A STUDY ON CENTRE OF GRAVITY

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ABSTRACT:

This paper proposes a simple and easy methods for determining the centre of gravity of a motionless object. A human's Centre of gravity can change as he takes on different position, but in many other objects, it is a fixed location. Here the uniqueness of centre of gravity is discussed. Moreover,the general formula for determining the centre of gravity, centre of gravity by integration, centre of gravity of a compound body and remainder is also determined.

KEYWORDS:

Centre of Gravity, Force, Moment, Weight, Mass

INTRODUCTION

The concept of a centre of gravity was first introduced by Archimedes of Syracuse. He worked with simplified assumptions about gravity that quantity to a consistent field, thus arriving at the mathematical properties of what we now call the "centre of mass". The terms "centre of mass" and "centre of gravity" are used synonymously during a uniform gravity field to represent the unique point in an object or system which may be wont to describe the system's response to external forces and torques. So, we can say centre of gravity is a point at which all of the weight (or mass) of an object appears to be concentrated.

The centre of gravity is the point on the body through which the resultant of all gravity vectors passes, that's why it got its name centre of gravity. The centre of mass and therefore the centre of gravity of an object are within the same position if the field during which the thing exist is uniform. Its symbol is COG or CG.

The centre of gravity are often located within or outside the body counting on the body's configuration and position; it's inside an object when the thing is uniform and outside the object when it is not uniform.

BASIC DEFINITIONS

MASS:

Mass may be a measure of the quantity of matter in an object. An object's mass is constant altogether circumstances; contrast with its weight, a force that depends on gravity.

WEIGHT:

The weight of an object is defined because the force of gravity on the thing and should be calculated because the mass times the acceleration thanks to gravity.
Weight can be calculated by the formula,
$W = mg$
W = Weight, m = mass of an object, g = acceleration due to gravity
FORCE:
Force is any cause which produces or tends to produce a change in the existing state of rest of a body or of its uniform motion in a straight line.
The formula for force is,
$F = m.a$
F = Force
m = mass of an object
a = acceleration

MOMENT OF FORCE:

Moment of force about a point is defined to be the product force and the perpendicular distance of the point from the line of action of the force.

EQUILIBRIUM:

When a number of forces act on a body and keep at rest, the forces are said to be in equilibrium.

PARALLEL FORCES:
Two parallel forces are said to be like once they act in same direction; once they act in opposite parallel directions they're said to be unlike.

COPLANAR:

If three forces working on a rigid body are in equilibrium, they need to be coplanar.

CENTRE OF GRAVITY:

The centre of gravity (CG) of a body is that point through which the line of action of the weight of the body always passes in whichever position of the object.

DETERMINATION OF CENTRE OF GRAVITY

THE CENTRE OF GRAVITY OF A BODY IS UNIQUE:

We can show that a body can have only one centre of gravity.
For, if possible, let it have two centers of gravity, G and G1. This means that, in all positions of the body, its weight acts through G as well as G1. Hence the weight acts along GG1.

Mass is a measure of the amount of matter in an object. An object's mass is constant in all circumstances; contrast with its weight, a force that depends on gravity.

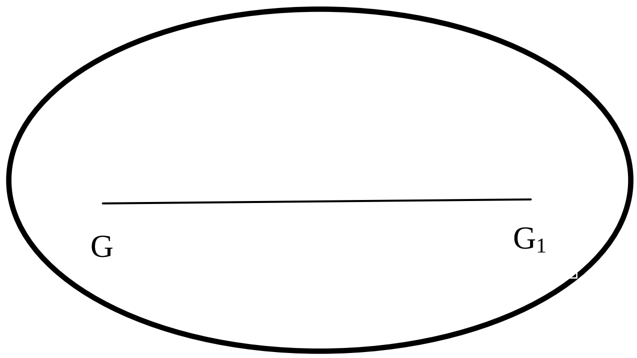

Fig 3.1

Now, hold the body such that GG_1 is horizontal. In this position, the weight is a vertically downward force acts along a horizontal line GG_1, which is absurd. Hence the body cannot have centres of gravity. i.e. the C.G of the body is unique.

GENERAL FORMULA FOR DETERMINATION OF THE CG

We will now obtain formulae for determining the C.G of any system of particles, whose positions and weights are known.

Let a number of particles of weights $W_1, W_2 .... W_n$ be placed in a plane at points $A_1, A_2, A_3 .... A_n$ and let the coordinates of these points referred to two rectangular axes OX, OY in the plane be (x_1, y_1), (x_2, y_2) … (x_n, y_n). Let G, the C.G of the system, be the point $(\bar{x}, \bar{y})$.

Now the position of G relative to the plane of the particles does not depend on the position of the plane.

Hence to find $\bar{x}$, we can assume that the plane is vertical and is so placed that the x-axis is horizontal. Hence the weights W_1, W_2, W_3etc, will be all like parallel forces, parallel to the y-axis. The resultant of the weights is a force $R = W_1+W_2+W_3+ \ldots$ which is also parallel to the y-axis, acting through the point G $(\bar{x}, \bar{y})$.

Hence taking moment about O, we have

$R. \bar{x} = W_1 x_1+W_2 x_2+W_3 x_3+ \ldots$

i.e. $\bar{x}=\frac{\text{W1 x1+W2 x2+W3 x3+ ...}}{\text{W1+W2+W3+ ...}} = \frac{\sum Wx}{\sum W}$....... (1)

For finding $\bar{y}$, we assume that the plane is vertical and is so placed that the y-axis is horizontal. The weights W_1, W_2 etc. will be all like parallel forces, parallel to the x-axis.

Taking moments about G, we get

i.e. $\bar{y} =\frac{\text{W1 y1+W2 y2+W3 y3+ ...}}{\text{W1+W2+W3+ ...}} = \frac{\sum Wy}{\sum W}$ .... (2)

Equations (1) and (2) gives the coordinates of G.

NOTE:

Since W= mg. where m is the mass of the particles, the above formula can also be written as

$\bar{x} = \frac{\sum mx}{\sum m}$ and $\bar{y} = \frac{\sum my}{\sum m}$

CENTRE OF GRAVITY BY INTEGRATION

From the above note, we know that if masses m_1, m_2... are situated at the points

(x_1, y_1), (x_2, y_2) ... then $(\bar{x}, \bar{y})$, the coordinates of their C.G. are given by

$\bar{x} = \frac{\sum mx}{\sum m}$ and $\bar{y} = \frac{\sum my}{\sum m}$

When we are given a finite number of particles whose masses and positions are known, the above summations are affected by ordinary additions.

But in the case of a rigid body, the number of particles is infinite. Hence, to find the C.G. of a rigid body or lamina (involving continuous distribution of matter) we divide the body into a large number of elementary strips or slices, the positions of whose centres of gravity are known.

We take an element of mass "dm" at the point (x, y) and apply the above formulae to get $(\bar{x}, \bar{y})$, the C.G. of the whole body. In that case, the summations $\sum mx$, $\sum mx$, $\sum m$ will be replaced by the corresponding definite integrals.

Then $\bar{x}=\frac{\int xdm}{\int dm}$ and $\bar{y} =\frac{\int ydm}{\int dm}$

In this formula, "dm" is the mass of an elementary portion of the body situated at (x, y) its centre of gravity.

CENTRE OF GRAVITY OF A COMPOUND BODY:

To find the C.G. of a body consisting of two parts, the C.G. of each part being given.

Let G_1, G_2 be the centres of gravity of the two parts of a body, W_1 and W_2 being their weights.

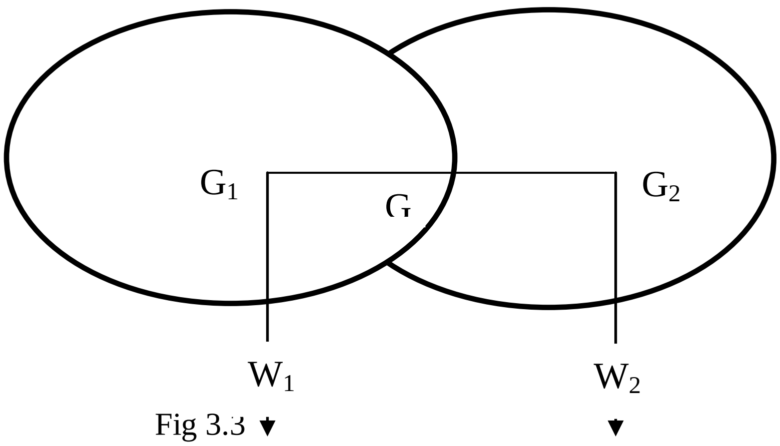

Fig 3.3

These weights are like parallel forces acting at G_1 and G_2 and their resultant is equal to W_1+ W_2 acting at the point G on G_1G_2 such that $\frac{G1G}{GG2} = \frac{W2}{W1}$

i.e. $\frac{G1G}{W2} = \frac{GG2}{W1}$

$\frac{G1G+GG2}{W2+W1} = \frac{G1G2}{W2+W1}$

$\therefore G1G = \frac{W2}{W1+W2}. G1G2 \text{and} GG2 = \frac{W1}{W1+W2}. G1G2$

The point G is the centre of gravity (C.G) of the

CENTER OF GRAVITY OF A REMAINDER:

To find C.G. of the remaining part of a body, when some portion of it is removed.

Let W be the weight of the body and G its C.G. and let W_1 be the weight of the portion removed and G_1 its C.G.

Then the weight of the remaining part is W- W_1 and let its C.G. be at G_2.

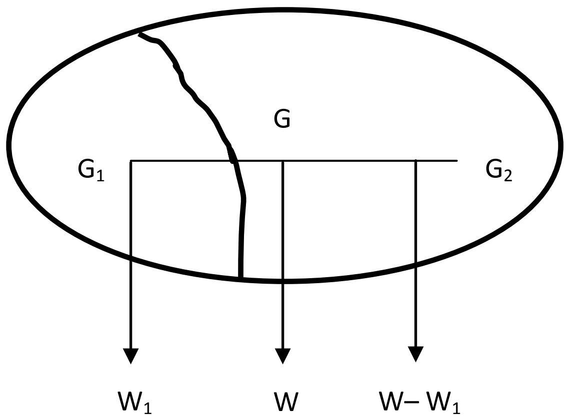

Fig 3.4

Clearly G_2 must be on the line G_1G, on the side of G opposite to G_1. Now the resultant of W_1 at G_1 and W- W_1 at G_2 is the force W at G.

$\therefore W1. G1G = (W - W1)G2G.$

or$G2G = \frac{W}{W - W1} . G1G$

This gives the position of G_2, the required centre of gravity of the remaining part.

CONCLUSION

In this paper, several simple methods for determining the centre of gravity for a motionless object had been discussed. Using this method, we can solve problems in statics. Centre of gravity plays a vital role in the field of space electronics, automotive industry, construction works etc. For example, aligning the direction of thrust of a rocket motor so that it pushes exactly through the centre of gravity of the rocket is essential to achieving a straight flight.

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