by same chord QR) $d = 105^0$ (tan - chord thm and $\angle_s$ in a straight line)	S/R✓✓ S/R✓✓ S/R✓✓ S/R✓✓(8)

Question 4		
4.1	 Construct OP and OS and label O_1 and O_2 $O_1 = 2\angle SRP$ ($\angle$ at the centre) $O_2 = 2\angle STP$ ($\angle$ at the centre) and $O_1 + O_2 = 360^0$ ($\angle$s around a point) $2\angle SRP + 2\angle STP = 360^0$ $\angle SRP + \angle STP = 180^0$	✓construction ✓O_1 and reason ✓O_2 and reason ✓reason ✓conclusion (5)
	$\angle OAT = 90^0 (OA \perp AT)$ $\angle OBT = 90^0$ $(OB \perp BT)$ $\angle OAT + \angle OBT = 180^0$ $90^0 + 90^0 = 180^0$ $180^0 = 180^0$ $AOBT$ is a cyclic quadrilateral (opp. $\angle$s are supp.)	✓✓S/R ✓✓S/R ✓R (5)
4.2.2	$A_1 = C$ (tan - chord thm) $C = K_3$ (corresp. $\angle$s, CA//PT) $A_1 = K_3$ (both = C)	✓✓S/R ✓S/R ✓S (4)
4.2.3	$K_3 = A_1$ (sub. by chord BT) $AKBT$ is a cyclic quadrilateral ($K_3 = A_1$)	✓S ✓R (2)
	MARKS	**[50]**

APPENDIX I: PILOT STUDY MARKSHEET

No.	Learner code	Pre-Test Mark	Post-Test Mark
1	PS001	02	04
2	PS002	02	10
3	PS003	02	13
4	PS004	01	08
5	PS006	04	08
6	PS007	03	17
7	PS008	03	20
8	PS009	00	06
9	PS010	04	17
10	PS011	02	12
11	PS012	04	02
12	PS013	01	30
13	PS014	04	08
14	PS017	03	12
15	PS018	00	33
16	PS019	00	22
17	PS020	00	15
18	PS021	02	07
19	PS022	00	09
20	PS023	05	05
21	PS024	11	14
22	PS025	01	00
23	PS026	06	02
24	PS027	00	14
25	PS028	03	07
26	PS031	07	23
27	PS032	00	10
28	PS033	00	22
29	PS034	08	41
30	PS035	04	02
31	PS036	06	08
32	PS037	06	01
33	PS038	01	03
34	PS039	04	08
35	PS040	01	02
36	PS041	01	44
37	PS042	03	09
38	PS043	05	27
39	PS044	03	13
40	PS045	01	01
41	PS046	01	16
42	PS047	02	06
43	PS048	05	13

APPENDIX J: EXPERIMENTAL GROUP MARKSHEET

No.	Learner code	Pre-Test Mark	Post-Test Mark
1	EG001	00	00
2	EG002	01	02
3	EG003	02	12
4	EG004	02	04
5	EG005	01	01
6	EG006	00	13
7	EG007	01	06
8	EG008	01	02
9	EG009	00	12
10	EG010	00	18
11	EG011	01	03
12	EG012	03	29
13	EG013	00	01
14	EG014	08	30
15	EG015	00	01
16	EG016	00	01
17	EG017	00	03
18	EG018	00	17
19	EG019	01	37
20	EG020	00	15
21	EG021	00	02
22	EG022	00	25
23	EG023	00	02
24	EG024	00	03
25	EG025	00	08
26	EG026	01	06
27	EG027	00	04
28	EG028	00	04
29	EG029	02	08
30	EG030	00	07
31	EG031	00	06
32	EG032	00	05
33	EG033	00	04
34	EG034	03	24
35	EG035	00	18
36	EG036	04	27
37	EG037	01	06
38	EG038	00	10
39	EG039	04	24
40	EG040	00	15
41	EG041	00	27
42	EG042	01	02
43	EG043	00	09

44	**EG044**	**00**	**11**
45	**EG045**	**02**	**18**
46	**EG046**	**01**	**05**
47	**EG047**	**00**	**12**
48	**EG048**	**01**	**32**
49	**EG049**	**00**	**07**
50	**EG050**	**00**	**30**
51	**EG051**	**02**	**16**
52	**EG052**	**00**	**04**
53	**EG053**	**01**	**01**
54	**EG054**	**03**	**30**
55	**EG055**	**00**	**07**
56	**EG056**	**00**	**08**
57	**EG057**	**00**	**15**
58	**EG058**	**00**	**05**
59	**EG059**	**02**	**23**
60	**EG060**	**01**	**03**

APPENDIX K: CONTROL GROUP MARKSHEET

No.	Learner Code	Pre-Test Marks	Post-Test Marks
1	CG01	00	02
2	CG02	00	02
3	CG03	01	02
4	CG04	00	09
5	CG05	00	02
6	CG06	00	00
7	CG07	00	08
8	CG08	00	03
9	CG09	00	03
10	CG10	00	03
11	CG11	00	05
12	CG12	00	02
13	CG13	00	08
14	CG14	01	01
15	CG15	00	03
16	CG16	01	00
17	CG17	00	05
18	CG18	08	01
19	CG19	00	31
20	CG20	00	00
21	CG21	00	10
22	CG22	00	03
23	CG23	00	02
24	CG24	00	03
25	CG25	00	01
26	CG26	02	06
27	CG27	00	06
28	CG28	00	02
29	CG29	00	02
30	CG30	00	01
31	CG31	00	03
32	CG32	00	06
33	CG33	00	01
34	CG34	01	01
35	CG35	00	06
36	CG36	00	00
37	CG37	00	07
38	CG38	00	01
39	CG39	00	01
40	CG40	00	02
41	CG41	01	03
42	CG42	00	08
43	CG43	02	03

44	**CG44**	**00**	**05**
45	**CG45**	**00**	**00**
46	**CG46**	**00**	**03**
47	**CG47**	**00**	**03**

APPENDIX L: EXPERIMENTAL GROUP'S QUESTIONNAIRE

Learners' perceptions towards GeoGebra as a tool for learning Circle Geometry

TOPIC: CIRCLE GEOMETRY	PARTICIPANT No.: EG

The researcher is interested in your opinion of using GeoGebra as a tool to learn circle geometry.

Part A – Demographic data (Use ✓in the appropriate box)

Gender	Female	
	Male	

Age	13-15	
	16-17	
	18+	

Race	Black	
	Coloured	
	White	
	Indian	

Part B – Usability of the GeoGebra in the learning of Circle Geometry

This questionnaire is not something to be graded and your answers are completely anonymous. Please tell the researcher what you *really* think by putting an ✓ in the box corresponding to **Strongly Agree (SA), Agree (A), May Agree (MA), Disagree (D), or Strongly Disagree (SD)**. Thank you for your help!

(i)	Learner's views of mathematics					
		SA	A	MA	D	SD
1	I like mathematics					
2	I like the section of Circle Geometry in mathematics					
3	In Circle Geometry you can be creative and discover things by yourself					
4	Memorisation helps me understand theorems of Circle Geometry					

5	I prefer to learn Circle Geometry using GeoGebra					
(ii)	**Learners' views of GeoGebra advantages in learning Circle Geometry**					
		SA	A	MA	D	SD
6	I think GeoGebra is the best tool to learn Circle Geometry					
7	I participate more in class when learning Circle Geometry using GeoGebra.					
8	GeoGebra has motivated me to learn more in Circle Geometry					
9	My teacher is competent to teach Circle Geometry using GeoGebra					
10	I think learning Circle Geometry with GeoGebra can improve my marks in Geometry					
11	I had problems with Circle Geometry before exposed to GeoGebra but now I think I will approach the topic with confidence					
12	I enjoyed myself when learning Circle Geometry using GeoGebra					
13	I recommend GeoGebra when learning Circle Geometry					
14	After learning Circle Geometry using GeoGebra, I think I have gained confidence in Geometry					
15	My teacher is knowledgeable in Circle Geometry using GeoGebra					
16	Calculations of unknown angles using GeoGebra helped me grasp the geometric concepts in Circle Geometry					
(iii)	**Learners' experiences of learning Circle Geometry using GeoGebra**					
		SA	A	MA	D	SD
17	When I started I had a negative attitude and did not really understand Circle Geometry					
18	I am positive that I will change my attitude towards Circle Geometry					
19	I solve Circle Geometry problems at home because there is somebody to guide me					
20	Teacher's attitude towards Circle Geometry contribute to my academic performance					
21	Extra support from the teacher affected my performance in Circle Geometry					

22	I am willing to learn on my own when learning Circle Geometry					
23	Working in small groups can improve my performance in Circle Geometry					
24	Hands-on activities in Circle Geometry helped me understand the topic					
25	Identification of properties of Geometrical figures improved my performance in Circle Geometry					
26	I'm able to form better connections between previous learning and new learning					
27	I benefited a lot from the teacher-student interaction					

APPENDIX M: CONTROL GROUP'S QUESTIONNAIRE

<u>Learners' perceptions towards learning Circle Geometry using conventional methods</u>

TOPIC: CIRCLE GEOMETRY	**PARTICIPANT No.: CG**

The researcher is interested in your opinion of using Traditional approach when learning Circle Geometry.

Part A – Demographic data (Use ✓in the appropriate box)

Gender	Female	
	Male	

Age	13-15	
	16-17	
	18+	

Race	Black	
	Coloured	
	White	
	Indian	

Part B – Usability of the conventional methods in the learning of Circle Geometry

This questionnaire is not something to be graded and your answers are completely anonymous. Please tell the researcher what you *really* think by putting an ✓ in the box corresponding to **Strongly Agree (SA), Agree (A), May Agree (MA), Disagree (D), or Strongly Disagree (SD)**. Thank you for your help!

(i)	**Learner's views of circle geometry**					
		SA	A	MA	D	SD
1	I like mathematics					
2	I like the section of Geometry of Circle in mathematics					
3	In Circle Geometry you can be creative and discover things by yourself					
4	Memorisation helps me understand theorems of Circle Geometry					

5	I prefer to learn Circle Geometry using conventional method and not alternative methods					
(ii)	**Learners' views of conventional methods in learning Circle Geometry.**					
		SA	A	MA	D	SD
6	I think the conventional method is the best method to learn Circle Geometry					
7	I participate more in class when learning Circle Geometry using conventional methods					
8	Conventional teaching methods have motivated me to learn more in Circle Geometry					
9	My teacher is competent to teach Circle Geometry					
10	I think continuing to learn Circle Geometry using conventional methods will improve my marks in Geometry					
11	I had problems with Circle Geometry but now I think I will approach the topic with confidence					
12	I enjoy myself when learning Circle Geometry using conventional methods					
13	I recommend current teacher-centred approach when learning Circle Geometry					
14	Learning Circle Geometry using conventional methods helps me gain confidence in Geometry					
15	My teacher is knowledgeable in Circle Geometry					
16	Calculations of unknown angles using conventional methods helps me grasp the geometric concepts in Circle Geometry					
(iii)	**Learners' experiences of learning Circle Geometry using conventional methods**					
		SA	A	MA	D	SD
17	I have a negative attitude and do not really understand Circle Geometry					
18	I am positive that I will change my attitudes towards Circle Geometry					
19	I solve Circle Geometry problems at home because there is somebody to guide me					
20	Teacher's attitude towards Circle Geometry contribute to my academic performance					
21	Extra support from the teacher affect my performance in Circle Geometry					

22	I am willing to learn on my own when learning Circle Geometry					
23	Working in small groups can improve my performance in Circle Geometry					
24	Hands-on activities in Circle Geometry help me understand the topic					
25	Identification of properties of Geometrical figures improves my performance in Circle Geometry					
26	I am able to form better connections between previous learning and new learning					
27	I benefit a lot from the teacher-student interaction					