Spatial Markov Chains Implemented in GIS

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***Abstract*— This paper presents a novel approach that simulates the expansion of urban sprawl based on the Spatial Markov Chains and a random diffusion rule at the pixel level. Goodness fit metrics are selected to compare the accuracy of the satellite images against the model's simulation results. Our approach is evaluated using Landsat 8 satellite's raster images from the Toluca Metropolitan Area. Experimental results indicate that our approach is robust for spatial analysis, providing a more suitable alternative model to the traditional urban sprawl approach.**

Keywords—: Urban expansion, Spatial Markov Chains, random diffusion rule, goodness fit metrics

I. INTRODUCTION

This paper presents a methodology that combines mathematical methods, programming and Geographic Information Systems (GIS) to simulate the expansion of the urban sprawl based on novel spatial variation of the Markov Chains (MC). This methodology is tested with goodness fit metrics to assess the accuracy of the simulation performance. That is, to use indicators that measure how similar the results of the model are with respect to what happens in reality. In this paper four goodness fit metrics are used: Cohen's Kappa Index, Jaccard Index, Fractal Dimension and Shannon's Entropy. If the model acceptable resemble the real sprawl, then it can be used as an instrument that generates relevant information about the future of the city's growth.

In fact, our method is a procedure in discrete time, where the value in time t_1 depends on the values of the times t_0 and t (i.e. second order markovian chain). The algorithm compares and counts data of two raster-based maps, where the occupied floor is coded as one and the unoccupied one as zero, these data estimate and configure a transition probability matrix. In our case study, we are using maps of the urban sprawl of the city of Toluca from 2003 and 2017. The prediction is done in a map of the city of Toluca of 2031, where there is occupied land and unoccupied ground at the pixel level that expresses the probability of changing or belonging to the analyzed category (i.e. binary category). The results are series of binary probability maps for time t_1, as a projection from t. To do this, we consider the number of temporary units (years in our case) elapsed between t_0 and t, assuming that it is a linear evolution. In the case of the metropolitan area of Toluca, it has been projected to 2031 from the years 2003 and 2017. By having the projections of the metropolitan area and analyzing the goodness fit indicators we observed that our Spatial Markov Chains' model resembles the real sprawl with an acceptable accuracy.

An analytical approach of Socially Integrated Social Sciences is applied. This approach integrates mathematical and technological model, such as satellite photos, Geographic Information Systems, automated computer developments (had-hoc programs) and a spatial vision to analyze social processes. Our methodology is novel since MC are computed at the pixel scale, in comparison to the typical spatial Markov Chains that only incorporates the spatial interaction between regions. Contrarily to the search for intra-distributional dynamics at the municipal level through spatial Markov Chains, our work do analyze the relationship between pixels within a map [1, 2].

Several investigations combinate MC, cellular automata (CA) and GIS's, where the MC determines the potential transition of the states in the neighborhood. While the CA controls the spatial change through the global rules, considering the configuration of the neighborhood and the GIS show the input and output maps of the model [3, 4]. In this work, MC and GIS are combined, leaving aside CAs, since only a local random growth rule is used. By using this scheme, an spatial Markov Chains Model is generated, which is strong in time and space.

II. MARKOV CHAINS

The MC shows the transition from one state to another within a finite number of possible states. It is the most useful method for modeling stochastic processes and probabilistic evolution, knowing only the present situation. The growth of urban sprawl and many other processes that can be observed over time are modeled by stochastic processes, like any random variable collection $\{X(t)\}$ that depends only on time.

A stochastic process X has the Markovian property if the conditional of any future event t_1 is independent of the past event, it only depends on the current state of the process. In this case, the process has no memory, if for all integers $n>=0$ and all states $(i_0, i_1, \ldots, i_{n-1}, i, j)$:

$$P\,(X_{n+1} = j \mid X_n = i,\ X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) = P\,(X_{n+1} = j \mid X_n = i) \quad (1)$$

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Equation 1, indicates the process in state i at time n, and $\{X_n\}_{n>=0}$ a discrete stochastic process with state space $E=\{i,j,k,..\}$.

Note that the typical MC's method considers the probabilistic situation of temporal change, but not spatial; therefore, within our approach the image in t is used to apply the transition probability and incorporates a random diffusion rule that spatially locates pixels that have the highest probability of change in each category. With this procedure the probability of change depends on the number of pixels and their relationship within a neighborhood. Hence, an Spatial Markov Chain is generated based on time and space.

III. Model for GIS

Our developed Markov chains tool is an extension coded in the programming language Python for GIS environments. This tool calculates the transition matrix from two input images in raster file format with the projection based on the transition matrix and a secondary raster file, which shows only the new pixels that are part of the projection, thus giving the possibility to generate adequate explanations from a particular phenomenon. Figures 1a and 1b show images of the city of Toluca where the urban area is compared in two years, 2003 and 2017. Figure 1c presents the area's projection in year 2031, whereas Figure 1d depicted projection pixels from the 2031 raster file.

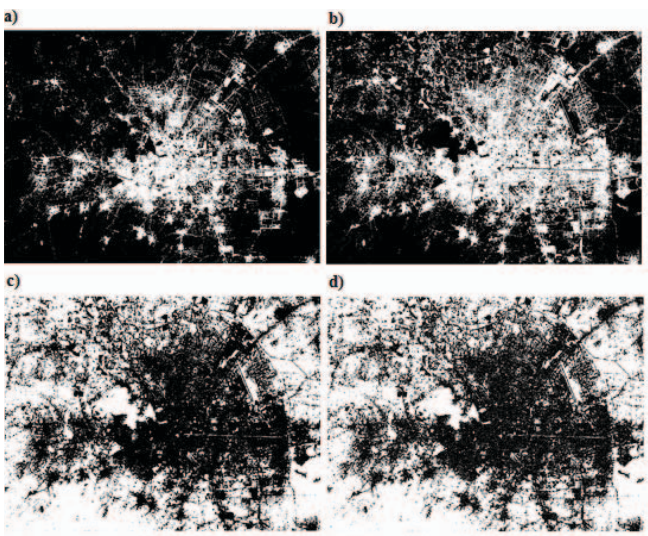

Figure 1: Images from the city of Toluca in different years: a) 2003, b) 2017, c) 2031 projection, and d) 2031 projection pixels.

IV. Random Diffusion Rule

In this section, we present our diffusion rule methodology. To better understand our method, a case of study is used to show how projection pixels are computed by using the Spatial Markov Chains Model. In Fig. 2, the growth cases covered by the diffusion rule are shown when going through the study image, as well as its algorithm.

Programmed expansion-diffusion rules are based on the conventional transition matrix calculation. After that, following algorithm is executed:

1) A point in the grid is randomly selected.
2) If selected point is a black pixel or cell, its eight surrounding neighbors are turned into black pixels or cells. Note that a black pixel can be rewrote in this step.
3) Number of pixels or cells in the transition matrix is decreased.
4) If selected point is a white pixel or cell, it is not considered.
5) Go back to step one until the transition matrix is completed.

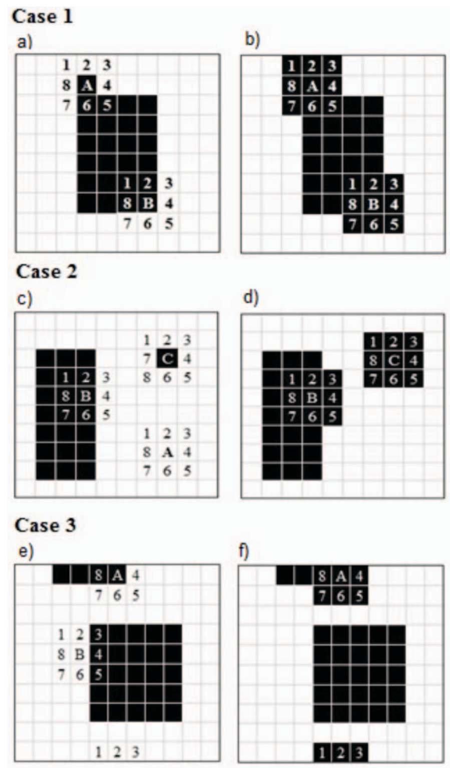

Figure 2: Three growing cases based on the central pixel: A, B, or C, respectively.

Our random diffusion rules proposed in this paper define a neighborhood for each pixel or point in the territory, assuming that what happens to one pixel, also affects its neighbors.

Other investigations have used a limited aggregation model that is similar to our proposed random diffusion rule, in which urban pixels are randomly spread over a probabilistic field until they join another pixel [5, 6, 7].

V. Goodness Adjustment Method

For the validation of the method it is necessary to compare real against simulated maps. That is, compare the results generated by the proposed technique, with respect to the real data to decide how accurate the model is to resemble real maps. The comparison is not simple, and it is essential to adopt feasible methods or techniques [8,9,10], for that reason,

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four metrics are selected in this research: Cohen's Kappa Index, Jaccard index, Fractal dimension and Shannon entropy.

A. Cohen's Kappa Index

Cohen's Kappa Index (k) is a map comparison measure that adjusts the effect of change. The value k can take values between -1.0 and +1.0 and is interpreted in a similar way to the Pearson correlation index. The closer to +1.0, the greater the degree of agreement between the maps is. The closer to -1.0, the higher the degree of discordance between the maps is. An index equals to 0.0 indicates absence of similarity between the maps. The important thing about the results of Cohen's Kappa Index is that they control the effect of change and it is possible to know if their results are statistically significant [11,12,13]:

$$K = \frac{P_0 - P_e}{1 - P_e} \tag{2}$$

Where P_0 is the observed similarity ratio, P_e is the expected similarity ratio and $1 - P_e$ represents the similarity or maximum match. If the Kappa index is equal to 1.0, it means that the similarity between the maps is perfect, when that index is 0.0 the maps have zero similarity (i.e. they are completely different). Finaly, a Kappa index equals to -1.0 means that one map is the inverse of the other [13,14].

Thresholds for Kappa Index have been proposed by [13] and they are shown in Table 1. Note that those values are note generalized among researchers in the field. However, this is suitable guide when experimental images' resolutions is high [14], which is our case with about 4.0 million pixel images.

Kappa Index	Similarity
<0	Not equal
0.0 - 0.2	Not relevant
0.2 - 0.4	Low
0.4 - 0.6	Moderate
0.6 - 0.8	Good
0.8 - 1.0	Very Good

Table 1: Similarity values for Kappa Index [13].

B. Jaccard Index

The Jaccard Similarity Index (I_j) measures the similarity degree between two images (e.g. maps), considering both the number of pixels as well as their position within the maps [15]. The Jaccard index value goes from 0.0 to 1.0, a 0.0 index value means a total dissimilarity between the maps, contrarily, an index value equals to 1.0, means a complete similarity. The Jaccard Index is computed as:

$$I_j = \frac{T_{11}}{T_{21} + T_{12} + T_{11}} \tag{3}$$

The Jaccard Index is easy to analyze using set theory [14]. Let us name elements in set A as T_{21} and elements in set B as T_{12}. Then, the union between A and B ($A \cup B$) are denominated as T_{11}, whereas elements outside the union are labeled as T_{22}.

Remember that Jaccard Index (Equation 3) calculates two key aspects to compare a pair of maps: the equality of the raster maps and the pixels' *position* correspondence between the maps.

C. Fractal Dimension

Fractals were introduced by [16], in a study of irregular and fragmented structures presented at different scales (e.g. as the coasts or mountains). The appearance of a structure at different scales is called self-similarity, since each part, whatever its degree of approach is, it is presented as the original figure. If we observed at the microscopic level an object it could be noted its geometric characteristics, which would be in somehow the fractal's theory fundamentals.

Then, using the object's degree of irregularity and fragmentation in its form (e.g. in this work maps), it is possible to be measured using Fractal Dimension (*D*), which is a expressed value from a geometric point of view. When measuring irregular objects, the Fractal Dimension is a non-integer value (i.e. is a fractional number), whereas in the Euclidean space, values are well defined as $D=0$ (point), $D=1$ (line), $D=2$ (two-dimensional plane) and $D=3$ (volume) [17].

Note that considered dimension is also the fractal's growth dimension, since $D = 1$ is a line and $D = 2$ is a plane. Therefore, the fractal dimension used in this research would be a real value between 1 and 2. If the metric is close to one, we can say that the projection has no points, it is empty (i.e. a decrease in the model's projection). The other case, if the measurement is close to two, we can say that the projection is almost full of points (i.e. an increase in the model's projection).

The calculation of *D* is based on the corresponding measurement of the number of black pixels that cover a certain image. The fractal dimension, *D*, is computed by:

$$D = \frac{Log\,(N(L))}{\text{Log}\left(\frac{1}{L}\right)} \tag{4}$$

Where L is the scale factor and N is the number of similar objects. The relationship between N(L) and 1/L is referred as potential relationship, which is also called the box counting method and it measures the growth or contraction of the urban sprawl. When using the fractal concept many fractal objects have been found not only in different natural systems (irregular and fragmented systems), but also in social systems and in socio-spatial structures [18]. With this we can differentiate between perfectly self-similar fractals (generated through iterative processes in a regularly way) and fractals whose self-similarity is basically statistical (non-deteministic or generated through a stochastic process) [7].

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D. Shannon Entropy

Shannon Entropy is a concept that has been used to describe the structure and behavior of different systems [19]. In this paper, the application of the entropy measure in urban expansion is proposed to determine the spatial concentration and dispersion.

Shannon's entropy indicates the proportion of the maximum possible dispersion in which a variable is distributed among categories or spatial zones. That is, it has a value of 1.0, the variable is evenly distributed among all the zones, and if it approaches 0.0, the variable is concentrated in a small number of zones [20]. It is expressed by equation:

$$E_n = \frac{\sum_1^n p_1 \log(1/p_1)}{\log n} \quad (5)$$

Where E_n is the relative entropy, $p(x_i)$ is the probability that the variable x is in one of the zones, classes or categories.

The calculation of Shannon Entropy is an index of urban expansion that uses data obtained remotely (e.g. with GIS), which can identify and efficiently characterize the degree of spatial concentration or dispersion in a specific area [22,10].

With the value of entropy, we can see that, for all the dimensions of urban systems, there is a range of values in which entropy could be tolerated without compromising its efficiency and/or resilience. If the entropy has a value near zero, then the urban area is too uniform, therefore, vulnerable to changes or disasters. If the entropy has a value close to one, then the urban system will not be able to allocate efficiently the resources necessary for the system to work [19,20].

Therefore, entropy must be maintained within a range defined by the minimum value, below which the system becomes vulnerable and unstable, and at the maximum value, above which the system becomes unsustainable [10].

Goodness fit metrics presented in this section are calculated by the *Territorial Intelligence Station*: CHRISTALLER®.

VI. Results

The classification of the occupied and empty space gives a sample of the growth of the urban spot in a determined period. Consequently, the MC allow us to know the probability that the pixels are kept in one or another classification of occupied or empty space. A simple and efficient way to discretize the categories is to label or set as 1 to the occupied space and 0 to the empty space. Table 2 shows the states or categories in which the urban space of each city is classified. *Total Initial Vectors (s)* were constructed from the binary pixel count in each category per year, in relation to the total.

CITY	TOLUCA		
	1	0	Total
2003	736513	2946179	3682692
%	20.0	80.0	100
2017	1220632	2462060	3682692
%	33.1	66.9	100

Table 2: Initial state vectors, 2003-2017, computed by CHRISTALLER®.

Initial state vectors show the increase or change of pixel status in 2003 and 2017. Those that remain in 1 went from 736,513 to 1,220,632 (an increase of 11.1%) and those that remained in 0 decreased from 2,946,179 to 2,462,060 (a decrease of 13.1%). In total in the city of Toluca, the pixels that change their state (i.e. the urban area that is modified or grows) are 484,119 in 14 years (an increase of 13.1%).

1) *Transition matrix:2003-2017*

One way to observe the changes within the distribution over time are the MCs where the main input is the matrices of transition probabilities, which represent the probability of being in a k state in the t_1 period, starting with the distribution in the t period [21].

The construction of the transition matrix for 2003-2017 is generated by a count of changes between the two categories of pixels (0 and 1) from one year to the next. This is done by using CHRISTALLER® applications to compute the transition matrix as shown in Table 3.

		2017		
		0	1	n
2003	0	80	20	2946179
	1	15	85	736513
				3682692

Table 3: Transition Matrix 2003-2017 computed by CHRISTALLER®.

Table 3 shows that, in 14 years, when measuring the growth of cities, 2,349,918 remain at 80% at zero (no occupied land). There is a transition from zero to one (change to occupied land) 596,261 equals to 20%. Change from one to zero 112,142 equals to 15%, land that became unoccupied; and finally land that remains in occupied land status 624,371 equals to 85%. Therefore, it can be inferred that there is a large amount of land that has not been occupied or that has changed its status.

2) *Second transition state: 2017-2031*

Table 4 is the new transition matrix that is a *projection* of binary data that will be in the 2017-2031 map of Toluca and that maintained the conditions for counting.

		2031		
		0	1	n_1
2017	0	67	33	1972089
	1	25	75	1710603
				3682692

Table 4: Projection matrix, 2017-2031 computed by CHRISTALLER®.

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Main changes in the period 2003-2017 are: 3,249,918 pixels that were in the category of unoccupied land (pixels in zero that remain zero), which was equivalent to 80% of the total. Projection in Table 4 indicates that by 2031 the probability that the pixels remain in that category is reduced to 67%. The probability of transiting from zero to one would be increased from 20% to 33%, whereas the probability of transiting from one to zero value goes from 15% to 25%. In short, the projection suggests an important progressive redistribution of living or working conditions, therefore there is more occupied than empty land.

3 *Results of the urban expansion model with Spatial Markov Chains*

The satellite images that we use as a reference for the results of the Space Markov Chains Model were obtained from Landsat 8. The image of 2003, together with the image of 2017, are the maps with which the transition matrix is made, which is the entry for the Markov Chains, used as the input data for the model in CHRISTALLER®.

Using the tools of CHRISTALLER®, the comparison indicators are estimated, in addition to the growth indicator and the distribution indicator that validates the random growth rule used in the Markov Space Chains, which shows how effective the methodology is in the expansion of the urban sprawl in the city of Toluca during the period of experimentation.

To make a model test bench it is necessary to make a projection using the Toluca map as a reference in 2003 to project the map in 2017 and thus make the comparison with the real map of 2017. To carry out the diffusion or expansion, values are taken of Table 3, which correspond to the transition matrix. From the matrix, only the values that change from zero to one are 596,261 pixels and those that remain in one are 624,371 pixels, giving a total of 1,220,632 pixels that will change in the 2003 map.

With the projection of the city of Toluca for 2017, by means of the random diffusion rule we can observe how efficient is the method of pixel distribution. As mentioned, the maximum value of Cohen and Jaccard's Kappa indices is 1.0. Cohen's Kappa index for Toluca is 0.78, indicating that the diffusion rule is *good* in the process of urban sprawl expansion.

The Jaccard index shows even more encouraging results for the random rule. Toluca registered a Jaccard of 0.85, which indicates a high capacity of the rule to distribute the pixels on the map with high efficiency. The values of the comparison indicators are good, the rule allows us to distribute the pixels with less uncertainty in the expansion of the urban spot for the city of Toluca, for a period of fourteen years.

By joining the *random diffusion rule* to Markov Chains, the Spatial Markov Chaining Model is generated, and the Fractal Dimension is calculated to determine how much the urban stain grew, showing good results (Table 5). The differences in growth between the satellite image of 2017 and the projection using the Space Markov Chaining Model made by CHRISTALLER® register a variation of less than one tenth, which can be considered a very good adjustment.

City	2003	2017	Projection 2017	Projection 2031
Toluca	1.74	1.85	1.79	1.92

Table 5: Fractal Dimension Indices computed by CHRISTALLER®.

On the other hand, Table 6 shows the Shannon Entropy index for the city of Toluca, remembering that the maximum value of this index is 1.0. The entropy index shows a uniform distribution of the concentration of the pixels in the metropolitan area. This shows a growth of urban sprawl.

City	2003	2017	Projection 2017	Projection 2031
Toluca	0.72	0.91	0.89	0.99

Table 6: Shannon Entropy indices computed by CHRISTALLER®.

The Shannon Entropy index for the city of Toluca in 2003-2017 and the projection 2017, indicates a good distribution of pixels with a difference of almost two tenths, indicating that the urban spot is growing adequately (e.g. it does not grow in just one area, but in all areas of the map). In the projection made by the Spatial Markov Chains model when calculating the entropy results in very close to one, this is interesting, because, apparently, the speed of growth of the urban sprawl is high, therefore, the value indicates that growth is very well distributed, this is a conjecture that should be tested considering more cities.

Despite used files and images are massive and highly complex, CHRISTALLER® was able to handle all this information. The satellite images of the city of Toluca contain around four million pixels. Considering the magnitude of the information, the complexity of the calculations, the use of the random diffusion rule, the iterative estimation of the urban comparison or expansion indices, and the automatic generation of the simulation map, we can consider that the Space Markov Chains Model performance by using CHRISTALLER® is very fast and good in the projection, as it was expected. On average, CHRISTALLER® requires a minute to generate the results and projections of the city, these are very good results in the application.

Now having a certainty that the proposed model of Space Markov Chains is efficient and has good results, we can generate a projection of the city of Toluca to 2031. This map shows a balance in the values that changed from one to zero and conversely, therefore, it can be said that in a period of 14 years the growth speed of the city of Toluca decreases, but it is necessary to verify this result with tests that are not discussed in this paper.

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The visual comparison showed how the model developed with Spatial Markov Chains in the growth of urban sprawl, responds to the basic pattern that occurs, but showing a greater and more compact growth. In addition, with this model the new growth of the urban sprawl not only adds existing buildings, but also disappears new isolated urbanizations due to lack of neighborhood or space constraints.

VII. CONCLUSIONS

In this work, an *innovative* technique was adopted to explore the growth of urban sprawl in the city of Toluca: *Space Markov Chains*. The objective is to show how this growth-projection technique works, taking as an example the metropolitan area of the city of Toluca that can be said to have a high growth rate because it is a millionaire city in relation to its inhabitants. The results show that the analysis approach using Spatial Markov Chains offers valuable information and has an alternative vision to the traditional approach of growth with regions, focusing mainly on the growth of the urban spot with pixels in binary maps in raster files.

In Spatial Markov Chains, the most remarkable thing is its Spatial-Temporal analysis, which is very difficult to find in techniques that project maps or perform growth on maps. The power that the Markov Chains have is the time, is what is used to only add the space with a *random distribution rule* in the distribution, which is the main contribution of this work. Projections are generated over 14 years, where time is a constant, dependent on the maps of the city that we have available.

The techniques of comparison of maps and comparison-position of the urban spot leads us to use statistical comparison techniques, such as *Cohen's Kappa Index* and the *Jaccard Index*. Where it is observed that the results were encouraging and precise.

The procedures of comparison of maps, led us to explore techniques, which are rarely used in cartographic research. These techniques are the *fractal dimension* and the *entropy of Shannon*. Which is an important contribution in our research by showing the growth-diffusion of the urban sprawl.

With the results shown by the Spatial Markov Chains and GIS, we can say that it is possible to build a predictive model. The projection of the data is done with Markov Chains, which is a technique already proven in related literature. This technique gives us a projection in certain years, it depends on the construction of the transition matrix. Also, it is important to remark that to generate the projection in space we add a rule that we baptize it as a *rule of random diffusion* to get Spatial Markov Chains.

It can be concluded that the combination of mathematical tools and GIS is a powerful tool for analysis, supervision and control of urban phenomena; especially, having maps or images with high spatial and temporal resolution will improve indices and cities' projection accuracy.

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