Power Allocation, Relay Selection, and User Pairing for Cooperative NOMA Systems with Rate Fairness

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***Abstract*—Assuming a cooperative non-orthogonal multiple access (NOMA) system with rate fairness in a scenario with multiple users and arbitrary relays, this paper investigates adaptive power allocation (PA), relay selection (RS), and user pairing (UP) policies. Specifically, two adaptive PA optimization problems are formulated, one at the base station (BS) and another at the selected relays. Closed-form expressions for the power allocation factors are derived as well as an algorithm that provides the optimal solution at the BS. In order to show the superiority of the proposed study, our results are compared with other benchmark schemes in terms of outage probability, Jain's fairness index, and average sum rate.**

***Index Terms*—NOMA, power allocation, rate fairness, relay selection, user pairing.**

I. Introduction

Non-orthogonal multiple access (NOMA) has been consensually regarded as a promising multiple access scheme for the next generation of wireless networks [1]. In order to further improve its performance, NOMA has been integrated with cooperative communications, in which the first cooperative NOMA (C-NOMA) scheme was studied in [2]. To reduce system complexity in a multi-relay C-NOMA scenario, relay selection (RS) techniques were proposed along the years. The work in [3] presented a two-stage max-min RS scheme, while [4] investigated NOMA in amplify-and-forward (AF) relay systems with partial relay selection (PRS). In addition, to alleviate the system complexity behind NOMA, user pairing (UP) schemes were proposed in [5] and [6]. On the other hand, to achieve user fairness, power allocation (PA) policies in NOMA systems were studied in [7]–[9], whereas sum rate maximization was considered in [10] and [11].

Differently from previous works, this paper aims to investigate adaptive PA, RS, and UP policies in C-NOMA systems with rate fairness. Closed-form expressions for the power allocation factors are derived as well as an algorithm that provides the optimal solution at the BS. Our results are compared with other benchmark schemes (e.g., fixed PA, random RS, random UP, among others) in terms of outage probability, Jain's fairness index, and average sum rate, along with insightful discussions, and the superiority of the proposed study is shown.

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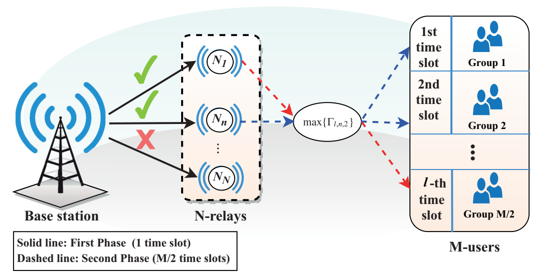

Fig. 1. System model.

II. System Model and Functionality

We consider a downlink cooperative NOMA scenario with arbitrary half-duplex decode-and-forward (DF) relays and multiple users, as illustrated in Fig. 1. The system is composed by one base station (BS), one cluster of N relays (N_n, with $n \in \mathcal{R} = \{1, \ldots, N\}$), and M users (M_m, with $m \in \mathcal{M} = \{1, \ldots, M\}$), with different channel conditions. Without loss of generality, it is assumed that M is an even number. Due to severe fading, we consider there is no direct links between the BS and users. In addition, we consider that all nodes are equipped with single antenna and that the users subscribe to the same traffic service that requires a minimum data rate to be acceptably provided. All channels exhibit non-frequency-selective Rayleigh-block fading and are disturbed by additive white Gaussian noise (AWGN) normalized with zero mean and unit variance.

As shown in Fig. 1, the communication consists basically of two phases. In the first phase, the BS transmits a superposed signal to all relays using superposition coding (SC). Then, all relays employ successive interference cancellation (SIC) technique to decode the respective users' information. Specifically, the n-th relay first decodes the signals of the weakest m-th users by treating the messages of the strongest k-th users,

$m < k$, as interference, then it removes the decoded messages from its observation in a successive manner. The message for the k-th user, $k > m$, will be treated as noise at the n-th relay when the message from the m-th user is being decoded. Therefore, the signal-to-interference-plus noise ratio (SINR) for m-th user, $1 \leq m \leq (M-1)$, at the n-th relay can be written as

$$\gamma_{n,m} = \frac{\rho |h_n|^2 \omega_{n,m}}{\rho |h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k} + 1}, \tag{1}$$

where $\rho = P/\sigma^2$ denotes the transmit signal-to-noise ratio (SNR) at the BS, P means the transmit power of the BS, $\omega_{n,m}$ denotes the power allocation factor for the m-th user at the n-th relay, with $\sum_{m=1}^{M} \omega_{n,m} = 1$ and $\omega_{n,m} > 0$, $h_n \sim \mathcal{CN}(0, d_n^{-\nu})$ denotes the channel coefficients of the BS $\rightarrow \mathcal{R}$ link, d_n is the distance between BS and the n-th relay, and ν stands for path loss exponent. By its turn, the SNR for the user with the best channel conditions, i.e., when $m = M$, is given by $\gamma_{n,M} = \rho |h_n|^2 \omega_{n,M}$.

Next, the UP and RS policies of this work are presented.

User Pairing Policy: The user pairing scheme is formulated in order to avoid the pairing between users with similar channel conditions. With this aim, users are sorted out according to their channel conditions and then the weakest user is paired with the strongest one. The selected users are removed and among the remaining users, we pair the next weakest user with the next strongest one. This step is done until there is no user left to pair. In this way, one can ensure that users with dissimilar channel conditions are paired with each other. At the end, we will have G_l groups, $l \in \mathcal{G} = \{1, \ldots, \frac{M}{2}\}$, with each group having two users ($\mathrm{U}_{l,i}$), $i \in \{1, 2\}$. The channel coefficient of the $\mathcal{R} \rightarrow \mathrm{U}_{l,i}$ link is denoted by $g_{l,n,m} \sim \mathcal{CN}(0, d_{l,n,m}^{-\nu})$, where $d_{l,n,m}$ means the distance between the n-th relay and m-th user of the l-th group. For each formed group, we sort the users in descending order based on their channel conditions and denote $\mathrm{U}_{l,1}$ as the strongest user and $\mathrm{U}_{l,2}$ as the weakest user, i.e., $(|g_{l,n,1}|^2 \geq |g_{l,n,2}|^2)$.

Relay Selection Policy: The proposed RS scheme consists of two stages, in which the first one consists in selecting a subset $\mathcal{S}$ of relays that are able to decode correctly the incoming message, i.e.,

$$\mathcal{S} = \Big\{ n \in \mathcal{R},\ \log_2(1 + \gamma_{n,m}) \geq \tilde{R} \Big\}, \tag{2}$$

where $\tilde{R}$ denotes the target rate of the users. Once the users are paired, the relays in subset $\mathcal{S}$ decode the users' signals simultaneously and transmit the superposed signal of the $\mathrm{U}_{l,i}$ to l-th group based on NOMA principle. Then, $\mathrm{U}_{l,2}$ decodes its own message by treating the message of $\mathrm{U}_{l,1}$ as noise. Thus, the SINR at $\mathrm{U}_{l,2}$ can be expressed as

$$\Gamma_{l,n,2} = \frac{\rho \alpha_{l,n,2} |g_{l,n,2}|^2}{\rho \alpha_{l,n,1} |g_{l,n,2}|^2 + 1}, \tag{3}$$

where $\alpha_{l,n,i}$ denotes the power allocation coefficient provided at the n-th relay to the i-th user of the l-th group, such that $\alpha_{l,n,1} + \alpha_{l,n,2} \leq 1$ and $\alpha_{l,n,1} < \alpha_{l,n,2}$. We consider $\alpha_{l,n,2} = 1 - \alpha_{l,n,1}$. By its turn, the user $\mathrm{U}_{l,1}$ firstly decodes $\mathrm{U}_{l,2}$'s data and then cancels it from the received signal and detects its own data. Therefore, the received SINR at $\mathrm{U}_{l,1}$ related to $\mathrm{U}_{l,2}$'s data can be written as

$$\Gamma_{l,n,1\rightarrow 2} = \frac{\rho \alpha_{l,n,2} |g_{l,n,1}|^2}{\rho \alpha_{l,n,1} |g_{l,n,1}|^2 + 1}. \tag{4}$$

If $\mathrm{U}_{l,1}$ perfectly cancels the $\mathrm{U}_{l,2}$'s signal, the SNR at $\mathrm{U}_{l,1}$ for detecting its message is given by

$$\Gamma_{l,n,1} = \rho \alpha_{l,n,1} |g_{l,n,1}|^2. \tag{5}$$

The second stage of relay selection consists in selecting the relay-destination link among the available ones in subset $\mathcal{S}$, which maximizes the SINR of $\mathrm{U}_{l,2}$ at the l-th group, i.e.,

$$n_l^* = \arg\max_{n \in \mathcal{S}} \{\Gamma_{l,n,2}\}, \tag{6}$$

which is shown in Fig. 1. It is worth mentioning that a relay can be chosen to transmit to more than one group of users. The achievable rates for $\mathrm{U}_{l,1}$ and $\mathrm{U}_{l,2}$ are given, respectively:

$$R_{l,1} = \log_2(1 + \Gamma_{l,n_l^*,1}), \tag{7}$$

$$R_{l,2} = \log_2(1 + \Gamma_{l,n_l^*,2}). \tag{8}$$

III. Adaptive Power Allocation with Rate Fairness

This section investigates an adaptive PA problem to maximize the minimum users' achievable rate assuming rate fairness. For each phase, it is formulated a power allocation optimization problem.

A. Power Allocation Formulation: First Phase

Based on the attained channel state information (CSI), the problem is formulated as follows

$$\max_{\omega_{n,m}} \min_{m \in \mathcal{M}} \left\{ \frac{1}{2} \log_2(1 + \gamma_{n,m}) \right\}, \tag{9}$$

subject to

$$\sum_{m=1}^{M} \omega_{n,m} \leq 1, \tag{10}$$

$$0 \leq \omega_{n,m}, \quad \text{for } n \in \mathcal{R} \text{ and } m \in \mathcal{M}. \tag{11}$$

One can observe that problem (9) is not convex due to the non-convexity of the objective function. Therefore, it is not possible to obtain the optimal solution through standard optimization solvers. To circumvent this, the following lemma is proposed.

Lemma I: The optimization problem (9) is quasi-concave.

Proof: This is equivalent to demonstrate that the objective function is quasi-concave and the constraints are convex. The constrains (10) and (11) are convex due to their linearity. To demonstrate the quasi-concavity of the objective function, all its superlevel sets are $S_r = \{\omega_{n,m} | \min(\frac{1}{2}\log_2(1 + \gamma_{n,m})) \geq r\}$, for $r \in \mathbb{R}$. After algebraic manipulations, the constraints in set S_r can be equivalently rewritten as the following inequalities

$$\rho |h_n|^2 \omega_{n,m} \geq \beta \Big(\rho |h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k} + 1 \Big), \tag{12}$$

where $\beta = 2^{2r}-1$. The superlevel sets are convex because they can be expressed as the convex set. Therefore, the objective function is quasi-concave. ■

Due to the quasi-concavity of problem (9), one can convert it into a convex feasibility problem. Let ϵ^* denote the optimal value of (9). For a given $r > 0$, if the following feasibility problem is feasible

$$\text{Find } \omega_{n,m}, \text{ subject to (10), (11), and (12),} \tag{13}$$

then we have $\epsilon^* \leq r$. Otherwise, the problem is infeasible if $\epsilon^* > r$. Equivalently, we can solve

$$\min_{\omega_n} \left\{ \sum_{m=1}^{M} \omega_{n,m} \right\} \text{ subject to (11) and (12),} \tag{14}$$

and check if the solution for (14) satisfies the constraint (10). Based on this, the following theorem is presented to obtain the optimal closed-form solution for (14).

Theorem I: The optimal closed-form solution to (14) is

$$\omega^*_{n,m} = \frac{\beta}{\rho|h_n|^2}\left(\rho|h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k} + 1\right). \tag{15}$$

Proof: The Karush-Kuhn-Tucker (KKT) conditions provide the necessary and sufficient conditions to verify the existence of a optimal solution. The Lagrange function corresponding to optimization problem (14) is given by

$$\mathcal{L}(\omega_{n,m}, \lambda_{n,m}, \delta_{n,m}) = -\omega_{n,m} + \lambda_{n,m}\left(\beta(\rho|h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k} + 1) - \rho|h_n|^2\omega_{n,m}\right) + \delta_{n,m}(\omega_{n,m}), \tag{16}$$

where $\lambda_{n,m}$ and $\delta_{n,m}$ are the Lagrangian multipliers. Then, using KKT conditions, we obtain the following equations:

$$\nabla\mathcal{L}(\omega_{n,m}, \lambda_{n,m}, \delta_{n,m}) = 0, \tag{17}$$

$$\omega_{n,m}\rho|h_n|^2 \geq \beta\left(\rho|h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k} + 1\right), \tag{18}$$

$$\omega_{n,m} \geq 0, \tag{19}$$

$$\lambda_{n,m} \geq 0, \ \delta_{n,m} \geq 0, \tag{20}$$

$$\lambda_{n,m}\left(\beta(\rho|h_n|^2 \sum_{k=m+1}^{M} \omega_{n,k}+1) - \omega_{n,m}\rho|h_n|^2\right) = 0, \tag{21}$$

$$\delta_{n,m}\omega_{n,m} = 0. \tag{22}$$

The symbol ∇ denotes the gradient operator. One can see that (17) is strictly positive, $\forall\ n \in \mathcal{R}$ and $m \in \mathcal{M}$, which implies that $\lambda_{n,m} > 0$ and $\delta_{n,m} = 0$, due to (22). Similarly, (18) is strictly positive which implies that $\omega_{n,m} > 0$ and $\delta_{n,m} = 0$. According to (21), (22), and $\lambda_{n,m} > 0$, one can conclude that all constraints (18) should be equalities. Based on this, the closed-form solution to the power coefficient $\omega_{n,m}$ only depends on the power allocated to the stronger channels $(m+1, \ldots, M)$. In the other words, by allocating power from the strongest user to the weakest user, the optimal closed-form solution to problem (14) can be obtained as (15), which completes the proof. ■

Algorithm 1: Adaptive power factor allocation at the BS for M-users and N-relays.

```
for n=1:N do
    Set r_LB = 0, r_UB = (1/2) log2(1 + ρ|h_n|^2)
    while (r_UB − r_LB) ≥ τ do
        Set r = (r_UB + r_LB)/2; Obtain ω*_{n,m} in (15).
        if Σ_{m=1}^{M} ω*_{n,m} ≤ 1 then
            Set r_LB = r; ε* = r; ω_{n,m} = ω*_{n,m}.
        else
            Set r_UB = r.
        end
    end
end
```

Relying on bisection method, similar to [9], and using the optimal closed-form expression obtained in (15), the solution for the problem (9) is shown in Algorithm 1.

B. Power Allocation Formulation: Second Phase

Once the UP procedure is concluded, the PA problem at the relays pertaining to $\mathcal{S}$ is formulated in which the optimal power coefficient to the messages of the i-th user of the l-th group is obtained. From the fairness standpoint, the optimization problem can be formulated as

$$\max_{\alpha_{l,n,1},\alpha_{l,n,2}} \left\{ \min\left(\frac{1}{2}\log_2(1+\Gamma_{l,n,1}), \frac{1}{2}\log_2(1+\Gamma_{l,n,2})\right) \right\}, \tag{23}$$

subject to

$$\alpha_{l,n,1} + \alpha_{l,n,2} = 1, \tag{24}$$

$$\alpha_{l,n,1} \geq 0 \text{ and } \alpha_{l,n,2} \geq 0. \tag{25}$$

The problem (23) can be reformulated so that the power set to the strongest user of the l-th group should be enough to guarantee a successful SIC, i.e.,

$$\max_{\alpha_{l,n,1},\alpha_{l,n,2}} \left\{ \frac{1}{2}\log_2(1+\Gamma_{l,n,1}) \right\}, \tag{26}$$

subject to (24), (25) and

$$\frac{1}{2}\log_2(1+\Gamma_{l,n,1}) \leq \frac{1}{2}\log_2(1+\Gamma_{l,n,2}). \tag{27}$$

Because the function $\log_2(\cdot)$ is monotonically increasing, and inserting (3) and (5) into (27), the constraint can be rewritten as $\rho\alpha_{l,n,1}|g_{l,n,1}|^2 \leq \frac{\rho\alpha_{l,n,2}|g_{l,n,2}|^2}{\rho\alpha_{l,n,1}|g_{l,n,2}|^2+1}$, or, equivalently,

$$\rho\alpha_{l,n,1}|g_{l,n,1}|^2(\rho\alpha_{l,n,1}|g_{l,n,2}|^2 + 1) \leq \rho\alpha_{l,n,2}|g_{l,n,2}|^2. \tag{28}$$

Thus, problem (26) will be constrained to (24), (25) and (28). Since $\alpha_{l,n,2} = 1 - \alpha_{l,n,1}$, and as a result of algebraic manipulations in (28), it follows that

$$\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha^2_{l,n,1} + \rho(|g_{l,n,1}|^2 + |g_{l,n,2}|^2)\alpha_{l,n,1} - \rho|g_{l,n,2}|^2 \leq 0. \tag{29}$$

One can see that the problem can be converted into a function of $\alpha_{l,n,1}$. Hence, the problem (26) can be reformulated as

$$\max_{\alpha_{l,n,1}} \left\{ \frac{1}{2} \log_2(1+\gamma_{l,n,1}) \right\}, \tag{30}$$

subject to $\alpha_{l,n,1} \geq 0$ and (29). As the transmit power of the n-th relay in $\mathcal{S}$ increases, the max-min achievable rate also increases monotonically. Ensuring that $\alpha_{l,n,1}$ contains enough power to perform a perfect SIC, the objective function can be simplified to $\max_{\alpha_{l,n,1}}\{\alpha_{l,n,1}\}$. Rewriting this optimization problem in the standard form, we obtain

$$\min_{\alpha_{l,n,1}} \{-\alpha_{l,n,1}\}, \;\text{ subject to } -\alpha_{l,n,1} \leq 0 \;\text{ and (29)}. \tag{31}$$

Based on (31), we verify the existence of a global optimal solution. For this purpose, the following lemma is formulated.

Lemma II: The optimization problem (31) is convex.

Proof: A problem is convex when its objective function is convex, the space of feasible solutions is convex, and equality constraints are affine. The objective function is convex because it is a linear function. Since the function that maps $\alpha_{l,n,1}$ to $-\alpha_{l,n,1}$ is linear, and the polynomial function related to condition (29) is convex, it follows that the inequality constraints are convex. Therefore, problem (31) is convex. ■

Since the optimization problem is convex, the following theorem is formulated to obtain the global optimum solution.

Theorem II: The optimal power coefficient allocated by the n-th relay to the user $\mathrm{U}_{l,1}$ of the l-th group is given by

$$\alpha_{l,n,1} = \frac{-b+\sqrt{b^2-4ac}}{2a}, \quad \text{for } n \in \mathcal{S}, \tag{32}$$

where $a = \rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2$, $b = \rho(|g_{l,n,1}|^2 + |g_{l,n,2}|^2)$, and $c = -\rho|g_{l,n,2}|^2$.

Proof: The proof is a consequence of Lemma II. Because the problem is convex, the KKT conditions are sufficient to verify the existence of a global optimum solution. The Lagrange function corresponding to optimization problem (31) can be formulated as

$$\begin{aligned}\mathcal{L}(\alpha_{l,n,1}, \mu_{l,1}, \mu_{l,2}) &= -\alpha_{l,n,1} \\ + \mu_{l,1}\Big(\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1}^2 &+ \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2)\alpha_{l,n,1} \\ &- \rho|g_{l,n,2}|^2\Big) + \mu_{l,2}(-\alpha_{l,n,1}),\end{aligned} \tag{33}$$

where $\mu_{l,1}$ and $\mu_{l,2}$ are Lagrange multipliers. Using KKT conditions, we can obtain the following equations:

$$\nabla\mathcal{L}(\alpha_{l,n,1}, \mu_{l,1}, \mu_{l,2}) = 0. \tag{34}$$

$$\begin{aligned}\Big(\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1}^2 + \rho(|g_{l,}|^2+|g_{l,n,2}|^2)\alpha_{l,n,1} \\ - \rho|g_{l,n,2}|^2\Big) \leq 0,\end{aligned} \tag{35}$$

$$-\alpha_{l,n,1} \leq 0, \; \mu_{l,1} \geq 0, \; \mu_{l,2} \geq 0. \tag{36}$$

$$\begin{aligned}\mu_{l,1}\Big(\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1}^2 + \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2)\alpha_{l,n,1} \\ - \rho|g_{l,n,2}|^2\Big) = 0,\end{aligned} \tag{37}$$

$$\mu_{l,2}(-\alpha_{l,n,1}) = 0. \tag{38}$$

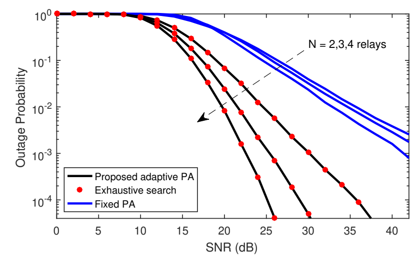

Fig. 2. Outage probability versus SNR for different number of relays and $M = 4$ users.

From (31) and (34), we have

$$\begin{aligned}\frac{\partial}{\partial\alpha_{l,n,1}}\mathcal{L}(\alpha_{l,n,1}, \mu_{l,1}, \mu_{l,2}) = -1 \\ + \mu_{l,1}\Big(2\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1} + \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2)\Big) \\ - \mu_{l,2} = 0.\end{aligned} \tag{39}$$

Based on condition (38) and assuming $\alpha_{l,n,1} > 0$, it follows that $\mu_{l,2} = 0$. Using this result into (39), we obtain

$$\mu_{l,1} = \Big(2\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1} + \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2)\Big)^{-1}. \tag{40}$$

Since that $2\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1} + \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2) \neq 0$, we have that $\mu_{l,1} \neq 0$ in (40). This result combined with the condition (37) implies that the solution is found by solving

$$\begin{aligned}\rho^2|g_{l,n,1}|^2|g_{l,n,2}|^2\alpha_{l,n,1}^2 + \rho(|g_{l,n,1}|^2+|g_{l,n,2}|^2)\alpha_{l,n,1} \\ - \rho|g_{l,n,2}|^2 = 0.\end{aligned} \tag{41}$$

Retaining the feasible root of equation (41), we obtain expression (32) for the power allocated to the stronger user for the l-th group that guarantees rate fairness in the second phase. Thus, the proof of Theorem II is completed. ■

IV. Simulation Results and Discussions

In the simulations, 10^5 Monte Carlo experiments are employed and it is assumed $\tilde{R} = 0.5$ bits/s/Hz, $\nu = 2$, $d_n = 1$ m, and $\tau = 10^{-4}$ (desirable accuracy). The users are randomly distributed on a disc with diameter 1 m whose center is located to 2 m from cluster of relays. When fixed PA is considered, the values of power coefficients employed by BS are $\omega_{n,1} = 0.52$, $\omega_{n,2} = 0.25$, $\omega_{n,3} = 0.13$, and $\omega_{n,4} = 0.1$, while it is assumed $\alpha_{l,n,1} = 0.2$ and $\alpha_{l,n,2} = 0.8$ in the second phase of the communication. The outage probability, Jain's fairness index, and average achievable sum rate are used as performance metrics. In particular, for the l-th group, the Jain's fairness index can be expressed as $J_l = (R_{l,1} + R_{l,2})^2/[2((R_{l,1})^2 + (R_{l,2})^2)]$,

In Fig. 2, the outage probability versus transmit SNR is plotted assuming $N = 2, 3, 4$ relays, and $M = 4$ users. It is compared the performance of the proposed analysis with the solution achieved by exhaustive search and with the fixed PA scheme. One can observe that the proposed adaptive PA with UP and RS scheme matches to the optimal solution obtained by exhaustive search, which verifies the correctness of Theorem II. For the exhaustive search, we discretized $\alpha_{n,1}$ with accuracy τ looking for the optimal solution, requiring

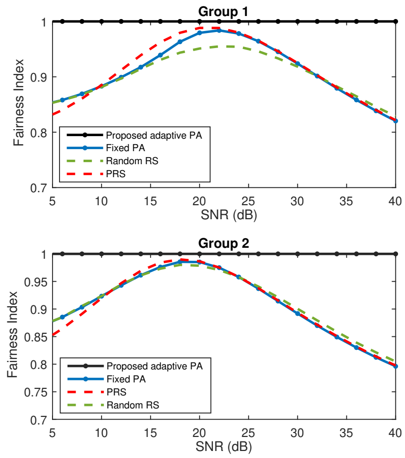

Fig. 3. Jain's fairness index versus SNR for $N = 2$ relays and $M = 4$ users.

high computational complexity. One can see that the proposed adaptive PA scheme can provide better outage performance when compared to the fixed PA with UP and RS scheme. This is because the power factors are adaptively calculated to maximize the minimum rate among users and among the user groups, ensuring that all users are satisfied with the traffic service provided. Meanwhile, the fixed PA ensures that the BS and the relays provide static power factor within each fading block. In addition, when the number of relays increases, the outage probability decreases. This is a result of the spatial diversity that each relay has in allocating different power factors to users groups. Another observation is that the proposed scheme provides a diversity order equals to the number of relays.

In Fig. 3, it is investigated the relation between the resource allocation fairness and the SNR for each user group. As a performance benchmark, we used the random RS and the PRS schemes [4]. All of them were plotted with fixed PA, but adopting our UP scheme based on CSI-sorting. The adaptive PA of the first phase provides the achievable fairness users' rate at the relays, while the adaptive PA scheme of the second phase, once UP is done, allows to adapt the power factor to ensure perfect SIC on the user with better channel conditions and to maximize the user rate with worst channel. This scheme ensures that all same group users are served equally throughout the communication process. The performance benchmark curves were similar for group 2, since the pair consists of two users with intermediate channel gains, their rates achieved are similar. Moreover, the proposed scheme achieved maximum level of fairness. The developed strategy favors all users of the system, ensuring the necessary resources for their messages to be decoded correctly.

Fig. 4 depicts the average sum rate versus transmit SNR. As performance benchmark, we use UP with random pairing and pairing based on minimum distance between the users, where users are paired with the nearest neighbor. All cases considered the proposed adaptive PA and RS scheme. The proposed UP scheme provides performance improvements compared to all the simulated benchmark schemes for intermediate and high SNR regions. The better performance comes from the fact that

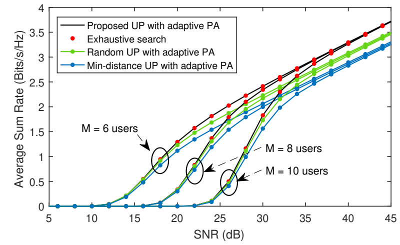

Fig. 4. Average sum rate for different UP schemes with $N = 4$ relays.

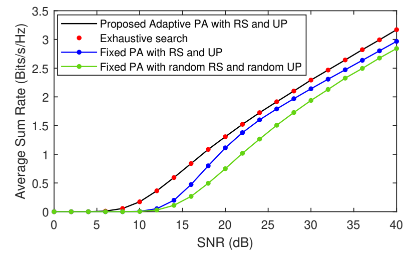

Fig. 5. Average sum rate for different power allocation schemes ($N = 2$ relays and $M = 4$ users).

proposed UP can select the pairs by exploiting the advantages of their respective channels, avoiding that the intermediate user be paired with a stronger user or with the weaker user. In this way, more power can be allocated to the weakest users to maximize the minimum rate and ensures the maximum level of rate fairness. Fig. 5 compares the average achievable rate with results obtained by exhaustive search and fixed PA scheme. The performance gap between adaptive PA and fixed PA demonstrates the importance of adaptively allocating power factors in order to ensure maximum rate and fairness among users.

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