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Multi-objective models for the forest harvest scheduling problem in a continuous-time framework

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A R T I C L E I N F O

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A B S T R A C T

In this study we present several multi-objective models for forest harvest scheduling in forest with single-species, even-aged stands using a continuous formulation. We seek to maximize economic profitability and even-flow of timber harvest volume, both for the first rotation and for the regulated forest. For that, we design new metrics that allow working with continuous decision variables, namely, the harvest time of each stand. Unlike traditional combinatorial formulations, this avoids dividing the planning horizon into periods and simulating alternative management prescriptions before the optimization process. We propose to combine a scalarization technique (weighting method) with a gradient-type algorithm (L-BFGS-B) to obtain the Pareto frontier of the problem, which graphically shows the relationships (trade-offs) between objectives, and helps the decision makers to choose a suitable weighting for each objective. We compare this approach with the widely used in forestry multi-objective evolutionary algorithm NSGA-II. We analyze the model in a *Eucalyptus globulus* Labill. forest of Galicia (NW Spain). The continuous formulation proves robust in forests with different structures and provides better results than the traditional combinatorial approach. For problem solving, our proposal shows a clear advantage over the evolutionary algorithm in terms of computational time (efficiency), being of the order of 65 times faster for both continuous and discrete formulations.

1. Introduction

The core of forest management planning is deciding the silvicultural prescription that should be applied to each stand to best meet the objectives of the forest owners. In traditional harvest scheduling, it implies to set where, when and how much to cut (Ware and Clutter, 1971). Many models have been used to formulate this basic problem. The most frequent approach (Başkent, 2001; Kurttila, 2001; Pukkala, 2019) has conceptually subdivided it into two computation stages: one for generating alternative management prescriptions for each stand, and other for selecting the combination of management prescriptions that best meets the objectives established for the forest while satisfying the possible restrictions. This approach has been formulated within the linear programming (e.g., Johnson and Scheurman, 1977), binary linear programming (e.g., McDill et al., 2002), and non-linear integer programming (e.g., Boston and Bettinger, 1999) frameworks. An alternative to avoid having to divide the problem into two stages is to work with continuous (non-discrete) decision variables. This approach is very interesting, since it does not require the temporal discretization, but it is less common, possibly because a rigorous mathematical study must be done to design a numerical method that provides a solution effectively and efficiently. Within this continuous-time approach, we can highlight the studies of Heaps (1984, 2015), who used optimal control theory to formulate and study the harvest scheduling problem under suitable hypothesis; Roise (1990), who proposed a non-linear programming formulation; and, more recently, the previous work of the authors of this study (González-González et al., 2021), who studied the problem in a multi-objective framework considering economic and volume control objectives. This latter paper includes some equivalences and transformations between different formulations of the forest harvest scheduling problem.

From an economic point of view, the best management plan of a

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forest would be the sum of the optimal plans at the stand level. Nevertheless, in some cases forest managers face the problem of providing a reasonably steady and continuous flow of products due to economic, environmental or social issues (Diaz-Balteiro and Romero, 2008). In fact, the aim of traditional forest regulation procedures was to achieve a "target forest" structure that would provide an even flow of products in perpetuity. Although the achievement of this ideal forest may be desirable, in most situations it will generally be preferable to dynamically manage the forest with intermediate structures between the optima at stand level and a perfect regulation (Ware and Clutter, 1971; Clutter et al., 1983, pp. 238–239). This makes the design of forest management plans a difficult task that may be approached within the framework of multi-objective optimization (Kangas and Pukkala, 1992), as long as there is only one decision maker. Problems with more decision makers with generally different interests are studied by game theory (Ungureanu, 2018, pp. 3–4). Examples of this type of problems in forestry can be found in Paradis et al. (2018).

The objectives of a multi-objective optimization problem (MOP) in the context of forest management are generally related to economic (e.g., maximize net present value or minimize costs), temporal (e.g., even-flow of timber harvested over time) or spatial (e.g., maximum area of adjacent units treated, minimum old-growth forest area) concerns (Murray, 1999). In a cooperative situation (Ungureanu, 2018, Section 1.1), e.g., when there is only one rational decision maker, solving a MOP implies to choose a satisfactory solution within the set of non-dominated feasible solutions, generally known as Pareto optimal solutions or efficient solutions. A satisfactory solution is the best compromise solution for the preferences of the decision maker. These preferences may be articulated prior to the analysis (therefore avoiding the generation of the set of efficient solutions; e.g., when the MOP is reformulated into a single objective problem through weights or/and treating some objectives as constraints), after it (which requires to obtain the complete set, also known as Pareto frontier), or interactively in a progressive way (trying to reduce the set) (Miettinen, 1998, pp. 61–65).

In recent papers, Arias-Rodil et al. (2017) and Gonzáez-González et al. (2020) presented rigorous mathematical analyses of two continuous metrics for measuring, respectively, land expectation value (LEV) and even flow (EF) of timber harvested using even-aged management. Later, González-González et al. (2021) appropriately modified these metrics to deal with the harvest scheduling problem of a forest, considering land and timber value (LTV) as the economic metric in which future rotations are based on economically optimal management prescriptions at stand level. These modifications included a new way to estimate the planning horizon in an automatic manner, and the possibility that a stand is harvested several times. The combination of these new metrics lead to a non-linear bi-objective model which could be efficiently solved by using gradient-type techniques.

In this work we go one step further on the continuous-time approach of the harvest scheduling problem, and focus on the concept of the regulated forest, which is one of the most widely used objectives in forest level planning. Based on the ideas developed in previous works, we present a new metric to measure forest regulation, and we adapt the previous ones introduced by the authors for measuring LTV and EF (González-González et al., 2021) to the hypothesis that must be assumed when looking for a regulated structure. The new metric has some similarities with the EF metric, but the aim pursued by both is different: the latter seeks to obtain a regular flow of timber during the planning horizon, while the first tries to achieve a forest structure that provides a constant flow of timber in perpetuity, cutting all the stands in the economically optimal rotation for forest level regulation. The main novelties of this manuscript are:

- A new metric is presented to measure the deviation from the fully regulated forest, and the continuous LTV and EF metrics are adapted to the needs of looking for a regulated forest. Gradient type methods are tested for optimizing these three metrics simultaneously.
- The bi-objective problem (economic vs. regulation) is formulated with continuous variables. In addition, the classic discrete formulation of this problem is obtained simply by adding constraints to the continuous formulation. The fact that the continuous formulation provides better results that the discrete one is verified on a real case study.
- The new metric proposed to deal with the concept of regulated forest is also used in the classic discrete formulation and the results obtained are analyzed.
- A three-objective problem (LTV-EF-regulation) is formulated and solved to generate a complete 3D Pareto frontier.
- The compatibility between EF and regulation is studied.

The rest of the paper is organized as follows. In Section 2, the new metric is detailed in a suitable mathematical framework, and the forest planning problem is formulated by means of novel MOPs, which are studied considering only one rational decision maker, by proposing a numerical method to obtain their Pareto-optimal frontiers. The efficiency and accuracy of the numerical method is shown in Section 3, where the model implementations over a *Eucalyptus globulus* Labill. showcase area are presented and discussed. Finally, the most interesting conclusions are summarized in Section 4.

2. Methods

In this section we detail a mathematical model designed to be a useful tool in harvest scheduling, one of the most frequent forest management planning problems. First, we establish starting hypotheses and introduce the notation used, next give rigorous definitions of some important (classical and original) concepts, and finally formulate the problem in the framework of multi-objective optimization and show how it can be solved successfully.

2.1. Starting hypotheses and notation

Consider a forest consisting of n_s stands. For each stand j, we assume to know its age at inventory (t_j^0, in years), area (A_j, in hectares), and measurements at age t_j^0 of the following state variables: dominant height (H_j^0, in meters, defined as the mean height of the dominant trees), number of trees (N_j^0, in a per hectare basis), and stand basal area (G_j^0, in m^2/ha, defined as the total cross-sectional area of all stems in a stand measured at breast height). We also assume that, from these values, for each stand we know smooth functions h_j, n_j, and g_j which provide, for every age $t > 0$, estimates of those state variables. That is, the state variables are given by $H_j(t) = h_j(t_j^0, H_j^0, t)$, $N_j(t) = n_j(t_j^0, N_j^0, t)$ and $G_j(t) = g_j(t_j^0, G_j^0, t)$. Finally, we also accept to know smooth functions $v_j(H, N, G)$ giving the total stand volume (in m^3/ha) in terms of the stand state variables.

To avoid confusion, it is important to distinguish between stand age, denoted by t, and the time from the beginning of the planning horizon (inventory), which is denoted by y. Both are given in years, and if the age of a stand at inventory is t_0, then $y = t - t^0$. The ultimate objective of forest planning is to determine the management prescription that will be applied to each stand. In this study, to alleviate notation, we seek only the times of clearcutting from inventory, that is, the decision vector $\mathbf{y} = (y_1, \ldots, y_{n_s}) \in \mathbb{R}^{n_s}$. Obviously, finding $\mathbf{y}$ is equivalent to finding $\mathbf{t} = \mathbf{y} + \mathbf{t}^0$. From now on, we will give the definitions in terms of $\mathbf{y}$ or $\mathbf{t}$ depending on how it is more appropriate. For instance, if stand j is clearcut at time y_j (when it is $t_j = y_j + t_j^0$ years old), the timber volume is given by $V_j(y_j) = v_j(H(t_j), N(t_j), G(t_j))$. On the contrary, present values of revenues and management costs if stand j is clearcut at age t_j will be denoted by functions $R_j(t_j)$ and $C_j(t_j)$, respectively, which are also defined from the state variables.

2.2. Main definitions

To correctly formulate the problem we must previously define some important concepts.

2.2.1. Land expectation value

The land expectation value (LEV, Faustmann, 1849) is one of the most important financial concepts in timberland management, as it allows to adequately compare even-aged stand management alternatives of different rotation ages. For a given stand j, the LEV can be defined as the present value, per unit area, of the projected costs and revenues from an infinite series of identical even-aged forest rotations, starting initially from bare land (Bettinger et al., 2009, pp. 40–42). If we assume a real interest rate of $r \in (0,1]$, then it is given by

$$J_j^{LEV}(t_j) = \frac{(1+r)^{t_j}\left(R_j(t_j) - C_j(t_j)\right)}{(1+r)^{t_j} - 1} \tag{1}$$

The LEV of the forest is the sum of the LEV of all stands, that is,

$$J^{LEV}(\mathbf{t}) = \sum_{j=1}^{n_s} J_j^{LEV}(t_j) \tag{2}$$

From a mathematical point of view, it should be noted that function J^{LEV} has the same smooth properties as functions h_j, n_j, g_j, and v_j (Arias-Rodil et al., 2017).

2.2.2. Optimal rotation at stand-level

Taking the LEV as indicator of profits, the optimal economic rotation for an individual stand j is given by the age of clearcutting corresponding to the solution of the problem

$$\begin{array}{ll} \mathit{maximize} & J_j^{LEV}(t_j) \\ \text{subject to} & t_j \geq 0, \end{array} \tag{3}$$

which is denoted by $\overline{t}_j$. This optimal rotation is generally known in the forest economics literature as "Faustmann formula", although this author only proposed Eq. (1) and did not maximize it (Cairns, 2017). The vector $\overline{\mathbf{t}} = (\overline{t}_1, \ldots, \overline{t}_{n_s})$ is named optimal rotation vector at stand-level.

2.2.3. Optimal rotation for the regulated forest

The regulated forest is a simplistic concept of an ideal forest with a relatively steady-state structure which provides an even, sustained flow of desired forest outcomes over time (Davis et al., 2001, p. 93). In even-aged management, it implies to use the same clearcutting age for all the stands and, if the primary objective is to maximize forest profitability, the optimal rotation for the regulated forest is the one that maximizes the LEV of the entire forest (Hoganson and McDill, 1993). It is given by the solution of the problem

$$\begin{array}{ll} \mathit{maximize} & \sum_{j=1}^{n_s} J_j^{LEV}(t) \\ \text{subject to} & t \geq 0, \end{array} \tag{4}$$

and hereinafter will be denoted by R.

2.2.4. Land and timber value

The land and timber value (LTV) is a generalization of the LEV which gives the value of a piece of forest land with an existing stand of trees, whatever its stage of development. It is useful for determining when the existing stand should be harvested. The LTV of stand j is the sum of the present value of the projected costs and revenues for the remainder of the current rotation, plus the present value of the LEV which accounts for all future rotations of timber that will be grown on the land (Clutter et al., 1983, pp. 226–228). Let p_j (in €/m^3) be the expected stumpage price. Then, for each $y_j \geq 0$:

(i) If we assume that next rotations will be based on the optimal rotation for the individual stand j, the LTV is given by

$$J_j^{LTV}(y_j) = \frac{p_j V_j(y_j) + J_j^{LEV}(\overline{t}_j)}{(1+r)^{y_j}} \tag{5}$$

(ii) If we assume that next rotations will be based on the optimal rotation for the regulated forest, the LTV is given by

$$J_j^{LTV}(y_j) = \frac{p_j V_j(y_j) + J_j^{LEV}(R)}{(1+r)^{y_j}}. \tag{6}$$

In both cases, the LTV of the forest is the sum of the LTV of all stands, that is,

$$J^{LTV}(\mathbf{y}) = \sum_{j=1}^{n_s} J_j^{LTV}(y_j).$$

As with J^{LEV}, the regularity of J^{LTV} is given by the properties of functions h_j, n_j, g_j, and v_j, and we can assume that it is smooth enough.

2.2.5. Regulation metric

If the primary goal is to obtain commercial timber outcomes, then we could think of a regulated forest as that with all stands harvested at the age of R years and producing a perfect even flow (EF) of timber volume in perpetuity, which implies that the harvest rate for the next R years (i.e., after the transition period from the unregulated structure to the regulated condition) is constant. Looking for the regulated forest, it is necessary to define a function (hereinafter the regulation metric) which provides, for every admissible planning strategy $\mathbf{y}$, the degree of achievement of that objective. With this aim we consider:

- The minimum harvest age for each stand, $l_j > 0$ (years), which leads to the constraint $t_j \geq l_j$.
- The instant $a \geq 0$ (years) when harvests can begin in the forest. This value is given by the first stand which can be harvested: it will be zero if any stand can be harvested now, otherwise it will be the minimum of the differences $l_j - t_j^0$. Specifically:

$$a = \begin{cases} \underset{j=1,\ldots,n_s}{\mathit{min}} \left\{ l_j - t_j^0 \right\} & \text{if } \ l_j - t_j^0 \geq 0, \quad \forall j = 1, \ldots, n_s, \\ 0 & \text{otherwise.} \end{cases}$$

- The set of admissible planning strategies $Y^{ad} \subset \mathbb{R}^{n_s}$, given by

$$Y^{ad} = \left\{ \mathbf{y} \in \mathbb{R}^{n_s}, \ \text{ such that } \ \mathit{max}\left\{0, l_j - t_j^0\right\} \leq y_j \leq a + R, \forall j = 1, \ldots, n_s \right\}.$$

- The goal harvest rate for the regulated forest, m^R. The current planning horizon (where the management prescriptions are chosen) is $[a, a+R]$, but to measure how the forest is regulated we must check the EF for the next planning horizon ($[a+R, a+2R]$), assuming that the second harvest is done when the stand is R years old. The total harvest volume is $\sum_{j=1}^{n_s} V_j(R)$ and, if a constant harvest rate is required, it must be

$$m^R = \frac{\sum_{j=1}^{n_s} V_j(R)}{R}. \tag{7}$$

- The real harvest rate for the next planning horizon ($[a+R, a+2R]$). At the beginning of that next planning horizon ($a+R$), the age of stand j will be $a+R-y_j$ and, consequently, the time from inventory

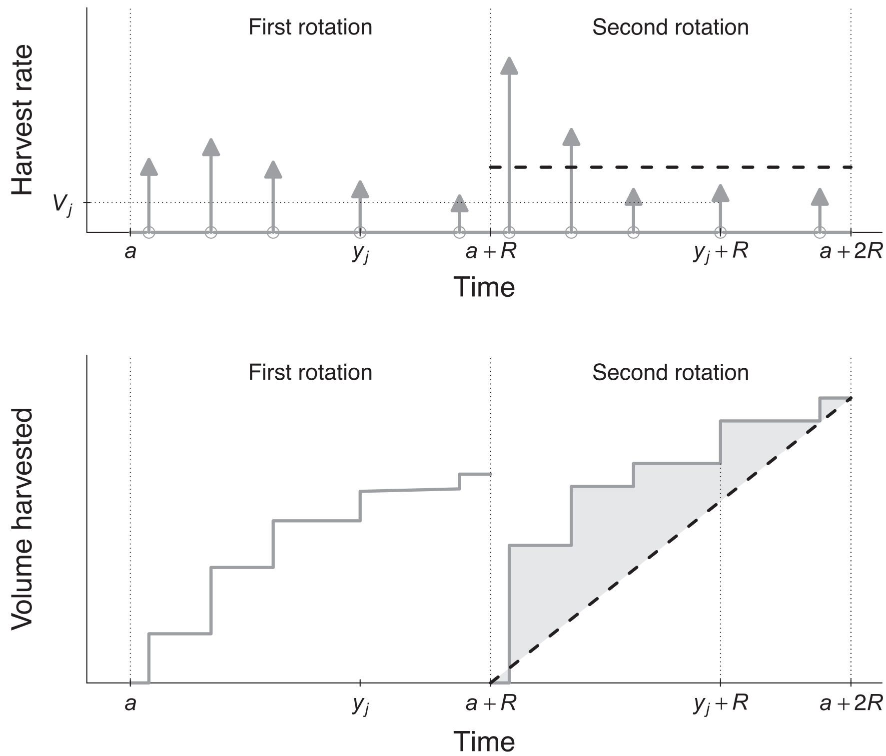

Fig. 1. Example of harvest rate (top) and volume harvested (bottom). The grey solid lines represent real instant harvests (top, Eq. (8)) and real volumes harvested (bottom, Eq. (10)), while the black dashed lines represent the goal harvest rate function (top, Eq. (7)) and the goal volume harvested function (bottom, Eq. (9)).

to the second clearcutting of stand j is $y_j + R$. Assuming instant harvests, the real harvest rate must be written in terms of the Dirac's delta function $\delta(y)$ (Oldham et al., 2009, Ch. 9). Specifically, for the period $[a + R, a + 2R]$, it is given by (see Fig. 1)

$$\frac{\partial V^n}{\partial y}(\mathbf{y}, y) = \sum_{j=1}^{n_s} V_j(R)\delta\left(y - \left(y_j + R\right)\right) \quad y \in [a + R, a + 2R]. \tag{8}$$

- A function given the real volume harvested for the next planning horizon (V^n) and a function given the desired harvested volume for each rotation of the regulated forest (V^R). The Dirac delta is not a function in the traditional sense, so the comparison between (8) and the constant function for the period $[a + R, a + 2R]$ given by (7) is not appropriate. Instead of comparing harvest rates, it is more suitable to compare those volumes, which are obtained by integrating those expressions (7) and (8):

$$V^R(\mathbf{y}, y) = \int_{a+R}^{y} m^R\, d\tau = m^R(y - (a + R)), y \in [a + R, a + 2R], \tag{9}$$

$$V^n(\mathbf{y}, y) = \int_{a+R}^{y} \frac{\partial V^n}{\partial \tau}(\mathbf{y}, \tau)\, d\tau = \sum_{j=1}^{n_s} V_j(R) H\left(y - \left(y_j + R\right)\right), y \in [a + R, a + 2R], \tag{10}$$

where $H(y)$ is the Heaviside step function (Oldham et al., 2009, Ch. 9).

The definition of a regulated forest given at the beginning of this section suggests that, at the end of the first planning horizon, a forest will be more regulated the more closely functions $V^R(\mathbf{y}, .)$ and $V^n(\mathbf{y}, .)$ resemble each other. Following Gonzáez-González et al. (2020), in this work we propose the distance associated to the L^2 norm to measure the similarity of these two functions, and establish the definition that, given a planning strategy $\mathbf{y} \in Y^{ad}$, the regulation metric associated with that strategy is the value of

$$J^R(\mathbf{y}) = -\int_{a+R}^{a+2R} \left(V^n(\mathbf{y}, y) - V^R(\mathbf{y}, y)\right)^2 dy$$

where the minus sign in the definition is included precisely so that a higher value of $J^R(\mathbf{y})$ corresponds to a greater similarity of the function $V^n(\mathbf{y}, .)$ to the goal function $V^R(\mathbf{y}, .)$, which is what is meant by a higher regulation.

Function J^R has not the same smooth properties as J^{LEV} or J^{LTV}, although it is continuous and has continuous derivatives in almost all points. The explicit expression of its gradient can be obtained in a way similar to that of González-González et al. (2020).

2.3. Multi-objective optimization problems (MOPs)

Initially, the forest management planning problem we are dealing with consists of selecting the set of silvicultural treatments to be applied to each stand (i.e., to determine $\mathbf{y} \in Y^{ad}$) to obtain maximum profitability (i.e., to maximize J^{LTV}) with the highest regulation possible (i.e., to maximize J^R). Therefore, this problem may be formulated as the following MOP:

$$\begin{array}{ll} \textit{maximize} & \mathbf{J}(\mathbf{y}) = \left(J^R(\mathbf{y}), J^{LTV}(\mathbf{y})\right) \\ \text{subject to} & \mathbf{y} \in Y^{ad}, \end{array} \tag{11}$$

where function J^{LTV} must be calculated assuming that next rotations will be based on the optimal rotation for the regulated forest R (by using Eq. (6)).

The idea is simple but, unfortunately, both objectives are in conflict and, consequently, it will not be possible to find an ideal (or perfect) element $\mathbf{y}^I \in Y^{ad}$ maximizing them simultaneously. Problem (11) must be studied assuming only one rational decision maker, by examining some distinguished elements of the admissible set (the non-dominated ones). Such vectors are those for which none of the objectives can be improved without a deterioration of the other, and are generally known

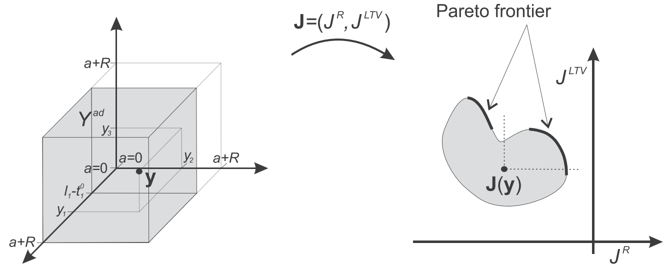

Fig. 2. Example of an admissible set $Y^{ad} \subset \mathbb{R}^3$ and the corresponding image set $\mathbf{J}(Y^{ad}) \subset \mathbb{R}^2$, where the Pareto frontier is highlighted.

as Pareto optimal solutions or efficient solutions (for a more formal definition see, for instance, Miettinen, 1998, pp. 11, 19). The corresponding objective vector is also known as Pareto optimal, while the set of these objective vectors is known as Pareto frontier (see Fig. 2).

From a mathematical point of view, MOP (11) is considered to be solved when the Pareto optimal set is found. However, what is desired in practice is to select a satisfactory solution among all the efficient solutions, for which we must know the preferences of the decision maker. The techniques for articulating these preferences in multi-objective optimization can be classified in (Miettinen, 1998, pp. 61–65): a priori methods (the decision maker first acts clearly establishing his preferences), a posteriori methods (all efficient solutions to the problem are sought, so that the decision maker chooses, among all of them, the one that suits him best), and interactive methods (based on the preferences indicated by the decision maker, one goes from one efficient solution to another, until one is achieved that, in addition to being efficient, is

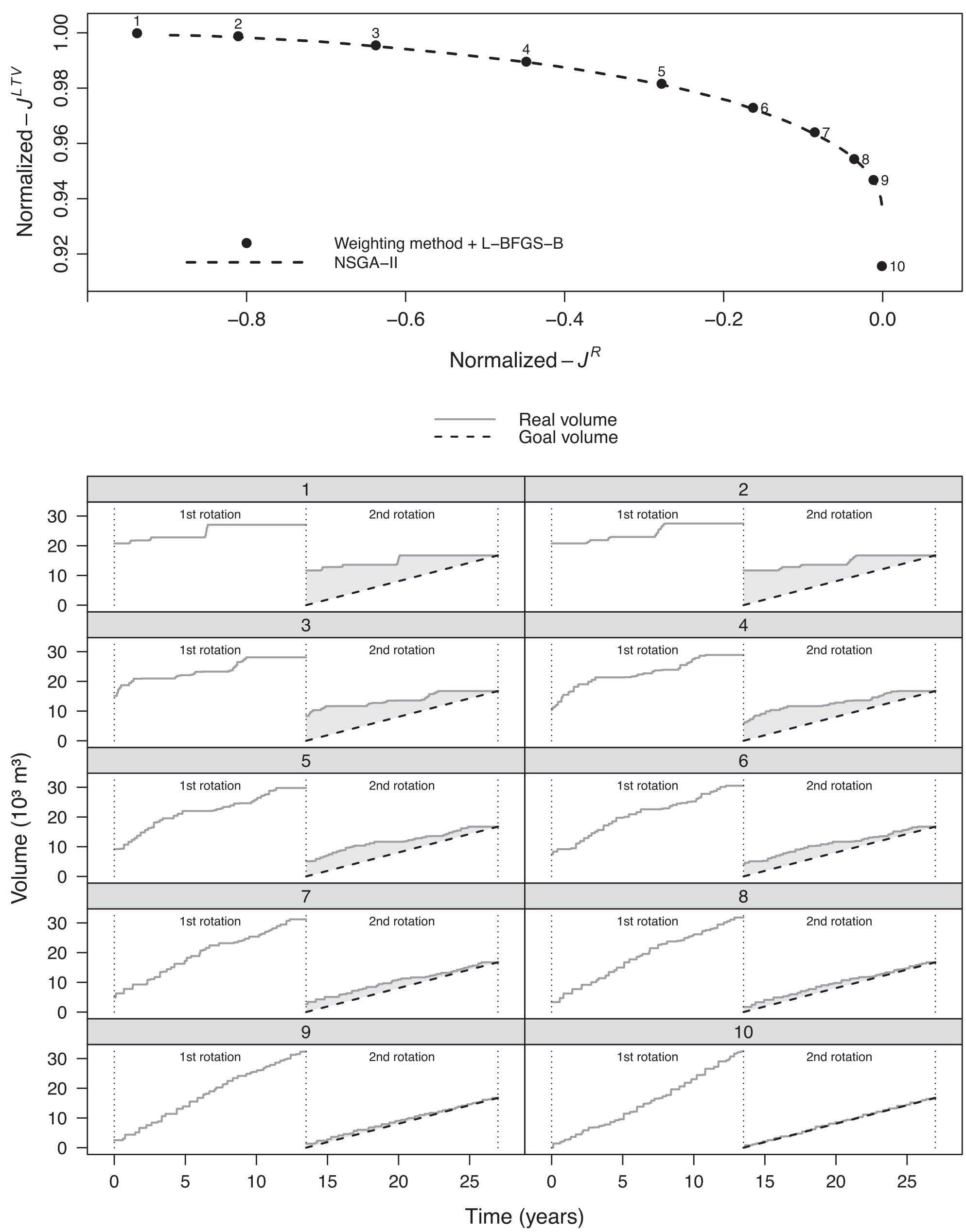

Fig. 3. Results for the real forest: normalized Pareto frontiers obtained for the continuous formulation with the weighting method + L-BFGS-B (points obtained with equally spaced weights over the Normalized-J^{LTV} axis) and with NSGA-II (top), and comparison between real and goal volumes for the second rotation corresponding to the numbered points (bottom).

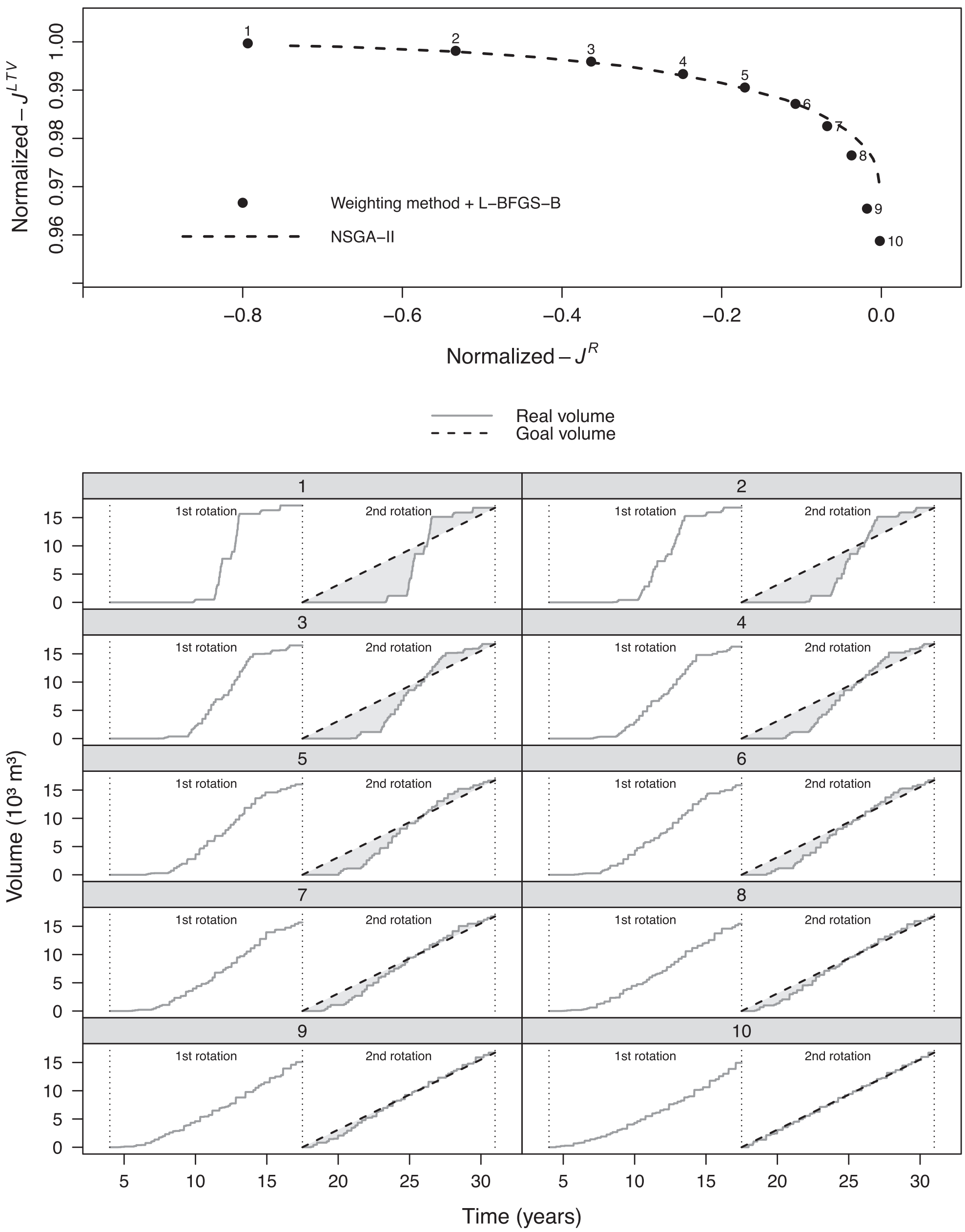

Fig. 4. Results for the simulated young forest: normalized Pareto frontiers obtained for the continuous formulation with the weighting method + L-BFGS-B (points obtained with equally spaced weights over the Normalized-J^{LTV} axis) and with NSGA-II (top), and comparison between real and goal volumes for the second rotation corresponding to the numbered points (bottom).

Table 1
Payoff tables for the case studies. Row *i* displays the values of all the objective functions calculated at the point where the *i*th-objective obtained its maximum.

	Real forest			Simulated young forest		
Objective	J^{LTV}	J^{R}	J^{EF}	J^{LTV}	J^{R}	J^{EF}
J^{LTV}	**1,134,430**	−778,540,270	−1,547,349,136	**548,181**	−270,743,037	−313,203,983
J^{R}	1,037,008	**−344,447**	−20,157,105	519,864	**−344,402**	−39,573,139
J^{EF}	1,047,370	−7,524,763	**−1,566,503**	504,651	−73,675,692	**−433,047**

satisfactory). In Section 2.4 we propose an a posteriori method to draw the Pareto frontier of the problem as the basic tool in forest planning.

Although it may be obvious, it should be noted that any requirement known in advance (other than the minimum thresholds for the objectives J^{LTV} and J^{R}) must be present in the formulation phase of the model, and not in the subsequent decision-making process (made from the Pareto frontier obtained). These requirements mean that MOP (11) must be modified (generally including new constrictions), causing a new Pareto frontier from which decision-making must be carried out. An example of this situation is the traditional combinatorial approach of, first, simulating alternative management prescriptions for each stand (collected in a discrete set Ω) and, second, using combinatorial

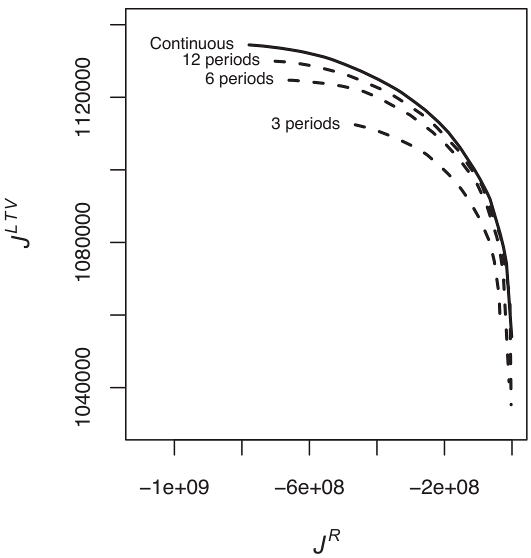

Fig. 5. Pareto frontiers obtained with NSGA-II for continuous and combinatorial formulations of different number of periods keeping the same planning horizon for the real forest.

optimization to seek the optimal prescriptions in these sets (Pukkala, 2019). In that case, MOP (11) must be replaced by

$$\begin{array}{ll} \textit{maximize} & \mathbf{J}(\mathbf{y}) = \left(J^R(\mathbf{y}), J^{LTV}(\mathbf{y})\right) \\ \text{subject to} & \mathbf{y} \in Y^{ad}, \\ & y_j \in \Omega, \end{array} \tag{12}$$

and the decision-making process must be carried out based on the Pareto frontier of this new problem.

On the other hand, MOP (11) must also be modified if the objectives of the landowner change. For example, if there is also interest in maximizing the even flow (EF) for the current planning horizon, the problem must be completed with a new function (J^{EF}) measuring the achievement of this objective (Kao and Brodie, 1979, analyzed a problem with similar objectives by goal programming). In this case, the MOP to solve is

$$\begin{array}{ll} \textit{maximize} & \mathbf{J}(\mathbf{y}) = \left(J^R(\mathbf{y}), J^{EF}(\mathbf{y}), J^{LTV}(\mathbf{y})\right) \\ \text{subject to} & \mathbf{y} \in Y^{ad}, \end{array} \tag{13}$$

where J^{EF} is defined as in González-González et al. (2021), replacing the automatically computed planning horizon by the fixed planning horizon $[a, a + R]$.

2.4. Numerical solution

The first step is to solve problem (3), which provides the optimal management prescription for each stand $\overline{\mathbf{y}} = (\overline{y}_1, \dots, \overline{y}_{n_s})$, and problem (4), which indicates the optimal rotation for the regulated forest R. These values are used to obtain the bounds of the set of admissible planning strategies. Additionally, R is also used to calculate the desired harvest rates and, in turn, functions J^R and J^{EF}, as well as the land and timber value function J^{LTV}. Because of the regularity properties of the functions involved, both problems can be solved by any gradient-type method. In this work we used the L-BFGS-B algorithm (Zhu et al., 1997) implemented in the free and open-source Python library SciPy 1.0 (Virtanen et al., 2020).

To take advantage of the regularity properties of the J^{LTV}, J^R and J^{EF} functions, we propose to convert MOPs (11) and (13) into differentiable single-objective problems (SOPs) using an scalarization technique and to solve them with a gradient-type method combined with a random multi-start if local minima are detected. In this study, to obtain the Pareto frontier, we used the weighting method (Miettinen, 1998, pp. 78–85) and, again, the L-BFGS-B algorithm. For the results presented in the next section, we took equally spaced weights and, using the ideal objective vector (that obtained by maximizing each of the objective functions individually subject to the constraints of the problem) and the approximate nadir objective vector estimated from a payoff table (Miettinen, 1998, pp. 15–19), we normalized the objective functions so that their objective values were of approximately the same magnitude.

The multi-objective optimization problem (12) was formulated using the traditional combinatorial approach in which the planning horizon $[a, a + R]$ is divided into P periods of $\Delta = R/P$ years, and only one harvest instant (the middle of the period) is assumed for each period. In this case the harvest instants are $c_i = a + (i - 1/2)\Delta$, and the discrete set is given by $\Omega = \{a + \Delta/2, \dots, a + R - \Delta/2\}$. These additional constraints prevent the use of gradient-based techniques and, therefore, we solved MOP (12) using the evolutionary algorithm NSGA-II (Deb et al., 2002), widely used in multi-objective optimization, which gradually approaches a well-distributed set of Pareto optimal solutions across the Pareto frontier and can be applied to continuous as well as to combinatorial search spaces (Emmerich and Deutz, 2018). Specifically, we used the implementation of this algorithm in the Python library Inspyred 1.0 (Tonda, 2020), with appropriate mutation and recombination operators for integer representation (Eiben and Smith, 2015, pp. 52–56). Finally, we also used the NSGA-II algorithm to check the numerical method proposed above for solving continuous MOPs (11) and (13).

3. Results and discussion

For testing the usefulness of our approach, we used the real forest of *Eucalyptus globulus* Labill. in the municipality of Xove (Galicia, NW Spain) of the case study analyzed by González-González et al. (2021). It comprises $n_s = 51$ stands and assumes that none of them can be cut before they are 5 years old ($l_j = 5$). Additionally, we considered immediate regeneration after clearcutting with a constant plantation density of 1333 trees/ha for all stands, and that there are no changes on site quality (site index). The inventory data and the transition functions of a dynamic growth model appropriate for the species (García-Villabrille, 2015) were used for computing the state variables and outcomes (for details, see González-González et al., 2021). After solving problem (4), we obtained an optimal rotation for the regulated forest of $R = 13.1$ years. To further test our approach in a forest with a different structure, we also used these data for simulating a hypothetical forest (hereinafter the simulated young forest) with 51 stands of the same species, area and site index, but all at one-year age.

For MOP (11), the results corresponding to the real forest of the case study and to the simulated young forest are presented, respectively, in Figs. 3 and 4. They show the Pareto frontiers obtained with the weighting method + L-BFGS-B and with the NSGA-II algorithm. The corresponding payoff tables used for scaling the problems as described in previous section are collected in Table 1. Additionally, these figures also exhibit the real volume harvested for the first rotation and the comparison between real and goal volumes for the second rotation corresponding to all (ten) Pareto optima obtained with the first of these two approaches. In view of these results, it can be highlighted that:

- Both methods, although using different techniques, provide the same Pareto fronts, which warrants the appropriateness of our approach.
- The continuous approach works well for forests with very different stand structures, as suggested from the results of the real forest (which has young and mature stands of different ages) and the simulated young forest (in which all stands are 1-year old).
- Figs. 3 and 4 are very useful for the decision-making process, as they allow to graphically analyze the trade-offs between objectives:
  - In the top graphs, the normalized-J^{LTV} is $J^{LTV}/J^{LTV}(\mathbf{y}^*)$, i.e., the LTV divided by its best value. This way of proceeding allows an intuitive

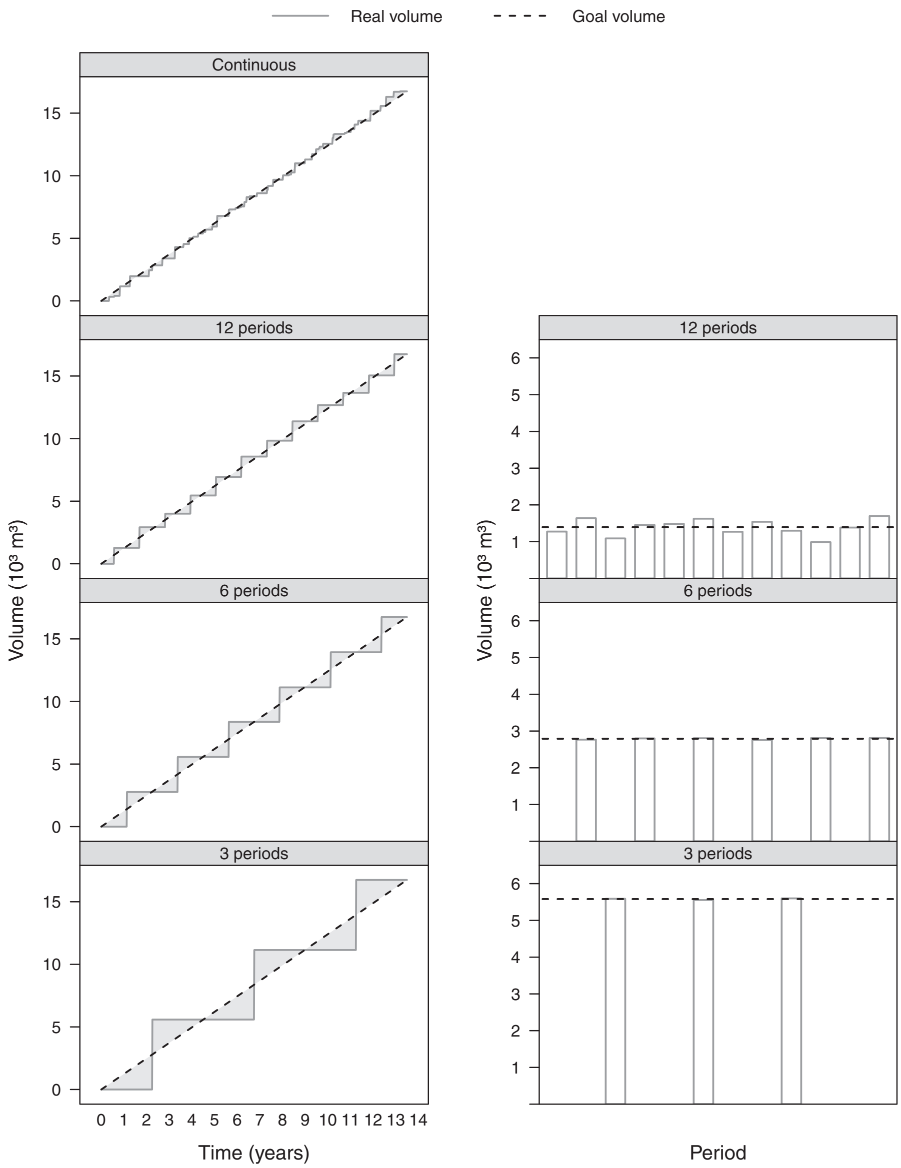

Fig. 6. Real and goal volumes for the second rotation (left) corresponding to the points of maximum J^R in Fig. 5, and equivalent volume per period (right).

comparison of the different optimal management strategies: for the real forest, there was a reduction of about 8.5% in LTV between the best solutions from the economic and regulation perspectives (points 1 and 10 in the Pareto front, respectively), while for the simulated young forest it was of only 5.2%; intermediate solutions can also be analyzed in the same manner.

- The regulation metric has not such an easy interpretation in terms of numerical values, but the grey area of the bottom graphs of the figures clearly indicates the achievement of the regulation objective as well as the moment in which the clearcuts are done. Additionally, these graphs also show the total volume harvested of each Pareto optimum, which deserves a separate analysis for each forest: in the real one, it is higher for the first rotation than for the regulated condition due to its current structure towards mature stands, while for the simulated young forest it is slightly lower in most cases (except for the Pareto optima with higher weights of J^{LTV}) because of its initial structure of young stands.

As shown, the results of problem (11) reveal the cost of regulation assuming that next rotations will be based on the optimal rotation for the regulated forest R (a low cost corresponds to a highly regulated forest at the beginning of planning). Nevertheless, the real cost of regulation should be measured in terms of lost net discounted returns compared to managing all stands indefinitely following their Faustmann rotation (i.e., $\bar{t}_j$) to maximize financial returns (Hoganson and McDill, 1993). The objectives implicit in regulation may be inappropriate for many landowners, and current management decisions should not generally be controlled by a long-term forest structure objective because of, among other reasons, highly probably changes in markets, technology, prices, and costs (Ware and Clutter, 1971). Nevertheless, the results of the regulated forest can serve as a baseline against which to compare alternative management scenarios. This role may become more important as interest in sustainable forest management increases. (Davis et al., 2001, pp. 435–436).

Regarding MOP (12), which considers the combinatorial formulation with different number of periods keeping the same planning horizon, Fig. 5 shows the Pareto frontiers of the real forest obtained with NSGA-II. As expected, the continuous formulation (MOP (11)) provided better results than the combinatorial one, since the latter implies additional constraints given by the discrete set Ω. Besides, the continuous formulation is much faster to solve when gradient-type methods are used. For example, for the real forest, computation times for the weighting

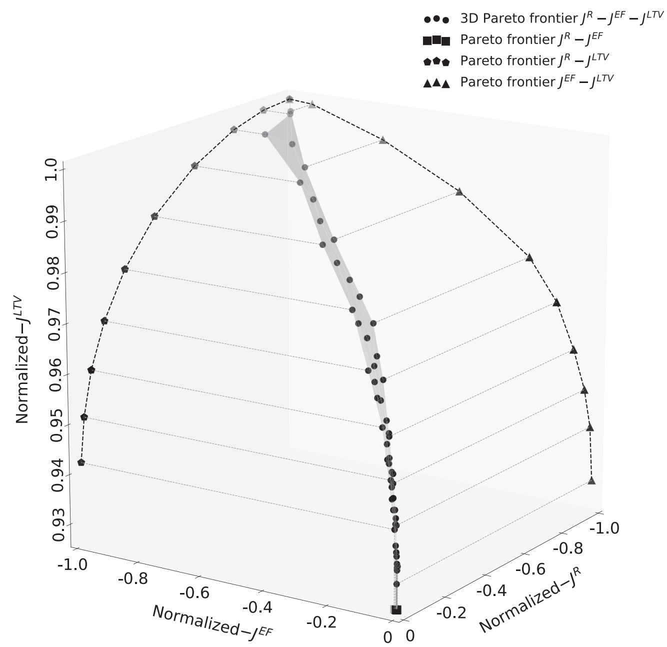

Fig. 7. Normalized 3D Pareto frontier obtained with the weighting method + L-BFGS-B with a continuous formulation for the real forest.

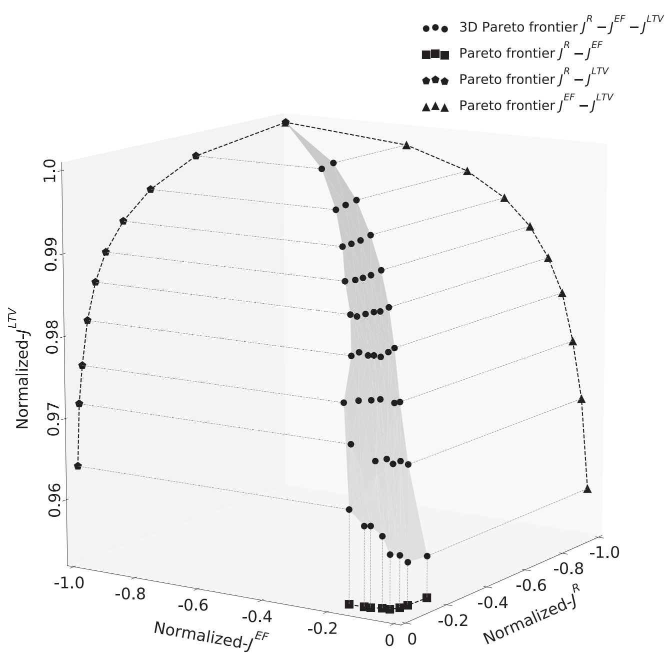

Fig. 8. Normalized 3D Pareto frontier obtained with the weighting method + L-BFGS-B with a continuous formulation for the simulated young forest.

method + L-BFGS-B were of the order of 65 times faster than for NSGA-II (for both continuous and discrete formulations). This ratio is expected to increase as the dimension of the problem (number of stands) increases (González-González et al., 2021). To graphically analyze the goodness of the new J^R metric, the left column of Fig. 6 shows the real and goal volumes corresponding to the best points for regulation, i.e., those which maximize function J^R (that measures the even flow in the second rotation). The right column shows, for these same points, the equivalent volume harvested per period. It can be seen that maximizing J^R leads to small variations in the volume harvested between periods, that is, good results with the classic ways of measuring even flow in combinatorial formulations (e.g., Kao and Brodie, 1979; Brumelle et al., 1998; Ducheyne et al., 2004).

Finally, with respect to MOP (13), Figs. 7 and 8 show the 3D Pareto frontier for the real and the simulated young forests, respectively, which provide all the information necessary to choose a compromise solution and may help decision makers with unclear priorities (Diaz-Balteiro et al., 2009). To improve the understanding of the trade-off information it is also useful to look at the Pareto frontier in a pairwise comparison of objectives (Couture et al., 2021), which can be seen projected in each plane (i.e., $J^R - J^{EF}$, $J^R - J^{LTV}$, and $J^{EF} - J^{LTV}$) of the aforementioned figures. The analysis of these frontiers shows a certain compatibility between objectives J^R and J^{EF}, which is more evident in the real forest because it is mainly formed by mature stands, which are in a rather slow growth phase. This compatibility can also be observed in Figs. 3 and 4, which indicate that when we increase the weight of the regulation objective J^R, we indirectly obtain a better even flow in the first rotation (values of J^{EF_R} closer to zero). This MOP (13) is more interesting for the planning process of young-stand forests, in which the best solutions for J^{EF} during the first rotation are clearly in conflict with those that maximize J^R for forest level regulation.

As we have seen, if two or three objectives are involved, it is advisable to obtain the Pareto frontier to graphically analyze the relationships (trade-offs) between objectives, helping the decision makers (forest owners and/or forest managers) to choose a suitable weighting for each objective (e.g., Pascual, 2021). For problems with more objectives, despite a posteriori techniques combined with special visualization methods can be used (e.g., Borges et al., 2014), the rationale for approximating of the whole Pareto frontier is questionable (Alvarez-Vázquez et al., 2015), and interactive techniques (e.g., Diaz-Balteiro, 1998) may be more appropriate.

Although the case study analyzed is located in Galicia (NW Spain), the proposed models can be applied to the management of any forest consisting of single-species, even-aged stands for which there exist differentiable stand growth models and in which the optimization objectives are related to economic and even-flow aspects. In the specific case of this region and for the species used, *Eucalyptus globulus*, most harvest scheduling plans are based on the area control method, which aims to achieve a regulated forest after the first rotation (Davis et al., 2001, pp. 528–529), although without relying on any mathematical model for its optimization (Diaz-Balteiro et al., 2009). Another approach used to optimize the planning of these stands within the multi-criteria framework has been goal programming (e.g., Bertomeu et al., 2009; Diaz-Balteiro et al., 2009; Giménez et al., 2013).

The specific improvements of our proposal with respect to the harvest scheduling methods commonly applied to this type of stands are: (i) it uses continuous decision variables for which it is not necessary to simulate in advance the silvicultural programs potentially applicable to each stand, (ii) it makes possible to use gradient-based methods for numerical resolution, which have shown to be more efficient, effective and robust than heuristic techniques (González-González et al., 2021), and (iii) the preferences of the decision maker are incorporated into the process after optimization, who benefits from the trade-off information between objectives to select the satisfactory solution. This model could be completed with two very relevant aspects in harvest scheduling: the consideration of spatial restrictions (Murray, 1999) and clustering. The latter could be desirable for reducing both road entry and maintenance costs. In addition, if the harvests are clustered, more interior forests are preserved in the landscape (Öhman and Lämås, 2003).

4. Conclusions

We present several multi-objective models for forest harvest scheduling in single-species, even-aged stands using a continuous formulation. We seek to maximize economic profitability and even-flow of timber harvest volume, both for the first rotation and for the regulated forest. For that, we have designed new metrics that allow working with continuous decision variables. To obtain the Pareto frontier, we propose to combine an scalarization technique (weighting method) with a gradient-type algorithm (L-BFGS-B), which we compare with the widely used in forestry multi-objective evolutionary algorithm NSGA-II. The main conclusions of this study are:

- Our proposal has been shown to be very effective, providing results at least as good as to those obtained with the evolutionary algorithm. Besides, in terms of efficiency, the use of the gradient-based algorithm has required much less computing time to obtain the complete Pareto frontier. This difference could be extended considering that our approach is easily parallelizable, which could take advantage of the potential of multi-core processors.
- In this work, the classical combinatorial formulation is obtained by adding additional constraints to the continuous model. This guarantees that the results of the latter are always better, as has been shown in the case study of a real *E. globulus* forest.
- The proposed metric to measure regularity, designed to work with continuous formulations, has been shown to be similar to the traditional metrics used in combinatorial problems. Planning strategies that maximize the new metric also lead to constant flows of products by period in the regulated forest.
- The graphical display of trade-off information between objectives using the Pareto frontier allows an a posteriori articulation of preferences in an intuitive way, therefore being a very interesting tool for the decision making process in forest planning.
- In a previous study we formulated a continuous model to maximize economic profitability and even-flow of timber harvested for a given planning horizon, which did not require to set management prescriptions in advance and avoided the division of the planning horizon into periods. In this study we show how this formulation can be extended to more objectives, specifically we deal with the problem of forest regulation. More sophisticated objectives and constraints can also be incorporated into the model, such as those that consider the spatial location of the harvests, which is being analyzed in an ongoing research.

Author statement

All authors have contributed similarly in research planning, analysis, and writing of the manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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