MATRIX POLYNOMIAL AND LAMBDA MATRIX

K.CHENDRAN[1],Dr.R.G.BALAMURUGAN[2],Dr.S.SANGEETHA[3],R.RAMYA[4]

Department of Mathematics

Dhanalakshmi Srinivasan College of

Arts and Science for Women (Autonomous)

Perambalur

ABSTRACT:

A matrix x is a solvent of the matrix polynomials. $f(x)=A_o x^m +\ldots..+A_m$ if $f(x) = 0$.Where $A_{o,}$ x, f are square matrices. In this paper we develop the linear algebra of matrix polynomials and solvents. we define division and interpolation the properties of matrices and define and study the existence of a complete set of solvents. we study the relation between the matrix polynomials problems and Theorems and lambda matrix problem, Which is to find scalar λ such that

$$A_o\lambda^m+A_1 \lambda^{m-1}+ \ldots\ldots. +A_m \text{ is singular.}$$

In a future paper we extend Cayley Hamilton Theorem for calculating of scalar polynomials to matrix polynomials. And lambda matrix for column and row Hermite calculating of lambda matrix.

KEYWORDS:

Matrix polynomial, n square matrix, characteristic polynomial, scalar variable

INTRODUCTION

A polynomial in the scalar variable x having coefficients which are scalars is known as a **scalar polynomial.** But there are polynomials where the variables, instead of being a scalar is an n-square matrix *A,* but the coefficients remain scalars. Then, these polynomials are called **matrix polynomials**. On the other hand, if matrices take the place of the scalar coefficients in the original scalar polynomial, then we have a polynomial in the scalar variable x having coefficients which are matrices, and we call it a **polynomial matrix (or) lambda matrix.** These two different types of polynomials involving matrices and their operational properties are the subject-matter of this section.

DEFINITION 1.1:

The matrix polynomial of degree **n** given by |A-XI | is called the “**Characteristic matrix”** of A.

DEFINITION 1.2:

The determinant **|** A-XI **|** which is ordinary polynomial in x of degree n over the field F is called the “**Characteristic polynomial”** of A.

The equation **|** A-XI **|=0** is called characteristic equation of A.

Definition 1.3:

The “**Rank of matrix”** A is common value of its row and column rank.

Matrix polynomial and lambda matrix

DEFINITION 2:1:

Considering x to be scalar variables the m^{th}degree polynomial in x denoted by f (x) is the functional relation.

$$f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0$$

Where $a_m \neq 0$ is positive integer and the coefficient ofa_0 is x^0

we shall call f(x) as a monic polynomial if the leading coefficient $a_m = 1$.

DEFINITION 2:2:

An n– square matrix A (λ) which is a polynomial in the scalar variable λ from a field F represented by

$$A(\lambda) = A_m \lambda^m + A_{m-1}\lambda^{m-1} + \ldots\ldots + A_1 \lambda + A_0$$

Where the leading co efficient $A_m \neq 0$, Ai ‘s are square matrices in $v_{n \times n}$ is a defined as a polynomial or a **lambda matrix** or simply a λ – matrix of order n and of degree m over a field F.

If the determinant of the leading coefficients is not zero. if det $A_m \neq 0$,

Then the lambda matrix is said to be regular.

THEOREM 2:1

Let two λ - matrices A (λ) and B (λ) of the same order with respective degrees m and p, $m \geq p$, be given A (λ) and B (λ) with B (λ) regular. Then the right quotient Q (λ) and the right remainder R (λ) Of A (λ) on dividing by B (λ).

$$A(\lambda) = Q(\lambda)\, B(\lambda) + R(\lambda)$$

There exist and are Unique. Similar assertion hold for the left quotient and the left remainder.

PROOF:

To prove the existence, we shall proceed as we do in the case of long division of one polynomial by another lower or equal degree.so, we pre -multiply both side B(λ) by $A_m B_p^{-1} \lambda^{m-p}$ to get

$$A(\lambda) = A_m\lambda^m + A_{m-1}\lambda^{m-1} + \cdots + A_1\lambda + A_0 \text{.......... (1)}$$

$$B(\lambda) = B_p\lambda^p + B_{p-1}\lambda^{p-1} + \cdots + B_1\lambda + B_0 \text{ .... (2)}$$

$$A_m B_p^{-1} \lambda^{m-p} B(\lambda) = A_m\lambda^m + \left(A_m Bp - 1B_{p-1}\right)\lambda^{m-1} + \cdots + A_m B_p^{-1} B_0 \lambda^{m-p} \text{ .... (3)}$$

Subtraction (3) from (1) we get,

$$A(\lambda) - A_m B_p^{-1} \lambda^{m-p} B(\lambda) = (A_{m-1} A_m B_p^{-1} B_{p-1})\, \lambda^{m-1} + \text{.............. (4)}$$

Since the first term on the right hand side (4) may be identically zero,

We get from (4),

$$A(\lambda) - A_m B_p^{-1} \lambda^{m-p} B(\lambda) = A^{(1)}(\lambda) \text{ .... (5)}$$

Where $A^{(1)}(\lambda)$ is an n-square λ - matrix with degree $m_1 < m$ and represented by,

$$A^{(1)}(\lambda) = A_{m1}^{(1)} \lambda^{m1} + \ldots\ldots + A_o^{(1)}.$$

If $m_1 \geq p$, we repeat the above process on $A^{(1)}(\lambda)$ rather than on A (λ) to get,

$$A^{(1)}(\lambda) - A_{m\,1}^{(1)} B_p^{-1} \lambda^{m1-p} B(\lambda) = A^{(2)}(\lambda) \ldots\ldots\ldots (6)$$

Where,

$A^{(2)}(\lambda) = A_{m2}^{(2)} \lambda^{m2} + \ldots\ldots\ldots\ldots + A\,o^{(2)}$. Is again an n – square λ - matrix with degree $m_2 < m_1$. Since the degree of $A(\lambda)$, $A^{(1)}(\lambda)$, $A^{(2)}(\lambda)$ are the decreasing progressively, we finally arrive after a finite number of (r + 1) steps at a polynomial $A^{(r+1)}(\lambda)$ with degree $m_{r+1} < p$ satisfying

$$A^{(r)}(\lambda) - A_{mr}^{(r)} B_p^{-1} \lambda^{m_r - p} B(\lambda) = A^{(r+1)}(\lambda) \ldots\ldots\ldots\ldots (7)$$

Where $A_{mr}^{(r)}$ is the leading coefficients of $A^{(r)}(\lambda)$ with degree $m_r \geq p$.

Adding (5) to (6) we get

$$A(\lambda) - \{[A_m \lambda^m + A_{m1}^{(1)} \lambda^{m1} + \ldots + A_{mr}^{(r)} \lambda^{m_r}] \lambda^{-p} B_p^{-1}\} B(\lambda) = A^{(r+1)}(\lambda),$$

Where it is not difficult to identify the expression within the braces with the right quotient $Q(\lambda)$ and $A^{(r+1)}(\lambda)$ with the right remainder $R(\lambda)$ in,

$$A(\lambda) = Q(\lambda) B(\lambda) + R(\lambda) \ldots\ldots\ldots\ldots (8)$$

To prove, Uniqueness, We assume that $Q_1(\lambda)$ is the another right Quotient and $R_1(\lambda)$ another right remainder so that,

$$A(\lambda) = Q_1(\lambda) B(\lambda) + R_1(\lambda) \ldots\ldots\ldots\ldots (9)$$

Subtracting (8) and (9), we obtain,

$$[Q(\lambda) B(\lambda) + R(\lambda)] = [Q_1(\lambda) B(\lambda) + R_1(\lambda)]$$

$$[Q(\lambda) B(\lambda) - Q_1(\lambda) B(\lambda)] = [R_1(\lambda) - R(\lambda)]$$

$$[Q(\lambda) - Q_1(\lambda)] B(\lambda) = [R_1(\lambda) - R(\lambda)] \ldots\ldots\ldots\ldots (10)$$

Where the degree of λ -matrix on the right hand side is the than that of $B(\lambda)$.

So if $Q(\lambda) \neq Q_1(\lambda)$.

We arrive at a contradiction in view of the **Equality of two lambda matrices.** Regarding the equality of two λ -matrix in (10)

$$Q(\lambda) = Q_1(\lambda).$$

Consequently, $R_1(\lambda) = R(\lambda)$.

Similar proof can be given for the left quotient and left remainder.

In general, the left quotient and the right quotient and the left remainder and right remainder are different.

THEOREM 2:2

REMANIDER THEOREM:

Statement:

Suppose A (λ), an n – square λ - matrix of degree m described by,

$$A(\lambda) = A_m\lambda^m + A_{m-1}\lambda^{m-1} + \cdots + A_1\lambda + A_0 \text{……….} \quad (1)$$

And B an n – square matrix, are given. Then there exist a unique n – square λ - matrix Q (λ) of degree m-1 represented by,

$$Q(\lambda) = Q_{m-1}\lambda^{m-1} + Q_{m-2}\lambda^{m-2} + \text{………} + Q_1(\lambda) + Q_0 \text{ ....} \quad (2)$$

And an n – square matrix R such that

$$A(\lambda) = Q(\lambda)(\lambda I - B) + R \quad \text{……………….} \quad (3)$$

Where R = A_r(B) is the right hand value of A (λ) at B,

(OR)

Where R= A_r(B) is the right value of $A(\lambda)$ at B,

(OR)

Where R = A_L (B) is the left value of $A(\lambda)$ at B, and Q (λ) is an n – square λ - matrix of degree (m-1).

PROOF:

Subtracting in (3) equation the expressions for A (λ)and Q (λ) from (1) and (2) respectively, we get

$A_m \lambda^m + A_{m-1}\lambda^{m-1} + \cdots + A_1 \lambda + A_0 =$ $Q_{m-1} \lambda^{m-1} +$ $Q_{m-2} \lambda^{m-2} +$ ……… $+ Q_1 (\lambda) + Q_0 \quad (\lambda I - B) + R.$

In which equating coefficients of like powers of λ in view of definition $(\lambda I - A) X = \theta$ for equality of two λ - matrices we have,

$A_m = Q_{m-1}$

$A_{m-1} = - Q_{m-1} B + Q_{m-2}$

$A_1 = - Q_1 B + Q$ …………

$A_O = - Q_O B + R$ ……………… (4)

Hence , since A_i's and B are given, Q_i's can be determined uniquely from the first m equations .post multiplying equation (4) successively from below by $I, B, B^2 \ldots B^{m-1}$ B^m and adding we get,

$A_m B^m + A_{m-1} B^{m-1} + \cdots + A_1 B + A_0 = R$

Hence, $R = A_r(B)$.

Similarly, the left quotient $Q(\lambda)$ can be uniquely determined with $R = A_L(B)$.

CONCLUSION:

In the project briefly discussed matrix polynomial and λ matrix. Also we have discussed some basic definition and some important derivates, Examples related theorem, and then we have discussed about the solution of the matrix polynomial equations, λ matrix equation and confluent matrix polynomials and λ matrix.

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