OPTIMIZATION OF INVENTORY MODEL COST PARAMETERS AS FUZZY NUMBERS

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ABSTRACT

In general, the demand rate and the unit cost of the items remains constant inspite of lot size in inventory models. But in reality, the demand rate and the unit cost of the items are connected together. In this research, demand dependent unit cost inventory model is considered where different cost parameters, maximum inventory and the lot size of the model are taken under fuzzy environment. First an analytic solution of the crisp model is obtained by the method of calculus where the inventory parameters are exact and deterministic. Later, the problem is developed with fuzzy parameters where inaccuracy has been introduced through triangular membership function.

KEYWORDS

Demand dependent on unit cost,Triangular fuzzy number,Trapezoidal.

INTRODUCTION

Inventory management is an efficient approach to sourcing, storing, and selling inventory. Most commonly, an optimum Economic order quantity model is assumed with fixed demand rate.But in reality, this assumption is quite impossible. If the demand is high, the production increases and hence the price of an item will reduce in nature. Hence the demand rate is inversely related to the unit prize of an item. With this assumption, Cheng (2) developed an inventory model and applied Geometric Programming approach to optimize it. Jung and Klein (4) also used the same concept to solve a profit maximization EOQ model using GP technique.

Manna and Chaudhuri(7) developed EOQ models with demand rate depending on both items availability and advertising expenditures. An EOQ model with demand dependent on stock for deteriorating items with partial backordering is proposed by Yang, Zeng and Cheen.(10). Lee and Yao (5) also formulated a demand dependent on stock with partial backordering and a controllable deterioration rate. Mandal, Bhunia and Maiti(6) also formulated inventory model with deteriorating products.

In most of the research works, the demand rate is assumed to be dependent on both the stock level and the selling price. In this investigation demand dependent on unit cost is considered.

FUZZY PRELIMINARIES

The uncertainty of a decision making process can be portrayed by a tool called Fuzzy set theory. Triangular and Trapezoidal fuzzy numbers can be used to fuzzify the input parameters and decision variables.

DEFINITION 2.1

A fuzzy set $\bar{A}$ in X (Universe set) is characterized by a membership function, which is associated with each element $x \in [0,1]$. The function value of $\mu_{\bar{A}}(x)$ is termed as the grade of membership of x in $\bar{A}$.

DEFINITION 2.2

The membership function of a triangular fuzzy number A, which is determined by (a_1,a_2,a_3) of crisp numbers such that $a_1<a_2<a_3$ is defined by

$$\mu_A(x) = \left\{ \begin{array}{l} \frac{x-a_1}{a_2-a_1} = m(x), a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} = n(x), a_2 \le x \le a_3 \\ 0, otherwise \end{array} \right\} \text{----------------}\rightarrow 1$$

DEFINITION 2.3

Let $A = (a_1,a_2,a_3)$ and $B = (b_1,b_2,b_3)$ with the following properties.

$$\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3)$$
$$-\tilde{B}=(-b_3, -b_2, -b_1)$$
$$\tilde{A}-\tilde{B}=(a_1-b_3, a_2-b_2, a_3-b_1)$$
$$\tilde{A}\bullet\tilde{B}=[\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)]$$
$$\frac{\tilde{A}}{\tilde{B}}=\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

GRADED MEAN INTEGRATION (GMI) METHOD

Decision makers mostly use crisp values rather than fuzzy values. Therefore, in order to defuzzify the fuzzy values to crisp values, Graded Mean Integration (GMI) representation method has been introduced by Chen and Hseih.

Ler A = (a_1,a_2,a_3) be a triangular fuzzy number and m^{-1}, n^{-1} are respectively the inverse functions of m and n. The graded $\boldsymbol{\alpha}$ level value of A is represented as $\frac{\alpha[m^{-1}(\alpha)+n^{-1}(\alpha)]}{2}$. Then the GMI representation of fuzzy number A can be found as

$$\theta(A)=\frac{\int_0^1 \frac{\alpha[m^{-1}(\alpha)+n^{-1}(\alpha)]}{2}d\alpha}{\int_0^1 \alpha d\alpha}$$

$$=\int_0^1 \alpha[m^{-1}(\alpha)+n^{-1}(\alpha)]d\alpha \text{ ---------------------} \rightarrow 2$$

Hence $\theta(A)=\frac{1}{6}(a_1+4a_2+a_3)$

CRISP AND FUZZY INVENTORY MODEL

The scope of this research work is to minimize the total annual cost of an inventory model by considering the following characteristics.

The setup costs, holding costs and the penalty costs are the input parameters.

Lot size, maximum inventory level and the unit price of an item are the decision variables.

NOTATIONS

TC – Total annual cost
Q –Order size
I – Maximum inventory level
S – Set up cost per cycle
D – Demand rate per cycle
H – Unit holding cost per item
m – Shortage cost per time
p-unit price per unit of item

EOQ MODEL

An EOQ model is formulated and solved using Karush Kuhn-Tucker conditions technique. Let p, Q and I are the decision variables to be determined, then the total annual cost of an item is given by

Total cost = Production cost + Setup cost + Holding cost + Penalty cost

$$TC = pD+\frac{SD}{Q}+\frac{I^2H}{2Q}+\frac{(Q-I)^2m}{2Q} \text{ ------------------------------} \rightarrow (1)$$

$$TC = Ap^{1-\beta} + \frac{SAp^{-\beta}}{Q} + \frac{I^2 H}{2Q} + \frac{(Q-I)^2 m}{2Q} \quad \text{-----------------------}\rightarrow(2)$$

SOLUTION ALGORITHM

This section involves the procedure to optimize the proposed model under crisp and fuzzy environment provided the real constant β is assumed to be fixed.

CRISP MODEL

An analytical solution of the crisp model is obtained by solving the equations

$$\frac{\partial T}{\partial p} = 0; \frac{\partial T}{\partial Q} = 0; \frac{\partial T}{\partial I} = 0$$

The optimum values of the unknowns are given by equations

$$p = \left[\frac{mHS\beta^2}{2A(H+m)(1-\beta)^2}\right]^{\frac{1}{2-\beta}} \quad \text{-------------------------}\rightarrow(4)$$

$$Q = \frac{S\beta}{p(1-\beta)} \quad \text{---------------------------------}\rightarrow(5)$$

$$I = \frac{Qm}{H+m} \quad \text{--}\rightarrow(6)$$

FUZZY EOQ MODEL

The input parameters namely the order cost, holding cost, penalty cost and the decision variables namely the maximum inventory level, lot size and the unit cost are assumed as triangular fuzzy numbers as follows.

Order cost: $\tilde{S} = (S-\delta_1, S, S+\delta_2), S > \delta_1$

Holding cost: $\tilde{H} = (H-\delta_3, H, H+\delta_4), H > \delta_3$

Shortage cost: $\tilde{m} = (m-\delta_5, m, m+\delta_6), m > \delta_5$

<u>Decision Variables</u>

Maximum Inventory level: $\tilde{I} = (I-\delta_7, I, I+\delta_8), I > \delta_7$

Order size: $\tilde{Q} = (Q-\delta_9, Q, Q+\delta_{10}), Q > \varphi_9$

Unit cost: $\tilde{p} = (p-\delta_{11}, p, p+\delta_{12}), p > \delta_{11}$

$$\tilde{TC} = (C_1, C_2, C_3)$$

where

$$C_1 = A(p-\delta_{11})(p+\delta_{12})^{-\beta} + \frac{A(S-\delta_1)(p+\delta_{12})^{-\beta}}{Q+\delta_{10}} + \frac{(H-\delta_3+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})} + \frac{(Q-\delta_9)(m-\delta_5)}{2}$$
$$-(I+\delta_8)(m+\delta_6)$$

$$C_2 = Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - \mathrm{Im}$$

$$C_3 = A(p+\delta_{12})(p-\delta_{11})^{-\beta} + \frac{A(S+\delta_2)(p-\delta_{11})^{-\beta}}{Q-\delta_9} + \frac{(H+\delta_4+m+\delta_6)(I+\delta_8)^2}{2(Q-\delta_9)} + \frac{(Q+\delta_{10})(m+\delta_6)}{2} - (I-\delta_7)(m-\delta_5)$$

The method of graded mean integration (GMI) is then introduced to remove the impreciseness of the total annual cost function which is given by

$$\theta(\tilde{T}C) = \frac{1}{6}\left[C_1 + 4C_2 + C_3\right]$$

$$(\tilde{T}C(\tilde{p},\tilde{Q},\tilde{I})) = \frac{1}{6}\left[\begin{array}{l} A(p-\delta_{11})(p+\delta_{12})^{-\beta} + \frac{A(S-\delta_1)(p+\delta_{12})^{-\beta}}{Q+\delta_{10}} + \frac{(H-\delta_3+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})} + \\ \frac{(Q-\delta_9)(m-\delta_5)}{2} - (I+\delta_8)(m+\delta_6) \end{array}\right]$$

$$+\frac{2}{3}\left[Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - \mathrm{Im}\right] + \frac{1}{6}\left[\begin{array}{l} A(p+\delta_{12})(p-\delta_{11})^{-\beta} + \frac{A(S+\delta_2)(p-\delta_{11})^{-\beta}}{Q-\delta_9} + \\ \frac{(H+\delta_4+m+\delta_6)(I+\delta_8)^2}{2(Q-\delta_9)} + \frac{(Q+\delta_{10})(m+\delta_6)}{2} \\ -(I-\delta_7)(m-\delta_5) \end{array}\right]$$

Now let us choose $I_1=I-\boldsymbol{\delta}_7$, $I_2=I$, $I_3=I+\boldsymbol{\delta}_8$; $Q_1=Q-\boldsymbol{\delta}_9$, $Q_2=Q$, $Q_3=Q+\boldsymbol{\delta}_{10}$; $p_1=p-\boldsymbol{\delta}_{11}$, $p_2=p$, $p_3=p+\boldsymbol{\delta}_{12}$ so that the above equation becomes

$$(\tilde{T}C(\tilde{p},\tilde{Q},\tilde{I})) = \frac{1}{6}\left[\begin{array}{l} Ap_1p_3^{-\beta} + \frac{A(S-\delta_1)p_3^{-\beta}}{Q_3} + \frac{(H-\delta_3+m-\delta_5)I_1^2}{2Q_3} + \\ \frac{Q_1(m-\delta_5)}{2} - I_3(m+\delta_6) \end{array}\right]$$

$$+\frac{2}{3}\left[Ap_2^{1-\beta} + \frac{ASp_2^{-\beta}}{Q_2} + \frac{(H+m)I_2^2}{2Q_2} + \frac{Q_2m}{2} - \mathrm{I_2m}\right] + \frac{1}{6}\left[\begin{array}{l} Ap_3p_1^{-\beta} + \frac{A(S+\delta_2)p_1^{-\beta}}{Q_1} + \\ \frac{(H+\delta_4+m+\delta_6)I_3^2}{2Q_1} + \frac{Q_3(m+\delta_6)}{2} \\ -I_1(m-\delta_5) \end{array}\right]$$

where $0 \le I_1 \le I_2 \le I_3$, $0 \le p_1 \le p_2 \le p_3$ and $0 \le Q_1 \le Q_2 \le Q_3$

The constraints under consideration are $I_1 - I_2 \le 0, I_2 - I_3 \le 0, -I_1 < 0$, $p_1 - p_2 \le 0, p_2 - p_3 \le 0, -p_1 < 0$ and $Q_1 - Q_2 \le 0, Q_2 - Q_3 \le 0, -Q_1 < 0$

The proposed model with the above mentioned constraints is then investigated by KKT conditions technique.

The solution of the decision variables that optimizes the objective function afterdefuzzification is

$$p=\left[\frac{\beta^2(H-\delta_3+4H+H+\delta_4)(S+\delta_2+4S+S-\delta_1)(m-\delta_5+4m+m+\delta_6)}{72A(1-\beta^2)(H-\delta_3+m-\delta_5+4(H+m)+H+\delta_4+m+\delta_6)}\right]^{\frac{1}{(2-\beta)}}$$

$$Q=\left[\frac{\beta(S+\delta_2+4S+S-\delta_1)}{6p(1-\beta\ )}\right]$$

$$I=\left[\frac{\beta(S+\delta_2+4S+S-\delta_1)(m-\delta_5+4m+m+\delta_6)}{6p(1-\beta\ )(H-\delta_3+m-\delta_5+4(H+m)+H+\delta_4+m+\delta_6)}\right]$$

OPTIMAL SOLUTION CALCULATION

The following values are assumed $\beta = 0.86$ and $A = 100$.

By choosing the input parameters like the setup cost, holding cost and the penalty cost as triangular fuzzy numbers, the rate of the decision variables (maximum inventory, lot size and unit cost) are calculated. The total annual cost function for different values of S, H and m as triangular numbers are calculated and the optimum solution table is given below.

CONCLUSION

The input and decision variables are considered as triangular fuzzy numbers. The objective function is defussified using GMI representation and then the model is resolved by KKT conditions technique. An efficient soluyion of the model is obtained by varying the triangular fuzzy numbers and the impact of the cost parameters over the decision variables and the objective function is noted. The most appropriate total annual cost is attained at the lowest unit price value.

A better solution can also be obtained by using trapezoidal membership function and the Gaussina membership function.

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