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Crystal structure optimization approach to problem solving in mechanical engineering design

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Abstract

Purpose – In this paper, the authors aim to examine and comparatively evaluate a recently-developed metaheuristic called crystal structure algorithm (CryStAl) – which is inspired by the symmetries in the internal structure of crystalline solids – in solving engineering mechanics and design problems.

Design/methodology/approach – A total number of 20 benchmark mathematical functions are employed as test functions to evaluate the overall performance of the proposed method in handling various functions. Moreover, different classical and modern metaheuristic algorithms are selected from the optimization literature for a comparative evaluation of the performance of the proposed approach. Furthermore, five well-known mechanical design examples are utilized to examine the capability of the proposed method in dealing with challenging optimization problems.

Findings – The results of this study indicated that, in most cases, CryStAl produced more accurate outputs when compared to the other metaheuristics examined as competitors.

Research limitations/implications – This paper can provide motivation and justification for the application of CryStAl to solve more complex problems in engineering design and mechanics, as well as in other branches of engineering.

Originality/value – CryStAl is one of the newest metaheuristic algorithms, the mathematical details of which were recently introduced and published. This is the first time that this algorithm is applied to solving engineering mechanics and design problems.

Keywords Metaheuristic, Optimization, Algorithm, Statistical analysis, Crystal structure, Lattice

Paper type Research paper

1. Introduction

Optimization is described as the art of searching for the best solution among the existing ones. It is widely used to reduce the cost of design, production and maintenance of engineering, economic and social systems. Due to their vast applications in various fields of science, engineering and finance, optimization procedures have been extensively developed in recent years. Also, different titles such as "Mathematical Programming" or "Operations Research" may be used to refer to optimization. Among different kinds of optimization methods, meta-heuristic methods are more popular in engineering as a result of their practicality and efficiency for engineering purposes.

Nature is a frequently utilized source of inspiration for metaheuristic specialists. It has turned out that many successful metaheuristic algorithms which have demonstrated convincing performance in dealing with difficult optimization problems are nature-inspired. Some well-known examples of such algorithms are as follows. Fogel *et al.* (1966) proposed the evolutionary algorithm which mimics artificial intelligence through simulated evolution. Holland (1992) proposed the genetic algorithm (GA) which is inspired by Darwin's theory of evolution. Glover and Laguna (1998) presented taboo search which is based on the mechanism of the direct inhibition of some inaccessible areas of the search space. Also, simulated annealing was proposed by Kirkpatrick *et al.* (1983) which mimics the detailed analogy of annealing in solids. Eberhart and Kennedy (1995) formulated Particle Swarm Optimization (PSO) which is inspired by the simulation of mass flight of birds. Dorigo *et al.* (1996) proposed ant colony optimization (ACO) which mimics the real behavior of ants in nature. Geem *et al.* (2001) presented the harmony search (HS) algorithm which is obtained by imitating the process of finding the best combination of notes and composing music. Yang (2012) presented flower pollination algorithm which mimics the pollination process in the flowers. Some other well-known approaches can also be mentioned including the chaos game optimization (Talatahari and Azizi, 2020a, 2021), atomic orbital search (Azizi, 2021) and crystal structure algorithm (CryStAl) (Talatahari *et al.*, 2021a, Khodadadi *et al.*, 2021). In addition, many other challenges have also been introduced and investigated in recent years (see, e.g. Arora *et al.*, 1994, Arora and Wang, 2005, Talatahari and Azizi, 2020b,; Talatahari *et al.*, 2021b, Yazdchi *et al.*, 2021, Azizi *et al.*, 2019, Chen *et al.*, 2020a,b and Sareh and Chen, 2020). Notably, the uncertainty effects in various optimization procedures have been thoroughly studied (see, e.g. Wang *et al.*, 2019, Wang *et al.*, 2021, Xiong *et al.*, 2019, Beck and De Santana Gomes, 2012, Daskilewicz *et al.*, 2011, Mukherjee *et al.*, 2019 and Diwekar and Kalagnanam, 1997).

In this paper, a recently proposed metaheuristic algorithm called CryStAl is utilized as an optimization technique for the optimum design of engineering problems (see Talatahari *et al.* (2021a) for a more extensive description of the theoretical details of this method). A total number of 20 numerical examples are utilized as test functions to evaluate the overall performance of the proposed method. To validate the results of CryStAl, different metaheuristic algorithms are selected from the literature for comparative purposes. Besides, five of the well-known engineering design examples are also selected to test the overall behavior of CryStAl in dealing with difficult optimization problems.

2.. Crystal structure optimization

2.1 Background

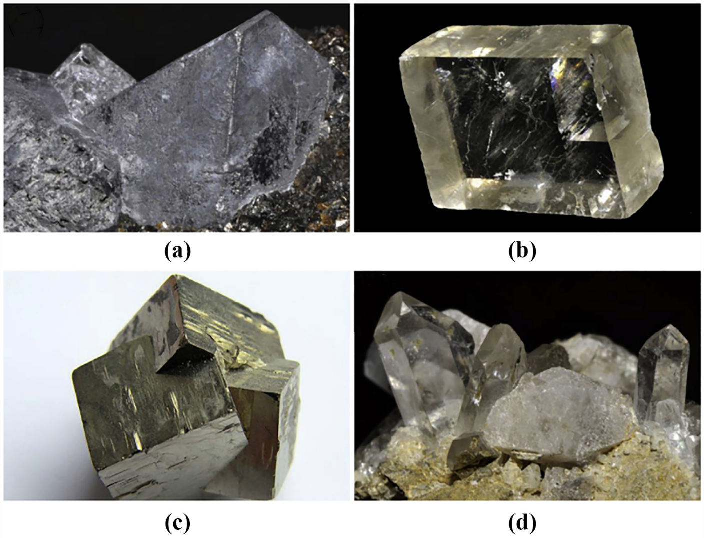
Historically, crystal science and engineering started with the study of minerals (Dhanaraj *et al.*, 2010, Brown, 1982 and Eberl *et al.*, 1998). By definition, crystals are solid minerals, the molecules, atoms or ions of which have a crystallographic order, i.e. they are symmetrically arranged in the three-dimensional space. Inspired by the vivid symmetries in the structure of natural crystalline solids, designers and engineers have created artworks (see, e.g. Bodner, 2013, Grünbaum, 2006, Necefoglu, 2003 and De Las Peñas *et al.*, 2018), engineering structures (Zingoni, 2015a-e, Sareh and Guest, 2015a-d, Chen *et al.*, 2019 and Sareh, 2019) and nano-objects (see, e.g. Ma *et al.*, 2014, Takiguchi *et al.*, 2020, Yu *et al.*, 2007, Kee *et al.*, 2007, Sun *et al.*, 2007, Yokoo *et al.*, 2017, Ra and Lee, 2021, Sergent *et al.*, 2019, Gorshkov *et al.*, 2020a, b and Gorshkov *et al.*, 2021a, b) with highly symmetric geometries. Natural samples of some well-known crystalline minerals are depicted in Figure 1.

A fundamental component of a typical crystal is the "lattice", which represents the parodic array of points in the space known as "lattice points". Besides, the location of atoms in the structure of a crystal is determined by another characteristic called the "basis". By adding the basis to the lattice points, we obtain the complete crystal structure, i.e.

$$\text{Crystal} = \text{Lattice} + \text{Basis} \tag{1}$$

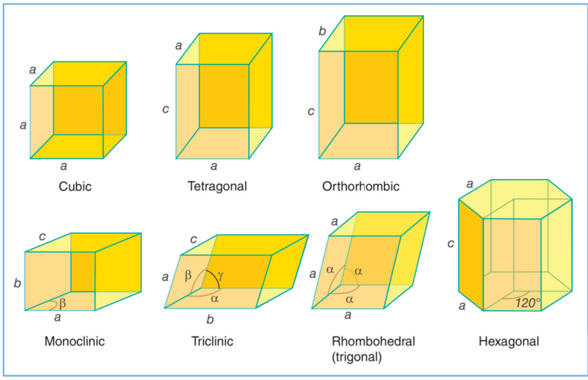
Figure 2 illustrates the lattice configurations corresponding to various crystal systems; on the right-hand side of this figure, the relationships between lattice parameters.

Conventionally, the arrangement of atoms in solid structures is represented by different spatial distributions of spherical elements within a unit cell of the solid. The unit cell is the smallest volume that contains the fundamental structural information which is necessary to



Source(s): Dhanaraj *et al.*, 2010

Figure 1.
Samples of crystalline minerals: (a) Galena, (b) Calcite, (c) Pyrite and (d) Quartz



System	Relationships between lattice parameters
Triclinic	$a \neq b \neq c$, $\alpha \neq \beta \neq \gamma \neq 90°$
Monoclinic	$a \neq b \neq c$, $\alpha = \gamma = 90°$, $\beta \neq 90°$
Orthorhombic	$a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$
Rhombohedral	$a = b = c$, $\alpha = \beta = \gamma \neq 90°$
Tetragonal	$a = b \neq c$, $\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$, $\alpha = \beta = 90°$, $\gamma = 120°$
Cubic	$a = b = c$, $\alpha = \beta = \gamma = 90°$

Source(s): Adapted from Shriver *et al.*, 2014

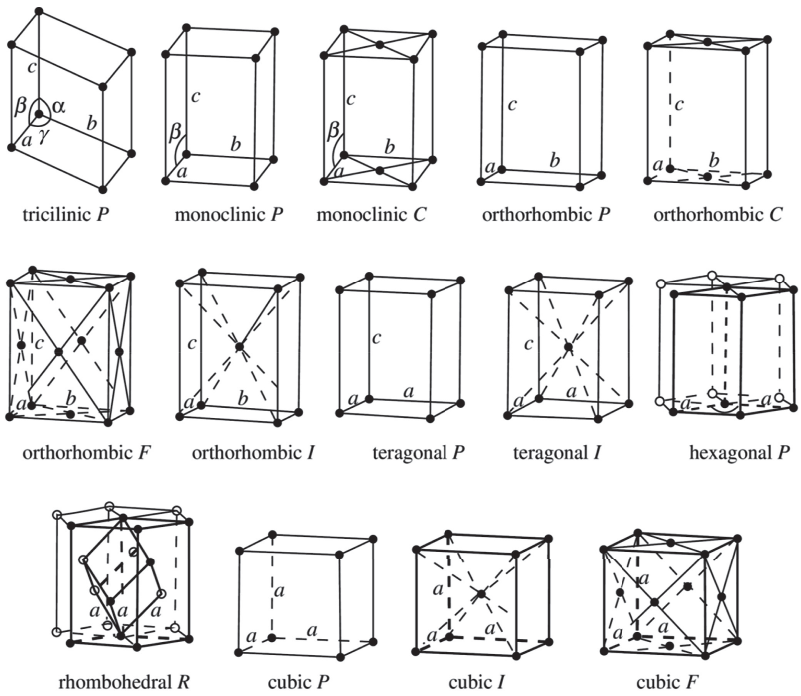
Figure 2.
Lattice configurations corresponding to various crystal systems

identify the crystal structure. In the middle of the 19th century, the French physicist Auguste Bravais proved that all three-dimensional lattices can be classified into 14 distinct types, known nowadays as "Bravais lattices". The unit cells corresponding to these 14 types are illustrated in Figure 3 (Li *et al.*, 2008).

The Bravais model is used for the mathematical representation of crystals in which lattice points are described by vectors as follows:

$$r = \sum n_i a_i, \tag{2}$$

Where n_i is an integer, a_i is the shortest vector along the principal crystallographic directions, and i is the number of crystal corners.



Source(s): Adapted from Li *et al.*, 2008

Figure 3. Lattice points describing the translational symmetry of (a) primitive, (b) body-centered and (c) face-centered cubic unit cells

2.2 Model description

This section describes the mathematical model of the CryStAl In this model, each candidate solution of the optimization problem is considered as a single crystal in the space and the basic concepts of crystallography are employed with required modifications. Moreover, in order to initialize the iterative process of computation, a number of crystals are randomly determined as follows

$$Cr = \begin{bmatrix} Cr_1 \\ Cr_2 \\ \vdots \\ Cr_i \\ \vdots \\ Cr_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ x_n^1 & x_n^2 & \cdots & x_n^j & \cdots & x_n^d \end{bmatrix}, \quad \begin{cases} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, d \end{cases} \tag{3}$$

Where, n is the number of crystals (i.e. candidate solutions) and d is the dimension of the problem. The initial positions of these crystals are determined randomly in the search space as follows:

$$x_i^j(0) = x_{i,min}^j + r\ and.\left(x_{i,max}^j - x_{i,min}^j\right), \quad \begin{cases} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, d \end{cases} \tag{4}$$

Where, $x_i^j(0)$ determines the initial position of the crystals; $x_{i,min}^j$ and $x_{i,max}^j$ are respectively the minimum and maximum allowable values for the *j*th decision variable of the *i*th solution candidate; and *rand* is a random number in the interval [0,1].

Based on the concept of "basis" explained in the previous section, all the crystals at the corners are considered as the main crystals, Cr_{Main}, determined randomly by considering the initially-created crystals (candidate solutions). The crystal with the *best* configuration is determined as Cr_b while the mean values of randomly selected crystals are denoted by F_C.

To update the positions of the candidate solutions in the search space, basic lattice principles are considered in which four kinds of updating process as detailed in Table 1.

In order to deal with the solution variables (x_i^j) violating the boundary conditions of the variables, a mathematical flag is defined in which for the x_i^j outside the variables range, the flag orders a boundary change for the violating variables. The terminating criterion is considered based on the maximum number of iterations in which the optimization process is terminated after a fixed number of iterations. The pseudo-code of the algorithm is presented in Figure 4.

3. Representative design examples

In order to verify the capabilities of the proposed method, i.e. CryStAl, in solving various optimization problems, 20 benchmark mathematical test functions solved by six widely-used metaheuristic algorithms are considered for comparison purposes. To this end, some well-established constrained and engineering optimization problems from the optimization literature are employed to demonstrate the performance of this new method in dealing with such problems in comparison with some previously reported results of other studies in the literature.

3.1 Mathematical test functions

The mathematical formulation and general characteristics of the considered mathematical functions are demonstrated in this section (Table 2) while the complete description of these problems is accessible in Karaboga and Akay (2009), Cheng and Lien (2012) and Cheng and Prayogo (2014). The first nine functions are two-dimensional (2D) whereas functions 10 to 20

System	Updating process	Notes
Simple cubicle	$Cr_{new} = Cr_{Old} + r.Cr_{Main}$	Cr_{new} is the new position, Cr_{Old} is the old position and r is a random number
Cubicle with best crystals	$Cr_{new} = Cr_{Old} + r1.Cr_{Main} + r2.Cr_b$	Cr_{new} is the new position, Cr_{Old} is the old position and $r1$ *and* $r2$ are random numbers
Cubicle with mean crystals	$Cr_{new} = Cr_{Old} + r1.Cr_{Main} + r2.F_c$	Cr_{new} is the new position, Cr_{Old} is the old position and $r1$ *and* $r2$ are random numbers
Cubicle with best and mean crystals	$Cr_{new} = Cr_{Old} + r1.Cr_{Main} + r2.Cr_b + r3.F_c$	Cr_{new} is the new position, Cr_{Old} is the old position and $r1$ *to* $r3$ are random numbers

Table 1. Descriptions of the mathematical benchmark functions

```
procedure Crystal Structure Algorithm (CryStAl)
    Create random values for initial positions (x_i^j) of initial crystals (Cr_i)
    Evaluate fitness values for each crystal
    while (t < maximum number of iterations)
        for i = 1: number of initial crystals
        Create Cr_Main
        Create new crystals by Eq. 4
        Create Cr_b
        Create new crystals by Eq. 5
        Create F_c
        Create new crystals by Eq. 6
        Create new crystals by Eq. 7
            if new crystals violate boundary conditions
                Control the position constraints for new crystals and amend it
            end if
        Evaluate the fitness values for new crystals
        Update Global Best (GB) if a better solution is found
        end for
        t = t+1
    end while
    return GB
end procedure
```

Figure 4.
The pseudo-code of CryStAl

are 50-dimensional (50D). These mathematical test functions are some kinds of unimodal (*U*), multimodal (*M*), separable (*S*) and non-separable (*N*) functions.

For the mentioned alternative metaheuristic algorithms utilized for comparative study, the specific parameters of the algorithms are presented in Table 3.

3.2 Classical constrained optimization problems

In this section, three constrained optimization problems are considered to evaluate the effectiveness and capability of the proposed method. These examples have been previously studied by utilizing different metaheuristic algorithms. A simple penalty approach for handling the problem constraints is selected.

The tension/compression spring design problem (Figure 5a) is the first constrained problem in this paper in which the wire diameter $d\,(=x_2)$, the mean coil diameter $D\,(=x_1)$ and the number of active coils $N\,(=x_3)$ are considered as design variables while the boundaries are $0.05 \leq x_1 \leq 2$, $0.25 \leq x_2 \leq 1.3$ and $2 \leq x_3 \leq 15$. The complete mathematical formulation of this problem is as follows:

$$f_{cost}(X) = (x_3 + 2)x_2x_1^2, \tag{5}$$

$$g_1(X) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0, \tag{6}$$

$$g_2(X) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0, \tag{7}$$

$$g_3(X) = 1 - \frac{140.45x_1}{x_2x_1^3} \leq 0, \tag{8}$$

Table 2. Descriptions of the mathematical benchmark functions

No	Name	Type	R	D	Formulation	Min
1	Beale	UN	[−4.5,4.5]	2	$f(x) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2 + (2.625 - x_1 + x_1x_2^3)^2$	0
2	Easom	UN	[−100,100]	2	$f(x) = -\cos(x_1)\cos(x_2)exp(-(x_1-\pi)^2 - (x_2-\pi)^2)$	−1
3	Matyas	UN	[−10,10]	2	$f(x) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$	0
4	Bohachevsky1	MS	[−100,100]	2	$f(x) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$	0
5	Booth	MS	[−10,10]	2	$f(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	0
6	Michalewicz2	MS	[0,π]	2	$f(x) = -\sum_{i=1}^{D} sin(x_i) + (sin(ix_i^2/\pi))^{20}$	−1.8013
7	Six hump Camel back	MN	[−5,5]	2	$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1\ x_2 - 4x_2^2 + 4x_2^4$	−1.03163
8	Boachevsky2	MN	[−100,100]	2	$f(x) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1)(4\pi x_2) + 0.3$	0
9	Boachevsky3	MN	[−100,100]	2	$f(x) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1 + 4\pi x_2) + 0.3$	0
10	Zakharov	UN	[−5,10]	50	$f(x) = -\sum_{i=1}^{D} x_i^2 + (\sum_{i=1}^{D} 0.5ix_i)^2 + (\sum_{i=1}^{D} 0.5ix_i)^4$	0
11	Step	US	[−5.12,5.12]	50	$f(x) = \sum_{i=1}^{D} (x_i + 0.5)^2$	0
12	Sphere	US	[−100,100]	50	$f(x) = \sum_{i=1}^{D} x_i^2$	0
13	SumSquares	US	[−10,10]	50	$f(x) = \sum_{i=1}^{D} ix_i^2$	0
14	Schwefel 2.22	UN	[−10,10]	50	$f(x) = \sum_{i=1}^{D} \lvert x_i\rvert + \prod_{i=1}^{D} \lvert x_i\rvert$	0
15	Schwefel 1.2	UN	[−100,100]	50	$f(x) = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	0
16	Rosenbrock	MN	[−30,30]	50	$f(x) = \sum_{i=1}^{D-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2$	0
17	Dixon–Price	UN	[−10,10]	50	$f(x) = (x_1 - 1)^2 + \sum_{i=2}^{D} i(2x_i^2 - x_i - 1)^2$	0
18	Rastrigin	MS	[−5.12,5.12]	50	$f(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$	0
19	Griewank	MN	[−600,600]	50	$f(x) = \frac{1}{4000}(\sum_{i=1}^{D} (x_i - 100)^2) - \left(\prod_{i=1}^{D} \cos\left(\frac{x_i - 100}{\sqrt{i}}\right)\right) + 1$	0
20	Ackley	MN	[−32,32]	50	$f(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{D} cos(2\pi x_i)\right) + 20 + e$	0

Note(s): R: variable range; D: dimension; M: multimodal; U: unimodal; S: separable; N: non-separable

Metaheuristic	Parameter	Description	Value
ACO	N_{pop}	Archive size	50
	N_s	Sample size	50
	q	Intensification factor	0.5
	ζ	Deviation-distance ratio	1
DE	N_{pop}	Number of scout bees	50
	p_c	Crossover probability	0.2
	β_{min}	Lower bound of scaling factor	0.2
	β_{max}	Upper bound of scaling factor	0.8
GA	N_{pop}	Population size	50
	p_c	Crossover percentage	0.8
	p_m	Mutation percentage	0.3
	μ	Mutation rate	0.02
	β	Roulette wheel selection pressure	1
HS	HMS	Harmony memory size	50
	N_{new}	Number of new harmonies	20
	$HMCR$	Harmony memory consideration rate	0.9
	PAR	Pitch adjustment rate	0.1
	FW	Fret width (bandwidth)	±0.02
	FW_{damp}	Fret width damp ratio	0.995
PSO	N_{pop}	Swarm size	50
	w	Inertia weight	1
	w_d	Inertia weight damping ratio	0.99
	c_1	Personal learning coefficient	2
	c_2	Global learning coefficient	2
ICA	N_{pop}	Population size	50
	N_{emp}	Number of empires/imperialists	10
	α	Selection pressure	1
	β	Assimilation coefficient	1.5
	p_r	Revolution probability	0.05
	μ	Revolution rate	0.1
	ζ	Colonies mean cost coefficient	0.2

Table 3.
Parameter summary of the alternative metaheuristic algorithms

$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \leq 0. \tag{9}$$

The second constrained problem of this paper is the welded beam design problem (Figure 5b) in which the shear stress (τ), bending stress (σ), buckling load (P_c), end deflection (δ) and some side constraints are the design constraint *f* this problem while the design variables, namely h $(= x_1)$, $l (= x_2)$, $t (= x_3)$ and $b (= x_4)$ are utilized accordingly regarding the $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$ and $0.1 \leq x_4 \leq 2$ as boundaries. The complete mathematical formulation of this problem is as follows:

$$f_{cost}(X) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2). \tag{10}$$

The optimization variables and their respective boundaries are $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$ and $0.1 \leq x_4 \leq 2$. The constraints are defined as follows.

$$g_1(X) = \tau(\{x\}) - \tau_{max} \leq 0, \tag{11}$$

$$g_2(X) = \sigma(\{x\}) - \sigma_{max} \leq 0, \tag{12}$$

$$g_3(X) = x_1 - x_4 \leq 0, \tag{13}$$

$$g_4(X) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \leq 0, \tag{14}$$

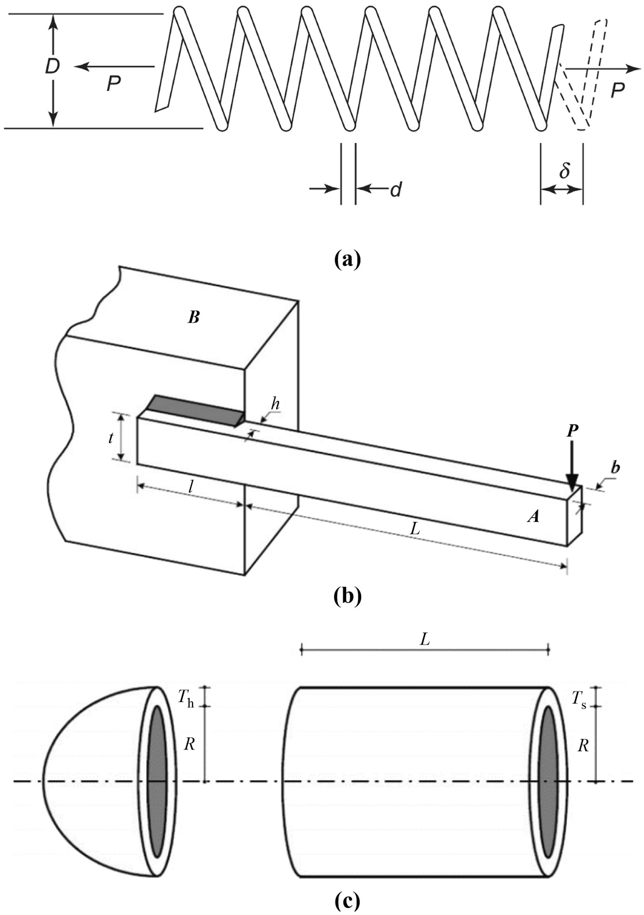


Figure 5.
Schematics of the design problems: (a) a conventional tension/compression spring; (b) a welded beam; (c) a pressure vessel

$$g_5(X) = 0.125 - x_1 \le 0, \tag{15}$$

$$g_6(X) = \delta(\{x\}) - \delta_{max} \le 0, \tag{16}$$

$$g_7(X) = P - P_c(\{x\}) \le 0, \tag{17}$$

$$\tau(X) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}, \tag{18}$$

Where

$$\tau' = \frac{P}{\sqrt{2}x_1x_2}, \ \tau'' = \frac{MR}{J}, M = P\left(L + \frac{x_2}{2}\right), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, \tag{19}$$

$$J = 2\left\{\sqrt{2}\,x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}, \tag{20}$$

$$\sigma(X) = \frac{6PL}{x_4 x_3^2}, \quad \delta(X) = \frac{4PL^3}{E x_3^3 x_4}, \tag{21}$$

$$P_c(X) = \frac{4.013E\sqrt{\frac{x_3^2 x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right), \tag{22}$$

$$P = 6000\,\text{lb},\ L = 14\,\text{in},\ E = 30 \times 10^6\,psi,\ G = 12 \times 10^6\,psi. \tag{23}$$

The pressure vessel design problem (Figure 5c) is the third constraint problem in this paper in which x_1 as the thickness of the shell (T_s), x_2 is the thickness of the head (T_h), x_3 is the inner radius (R) and x_4 is the length of the cylindrical section of the vessel (L), not including the head. The boundaries are $0 \le x_1 \le 99$, $0 \le x_2 \le 99$, $10 \le x_3 \le 200$ and $10 \le x_4 \le 200$. The complete mathematical formulation of this problem is as follows:

$$f_{cost}(X) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3 \tag{24}$$

$$g_1(X) = -x_1 + 0.0193 x_3 \le 0, \tag{25}$$

$$g_2(X) = -x_2 + 0.00954 x_3 \le 0, \tag{26}$$

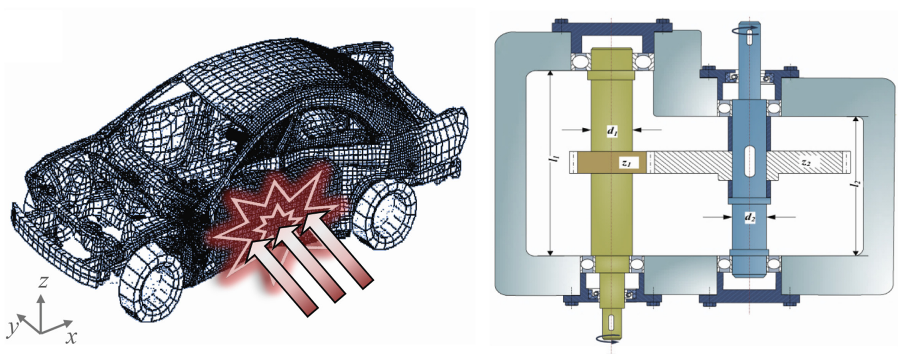
$$g_3(X) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \tag{27}$$

$$g_4(X) = x_4 - 240 \le 0. \tag{28}$$

3.3 Practical engineering design problems

This section examines and evaluates the performance of CryStAl in providing solutions for some typical engineering optimization problems, in comparison with other metaheuristics. To this end, here we consider two engineering design problems that were previously solved using other metaheuristic algorithms, now to be solved using CryStAl, employing a simple penalty approach to handle the constraints.

The car side-impact problem is the first practical problem in this paper which has a mathematical presentation as follows while the schematic view of this problem is demonstrated in Figure 6a.



(a) (b)

Figure 6.
Schematic views of (a) a car under side impact and (b) a speed reducer

$$f_{cost}(X) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \tag{29}$$

$$g_1(X) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} \leq 1, \tag{30}$$

$$\begin{aligned} g_2(X) = {} & 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} \\ & + 0.080405x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} \leq 0.32, \end{aligned} \tag{31}$$

$$\begin{aligned} g_3(X) = {} & 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 \\ & + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.000535x_6x_{10} \\ & + 0.00121x_8x_{11} \leq 0.32, \end{aligned} \tag{32}$$

$$g_4(X) = 0.074 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 \leq 0.32, \tag{33}$$

$$g_5(X) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} \leq 32, \tag{34}$$

$$\begin{aligned} g_6(X) = {} & 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} \\ & - 9.98x_7x_8 + 22.0x_8x_9 \leq 32, \end{aligned} \tag{35}$$

$$g_7(X) = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} \leq 32, \tag{36}$$

$$g_8(X) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 \leq 4, \tag{37}$$

$$g_9(X) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} \leq 9.9, \tag{38}$$

$$\begin{aligned} g_{10}(X) = {} & 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} \\ & - 0.000786x_{11}^2 \leq 15.7, \end{aligned} \tag{39}$$

Where $0.5 \leq x_1 - x_7 \leq 1.5$, x_8 and $x_9 \in (0.192, 0.345)$, and. $-30 \leq x_{10} - x_{11} \leq 30$.

The speed reducer problem is the second practical problem in this paper which has a mathematical presentation as follows while the schematic view of this problem is displayed in Figure 6b.

$$\begin{aligned} f_{cost}(X) = {} & 0.7854bm^2(3.3333z^2 + 14.9334z - 43.0934) - 1.508b(d_1^2 + d_2^2) \\ & + 7.4777(d_1^3 + d_2^3) + 0.7854(l_1d_1^2 + l_2d_2^2) \end{aligned} \tag{40}$$

$$g_1(X) = \frac{27}{bm^2z} - 1 \leq 0, \tag{41}$$

$$g_2(X) = \frac{397.5}{bm^2z^2} - 1 \leq 0, \tag{42}$$

$$g_3(X) = \frac{1.93 l_1^3}{mzd_1^4} - 1 \leq 0, \tag{43}$$

$$g_4(X) = \frac{1.93 l_2^3}{mzd_2^4} - 1 \leq 0, \tag{44}$$

$$g_5(X) = \frac{\sqrt{\left(\frac{745 l_1}{mz}\right)^2 + 16.9 \times 10^6}}{(110 d_1^3)} - 1 \leq 0, \tag{45}$$

$$g_6(X) = \frac{\sqrt{\left(\frac{745 l_2}{mz}\right)^2 + 157.5 \times 10^6}}{(85 d_2^3)} - 1 \leq 0, \tag{46}$$

$$g_7(X) = \frac{mz}{40} - 1 \leq 0, \tag{47}$$

$$g_8(X) = \frac{5m}{b} - 1 \leq 0, \tag{48}$$

$$g_9(X) = \frac{b}{12m} - 1 \leq 0, \tag{49}$$

$$g_{10}(X) = \frac{1.5 d_1 + 1.9}{l_1} - 1 \leq 0, \tag{50}$$

$$g_{11}(X) = \frac{1.1 d_2 + 1.9}{l_2} - 1 \leq 0, \tag{51}$$

Where
$2.6 \leq b \leq 3.6$, $0.7 \leq m \leq 0.8$, $17 \leq z \leq 28$, $7.3 \leq l_1 \leq 8.3$, $7.3 \leq l_2 \leq 8.3$, $2.9 \leq d_1 \leq 3.9$, and $5 \leq d_2 \leq 5.5$.

4. Computational results and statistical analyses

In Table 4, the best and statistical results of 100 optimization runs by means of multiple metaheuristic algorithms alongside the CryStAl are presented in dealing with the mathematical test functions. Based on the results, CryStAl calculated the global optimum value for 20 of the 20 functions, outperforming all the other metaheuristic approaches.

By conducting the Kruskal–Wallis (K-W) test as one of the well-known statistical analyses, the capability of CryStAl in competing with other algorithms is demonstrated. By referring to Table 5, it can be shown that the CryStAl has the lowest mean of ranks which makes this algorithm have the first ranking.

Considering the tension/compression spring design problem, Tables 6 and 7 present the results obtained from CryStAl, as well as those produced by different metaheuristic algorithms, some of which are extracted from the literature. As can be seen from the tables, CryStAl provides better results in this case which demonstrates its capability in dealing with such a constrained optimization problem.

Table 4.
Results obtained using CryStAl and the other metaheuristic algorithms

No	Name	Response	ACO	DE	GA	HS	ICA	PSO	CSS	CryStAl
1	Beale	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	0.00764	0.00010	0	0.06096	0	0
		Std	0	0	0.07620	0.00104	0	0.20778	0	0
2	Easom	Minimum	−1	−1	−1	−1	−1	−1	−1	−1
		Mean	−1	−1	−0.95	−1	−1	−1	−0.75	−1
		Std	0	0	0.21902	0	0	0	0.43519	0
3	Matyas	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	1.87E-06	0	0	0	0	0
		Std	0	0	2.64E-06	0	0	0	0	0
4	Bohachevsky1	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	1.78E-08	0	0	0	0	0
		Std	0	0	1.01E-07	0	0	0	0	0
5	Booth	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	1.36E-06	0	0	0	0	0
		Std	0	0	2.59E-06	0	0	0	0	0
6	Michalewicz2	Minimum	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013
		Mean	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013
		Std	0	0	0	0	0	0	0	0
7	Six hump camel back	Minimum	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316
		Mean	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316
		Std	0	0	0	0	0	0	0	0
8	Boachevsky2	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	0.01091	0.01746	0	0	0	0
		Std	0	0	0.04782	0.05952	0	0	0	0
9	Boachevsky3	Minimum	0	0	0	0	0	0	0	0
		Mean	0	0	0.00102	0	0	0	0	0
		Std	0	0	0.00254	0	0	0	0	0

(*continued*)

Table 4.

No	Name	Response	ACO	DE	GA	HS	ICA	PSO	CSS	CryStAl
10	Zakharov	Minimum	1657.16	225.769	331.061	94.1672	1.04646	0.00104	1.36513	1.01212
		Mean	11,223.9	323.458	536.616	143.909	8.28350	0.00590	6.44249	3.22223
		Std	10,096.5	35.0522	103.927	18.4680	5.15671	0.00471	3.75593	0.00002
11	Step	Minimum	69	0	0	16	0	0	23	0
		Mean	130.78	0	14.43	26.48	0	4.64	73.43	0.0101
		Std	20.7323	0	14.2327	5.57316	0	4.69799	25.0447	0.00201
12	Sphere	Minimum	22.3774	0	0	1.40240	0	0	5.92E-12	0
		Mean	41.8286	0	0	2.56322	0	0	2.99E-10	0
		Std	10.7586	0	0	0.49858	0	0	4.50E-10	0
13	SumSquares	Minimum	837.043	0	0.05683	26.1829	0	0	1.21E-05	0
		Mean	1596.00	0	1.20234	47.8878	0	8.64E-07	0.05046	5.61E-10
		Std	402.959	0	1.18362	10.1045	0	5.74E-06	0.17949	1.12E-15
14	Schwefel 2.22	Minimum	7.77E+5	0	4.26234	14.1276	2.80E-12	0.43045	0.00586	0
		Mean	7.86E+6	0	60.8489	20.6511	1.11E-10	312.796	128.241	0
		Std	3.36E+6	0	36.7611	3.38388	3.05E-10	388.997	74.3514	0
15	Schwefel 1.2	Minimum	8.49E-07	0	0	1.08E-09	0	0	0	0
		Mean	0.00298	9.86E-06	6.77E-07	1.72E-05	0	3.96E-10	2.26E-07	0
		Std	0.00770	1.83E-05	3.86E-06	3.90E-05	0	1.92E-09	1.34E-06	0
16	Rosenbrock	Minimum	1.97E+0	43.7887	80.2630	5691.21	2.24650	0.34071	13.3975	0.01265
		Mean	3.72E+0	51.4939	412.954	14,710.0	139.590	70.0159	109.075	1.2306
		Std	8,125,136	15.4712	462.834	4509.79	118.449	39.1339	65.9925	0.15986
17	Dixon–Price	Minimum	604,972	0.66666	1.09624	80.4043	0.66666	0.66666	0.66666	0.66666
		Mean	2,542,555	0.66666	12.0758	204.414	3.94902	0.71549	2.45832	0.66671
		Std	769,758	5.77E-06	7.18987	59.7066	2.79369	0.34134	2.54113	0.0001256
18	Rastrigin	Minimum	511.806	118.613	4.17363	15.9600	54.7231	49.7479	44.4388	2.2651
		Mean	587.608	140.499	8.17467	21.7294	91.8945	86.5414	96.7457	3.26533
		Std	25.1807	8.66061	1.56274	2.78111	15.1828	18.9382	24.9004	0.96585
19	Griewank	Minimum	2.11618	0	0.00047	1.03750	0	0	3.22E-06	0
		Mean	3.11711	0	0.01330	1.06428	0.02202	0.00911	0.02375	1.26E-15
		Std	0.49139	0	0.01620	0.01143	0.02691	0.01446	0.03303	3.265E-20
20	Ackley	Minimum	20.7046	0	0.22636	3.65534	5.04E-11	1.63E-08	1.80320	0
		Mean	20.9724	1.27E-13	1.11168	4.37289	1.12E-08	2.10471	3.44532	2.658E-16
		Std	0.08651	3.64E-13	0.2602	0.31331	7.26E-08	0.63722	1.02822	1.333E-22

Rankings	Min Algorithms	Min Mean of ranks	Mean Algorithms	Mean Mean of ranks	Std Algorithms	Std Mean of ranks
1	*CryStAl*	*66.6*	*CryStAl*	*64.925*	*CryStAl*	*53.6*
2	DE	70.8	DE	65.575	DE	59.55
3	PSO	71.75	ICA	67.625	ICA	64.75
4	ICA	72.35	PSO	79.725	PSO	82.2
5	GA	81.45	CSS	82.85	CSS	89.1
6	CSS	84.9	HS	93.275	HS	94.2
7	HS	95.375	GA	94.6	ACO	94.6
8	ACO	100.775	ACO	95.425	GA	106
Chi-sq	11.3046		11.8179		24.9141	
Prob > Chi-sq	0.1258		0.1067		7.8587e-04	

Table 5. The K-W test results (mean of the ranks) for the mathematical functions

Method	$x_1(d)$	$x_2(D)$	$x_3(N)$	f_{cost}
	Optimal Design Variables			
Belegundu (1982)	0.050000	0.315900	14.250000	0.0128334
Arora (1989)	0.053396	0.399180	9.185400	0.0127303
Coello (2000)	0.051480	0.351661	11.632201	0.0127048
Coello and Montes (2002)	0.051989	0.363965	10.890522	0.0126810
He and Wang (2007)	0.051728	0.357644	11.244543	0.0126747
Mezura-Montes and Coello (2008)	0.051643	0.355360	11.397926	0.012698
CryStAl (*present work*)	0.0517100	0.3571427	11.2670241	0.0126696

Table 6. Optimum results for the tension/ compression spring design

Method	Best	Mean	Worst	Standard deviation
Belegundu (1982)	0.0128334	N/A	N/A	N/A
Arora (1989)	0.0127303	N/A	N/A	N/A
Coello (2000)	0.0127048	0.012769	0.012822	3.9390e-5
Coello and Montes (2002)	0.0126810	0.0127420	0.012973	5.9000e-5
He and Wang (2007)	0.0126747	0.012730	0.012924	5.1985e-5
Mezura-Montes and Coello (2008)	0.012698	0.013461	0.16485	9.6600e-4
CryStAl (*present work*)	0.0126696	0.0127136	0.0127759	1.0365e-5

Table 7. Statistical results of different methods for the tension/ compression spring design

Method	$x_1(h)$	$x_2(l)$	$x_3(t)$	$x_4(b)$	f_{cost}
	Optimal Design Variables				
APPROX (Ragsdell and Phillips, 1976)	0.2444	6.2189	8.2915	0.2444	2.3815
David (Ragsdell and Phillips, 1976)	0.2434	6.2552	8.2915	0.2444	2.3841
SIMPLEX (Ragsdell and Phillips, 1976)	0.2792	5.6256	7.7512	0.2796	2.5307
RANDOM (Ragsdell and Phillips, 1976)	0.4575	4.7313	5.0853	0.6600	4.1185
Deb (1991)	0.248900	6.173000	8.178900	0.253300	2.433116
Coello (2000)	0.208800	3.420500	8.997500	0.210000	1.748309
Coello and Montes (2002)	0.205986	3.471328	9.020224	0.206480	1.728226
He and Wang (2007)	0.202369	3.544214	9.048210	0.205723	1.728024
Mezura-Montes and Coello (2008)	0.199742	3.612060	9.037500	0.206082	1.737300
CryStAl (*present work*)	0.185658	3.635274	9.190371	0.199565	1.694518

Table 8. Optimum results of the welded beam design problem

Similarly, the results of CryStAl alongside several other metaheuristic algorithms in solving the welded beam (Tables 8 and 9) and pressure vessel (Tables 10 and 11) design problems are tabulated below, where some of the results are extracted from previous studies. Again, CryStAl turns out to produce better results in comparison with those of its competitors for both constrained optimization problems.

Tables 12 and 13 present the results of CryStAl and other algorithms in dealing with the car side impact problem in which the CryStAl provides outstanding results.

The results of CryStAl alongside other algorithms regarding the speed reducer problem are shown in Tables 14 and 15. It can be concluded that CryStAl provides better results in

Table 9. Statistical results of different methods for the welded beam design problem

Method	Best	Mean	Worst	Standard deviation
Ragsdell and Phillips (1976)	2.3815	N/A	N/A	N/A
Deb (1991)	2.433116	N/A	N/A	N/A
Coello (2000)	1.748309	1.771973	1.785835	0.011220
Coello and Montes (2002)	1.728226	1.792654	1.993408	0.074713
He and Wang (2007)	1.728024	1.748831	1.782143	0.012926
Mezura-Montes and Coello (2008)	1.737300	1.813290	1.994651	0.070500
CryStAl (*present work*)	1.694518	1.753322	1.810659	0.008365

Table 10. Optimum results for the pressure vessel design problem

Method	Optimal Design Variables				
	$x_1(T_s)$	$x_2(T_h)$	$x_3(R)$	$x_4(L)$	f_{cost}
Sandgren (1988)	1.125000	0.625000	47.700000	117.701000	8,129.1036
Kannan and Kramer (1994)	1.125000	0.625000	58.291000	43.690000	7,198.0428
Deb (1997)	0.937500	0.500000	48.329000	112.679000	6,410.3811
Coello (2000)	0.812500	0.437500	40.323900	200.000000	6,288.7445
Coello and Montes (2002)	0.812500	0.437500	42.097398	176.654050	6,059.9463
He and Wang (2007)	0.812500	0.437500	42.091266	176.746500	6,061.0777
Mezura-Montes and Coello (2008)	0.812500	0.437500	42.098087	176.640518	6,059.7456
CryStAl (*present work*)	12.924693	7.0834174	42.098445	176.636595	6,059.7143

Table 11. Statistical results of different methods for the pressure vessel design problem

Method	Best	Mean	Worst	Standard deviation
Sandgren (1988)	8,129.1036	N/A	N/A	N/A
Kannan and Kramer (1994)	7,198.0428	N/A	N/A	N/A
Deb (1997)	6,410.3811	N/A	N/A	N/A
Coello (2000)	6,288.7445	6,293.8432	6,308.1497	7.4133
Coello and Montes (2002)	6,059.9463	6,177.2533	6,469.3220	130.9297
He and Wang (2007)	6,061.0777	6,147.1332	6,363.8041	86.4545
Mezura-Montes and Coello (2008)	6,059.7456	6,850.0049	7,332.8798	426.0000
CryStAl (*present work*)	6,059.7143	6,582.5273	6,370.7797	5.8426

this case which represents its ability in dealing with these kinds of difficult constrained problems.

Table 12. Optimum results for the car side impact problem

Algorithm	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{11}	x_{12}
ABC	0.5000000	1.0624205	0.5148211	1.4491503	0.5000000	1.5000000	0.5000000	0.345000	0.1920000	−29.34755	0.7410998
PSO	0.5000000	1.1165737	0.5000000	1.3018547	0.5000000	1.5000000	0.5000000	0.345000	0.345000	−19.52470	−0.019297
MFO	0.5000000	1.116539	0.5000000	1.301908	0.5000000	1.5000000	0.5000000	0.345000	0.345000	−19.5304	−0.000006
ALO	0.5000000	1.115960	0.5000000	1.302860	0.5000000	1.5000000	0.5000000	0.345000	0.192000	−19.6330	0.023649
ER-WCA	0.5000000	1.118688	0.5000000	1.298407	0.5000000	1.5000000	0.5000000	0.345000	0.192000	−19.1461	−0.01527
GWO	0.5000000	1.111484	0.5000000	1.312203	0.501214	1.5000000	0.500034	0.345000	0.192000	−20.6057	−25531
WCA	0.5000000	1.1155932	0.5000000	1.3034919	0.5000146	1.5000000	0.5000000	0.345000	0.192000	−19.69967	−0.023854
MBA	0.5000000	11.1172701	0.5000000	1.30008438	0.5000000	1.4999867	0.5000000	0.345000	0.345000	−19.40045	−0.379205
SSA	0.5000000	1.1093195	0.5000000	1.3148010	0.5000000	1.4999998	0.5000000	0.345000	0.192000	−20.821793	0.4412962
WOA	0.5000000	1.108001	0.534477	1.305770	0.5000000	1.473844	0.5000000	0.345000	0.192000	−19.69924	3.4816923
CryStAl (*present work*)	0.5000000	1.234118	0.5000000	1.187158	0.875000	0.892384	0.4000000	0.344682	0.196336	1.499999	0.5748396

Source(s): Competing algorithms are adapted from Yildiz *et al.* (2020)

Table 13. The comparative results of CryStAl for the car side impact problem

Algorithm	Best	Mean	Worst	Standard deviation
ABC	23.175889625990923	23.860680484086661	25.010762794496625	3.7642E-01
PSO	22.842984930697273	23.613571153685552	26.190640350882905	7.5252E-01
MFO	22.842970873572792	22.972834963056012	23.687547312526856	2.0794E-01
ALO	22.842980706120642	23.108402571838820	23.824366429288702	2.9093E-01
ER-WCA	22.843264619959352	23.069925342953958	24.455312800924212	3.5021E-01
GWO	22.852792762688743	22.992226614913008	23.347095471895521	1.6277E-01
WCA	22.843036481964047	22.975164427881293	23.370933765943949	1.9772E-01
MBA	22.843596400842499	22.936421047192962	23.488942174549098	1.5258E-01
SSA	22.846514099392973	23.253716124255313	23.829530847339793	3.0557E-01
WOA	23.042162202328310	24.814486173621617	27.360813682283315	9.6570E-01
CryStAl (*present work*)	23.561584764484730	23.561952853347831	23.562036172317104	1.3629E-01

Source(s): Competing algorithms are adapted from Yildiz *et al.* (2020)

Table 14. Optimum results for the speed reducer problem

Algorithm	b	m	z	l_1	l_2	d_1	d_2
ABC	3.5	0.7	17	7.3	7.71532	3.350214	5.286654
PSO	3.5	0.7	17	7.3	7.71532	3.350214	5.286654
MFO	3.5	0.7	17	7.3	7.71532	3.350214	5.286654
ALO	3.5	0.7	17	7.472705	7.735382	3.350541	5.286661
ER-WCA	3.5	0.7	17	7.3	7.715319	3.350214	5.286654
GWO	3.500881	0.700096	17.00101	7.302118	7.719974	3.350684	5.286708
WCA	3.5	0.7	17	7.3	7.715319	3.350214	5.286654
MBA	3.5	0.7	17	7.3	7.71532	3.350214	5.286654
SSA	3.5	0.7	17	7.36496	7.75803	3.35033	5.28666
WOA	3.500411	0.7	17	7.3	7.777372	3.352552	5.286675
CryStAl (*present work*)	3.5	0.7	17	7.3	7.7153	3.3505	5.2867

Source(s): Competing algorithms are adapted from Yildiz *et al.* (2020)

Table 15. The comparative results of CryStAl for the speed reducer problem

Algorithm	Best	Mean	Worst	Standard deviation
ABC	*2994.471067504619	2994.471075844169	2994.471115543837	9.2123E-06
PSO	*2994.471069674640	3070.655058796543	3209.297397650784	5.8657E+01
MFO	*2994.471066146822	2.994471066147108	2.994471066151665	7.3921E-10
ALO	2996.521745443848	3005.644279605541	3014.379001168207	4.7422E+00
ER-WCA	*2994.471066146826	2996.744541331202	3007.436552164085	4.3876E+00
GWO	*2995.704434912354	3001.556162056451	3009.944296784721	4.1218E+00
WCA	*2994.471066147307	2996.203773574547	3016.578575484153	4.8705E+00
MBA	*2994.471371019410	2944.744437623391	2994.484788566012	2.4195E-03
SSA	2996.021720467607	3005.574377149090	3015.662612037751	4.63871E+00
WOA	2996.604340024459	3.042915023571878	3233.598124214217	4.0888E+01
CryStAl (*present work*)	2994.424465756737	2996.852815956722	2994.844306772722	1.927E-03

Note(s): * These results do not satisfy the provided constraints of the speed reducer problem
Source(s): Competing algorithms are adapted from Yildiz *et al.* (2020)

5. Conclusions

This paper investigated the overall performance of a recently developed metaheuristic algorithm called CryStAl, which is inspired by the spatial symmetry in the structural configurations of crystalline solids, in dealing with mechanical engineering design problems. A total number of 20 mathematical functions were utilized as test functions to evaluate the overall performance of the proposed method. Furthermore, to validate the results of this algorithm, various classical and modern metaheuristic algorithms were selected from the literature for comparative purposes, followed by a statistical analysis of the outputs. Besides, three well-known engineering design examples were chosen to examine the capabilities of this algorithm in solving challenging optimization problems. The results obtained from the analyses demonstrated that CryStAl is superior to the other metaheuristics in most of the examined cases. It should be mentioned that this study was concerned with the applicability of this new algorithm to solving a well-known range of mechanical design optimization problems. As future work, further research is necessary to examine the utility of this method in dealing with challenging problems in other fields of science and engineering.

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