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Abstract
This paper evaluates the effects of redistribution policies on macroeconomic performance, income distribution and social welfare using three alternative social welfare criteria (Pareto's, Rawls' and a mixed one), and establishes the link between the above and the ruler's optimal policy. The main conclusions are the following: 1) a negative effect of redistribution through taxation and subsidies is the increase in the interest rate; 2) as a society advances in the process of wealth redistribution, there comes a time when the ruler faces a trade-off: to maintain this process or to avoid losses in social welfare.

Keywords
Income Distribution; Income Tax; Subventions; Interest Rate; Social Welfare Criteria

JEL classification:
D31, D33, D61, D63, E20, E25, E43, E62, I31

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I. Introduction

In countries with a relatively high-income concentration, the issue of redistribution policy is persistent, especially if it is common among a great fraction of the population to believe that the main determinants of such concentration are inheritance, political influence, undue exploitation of public resources and obstacles to legitimate access to individual development opportunities.

What can economists say about the scope and limitations of income redistribution policy and its social benefits and costs? Much, and in fact their contributions have been enormous, and not only in the theoretical field but also in applications to concrete cases for specific issues, regions and countries. Thus, what could be an additional contribution, such as contained in this document, is only marginal. This paragraph will be clarified in what follows.

From the academic point of view, the issues of income distribution and redistribution policy belong to the Welfare Economics. And it has been usual in this field to analyze and discuss its topics, including those of income distribution, based on alternative criteria of judgment on what leads to an improvement or worsening of social welfare, but without going into the consequences of redistributive policy on the macroeconomic intertemporal performance and, so, on its possible indirect negatives effects on the poor people.

On the other hand, in the field of Macroeconomics, the usual approach has been to evaluate the effects of changes in parameters and exogenous variables associated with redistributive policy on social welfare with a single welfare criterion (the Pareto´s one).

The objective of this paper is to contribute to reduce the gap between both approaches with a two-stage method: 1) to use a simple intertemporal macroeconomic model to evaluate the effects of redistributive policies on macroeconomic performance, income distribution and social welfare using three alternative criteria and, therefore, the social welfare functions associated with these criteria; and 2) to establish the link between stage 1 and the analysis of the optimal policy from the ruler's point of view. The achievement of such an objective would give some "added value" to this paper.

However, a word of caution is in order: the analysis omits a very important issue for the design of redistributive policies, but already well developed in several fields of economic theory: that of the effects of taxes and subsidies on the incentives influencing the labor and human capital supply, and on the optimal levels of physical capital. This omission is due to the convenience of being theoretically clear and easily quantifiable those effects of the redistributive policy that we try to capture, isolating them from the effects on incentives.

In what follows, these are the subjects: literature on social welfare review (section II); three criteria of social welfare (section III); comparative statics exercises with an intertemporal macroeconomic model and judgment of its results in the light of these three criteria (section IV); discussion on the relationship between the previous issues and what is optimal for the ruler (section V), and conclusions (section VI). Annex 1 consists of a specific model that describes an optimal situation from the point of view of the ruler. This is presented in order to offer the reader a simplified but concrete example of the analysis proposed in section V. Statistical information supporting a hypothesis provided in Section V is in Annex 2.

II. Review of the Literature on Social Welfare

In formal terms, social welfare is or could be something that has measurement and also the function that defines it and expresses its determinant variables. It can be a sum or a weighted average of individual welfare or utility indicators (Marshall [1922], Samuelson [1947], Harsanyi [1955], Ng [1984], Kaplow [2020]: the cardinal utility stream) or it can be something more general: a vector of such indicators (Pareto [1897, cited by Bergson, 1938], Bergson [1938], Lange [1942]; Arrow [1950]: the stream of ordinal utility or the ordering of individual preferences), so the function might not have conventional properties such as continuity properties in the space of real magnitudes and be derivable. But in the latter case it is also feasible to deduce quantitative implications and to make quantitative measurements on changes (null, positive or negative) of individual and social welfare or even on the possibility of not being able to conclude whether certain hypothetical individual changes imply an improvement or a worsening of social welfare (as Arrow [1950] showed it; see also Ng [1984, Note 7, p. 1039]).

The history of the theory is long: it begins with Dupuit ([1844], quoted by Hotelling [1938]), advances enormously with Pareto, and continues to be enriched with contributions from the different currents of the marginalist school, especially with Marshall, Pigou (1920) and other representatives of the neoclassical branch. But all economists, regardless of their affiliation to any theoretical current, are obliged to draw conclusions about what, in their opinion and according to their recommendations, would imply an improvement or a worsening of well-being or social convenience. If they leave it implicit, others will know how to make it evident, as Harsanyi (1955) put it well.

What is the criterion for judging whether some change in the material situation of a certain individual implies some alteration in social welfare? The relevance of this question arises from the possibility that such a change is only possible if actions are performed influencing another individual welfare.

The traditional criterion has been the Pareto´s one: if the change in the situation of one individual (consumer) does not entail a detriment to another, it can be said to improve social welfare (a "Pareto improvement"). If such a change implies a detriment to another, it can be said that the change implies abandoning a Pareto optimum or moving from a sub-optimal to another sub-optimal. The Pareto criterion is that of social efficiency; indeed, if at least one individual improves his (her) material welfare without detriment to that of another, it can be said that there was something that improved the efficiency of society made in such improvement possible (on this subject, see Konow, 2003, pp. 1201-2). Closely associated with this criterion is the traditional social welfare function, the so-called "Bergson-Samuelson", which only takes into consideration the (cardinal) utilities of individuals. We will use it throughout this paper.

There are other criteria; the main ones are two: 1) to judge the change in social welfare on the basis of a function in which individual utilities are weighted with some factor derived from some ethical criterion; and 2) to identify the variation in social welfare with the change in the situation of those individuals who were in the worst material situation without having done something consciously and voluntarily that would have led them to such an extreme. This is the Rawls´s criterion (1971; on Rawls, justice, and the problem of efficiency versus egalitarianism, and "injustice" associated with birth and luck, see Konow 2003).

III. Three Social Welfare Functions and their Criteria[1]

Under the assumption of cardinal utility, the traditional social welfare function (the Bergson-Samuelson) is the *B* function:

$$B = \sum_{i=1}^{P} u_i$$

Where *P* is the number of individuals in a certain society, and u_i the total utility perceived by individual *i* (itself a function of the quantities of goods whose consumption generates it), given its budget constraint.

The second function we consider is *S* (Foster-Sen), which originates from function *B*, but is weighted by an inequality index:

$$S = \left(\sum_{i=1}^{P} u_i\right) e^{-T_L}; \;\; T_L = \frac{1}{P}\sum_{i=1}^{N} ln\left(\frac{\mu}{x_i}\right); \;\; \mu = \frac{1}{P}\sum_{i=1}^{P} x_i$$

T_L being an index (Theil index) of the income (or wealth, etc.) inequality degree, and μ the average value of individual incomes (see: Sen [1997]).

Thus, the function *S* generates a lower value for social welfare the greater the income (or wealth) inequality by way of reducing the weight of the utility of those with relatively high incomes (the greater the negative weighting the greater their income relative to the average), and "reinforcing" with greater weight the utility of individuals with relatively lower incomes.

The third function is the one that would be the most compatible with Rawls's proposals, which we will call *R*:

$$R = min(u_1, \dots u_i, \dots, u_P\ )$$

[1] Several elements of this section are taken from *Wikipedia* (Terms: "Social welfare function" and "Theil index").

Therefore, the criteria corresponding to the *B* and *S* functions are to prefer the policy scenario that maximizes *B* or *S*, while the third criterion is to choose the scenario that maximizes *R*, that is, the scenario in which the least of all utilities is maximized (the "maximin" criterion) At this point it is interesting to read this statement by Konow (2003, p. 1235):

> "*equity (i.e., justice in the specific sense) guides but does not monopolize distributive preferences: people care about equity, but the allocations they prefer for themselves and consider right are also influenced by concerns for efficiency and need*".

The "need" dimension allows Konow to value (positively) Rawls' theses and proposals, but considering them as only one of the dimensions of justice.

IV. A Macroeconomic Model and Three Welfare Criteria

This section presents a model whose basic characteristics are the following: a) it contemplates two periods: present and future; b) there are three representative agents of individuals belonging to one of three social groups; these groups differ from each other by their level of wealth (understood this one as the present value of their income stream) and by their impatience (measured by their future utility or subjective discount rate). It is assumed that their information and forecasts are perfect, and they are optimizers; furthermore, it is assumed that the number of individuals and their aforementioned characteristics are the same in the present and in the future; that there is only one good; and that there is a government, i.e., public spending, subsidies and taxes; and these ones are different according to social groups. This model has its origin in one presented by Williamson (2014), but the current one is of heterogeneous agents, unlike the Williamson´s model.

The model has other characteristics that, without prejudice to the analysis of what is central to this paper, make it easier to use and clearer its results. These are: 1) closed economy; 2) absence of technical, demographic and preference changes, so in the steady state the total capital of the present is equal to that of the future (its level is constant in each scenario, but it turns out to be different between scenarios); 3) homogeneous, exogenous and fully employed labor supply; 4) production subject to constant returns to scale, so it is assumed, for simplicity, that there is only one firm in the economy; 5) there is only income tax and subsidies to individuals (there are none to the firm), and the tax rate is progressive; 6) the firm's shareholders receive as dividends all the profit (accounting profit), so the only saving made by the firm is that destined to capital replacement by depreciation; 7) the distribution of the population among the three social groups is exogenous and constant; 8) the three social groups are: (i) owners: their only source of income is dividends; (ii) mixed: individuals who receive dividends and salaries, and iii) employees (only receive salaries). Given that the shareholding concentration is assumed to be high, the income of the agent representing the people in the first group is substantially higher than that of the agent representing the people in the second group, and this agent, at his turn, has higher income than the agent who represents those which income is only wages.

Individual Consumption and Saving: The Microeconomic Level

Each individual i (and his representative agent) chooses optimally his present and future consumption levels taking into account his inter-temporal budget constraint. The problem it faces and solves is standard, namely: maximize its utility function subject to the above constraint:

$$Max_{(c_i,\ \ c_i')} \quad U_i\left(c_i, c_i'\right) = lnc_i + \beta_i lnc_i';\ \beta_i = \frac{1}{1+\theta_i}; \theta_i > 0;\ 0 < \beta_i < 1$$

$$subject\ to:$$

$$c_i + \frac{c_i'}{1+r} \leq y_i(1-t_i) + \frac{y_i'(1-t_i)}{1+r};\ \ -1 < t_i < 1$$

Where $c, \beta, \theta, r, y, t$ stands for consumption, discount factor, subjective discount rate, interest rate, income (dividends and/or wages) and income tax rate (equal in both periods); the superscript "comma" indicates the variable with this denotation belongs to the future (what is predicted today, with certainty, for tomorrow). In the event that a subsidy is granted, it is assumed to be allocated in the form of a negative tax rate. The subsidy, if any, is only received by the individual in the lowest income group.

From the above and from the first-order conditions (which, given the properties of the utility function, are both necessary and sufficient for optimality) the following follows:

$$(1) \qquad c_i' = c_i \beta_i (1+r);$$

$$(2) \qquad c_i = \frac{(y_i + \nu_i)(1-t_i) + \frac{(y_i' + \nu_i)(1-t_i)}{1+r}}{1+\beta_i}$$

$$Also:$$

$$(3) \quad s_i \equiv (y_i + \nu_i)(1-t_i) - c_i$$

Where s stands for saving (in the present period).

Production and Distribution: The Macroeconomic Level

The future gross total output (correctly predicted in the present) is:

$$(4) \quad Y' = Z' K'^{\alpha} N'^{1-\alpha}; 0 < \alpha < 1$$

From the optimal conditions of the firm:

$$\frac{\partial Y'}{\partial K'} - \delta = r; \qquad \frac{\partial Y'}{\partial N'} = w'$$

And from the assumptions of fully employed and exogenous labor supply we deduce the optimal levels of future capital and wage (per unit of labor time used)[2]:

$$(5) \quad K' = N' \left(\frac{\alpha Z'}{r+\delta}\right)^{\frac{1}{1-\alpha}}; \qquad (6) \quad w' = (1-\alpha) Z' K'^{\alpha} N'^{-\alpha}$$

Where $Y, Z, K, N, \alpha, \delta, r, w$ is output, the scale factor or total factor productivity (*TFP*), employed labor force, the output elasticity to capital, the capital depreciation rate, interest rate and wage[3].

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[2] This means the agents representing the individuals in groups 1 and 2 (the firm's shareholders) are in charge of the firm's optimizations.

[3] In a simplified model such as the present one the optimality conditions for consumption and capital are equivalent to a set of first-order conditions of a disaggregated general equilibrium model (multiple agents, multiple goods, multiple periods) Lange (1942) developed; this author summarized such a set as follows: "(such a set) implicitly determines the rate of capital accumulation that maximizes welfare over time ..."

As mentioned above, it is assumed that:

$$(7) \quad K = K^{'}; N = N^{'}; Z = Z^{'} \Rightarrow w = w'; Y = Y'$$

Further, what is distributed to the firm's shareholders (the dividends, $D, D^{'}$) are:

$$(8)\ D = Y - \delta K - wN = rK;$$

$$(9)\ D^{'} = Y^{'} - \delta K' - w^{'}N^{'};\ 0 < \delta \leq 1$$

And the capital ownership partake is equal to that of dividends, both in the present and in the future (and only for groups 1 and 2):

$$K_1 = \gamma K;\ K_2 = (1-\gamma)K;$$

$$(10) \quad D_1 = \gamma D;$$

$$(11)\ D_2 = (1-\gamma)D;\ 0 < \lambda < 1$$

γ being the proportion (exogenous) of capital belonging to group 1. The other equations of the model are as follows:

$$(12)\ Y = C + I + G;\ (13)\ Y^{'} = C^{'} + G^{'};$$

$$(14)\ C = \sum_{i=1}^{P} c_i = \sum_{j=1}^{3} \lambda_j c_j;\ C^{'} = \sum_{i=1}^{P} c_i^{'} = \sum_{j=1}^{3} \lambda_j c_j^{'};\ \lambda_j > 1$$

$$(15)\ I = \delta K = \delta K^{'};\ I' = 0$$

$$(16)\ G = T;\ G^{'} = T^{'};$$

$$(17)\ T = \sum_{i=1}^{P} t_i y_i = \sum_{j=1}^{3} \lambda_j y_j;\ T^{'} = \sum_{i=1}^{P} t_i y_i^{'} = \sum_{j=1}^{3} \lambda_j y_j^{'};$$

$$(18)\ Y - I - G - C = 0 = \sum_{j=1}^{3} \lambda_j s_j$$

I, λ_j, G being gross investment, the number of persons in group *j* who are represented by agent *j*, and the purchase of output by the government.

<u>The Personal Income Distribution</u>

To connect the factorial income distribution with its personal distribution we require some additional assumptions about the population break among groups receiving income (before taxes and subsidies) from labor and capital. Specifically, the following assumption is made:

5% of the population owns 70% of total capital and, therefore, receives, 70% of present and future dividends; this is group 1 or the capitalist one;

35% of the population owns 30% of the total capital and receives 30% of the dividends; in addition, it obtains a labor income; this is group 2 or "mixed”;

60% of the population receives only labor income (before taxes and subsidies); this is group 3 or "salaried".

Therefore, assuming that the population is 100 individuals, then: $\lambda_1 = 5$; $\lambda_2 = 35$; $\lambda_3 = 60$; $and\ \gamma = 0.7$.

With these assumptions, with the production and factorial distribution results of the macro sub-model, and given the values of personal income tax rates, the micro sub-model allows us to calculate individual income net of taxes (and after subsidies) and individual consumptions. Once these are estimated, the present and future aggregate consumption, the sum of personal savings, and the interest rate compatible with macroeconomic equilibrium are calculated. Since capital replacement (gross investment) in the first period is assumed to be equal to the part of gross output the firm reserves for depreciation, the function of the financial market is simply to balance the sum of individual savings with the sum of individual dis-savings, so the financial market and macroeconomic equilibrium involves setting an interest rate for which the aggregate net savings of individuals (i.e., agents) be 0, equal to net investment: 0.

The set of exogenous variables and parameters is: $\{\alpha, \gamma, \delta, \theta_j, \lambda_j, t_j, N, Z\},\ j = 1, 2, 3,$ while the core of endogenous variables is: $\{K, Y, D, w, y_j, c_j, r\},\ j = 1, 2, 3$

Knowing these endogenous variables, the others are immediately deduced.

The social welfare functions (in the framework of this model) are:

$$B = \sum_{j=1}^{3} \lambda_j U_j \left(c_j, c_j'\right)$$

$$S = \left[\sum_{j=1}^{3} \lambda_j U_j \left(c_j, c_j'\right)\right] e^{-TL};\ T_L = \frac{1}{3}\sum_{j=1}^{3} ln\left(\frac{\mu}{x_j}\right);\ \ \mu = \frac{1}{3}\sum_{j=1}^{3} x_j$$

$$Rawls\acute{}s\ Welfare\ Function = U_3$$

And, in all that follows, that which in the introduction and in section II was called the individual's income is: $x_{j\ (j=1,2,\ 3)}$.. This variable is the present value of agent j's income net of taxes (and subsidies).

Scenarios

Table 1 shows the numerical values of the parameters and exogenous variables of the scenario we call basic.

Table 1. Parameters and Exogenous Variables

Basic Scenario

α	γ	δ	$\theta_{j=1,2,3}$	$\lambda_1, \lambda_2, \lambda_3$	t_1, t_2, t_3	N	Z
0.47	0.7	1 (4.43% by year on 16 years)	0.53154 (2.7% by year on 16 years)	5; 35; 60	0.2; 0.1; 0.07	95	1200

Table 2 shows the characteristics of the other two scenarios in terms of their differences with respect to the basic scenario. It should be clarified that the last of the scenarios, scenario III, differs from the basic scenario by two features: a) it contemplates heterogeneity of the utility discount rates in a manner similar to the scenario II case[4], and b) a redistributive policy.

[4] Judd (1985) also considered the case of different discount rates among agents.

Table 2. Differences of other Scenarios vis-á-vis the Basic Scenario	
Scenario II	$\theta_1 = 0.53154;\ \theta_2 = 0.70737;\ \theta_3 = 0.99163$ [1]
Scenario III	The specifics of scenario II, and a redistribution policy: $t_1 = 0.25;\ t_3 = -0.05$
1. A very high discount rate of the poor agent utility is a way to take account of liquidity-credit constraints that make present consumption almost exclusively dependent on present income.	

The population and capital ownership distributions imposed on the model, and that of income (before taxes) generated by the basic scenario are presented in Table 3. This table also presents estimates of similar categories for Latin America as a whole made by the entity named *World Inequality Lab* and presented in its annual *World Inequality Report 2022* (WIR). From the comparison of the distributions of the model and those estimated by this entity, it can be deduced that what the model describes is not too far from the case of Latin America as a whole for population and capital, but it is for income participation of the poorest.

Table 3. Distributions (%)

	Population		Capital		Net Income (pre tax & subsidies)	
Group	*WIR*	Model	*WIR*	Model	*WIR*[1]	Model
The richer	10	5	77	70	56	21.3
Medium	40	35	22	30	34	34.8
The poorest	50	60	1	0	10	43.9
Total	100	100	100	100	100	100

1. Before taxes and some subsidies

As for the results (see Table 4), the first thing to observe is that related to the interest rate. If there is heterogeneity in subjective discount rates, being, as assumed, higher the less wealthy individuals are, then the redistributive policy generates an upward pressure on the interest rate, as will be made clear below. However, the heterogeneity of discount rates would explain an interest rate 24.4% higher than that of the basic scenario (compare scenario II with the basic scenario, according to Table 4), while the contribution of the redistribution policy to the increase in the interest rate is 4.5% (scenario II vis-á-vis scenario III).

But the magnitudes of the changes in the interest rate depend, of course, on we are assuming for the exogenous parameters and variables. In particular, if a more aggressive redistribution policy had been simulated in order to generate much larger and more significant magnitudes of the subsidy, the increase in the interest rate would have been much larger. On the other hand, if the redistributive policy had been aimed at favoring only 10% of the population (instead of the supposed 60%), then the impact on the interest rate would have been much less.

Table 4. Main Results (quantities)

Scenarios	r	Welfare Function: *B*	Welfare Function: *S*	Theil	Rawls: U_3
Basic	0.9763	1921.54	1338.38	0.3617	18.8502
II	1.2144	1775.60	1221.09	0.3744	16.9689
III	1.2695	1752.77	1266.71	0.3248	16.6155

The redistributive policy alters the structure of individual savings, thus: at an interest rate similar to that of the scenario II, the redistributive policy would induce falls in the savings of the richest by magnitudes greater than the increase it would generate in the savings of the poorest; an increase in the interest rate is therefore required for aggregate personal savings to be 0[5].

Table 5 presents welfare results in the form of an ordering according to the three criteria already mentioned: maximizing the *B* and *S* functions, or maximizing the welfare of the agent representing the poorest.

[5] The positive impact of a redistributive policy on the interest rate was also deduced by Benhabib *et al*. (2021) in a stochastic theoretical model in which the distribution of capital by income groups is endogenous.

Table 5. Main results (Scenarios ordering: Best:1; Worst: 3)			
Ordering criterium	**1**	**2**	**3**
B	Basic	II	III
S	Basic	III	II
$Rawls: U_3$	Basic	II	III

According to the three criteria, the best scenario is the basic one. But, assuming different types of discount, the best is scenario II. In other words, the redistributive policy simulated in this model does not maximize the welfare of the poorest, compared to scenario II. Why? Due to the negative indirect effect of the increase in the interest rate on wages. In fact, the wage corresponding to scenario III is 2.2% lower than of scenario II.

V. Discussion

Now we are going to suppose that, for some reason, the increase in the interest rate caused by the redistributive policy is so small that it does not prevent the subsidies or their increases in favor of the poorest group from having a positive impact on their welfare. In such a case, the question arises: what is the most appropriate policy? To answer, an approach neglecting the issue of the interest rate, that is, a static (not intertemporal) view such as the one presented in the following paragraphs, may be useful.

Also, it is now appropriate to discuss the issue of the choice of redistributive policy taking into account preferences and restrictions, and the search for an optimum. This section can therefore be framed within the public decisions theoretical stream inaugurated by Meltzer and Richard (1981; see: Stiglitz [1987]).

But before proceeding, one clarification should be made: An explicit distinction is made between the total amount of taxes collected (referred to here as *TB*) and the total amount of subsidies to be granted to the poorest (referred to as *V*).

To begin with, let us define an income (present value of income) distribution degree thus:

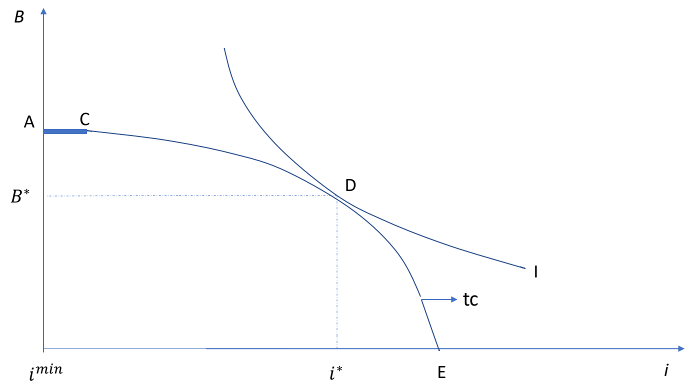
$$(20) \quad i \equiv e^{-T_L} = \frac{1}{e^{T_L}}$$

And recall that Theil's index (T_L) had been chosen as the index of inequality, thus: $T_L = \left(1/P\right) \sum_{i=1}^{N} \ln \left(\mu/x_i\right)$. The extreme case of perfect equality implies $\ln\left(\frac{\mu}{x_i}\right) = 0 \Rightarrow T_L = 0$, so $i = 1$. The other extreme is that of supreme inequality ($T_L \to \infty \Rightarrow i = 0$)

In all that follows, a distribution degree i will be understood as a degree such that: : $0 < i < 1$, i.e., it will be understood that *i* can be neither supreme inequality (*i*=0) nor perfect equality (*i*=1) but something between the extremes. And it will be assumed that whoever designs and implements the redistributive policy (say us: the ruler) only considers sensible that policy for which the increase in the distribution degree, *i*, goes always in the same direction as the increase in the income (present value of income) of the poorest, i.e. it will be assumed that the redistributive policy is subject to both the criterion of inequality aversion and the Rawlsian criterion of "maximin", but that the ruler also takes into account the criterion of social welfare (social efficiency: the Paretian one; the empirical evidence from experimental games seems to indicate that people adhere to the maximin criterion, in addition to two other criteria: efficiency and personal convenience; see: Engelmann and Strobel [2004]). The discussion on the choice of a redistributive policy will be made mainly in graphical terms.

Figure 1 represents the magnitude of the Bergson-Samuelson social welfare, *B*, on the vertical axis and the indicator of the distribution degree, *i*, on the horizontal axis. In this figure, a convex curve (looking at it from the origin) is represented as a set of pairs of values *B*, *i*. This curve, *I*, can be interpreted as an indifference curve, thus: the ruler, whether an autocrat or a representative agent of a majority of citizens who elected him, has preferences (or sticks to those of his constituents) on Bergson-Samuelson social welfare function and also on degrees of wealth distribution among his governed. Subsequently, the hypothesis about his utility function and the slope of the indifference curve for each social welfare – distribution degree pair is made explicit.

Figure 1. The ruler´s choice



For now, it will suffice to state that the ruler faces a trade-off between social welfare and income distribution and, therefore, a certain improvement in social welfare is indifferent for him if it is accompanied by a certain reduction in the distribution degree, and the other way around. And he really faces such a dilemma, and must seek an optimal solution to it, because he can modify both variables through fiscal policy (taxes and subsidies)[6].

In Figure 1 there is also a "quasi-curve" (*tc*, concave up to a certain point; then it becomes a straight line parallel to the horizontal axis). This quasi-curve is the transformation frontier of fiscal resources (present and future taxes) and their destinations (present and future subsidies) into outcomes that affect social welfare and the income distribution degree. The position of the transformation curve is conditional on the technology of production (on the parameters of the aggregate production function, including *TFP*), and on the technology of transforming taxes and subsidies into more or less social welfare and redistribution.

To understand precisely the meaning of the last three paragraphs, let us start from the government's inter-temporal budget constraint:

$$G + \frac{G^{\prime}}{1+r} = TB - V + \frac{TB^{\prime} - V^{\prime}}{1+r}$$

$$\Rightarrow V + \frac{V^{\prime}}{1+r} = TB - G + \frac{TB^{\prime} - G^{\prime}}{1+r}$$

[6] For a detailed analysis of this dilemma in the current case of China, see Ravallion and Chen, 2021.

We can continue to assume, as in the previous section, that public spending does not affect the welfare of any social group nor is it an instrument of redistribution. Therefore, we will assume that present and future public expenditures are constant magnitudes, while subsidies (to the poorest) and present and future taxes (or, in precise terms, their rates) are variable, and that they are instruments to modify the welfare of the different groups and the income distribution in the terms contemplated in the model in the previous section.

In such a case, the government's budget constraint implies the following:

$$\Delta^{+}\left(V + V^{'}\right) \Rightarrow \Delta^{+}\left(TB + TB^{'}\right)$$

And, from the results of the model, it can be stated that:

$$1)\ \ \Delta^{+}\left(V + V^{'}\right) \rightarrow \begin{cases} \Delta^{+} i \\ \Delta^{+}(welfare\ of\ the\ poorest) \end{cases}$$

$$\Delta^{+}\left(T + T^{'}\right) \rightarrow \{\Delta^{-}(welfare\ of\ the\ richest\ and\ the\ medium\ group)$$

The first assumption of the transformation (quasi-)curve is the following: once a certain wealth distribution degree is exceeded, increases in subsidies allow to continue increasing it (until reaching a ceiling); but the welfare of the richest is reduced, and, in addition, it becomes necessary to increase taxes borne by the intermediate group, and, therefore, to reduce the welfare of this one. In other words, the first assumption of the transformation quasi-curve is that, beyond a certain income distribution degree (i.e., beyond segment AC of the "quasi-curve" in Figure 1), the slopes of the quasi-curve are negative:

$$3)\ \Delta^{+}\left(V + V^{'}\right) \rightarrow \begin{cases} \Delta^{+} i \\ \Delta^{-}(\ social\ welfare\ [Bergson - Samuelson]) \end{cases}$$

The second assumption of the quasi-curve is that of concavity: from a certain point (when the zone of negative slopes begins) the policy of increasing subsidies has a cost in terms of social welfare that we can measure, for example, with the number 1 (loss of social welfare = 1), and has a benefit measured in terms of increasing the distribution degree; but the additional benefit falls for each unit of social cost; that is, the ratio between the marginal social cost and the marginal benefit increases as subsidies are increased, since it can be assumed that the income distribution degree grows less and less. At any given point on the transformation quasi-curve (*tc*, except in its straight part), its slope is the ratio between the marginal change in the income distribution degree and the marginal change in social welfare:

$$(21) \quad Transformation\ Curve\ Slope = \left.\frac{di}{dB}\right|_{ct} < 0$$

And the third assumption is the exhaustion of the redistributive strategy through greater subsidies: beyond a certain point (point E in Figure 1) it would no longer be possible to increase the income distribution degree through increases in subsidies, given the parameters of the production function and the technology of transformation of public resources into greater income for the poorest.

Therefore, the ruler's program is to reach an optimal point: to establish a policy of subsidies and taxes such that the ratio between the marginal variation of social welfare and the marginal variation of the wealth distribution degree corresponding to the transformation curve be equal to the marginal ratio of substitution between the income distribution degree and social welfare, which is the slope of his indifference curve.

Let us specify the above as follows: suppose that the ruler has a utility function with two arguments: (Bergson-Samuelson) social welfare, *B*, and income distribution degree, *i*. In such a case we can assume this:

$$\Omega = \Omega(B, i);\ \frac{\partial \Omega}{\partial B} > 0;\ \frac{\partial \Omega}{\partial i} > 0$$

$$d\Omega = \frac{\partial \Omega}{\partial B} dB + \frac{\partial \Omega}{\partial i} di;$$

$$d\Omega = 0 \Rightarrow$$

$$(22) \quad \left.\frac{di}{dB}\right|_{I} = -\frac{\frac{\partial \Omega}{\partial B}}{\frac{\partial \Omega}{\partial i}} < 0$$

Thus, the ruler would judge optimal the choice of a set of subsidies and taxes such that the slope of his indifference curve (the right-hand side of 22), corresponding to such a set, be equal to the slope of the transformation curve (the right-hand side of 21):

$$\left.\frac{di}{dB}\right|_{I} = \left.\frac{di}{dB}\right|_{ct}$$

Figure 1 shows the optimal point (D).

What is the ruler's utility function foundation? Many things, but a very important one is the set of beliefs of him and society (or his constituents) about the root causes of inequality: if the belief is that inequality is due to bad luck, birth, inheritance, godfathers, etc., it will be large the weight given by him to redistributive policies in his utility function (on the importance of such beliefs, see Alesina and Angeletos, 2005, and Alesina *et al.*, 2018).

But, according to the model depicted in Figure 1, the weight given to redistributive policies only affects the curvature of the utility function. Under this model, a redistributive policy that imposes the transition from a lower to a higher degree of optimal distribution would only be explained by the displacement of the quasi-transformation curve towards the northeast, allowing to achieve its tangency with a new indifference curve; and this would only be possible by exogenous improvements in production conditions or in the conditions of transformation of given fiscal resources (in order not to harm other groups) into more welfare for the poorest[7]. The model shown in the Annex 1 is simple but captures some basic features of the discussion carried out around what is illustrated by figure 1.

One more thing should be noted: in Figure 1 we observe a rectangle bounded by the points B^*, D, i^*, i^{min}. The area of this rectangle is equal to $B^* i^*$. And, according to the definition of i (equation 20), this is equal to what would be the optimal level of the Foster-Sen social welfare function: S.

The above can be expressed as follows:

Proposition 1: under certain conditions, the optimal choice made by a ruler of a pair B (Bergson-Samuelson social welfare) - i (wealth distribution degree) is equivalent to the choice of a maximum of the function S (Foster-Sen social welfare).

[7] The Alesina and Angeletos´s model predicts a negative or zero correlation between redistributive policy and income concentration; so does the model in section IV, and the model in this section. But the Meltzer and Richard´s model (1981) predicts a positive correlation: the greater the concentration of income, the greater the redistributive policy. Moreover, if the assumption of perfect information is rejected, the following could be found: a misperception among an important group of the population about the income distribution degree may deny empirical evidence to Meltzer and Richard's (1981) prediction of such a positive correlation (Stantcheva 2021).

Proposition 2: given certain conditions, if an individual calculates the value of the function S, and manages to find its maximum value (given some restrictions), he will thereby be finding the pair B, i that corresponds to the situation that would be optimal for a ruler.

Finally, what has been expressed in this section gives rise to a question about its empirical relevance. The following should be noted in this respect:

According to the production theory underlying section IV there is independence between the income distribution degree, *i*, and the level of aggregate output; therefore, *B* (the level the Bergson-Samuelson social welfare function reaches at a given time) can be assumed to be independent of *i* in the absence of redistributive policies. So, our hypothesis is the following: when aggregate output enlarges (above population increases) the rulers have greater opportunities, incentives and facilities to grant more subsidies to the poorest and, therefore, increase the income distribution degree, so it is optimal for them to look for a pair *B, i* further away from the origin in the northeast direction in terms of Figure 1.

Therefore, the verifiable hypothesis would be the following: in societies whose economies grow, one should expect a reduction in the Gini coefficient (supposing equivalent to an increase in the income distribution degree), either simultaneously or with some lag, assuming they have rulers with the characteristics and policies corresponding to those discussed in this section.

The empirical evidence suggests that the hypothesis could be not rejected according to the figures in the World Bank database (*World Development Indicators*, 2021). In precise terms, the following was found:

For each of 85 economies that database has 3 or more annual Gini coefficient figures for the period 2000-2019 (and none with figures for 2020 or years prior to 2000), and has all annual gross national income per capita (GNI) data for that same period (in dollars of comparable purchasing power; "Atlas method"). All of these 85 economies recorded per capita income (GNI) growth between 2000 and 2019, and, of those, 57 recorded persistent reductions in the Gini (i.e., increases in the degree of income distribution, i); that is 67% of the total, while in 28 economies (33% of the total) the Gini either increased or did not fall (see Annex 2, which includes details of the calculation and the list of the 85 countries).

But in 4 of the group that did not reduce the Gini, the magnitude of this coefficient in the initial period (2000-2008) was already very low, below 0.3 (Austria, Denmark, Malta and Norway), so for their rulers probably the redistribution policy would not be of major importance, i.e., they would have judged their societies were already with income distribution degrees very close to point E in Figure 1. Discounting these 4 economies, the proportion of those 57 economies that reduced the Gini amounts to 70%.

VI. Conclusions

Now it is the moment to taking stock of the previous sections and present the main conclusions. The following list presents them. But before looking at their content, it should be remembered that there could be negative effects of redistribution strategies through subsidies and taxes, such as the distortion in the structure of incentives to labor supply and human capital formation, which, despite their importance, were deliberately omitted in order to concentrate us only on a few issues[8].

1. A negative effect of redistributions via taxation in relatively large magnitudes is the increase in the interest rate, if the subjective discount rates (the impatience factors) are heterogeneous: higher the poorer the individuals.

2. As a society advances in the process of income redistribution, the moment comes when the ruler faces a trade-off: continue deepening this process or avoid losses of social welfare, understood these ones in Pareto terms (that is, losses measured with the Bergson-Samuelson welfare function). The rational solution the ruler finds is to choose an optimal combination of social welfare (Bergson-Samuelson) and wealth distribution degree taking into account the restrictions associated with the conditions of production, budgetary resources and the ability to transform them into efficient redistribution instruments. It seems to us the international empirical evidence after the 2000 year could be considered favorable to such a hypothesis.

3. After a certain income distribution degree, it is no longer possible to redistribute through taxes and subsidies; what is that level? It depends on many conditions associated

[8] According to Meltzer and Richard (1981) the disincentive effect of taxes on labor supply is significative. In their model, a such effect is the only factor putting a cap on income redistribution policy. Chamley (1986) and Judd (1985) conducted social welfare analyses on the effects of redistributive policies using macroeconomic models taking into account tax disincentives that affect the optimal (physical) capital in the long run.

with preferences, technology, information, etc., which correspond to each specific situation of a society. The important thing here is not to try to make conjectures about the "ceilings" of the income distribution degree but to understand that by granting subsidies we could reach situations in which it is no longer possible, at least not in a sensible way, to achieve additional improvements in wealth distribution.

4. Therefore, it is important, from a conceptual point of view, to separate social welfare (Bergson-Samuelson) and income distribution, even if it is practical, sometimes, to use a non-traditional welfare function, such as the Foster-Sen, which weights individual utilities by a factor that measures the wealth concentration degree.

5. In a small open economy, the external interest rate prevails so the upward effects of fiscal measures on the interest rate looking for redistributing income to the poorest people may not be observed, despite the heterogeneity of the subjective discount rates, if the redistribution strategy is flawless. But if it is not, the uncertainty it generates among external agents may lead to significant increases in risk premiums and, therefore, in the local interest rate.

6. In the real world, economies (with very few exceptions) are in intermediate situations between the theoretical cases of closed and open economies; therefore, some increase in the interest rate should be expected as a response to the redistributive process through the fiscal channel (remaining constant other determinants of the interest rate) even if the strategy does not generate fears among potential external lenders and investors.

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Annex 1. The ruler's problem and its solution

The following model allows a simple approximation to the solution of the ruler´s supposed problem: to achieve an optimal combination of social welfare (Bergson - Samuelson) and income distribution degree, given a set of restrictions. For simplicity, the model is static (so there is no public debt nor interest rate).

The model has two blocks (and is recursive): the first is a hypothesis about a positive nonlinear relationship between the degree of income distribution, i, and the subsidy by person of the poorest group, v, so

$$(A.1)\quad I \equiv 1 + 1 = \varphi_0 + \varphi_1 v - \varphi_2 v^2;\ \varphi_{i(i=0,1,2)} > 0$$

Equation A. 1 only applies, in our case, to its ascending segment. The constant φ_0 is associated with what would be the income distribution degree in the absence of subsidies: $\varphi_0 - 1 = i$, for v = 0.

The second block consists of an objective function that the ruler maximizes, and the constraints to which he is subject, and what results from such a program, thus:

$$Max \quad \Omega = \chi C^{\eta} V^{1-\eta}; \; \chi > 0, \qquad 0 < \eta < 1$$

subject to:

$$C = Y - \delta K - G;$$

$$V + G = t(Y - \delta K); t > 0$$

Where *t* is the average tax rate (not including subsidies). So, it is assumed that the ruler tries to maximize (subject to constrains) the value of a Cobb-Douglas function of two variables: the total consumption of society (which can be considered as an approximation to Bergson-Samuelson social welfare under certain assumptions, as will be pointed out below) and the total amount of subsidies to be granted to the poorest group (which has a number M of people). The constraints are the fundamental macroeconomic equation and the public finances balance. The source of the tax revenue is the product (net of capital depreciation), and *t* is the weighted average of the tax rates corresponding to the different social groups (a weighting that takes into account the size of each group and its income, which is the basis on which the only tax is applied: the income tax).

The Lagrangian function corresponding to the problem is:

$$\mathfrak{L} = \chi C^{\eta} V^{1-\eta} + \lambda_1 (Y - \delta K - G - C) + \lambda_2 [t(Y - \delta K) - G - V]$$

The first order conditions (which are also sufficient for maximization) are:

$$\frac{\partial \mathfrak{L}}{\partial C} = \eta \chi C^{\eta-1} V^{1-\eta} = \lambda_1;$$

$$\frac{\partial \mathfrak{L}}{\partial V} = (1-\eta) \chi C^{\eta} V^{-\eta} = \lambda_2;$$

$$\frac{\partial \mathfrak{L}}{\partial \lambda_1} = Y - \delta - G = C;$$

$$\frac{\partial \mathfrak{L}}{\partial \lambda_2} = t(Y - \delta K) - G = V$$

For: $\lambda_1 = \lambda_2$, it results:

$$\left(\frac{\eta}{1-\eta}\right) V = C = Y - \delta K - G$$

$$\Rightarrow$$

$$(A.2) \quad V = \left(\frac{1-\eta}{\eta}\right)(Y - \delta K - G\ ) = V^*;\ v^* = V^*/M$$

Where M is the number of individuals to subsidy. The intuition is as follows: given as exogenous *G, Y, δ, K, η*, and since C fits with the fundamental macroeconomic equation, this model determines the optimal level of the total subsidy (= V^*). The tax rate (weighted average) is a variable that adjusts to the exogenous variables and to the optimal level of the subsidy:

$$t = \frac{G + V^*}{Y - \delta K}$$

Knowing the optimal level of the total subsidy, V^*, and given the number of people to benefit from it, we immediately derive the optimal value of the subsidy by person, v^*, and, therefore, the income distribution degree, according to equation *A.1*. This degree can be considered optimal for the ruler, given the constraints he faces.

The above can be summarized as :

$$i = i(v), i' > 0, given\ \varphi_{k(k=0,1,2)};$$

$$v = v(Y, \delta, G, M), \frac{\partial v}{\partial Y} > 0, \frac{\partial v}{\partial \delta} < 0;\ \frac{\partial v}{\partial G} < 0;\ \frac{\partial v}{\partial M} < 0;$$

$$t = t(v(Y, \delta, G, M), Y, \delta, G, M)$$

In the case of the tax rate, *t*, in the absence of specific functions, we can only state the following:

$$\frac{dt}{dY} = \left(\frac{\partial t}{\partial v}\frac{\partial v}{\partial Y} + \frac{\partial t}{\partial Y}\right) \lesseqgtr 0;\ \frac{dt}{d\delta} = \left(\frac{\partial t}{\partial v}\frac{\partial v}{\partial \delta} + \frac{\partial t}{\partial \delta}\right) \lesseqgtr 0;$$

$$\frac{dt}{dG} = \left(\frac{\partial t}{\partial v}\frac{\partial v}{\partial G} + \frac{\partial t}{\partial G}\right) \lesseqgtr 0;\ \frac{dt}{dM} = \left(\frac{\partial t}{\partial v}\frac{\partial v}{\partial M} + \frac{\partial t}{\partial M}\right) < 0$$

All in all, we can be sure of the following: (a) over a certain range of subsidies, their increase increases the income distribution degree; (b) an increase in the efficiency with which subsidies are applied (an increase in the parameter φ_1) increases the income distribution degree; (c) increases in total output increase the subsidy; (d) increases in the depreciation rate and in government spending reduce the subsidy; (e) increases in the population that merits subsidies increase the total subsidy but reduce the subsidy per person benefited; f) the tax rate adjusts to changes in the total subsidy but the impact on the rate of three of the factors that we consider exogenous (output, the depreciation rate and public spending) can be positive, zero or negative, depending on the magnitudes of different direct and indirect "partial reactions" (function´s slopes); g) the increase in the population benefiting from the subsidy reduces the tax rate.

But what can happen with the couple *B* (social welfare [Bergson-Samuelson]), *i* according to the above and as it was discussed with the help of Figure 1? An increase in the efficiency in the application of the subsidy (increase in the parameter φ_1) and a decrease in the population benefiting from it simultaneously increase *B* and *i* (shift northeastward the *tc* curve and the equilibrium point). The increase in total output would also have such effects unless it generated such a large increase in the subsidy per beneficiary as to force an increase in the tax rate; in the latter case the improvement in the welfare of the poorest would be accompanied by welfare losses of the richest, but this seems unlikely. And, finally, increases in the capital depreciation rate and in public spending most likely have a negative impact on the subsidy per person and the income distribution degree, and it is possible that they have a positive effect on the tax rate and, then, a negative one on *B*.

These results confirm what was noted in section V: achieving higher levels of social welfare (Bergson-Samuelson) or higher degrees of income distribution (associated with greater wealth of the poorest) in an optimal (and, therefore, sustainable) way require improving the technological conditions of production or those of transformation of fiscal resources into greater welfare of the poorest (the latter conditions are synthesized in this model by increasing the parameter φ_1).

Annex 2. Variations in the Gini Coefficient and Economic Growth

Changes in Annual Per Capita Gross National Income (GNI) and in the Gini Coefficient	
Change in GNI: annual geometric rate of increase in per capita GNI between 1999 and 2019 (in all countries in the sample this rate was greater than 0). Gini coefficient change: difference between the median of the 2014-2019 annual Gini and its 2000-2008 median. Question: these variations were going in the opposite direction?	
Response on cases from 57 countries	Yes
Response on cases from 28 countries:	No
Source: *World Development Indicators* (2021); World Bank	

Countries: Positive Response (57)						
Argentina	Cote d'Ivoire	Georgia	Kyrgyz Republic	Namibia	Paraguay	Uganda
Armenia	Colombia	Guatemala	Lesotho	Nigeria	Romania	Ukraine
Bangladesh	Comoros	Honduras	Latvia	Nicaragua	Russian Federation	Uruguay
Brazil	Costa Rica	Ireland	Moldova	Netherlands	Rwanda	
Bhutan	Czech Republic	Iran, Islamic Rep.	Maldives	Panama	Sudan	
Botswana	Dominican Republic	Iceland	Mexico	Peru	Sierra Leone	
Canada	Ecuador	Israel	Mongolia	Philippines	Slovak Republic	
Chile	Finland	Kazakhstan	Mauritania	Poland	Thailand	
China	Gabon	Kenya	Malaysia	Portugal	Tunisia	

Countries: Negative Response (28)						
Angola	Cyprus	Ethiopia	Italy	Mozambique	Sweden	Turkey
Albania	Germany	France	Lithuania	Norway	Eswatini	Tanzania
Australia	Denmark	Ghana	Luxembourg	Pakistan	Togo	United States
Austria	Egypt, Arab Rep.	Indonesia	Malta	Slovenia	Tonga	