Nazanin Vafaei

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Orientador: Doutora Maria Rita Sarmento de Almeida Ribeiro, Professora Associada Convidada, NOVA School of Science and Technology

Co-orientador: Doutor Luís Manuel Camarinha de Matos, Professor Catedrático, NOVA School of Science and Technology

Júri:

Presidente: Doutor João Carlos da Palma Goes, Professor Catedrático, NOVA School of Science and Technology

Arguentes: Doutor Carlos Henggeler Antunes, Professor Catedrático, Universidade de Coimbra
Doutora Ana Luisa do Carmo Correia Respício, Professora Auxiliar, Universidade de Lisboa

Vogais: Doutora Maria Leonilde Rocha Varela, Professora Auxiliar, Universidade do Minho
Doutor João Carlos da Palma Goes, Professor Catedrático, NOVA School of Science and Technology
Doutor José Manuel Matos Ribeiro da Fonseca, Professor Associado com Agregação, NOVA School of Science and Technology
Doutora Maria Rita Sarmento de Almeida Ribeiro, Professora Associada Convidada, NOVA School of Science and Technology
Doutor Javad Jafarali Jassbi, Professor Auxiliar Convidado, NOVA School of Science and Technology

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To whom I love

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Abstract

With the fast-growing of data-rich systems, dealing with complex decision problems is unavoidable. Normalization is a crucial step in most multi criteria decision making (MCDM) models, to produce comparable and dimensionless data from heterogeneous data. Further, MCDM requires data to be numerical and comparable to be aggregated into a single score per alternative, thus providing their ranking.

Several normalization techniques are available, but their performance depends on a number of characteristics of the problem at hand i.e., different normalization techniques may provide different rankings for alternatives. Therefore, it is a challenge to select a suitable normalization technique to represent an appropriate mapping from source data to a common scale. There are some attempts in the literature to address the subject of normalization in MCDM, but there is still a lack of assessment frameworks for evaluating normalization techniques.

Hence, the main contribution and objective of this study is to develop an assessment framework for analysing the effects of normalization techniques on ranking of alternatives in MCDM methods and recommend the most appropriate technique for specific decision problems. The proposed assessment framework consists of four steps: (i) determining data types; (ii) chose potential candidate normalization techniques; (iii) analysis and evaluation of techniques; and (iv) selection of the best normalization technique. To validate the efficiency and robustness of the proposed framework, six normalization techniques (Max, Max-Min, Sum, Vector, Logarithmic, and Fuzzification) are selected from linear, semi-linear, and non-linear categories, and tested with four well known MCDM methods (TOPSIS, SAW, AHP, and ELECTRE), from scoring, comparative, and ranking methods. Designing the proposed assessment framework led to a conceptual model allowing an automatic decision-making process, besides recommending the most appropriate normalization technique for MCDM problems. Furthermore, the role of normalization techniques for dynamic multi criteria decision making (DMCDM) in collaborative networks is explored, specifically related to problems of selection of suppliers, business partners, resources, etc.

To validate and test the utility and applicability of the assessment framework, a number of case studies are discussed and benchmarking and testimonies from experts are used. Also, an evaluation by the research community of the work developed is presented. The validation process demonstrated that the proposed assessment framework increases the accuracy of results in MCDM decision problems.

Keywords: Multi Criteria Decision Making, MCDM, Normalization, Dynamic MCDM, Data Fusion, Aggregation.

Resumo

Com o rápido crescimento dos sistemas ricos em dados, lidar com problemas de decisão complexos é inevitável. A normalização é uma etapa crucial na maioria dos modelos de tomada de decisão multicritério (MCDM), para produzir dados comparáveis e adimensionais a partir de dados heterogéneos, porque os dados precisam ser numéricos e comparáveis para serem agregados em uma única pontuação por alternativa. Como tal, várias técnicas de normalização estão disponíveis, mas o seu desempenho depende de uma série de características do problema em questão, ou seja, diferentes técnicas de normalização podem resultar em diferentes classificações para as alternativas. Portanto, é um desafio selecionar uma técnica de normalização adequada para representar o mapeamento dos dados de origem para uma escala comum. Existem algumas tentativas na literatura de abordar o assunto da normalização, mas ainda há uma falta de estrutura de avaliação para avaliar as técnicas de normalização sobre qual técnica é mais apropriada para os métodos MCDM.

Assim, a principal contribuição e objetivo deste estudo são desenvolver uma ferramenta de avaliação para analisar os efeitos das técnicas de normalização na seriação de alternativas em métodos MCDM e recomendar a técnica mais adequada para problemas de decisão específicos. A estrutura de avaliação da ferramenta proposta consiste em quatro etapas: (i) determinar os tipos de dados, (ii) selecionar potenciais técnicas de normalização, (iii) análise e avaliação de técnicas em problemas de MCDM, e (iv) recomendação da melhor técnica para o problema de decisão. Para validar a eficácia e robustez da ferramenta proposta, seis técnicas de normalização (Max, Max-Min, Sum, Vector, Logarithmic e Fuzzification) foram selecionadas - das categorias lineares, semilineares e não lineares- e quatro conhecidos métodos de MCDM foram escolhidos (TOPSIS, SAW, AHP e ELECTRE). O desenho da ferramenta de avaliação proposta levou ao modelo conceptual que forneceu um processo automático de tomada de decisão, além de recomendar a técnica de normalização mais adequada para problemas de decisão. Além disso, é explorado o papel das técnicas de normalização para tomada de decisão multicritério dinâmica (DMCDM) em redes colaborativas, especificamente relacionadas com problemas de seleção de fornecedores, parceiros de negócios, recursos, etc.

Para validar e testar a utilidade e aplicabilidade da ferramenta de avaliação, uma série de casos de estudo são discutidos e benchmarking e testemunhos de especialistas são usados. Além disso, uma avaliação do trabalho desenvolvido pela comunidade de investigação também é apresentada. Esta validação demonstrou que a ferramenta proposta aumenta a precisão dos resultados em problemas de decisão multicritério.

Palavras-chave: Tomada de decisão multicritério, MCDM, normalização, MCDM dinâmico, fusão de dados, agregação.

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List of Acronyms

AI	Artificial Intelligence
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
AV	Alternative's value
BN	Bayesian Networks
CN	Collaborative Networks
CP	Collaborative Planning
CPS	Cyber-Physical Systems
CSR	Corporate Social Responsibility
cDSP	compromise Decision Support Problem
CW	Consistency Weight
CRm	Constructive Research method
DS	Dampster Shafer
DMCDM	Dynamic Multiple Criteria Decision Making Method
DDN	Dynamic Decision Networks
DV	Data Visualization
ELECTRE	ELimination Et Choice Translating REality
ES	Expert Systems
FAHP	Fuzzy Analytic Hierarchy Process
FL	Fuzzy Logic
FSD	Fuzzy Synthetic Decision
GA	Genetic Algorithm
IoT	Internet of Things
KB	Knowledge Base
MADM	Multi Attribute Decision Making
MCDM	Multi Criteria Decision Making
ML	Machine Learning
MODM	Multi Objective Decision Making
MECDSS	Multi Enterprise Collaborative Decision Support System

MSE	Mean Squared Error
NIS	Negative Ideal Solution
NN	Neural Network
PIS	Positive Ideal Solution
POMDP	Partially Observable Markov Decision Processes
PROMETHEE	Preference Ranking Organization METHod for Enrichment Evaluation
PV	Plurality Voting
RCI	Ranking Consistency Index
SA	Simulated-annealing
SAW	Simple Additive Weighting
SC	Supply Chain
Sig	Significant test
STD	Standard Deviation
SOM	Self-Organizing-Map
TS	Tabu-Search
Target-AVG	Target-based normalization technique with Average
Target-Med	Target-based normalization technique with Med
TOPSIS	Technique for Order Performance by Similarity to Ideal Solution
VE	Virtual Enterprise
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje
WASPAS	Weighted Aggregated Sum Product Assessment Method
WP	Weighted Product method

1

1 Introduction

This chapter introduces the main topic of this research. The problem statements and motivations to do this research are described and then the research question and related hypothesis are explained as well. The summary of the research method and adopted works from the literature are presented shortly. The outline of this dissertation ends this chapter.

1.1 Problem Statement and Motivation

Human beings use multi-criteria decision-making methods (MCDM), sometimes also called multiple attribute decision-making (MADM), in many daily activities to solve decision problems and find the best decision, in face of several criteria and alternatives (Zavadskas and Turskis, 2010). A multi-criteria decision-making (MCDM) problem can be defined by a decision matrix, composed of a finite set of alternatives A_i (i=1, …, m), a set of criteria C_j (j=1,…, n), the relative importance of the criteria (or weights) W_j, and the matrix cell elements, r_{ij}, representing the rating for alternative A_i with respect to criteria C_j (Jahan et al., 2016; Triantaphyllou, 2000). Each criterion may be measured in different units and can be expressed either as a qualitative or quantitative value, for example, size (qualitative), degrees, kilograms or meters, which is an obstacle for the aggregation/ranking process (Zavadskas and Turskis, 2010). Hence, there is a need to use normalization to prepare dimensionless and comparable criteria values to allow their aggregation into a final score (Jahan et al., 2016; Triantaphyllou, 2000; Zavadskas and Turskis, 2010).

Summarizing, the first step for modeling and applying most MCDM methods, to solve decision problems, is to choose a suitable normalization technique for the problem at hand. There are several normalization techniques introduced in the literature. Jahan and Edwards (2015) identified thirty-one normalization methods for transforming raw data into dimensionless criteria to be used in MCDM decision problems. However, so far, there is no consensus in the literature about which normalization technique is more suitable for many well-known MCDM methods (Vafaei et al., 2015). The process of normalization maps criteria values into the interval [0-1] by keeping approximately the same magnitude, so, different normalization techniques may address different ranking for alternatives. Thus, it causes deviation from initial ordering/ranking (Chatterjee and Chakraborty, 2014). In other words, if the normalization technique is not suitable for a specific decision problem or for the chosen MCDM method, the best decision solution may be overlooked (Chatterjee and Chakraborty, 2014; Vafaei et al., 2018a). Therefore, the idea of recommending appropriate normalization techniques for usage with MCDM methods will improve the accuracy of decision solutions.

There are many performance metrics to assess classification problems (see for example (Eftekhary et al., 2012) but unfortunately, there are very few studies on assessing normalization techniques for MCDM methods and the question of how to choose and recommend the best technique, i.e., the one which better represents the input/raw data, still remains. For instance, Chakraborty and Yeh (2007) explored the effects of four normalization techniques (vector, Max-Min, Max, and Sum) in the SAW method, using a Ranking Consistency Index (RCI), to assess the best normalization technique. In another study, Celen (2014) discussed the impacts of four normalization techniques (Vector, Max-Min, Max and Sum) in the TOPSIS method and evaluated the suitability of these techniques by using consistency conditions. There are other contributions on assessing normalization techniques in MCDM which will be explained in the next chapter.

Although related approaches can be found in the literature, there is a lack of a comprehensive assessment framework that manages the process of choosing the best normalization technique for MCDM decision problems. So, this is the main motivation for this research. Specifically, the motivation for carrying out this study includes three important issues, listed below (Vafaei et al., 2016a):

- ✓ The crucial importance of data normalization for decision making problems where we need to fuse/aggregate multiple data to obtain a final score per alternative.
- ✓ The absence of a general assessment framework to recommend the best normalization technique for each well-known MCDM method.
- ✓ The fact that there are very few research studies in the literature about data normalization and their effects on MCDM decision problems.

In summary, the main aim of this thesis is to develop an assessment framework to evaluate different normalization techniques, using several metrics, and to recommend appropriate techniques for decision makers to handle MCDM problems.

1.2 Research Question and Hypothesis

As mentioned above, there are few studies about normalization techniques for MCDM methods, and most methods use the simple 1/N (where N is the biggest value for one criterion), without further consideration on how it might affect the final ranking, if other normalization technique is used instead of it. Therefore, the general open question to be answered in this thesis, is "which are the best normalization techniques for well-known MCDM methods?". Specifically, in this thesis we will try to answer the following research question:

> What are the characteristics and different steps of an evaluation framework to assess and recommend appropriate normalization techniques for well-known MCDM methods, namely SAW, TOPSIS, AHP, and ELECTRE?

In order to better analyze, interpret and answer the above research question, some sub-research questions require attention, as follows:

1. How can we effectively enhance the robustness of the assessment framework?
2. How can we classify normalization techniques based on their effects and behaviors on normalized values?
3. What are the effects of different input data on normalized values and thereby on ranking of alternatives in decision problems?
4. Which are the suitable metrics for building an assessment framework?
5. Which metrics guarantee covering several categories of mathematical measurements?
6. How to combine the results from different metrics to recommend the best normalization technique?

The following hypothesis is proposed to address the mentioned research question:

If we build a strong assessment framework to identify the best normalization technique for decision problems, using well-known MCDM methods, then we can ensure more robust results for ranking alternatives in decision problems. In other words, this assessment framework should support decision makers by recommending which normalization technique is more appropriate to solve their decision problems.

1.3 Research Method

This section discusses the research method adopted to answer the research questions. We opt for a classical research method (Camarinha-Matos, 2015), which consists of seven steps , as illustrated in Figure 1.

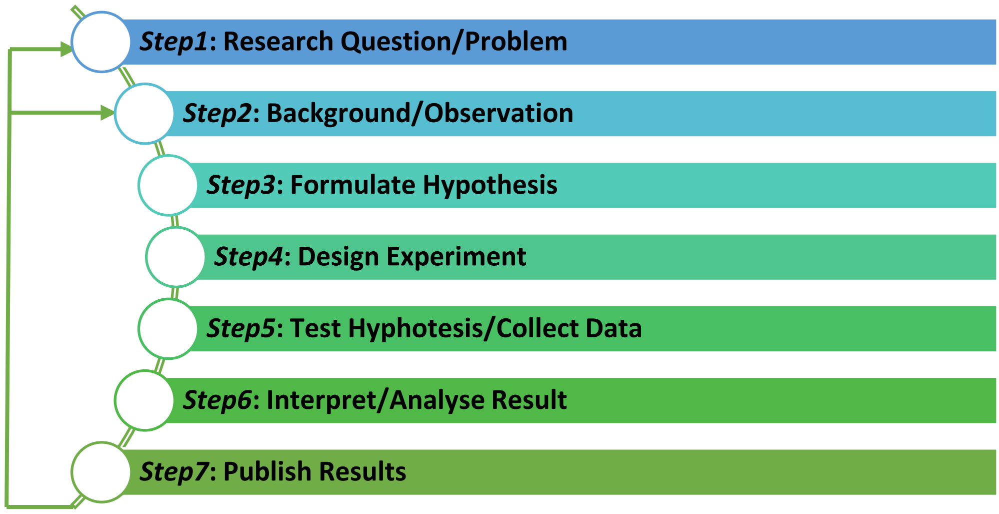


Figure 1: Classical Research Methodology adopted from (Camarinha-Matos, 2015).

Specifically, the steps of this classical research method when tailored for this thesis research work are the following:

Step 1- Research Question / Problem: Defining the research question considering the role of normalization in decision problems and motivation to formulate the research questions.

Step 2- Background / Observation: Collecting background information about the problem. In this observation some main topics are addressed, namely: multi criteria decision making (MCDM) methods, the role of normalization techniques in MCDM, taxonomy of MCDM methods and normalization techniques, different metrics for evaluating normalization techniques in MCDM problems, and glance at dynamic systems in collaborative networks.

Step 3- Formulate Hypothesis: Formulating potential solutions, to build the hypothesis, to answer the research question based on previous knowledge, state of the art, and gathered data.

Step 4- Design Experiment: Three evolutionary phases are defined to develop the proposed hypothesis, which leads to design the conceptual model of the assessment framework.

Step 5- Test Hypothesis / Collect Data: Different validation scenarios are applied to test and validate the proposed assessment framework, namely: case studies (with different scaling), benchmarking, and reviewers' testimonies. The results are summarized to support interpretation.

Step 6- Interpret / Analyse Results: Analysis and interpretation of obtained results from validation scenarios for the proposed framework.

Step 7- Publish findings: Publishing of intermediate findings were done since the beginning of this research work with the aim to obtain expert comments for the next thesis phases developments. The research work was published in indexed and recognized international journals as well as peer-reviewed conferences.

1.4 Thesis Outline

This thesis is organized into five chapters. A brief overview of each chapter is:

Chapter 1- Introduction: Describes the problem statements and motivations for the research work and defines the research question and hypothesis. Then the research method is described and the chapter finishes with the outline of this thesis.

Chapter 2- Background and Literature Review: This chapter describes the literature background for this research, as well as the required information to address challenges related to this thesis. It introduces and discusses the main concepts of multi criteria decision making (MCDM), the main existing normalization techniques and their role in MCDM, and presents a taxonomy for MCDM methods and other for normalization techniques. The literature review also discusses previous works about evaluation of normalization techniques in MCDM and recommendations of normalization techniques for specific problems. Further, some insights about dynamic systems and collaborative networks are introduced.

Chapter 3- Assessment Framework: This chapter presents and discusses the proposed framework and uses case studies and illustrative examples to enhance readability and deepen the explanation of the used methods. This chapter also describes well-known assessment tools and metrics, with the reasoning for selecting them to develop the proposed assessment framework. Furthermore, the framework's conceptual model for recommending the best normalization technique in MCDM methods and its respective automatic decision process, are addressed.

Chapter 4- Evaluation and Validation: This chapter discusses the validation and testing process of the proposed assessment framework to recommend the best normalization technique in MCDM method. The evaluation's method focuses on case studies, benchmarking, and expert testimonies; as well as accepted research work by the research community, such as panels, presentations and publications.

Chapter 5- Conclusion and Future Work: This chapter includes the main findings and analysis of the obtained results using the proposed assessment framework. Also, this chapter concludes this thesis research work and indicates open issues for future research.

2

2 Background and Literature Review

This chapter provides a concise literature review on established approaches as well as the state-of-the-art research on multi-criteria decision making (MCDM) and normalization techniques. It is organized as follows: Section 2.1 introduces the underlying concepts of decision making and discusses the background of multi-criteria decision making (MCDM) methods as well as their taxonomy. Section 2.2 introduces dynamic multi criteria decision making and collaborative networks. Section 2.3 presents and discusses existing normalization techniques, then it introduces a taxonomy for normalization techniques, explored in this thesis, and finishes with a review of previous works about evaluation of normalization techniques in MCDM methods.

2.1 Multiple Criteria Decision Making (MCDM)

2.1.1 Overview and taxonomy

Everybody makes decisions in their daily lives, as for example: "Should I take an umbrella today"? "Where should I go for lunch"? To make decisions we need access to information (or data) and to reach a decision we need to combine the retrieved data (e.g. prices and service of potential restaurants for having lunch) to obtain a final score for the candidate decision alternatives (Triantaphyllou, 2000).

In general, the aim of MCDM (sometimes also called Multiple Attribute Decision Making - MADM) is to select the best decision alternative, i.e. the one with the highest degree of satisfaction for all relevant attributes or criteria (Kazimieras Zavadskas *et al.*, 2014; Ribeiro, 1996). Decision makers' goals, preferences, alternatives and outcomes could be represented by five components of any MCDM problem, as described below (Kumar *et al.*, 2017; San Cristobal Mateo, 2012; Triantaphyllou, 2000; Wang *et al.*, 2009; Yoon and Hwang, 1995):

Alternative: The set of candidate solutions for any decision-making problem.

Criteria: the set of independent attributes/elements chosen to rate the alternatives.

Units: Criteria may be expressed in different units, so, usage of normalization techniques is obligatory to ensure comparable and dimensionless units for rating and ranking all criteria/attributes in decision making problems.

Weights: The relative importance of each criterion is expressed by a weight. It may be assigned directly by analysts or decision makers or by using trade-off methods.

Decision Matrix: it is a graphical representation for decision problems, where columns include the set of criteria and rows depict alternatives and the cells include the score for each criterion per alternative.

$$D = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nm} \end{bmatrix} \quad (2\text{-}1)$$

Where i=1,..,m denotes the alternatives and j=1,…,n refers to the attributes/criteria; rij represents value of the j^{th} attribute/criteria related to i^{th} alternative.

The main goal of MCDM methods is to help decision makers evaluate real-word situations based on qualitative/quantitative criteria/objectives in certain/uncertain/ risky environments, namely to find the most suitable course of action/choice/strategy/policy among several available options (Kazimieras Zavadskas *et al.*, 2014). Usually, MCDM methods could be classified in two major categories: Multi Attribute Decision Making (MADM) - used in discrete problems; and Multi Objective Decision Making (MODM) – used in continuous problems (see for example: (Belton and Stewart, 2002; Chen and Hwang, 1992; Hayashi, 2000; Kazimieras Zavadskas *et al.*, 2014; Korhonen *et al.*, 1992; Ribeiro, 1996; Zimmermann, 1986; Zimmermann and Sebastian, 1994). Figure 2 depicts this general classification of MCDM.

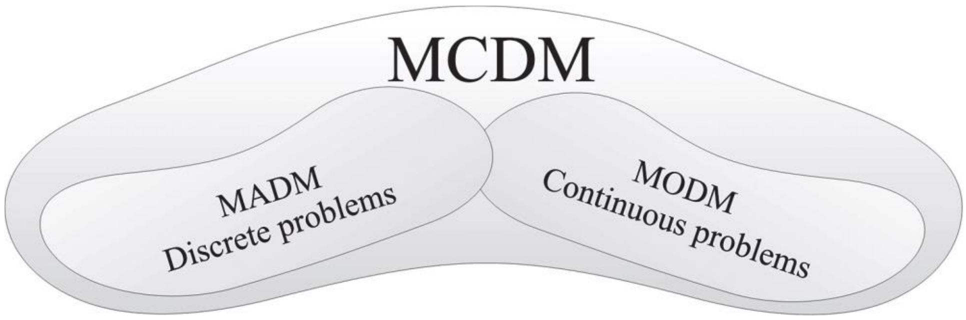


Figure 2: Broad classification of MCDM methods (adopted from (Kazimieras Zavadskas *et al.*, 2014))

MODM methods are utilized when the number of alternatives is within a domain (continuous domain) and alternatives are non-predetermined, i.e. the choice is made within a limited continuous space of alternatives. The goal of MODM is to find the optimal alternative/objective by considering a set of constraints and a set of quantifiable objectives (Jahan and Edwards, 2013; Kazimieras Zavadskas *et al.*, 2014).

On the other hand, MADM methods assume a limited set of alternatives from where the best one will be selected, by comparing the alternatives with respect to each criteria (attributes) (Kumar et al., 2017). In general, MADM methods include a pre-defined set of criteria to be assessed (i.e., rating process) and then rank (order) the pre-defined set of alternatives.

Other differences between these two types of MCDM methods are summarized in the Table 1.

Table 1: Comparison of MODM and MADM approaches (adopted from (Malczewski, 1999; Mendoza and Martins, 2006))

Criteria for comparison	MODM	MADM
Objectives defined	Explicitly	Implicitly
Criteria/Attributes defined	Implicitly	Explicitly
Constraints defined	Explicitly	Implicitly
Alternatives defined	Implicitly	Explicitly
Number of alternatives	Continuous domain	Discrete domain
Decision maker's control	Significant	Limited
Decision modelling paradigm	Process-oriented	Outcome-oriented
Relevant to	Design/search	Evaluation/choice

It should be noted that in this thesis we use the MCDM acronym instead of MADM because most literature does not distinguish MCDM and MADM and just uses the general MCDM nomenclature for discrete decision problems. Furthermore, we use this form because MCDM acronym is more commonly known in the addressed research area.

As mentioned above, the goal of MCDM methods is to support decision makers to make the best decision possible, based on pre-selected alternatives and criteria with relative associated weights (Yoon and Hwang, 1995). Usually, MCDM methods define the priority of alternatives by using mathematical algorithms based on experts' judgments to rank alternatives (Triantaphyllou, 2000). MCDM methods are applied in a wide range of problems such as financial application, software engineering, sports, e-business, site selection, supplier selection, etc. (see for example: (Ahmad and Laplante, 2006; Chen and Hwang, 1992; Golden *et al.*, 1989; Lee and Kozar, 2006; Mead, 2006; Pais *et al.*, 2010; Srdjevic, 2007)).

There are many different MCDM methods proposed in the literature (Cinelli *et al.*, 2020; Hwang and Yoon, 1981b; Tzeng and Huang, 2011; Yoon and Hwang, 1995) and Figure 3 shows a summarized taxonomy by the type of logical representation methods (from (Ribeiro *et al.*, 2011)). As Figure 3 depicts MCDM methods can be classified into two main categories Compensatory and Non-Compensatory methods. The Non-compensatory category can be sub-divided in three MCDM methods: Dominance, MaxMin, and MaxMax. Compensatory methods can be sub-divided in further three sub-classes: Scoring, Ordering, and Comparative methods. And the compensatory sub-class includes: (1) Weighted Average, Weighted Product, and Weighted Aggregated Sum Product Assessment Method (WASPAS) methods which belong to the Scoring sub-class; (2) TOPSIS, VIKOR, and Lexicographic methods, which belong to Ranking sub-class; and (3) AHP and ELECTRE, which belong to the Comparative sub-class.

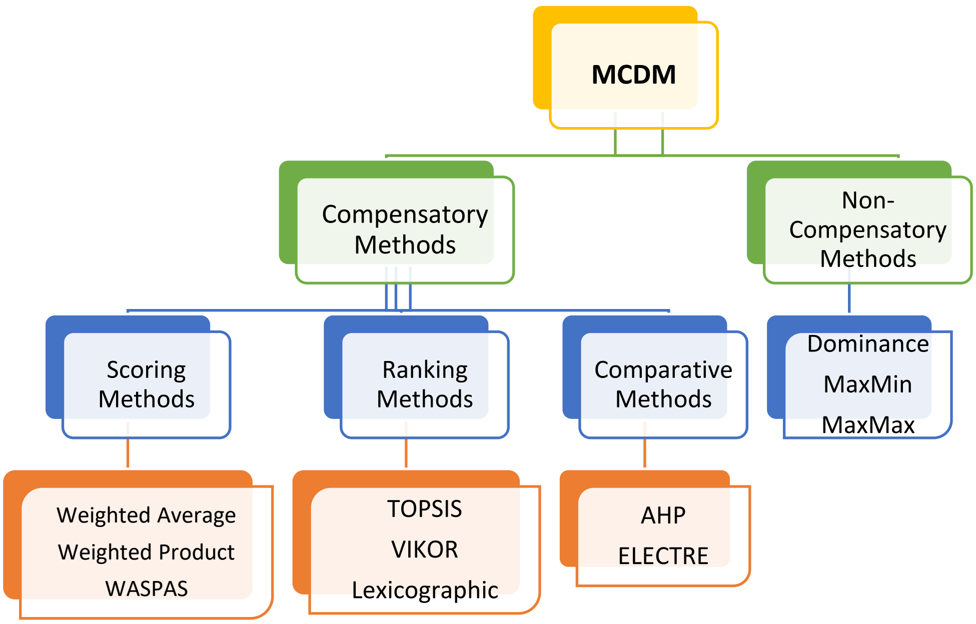


Figure 3: Taxonomy of MCDM methods (adapted from (Ribeiro *et al.*, 2011))

The main advantages of using compensatory methods is their ability to allow trade-offs between good and bad performance of different criteria, through compensation between those two types of criteria (Bhole and Deshmukh, 2018). For instance, in a car selection problem, the poor performance of internal design of the car could be compensated by good performance of fuel consumption. Bhole and Deshmukh (2018) stated about compensatory methods: "The mathematical calculation of MCDM can give better decision for one criterion and poor for another criterion this is an obligatory thing". Most well-known MCDM methods belong to this high-level compensatory category, so, the focus of this thesis is on compensatory methods and from each sub-class (scoring, raking and comparative) at least one MCDM method is chosen to evaluate their most adequate normalization procedure. In summary, the chosen MCDM methods to be studied in this thesis are: Sum Weighted Average (which is often known as Simple Additive Weighting (SAW) method), TOPSIS, AHP, and ELECTRE. A brief explanation about the chosen four MCDM methods (SAW, TOPSIS, AHP, ELECTRE) is described below. Also, a brief overview of other relevant methods namely VIKOR, WP, and WASPAS is mentioned in the following.

2.1.2 MCDM Scoring Methods: -Simple Additive Weighting (SAW)

The Simple Additive Weighting (SAW) method was first defined by Churchman and Ackoff (1945) for the portfolio selection problem. SAW is nowadays the most well-known and widely used method in dealing with multiple criteria decision problems (Bendra Wardana *et al.*, 2020; Tzeng and Huang, 2011; Yoon and Hwang, 1995; Zhou *et al.*, 2006). The general SAW method includes the following steps:

Step 1: Define a decision matrix (Equation (2-1)).

Step 2: Normalize the criteria scores.

Step 3: Calculate all alternatives' rating by aggregating the criteria scores for each alternative.

$$AV_i = \sum_{j=1}^{n} w_j \, n_{ij} \tag{2-2}$$

where n_{ij} are the normalized values per alternative, w_j is the weight of criterion j, and j =1,…,n and i = 1,…,m.

2.1.3 MCDM Ranking methods: Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS is a technique for order performance by similarity to an ideal solution, developed by Hwang and Yoon (1981b) and it is one of the most well-known MCDM methods. It ranks alternatives based on the shortest distance from a positive ideal solution (PIS) and the farthest distance from a negative ideal solution

(NIS). PIS is the most beneficial and lowest cost of alternatives and NIS is the lowest benefit and highest cost (Cheng *et al.*, 1999). TOPSIS is a classical MCDM method used in many different areas such as Supply Chain Management and Logistics; Design, Engineering and Manufacturing Systems; Business and Marketing Management; Health, Safety and Environment Management, and so forth (Alimoradi *et al.*, 2011; Behzadian *et al.*, 2012; Kahraman *et al.*, 2009; Khorshidi and Hassani, 2013; Krohling and Campanharo, 2011; Kwong and Tam, 2002; Mahdavi *et al.*, 2008; Meshram *et al.*, 2020). In general, the TOPSIS method includes the following steps (Joshi *et al.*, 2011; Tzeng and Huang, 2011; Yoon and Hwang, 1995):

Step 1: Define the decision matrix (Equation (2-1)).

Step 2: Normalize the scores per criterion. The preferred normalization technique for TOPSIS method is Vector normalization and the formula is below.

$$n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{n} r_{ij}^2}} \tag{2-3}$$

where j =1,…,n; and i = 1,…,m.

Step3: Calculate the weighted normalized scores in the decision matrix by multiplying the normalized criterion values by its associated weight:

$$V_{ij} = w_j \, n_{ij} \tag{2-4}$$

where w_j represents the weight of the j^th^ criterion and n_{ij} is the normalized value of the i^th^ alternative related to the j^th^ criterion.

Step 4: Determine the positive ideal solution (PIS) and negative ideal solution (NIS).

$$\begin{aligned} \text{PIS} &= \{V_1^+, \dots, V_n^+\} = \{(Max\ V_{ij} | j\epsilon J), (Min\ V_{ij} | j\epsilon J')\} \\ \text{NIS} &= \{V_1^-, \dots, V_n^-\} = \{(Min\ V_{ij} | j\epsilon J), (Max\ V_{ij} | j\epsilon J')\} \end{aligned} \tag{2-5}$$

where j=1,…,n and J represents the positive factors and J' are the negative factors (e.g., in the car selection example, fuel consumption and price are negative factors or criteria and comfort and safety are positive criteria, so, we should minimize the Vij for PIS and maximize Vij for NIS, while comfort and safety are positive criteria and we should maximize Vij for PIS and minimize Vij for NIS.)

Step 5: Calculate the distances of all alternatives to the positive ideal solution (D_i^+) and to the negative ideal (D_i^-) solution.

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(V_{ij} - V_j^+)^2}$$
$$D_i^- = \sqrt{\sum_{j=1}^{n}(V_{ij} - V_j^-)^2} \qquad (2\text{-}6)$$

Step 6: Calculate the relative closeness of each alternative as follow:

$$C_i^* = \frac{D_i^-}{D_i^+ + D_i^-} \qquad (2\text{-}7)$$

where C^*_i relies between 0 and 1 and the higher value corresponds to better performance.

2.1.4 MCDM Comparative Methods: Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) was introduced by Saaty (1977, 1980) to solve unstructured problems, mostly in economics, social sciences, and management (Cheng *et al.*, 1999). For example, AHP has been used in a vast range of problems from simple ones (e.g. selecting a school) to harder ones (e.g. allocating budgets and energy domains) (Cheng *et al.*, 1999). When applying the AHP method, the decision maker has to structure the decision problem and break it into a hierarchical top-down process. Then, he/she will perform a pairwise matrix comparison of criteria using a [1-9] scale (corresponding to semantic interpretations such has "A is much more important than B" regarding a criterion). Then a normalization is performed, dividing each criterion score per its sum (column). After, the rating of criteria (in AHP terminology denoted as priorities) is determined using either Eigen vectors or a simplified version with a weighted sum (SAW) (Gaudenzi and Borghesi, 2006; Zahedi, 1986). AHP involves five main steps (Tzeng and Huang, 2011; Yoon and Hwang, 1995):

Step 1: Decompose the problem into a hierarchical structure;

Step 2: Employ pairwise comparisons to establish the priority amongst criteria.

Notice that, a pairwise comparison is the process of comparing the relative importance, preference, or likelihood of two elements (objectives) with respect to another element (the goal). Pairwise comparisons are carried out to establish priorities. Decision elements at each hierarchy level are compared pairwisely and then the reciprocal matrix is completed.

Step 3: Determine the logical consistency and if the consistency index CI> 10%, revise the pairwise classifications until the consistency index is below 10%. When using AHP, we may face inconsistent judgments of input data which may cause some bad effects on the decision process. For example, A1 may be preferred to A2 and A2 to A3, but A3 may be preferred to A1. To deal with such situations, Saaty (Saaty, 1980) defined a measure of deviation that is called a consistency index as shown in equation (2-8):

$$CI = \frac{\lambda_{max} - N}{N - 1} \tag{2-8}$$

Where N is the dimension of the matrix and λ_{max} is the largest eigenvalue of the matrix A. Saaty (1980) suggested that the value of the CI should not exceed 0.1 for a confident results.

Step 4: Estimate the relative weights by combining the individual subjective judgments of a team of decision makers. We can use the eigenvalue method to estimate the relative weights of the decision elements. In order to estimate the relative weight of the decision elements in a matrix, we can use the formula (Tzeng and Huang, 2011):

$$A.W = \lambda_{max} \tag{2-9}$$

Where W is the weight of criteria and λ_{max} is the largest eigenvalue of the decision matrix A.

Step 5: Determine the priority of alternatives by aggregating relative weights obtained by combining the criterion priorities and priorities of each decision alternatives relative to each criterion.

Since in our work we discuss the suitability of normalization techniques for the AHP method, we will only focus on step 4 and 5 of AHP. Step 4 needs normalizing process and using different normalization techniques leads to different ranking of alternatives in step 5.

2.1.5 MCDM Comparative methods: ELimination Et Choice Translating REality (ELECTRE)

ELimination Et Choice Translating REality (ELECTRE) was first devised by B. Roy (1968). It is based on outranking relations and dichotomizing preferred and non-preferred alternatives. ELECTRE finds outranking relationships, then renders a set of preferred alternatives by forming a kernel (Tzeng and Huang, 2011). A variety of ELECTRE methods, such as ELECTRE I, II, III, IV, IS and TRI were developed for different purposes (Tzeng and Huang, 2011). However, each type is based on the same concept, although operating in different ways.

This method uses concordance and discordance representing the satisfaction or dissatisfaction of the decision maker. In this study we use ELECTRE I method formalization (Tzeng and Huang, 2011; Yoon and Hwang, 1995), which includes the following steps:

Step 1: Define the decision matrix (Equation (2-1)).

Step 2: Normalize the values of the decision matrix. The preferred normalization technique for the ELECTRE method is Vector (please see equation (2-3)). In this thesis, different normalization techniques to be used and their effects on ELECTRE method are analysed.

Step 3: Calculate the weighted normalized decision matrix (v_{ij}) by multiplying the normalized decision matrix by its associated weights.

$$v_{ij} = w_j \, n_{ij} \tag{2-10}$$

where w_j represents the weight of the j^{th} criterion and n_{ij} is the normalized value of the i^{th} alternative related to the j^{th} criterion.

Step 4: Determine the concordance (C) and discordance (D) sets for each pair of alternatives A_p and A_q using the formula below:

$$\begin{aligned} C(p,q) &= \{j \mid v_{pj} \geq v_{qj}\} \\ D(p,q) &= \{j \mid v_{pj} < v_{qj}\} \end{aligned} \tag{2-11}$$

It should be mentioned that v_{pj} and v_{pj} are the weighted normalized value of alternative A_p and A_q with respect to the j^{th} criterion. In other words, *C(p, q)* is the collection of criteria where A_p is better than or equal to A_q and $D(p, q)$ is the collection of criteria where A_p is worse than A_q.

Step 5: Calculate the concordance and discordance indexes. Each concordance and discordance set is measured by concordance (C_{pq}) and discordance (D_{pq}) indices as below:

$$C_{pq} = \sum_{j^*} w_{j^*}$$
$$D_{pq} = \frac{\sum_{j^+} \left| v_{pj^+} - v_{qj^+} \right|}{\sum_{j} \left| v_{pj} - v_{qj} \right|} \tag{2-12}$$

Where j^* and j^+ are attributes contained in the concordance set $C(p, q)$ and discordance set of $D(p, q)$ respectively. Also, v_{pj^+} and v_{qj^+} are weighted normalized values of alternative A_p and A_q with respect to the j^{th} criterion in discordance set of $D(p, q)$; and v_{pj} and v_{pj} are the weighted normalized value of alternative A_p and A_q with respect to the j^{th} criterion in concordance set of $C(p, q)$. Moreover, w_{j^*} is the weight of criterion j in concordance set of $C(p, q)$.

Step 6: Indicate outranking relationships. ELECTRE shows that alternative A_p is over the alternative A_q when:

$$C_{pq} \geq \bar{C} \text{ and } D_{pq} < \bar{D} \tag{2-13}$$

Where, $\bar{C}$ and $\bar{D}$ are the averages of C_{pq} and D_{pq}.

In other words, a higher concordance index C_{pq} and a lower discordance index D_{pq} cause stronger dominance relationship for the alternative A_p over the alternative A_q.

Step 7: Calculate the net concordance (C^{net}) and net discordance (D^{net}) indexes using following formula:

$$C_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{kp}$$
$$D_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{kp} \tag{2-14}$$

C_p measures the degree of dominance of A_p with respect to the other alternatives. Also, A_p measures the relative weakness of alternative A_p with respect to the other alternatives.

The final selection and sorting of alternatives is dependent on holding both the maximum C_p and the minimum D_p indexes. If both indexes are not satisfied, the alternative with highest average rank could be considered as the best alternative.

2.1.6 Other relevant MCDM methods

There are some other MCDM methods in the literature that are quite well-known namely VIKOR, WP, and WASPAS. Figure 3 shows that VIKOR belongs to ranking classification, and WP and WASPAS are classified as scoring methods. It should be noticed that we did not focus on these three methods and just used VIKOR in the small case study in section 3.1.1.3 and WASPAS in section 4.2.2.2. Also, WP is used by one of the mentioned paper in benchmarking part in section 4.2.4.4. So, we prefer to have a brief description of these three well-known methods in this thesis in the following.

2.1.6.1 VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)

The VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was developed by Opricovic (1998) for multi criteria optimization of complex systems. This method calculates the compromise ranking list, the compromise solution, and the weight stability intervals which lead to rank and select alternatives when there are conflicts between criteria. The VIKOR method measures the closeness to the ideal solution (Tzeng and Huang, 2011).

Step 1: Define decision matrixes (Equation (2-1)).

Step 2: Normalize the value of decision matrixes. In this thesis, different normalization techniques to be used and their effects on VIKTOR method are analyzed.

Step 3: Calculate the positive ideal solution (f_i^*) and the negative ideal solution (f_i^-).

$$f_j^* = \{(Max\ f_{ij} | j\epsilon J), (Min\ f_{ij} | j\epsilon J')\} \tag{2-15}$$

$$f_j^- = \{(Min\ f_{ij} | j\epsilon J), (Max\ f_{ij} | j\epsilon IJ')\}$$

where j=1,…,n and J represents the positive factors and J' represents the negative factors.

Step 4: Calculate the values of S_k and R_k using the equations bellow:

$$S_k = \sum_{j=1}^{m} w_j \left|f_j^* - f_{kj}\right| / \left|f_j^* - f_j^-\right|$$
$$R_k = max_j\{w_j \left|f_j^* - f_{kj}\right| / \left|f_j^* - f_j^-\right|\} \quad (2\text{-}16)$$

Where j=1,..,n and k=1,…,m and w_j are the weights of criteria which demonstrate their relative importance.

Step 5: Calculate Q_k that is the values of interests ratio brought by scheme, j=1,2, …, n using bellow formula:

$$Q_k = v\,(S_k - S^*)/(S^- - S^*) + (1 - v)(R_k - R^*)/(R^- - R^*) \quad (2\text{-}17)$$

Where k=1,…,m (alternatives) and v represents the weight of "the majority of criteria" strategy or the largest group's utility value, here v=0.5 that is borrowed from (Tzeng and Huang, 2011). Also, S^*=min S_k, S^- =max S_k, R^*=min R_k, and R^- =max R_k.

Step 6: Rank alternatives based on obtained Q, R, and S in decreasing orders (i.e., three tables for ranking are available).

Step 7: Alternative (a′) could be ranked as the best with the minimum Q if the following two condition are met simultaneously:

- C1. "Acceptable advantage":

  $$Q(a'') - Q(a') \geq DQ$$
  $$DQ=1/(J\text{-}1) \quad (2\text{-}18)$$

  Where a′′ is the alternative with the second position and J= number of alternatives.
- C2. "Acceptable stability in decision making"

  Alternative a′ must be the best rank for S or R. This compromise solution is stable within a decision-making process, which could be: “voting by majority rule” (when v > 0.5 is needed), “by consensus” v ≈ 0.5, or “with vote” (v < 0.5). Here, v is the weight of the decision-making strategy “the majority of criteria” (or “the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternative a′ and a″ if only condition C2 is not satisfied, or
- Alternative a′, a″,…, a^n if condition C1 is not satisfied; and a^n is determined by the following relation:

$$Q(a^n) - Q(a') < DQ \qquad \text{DQ=1/(J-1)} \tag{2-19}$$

where, $Q(a^n)$ is for maximum n (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the "advantage rate".

2.1.6.2 Weighted Product (WP)

The Weighted Product method (WP) can rank alternatives without normalization of criteria values because the weights may provide scores on the [0,1] scale. We did not address this method but one of the borrowed case studies in section 4.2.4.4 used this method in their paper. So, it could be helpful to have an overview of this method in this thesis. The following formulation (equation (2-20)) presents the value of alternative A_i: (Hwang and Yoon, 1981a; Triantaphyllou, 2000)

$$V(A_i) = V_i = \prod_{j=1}^{n} x_{ij}^{w_j} \tag{2-20}$$

Where Vi is the value of alternative i, w_j is the weight of criterion j and i=1,…,m is the number of alternatives being scored.

In the above formula, multiplying the attribute values causes the association of weights in the exponents. Also, the positive power implies benefit criteria and the negative power represents the cost criteria. To rank alternatives, the equation (2-21) should apply:

$$R_i = \frac{V(A_i)}{V(A^*)} = \frac{\prod_{j=1}^{n} x_{ij}^{w_j}}{\prod_{j=1}^{n} (x_j^*)^{w_j}} \tag{2-21}$$

Where x_j*is the most favourable value for the j^{th} criterion (the maximum values for the benefit and the minimum values for the cost criteria).

It is clear that $0 \leq R_i \leq 1$ and for the ordering of the alternatives the higher value has the higher rank (Hwang and Yoon, 1981a; Triantaphyllou, 2000).

2.1.6.3 Weighted Aggregated Sum Product Assessment Method (WASPAS)

The Weighted Aggregated Sum Product Assessment Method (WASPAS) method, introduced by Chakraborty and Zavadskasthe (2014), is acombination of the Simple Additive Weighting (SAW) and the Weighted Product (WP) method to handle MCDM problems. The related formula to rank alternative is determined by equation (2-22).

$$Q_i = 0.5 \sum_{j=1}^{n} n_{ij}\, w_j + 0.5 \prod_{j=1}^{n} (n_{ij})^{w_j} \tag{2-22}$$

Where Q_i is the ranking of i^{th} alternative, n_{ij} is the normalized values of r_{ij}, and w_j is the weight of j^{th} criterion.

2.2 Dynamic Multi Criteria Decision Making (DMCDM) and Collaborative Networks

2.2.1 Dynamic Multi-Criteria Decision Methods

As mentioned above, the main goal of any MCDM problem is to select the best alternative, among a set of feasible choices, according to a pre-defined set of criteria (Triantaphyllou, 2000; Yoon and Hwang, 1995). However, many decisions are taken over time, which implies a dynamic process of combining current and past data/information (Campanella and Ribeiro, 2011; Lin *et al.*, 2008; Saaty, 2007). For example, when selecting a supplier among various potential candidates, their good or bad past behaviour should be considered in the final decision (Campanella and Ribeiro, 2011; Zulueta *et al.*, 2013). This type of problems are commonly called Dynamic Multi-Criteria Decision Making (DMCDM) problems because they consider past behaviour and/or changeable conditions of criteria ratings, or even addition or removal of alternatives/criteria over time (Arrais-Castro *et al.*, 2015b; Campanella *et al.*, 2011; Campanella and Ribeiro, 2011; Jassbi *et al.*, 2014b; Lin *et al.*, 2008; Saaty, 2007; Simoes *et al.*, 2012; Zulueta *et al.*, 2013). In other words, DMCDM rates and ranks alternatives based on information collected at multiple time periods (Campanella and Ribeiro, 2011; Xu, 2008; Xu and Yager, 2008; Yao, 2010; Zhang *et al.*, 2011; Zulueta *et al.*, 2013).

To solve DMCDM problems, some authors proposed approaches considering time or space. Campanella et al. (2012) designed a dynamic model for supplier selection with multiple input and output using historical information for a business-to-business system. Their model (Campanella and Ribeiro, 2011) was successfully tested in two very different applications, a hazard avoidance landing of Spacecraft (including a large set of alternatives (e.g. images)) in changeable periods of time and another of supplier selection (Arrais-Castro *et al.*, 2015b; Simoes *et al.*, 2012). Later, Campanella et al. (2011) applied the DMCDM model for supplier selection and combined it with an optimization process to select the optimum supplier in a network of collaborative businesses (B2B supplier selection). In a recent study, Arrais-Castro et al. (2015b) extended the concept by using a data fusion approach (Ribeiro *et al.*, 2014) in supplier selection and applied historical, current, and forecasting information in each iteration of a dynamic decision system. In addition, Jassbi et al. (2014a) proposed a new model for group DMCDM problems which is influenced by time, population, and location. Xu (2008) proposed a dynamic weighted averaging operator to solve DMCDM problems and introduced some methods to obtain associated weights, while, Xu and Yager (2008) applied uncertain dynamic intuitionistic fuzzy weighted averaging operator to DMCDM examples. Zulueta et al. (2013) proposed a novel aggregation process using associative bipolar operators to deal with DMCDM problems that show associativity property (that is the selected associative aggregation operator for the computation of dynamic ratings in their study). They (Zulueta *et al.*, 2013) mentioned that "*This feature allows to exploit the associativity property to represent the rating behaviour of alternatives over different periods as well as to model effects of rating changes above and below the neutral element on the final aggregated value*."

Some other authors developed optimization techniques in order to address dynamic decision processes. For example, one technique is the partially observable Markov decision process (POMDP), which is a special case of Markov decision process and used by (Astrgm, 1965; Da Costa and Buede, 2000; Monahan, 1982; Smallwood and Sondik, 1973; Sondik, 1971, 1978). Another technique is dynamic decision networks (DDN) which is a combination of Bayesian networks (BN), influence diagrams, and multi-attribute utility theory (Da Costa and Buede, 1999, 2000).

In the last decade the need for DMCDM emerged as important to achieve more flexible decision making models, where time (and/or space) is of importance, particularly to ensure support for changing environments or even forecasts (Jassbi *et al.*, 2014b). The vital step to solve DMCDM problems is the determination of a proper aggregation method to calculate the dynamic ratings (Ribeiro *et al.*, 2010; Yager and Rybalov, 1998; Zulueta *et al.*, 2013). The choice of aggregation operator can modify the computing cost and determine different outcomes (Ribeiro *et al.*, 2010; Yager and Rybalov, 1998; Zulueta *et al.*, 2013).

The DMCDM approach of (Campanella and Ribeiro, 2011), extended by (Jassbi *et al.*, 2014b), is shown in (Figure 4). Observing Figure 4 we see that: (i) the first decision matrix contains information related to past data; (ii) the second matrix includes current scores for the same or another alternative or criteria; while (iii) the third matrix includes forecast values for criteria. The dynamic process starts by determining the result vectors for each three matrixes and then aggregating them to obtain a final vector of scores per alternative. This resulting vector is passed as historical data for the next iteration and so on until a stopping criterion is reached (dynamicity of the process). In the past, present, and future decision matrixes, x_{ij} represent the value of criterion C_i with respect to alternative A_j.

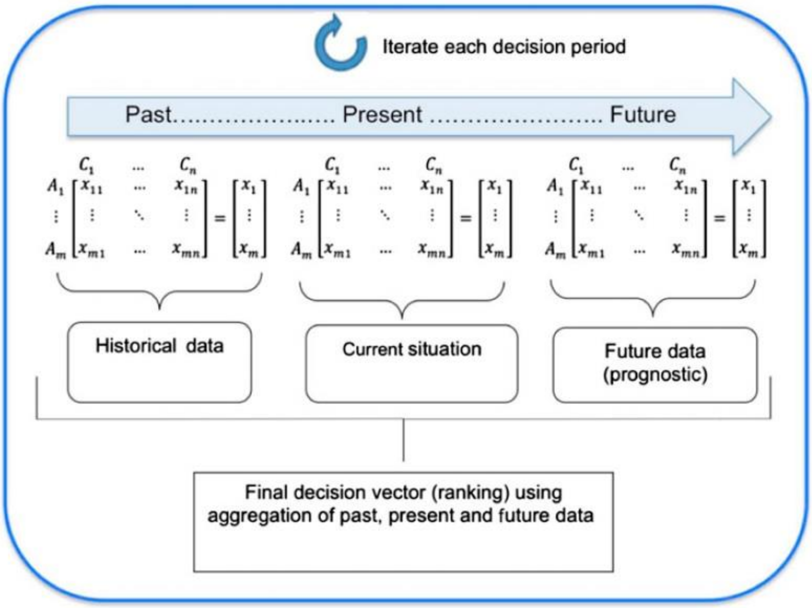


Figure 4: DMCDM model using past, present, and future information/data (borrowed from (Jassbi *et al.*, 2014b)).

More details about the dynamic process and the three matrixes and resulting vectors is available in (Campanella and Ribeiro, 2011; Jassbi *et al.*, 2014b). Notice that in this thesis the focus is on the effects of normalization techniques on both MCDM and DMCDM methods and recommending the most suitable technique for evaluation case studies. So, we only discuss the first iteration of the dynamic system (normalization) to simplify the explanation (please see section 3.2.1.1).

2.2.2 Collaborative Networks

Due to globalization and highly demanding contexts, companies are forced to increasingly integrate collaborative networks to overcome their difficulties and improve their competitiveness levels for survival (Camarinha-Matos *et al.*, 2011, 2013; Oliveira *et al.*, 2010).

There are several definitions for CN (Collaborative Networks) in the literature (Alves *et al.*, 2007; Camarinha-Matos and Afsarmanesh, 2005; Chituc and Azevedo, 2005; Parung and Bititci, 2008). Camarinha and Afsarmanesh (2005) defined CN as: "*a network consisting of a variety of entities (e.g. organizations, people, even intelligent machines) that are largely autonomous, geographically distributed, and heterogeneous in terms of their operating environment, culture, social* *capital and goals, but which decide to collaborate to better achieve common or compatible goals (e.g. problem solving, production, or innovation), and whose interactions are supported by computer networks*". Collaborative Networks are also defined as a group of enterprises that have their own goals and strategies but the aim of having better achievements persuades them to define compatible goals and jointly generate value together (Camarinha-Matos and Afsarmanesh, 2005). Andres and Poler (2016) said that "*In CN, each enterprise defines its own objectives and formulates its own strategies; therefore, distinct interests are involved, which may lead to conflictive situations that derive from disagreements in the selection of strategies*."

The development of collaborative networks, particularly in what concerns dynamic selection of business partners is based on effective and efficient normalization techniques to ensure appropriate and meaningful data processing and analysis. Some contributions in this direction can be found in (Addo-Tenkorang and Helo, 2016; Arrais-Castro et al., 2015a, 2018; Babar and Arif, 2017; Campanella et al., 2012; Golov and Rönnbäck, 2015; Ribeiro et al., 2014).

Collaborative systems are information systems tailored to overcome the obstacles inherent in the creation of value through joint effort (Moghaddam and Nof, 2018; Nikolic et al., 2017; Nunamaker and Briggs, 2015). As stated in (Li et al., 2014), due to business division and outsourcing, from design and manufacturing to sales, a complete business process is divided into a set of business process fragments. These are carried out within different enterprises' organizational boundaries and related business data sets, and, in this complex scenario, how to form a collaborative information system across heterogeneous infrastructures, rapidly and dynamically, determines whether the enterprise network can succeed or not.

It is noteworthy that in this thesis we will focus our research on collaborative networks with a decision making perspective, for selection problems (details in section 3.3.1.1) – i.e., where MCDM methods can be applied - such as selection of supplier, partner, resources, etc (Appio *et al.*, 2017; Arrais-Castro *et al.*, 2015a, 2018; Camarinha-Matos *et al.*, 2017; Camarinha-Matos and Afsarmanesh, 2005, 2006; Campanella *et al.*, 2012). Hence, it is out of scope to go deeper into other aspects of collaborative networks and here we only introduce specific related literature. Selim et al. (2008) used multi-objective decision making in order to develop collaborative production–distribution planning supply chain system. Alemany et al. (2011) developed an application to support decision makers in supply chain systems by collaborative planning (CP). Multi-enterprise collaborative decision support system (MECDSS) frameworks are used by Shafiei et al. (2012) to make seamless integration between all partners/members in collaborative environment without any dependency on the user's knowledge. Arrais-Castro et al. (2015a) developed a decision making model in a collaborative system by combining dynamic multi-criteria decision making (MCDM) and software agents for optimal selection of partners/suppliers. In their model, historical, current and future information are considered (Arrais-Castro et al., 2015a). Arrais-Castro et al. (2012) also proposed a new framework for Agile/Virtual Enterprises that share all resources such as knowledge, market, and customers in a competitive environment which is dynamic and reconfigurable (Arrais-Castro et al., 2012). Nematollahi et al. (2017) proposed a new collaborative method in supply chain (SC) networks in order to optimize the profit of the SC by considering corporate social responsibility (CSR) activities that can define the popularity of products and demands for these products. Verdecho et al. (2012) used the Analytic Network Process (ANP) to define the structures performance of elements under a performance management framework. The result would help collaborative networks to identify contributions of each participant to achieve the strategy of collaboration (Verdecho et al., 2012). Guillaume et al. (2014) proposed a decision problem in collaborative supply chain system to help decision makers to select the best partner (alternative) under imprecise criteria that is optimal for itself and for other partners in the collaborative network. Varela and Ribeiro (2014) developed a dynamic multi-criteria decision making model to support decision makers in distributed dynamic manufacturing scheduling problems.

Approaches using computational intelligence methods are also proposed in the literature within this topic. For example, Zha et al. (2008) proposed a new hybrid decision model and a multi-agent framework for collaborative decision which rely on algorithms like compromise decision support problem technique (cDSP), fuzzy cDSP, and fuzzy synthetic decision model (FSD). Dao et al. (2014) used a novel Genetic Algorithm (GA) to find optimal solutions for integration of partner selection and collaborative transportation scheduling in Virtual

Enterprise. Zhang et al. (2013) combined genetic algorithm with TOPSIS method and proposed a new multi-criterion optimization method for supply chain management. Che and Chiang (2012) combined the Rough Sets Theory with AHP in collaborative supply-chain to optimize the cycle-time estimation procedure. In the study of Lu et al. (2013) fuzzy analytic hierarchy process (FAHP) is proposed to define and analyze the key processes in SC networks with the defined weightings.

As mentioned above, a collaborative case study from (Arrais-Castro et al., 2015a, 2018) is used for research in this thesis (Section 3.2.1.1). It uses a Dynamic Multiple Criteria Decision Making Method (DMCDM) (Campanella et al., 2012; Ribeiro et al., 2014) mixed with a data fusion approach (Ribeiro et al., 2014). The case objective is the evaluation and selection of suppliers or business partners, through the use of software agents and a negotiation process to capture business opportunities, evaluate and select businesses (suppliers or partners), and process associated orders within a spatial-temporal changeable context (Vafaei et al., 2019).

2.3 Normalization

2.3.1 Introduction

There are several definitions for data normalization, depending on the study domain. For example, in Databases, data normalization is viewed as a process where data attributes, within a data model, are organized in tables to increase the cohesion and efficiency of managing data (Vafaei et al., 2016b). In statistics and its applications, the most common definition is the process of adjusting values measured on different scales to a common scale, often prior to aggregating or averaging them (Vafaei et al., 2016b). Many other definitions exist, depending on the context or study domain (see for example (Wikipedia contributors, 2004)).

In this thesis, since we focus on normalization techniques for MCDM, normalization is viewed as a transformation process of raw data to obtain numerical and comparable criteria by using a common scale (Vafaei et al., 2016b, 2018a). Furthermore, in MCDM, normalization techniques usually map attributes (criteria) with different measurement units to a common scale in the interval [0-1]

(Pavlicic, 2001; Vafaei et al., 2016b). In other words, normalization is a transformation process to obtain numerical and comparable input data by using a common scale. Table 2 illustrates a normalization process on input data that maps a criterion price to the interval [0-1] using two linear normalization techniques (N5 and N10 from Table 4 and Table 5). It is obvious the difference of obtained values - even if both from the linear class - because with Max technique (N5 – cost criteria) we never attain the upper limit of the normalized interval, i.e. 1.

Table 2: Normalization of input data for a cost criterion "car price"

Car selection	Price	Cost Normalization	
		Max (N5)	Max-Min (N10)
A	11,000.00 €	0.45	1.00
B	14,000.00 €	0.3	0.67
C	16,000.00 €	0.2	0.44
D	20,000.00 €	0	0.00

In recent years, data normalization is receiving considerable attention due to its essential role as a pre-processing step for complex decision-making problems based on large amounts of data. Especially, for developments in Big Data, Artificial Intelligence (AI), Machine Learning (ML), Data mining, Internet of Things (IoT), Cyber-Physical Systems (CPS), Data Visualization (DV), Optimization, etc., normalization's role is distinguished and deemed crucial. Relevant literature discussing the importance of normalization in these topics are: Big Data (Golov and Rönnbäck, 2017); Artificial Intelligence (Arabameri *et al.*, 2020; Perny and Pomerol, 1999; Sola and Sevilla, 1997); Machine Learning (Jo, 2019; Wahid *et al.*, 2018); Data mining (Al Shalabi *et al.*, 2006; Al Shalabi and Shaaban, 2006; Haque *et al.*, 2014); Internet of Things (IoT) (Liu *et al.*, 2019; Rathee *et al.*, 2021; Ray, 2016); Cyber-Physical Systems (Huang *et al.*, 2018; Junejo and Goh, 2016); Data Visualization (Mangat *et al.*, 2014); Optimization algorithms (Budiman *et al.*, 2020, 2021; Eftekhary *et al.*, 2012; Fayazbakhsh *et al.*, 2009; Migilinskas and Ustinovichius, 2007; Nayak *et al.*, 2014).

For instance the integration of MCDM and Artificial Intelligence techniques reached success in handling real-world problems (Doumpos and Grigoroudis,

2013) by enabling decision makers to better structure complex decision problems in static and distributed environments. Doumpos and Grigoroudisby (2013) mention that combining MCDM&AI is important for "handling of massive data sets, the modelling of ill-structured information, the construction of advanced decision models, and the development of efficient computational optimization algorithms for problem solving". Several AI techniques are introduced in the literature which are used in combination with MCDM, such as Fuzzy Logic (FL), Genetic Algorithm (GA), Neural Network (NN), Heuristic or meta-heuristics, Knowledge-Based (KB), Expert Systems (ES), Tabu-Search (TS), Simulated-annealing (SA), Dampster Shafer (DS), and Self-Organizing-Map (SOM). FL is perhaps the most popular technique to be used with MCDM methods (e.g., Fuzzy-AHP, Fuzzy-ANP, and Fuzzy-TOPSIS, etc.) (Aliasi et al., 2008). Ribeiro (Ribeiro, 1996) discussed the main theories and method is applied for Fuzzy MCDM problems. In another study, Ribeiro et al. (Ribeiro *et al.*, 2014) combined computational intelligence and MCDM and introduced FIF (Fuzzy Information Fusion) algorithm to aggregate data in presence of qualitative and quantitative criteria at the same time using particle swarm optimization (PSO) and Tabu search algorithms to select the optimal landing place for aircraft (Ribeiro *et al.*, 2014). There are some other research works on combining AI techniques with MCDM methods such as Ho (2008) that uses 8 meta-heuristics along with AHP, Pan (2008) that applies Fuzzy-AHP for bridge construction methods selection, Sheu (2008) that uses Fuzzy-AHP, Fuzzy-TOPSIS, and Fuzzy-MCDM for global logistic operational model, Kulturel-Konak et al. (2007) that apply TS for system redundancy allocation problem, Efendigil et al. (2008) that implement ANN and Fuzzy-AHP for third-party logistics providers selection, and Wu et al. (2009) that use Fuzzy-ANP for site selection problem. Kahraman et al. (Kahraman *et al.*, 2003) used Fuzzy-AHP for supplier selection to overcome the uncertainty regarding assessing the evaluation scores by humans in crisp AHP.

Another area where normalization and MCDM are being applied is the relationship between Cyber-Physical Systems and Internet of Things (Jeschke, 2013). Camarinha and Afsarmanesh (2014) mention that "there is a growing convergence between the two areas since CPSs are becoming more Internet-based". For example, in a smart car parking (section 3.1.1.2), data from the parking space is transferred to the car drivers with the help of CPS and IoT technologies. Data is collected from sensors, which are installed in the parking lot, and transferred to the data center to be processed with MCDM methods, to determine the ranking of alternatives (best parking spaces). The best parking spaces are provided to the car drivers to support them making more informed decisions. The smart car parking example shows a robust relationship between the cyber physical system (CPS), Internet of Thing (IoT) and multi-criteria decision making (MCDM) concepts, where normalization has an important role in preparing dimensionless data from heterogeneous input data sets from sensors (Vafaei et al., 2016a).

Summarizing, normalization in MCDM entails that after collecting input data, we must do some pre-processing to ensure comparability of data, thus making it useful for decision modelling (Etzkorn, 2012). This pre-processing should consider two important points (Etzkorn, 2012; Vafaei et al., 2018a):

1. All non-numeric data should first be converted into numerical data to allow normalization;
2. Choosing a suitable normalization technique to ensure a common scale and appropriate modeling representation (benefit or cost criteria) as well as comparability on criteria aggregation to obtain alternative ratings.

Next, we will provide a survey of normalization for MCDM methods.

2.3.2 Survey on normalization techniques for MCDM

Numerous normalization techniques have been proposed in the literature and most MCDM methods use one of these techniques. Jahan and Edwards (Jahan, 2018; Jahan and Edwards, 2015) pointed to some important features that have influencing effects on the capability of normalization techniques and should be considered when choosing, developing and evaluating them.

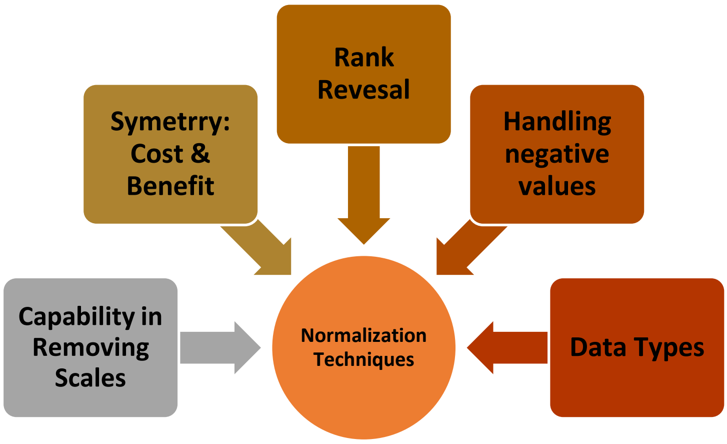


Figure 5: Relevant aspects for assessing normalization techniques (Jahan, 2018; Jahan and Edwards, 2015)

As Figure 5 shows, one important aspect is the capability of removing scales, because the basic role of any normalization technique is to convert different criteria measurement units (in MCDM models) into dimensionless units and making them comparable for aggregation in decision matrixes (Jahan, 2018; Jahan and Edwards, 2015). Symmetry is another feature present in some normalization techniques, that allows conversion of cost criteria into benefit ones (Jahan, 2018; Jahan and Edwards, 2015). The next property is rank reversal which may cause different rankings by adding or removing alternatives (Jahan, 2018; Jahan and Edwards, 2015). Rank reversal could happen when selecting a unsuitable normalization technique (Jahan, 2018; Jahan and Edwards, 2015). Handling negative values is also an important capability, when dealing with negative values in criteria of MCDM methods (Jahan, 2018; Jahan and Edwards, 2015). Figure 5 also includes data types as an important feature. Previous research done by the author of this thesis proved that the types of input data may have great influence in ranking alternatives for any MCDM method (Vafaei et al., 2019). For example, including zero or decimal numbers in the input data using Sum or Logarithmic normalization techniques are not recommended because it may produce undefined and infinite normalized values for the two mentioned techniques (Vafaei et al., 2019).

There are some specific works on normalization techniques for MCDM problems such as (Celen, 2014; Dehghan-Manshadi *et al.*, 2007; Fayazbakhsh *et* *al.*, 2009; Hwang and Yoon, 1981a; Jahan *et al.*, 2012; Jee and Kang, 2000; Milani *et al.*, 2005; Shih *et al.*, 2007). However, those studies are always performed on particular contexts and do not propose a general assessment process, and this is the challenge for this research work.

Jahan and Edwards (2015) published an interesting paper with an exhaustive survey on normalization techniques. These authors identified thirty-one normalization methods for transforming raw data into dimensionless criteria, classified them in 4 groups, and discussed some specific pros and cons for each technique.

Here we follow their classification (Jahan and Edwards, 2015) and the 4 groups are depicted in: Table 3 (Sum-based); Table 4 (Ratio-based); Table 5 (Max-min based); and Table 6 (Other). Further, Jahan and Edwards (2015) tried to divide all normalization techniques into benefit and cost criteria and discussed their pros and cons. Please notice that r_{ij} is the rating of alternative i with respect to criterion j and n_{ij} is the normalized value of r_{ij}.

Table 3 includes 4 sum-based normalization techniques: N1) linear-sum; N2) Vector; N3) Logarithmic; and N4) Enhanced accuracy. The first three normalization techniques (N1, N2, N3) are common in MCDM methods, while the last one (N4) is less common. Jahan and Edwards (2015) mentioned that the result of all sum based normalization techniques depends on the number of alternatives, so, rank reversal (add/remove alternatives) will clearly affect the results, i.e. the alternatives' ranking. The logarithmic normalization technique, proposed by Zavadskas and Turskis (2008), is usually used when criteria values display a clear monotonic increase or decrease.

Table 3: Sum-based normalization techniques for cost and benefit criteria (Jahan and Edwards, 2015)

Normalization technique	Condition of use	Formula
1- Linear: Sum normalization *(Jahan and Edwards, 2015; Wang and Luo, 2010)*	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$
	Cost criteria	$n_{ij}^{-} = \frac{1/r_{ij}}{\sum_{i=1}^{m} 1/r_{ij}}$
2- Vector normalization *(Delft and Nijkamp, 1977; Jahan and Edwards, 2015)*	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}$
	Cost criteria	$n_{ij}^{-} = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}$
3- Logarithmic normalization *(Jahan and Edwards, 2015; Kazimieras ZAVADSKAS and Turskis, 2008)*	Benefit criteria	$n_{ij}^{+} = \frac{\ln(r_{ij})}{\ln(\prod_{i=1}^{m} r_{ij})}$
	Cost criteria	$n_{ij}^{-} = \frac{1 - \frac{\ln(r_{ij})}{\ln(\prod_{i=1}^{m} r_{ij})}}{m-1}$
4- Enhanced accuracy method *(Jahan and Edwards, 2015; Zeng et al., 2013)*	Benefit criteria	$n_{ij}^{+} = 1 - \frac{r_{j}^{max} - r_{ij}}{\sum_{i=1}^{m}(r_{j}^{max} - r_{ij})}$
	Cost criteria	$n_{ij}^{-} = 1 - \frac{r_{ij} - r_{j}^{min}}{\sum_{i=1}^{m}(r_{ij} - r_{j}^{min})}$

Ratio-based normalization techniques are shown in Table 4. It includes five normalization techniques: N5) Max; (N6) Min; N7) Markovic normalization; N8) Tzeng & Huang normalization; and N9) Non-linear normalization. Max (N5) and Min (N6) are rather common and currently used in many fields of science; while N7, N8 and N9 are less common.

Table 4: Ratio-based normalization techniques for cost and benefit criteria (Jahan and Edwards, 2015)

Normalization technique	Condition of use	Formula
5- Linear: Max *(Celen, 2014; Jahan and Edwards, 2015)*	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{r_{max}}$
	Cost criteria	$n_{ij}^{-} = 1 - \frac{r_{ij}}{r_{max}}$
6- Linear: Min *(Jahan and Edwards, 2015; Zhou et al., 2006)*	Benefit/Cost	$n_{ij} = \frac{r_{min}}{r_{ij}}$
7- Markovic´ method *(Jahan and Edwards, 2015; Tzeng and Huang, 2011)*	Benefit/Cost	$n_{ij} = 1 - \frac{r_{ij} - r_j^{min}}{r_j^{max}}$
8- Tzeng and Huang method *(Jahan and Edwards, 2015; Tzeng and Huang, 2011)*	Benefit/Cost	$n_{ij} = \frac{1/r_{ij}}{1/r_j^{max}} = \frac{r_j^{max}}{r_{ij}}$
9- Non-linear normalization *(Jahan and Edwards, 2015; Kazimieras ZAVADSKAS and Turskis, 2008)*	Benefit criteria	$n_{ij}^{+} = \left(\frac{r_{ij}}{r_j^{max}}\right)^2$
	Cost criteria	$n_{ij}^{-} = \left(\frac{r_j^{min}}{r_{ij}}\right)^3$

Three max-min based normalization techniques are introduced in Table 5. Linear max–min (N10) produces dimensionless units within the straight line (∝r_ij+β), while the others (N11 & N12) do not have this advantage (Jahan and Edwards, 2015). The main disadvantage of Linear max–min (N10) is related to the effect of rank reversal (adding or removing alternatives) on the obtained results in MCDM methods because it causes changing the maximum and minimum values (Jahan and Edwards, 2015). Also, Jahan and Edward (2015) showed, with a numerical example, that Lai and Hwang normalization technique (N11) results on values greater than 1, hence, this can prove problematic for many MCDM methods.

Table 5: Max–min normalization techniques for cost and benefit criteria (Jahan and Edwards, 2015)

Normalization technique	Condition of use	Formula
10- Linear : Max-Min *(Jahan and Edwards, 2015; Kazimieras ZAVADSKAS and Turskis, 2008; Patro and Sahu, 2015; Tzeng and Huang, 2011)*	Benefit criteria	$n_{ij}^{+}=\frac{r_{ij}-r_j^{min}}{r_j^{max}-r_j^{min}}$
	Cost criteria	$n_{ij}^{-}=\frac{r_j^{max}-r_{ij}}{r_j^{max}-r_j^{min}}$
11- Lai and Hwang method (Jahan and Edwards, 2015; Lai and Hwang, 1994; Opricovic and Tzeng, 2004)	Benefit criteria	$n_{ij}^{+}=\frac{r_{ij}}{r_j^{max}-r_j^{min}}$
	Cost criteria	$n_{ij}^{-}=\frac{r_{ij}}{r_j^{min}-r_j^{max}}$
12- Zavadskas and Turskis normalization *(Jahan and Edwards, 2015; Kazimieras ZAVADSKAS and Turskis, 2008)*	Benefit criteria	$n_{ij}^{+}=1-\left\lvert\frac{r_j^{max}-r_{ij}}{r_j^{max}}\right\rvert$
	Cost criteria	$n_{ij}^{-}=1-\left\lvert\frac{r_j^{min}-r_{ij}}{r_j^{min}}\right\rvert$

Table 6 lists 6 unclassified normalization techniques, which are also suitable for use with benefit, cost, and target values criteria: N13) nominal-is-best; N14) Linear method-ideal; N15) Non-monotonic; 16) Comprehensive normalization technique; N17) Target-based; N18) Distance for target criteria; and N19) Z-transformation. Nominal-is-best normalization (N13) was introduced by (Wu, 2002) and applicable until target value is less than the maximum performance rating (Jahan and Edwards, 2015). N14 was introduced by Zhou et al. (2006) with three formulas for cost, benefit and target criteria. Jahan and Edwards (2015) mentioned that non-monotonic normalization (N15) has a higher concentration towards the values zero/one because of presence of e (Euler's number), and for Comprehensive normalization technique (N16) Jahan and Edwards (2015) claimed that this technique does not cover the whole numerical domain (0-1) because there are no values for less than 0.37. Jahan et al. (2011) also stated that the comprehensive normalization technique (N16) covers a wide range of criteria by using an exponential function. The target-based normalization technique (N17), proposed by Jahan et al. (2012), and they used numerical examples to show normalized values by Target-based (N17). In Table 6, Target value (Tj) is the desirable level of achievements for each attribute that is defined by decision makers (Stewart, 1992). Also, μ_j and σ_j that are used in the formulas of Table 6 are defined in the columns of N19. When a criterion is a benefit one, high values correspond to high normalized values (maximization - benefit); when it is a cost criterion high values will correspond to low normalized values (minimization - cost).

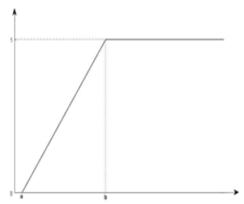
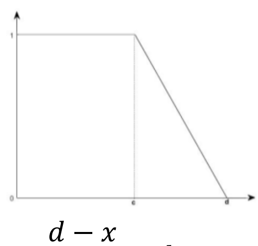
Another interesting normalization technique is Fuzzification (N20), as shown in Table 7 Fuzzification is the process of converting crisp values into linguistic terms by using membership functions (Schmid, 2005). Several functions can be used for fuzzification (e.g., Trapezoidal, Gaussian, Logarithmic, Triangular). Normalizing data with such functions is a mechanism for transforming raw data into fuzzy sets (functions), which appropriately represent concepts understandable for decision makers and enables dealing with alternatives and criteria in MCDM (Ribeiro *et al.*, 1995, 2014). In this thesis we chose trapezoidal membership functions because they are linear, but others such as sigmoid or gaussian could be used.

Table 6: Some other normalization techniques (Jahan and Edwards, 2015)

Normalization technique	**Condition of use**	**Formula**
13- Nominal-is-best method *(Jahan and Edwards, 2015; Wu, 2002)*	Benefit/Cost	$n_{ij} = \frac{\lvert r_{ij} - T_j \rvert}{r_{ij}^{max} - T_j}$
14- Linear method-ideal *(Jahan and Edwards, 2015; Zhou et al., 2006)*	Cost criteria	$n_{ij}^{-} = \frac{r_j^{min}}{r_{ij}}$
	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{r_j^{max}}$
	Target criteria	$n_{ij} = \frac{min\{r_{ij} - T_j\}}{max\{r_{ij} - T_j\}}$
15- Non-monotonic normalization *(Jahan et al., 2012; Shih et al., 2007)*	Benefit/Cost	$n_{ij} = e^{\frac{(r_{ij}-r_j)^2}{-2\sigma_j^2}}$
16- Comprehensive normalization technique *(Bahraminasab et al., 2014; Jahan and Edwards, 2015)*	Benefit/Cost	$n_{ij} = 1 - e^{\frac{\lvert r_{ij}-T_j \rvert}{min\{r_j^{inx}-T\}-max\{r_j^{max}-T_j\}}}$
17- Target-based normalization technique *(Jahan et al., 2012; Jahan and Edwards, 2015)*	Benefit/Cost	$n_{ij} = 1 - \frac{\lvert r_{ij} - T_j \rvert}{Max\{r_{ij}^{max}, T_j\} - Min\{r_{ij}^{min}, T_j\}}$
18- Distance for target criteria *(Jahan and Edwards, 2015; Zeng et al., 2013)*	If $r_{ij} > (\mu_j + 1.96\sigma_j)$	$n_{ij} = \frac{r_{ij} - (\mu_j + 1.96\sigma_j)}{\sum_{i=1}^{m}\left(r_{ij} - (\mu_j + 1.96\sigma_j)\right)}$
	If $(\mu_j - 1.96\sigma_j) > r_{ij}$	$n_{ij} = \frac{(\mu_j + 1.96\sigma_j) - r_{ij}}{\sum_{i=1}^{m}\left((\mu_j + 1.96\sigma_j) - r_{ij}\right)}$
	If $r_{ij} \in (\mu \pm 1.96\sigma)$	1
19- Z-transformation for which the scale would be around zero	$\mu_j = \frac{\sum_{i=1}^{m} r_{ij}}{m}$ $\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}(r_{ij} - \mu_j)^2}{m}}$	$n_{ij} = \frac{r_{ij} - \mu_j}{\sigma_j}$

An important issue on the variables/criteria "fuzzification" is to select suitable membership functions since we need to consider the context and objective (Ribeiro *et al.*, 2014). There are various proposals in the literature on how to fuzzify concepts/criteria (data) to normalize and allow comparable data (Ross, 2004; Tzeng and Huang, 2011), however, these studies do not formally recognize fuzzification as another normalization technique. Even without formally acknowledging fuzzification as a normalization technique, many authors applied fuzzification as a normalization technique in order to deal with dimensionless data in fuzzy multi-criteria decision making problems (see for example (Ribeiro *et al.*, 2014; Tzeng and Huang, 2011; Zhang *et al.*, 2014)). Other examples for decision problems include: Pires et al. (Pires *et al.*, 1996) and Ribeiro and Varela (Ribeiro and Varela, 2003) which used fuzzification for solving optimization problems.

Table 7: Fuzzification normalization techniques (adapted from (Ribeiro, 1996; Ross, 2004))

<table>
<tr><th>Normalization technique</th><th>Condition of use</th><th>Formula (chosen)</th></tr>
<tr><td rowspan="2">20-Fuzzification–membership functions (Ribeiro, 1996; Ross, 2004)</td><td>Benefit Criteria</td><td>Open right trapezoidal membership function:

$n_{ij} = \frac{x-a}{b-a} \quad where\ c = d$</td></tr>
<tr><td>Cost criteria</td><td>Open left trapezoidal membership function:

$n_{ij} = \frac{d-x}{d-c} \quad where\ a = b$</td></tr>
</table>

As observed on the above tables, most normalization techniques are divided in two formulas, one for benefit and another for cost criteria, to ensure that the final decision objective (rating) is correct, i.e., when it is a benefit criterion higher values correspond to high normalized values (maximization - benefit) and when it is a cost criterion high values correspond to low normalized values (minimization - cost). The same logic applies to fuzzification techniques, i.e., memberships functions can be monotonically increasing or decreasing to represent, respectively, benefit or cost (Table 7).

For this research work we selected at least one normalization technique from each of the above five tables (Table 3, Table 4, Table 5, and Table 6 and Table 7): Sum (N1), Vector (N2), and Logarithmic (N3) techniques from Table 3; Max (N5) technique from Table 4; Max-Min (N10) technique from Table 5; Target-based (N17) normalization technique form Table 6 and Fuzzification (N20) from Table 7. Along the research done for this thesis, N17 (Target-based) was only used in specific cases (section 3.3.1.1) and N20 was sometimes substituted by N3 (Logarithmic), because – as mentioned- any function can be used for normalizing with a fuzzification process.

As mentioned above, we selected 7 normalization techniques for our comparison study. The objective was to ensure we included normalization techniques from all categories (linear, semi-linear and non-linear) for evaluating the proposed assessment framework developed. Further, the chosen normalization techniques are also well-known in the literature and widely used by decision makers in decision problems. So, Figure 6 depicts the selected normalization techniques, divided in 3 categories, which are later studied in chapters 3 and 4 to analyse their effects on MCDM problems.

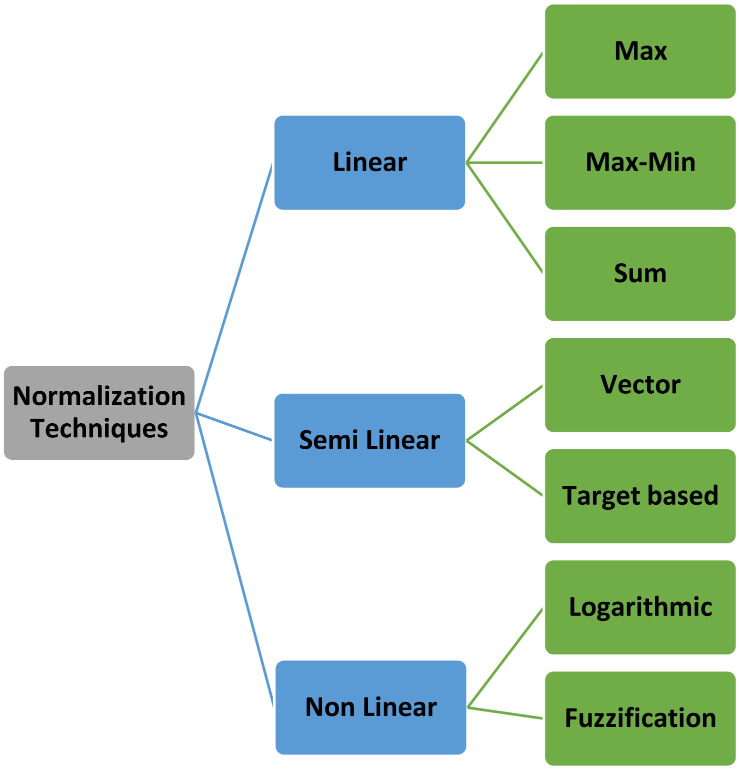


Figure 6: Well-known normalization techniques from three categories (adapted from (Jahan and Edwards, 2015)).

To simplify the discussion on the next sections of the thesis, about the selected normalization techniques' formulas, Table 8 summarizes them, with both the formulation for benefit and cost criteria.

Table 8: The seven chosen normalization techniques to be compared in this thesis.

Number	Normalization technique	Condition of use	Formula
N1	Linear: Max	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{r_{max}}$
		Cost criteria	$n_{ij}^{-} = 1 - \frac{r_{ij}}{r_{max}}$
N2	Linear: Max-Min	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij} - r_{min}}{r_{max} - r_{min}}$
		Cost criteria	$n_{ij}^{-} = \frac{r_{max} - r_{ij}}{r_{max} - r_{min}}$
N3	Linear: sum	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$
		Cost criteria	$n_{ij}^{-} = \frac{1/r_{ij}}{\sum_{i=1}^{m} 1/r_{ij}}$
N4	Semi-linear: Vector	Benefit criteria	$n_{ij}^{+} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}$
		Cost criteria	$n_{ij}^{-} = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}$
N5	Non-linear: Logarithmic	Benefit criteria	$n_{ij}^{+} = \frac{\ln(r_{ij})}{\ln(\prod_{i=1}^{m} r_{ij})}$
		Cost criteria	$n_{ij}^{-} = \frac{1 - \frac{\ln(r_{ij})}{\ln(\prod_{i=1}^{m} r_{ij})}}{m - 1}$
N6	Non-linear: Fuzzification – membership functions (see Table 7)	Benefit criteria	$n_{ij} = \frac{x - a}{b - a} \quad where\ c = d$
		Cost criteria	$n_{ij} = \frac{d - x}{d - c} \quad where\ a = b$
N7	Semi-linear: Target based	Benefit & Cost criteria	$n_{ij} = 1 - \frac{\lvert r_{ij} - T_j \rvert}{Max\{r_{ij}^{max}, T_j\} - Min\{r_{ij}^{min}, T_j\}}$

2.3.3 Assessing normalization techniques in MCDM Methods

As mentioned above, over the last decades, several normalization techniques have been proposed to produce dimensionless data from heterogeneous data sets (Jahan and Edwards, 2015). As such, selection of the best normalization technique is a crucial step in the aggregation/fusion process (i.e., finding which one better represents the input/raw data) to rank alternatives in MCDM problems.

There are many performance metrics to assess classification problems (see for example (Budiman *et al.*, 2020, 2021; Eftekhary *et al.*, 2012; Fayazbakhsh *et al.*, 2009; Migilinskas and Ustinovichius, 2007; Nayak *et al.*, 2014)) but unfortunately, there are few studies on specific metrics for assessing normalization techniques in MCDM methods and the question of how to choose the appropriate one is still an open one. In classification problems of the type "selecting features" or "classifying objects" we may have access to ground-truth results for comparison, however, in MCDM we only obtain a rating for the candidate alternatives and this rating depends both on the method and normalization technique used (Vafaei et al., 2018a).

Furthermore, if the normalization technique is not suitable for the decision problem or for the chosen MCDM method, the best decision solution may be overlooked, which may cause serious errors of judgement (Chatterjee and Chakraborty, 2014). As Chatterjee and Chakraborty (Chatterjee and Chakraborty, 2014) say "*In fact, while the normalization process scales the criteria values to be approximately of the same magnitude, different normalization techniques may yield different solutions and, therefore, may cause deviation from the originally recommended solutions*" (Vafaei et al., 2018a).

There are some interesting approaches for choosing normalization techniques in specific MCDM problems are: (Baghla and Bansal, 2014; Celen, 2014; Chakraborty and Yeh, 2007, 2009, 2012; Chatterjee and Chakraborty, 2014; Chawade *et al.*, 2014; Jahan, 2018; Krylovas *et al.*, 2018; Lakshmi and Venkatesan, 2014; Mathew *et al.*, 2017; Milani *et al.*, 2005; Opricovic and Tzeng, 2004; Papathanasiou *et al.*, 2016; Pavlicic, 2001; Peldschus, 2007; Podviezko and Podvezko, 2015; Vafaei *et al.*, 2016a, 2018a, 2019; Yazdani *et al.*, 2017; Zavadskas *et al.*, 2003, 2006).Some of these authors already discussed metrics/approaches for how to assess normalization techniques in decision problems, but our approach advances those studies by proposing what we believe are more appropriate metrics, i.e. an assessment framework to classify the best technique for MCDM. For instance, Pavlicic (2001) analyzed the effects of simple (divided by max), linear and vector normalization techniques on simulations results of TOPSIS, ELECTRE, and SAW MCDM methods. Specifically, he showed that results depend on the initial measurement units (e.g., temperature measured in Celsius or Fahrenheit) when using vector or simple normalization techniques. It should be noted that Pavlicic (2001) was an inspiration for also exploring other suitable normalization techniques for TOPSIS and to elaborate a more robust assessment method, due to the shortcomings of normalization techniques for MDCM methods.

Zavadskas et al. (2003) applied four linear and one non-linear normalization techniques in their study and showed that the non-linear one improved the quality of the transformation step and consequent final decision in their study.

Opricovic and Tzeng (2004) analyzed the effect of Vector normalization in TOPSIS and Linear normalization techniques (namely- Max-Min) in the VIKOR method, but without mentioning any conclusion about the preference for normalization techniques in respect to those MCDM methods.

Milani et al. (2005) analyzed the effects of different normalization techniques (Vector, Max, and Sum) on TOPSIS for a gear material selection case study. They just demonstrated the different rankings of alternatives using different normalization techniques, without discussing the normalization techniques comparison.

Zavadskas et al. (2006) measured the accuracy of determining the relative significance of alternatives as a function of the initial criteria values with TOPSIS method using Vector and Lai and Hwang normalization techniques. The proposed methodology calculated the error of obtained results from the initial criteria values. The authors mentioned that the final results of MCDM depend on errors of initial values as well as on the selection of MCDM methods and normalization techniques (Zavadskas *et al.*, 2006). However, this study did not include metrics such as Minkowski distance, Standard deviation, etc.

The study of Peldschus (2007) focused on the effects of normalization techniques on the optimal solution and discussed the suitability of linear and non-linear normalization techniques. The obtained results of the case study suggested that the non-linear normalization technique improved the quality of the evaluation (Peldschus, 2007). Again, this study was partial because the authors did not address, for example, distance metrics, or comparison metrics such as correlation.

Chakraborty and Yeh (2007) explored the effects of four normalization techniques (Vector, Max-Min, Max, and Sum) in SAW (Simple Additive Weighting) method using Ranking Consistency Index (RCI) to assess the best normalization technique. In another study, Chakraborty and Yeh (2009) evaluated the same normalization techniques in TOPSIS using the same metrics (RCI). In this thesis, RCI metric is selected because it presents similarity/dissimilarity of different normalization techniques. However, these studies did not evaluate other metrics such as mean squared error, Minkowski distance, etc.

Chakraborty and Yeh (2012) applied four normalization techniques namely Max, Max-Min, Sum, and Vector for SAW and TOPSIS methods to analyze the effect of the mentioned techniques on the ranking of alternatives. The authors used Spearman correlation and calculated the average of correlation (Mean ks values) for each normalization techniques for both SAW and TOPSIS methods and recommended using Max normalization technique for both SAW and TOPSIS methods. From this study we borrowed the correlation metric because it is important for comparing the consistency of different normalization techniques in MCDM methods.

Celen (2014) analyzed the impact of vector normalization and three linear normalizations (vector, max-min, max and sum) techniques in the TOPSIS method. They used a consistency process for assessing banks performance in Turkey, which included using Pearson correlation. The conclusion was that vector normalization is the best technique for TOPSIS, in the proposed application. Here, we advanced this work by testing other normalization techniques and also discussing a more general assessment approach to select the best normalization technique for TOPSIS.

Chawade et al. (2014) introduced an open-source tool, called "Normalizer", and used 12 different normalization techniques for the related data sets and displayed a comparative evaluation with several quantitative and qualitative plots. However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc.

Lakshmi and Venkatesan (2014) assessed five normalization techniques (Max, Max-Min, Sum, Vector, and Fuzzification (Gaussian membership function)) to analyze the effects of using different normalization techniques on the TOPSIS method. The authors of a related paper (Lakshmi and Venkatesan, 2014) calculated time complexity and space complexity for each normalization technique with the help of MATLAB and recommended the Sum normalization technique as the best technique for the case study using the TOPSIS method. However, this study did not include metrics such as correlation, mean squared error, Minkowski distance, standard deviation, etc.

Baghla and Bansal (2014) analyzed the effects of three normalization techniques namely Max, Max-Min, and Vector techniques, besides comparing the performance of AHP and ANP for determining the weights in VIKOR method. The simulation results in their research showed up Vector has the one giving the best results in their case study. However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc.

Chatterjee and Chakraborty (2014) used four different normalization techniques as Vector, Ma-Min, Jüttler's-Körth, and Non-linear normalization by Jahan and Edwards in TOPSIS, PROMETHEE II, and GRA (Grey Relational Analysis). They calculated Average of Spearman correlation (Mean ks values) for each normalization techniques and MCDM method. The authors concluded that TOPSIS is the most sensitive method using different normalization techniques while PROMETHEE II is the less sensitive method. Also, the results from the case study concluded that Vector normalization displayed better performance among the three tested MCDM methods. From this study we borrowed the average of Spearman correlation (Mean ks values metric for our assessment framework).

Mathew et al. (2014) compared the effect of six normalization techniques using Spearman correlation in weighted aggregated sum product assessment (WASPAS) method and recommended Max-Min as the best one for WASPAS method. However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc.

Podviezko and Podvezko (2015) showed the influence effects of data transformation (normalization process) on TOPSIS and SAW methods. They suggested to implement different normalization techniques as much as possible in MCDM methods and analyze their effects on the ranking results considering which technique covers utility of the decision maker. They suggest the evaluation should be done by decision makers intervention and we believe this process will have biases.

Papathanasiou et al. (2016) proposed a new web-based decision support system to rank alternatives and compare them using different normalization techniques on the MCDM methods TOPSIS and VIKOR. However, they did not compare the normalization techniques using suitable metrics.

Yazdani et al. (2017) implemented the Gray Complex Proportional Assessment (COPRAS) COPRAS-G method in material selection case study using five normalization techniques (Vector, Logarithmic, Sum, Max, and Non-linear by Turskis). The authors used two case studies to compare the results of initial normalization of COPRAS-G with the chosen five normalization techniques. For the first case, there were no differences between ranking of alternatives using different techniques and good correlation existed between all techniques and the initial technique of COPRAS-G, however, case 2 showed different results and considering the errors from the initial method of COPRAS-G non-linear normalization by Turskis had the accepted performance (Yazdani et al., 2017). However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc.

Jahan (2018) developed range target-based normalization technique in WASPAS method and implemented ANOVA to compare the efficiency of Non-monotonic, Comprehensive, and Target-based (point and range) normalization techniques. However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc. to have a robust comparison between different normalization techniques.

Krylovas et al. (2018) discussed the optimal parameters for normalization techniques by approximation of real data with appropriate probability distributions and compared the efficiency of normalization techniques in the final solution of MCDM problems. However, this study did not include metrics such as mean squared error, Minkowski distance, standard deviation, etc. to be ensure about the robustness of comparison between several normalization techniques.

Aytekin (2021) analysed and tested the effects of 23 normalization techniques on 14 different scenarios (decision matrices) and used the SAW method as an aggregation technique for checking rank reversal. The authors pointed out that several features have effects on the selection of normalization techniques such as rank reversal, the range of normalized values, obtaining the same optimization aspect for all criteria, and the validity of results (Aytekin, 2021). They calculated maximum and minimum of normalized values for each scenario while still there is a lack of assessment framework that is consisted of different metrics such as mean squared error, Minkowski distance, standard deviation, etc. to be ensure about the robustness of comparison between several normalization techniques and recommend the best techniques to decision makers.

As already mentioned in this section, there are several studies regarding the evaluation of normalization techniques for MCDM problems, however, they lack providing a general recommendation framework for choosing the most suitable normalization technique. Hence, in chapter 3 of this thesis we propose and discuss a developed evaluation framework to recommend the more suitable normalization technique for MCDM problems.

2.4 Summary

This chapter addressed a literature review on the main topics related to this thesis, multi criteria decision making (MCDM) methods and the definition and importance of normalization techniques in MCDM. Also, a taxonomy of MCDM from literature is presented and the selected MCDM methods to be used in this research are detailed. Furthermore, dynamic multicriteria decision making (DMCDM) and collaborative networks are briefly introduced in this chapter for MCDM methods.

Moreover, a complete survey of the most well-known normalization techniques is introduced as well as their importance and effects on the final results of the MCDM methods. Although, there are some studies on evaluating the effects of different normalization techniques on MCDM methods, including recommendation of the best technique for specific case studies, still there is a lack of a general assessment framework to be used in decision problems for recommending the best normalization technique. The existence of this gap is the main concern and motivation for developing an assessment framework (chapter 3) in this research work. Concluding, this chapter covers the fundamental concepts associated with the development of an assessment framework for recommending the best normalization techniques in MCDM problems. The contributions for this thesis chapter are supported by the following publications: Vafaei et al. (2016a, 2016b, 2018a, 2018b, 2019, 2020, 2022).

3

3 Assessment Framework

This chapter focuses on the proposal of a framework, developed during the research work for this thesis. The main aim of this framework is to offer a novel assessment tool to recommend which normalization technique is more suitable for usage with well-known MCDM methods. The framework development evolved along several improvement cycles, to achieve a consistent tool, proposing the necessary steps to select the best normalization technique. The chapter is divided into three research phases, each describing the framework evolution, towards the final goal of building a decision support tool to help decision makers and analysts on their work with MCDM methods. Furthermore, this chapter introduces an automatic process to recommend the best normalization technique for MCDM methods.

3.1 Phase 1 of assessment framework evolution

In the first phase of the framework's development and evolution, we tested three promising metrics (Vafaei et al., 2018a) for assessing normalization techniques in MCDM methods, divided as follows:

Step a) Determine Ranking Consistency Index (RCI) [from(Chakraborty and Yeh, 2009)]:

This index is borrowed from (Chakraborty and Yeh, 2009) and demonstrates how well a specific normalization technique produces rankings similar to other techniques. RCI is the ratio between the total number of times a specific normalization technique produces a similar/dissimilar ranking of alternatives with other techniques and the total number of simulation runs (Chakraborty and Yeh, 2009). Also, a consistency weight (CW) is calculated for each specific normalization technique to assign weights to the formula (Chakraborty and Yeh, 2009). This metric is chosen because of its ability to indicate the similarity/dissimilarity of each specific normalization technique, compared with other techniques. However, there is no general formula for this metric because of its characteristics, regarding decision problems and the number of normalization techniques. Hence, we calculate the RCI for each normalization technique and then we count the total number of times that these normalizations are similar or dissimilar in the ranking of alternatives in tested decision problems. In this study, the number of iterations for simulation runs is assumed 1. Details about the calculation process (RCI, CW, etc.) are provided in section 3.1.1. For interpretation of the RCI results (Chakraborty and Yeh, 2009), the normalization techniques with higher RCI are more desirable because they have more similarity and less dissimilarity with other normalization techniques, i.e. the higher the value of RCI the better.

Step b) Calculation of Pearson and Spearman correlations between ranking of alternative/alternatives' values to determine the mean ks values [from (Chatterjee and Chakraborty, 2014)]**:**

The reason to select Pearson correlation was that it is the first known correlational measure, developed by Karl Pearson in 1948, based on an idea from Sir Francis Galton in the late 1800s; further, all posterior correlation measures derived from the original Pearson formula (Chee, 2013; Cramer, 1998). Pearson correlation measures the linear relationship between two ratio/interval variables and its values lie between -1 and +1 (Chee, 2013; Cramer, 1998). Pearson correlation has some pros and cons such as its ability to define the strength of the relations between two variables in a simple way, while sharing variance (covary) (Chee, 2013). On the other hand, Pearson correlation weakness is its inability to measure relationships of non-linear variables (i.e. displaying correlation zero when there are non-linear relationships). Pearson correlation (P) formulation is (Chee, 2013; Cramer, 1998):

$$Pearson\_Corr = P = \frac{n(\sum xy) - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}} \tag{3-1}$$

Where, n= number of paired variables, x=the first variable, y=the second variable, and xy=the product of the two paired variables.

The reasoning for choosing Spearman's rank correlation coefficient was that it is a non-parametric test to define the degree of association between two variables without any assumption about the data distribution (Wang and Luo, 2010). Spearman's correlation coefficient is usually a good metric to define the association and strength of a relationship between two sets of data and variables with ordinal scales. Spearman's correlation coefficient (S) is defined as (Chakraborty and Yeh, 2009):

$$Spearman\_Corr = \mathrm{S} = 1 - 6\frac{\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \tag{3-2}$$

Where, Di is the difference between values/ranks r_i and $r_i'$and m is the number of alternatives; S value lies between –1 and +1.

Therefore, in this step b) of the first Phase, we start by calculating Pearson correlation (Celen, 2014) and Spearman correlation, both between ranking/values of alternatives, (Wang and Luo, 2010) and then we compare the results. In this evaluation, for all pairs of normalization techniques, we calculated their correlation and also the average k_s value to determine the mean ranking agreement among them, using ranking/values of alternatives with Pearson and Spearman correlations. The higher the value of the Mean k_s value the better. Because, it means that the method which has higher rank is more correlated with the other normalization techniques.

Step c) Analysis and evaluation of normalization techniques consistency with three conditions [borrowed from (Celen, 2014)]:

This step is inspired on Celen (2014) and consists of three conditions to analyze the effect of different normalization techniques on ranking alternatives using MCDM methods. Celen (2014) conditions are interesting to evaluate how normalization techniques behave on decision problems and which one is more similar to the others.

Condition 1*:* The result should consider a distributional property, location, of normalized values (metrics for measure of location) such as means, standard deviations, minimum and maximum values. However, since location is not enough to determine similarity in distributional properties, we proceed to also use conditions 2 and 3.

Condition 2*:* Check for normal distributions to ensure consistency using Kolmogorov- Smirnov test to measure Skewness and Kurtosis. Skewness is a measure of symmetry, or more precisely, the lack of symmetry and Kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution. If the amount of Skewness and Kurtosis is between (-2, 2), there is a possibility data have normal distributions. However, to be sure about normal distributions we also need to calculate the statistic test and significant level test (Sig) (Field, 2000; Trochim and Donnelly, 2006). The amount of statistical test should be less than 1 and the amount of significant level test (Sig) should be more than 0.05 (sig > 0.05)(Field, 2000; Trochim and Donnelly, 2006).

Condition 3*:* This condition encompasses the comparison between three best and worst ranking results for robustness purposes. When normalization techniques rank alternatives, mostly in the same order, we can say the results are more robust. The calculation process is explained below in section 3.1.1.

3.1.1 Test cases for Phase 1

In order to validate the robustness of this assessment framework's phase 1, we applied it to some case studies and illustrative examples using the chosen normalization techniques (Table 8): Max, Max-Min, Sum, Vector, Logarithmic, and Fuzzification except Target-based. Further, we tested benefit and cost criteria normalization formulas, related to the characteristics of criteria in those case studies and illustrative examples. The case studies and examples use well-known MCDM methods such as: TOPSIS, ELECTRE, SAW, VIKOR, and AHP. Further, to focus on the effect of different normalization techniques on the chosen MCDM methods we applied equal weights for criteria in all tested case studies.

It should also be noticed that there were some doubts in condition 2 of Step c for proving normal distribution of different normalization techniques regarding the Kolmogorov-Smirnov test. Generally, the amount of statistical test should be less than 1 and the amount of significant level test (Sig) should be more than 0.05 (sig > 0.05) to have normal distribution. However, some techniques (from Table 15) have statistic tests<1 and sig <0.05, i.e. normal distributions cannot be proven. Moreover, not all conditions of Step c produce numerical and comparable results to compare their numeric results with Steps a & b. So, we will implement Step a & b for the rest of the case studies in phase 1 and postpone further research about step c-phase 1 to a future stage.

3.1.1.1 TOPSIS Method [adapted from (Vafaei et al., 2018a)]

This numerical example is based on a project of autonomous landing of drones with hazard avoidance, where the criteria are hazard maps (http://www.ca3-uninova.org/project_iluv). We reduced the illustrative example to only 3 criteria (C1, C2, C3), corresponding to the hazard maps of illumination, reachability, and land texture; and 16 alternatives (A1, A2, …, A16) from the 90.000 real alternatives (300 pixels by 300 pixels hazard map), which correspond to candidate location sites for landing. This small subset is enough to analyze the effects of different normalization techniques and see the different rankings. In the original large case study it was impossible to see the different ranking of alternatives in presence of different normalization techniques easily. Table 9 shows the data used for assessing the framework where C1 and C2 are criteria of type benefit, i.e., the higher the raw values the better they should be on the normalization and C3 is a cost criterion for which low normalized values are desirable. After collecting the input data, we tested six (shown in Table 8) of the most well-known normalization techniques (Jahan and Edwards, 2015; Nayak et al., 2014; Peldschus, 2007; Zavadskas et al., 2003) and analyzed their effect on this case study to select the best location sites for landing.

Table 9: Decision matrix for landing drones

	C1 (illumination)	C2 (reachability)	C3 (land texture)
A1	138.6090	0.3349	6.4543
A2	154.7214	0.3395	11.4244
A3	158.3081	0.3441	11.4244
A4	157.3082	0.3487	6.8542
A5	144.5976	0.3301	11.2616
A6	138.5982	0.3346	11.2616
A7	131.5989	0.3391	11.1988
A8	132.5988	0.3437	11.1988
A9	144.5976	0.3252	11.2616
A10	138.5982	0.3297	11.2616
A11	132.5988	0.3342	11.1988
A12	135.9513	0.3387	6.8974
A13	119.7141	0.3204	11.2616
A14	112.7148	0.3248	11.1988
A15	112.7148	0.3292	11.1988
A16	128.9520	0.3337	6.8974

In step 1 of TOPSIS method, the decision matrix is defined (see Table 9).

In step 2, we calculate the normalized decision matrix. To illustrate this process, we use alternative A3 with respect to C1, with the six tested normalization techniques mentioned above (Max, Max-Min, Sum, Vector, Logarithmic, and Fuzzification). Below, we illustrate the numerical calculations, and the results are summarized in Table 10.

$$n_{Max,\ 3} = \frac{158.3081}{158.3081} = 1$$

$$n_{Max-Min,\ 3} = \frac{158.3081 - 112.7148}{158.3081 - 112.7148} = 1$$

$$n_{Linear,\ 3} = \frac{158.3081}{138.609 + 154.7214 + \cdots + 128.952} = 0.0725$$

$$n_{Vector,\ 3} = \frac{158.3081}{\sqrt{138.609^2 + 154.7214^2 + \cdots + 128.952^2}} = 0.2887$$

$$n_{Logarithmic,\ 3} = \frac{\ln (158.3081)}{\ln (138.609 * 154.7214 * \cdots * 128.952)} = 0.0645$$

For Fuzzification we used the proposed trapezoidal membership functions (N20-Table 7), which were adapted to this case study criteria domains, as illustrated in Figure 7. Notice the two benefit criteria, C1 and C2 with right open trapezoidal functions and cost criteria C3 with open left trapezoidal function.

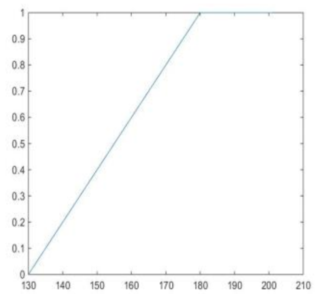
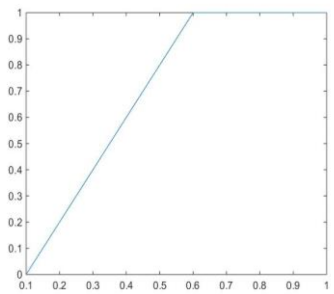
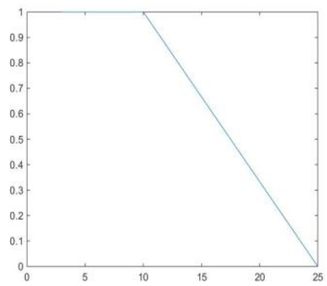
		
C1 domain [130, 180, 201, 201]	C2 domain [0.1, 0.6, 1, 1]	C3 domain [3, 3, 10, 25]

Figure 7: Fuzzification of Criteria for Landing Drones

In step 3 of TOPSIS, we calculate the weighted normalized decision matrix by multiplying weights to the criteria of the normalized decision matrix. As mentioned before, for simplicity purposes we considered all criteria of equal importance, hence, for the three criteria weights are 0.3333.

In step 4 of TOPSIS method, after choosing the maximum and minimum criteria value from the weighted normalized matrix, we calculated the positive-ideal and negative-ideal solutions based on the criterion's nature (cost or benefit ones). Illustrating for A3 alternative:

$$D^{+}_{Max,\ 3} = \sqrt{(0.333 - 0.333)^2 + (0.3286 - 0.333)^2 + (0 - 0.333)^2} = 0.0043$$

$$D^{-}_{Max,\ 3} = \sqrt{(0.333 - 0.2370)^2 + (0.3286 - 0.3059)^2 + (0 - 0.1453)^2} = 0.1755$$

For performing step 5 and step 6 of TOPSIS, we calculate the relative closeness and the ranking of alternatives using positive and negative ideal solution values. Table 10 shows the results of all tested normalization techniques for alternative A3, as follows:

$$C^*_{Max,\ 3} = \frac{0.1756}{0.1756 + 0.0044} = 0.9756$$

Table 10: Result of TOPSIS steps for A3

		Max (N1)	Max-Min (N2)	Sum (N3)	Vector (N4)	Loga-rithmic (N5)	Fuzzi-fica-tion (N6)
Normalized decision matrix	C1	1	1	0.0725	0.2888	0.0645	1
	C2	0.9868	0.8375	0.0643	0.2572	0.0932	0.4882
	C3	0	0	0.0528	0.7234	0.0622	0.9050
Positive and negative ideal solutions	D+	0.0044	0.0541	0.0003	0.0011	0.0021	0.0031
	D-	0.1756	0.5479	0.0153	0.0492	0.0015	0.3349
Relative closeness values [$D^-/(D^- + D^+)$]	C3*	0.9756	0.9101	0.9817	0.9772	0.4244	0.9908
Rank of normalization techniques	Rank	1	1	1	1	12	1

The relative closeness values and comparison of ranking results for the 16 alternatives with six normalization techniques are shown in Table 11. It is interesting to observe that A3 is considered the best candidate for five of the six normalization techniques: Max (N1), Max-Min (N2), Sum (N3), Vector (N4), and Fuzzification (N6) techniques; while for A9, N5 is the best for logarithmic normalization.

Table 11: Relative closeness (RC) and Ranking of alternatives (R)

	Max (N1)		Max-Min (N2)		Sum (N3)		Vector (N4)		Logarithmic (N5)		Fuzzification (N6)	
	RC	R	RC	R	RC	R	RC	R	RC	R	RC	R
A1	0.2704	15	0.3901	15	0.2252	15	0.2772	15	0.5031	8	0.8345	8
A2	0.9366	2	0.8196	2	0.9501	2	0.9380	2	0.4851	10	0.9820	2
A3	0.9756	1	0.9101	1	0.9817	1	0.9772	1	0.4244	12	0.9908	1
A4	0.4239	13	0.6030	8	0.3816	13	0.4325	13	0.3629	15	0.9131	5
A5	0.8202	3	0.6323	6	0.8553	3	0.8189	3	0.6718	3	0.9626	3
A6	0.7753	5	0.6508	4	0.8160	5	0.7703	5	0.5245	6	0.8577	6
A7	0.7173	9	0.6454	5	0.7668	9	0.7096	9	0.3751	14	0.1786	12
A8	0.7283	7	0.6927	3	0.7756	7	0.7204	7	0.2864	16	0.2715	10
A9	0.8087	4	0.5775	10	0.8470	4	0.8089	4	0.7999	1	0.9540	4
A10	0.7684	6	0.5944	9	0.8111	6	0.7645	6	0.6517	4	0.8557	7
A11	0.7213	8	0.6031	7	0.7710	8	0.7145	8	0.5026	9	0.2695	11
A12	0.2754	14	0.4305	12	0.2592	14	0.2806	14	0.3967	13	0.5888	9
A13	0.6225	10	0.4284	13	0.6850	10	0.6147	10	0.6784	2	0.0777	13
A14	0.5841	12	0.4255	14	0.6502	12	0.5749	12	0.5835	5	0.0745	15
A15	0.5866	11	0.4531	11	0.6520	11	0.5770	11	0.5098	7	0.0758	14
A16	0.2086	16	0.3253	16	0.2092	16	0.2120	16	0.4830	11	0.0259	16

Since there is no consensus about which is the best normalization technique just by looking at the results obtained we now apply the assessment framework metrics of phase 1, for selecting the best normalization technique. The calculations for the three steps of the on-going framework are as follows:

Step a) Determine the Ranking Consistency Index (RCI):

Since we have 6 normalization techniques, we start by defining the consistency weight (CW) as follows:

1- If a method is consistent with all other 5 methods, then CW = 5/5 = 1.

2- If a method is consistent with 4 of the other 5 methods, then CW = 4/5.

3- If a method is consistent with 3 of the other 5 methods, then CW = 3/5.

4- If a method is consistent with 2 of the other 5 methods, then CW = 2/5.

5- If a method is consistent with 1 of the other 5 methods, then CW = 1/5.

6- If a method is not consistent with any of the other methods, then CW = 0/5=0.

And then the ranking consistency index (RCI), for instance for N1, is calculated as:

RCI (N1) = [(T123456 * (CW=1))+ (T12345 * (CW=4/5))+ (T13456 * (CW=4/5))+ (T12456 * (CW=4/5))+ (T12346 * (CW=4/5))+ (T12356 * (CW=4/5))+ (T1234 * (CW=3/5))+ (T1235 * (CW=3/5))+ (T1236 * (CW=3/5))+ (T1245 * (CW=3/5))+ (T1246 * (CW=3/5))+ (T1256 * (CW=3/5))+ (T1346 * (CW=3/5))+ (T1356 * (CW=3/5))+ (T1345 * (CW=3/5))+ (T1456 * (CW=3/5))+ (T123 * (CW=2/5))+ (T124 * (CW=2/5))+ (T125* (CW=2/5))+ (T126 * (CW=2/5)) + (T134 * (CW=2/5))+ (T135 * (CW=2/5))+ (T136 * (CW=2/5))+ (T145 * (CW=2/5))+ (T146 * (CW=2/5))+ (T156 * (CW=2/5))+ (T12 * (CW=1/5))+ (T13 * (CW=1/5))+ (T14 * (CW=1/5))+ (T15 * (CW=1/5))+ (T16 * (CW=1/5))+ (TD123456 * (CW=0))/TS]

Where,

TS = Total number of times the iteration of simulation was run (in this study TS=1)

TD123456 = the Total number of times N1, N2, N3, N4, N5, and N6 produced different rankings.

T123456 = Total number of times N1, N2, N3, N4, N5, and N6 produced the same ranking.

T12345 = Total number of times N1, N2, N3, N4, and N5 produced the same ranking.

T1234 = Total number of times N1, N2, N3, and N4 produced the same ranking.

T123 = Total number of times N1, N2, and N3 produced the same ranking.

T12 = Total number of times N1 and N2 produced the same ranking.

The RCI (Ranking Consistency Index) for the other normalization techniques is calculated similarly to the above formula and the results are depicted in Table 12. As shown, RCI points to Max (N1), Sum(N3), and vector normalization (N4) as the best normalization techniques for this case study with TOPSIS method and the worst one is logarithmic (N5).

Table 12: Ranking consistency index of normalization techniques.

	RCI	**Rank**
N1	40	1
N2	24.2	5
N3	40	1
N4	40	1
N5	6.8	6
N6	28.6	4

Step b) We now calculate the Pearson and Spearman correlation between ranking alternative values from Table 11 and results are depicted in Table 13. Also, the results of Pearson and Spearman correlation between the relative closeness of alternatives are depicted in Table 14. Furthermore, for all pairs of normalization techniques, we calculate their Pearson and Spearman correlations and the average ks values to determine the mean ranking agreement among them, as shown in Table 13 and Table 14.

Table 13: Pearson (P) and Spearman (S) correlation between rankings of alternatives and their mean ks values.

	N1		N2		N3		N4		N5		N6		Mean k_s value		Rank	
	P	S	P	S	P	S	P	S	P	S	P	S	P	S	P	S
N1			0.808	0.808	1	1	1	1	0.244	0.244	0.714	0.714	0.753	0.753	1	1
N2	0.808	0.808			0.808	0.808	0.808	0.808	-0.323	-0.323	0.641	0.641	0.548	0.548	5	5
N3	1	1	0.808	0.808			1	1	0.244	0.244	0.714	0.714	0.753	0.753	1	1
N4	1	1	0.808	0.808	1	1			0.244	0.244	0.714	0.714	0.753	0.753	1	1
N5	0.244	0.244	-0.323	-0.323	0.244	0.244	0.244	0.244			0.032	0.032	0.088	0.088	6	6
N6	0.714	0.714	0.641	0.641	0.714	0.714	0.714	0.714	0.032	0.032			0.563	0.563	4	4

Table 14: Pearson (P) and Spearman (S) correlation between relative closeness of alternatives and their mean ks values.

	N1		N2		N3		N4		N5		N6		Mean k_s value		Rank	
	P	S	P	S	P	S	P	S	P	S	P	S	P	S	P	S
N1			0.835	0.995	0.992	0.999	0.999	1	0.234	0.998	0.366	0.996	0.685	0.9987	2	3
N2	0.835	0.995			0.776	0.999	0.844	0.999	-0.224	0.998	0.552	0.997	0.556	0.9989	4	2
N3	0.992	0.992	0.776	0.992			0.989	0.999	0.266	0.998	0.267	0.996	0.658	0.9985	3	4
N4	0.999	1	0.844	0.995	0.989	0.999			0.232	0.998	0.386	0.996	0.690	0.9989	1	1
N5	0.234	0.998	-0.224	0.998	0.266	0.998	0.232	0.998			0.170	0.996	0.136	0.998	6	5
N6	0.366	0.996	0.552	0.997	0.267	0.996	0.386	0.996	0.170	-0.096			0.348	0.996	5	6

Step c) Analysis and evaluation of normalization techniques consistency with three conditions:

Condition 1 & 2: For the first condition of the assessment approach, we determined the descriptive statistics for the six normalization techniques (see Table 15). By just looking at Table 15 we could not determine similarity in distributional properties, so, we also applied the Kolmogorov–Smirnov test (Condition 2) to check the consistency of normalization techniques for normal distribution.

Table 15: Condition 1 & 2. Descriptive statistics and, Kolmogorov-Smirnov test for normalization techniques.

		N1	N2	N3	N4	N5	N6
Statistics of closeness coefficient values (Condition 1)	**Mean**	**0.6389**	**0.5739**	**0.6648**	**0.6370**	**0.5149**	**0.5571**
	Std. deviation	0.2346	0.1588	0.2551	0.2321	0.1355	0.3966
	Minimum	**0.2086**	**0.3253**	**0.2092**	**0.2120**	**0.2864**	**0.0259**
	Maximum	0.9756	0.9101	0.9817	0.9772	0.7999	0.9908
Kolmogorov-Smirnov test statistics (Condition 2)	Skewness	-0.2164	0.4168	-0.5719	-0.8760	0.4381	-0.6273
	Kurtosis	-1.9673	-0.1026	-0.5665	-0.5441	-0.0649	-0.5574
	Statistic	0.193	0.152	0.227	0.185	0.159	0.258
	Sig.	0.112	0.200	0.027	0.145	0.200	0.006

Since for condition 2 (see Table 15) the amount of Skewness and Kurtosis is between (-2, 2), it is possible to have normal distributions. However, to be sure about normal distributions, the statistic test and significant level test (Sig) were calculated. The amount of statistical test should be less than 1 and the amount of significant level test (Sig) should be higher than 0.05 (sig > 0.05). In Table 15, for all normalization techniques in Kolmogorov-Smirnov test, the amount of statistic tests is less than 1 and the significant level test (Sig) for N1, N2, N4 and N5 is higher than 0.05. However, for cases N3 and N6, since their sig <0.05 the normal distributions could not be proven.

Condition 3: For the third condition, we examined the result of TOPSIS by choosing the highest three and the lowest three ranked alternatives for each normalization technique (see Table 16). As it is shown, the logarithmic normalization technique (N5) has very different scoring in comparison with the other techniques. Also, max-min (N2) and fuzzification (N6) have some different scores from the others (they are highlighted in Table 16 with the grey color). The other three techniques (N1, N3, and N4) have similar results, which seem to indicate these normalization techniques generate more consistent results.

Table 16: Condition 3 – comparison of best and worst normalization techniques.

	Rank	N1	N2	N3	N4	N5	N6
Three highest rank	1	A3	A3	A3	A3	A9	A3
	2	A2	A2	A2	A2	A13	A2
	3	A5	A8	A5	A5	A5	A5
Three lowest rank	14	A12	A14	A12	A12	A7	A15
	15	A1	A1	A1	A1	A4	A14
	16	A16	A16	A16	A16	A8	A16

The results from Table 15 and Table 16 reveal that Step c of phase1 of the assessment framework is not enough to judge the best normalization technique for MCDM methods. So, to recommend the best normalization technique for TOPSIS method, we also need to analyse the results from Steps a and b (Table 12, Table 13, and Table 14) of phase 1 and the summarized results are shown in Table 17.

Table 17: The results of Step a (RCI), Step C. (Pearson (P) and Spearman (S) correlation) for TOPSIS method

	RCI	Mean Ks (Ranking of Alt.)		Mean Ks (Alt. values)	
		P	S	P	S
Max (N1)	1	1	1	2	3
Max-Min (N2)	5	5	5	4	2
Sum (N3)	1	1	1	3	4
Vector (N4)	1	1	1	1	1
Logarithmic (N5)	6	6	6	6	5
Fuzzification (N6)	4	4	4	5	6

The results of Steps a), b) and c) of phase 1 of our assessment approach (Table 17), show there is a consensus between different metrics (RCI, Mean Ks for alternative values and Mean Ks for ranking of alternatives) regarding Vector normalization technique (N4), which shows being the best technique for this case study with TOPSIS method. Furthermore, it seems that Logarithmic normalization technique is the worst technique for this case study. Concluding, with our Phase 1 assessment framework approach, we not only validated Chakraborty & Yeh (2009) result of Vector normalization being the best technique as well as identified the worst solution (Logarithmic), which definitively should not be used as normalization technique for this case study.

3.1.1.2 AHP Method: case study [adapted from (Vafaei et al., 2016a)]

In this case study, we discuss the suitability of our phase 1 assessment framework for evaluating four normalization techniques from Table 8 (Max, Max-Min, Sum, Vector) with an illustrative example of smart car parking, using the AHP method. Here we do not use N5 (logarithmic), N6 (Fuzzification and N7 (target-based) from the selected techniques (see Table 8) due to AHP being a pairwise comparative method (step 2 of AHP method (see section 2.1.4)) where it is impossible to individually represent each criterion either by a membership function or defining a criterion target. Further, we discarded the logarithmic normalization technique because we obtained negative and infinite data (due to the characteristics of pairwise matrixes), hence it is not usable (appropriate) for the AHP method (likewise any other similar fuzzification technique). Notice that Logarithmic normalization technique could be seen as a fuzzification process because it uses a Sigmoid membership functions to represent criteria

This illustrative case consists of 3 criteria (C1, C2, C3), which correspond to time to park, distance, and size of the parking space, and 7 alternatives (A1, A2, …, A7), which correspond to candidate location sites for parking. Finding the best place for parking the car is the goal; C1 and C2 are cost criteria, where low values are better, and C3 is a benefit criterion, where high values are desirable. Following the AHP method we define three pairwise comparison matrices for each criterion (Table 18, Table 19, Table 20) and then one pairwise comparison matrix between criteria (Table 21). To these four matrixes, we apply the five normalization techniques, separately, to determine the ranking of alternatives and compare results.

Table 18: Pairwise Comparison matrix with respect to the time.

	A1	A2	A3	A4	A5	A6	A7
A1	1	1/3	1/2	3	1/3	2	1
A2	3	1	1	4	1	3	1
A3	2	1	1	2	1/2	3	2
A4	1/3	1/4	1/2	1	1/4	1	1/3
A5	3	1	2	4	1	3	1
A6	1/2	1/3	1/3	1	1/3	1	3
A7	1	1	1/2	3	1	1/3	1

Table 19: Pairwise Comparison matrix with respect to the distance

	A1	A2	A3	A4	A5	A6	A7
A1	1	1/2	2	4	1/3	1/6	1
A2	2	1	3	5	1/2	1/4	1
A3	1/2	1/3	1	5	1/4	1/4	2
A4	1/4	1/5	1/5	1	1/5	1/5	1/7
A5	3	2	4	5	1	1	3
A6	6	4	4	5	1	1	2
A7	1	1	1/2	7	1/3	1/2	1

Table 20: Pairwise Comparison matrix with respect to the size of the parking space

	A1	A2	A3	A4	A5	A6	A7
A1	1	4	4	5	1/2	2	5
A2	1/4	1	1	3	1/3	1/3	1/5
A3	1/4	1	1	1	1/4	1/5	1/4
A4	1/5	1/3	1	1	1/5	1/6	1/9
A5	2	3	4	5	1	1	2
A6	1/2	3	5	6	1	1	1/2
A7	1/5	5	4	9	1/2	2	1

Table 21: Pairwise Comparison matrix between criteria

	C1	C2	C3
C1	1	4	7
C2	1/4	1	4
C3	1/7	1/4	1

We start by testing the Sum normalization, the usual normalization technique for AHP (Saaty, 1980) because it ensures column sum per alternative is equal to one that is defined by Saaty (1980). The other normalization techniques do not include this characteristic and the sum of the normalized values can be bigger than 1; hence, for comparison purposes, we opted for re-normalizing the other four tested techniques using the Sum normalization technique. For illustrating the alternatives rating procedure, we show the calculation for vector normalization of alternative A1, and the final results for all alternatives are shown in Table 22 and Table 23:

$$P_{11} = \frac{x_{11}}{\sqrt{\sum_{j=1}^{7} x_{1j}}} = \frac{1}{\sqrt{(1^2) + (3^2) + (2^2) + \left(\frac{1}{3}^2\right) + (3^2) + (\frac{1}{2}^2) + (1)^2}} = 0.7974$$

$$Average\ P1 = \frac{0.7974 + 0.8390 + 0.8091 + 0.5991 + 0.8227 + 0.6524 + 0.7583}{7} = 0.7540$$

$$A_{11} = \frac{Average\ P1}{Sum} = \frac{0.7974}{4.8050} = 0.1659$$

$$Average\ A1 = \frac{0.1659 + 0.1814 + 0.1659 + 0.1304 + 0.1769 + 0.1393 + 0.1598}{7} = 0.1605$$

Table 22: Normalization results for vector normalization technique related to C1.

	P1	*P2*	*P3*	*P4*	*P5*	*P6*	*P7*	*Average*
P1	0.7974	0.8390	0.8091	0.5991	0.8227	0.6524	0.7583	0.7540
P2	0.3922	0.5169	0.6182	0.4655	0.4681	0.4786	0.7583	0.5283
P3	0.5948	0.5169	0.6182	0.7327	0.7341	0.4786	0.5165	0.5988
P4	0.9325	0.8792	0.8091	0.8664	0.8670	0.8262	0.9194	0.8714
P5	0.3922	0.5169	0.2365	0.4655	0.4681	0.4786	0.7583	0.4737
P6	0.8987	0.8390	0.8727	0.8664	0.8227	0.8262	0.2748	0.7715
P7	0.7974	0.5169	0.8091	0.5991	0.4681	0.9421	0.7583	0.6987
sum	4.8051	4.6247	4.7730	4.5946	4.6508	4.6829	4.7437	4.6964

Table 23: Re-normalization results for vector normalization technique related to C1.

	A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*Average*
A1	0.1659	0.1814	0.1695	0.1304	0.1769	0.1393	0.1598	0.1605
A2	0.0816	0.1118	0.1295	0.1013	0.1007	0.1022	0.1598	0.1124
A3	0.1238	0.1118	0.1295	0.1595	0.1578	0.1022	0.1089	0.1276
A4	0.1941	0.1901	0.1695	0.1886	0.1864	0.1764	0.1938	0.1856
A5	0.0816	0.1118	0.0495	0.1013	0.1007	0.1022	0.1598	0.1010
A6	0.1870	0.1814	0.1828	0.1886	0.1769	0.1764	0.0579	0.1644
A7	0.1659	0.1118	0.1695	0.1304	0.1007	0.2012	0.1598	0.1485
sum	1	1	1	1	1	1	1	1

Similar calculation processes are performed for the other comparison matrixes, using the remaining four normalization techniques. The global weights of alternatives and ranking results for the tested normalization techniques are shown in Table 24. As it can be seen in Table 24, there is consensus on which normalization techniques is better for alternatives A2, A3, A4, and A5 (i.e. they all have the same ranking), but for the other alternatives, there was no consensus.

Table 24: Global score (G) and Ranking (R) of alternatives for the smart parking example using AHP method.

	Max (N1)		*Max-Min (N2)*		*Sum (N3)*		*Vector (N4)*	
	G	R	G	R	G	R	G	R
A1	0.1972	2	0.1925	2	0.1505	4	0.1693	2
A2	0.0681	6	0.0634	6	0.0762	6	0.1165	6
A3	0.1143	5	0.1161	5	0.0993	5	0.1297	5
A4	0.2469	1	0.2658	1	0.2876	1	0.1755	1
A5	0.0460	7	0.0291	7	0.0749	7	0.1101	7
A6	0.1765	3	0.1869	3	0.1598	2	0.1450	4
A7	0.1509	4	0.1462	4	0.1517	3	0.1538	3

In phase 1, we calculated RCI, Pearson and Spearman correlation and mean ks values with the global weights of alternatives and also with the ranks of alternatives to assess the suitability of the four tested normalization techniques for the AHP method. It should be noticed that in the related paper (Vafaei et al., 2016a) only the Pearson correlation with the global weights of alternatives and Spearman correlation with the rank of alternatives were calculated. In this thesis, in order to show the complete calculation, we included calculations for Pearson and Spearman with both global weights and ranks of alternatives. Table 25 shows the results of step a & b-phase 1.

Table 25: The results of Step a & b-phase 1. RCI, Pearson (P) and Spearman (S) correlation for AHP method

			Correlation with rank of Alt.				Correlation with Alt. Values			
	RCI		P		S		P		S	
	Value	Rank	ks	Rank	ks	Rank	ks	Rank	Ks	Rank
Max (N1)	18	1	0.9524	1	0.7333	1	0.9606	1	0.9993	1
Max-Min (N2)	18	1	0.9524	1	0.7333	1	0.9564	2	0.9991	2
sum (N3)	16.3333	4	0.8810	4	0.3333	4	0.9029	4	0.9991	3
Vector (N4)	17.6667	3	0.9286	3	0.6	3	0.9263	3	0.9983	4

Hence, Table 25 displays that there exists a complete consensus between Pearson and Spearman correlation's results and RCI. It is clear that the best normalization technique is Max because it has the highest mean ks values for both Pearson and Spearman correlations and RCI.

Summarizing, although Max is elected as the most suitable normalization technique, it requires a re-normalization with Sum normalization technique because the sum of the normalized values has to be 1 (for more information about AHP method please see Section 2.1.4). Therefore, we may conclude that a combination of Max normalization with Linear-Sum seems the most appropriate for AHP.

3.1.1.3 Illustrative test for ELECTRE, SAW, VIKORs Methods [Adapted from (Vafaei et al., 2016b) and (Vafaei et al., 2018b)]

In this section we present a small numerical example with 7 alternatives and 3 criteria to discuss the results of step a & b of phase 1 (determining RCI and comparative study between ranking of alternatives using Pearson and Spearman correlation to determine the mean k_s value).

Table 26 shows the example's input data used for discussing the chosen six normalization techniques (N1, N2, N3, N4, N5, and N6 except N7 from Table 8) with three MCDM methods (ELECTRE, SAW, VIKORs), where C1 and C2 are benefit criteria, i.e. the higher the raw values the better they should be on the normalization, and C3 is a cost criterion, where low normalized values are desirable. Table 27, Table 28, and Table 29 depict the results of ELECTRE, SAW, and VIKOR methods using selected normalization techniques (see Table 8). For more information about ELECTRE, SAW, and VIKOR methods and their steps please see section 2.1.

Table 26: Decision matrix for the illustrative example.

	C1	*C2*	*C3*
A1	171.3068	0.3176	3.9516
A2	178.0288	0.3219	5.5274
A3	179.3276	0.3263	5.5274
A4	171.3068	0.3127	3.9516
A5	179.3276	0.3171	5.5274
A6	171.0295	0.3214	5.8126
A7	162.0905	0.3079	10.6341

Table 27: Ranking (R) of alternatives using ELECTRE method.

	Max	Max-Min	Sum	Vector	Logarithmic	Fuzzification
A1	3	2	3	3	4	1
A2	1	2	1	1	6	1
A3	1	2	1	1	6	1
A4	3	2	3	3	4	1
A5	3	2	5	3	1	1
A6	3	1	5	3	3	1
A7	7	7	5	7	1	1

Table 28: Alternatives' Values (AV) and Ranking (R) of alternatives using SAW method.

	Max (N1)		Max-Min (N2)		Sum (N3)		Vector (N4)		Logarithmic (N5)		Fuzzification (N6)	
	AV	R	AV	R	AV	R	AV	R	AV	R	AV	R
A1	0.8521	1	0.6876	4	0.1589	1	0.5038	1	0.1444	2	0.7540	5
A2	0.8195	4	0.8165	2	0.1430	4	0.4783	4	0.1426	4	0.8017	3
A3	0.8264	3	0.9212	1	0.1440	3	0.4810	3	0.1421	5	0.8133	1
A4	0.8471	2	0.5989	6	0.1581	2	0.5019	2	0.1450	1	0.7507	6
A5	0.8170	5	0.7547	3	0.1427	5	0.4774	5	0.1433	3	0.8071	2
A6	0.7970	6	0.6580	5	0.1388	6	0.4673	6	0.1421	6	0.7547	4
A7	0.6152	7	0	7	0.1145	7	0.3574	7	0.1405	7	0.6721	7

Table 29: Alternatives' Values (AV) and Ranking (R) of alternatives using VIKORs method.

	Max (N1)		Max-Min (N2)		Sum (N3)		Vector (N4)		Logarithmic (N5)		Fuzzification (N6)	
	AV	R	AV	R	AV	R	AV	R	AV	R	AV	R
A1	0.7636	4	0.8732	4	1	1	0.9754	1	0.3839	4	0.7900	5
A2	0.8669	3	0.9041	3	0.3906	4	0.8223	4	0.2562	5	0.9589	3
A3	0.9457	1	0.9985	1	0.4013	3	0.8574	3	0.1536	6	1	1
A4	0.6199	6	0.8251	5	0.9923	2	0.9501	2	0.7745	1	0.7786	6
A5	0.9259	2	0.9081	2	0.3868	5	0.8098	5	0.4553	3	0.9781	2
A6	0.7506	5	0.7229	6	0.3156	6	0.7140	6	0.0247	7	0.7928	4
A7	0	7	0	7	0	7	0	7	0.5665	2	0	7

Observing the results, it shows that there are different ranks for alternatives using selected normalization techniques. Hence, it is difficult to select the best normalization technique for these three MCDM methods and to choose the best technique for each MCDM method we apply the on-going evaluation framework. Table 30, Table 31, and Table 32 show the results of step a & b, for ELECTRE, SAW, and VIKOR methods. Notice that for ELECTRE method we could not use Pearson and Spearman correlation with the alternatives' values because this method requires preferences (elimination) between criteria and does not produce alternatives' values.

Table 30: The results of Step a & b-phase 1. RCI, Pearson (P) and Spearman (S) correlation and their Mean ks (ks) for ELECTRE method

			Correlation with rank of alternatives			
	RCI		P		S	
	Value	Rank	ks	Rank	ks	Rank
Max (N1)	7.8	1	0.3395	1	0.4107	1
Max-Min (N2)	1	6	0.2598	3	0.3821	3
sum (N3)	6.4	3	0.1558	4	0.2821	4
Vector (N4)	7.8	1	0.3395	2	0.4107	2
Logarithmic (N5)	1.6	5	-0.6439	6	-0.5179	6
Fuzzification (N6)	5.4	4	-0.0260	5	0.0250	5

Table 31: The results of Step a & b-phase 1. RCI, Pearson (P) and Spearman (S) and their Mean ks (ks) correlation for SAW method

			Correlation with rank of Alt.				Correlation with Alt. Values			
	RCI		P		S		P		S	
	Value	Rank	ks	Rank	ks	Rank	ks	Rank	ks	Rank
Max (N1)	24	2	0.65	2	0.65	2	0.8795	1	0.9738	4
Max-Min (N2)	17.8	5	0.4	5	0.4	5	0.7736	4	0.9786	2
sum (N3)	25.2	1	0.65	2	0.65	2	0.8125	3	0.9690	5
Vector (N4)	24	2	0.65	1	0.65	1	0.8755	2	0.9876	1
Logarithmic (N5)	21	4	0.5143	4	0.5143	4	0.6689	5	0.9688	6
Fuzzification (N6)	16.4	6	0.2500	6	0.2500	6	0.6653	6	0.9764	3

Table 32: The results of Step a & b-phase 1. RCI, Pearson (P) and Spearman (S) correlation and their Mean ks (ks) for VIKORs method

			Correlation with rank of Alt.				**Correlation with Alt. Values**			
	RCI		**P**		**S**		**P**		**S**	
	Value	**Rank**	**ks**	**Rank**	**ks**	**Rank**	**ks**	**Rank**	**ks**	**Rank**
Max	9.4	1	0.4	2	0.4	2	0.5440	4	0.9867	3
Max-Min	8.4	2	0.4857	1	0.4857	1	0.6309	2	0.9870	2
sum	8	4	0.3857	3	0.3857	3	0.4941	5	0.9811	5
Vector	8.2	3	0.3857	3	0.3857	3	0.6701	1	0.9885	1
Logarithmic	0.6	6	-0.2357	6	-0.2357	6	-0.1978	6	0.9635	6
Fuzzification	7.8	5	0.2929	5	0.2929	5	0.5831	3	0.9853	4

Table 30 results show that for ELECTRE method there is consensus between RCI, Pearson and Spearman results regarding Max being the best normalization technique and Logarithmic being the worst one (Table 30).

For SAW method, there is no consensus between RCI and correlation with the ranking of alternatives and correlation with the alternatives' values. The obtained results from correlation with the rank of alternatives reveal that Vector (N4) has the highest Mean ks value (both Pearson and Spearman) for SAW (Table 31). Hence, Vector (N4) normalization technique could be more appropriate for SAW method but this statement will need more metrics to be sure.

As Table 32 shows, there is a consensus between Pearson and Spearman correlation for both using rank of alternatives and alternative values for VIKORs method. Based on the obtained results, both Max-Min (N2) and Vector (N4) are suitable normalization techniques for VIKOR, however to decide which one is better will require to add more metrics.

Summarizing, the best technique for ELECTRE is Max (N1), for SAW method is Vector (N4), and for VIKORs method, Max-Min (N2) and Vector (N4) are both suitable normalization techniques. However, SAW and VIKOR methods will need to more metrics on the assessment framework to be sure about recommending techniques in the related case studies.

3.2 Phase 2 of assessment framework evolution

In phase 2 of the assessment framework development, we modified **Step c** of Phase 1 (section 3.1) Analysis and evaluation of normalization techniques consistency with three conditions (Celen, 2014)- because this step is rather limited and only analyses the behaviour of different normalization techniques and provides interpretative results for determining RCI and correlation. First, metrics in condition 2 assume datasets without outliers and having normal distributions, therefore, they cannot be generalized, i.e. those metrics are quite specific for Celen (2014) case study. Generally, for confirming normal distributions, using Kolmogorov-Smirnov test, the value of statistical test should be less than 1 and the value of significant level test (Sig) should be more than 0.05 (sig > 0.05) (Field, 2000; Trochim and Donnelly, 2006). So, we decided to eliminate condition 2 of step c (from phase 1).

Second, condition 3 - Checking similarity ranking of alternatives by comparison of best and worst ranking of three results/alternatives - is rather cumbersome for case studies with a large number of alternatives and the results do not seem to positively contribute to the selection of the best normalization technique. Therefore, we removed this condition in phase 2 of the assessment framework to improve the comparison results' robustness.

Furthermore, in this phase 2 we also modified **Step b)** of phase 1 (3.1)- Comparative study between ranking of alternative/alternatives' values using Pearson and Spearman correlations to determine the mean ks value - , because the observed results from Table 13, Table 25, Table 30, Table 31, and Table 32 show that Pearson and Spearman always produce the same results for calculating the correlation between ranking of alternatives. Spearman (Chakraborty and Yeh, 2009; Wang and Luo, 2010) is good for calculating the correlation between ranking of alternatives because it is a suitable non-parametric test to define the strength of a relationship between two sets of data and deals with ordinal scale variables; Pearson correlation is good for finding the relation strength between alternatives' values because of its ability to define the correlation between real numbers, in a simple way, by sharing variance (Chee, 2013). Furthermore, in this phase we questioned the usefulness of using both correlations comparison because, as mentioned before, Spearman formulation is derived from Pearson formula. So, in order to avoid any confusion for decision makers by producing different results using Pearson and Spearman, we propose to use just Pearson correlation in the framework.

In addition, besides modifying Step b) & c) we added a new step to the evaluation framework in **phase 2,** namely Step d, to develop the robustness of the assessment framework's results by measure the proximity (using Minkowski distances metrics) and data dispersion (using standard deviation metric) in MCDM decision problems. The novel four steps of **phase 2** are as follows:

Step a: Determine the Ranking Consistency Index (RCI) (Chakraborty and Yeh, 2009).

Step b: Calculate Pearson correlation (equations (3-1)) and its mean value (ks) (Chatterjee and Chakraborty, 2014).

Step c: Calculate the Standard Deviation (STD) of alternatives' values for each normalization technique to assess the spreading out of data set using alternatives' values (Bland and Altman, 1996; Rumsey, 2009; Yeh, 2003). STD provides a measure of the spread out of the dataset from its mean and its formulation (3-3) is expressed as:

$$STD = \sqrt{\frac{\sum_{i=1}^{q}(x_i - \bar{x})^2}{q-1}} \tag{3-3}$$

Where x_i are the observed values and $\bar{x}$ is the mean value of the observed values, and q is the number of observations. More explanations about the calculation process and interpretation of this metric can be seen in section 3.2.1.1.

Step d: Calculate Minkowski distances (Guo, 2004; Han *et al.*, 2012; Hassan *et al.*, 2014; Shih *et al.*, 2007) for three well-known distances (equations (3-5),(3-6), and (3-7)) to ensure consistency on evaluating normalization techniques through distances. The Minkowski distance formula is (3-4):

$$Minkowski\_d(x,y) = \sqrt[p]{\sum_{i=1}^{n}|(x_i - y_i)^p|} \tag{3-4}$$

As Minkowski distance formula (3-4) shows, it is a generalization of Euclidean, Manhattan, and Chebyshev distances, where p is a real number such that p≥1 and x_i and y_i are the observed alternatives' values and n is the number of alternatives.

This formula represents the Manhattan distance when p=1 (equation (3-5)) and Euclidean distance when p=2 (equation (3-6)):

- Manhattan (p=1):

$$Manhattan\_d(x, y) = \sum_{i=1}^{n} |(x_i - y_i)| \tag{3-5}$$

- Euclidean (p=2):

$$Euclidean\_d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \tag{3-6}$$

- Chebyshev (p=∞):

  In dealing with the limit case of p=∞, the equation (3-4) represents the supremum distance and it is called Chebyshev distance (equation(3-7)), which gives the maximum difference in values between the two objects.

$$Chebyshev\_d(x, y) = \lim_{p \to \infty} \left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{1/p} = \max_i(|x_i - y_i|) \tag{3-7}$$

In the above formulas, x_i and y_i are the observed alternatives' values and n is the number of alternatives. More explanation about the calculations process and interpretation of this step is described in section 3.2.1.1.

3.2.1 Test Cases for Phase 2

In this section, we test the evaluation framework applicability, to detect faults, to allow improving the efficiency of the framework. To simplify the testing, such as in Phase 1 we apply equal weights for criteria in all case studies.

3.2.1.1 SAW with DMCDM and CN [adapted from (Vafaei et al., 2019)]

In this phase we test the suitability of the chosen six normalization techniques (N1, N2, N3, N4, N5, and N6 except N7 from Table 8) applied to a borrowed case study (Arrais-Castro *et al.*, 2015a), which consists of evaluating and ranking a set of six business partners for product design service provision. The case includes six alternatives to design services, with three agencies and three independent businesses as alternatives to be evaluated and ranked.

The set of criteria for past and future evaluation, are (Table 33): cost per hour; on-time delivery performance; delay penalty, based on the number of days orders were delayed and performance evolution; quality rating, about work delivered; lack of quality penalty, consisting on a penalty based on the number of complains per order and performance evolution; and portfolio rating. Also, the set of criteria for present evaluation are (Table 33): quoted price; delivery time (quoted); lead time, portfolio rating, and strategic rating.

In the original case study (Arrais-Castro *et al.*, 2015a) the chosen data normalization technique was fuzzification (N6, from Table 8) and the method was DMCDM (Dynamic Multiple Criteria Decision Making) (see section2.2) . We reduced the original six alternatives in the past and future decision matrixes to 5 (A1, A2, …, A5) because there is no information related to A6 for past and future, and we merged 2 criteria (Delivery time and Lead time from the original paper) of present data because they have the same characteristics. Further, with this merge and reduction we could show the capacity of the dynamic model to handle adding or deleting alternatives or criteria.

Table 33 shows the example´s input data (Arrais-Castro *et al.*, 2015a), where Cost per hour, Delay penalty, Lack of quality, Quoted price, Delivery time, and Lead time are cost criteria - low normalized values are desirable - while the rest of the criteria are benefit ones - higher values are better. Furthermore, we only discuss the first iteration of the dynamic system to simplify the explanation.

Table 33: Input data adapted from (Arrais-Castro *et al.*, 2015a)

	Cost per hour (Average)		On time delivery performance		Delay penalty		Quality rating (work delivered)		Lack of Quality Penalty		Portfolio Rating		Quoted Price	Lead Time	Portfolio Rating	Strategic Rating
	Past	future	Past	future	Past	future	Past	future	Past	future	Past	future	Present			
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16
A1= Agency 1	72	82.5	80%	98%	15	10	80%	100 %	2	0	85%	90%	5760	15	100%	10
A2= Agency 2	65	95	95%	95%	10	5	90%	98%	4	1	90%	80%	8840	22	98%	5
A3= Agency 3	40	80	80%	98%	5	15	95%	98%	0	3	85%	85%	5760	22	98%	8
A4= Designer 1	32	50	90%	85%	25	20	85%	85%	6	5	80%	75%	6150	25	95%	8
A5= Designer 2	75	55	85%	80%	20	25	90%	90%	8	8	75%	80%	13200	32	80%	6
A6= Designer 3													6000	10	100%	0

Observing Table 33 we see that some criteria include zero values (eg., lack of quality penalty) and this prevents normalizing with Sum and Logarithmic formulations because of division by zero causes infinite and undefined results in the normalization process. So, we could immediately eliminate these two normalization techniques from our comparative study, since they are not suitable for decision problems with criteria´s scores of zero. Illustrating, the calculations to show this characteristic, using Sum normalization technique for Agency 3, for Lack of quality penalty criteria are:

$$n_{Agency3, lack\ of\ quality\ (past)} = \frac{\frac{1}{0}}{\left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{8}\right)} = NA$$

After eliminating the two mentioned normalization techniques (Sum and Logarithmic), we normalize the input values for the other three normalization techniques (Max, Max-Min, Vector) and add the already normalized values from the fuzzification technique (Arrais-Castro *et al.*, 2015a). Illustrating the calculation process for the three normalized techniques, for Agency 3, criterion "Quoted price" (present information), using the cost formula of the remaining three Normalization techniques (see Table 8 N1, N2, and N4), we have:

$$Max: \quad n_{Agency3, Quoted\ price\ (present)} = 1 - \frac{5760}{13200} = 0.564$$

$$Max - Min: \quad n_{Agency3, Quoted\ price\ (present)} = \frac{(13200 - 5760)}{(13200 - 5760)} = 1$$

$$Vector: \quad n_{Agency3, Quoted\ price\ (present)} = 1 - \left(\frac{5760}{\sqrt{(5760^2) + (8840^2) + \cdots + (6000^2)}}\right) = 0.709$$

Table 34 depicts the normalized results for all alternatives. To facilitate visualization, we only show the normalized values for present information. Further, since normalization with the fuzzification technique was calculated in Arrais-Castro et al. (2015a) it is not displayed in Table 34.

Table 34: Normalized values for present information.

	Max				Max-Min				Vector			
	*Quoted Price [€]	* Lead Time [min]	Portfolio Rating [%]	Strategic Rating [0-10]	*Quoted Price [€]	* Lead Time [min]	Portfolio Rating [%]	Strategic Rating [0-10]	*Quoted Price [€]	* Lead Time [min]	Portfolio Rating [%]	Strategic Rating [0-10]
A1	0.5636	0.5313	1	1	1	0.7727	1	1	0.7093	0.7235	0.4278	0.5882
A2	0.3303	0.3125	0.98	0.5	0.5860	0.4545	0.9	0.5	0.5538	0.5944	0.4193	0.2941
A3	0.5636	0.3125	0.98	0.8	1	0.4545	0.9	0.8	0.7093	0.5944	0.4193	0.4706
A4	0.5341	0.2188	0.95	0.8	0.9476	0.3182	0.75	0.8	0.6896	0.5391	0.4064	0.4706
A5	0	0	0.8	0.6	0	0	0	0.6	0.3338	0.4100	0.3423	0.3529

After obtaining the normalized scores for each matrix, past, present, and future, we need to aggregate the values for rating each alternative. For this process, we use as aggregation operator (i.e., fusion process) the SAW method with equal weights for all criteria for obtaining the final score (Alternatives' values) for all suppliers, as depicted in Table 35.

Table 35: Alternatives' Values (AV) and Ranking (R) of suppliers using four normalization techniques.

	Max (N1)		Max-Min (N2)		Vector (N4)		Fuzzification (N6)	
	AV	R	AV	R	AV	R	AV	R
A1	0.376	1	0.339	2	0.274	1	0.339	2
A2	0.287	4	0.317	4	0.253	3	0.317	4
A3	0.357	2	0.382	1	0.268	2	0.382	1
A4	0.320	3	0.338	3	0.241	4	0.338	3
A5	0.282	5	0.099	5	0.236	5	0.099	5

Observing the rankings of alternatives/suppliers in Table 35, it is impossible to choose which normalization technique is more appropriate for this case study which uses dynamic methods (DMCDM) in collaborative networks. Therefore, it is important to apply the four steps of our assessment framework (section

03.2) to select the most suitable normalization technique. The implemented steps are explained as below:

Step a: Determine the Ranking Consistency Index (RCI)

As mentioned in Phase I, the RCI is calculated with the total number of similarity/dissimilarity that each normalization technique produces in comparison with the other normalization techniques. So, we start by defining the consistency weight (CW) for the four normalization techniques (Max, Max-Min, Vector, and Fuzzification), as follows:

1- If one normalization technique ranking score is similar in the other 3 normalization techniques, then CW=3/3=1.
2- If one normalization technique ranking score is similar in the other 2 normalization techniques, then CW=2/3.
3- If one normalization technique ranking is only similar with another normalization technique, then CW=1/3.
4- If one normalization technique ranking is not similar with any of the other 3 normalization techniques, then CW=0/3=0.

To illustrate, the calculation of RCI for N1 (Max) normalization technique is:

RCI (N1) = [((T_{1234} * (CW=1))+ (T_{123} * (CW=2/3))+ (T_{124} * (CW=2/3))+ (T_{134}* (CW=2/3))+ (T_{12} * (CW=1/3))+ (T_{13} * (CW=1/3))+ (T_{14} * (CW=1/3))+ (TD_{1234} * (CW=0)))/TS].

T_{1246} = Total number of times N1, N2, N4 and N6 produced the same ranking.
T_{124} = Total number of times N1, N2 and N4 produced the same ranking.
T_{126} = Total number of times N1, N2 and N6 produced the same ranking.
T_{146} = Total number of times N1, N4 and N6 produced the same ranking.
T_{12} = Total number of times N1 and N2 produced the same ranking.
T_{14} = Total number of times N1 and N4 produced the same ranking.
T_{16} = Total number of times N1 and N6 produced the same ranking.
TD_{1246} = Total number of times N1, N2, N4 and N6 produced different rankings.
TS = Total number of times the simulation was run (in this study TS=1).

RCI (N1) = [((0 * 1) + (0 * 2/3) + … + (2 * 1/3) + (0 * 0))/1] = 4

Similar calculations are performed for the other three normalization techniques (Max-Min, Vector, and Fuzzification) to obtain their respective RCI. The results of Step a) are depicted in Table 36.

Table 36: Step a. RCI for the selected Normalization Techniques

	RCI	Rank Step A
Max (N1)	4	1
Max-Min (N2)	4	1
Vector (N4)	1.333	4
Fuzzification (N6)	4	1

Observing Table 36 we can see that there is a draw between the normalization techniques Max (N1), Max-Min (N2), and Fuzzification (N6) – all ranked first – but we can discard Vector normalization technique (N4) as not appropriate for the case study.

Step b: Calculate the Pearson correlation and determine the mean value (ks)

In this step, we calculate the Pearson correlation (equation (3-1)) for each pair of normalization techniques and then their mean ks values (the average of correlation) for each technique using the rank of alternatives. Table 37 shows the results of Pearson correlation and mean ks values (Step b) using data from Table 35.

Table 37: Step b. Pearson correlation between alternatives' values and Mean ks values for each normalization technique

	Max	Max-Min	Vector	Fuzzification	k_s	Rank
Max (N1)		0.8857	0.9429	0.8857	0.9048	1
Max-Min (N2)	0.8857		0.8286	1	0.9048	1
Vector (N4)	0.9429	0.8286		0.8286	0.8667	4
Fuzzification (N6)	0.8857	1	0.8286		0.9048	1

The results of Step b), displayed in Table 37, are similar to the ones obtained in Step a) (Table 36), and still there is no discrimination between three normalization techniques (Max (N1), Max-Min (N2), and Fuzzification (N6)), because they have the same mean ks values, i.e. neither Step a) nor Step b) can discriminate between N1, N2, and N6.

Step c: Calculate the Standard Deviation (STD) using alternatives' values

In Phase 2 third step of the assessment framework, we calculate the standard deviation (STD) of alternatives' values (using Table 35) for each normalization technique. STD (equation(3-3)) provides a measure of the spread out of the dataset from its mean. Usually, a lower STD indicates that data are close to the mean value, while a higher STD shows that data are further away from the mean and spread out within the data range (Bland and Altman, 1996; Rumsey, 2009). However, a small STD value is not always favourable and its interpretation depends on the case study and its properties (Investopedia, 2018; J. Rumsey, 2018). To clarify the interpretation of STD in our case study, we use partial data from Table 33, and calculate the STD for Max-Min normalization technique (C´7 and C´9), as shown in Table 38.

Table 38: Step c and d for normalized values C´7 and C´9 using Max-Min normalization technique.

	Raw data		Normalized values (using Max-Min normalization technique)	
	C7	C9	C´7	C´9
A1	80%	2	0	0.75
A2	90%	4	0.6667	0.5
A3	95%	0	1	1
A4	85%	6	0.3333	0.25
A5	90%	8	0.6667	0
Phase 2- Step c) STD			0.3801	0.3953

Table 38 shows that C9 range is [0-8] while C7 values are concentrated on the interval [80%-90%]. The STD results (Step c) in Table 38) should be interpreted as the higher values are the better and vice versa the lower STD value are less desirable, i.e. worst choices. The reason for this interpretation is that we intend to choose the best normalization technique which produces the discriminative normalized values to rank alternatives and avoid to produce the same rank for more than one alternative. This goal could be reached if normalized values are far enough from each other and respectively, the mean value for the normalized values range [0-1] in decision problems should be far enough from each other. So, the higher STD in the limited range would guaranty more discriminative results to order/rank alternatives in decision problems.

The final results of Step c) for the four tested normalization techniques (Max, Max-Min, Vector, Fuzzification) are shown in Table 39. Observing these results, we can now say that Max-Min (N2) and Fuzzification (N6) are good normalization techniques (based on their higher STD), while N1 and N6 (N6 was already discarded in Step c) are not good for the problem at hand.

Table 39: Step c) STD for the selected Normalization Techniques

	STD	Rank
Max (N1)	0.0940	3
Max-Min (N2)	0.1269	1
Vector (N4)	0.0550	4
Fuzzification (N6)	0.1269	1

Step d: Calculate Minkowski distances (Guo, 2004; Hassan et al., 2014)

In the last step of the on-going evaluation framework, we use Minkowski distances (equation (3-4)) to assess which normalization technique is better (Max, Max-Min, Vector, Fuzzification). We decided to use three well-known types of Minkowski distances (Manhattan, Euclidean, and Chebyshev) (equations (3-5), (3-6), and (3-7)) and the same reasoning of Step c) to interpret distance values, i.e. the higher distance value the better is the normalization technique. The results are depicted in Table 40.

Table 40: Step d. Manhattan, Euclidean, and Chebyshev distances for selected normalization techniques.

	Manhattan		Euclidean		Chebyshev	
	Distance	Rank	Distance	Rank	Distance	Rank
Max (N1)	1.633	3	0.515	3	0.264	4
Max-Min (N2)	2.040	1	0.695	1	0.345	1
Vector (N4)	0.865	4	0.301	4	0.150	3
Fuzzification (N6)	2.040	1	0.695	1	0.345	1

It should be noticed that step d) did not provide any distinction between Max-Min (N2) and Fuzzification (N6) (see Table 40) as the best normalization technique for the dynamic collaborative network case study. However, since Step c) and Step d) concur on the results, one may say that both normalization techniques are suitable for usage in this problem.

Table 41 summarizes the obtained results of applying the evaluation framework, for recommending which normalization technique is more appropriate for a case of dynamic collaborative networks for ranking suppliers/businesses. Through the comparison carried out with the case study it was possible to say that two of the four normalization techniques considered (Max-Min and Fuzzification) are the best suited for ranking suppliers/businesses (see Table 41).

Table 41: Summary of ranked for selected normalization techniques obtained from the four steps (a, b, c, and d) evaluation assessment.

	Step a	Step b	Step c	Step d		
	RCI	Mean ks	STD	Minkowski distances		
				Manhattan	Euclidean	Chebyshev
Max (N1)	1	1	3	3	3	4
Max-Min (N2)	1	1	1	1	1	1
Vector (N3)	4	4	4	4	4	3
Fuzzification (N4)	1	1	1	1	1	1

The final ordering of the assessed normalization techniques (Table 41) is: first, both Max-Min and Fuzzification normalization techniques are appropriate for normalization of this case study; second best is Max normalization, and the worst is the Vector normalization technique. The other two initially chosen techniques, Sum and logarithmic, are definitively not suitable for usage in this case study because of the infinite and undefined results obtained when any criterion includes a zero value.

Concluding, normalization techniques influence the score and ranking of alternatives in any DMCDM model, in presence of collaborative networks and highlights the need for having a sound evaluation process/framework to recommend the most suitable normalization technique for Dynamic Multiple Criteria Decision Making in collaborative networks.

3.2.1.2 TOPSIS Method

In this section, we discuss the suitability of six normalization techniques from Table 8 applied to the case study from section 3.1.1.1. In order to facilitate the understanding of the decision problem, we demonstrate the results after the aggregation process with the TOPSIS method for the six normalization techniques (Table 11).

Table 11: Relative closeness (RC) values and Ranking of alternatives (R) (adapted from section 3.1.1.1)

	Max (N1)		Max-Min (N2)		Sum (N3)		Vector (N4)		Logarithmic (N5)		Fuzzification (N6)	
	RC	R	RC	R	RC	R	RC	R	RC	R	RC	R
A1	0.2704	15	0.3901	15	0.2252	15	0.2772	15	0.5031	8	0.8345	8
A2	0.9366	2	0.8196	2	0.9501	2	0.9380	2	0.4851	10	0.9820	2
A3	0.9756	1	0.9101	1	0.9817	1	0.9772	1	0.4244	12	0.9908	1
A4	0.4239	13	0.6030	8	0.3816	13	0.4325	13	0.3629	15	0.9131	5
A5	0.8202	3	0.6323	6	0.8553	3	0.8189	3	0.6718	3	0.9626	3
A6	0.7753	5	0.6508	4	0.8160	5	0.7703	5	0.5245	6	0.8577	6
A7	0.7173	9	0.6454	5	0.7668	9	0.7096	9	0.3751	14	0.1786	12
A8	0.7283	7	0.6927	3	0.7756	7	0.7204	7	0.2864	16	0.2715	10
A9	0.8087	4	0.5775	10	0.8470	4	0.8089	4	0.7999	1	0.9540	4
A10	0.7684	6	0.5944	9	0.8111	6	0.7645	6	0.6517	4	0.8557	7
A11	0.7213	8	0.6031	7	0.7710	8	0.7145	8	0.5026	9	0.2695	11
A12	0.2754	14	0.4305	12	0.2592	14	0.2806	14	0.3967	13	0.5888	9
A13	0.6225	10	0.4284	13	0.6850	10	0.6147	10	0.6784	2	0.0777	13
A14	0.5841	12	0.4255	14	0.6502	12	0.5749	12	0.5835	5	0.0745	15
A15	0.5866	11	0.4531	11	0.6520	11	0.5770	11	0.5098	7	0.0758	14
A16	0.2086	16	0.3253	16	0.2092	16	0.2120	16	0.4830	11	0.0259	16

As discussed in Phase 1, it is difficult to assess which is the best normalization technique just by looking at the results obtained in step a), therefore, we will apply steps c) and d) of phase 2 from the assessment framework for selecting the best normalization technique (Table 42).

Table 42: Step c & d: STD, Manhattan, Euclidean, and Chebyshev distances for selected normalization techniques.

	STD		Manhattan		Euclidean		Chebyshev	
	Distance	Rank	Distance	Rank	Distance	Rank	Distance	Rank
Max (N1)	0.2346	3	32.1911	3	3.6340	3	0.7670	3
Max-Min (N2)	0.1588	5	21.7828	5	2.4594	5	0.5848	5
Sum (N3)	0.2551	2	33.9894	2	3.9515	2	0.7725	2
Vector (N4)	0.2321	4	32.0058	4	3.5958	4	0.7652	4
Logarithmic (N5)	0.1355	6	18.7598	6	2.0990	6	0.5135	6
Fuzzification (N6)	0.3966	1	53.9947	1	6.1436	1	0.9650	1

The above results plus the results of RCI (Table 12) and mean ks values (Table 13) from section 3.1.1.1 are summarized in Table 43.

Table 43: Summary of ranked for selected normalization techniques obtained from the four steps (a, b, c, and d) assessment of phase 2.

	Step a	Step b	Step c	Step d		
	RCI	Mean ks	SDT	Minkowski distances Manhattan	Euclidean	Chebyshev
Max (N1)	2	1	3	3	3	3
Max-Min (N2)	4	5	5	5	5	5
Sum (N3)	3	1	2	2	2	2
Vector (N4)	1	1	4	4	4	4
Logarithmic (N5)	6	6	6	6	6	6
Fuzzification (N6)	5	4	1	1	1	1

As Table 43 shows, there is still no consensus from the four steps of Phase 2. So, clearly, there is a need for adding more metrics to ensure that the evaluation framework is robust for selecting the most appropriate technique. In Phase 3 we will describe the proposed additions to the framework.

3.2.1.3 AHP Method

In this section, we discuss the suitability of the four tested normalization techniques (see Table 8) applied to the case study from section 3.1.1.2. In order to facilitate the understanding of the decision problem, we demonstrate the results after the aggregation process with AHP method (Table 24).

Table 24: Global weight (G) and Ranking (R) of alternatives for the smart parking example (adapted from section 3.1.1.2)

	Max (N1)		*Max-Min (N2)*		*Sum (N3)*		*Vector (N4)*	
	G	R	G	R	G	R	G	R
A1	0.1972	2	0.1925	2	0.1505	4	0.1693	2
A2	0.0681	6	0.0634	6	0.0762	6	0.1165	6
A3	0.1143	5	0.1161	5	0.0993	5	0.1297	5
A4	0.2469	1	0.2658	1	0.2876	1	0.1755	1
A5	0.0460	7	0.0291	7	0.0749	7	0.1101	7
A6	0.1765	3	0.1869	3	0.1598	2	0.1450	4
A7	0.1509	4	0.1462	4	0.1517	3	0.1538	3

Again, since it is difficult to assess which is the best normalization technique just by looking at the results obtained, we now apply steps c) and d) of phase 2 and borrow results of step a) and b) from section 3.1.1.2. for selecting the best normalization technique. Table 44 shows the results obtained with these steps.

Table 44: Steps c & d-phase2: STD, Manhattan, Euclidean, and Chebyshev distances for tested normalization techniques.

	Mean ks values		STD		Manhattan		Euclidean		Chebyshev	
	Value	Rank	Distance	Rank	Distance	Rank	Distance	Rank	Distance	Rank
Max (N1)	0.9606	1	0.0716	3	1.7155	3	0.4642	3	0.2010	3
Max-Min (N2)	0.9564	2	0.0811	1	2.0783	1	0.5258	1	0.2367	1
Sum (N3)	0.9029	4	0.0734	2	1.8468	2	0.4758	2	0.2127	2
Vector (N4)	0.9263	3	0.0253	4	0.6520	4	0.1638	4	0.0655	4

The above results plus the results of mean ks values (Table 25) from section 3.1.1.2 are depicted in Table 45.

Table 45: Summary of ranked normalization techniques obtained from Phase 2 steps (a, b, c, and d).

	Step a	Step b	Step c	Step d		
	RCI	Mean ks	STD	Minkowski distances Manhattan	Euclidean	Chebyshev
Max (N1)	1	1	3	3	3	3
Max-Min (N2)	1	2	1	1	1	1
Sum (N3)	4	4	2	2	2	2
Vector (N4)	3	3	4	4	4	4

Anyway, as Table 45 shows, still there is no consensus between the results using the four proposed steps. Definitively, there is a need for adding other metrics to the framework, to provide more robust and effective recommendations when selecting the most proper technique.

3.2.1.4 ELECTRE Method

In this section, we discuss the suitability of six normalization techniques (see Table 8, N1, N2, N3, N4, N5, and N6 except N7) applied to the case study of section 3.1.1.3. In order to facilitate the decision problem understanding, we discuss the results after the aggregation process with ELECTRE method (Table 27).

Table 27: Ranking (R) of alternatives using ELECTRE method.

	Max (N1)	Max-Min (N2)	Sum (N3)	Vector (N4)	Logarithmic (N5)	Fuzzification (N6)
A1	3	2	3	3	4	1
A2	1	2	1	1	6	1
A3	1	2	1	1	6	1
A4	3	2	3	3	4	1
A5	3	2	5	3	1	1
A6	3	1	5	3	3	1
A7	7	7	5	7	1	1

In this example, as well as in the previous 2 examples, it is difficult to assess which is the best normalization technique just by looking at the results obtained with the aggregation process, therefore we again applied steps c) and d) of phase 2 and borrow the results of step a) and b) from Table 30 of section 3.1.1.2. Table 46 shows the results of these steps.

Table 46: Steps c& d: STD, Manhattan, Euclidean, and Chebyshev distances for selected normalization techniques.

	STD		Manhattan		Euclidean		Chebyshev	
	Distance	Rank	Distance	Rank	Distance	Rank	Distance	Rank
Max (N1)	2	2	44	2	12.9615	2	6	1
Max-Min (N2)	1.9881	4	36	5	12.8841	4	6	1
Sum (N3)	1.7995	5	44	2	11.6619	5	4	5
Vector (N4)	2	2	44	2	12.9615	2	6	1
Logarithmic (N5)	2.0702	1	52	1	13.4164	1	5	4
Fuzzification (N6)	0	6	0	6	0	6	0	6

The complete results from Phase 2, joining Table 46 ranking plus the results obtained from step a) and b) (RCI and mean ks values - Pearson correlation (Table 30) section 3.1.1.3)) are shown in Table 47.

Table 47: Summary of ranked normalization techniques obtained with Phase 2 steps (a, b, c, and d).

	Step a	Step b	Step c	Step d		
	RCI	Mean ks	STD	Minkowski distances		
				Manhattan	Euclidean	Chebyshev
Max (N1)	1	1	2	2	2	1
Max-Min (N2)	6	3	4	5	4	1
Sum (N3)	3	4	5	2	5	5
Vector (N4)	1	2	2	2	2	1
Logarithmic (N5)	5	6	1	1	1	4
Fuzzification (N6)	4	5	6	6	6	6

As Table 47 shows, still there is no consensus using framework Phase 2 - four steps. Hence, we may conclude that the metrics used in this Phase are clearly not enough to discriminate which is the best normalization technique. However, from this experience we observe that what we need to improve is a process to aggregate the rankings into a single value, from where the highest will be the best. To this aim we propose the plurality voting method ((d'Angelo *et al.*, 1998; Vafaei et al., 2022)), which is described in the next phase 3.

3.3 Phase 3 of Framework evolution

For **phase 3** of the assessment framework development, we decided to build a conceptual model (Figure 8) to support decision makers using the framework in a user-friendly way. The proposed conceptual model includes four numerical steps, but instead of just referring to different metrics they describe a complete process to recommend the best normalization technique for MCDM methods. Please notice that Step 3 includes most metrics tested in Phase 1 and 2, but organized in a taxonomy of types of metrics, to demonstrate the wide coverage of our chosen evaluation metrics. Besides the proposed metrics, the conceptual model includes determining the types of data (Step 1) and then choosing the types of normalization techniques to be evaluated (Step 2). Finally, we added a voting process step (step 4), which combines the ranking of normalization techniques, obtained when using the metrics. With these four steps, we believe that our proposed framework is now more complete, clearer, and helpful for decision makers to make informed decisions about choosing normalization techniques for MCDM methods. Figure 8 depicts the conceptual model of the developed assessment framework.

Step 1: Data types

Determine the type of criteria values:

- Benefit or cost criteria
- Ordinal number
- Natural number
- Skewed data with outlier
- Real number
    - Float numbers in the unit interval [0-1]

Step 2: Selection of Normalization Techniques

Candidate well-known normalization techniques:

- **Linear:** Max; Max-Min; Sum
- **Semi-Linear:** Vector; Target-based
- **Non-Linear:** Logarithmic, Fuzzification

Data preparation:
(1) Sum is not applicable if there is any zero in criteria values (using cost formula)
(2) Logarithmic is not applicable when there are zeros in any criterion

Step 3: Evaluation of Techniques

Chosen metrics:

a) **Outlier detection:** Line chart/ Histogram/Scatter/ Box plot
b) **Measures of location:** Maximum, Minimum
c) **Measures of data dispersion:** STD, ANOVA / Regression analysis (P-value, Standard error, T-stat)
d) **Measures of proximity:** Minkowski distances (Manhattan, Euclidean, Chebyshev)
e) **Comparison metrics:** Mean Ks values (from Spearman/ Pearson Correlation); Ranking Consistency Index (RCI); Mean squared error (MSE)

Step 4: Selection of the Best Technique

Selection of the best technique with:

- **Plurality voting**: Selection of best normalization technique with the large number of first order/rank, in the different used metrics

Figure 8: Phase 3- Conceptual model of the evaluation framework

In the following, we provide details for each step, particularly for the new step included in this final conceptual model:

Step 1: In this first step, we explore the type of input data. Further, we identify the benefit and cost criteria, and then we determine the criteria's type of values, such as: Ordinal numbers, Real numbers, Natural numbers, etc. For instance, as mentioned in section 3.2.1.1, some criteria might include zero values and this prevents normalizing with Sum (using cost formula) and Logarithmic formulations because of division by zero causes infinite and undefined results in the normalization process (Vafaei et al., 2019). This step helps to estimate these critical situations and detect inappropriate techniques before starting the normalization process.

Step 2: In the second step we chose the seven normalization techniques, defined in Table 8, i.e. we included target normalization (N7) to ensure testing at least two techniques from each class: linear, semi-linear, non-linear.

However, target-based technique requires human intervention to define the target value ***T***j (Table 8). We tried to counteract this shortcoming by using as target value (Tj) both: i) average, and ii) median of criterion; and we called them Target-Avg and Target-Med.

In step 2, we also eliminate normalization techniques which are not suitable for the input data set due to some shortcoming. For instance, Sum normalization is not appropriate (in presence of cost criterion and using cost formula) when there is any zero in the input data set because it will produce an infinite output – division by zero (section 3.2.1.1) (Vafaei et al., 2019). Also, the Logarithmic normalization technique produces undefined and negative outputs when input data includes zero and decimal numbers (section 3.2.1.1) (Vafaei et al., 2019).

Step 3: In the third step, we introduce a taxonomy for the framework metrics to compare and differentiate normalization techniques' results. The proposed taxonomy includes some metrics from Phase 1 and Phase 2 (Note: in each phase they were denoted as steps) plus new ones. We classified the metrics into different categories based on their usage and characteristics (Filliben and Heckert, 2012; Han *et al.*, 2012) , such as:

a) Outlier detection - The first category of metrics proposes tools for data set visualization to allow finding any outlier which may influence the aggregation process. The most common graphics for outlier detection are Line chart or Histogram or scatter or Box plot (Filliben and Heckert, 2012; Han *et al.*, 2012). Notice that the existence of outliers in the classic MCDM problems (such as site selection, partner selection, etc.) is very rare but to ensure full coverage of metrics we added this step to the assessment framework.

b) Measures of Location -This category of metrics measures the central tendency of input data as Maximum and Minimum. Then, by defining the maximum and minimum of normalized values we checked the role of normalization techniques in keeping the dominance of alternatives (Filliben and Heckert, 2012; Han *et al.*, 2012). For instance, if A1 is the maximum value of both input data and normalized values using N1 as a normalization technique, it means that normalization technique N1 keeps its dominancy on all alternatives.

   Other Location metrics such as Average/Mean, Median, Mode are eliminated from the assessment framework. Mean is a good metric but it is very sensitive to the presence of high and low extreme values (outliers). Hence, we avoided using Mean in this class but considered a similar metric in the class of comparison measurements, with correlation (Mean ks value). Median is the middle of the data set that deals with two situations: (i) if the data set number is odd, the median is its middle value (ii) if the data set number is even, the median is the average of its two middlemost values (Han et al., 2012). Further, the median in decision matrixes with an even number of alternatives is the average of the two middle alternatives, and this is not meaningful in MCDM. A number that appears most often in a data set is the mode. To calculate the mode, or modal value, we order the numbers and then count how many of each exists. It is possible to have more than one number with the same count which is called multimodal. In a data set without value's repetition (i.e. they occur just once), the data set does not have mode (Han et al., 2012). Therefore, the two last metrics (Median and Mode) do not seem useful for comparison of normalization techniques in MCDM. So, Maximum and Minimum are used as measure of location category in step 3- Phase 3 (Figure 8).

c) Measures of data dispersion – This category includes metrics to measure the dispersion of input data and normalized values. These measurements are related to statistical metrics such as STD and Regression analysis/

ANOVA (Analysis of Variance). The STD is used to measure the data spreading out using alternatives' values, as explained in sections 0 and 3.2.1.1. Furthermore, we include other well-known statistical metrics, such as: P-value, T-stat, and Standard Error (Cameron, 2009; Filliben and Heckert, 2012; Stephanie, 2014a) to measure the normalized values dispersion. The T-statistic and P-value are for the null hypothesis versus the alternate hypothesis (Filliben and Heckert, 2012; Stephanie, 2013b). The P-value is the probability that the sample data results occur by chance (Filliben and Heckert, 2012; Stephanie, 2014b). A small P-value, is stronger evidence that the sample data occur by chance and the null hypothesis should be rejected (Filliben and Heckert, 2012; Stephanie, 2014b). The P-value is the evidence against a null hypothesis (Filliben and Heckert, 2012; Stephanie, 2014b). Also, T-statistic is used in the T-test (Student's T-test) to support/reject the null hypothesis. Calculating T-statistic and P-value (probability value) provide evidence of significant differences between the results of sample data average and input data average (Filliben and Heckert, 2012; Stephanie, 2013b). The higher T is, more evidence that normalized values are significantly different from the mean (Stephanie, 2013b). A higher T-statistic is more desirable to ensure discriminative results between different normalization techniques (see section 0). Therefore, a high T-statistic value with a P-value less than 0.05, is a more reliable result with fewer odds due to chance (Filliben and Heckert, 2012; Stephanie, 2013b). Another chosen metric from this category is Standard Error which enables us to measure the deviation of the actual mean from sample/predicted mean of data sets. A lower Standard Error is a good measure to ensure choosing a normalization technique with minimum error (Filliben and Heckert, 2012; Stephanie, 2013c). Thus, STD and Regression analysis (P-value, Standard error, T-stat) are used from measure of data dispersion category in step 3- Phase 3 (Figure 8).

d) <u>Measures of proximity</u>- This category includes metrics to compute the proximity between two objectives (see sections 3.2 and 3.2.1.1). The most well-known metrics are Manhattan, Euclidean, and Chebyshev (equations (3-5),(3-6), and (3-7)), which belong to the general distance metric called Minkowski (equation (3-4)) (Guo, 2004; Hassan *et al.*, 2014; Shih *et al.*, 2007).

e) Comparison metrics. This category includes metrics such as: Mean ks (average of Pearson correlation), RCI, and Mean Squared Error (MSE). For calculating Mean ks values, after measuring Pearson/Spearman correlation, we calculate the correlation average for each normalization technique and then rank them (Chatterjee and Chakraborty, 2014). The higher value of the Mean ks values are the better (Chatterjee and Chakraborty, 2014) (see sections 3.1 and 3.1.1.1). Moreover, the RCI is computed by using the total number of times that these normalizations have similarity or dissimilarity in the ranking of alternatives in decision problems (Chakraborty and Yeh, 2009) (see sections 3.1 and 3.1.1.1). Also, MSE is an estimator to measure the average squared difference between the actual and estimated values (Filliben and Heckert, 2012; Stephanie, 2013a). In other words, MSE shows how close the regression line is to the data set , hence smaller the better (Filliben and Heckert, 2012; Stephanie, 2013a). Summarizing, the interpretations of the above metrics are as (a) for P-value, Standard Error, and Mean squared Error (MSE), lower values are better because we want to minimize the error as much as possible; (b) for T-stat, Minkowski distances (Manhattan, Euclidean, and Chebyshev), Mean ks, RCI, and STD, higher values are better. Comparison metrics which are described above namely Mean Ks values, RCI, and MSE are implemented in Step 3- Phase 3 (Figure 8).

Step 4: In this new step we propose to apply Plurality Voting (PV), from social choice methods (d'Angelo *et al.*, 1998) to recommend the most appropriate normalization technique i.e. the one with the largest number of first order/rank.

Plurality Voting (PV) is defined mathematically by (d'Angelo *et al.*, 1998) by the equations:

$$f(a_{ij}) = \begin{cases} 1, & \text{if } a_{ij} = 1 \\ 0, & \text{otherwise} \end{cases} \tag{3-8}$$

for all i and j in the decision matrix and for each alternative j, let define Aj using formulated as follows:

$$A_j = \sum_{i=1}^{m} f(a_{ij}) \quad (3\text{-}9)$$

The obtained Aj from equation (3-9) represents the total number of times alternatives j has been selected as the first rank/order. Then social choice using plurality voting (PV) is defined with the following formula:

$$\mathrm{PV} = \max_{j}\{A_j\} \quad (3\text{-}10)$$

As equation (3-10) shows, PV determines the alternative with the highest Aj value.

The numerical example from section 3.1.1.3 will clarify the usage of PV in this thesis work. Below, we repeat Table 27 from example to facilitate readability.

Table 27: Ranking (R) of alternatives using ELECTRE method.

	Max (N1)	Max-Min (N2)	Sum (N3)	Vector (N4)	Logarithmic (N5)	Fuzzification (N6)
A1	3	2	3	3	4	1
A2	1	2	1	1	6	1
A3	1	2	1	1	6	1
A4	3	2	3	3	4	1
A5	3	2	5	3	1	1
A6	3	1	5	3	3	1
A7	7	7	5	7	1	1

Based on equation (3-8), $f(a_{ij})$ is calculated as (for all i and j):

A1: $f(a_{11}) =0$; $f(a_{12})=0$; $f(a_{13})=0$; $f(a_{14})=0$; $f(a_{15})=0$; $f(a_{16})=1$

A2: $f(a_{21}) =1$, $f(a_{22})=0$; $f(a_{23})=1$; $f(a_{24})=1$; $f(a_{25})=0$; $f(a_{26})=1$

Similarly, $f(a_{ij})$ for A3, A4, …, and A7 is calculated. Then Aj is determined using equation (3-9) for each alternative:

A_1=1+0+0+0+0+1=2

A_2=1+0+1+1+0+1=4

And A3, A4, …, and A7 are, also, calculated:

A_3=4; A_4=1; A_5=2; A_6=2; A_7=2

Now, using Plurality voting (PV) we can select the alternative with the largest number of times being the first order/rank (equation (3-10)):

PV=max {A_1, A_2, A_3, A_4, A_5, A_6, A_7} = max {2,4,4,1,2,2,2}={4}= A_2 and A_3

As PV calculation shows, A2 and A3 are both selected as the social choice because of having the largest number of first ranking.

3.3.1 Test case studies for phase 3

In this section, we test the applicability of Phase3 steps of the evaluation framework to ensure the framework is now a good tool to recommend normalization techniques in MCDM. Likewise in all above phases' case studies, equal weights for criteria are used.

3.3.1.1 SAW Method [adapted from (Vafaei et al., 2022)]

For the Simple Additive Weighted (SAW) MCDM method, we use a numerical example with benefit criteria that contains outliers, to compare the effects of six normalization techniques (Max, Max-Min, Sum, Vector, Target-Based (both for Target-Avg and Target-Med) , Fuzzification).

The reason for using this numerical example with outliers is that in recent years, with the advent of data science and data analysing contexts (Chen *et al.*, 2012), many datasets with outliers emerged (i.e. criterion values skewness), which may greatly influence the aggregation/ranking process. Barnett and Lewis (Barnett and Lewis, 1974) defined outlier as "*an observation (or subset of observations) which appears to be inconsistent with the remainder of the data set*". Kennedy et al. (Kennedy *et al.*, 1992) stated that "*an outlier is not an "incorrect" observation but is a realization from a distribution that is in general highly skewed…. One reason for these extreme observations is that some popular variables, such as size, have skewed distributions.*" Therefore, we discuss the effect of outliers in criteria values and recommended the most suitable normalization technique for MCDM problems (using SAW method) that contain skewed criteria values.

The numerical example includes 2 criteria (C1 and C2) and 5 alternatives (A1, A2, …, A5). C1 and C2 input values include ordinal and large numbers and have as outliers small decimal numbers (0.01 and 0.3), as shown in Table 48.

Table 48: Input data and related parameters for implementing the initial eight normalization techniques.

Alternatives	C1	C2
A1	30	0.3
A2	50	3000
A3	70	6000
A4	80	8000
A5	0.01	9000
Parameters	**C1**	**C2**
Maximum	80	9000
Minimum	0.01	03
Average	46.002	6000
Median	50	5200.06
Fuzzification [a b c d]	[0 30 70 80]	[0.3 7500 9000 9000]

Phase3, Step 1 - Determine the type of input data. In Table 48 we see that criteria values are real numbers which contain two small outliers, the decimal numbers C1-0.01 and C2- 0.3.

Phase3, Step 2- Choosing the normalization techniques. The normalization techniques evaluated in this example are: Max (N1), Max-Min (N2), Sum (N3), Vector (N4), Fuzzification (N6), Target-AVG (N7a), and Target-Med (N7b) (please see Table 8). The related parameters for implementing the selected normalization techniques are shown in the bottom part of Table 48.

Since C1 and C2 include small decimal numbers (Table 48), the logarithmic normalization technique (N5) is not appropriate because it produces negative values (Vafaei et al., 2019). Therefore, we eliminated the logarithmic technique from our comparative study, but still evaluated seven normalization techniques because we use two versions for Target-based technique, the average and median.

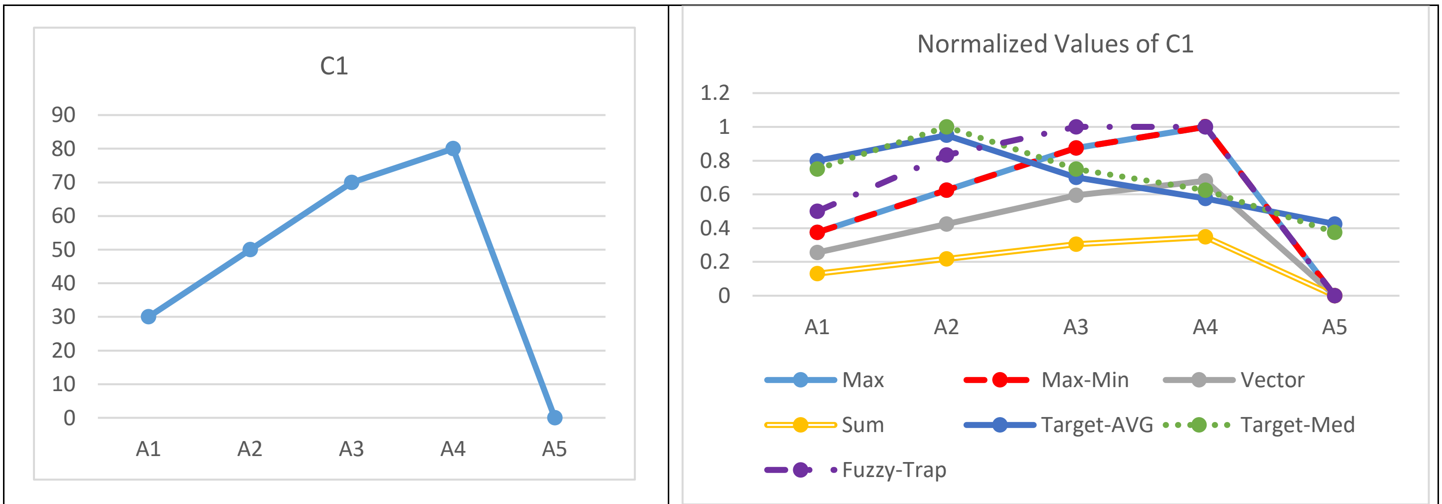
The criteria data sets and their respective normalized values are shown in Table 49.

Table 49: Normalized values using seven selected normalization techniques.

	C1								C2							
	Input Data	Max	Max-Min	Vector	Sum	Target-Avg	Target-Med	Fuzzification	Input Data	Max	Max-Min	Vector	Sum	Target-Avg	Target-Med	Fuzzification
A1	30	0.375	0.37492	0.25510	0.13043	0.79995	0.74997	0.5	0.3	3.33333E-05	0	2.17643E-05	1.15383E-05	0.42223	0.33331	0
A2	50	0.625	0.62495	0.42517	0.21738	0.95002	1	0.833	3000	0.33333	0.33331	0.21764	0.11538	0.75554	0.33335	0.4
A3	70	0.875	0.87498	0.59523	0.30433	0.69999	0.74997	1	6000	0.66667	0.66666	0.43529	0.23077	0.91111	0.33339	0.8
A4	80	1	1	0.68027	0.34781	0.57497	0.62495	1	8000	0.88889	0.88889	0.58038	0.30769	0.68889	0.33341	1
A5	0.01	0.0000125	0	8.50333E-05	4.34764E-05	0.42503	0.37505	0.0002	9000	1	1	0.65293	0.34615	0.57777	0.33342	1

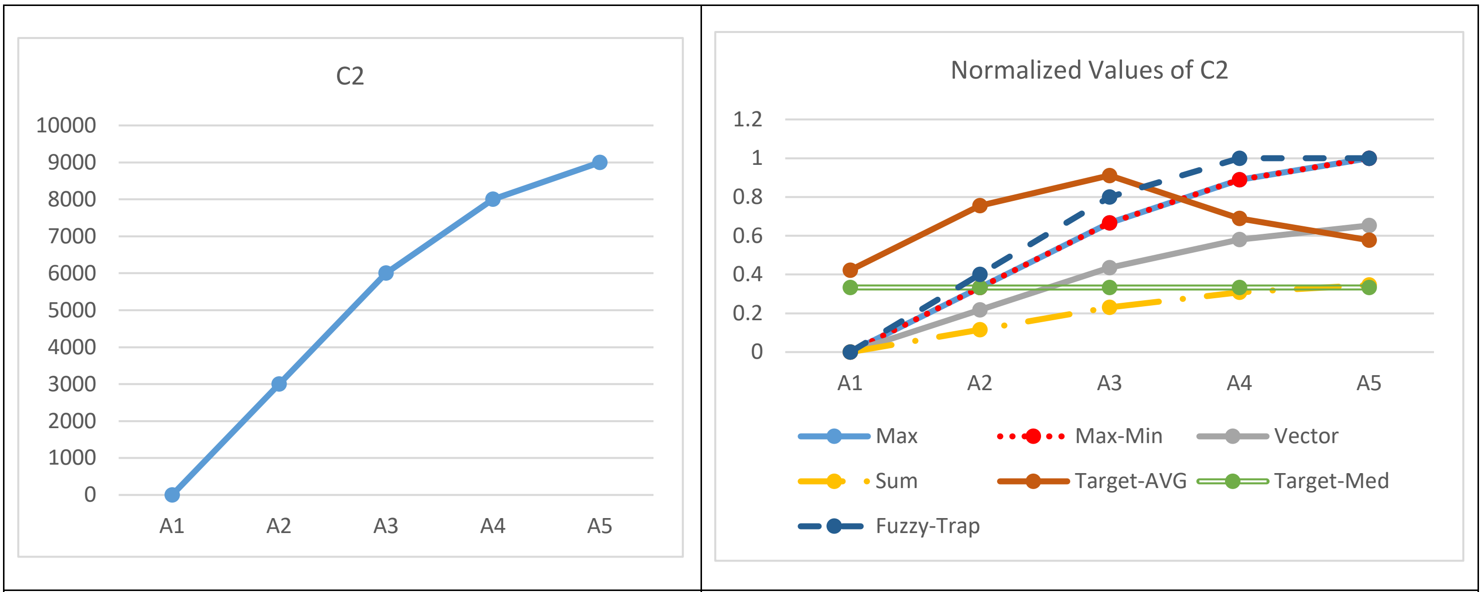
Phase 3, Step 3- for single criterion example– a) measures of location:

Figure 9a) and Figure 10a) show the input data for C1 and C2 with their decimal outliers for A5 (0.01) and A1 (0.3) respectively. Figure 9b) and Figure 10b) show the normalized values where Max and Max-Min produced almost identical normalized results (i.e., they are on top of each other in the figures) for both criteria. Target-Avg and Target-Med produced quite different normalized values from the input values of C1 and C2. For instance, A4 is classified as the best alternative for C1 with Max, Max-Min, Vector, Sum, and Fuzzification while Target-Avg and Target-Med show A2 as the best. So far, it is not possible to recommend the best normalization technique, then we proceed to step 3, assessing each data set individually and then we assess their aggregated value for the SAW MCDM method.



a) Input data of C1 | b) Normalized values of C1

Figure 9: Visual behavior of C1: input data and its normalized values



a) Input data of C2	b) Normalized values of C2

Figure 10: Visual behavior of C2: input data and its normalized values.

For measures of location, we use Maximum and Minimum and Table 50 shows the maximum of C1 using Max, Max-Min, Vector, and Sum is A4, while Target-Avg, Target-Med, and Fuzzification have different Maximum(s). As Table 51 represents Max, Max-Min, Vector, Sum, and Target-Med the best alternative for C2 is A5. Target-Avg chooses A3 as the best alternative and Fuzzification considers both A4 and A5 as the best alternatives (Table 51). For the worst alternatives, the results are similar for each normalization technique (Table 50 and Table 51).

Table 50: Measures of location with Maximum and Minimum for C1

	Input Data	Max	Max-Min	Sum	Vector	Fuzzification	Target-Avg	Target-Med
Maximum	80	1	1	0.34781	0.68027	1	0.95002	1
Related Alternatives	A4	A4	A4	A4	A4	A3,A4	A2	A2
Minimum	0.01	0.000125	0	4.34764E-05	8.50333E-05	0.0002	0.42503	0.37505
Related Alternatives	A5	A5	A5	A5	A5	A5	A5	A5

Table 51: Measures of location with Maximum and Minimum for C2

	Input Data	Max	Max-Min	Sum	Vector	Fuzzification	Target-Avg	Target-Median
Maximum	9000	1	1	0.65293	0.34615	1	0.91111	0.33342
Related Alternatives	A5	A5	A5	A5	A5	A4,A5	A3	A5
Minimum	0.3	3.33333E-05	0	2.17643E-05	1.15383E-05	0	0.42223	0.33331
Related Alternatives	A1	A1	A1	A1	A1	A1	A1	A1

Regarding measures of data dispersion of assessment framework (Figure 8), we calculated the STD, as well as P-value, Standard error, T-stat from the regression analysis. For measures of proximity, we calculate Minkowski distances (Manhattan, Euclidean, and Chebyshev). Also, we determine Mean Ks values (from Pearson Correlation) and Mean squared error (MSE) for comparison metrics. Notice that, we start by evaluating just C1 criterion, hence RCI could not be calculated. Table 52 and Table 53 depict the results.

Table 52: Phase 3, Step 3 evaluation for C1

	Regression Analysis				**Minkowski distances**			**Pearson correlation**	**Standard deviation**
	P-values	Standard Error	T-stat	MSE	Manhattan	Euclidean	Chebyshev	Mean ks	STD
Max	1.628E-48	7.2316E-15	1.10626E+16	2866.959	4.9995	1.7939	0.9999	0.7931	0.4011
Max-Min	1.124E-47	1.37668E-14	5.81034E+15	2866.961	5.0001	1.7941	1	0.7931	0.4012
Sum	8.048E-49	1.64373E-14	1.39932E+16	2914.491	1.7389	0.6239	0.3478	0.7931	0.1395
Vector	6.438E-48	1.68084E-14	6.99658E+15	2890.213	3.4010	1.2203	0.6802	0.7931	0.2729
Fuzzification	0.0040	9.1448	8.0768	2857.928	4.9992	1.8998	0.9998	0.8406	0.4248
Target-Avg	0.6231	87.4693	0.5460	2873.908	2.5499	0.9034	0.5250	0.4436	0.2020
Target-Med	0.4131	71.5776	0.9480	2870.532	2.7498	1.0154	0.6250	0.5903	0.2271

Table 53: Phase 3, Step 3 evaluation for C2

	Regression Analysis				**Minkowski distances**			**Pearson correlation**	**Standard deviation**
	P-values	Standard Error	T-stat	MSE	Manhattan	Euclidean	Chebyshev	Mean ks	STD
Max	1.10007E-48	7.13771E-13	1.26091E+16	37991556.04	5.1110	1.8392	0.9999	0.8926	0.4112
Max-Min	1.10007E-48	7.13748E-13	1.26091E+16	37991556.11	5.1111	1.8392	1	0.8926	0.4113
Sum	2.22639E-48	2.60828E-12	9.96835E+15	37997077.03	1.7692	0.6366	0.3461	0.8926	0.1424
Vector	9.46879E-48	2.24036E-12	6.1526E+15	37994486.6	3.3371	1.2008	0.6529	0.8926	0.2685
Fuzzification	0.00107	669.3540	12.6349	37990800.58	5.2	1.9391	1	0.9040	0.4336
Target-Avg	0.5457	10801.18729	0.67914381	37992622.71	2.3111	0.8237	0.4889	0.3826	0.1842
Target-Med	5.21947E-38	2.32553E-05	3.48296E+12	37996532.72	0.0006	0.0002	0.0001	0.8926	0

Table 54 and Table 55 show the results obtained for the metrics used and their interpretation is as follows: (a) since P-value, Standard Error, and Mean squared Error (MSE) present low values are the better (minimization); (b) for T-stat, Minkowski distances (Manhattan, Euclidean, and Chebyshev), Mean ks, and STD, since higher values are better. So, the used metrics represent the best normalization techniques for C1 as Sum is shown by P-value and T-test; Max is represented by Standard error; Max-Min is demonstrated by Manhattan and Chebyshev; Fuzzification is addressed by MSE, Euclidean, Mean ks, and STD. Moreover, the best techniques for C2 are as Max is presented by P-value; Max-Min is addressed by P-value, Standard error, T-test, and Chebyshev; Fuzzification is presented by MSE, Manhattan, Euclidean, Chebyshev, Mean ks, and STD.

From this Step 3 the assessment for C1 and C2, it is not possible to say which normalization technique is better for skewed data sets with outliers, because with some measures some techniques are better and with others they are worse. Therefore, we need to perform Step 4 of Phase 3, application of Plurality Voting (PV), to select the alternative which has the large number of the first order/rank (d'Angelo *et al.*, 1998).

Phase 3, Step 4- for single criterion example - Plurality Voting for single C1 and C2

Based on the results of applying plurality voting for the combined voting on C1 and C2, Fuzzification gets the highest score (Table 54 and Table 55) and should be recommended as the best normalization technique for both input data. The second best technique for the combined criteria is Max-Min. It should be noted that if we observe each criterion, individually, for C1 there is a draw between the second best techniques Max-Min, Vector and Sum, while for C2 Max-Min is clearly the second best technique. Further, we can also conclude that Target-Avg and Target-Med are the worst techniques for dealing with outliers. To illustrate how we obtained the recommended normalization technique, fuzzification for C1, we performed the following plurality voting calculation:

For Max: $f(a_{Max,1})=0$, $f(a_{Max,2})=1$; $f(a_{Max,3})=0$; $f(a_{Max,4})=0$; $f(a_{Max,5})=0$; $f(a_{Max,6})=0$; $f(a_{Max,7})=0$; $f(a_{Max,8})=0$; $f(a_{Max,9})=0$

For Fuzzification: $f(a_{Fuz,1})=0$, $f(a_{Fuz,2})=0$; $f(a_{Fuz,3})=0$; $f(a_{Fuz,4})=1$; $f(a_{Fuz,5})=0$; $f(a_{Fuz,6})=1$; $f(a_{Fuz,7})=0$; $f(a_{Fuz,8})=1$; $f(a_{Fuz,9})=1$

Similarly, $f(a_{ij})$ for Max-Min, Vector, Sum, Target-AVG, and Target-Med are determined. Then Aj is determined using equation (3-9) for each normalization techniques:

A_{Max}=0+1+0+0+0+0+0+0+0=1

A_{Fuzz}=0+0+0+1+0+1+0+1+1=4

Similarly, Aj is calculated for Max-Min, Vector, Sum, Target-AVG, and Target-Med:

$A_{Max-Min}$=2; A_{Vector}=2; A_{Sum}=2; $A_{Tar-AVG}$=0; $A_{Tar-Med}$=0

Now, Plurality voting (PV) selects the normalization technique which has the largest number of times being the first order/rank respect to the applied metrics (using equation (3-10)):

PV=max {$A_{Max,}$ $A_{Max-Min,}$ A_{Vector}, A_{Sum}, $A_{Tar-AVG}$, $A_{Tar-Med}$, A_{Fuzz}}= max {1,2,2,2,0,0,4}= A_{Fuzz}

As PV calculation shows, A_{Fuzz} is selected as the social choice because of having the largest number of first ranking regarding the used metrics.

Table 54: Phase 3, Step 4 - plurality voting for C1

	P-values ↓	Standard Error ↓	T-stat ↑	MSE ↓	Manhattan ↑	Euclidean ↑	Chebyshev ↑	Meanks ↑	STD ↑	PV
Max	2	1	2	2	2	3	2	2	3	1
Max-Min	4	2	4	3	1	2	1	2	2	2
Sum	1	3	1	7	7	7	7	2	7	2
Vector	3	4	3	6	4	4	4	2	4	2
Fuzzification	5	5	5	1	3	1	3	1	1	4
Target-Avg	7	7	7	5	6	6	6	7	6	0
Target-Med	6	6	6	4	5	5	5	6	5	0

Table 55: Phase 3, Step 4 **-** plurality voting for C2

	P-values ↓	Standard Error ↓	T-stat ↑	MSE ↓	Manhattan ↑	Euclidean ↑	Chebyshev ↑	Meanks ↑	STD ↑	PV
Max	1	2	2	2	3	3	3	5	3	1
Max-Min	1	1	1	3	2	2	1	3	2	4
Sum	3	4	3	7	6	6	6	3	6	0
Vector	4	3	4	5	4	4	4	5	4	0
Fuzzification	6	6	6	1	1	1	1	1	1	6
Target-Avg	7	7	7	4	5	5	5	7	5	0
Target-Med	5	5	5	6	7	7	7	2	7	0

Summarizing, the Fuzzification normalization technique (N7) seems the best normalization technique for this illustrative example when assessing each criterion separately and using a plurality voting scoring (Table 54 and Table 55).

So far, we only analysed each criterion normalization behaviour individually, but there is a need to consider their aggregation to obtain the best ranking for the SAW MCDM method. The average aggregation scores and rankings of alternatives are shown in Table 56.

Table 56: Final score and rank of alternatives using SAW with the seven normalization techniques

	Max		**Max-Min**		**Vector**		**Sum**		**Target-AVG**		**Target-Med**		**Fuzzy-Trap**	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
A1	0.1875	5	0.1875	5	0.1276	5	0.0652	5	0.6111	4	0.5416	3	0.25	5
A2	0.4792	4	0.4791	4	0.3214	4	0.1664	4	0.8528	1	0.6667	1	0.6165	3
A3	0.7708	2	0.7708	2	0.5153	2	0.2676	2	0.8056	2	0.5417	2	0.9	2
A4	0.9444	1	0.9444	1	0.6303	1	0.3277	1	0.6319	3	0.4792	4	1	1
A5	0.5001	3	0.5000	3	0.3265	3	0.1731	3	0.5014	5	0.3542	5	0.5001	4

Similarly, by just looking at the results in Table 56, it is impossible to select the best normalization technique. However, as mentioned above, some metrics are just usable per criterion at a time, for instance, regression analysis cannot be used for assessing aggregated data. Therefore, we did not apply all metrics from Step 3 of our framework to evaluate the aggregation/ranking process with MCDM (SAW method) and recommend the best normalization technique for the illustrative example.

It is noticeable that the calculation procedure for MSE of aggregated data is different from MSE for a single criterion. For a single criterion, we have the binary comparison between input data and normalized values, however, MSE for aggregated data set is the average of mean squared error for each normalization technique with other normalization techniques using the ranking of alternatives (Aires and Ferreira, 2019; Felinto de Farias Aires *et al.*, 2018). The results of MSE are depicted in Table 57.

Table 57: MSE for SAW aggregated data set

	Max	Max-Min	Vector	Sum	Target-AVG	Target-Med	Fuzzy-Trap	MSE	Rank
Max		0	0	0	3.6	5.2	0.4	1.5333	2
Max-Min	0		0	0	3.6	5.2	0.4	1.5333	2
Vector	0	0		0	3.6	5.2	0.4	1.5333	2
Sum	0	0	0		3.6	5.2	0.4	1.5333	2
Target-AVG	3.6	3.6	3.6	3.6		0.4	2	2.8	6
Target-Med	5.2	5.2	5.2	5.2	0.4		3.6	4.1333	7
Fuzzy-Trap	0.4	0.4	0.4	0.4	2	3.6		1.2	1

The results from the usable Step 3 metrics with MCDM (SAW method) and recommend the best normalization technique for the illustrative example are shown in Table 58.

Table 58: Phase 3, Step 3 - Results of applied metrics for SAW aggregation method

	Manhattan	Euclidean	Chebyshev	STD	Mean Ks	RCI	MSE
Max	3.6110	1.3034	0.7569	0.29154	0.6673	45.6667	1.5333
Max-Min	3.6113	1.3035	0.7570	0.2915	0.6673	45.6667	1.5333
Sum	1.2525	0.4522	0.2625	0.1011	0.6680	45.6667	1.5333
Vector	2.3988	0.8686	0.5028	0.1942	0.6728	45.6667	1.5333
Fuzzification	3.7998	1.3575	0.7500	0.3035	0.7181	41.6667	1.2
Target-Avg	1.7944	0.6503	0.3514	0.1454	0.3283	31.3333	2.8
Target-Med	1.3747	0.5076	0.3124	0.1135	0.0480	31.3333	4.1333

Phase 3, Step 4 – Plurality Voting for SAW method example

Table 59 shows the final ordering of the selected normalization techniques and using the plurality voting (PV) process. Table 59 reveals that Fuzzification is (again) the best technique for this example while Target-Avg and Target-Med are the worst techniques. These results show that both single criterion evaluation and the SAW evaluation agree on recommending the best and worst normalization techniques when in presence of skewness data with outliers. Also, Max-Min is the second best technique (Table 59).

Table 59: Phase 4, Step 4- Plurality voting for SAW aggregated data set

	Manhattan ↑	Euclid-ean ↑	Cheby-shev ↑	STD ↑	Mean Ks ↑	RCI ↑	MSE ↓	PV
Max	3	3	2	3	5	1	2	1
Max-Min	2	2	1	2	4	1	2	2
Sum	7	7	7	7	3	1	2	1
Vector	4	4	4	4	2	1	2	1
Fuzzifica-tion	1	1	3	1	1	5	1	5
Target-Avg	5	5	5	5	6	6	6	0
Target-Med	6	6	6	6	7	6	7	0

It should also be noticed that although Fuzzification is the recommend technique it has the drawback of requiring the definition of an appropriate membership function by analysts or experts for each criterion (Ribeiro, 1996), which is a cumbersome manual process.

3.4 Phase 4 of Framework Evolution (Final Framework)

Analysing the results of phase 3 made us think about simplifying the assessment framework and make it more user-friendly for decision makers dealing with common decision problems such as supplier selection, partner selection, etc. For instance, the existence of outliers in those kinds of decision problems is very rare. So, we performed some improvements on Step 2 and Step 3, to achieve a final version of a useful tool to assist decision makers and recommend the best normalization technique for MCDM methods problems, such as:

Phase 4, Step 2: We eliminated Target based normalization technique (N7) from our chosen normalization techniques (Table 8) because it needs human interference for defining the target values. Further, as discussed in the SAW case study, the use of Average and Median as target values did not lead to accurate results.

Phase 4, Step 3:

a) Outlier detection: To the best of our knowledge from the literature the presence of outliers in common MCDM problems is very rare. Thus, to make the assessment framework simpler and more user-friendly for decision makers, metrics related to the outlier detection are deleted.

b) Measure of location: Calculation of the maximum and minimum of input data and comparing them with the maximum and minimum of the normalized values proved not to have direct effects on the numerical results of the framework (like RCI, Mean Ks, etc.). They just analyse the effects of normalization techniques on the input data and just indicate us to predict the type of normalization techniques (linear or non-linear). As mentioned in section 3.3.1 the linear normalization techniques do not change the dominancy of alternatives with respect to each criterion while the non-linear techniques change the dominancy. So, these metrics are also eliminated from the final assessment framework to avoid producing irrelevant information.

c) Measures of data dispersion: We removed ANOVA/ Regression Analysis because related metrics such as P-value, T-stat, and Standard Error are ONLY usable for single data sets, i.e. single criterion, while the objective of this thesis is recommending the best normalization technique for aggregated data sets, basis of any MCDM method. So, only STD is chosen for this category of metrics because it is usable for aggregated data sets.

d) Measure of proximity: In this category of metrics, by observing the obtained results from Table 41, Table 43, Table 45, Table 47, Table 55, and Table 59 the ordering of the normalization techniques using Euclidean and Manhattan are identical. Since there is no difference between using Euclidean and Manhattan and to avoid repetitive information, Euclidean is selected to be used in the final framework. Also, comparing Chebyshev with Euclidean, the results for case studies (Table 43, Table 45, and Table 55) are identical and only tiny differences were found for those cases (Table 41, Table 47, and Table 59). Since, differences between Euclidean and Chebyshev distances are very small in the ordering of normalization techniques, and identical to Manhatten and Euclidean we decided just to keep Euclidean distance. Further, Euclidean distance is more intuitive and well-known than Chebyshev and Manhattan for decision makers and most of the programming tools (e.g. MATLAB) have codes for this metric which cause easy usage for the users. So, removing Manhattan and Chebyshev distances from the evaluation framework avoids repetitive results and makes it quicker and user friendly for decision makers.

Based on the above explanation, the final proposed assessment framework (Phase 4) is depicted in Figure 11.

Step 1: Data types

Determine the type of criteria values:

- Benefit or cost criteria
- Ordinal number
- Natural number
- Skewed data with outlier
- Real number
  - Float numbers in the unit interval [0-1]

Step 2: Selection of Normalization Techniques

Candidate well-known normalization techniques:

- **Linear:** Max; Max-Min; Sum
- **Semi Linear:** Vector
- **Non Linear:** Logarithmic, Fuzzification

Data preparation:
(1) Sum is not applicable if there is any zero in criteria values (using cost formula)
(2) Logarithmic is not applicable when there are zero in any criterion

Step 3: Evaluation of Techniques

Chosen metrics:

a) **Measures of data dispersion:** STD
b) **Measure of proximity:** Minkowski distances (Euclidean)
c) **Comparison metrics:** Mean Ks values (from Pearson Correlation); Ranking Consistency Index (RCI); Mean squared error (MSE)

Step 4: Selection of the Best Technique

Selection of the best technique with:

- **Plurality voting:** Selection of best normalization technique with the large number of first order/rank, in the different used metrics

Figure 11: Phase 4 conceptual model of the final assessment framework

Next, we are going to test the applicability of the final evaluation framework and try to find the probable faults and errors in case studies to develop the efficiency of the framework. Again, to focus on the effects of different normalization techniques on MCDM methods we use equal weights for criteria in all evaluated case studies.

3.4.1 Test case studies for phase 4

3.4.1.1 **SAW Method** (adapted from section 3.2.1.1)

The purpose of this test is to recommend the best normalization technique for SAW method among four normalization techniques (Max, Max-Min, Vector, and Fuzzification). This test uses the case study borrowed from 3.2.1.1. Regarding Step 1 & 2 (Phase 3) and observing Table 33, the results are identical for Step 1 and 2 of Phase 4. After the aggregation process using the SAW method (Table 35) we proceed with the evaluation framework applying metrics of the new Step 3 (Phase 4). The results of RCI, Mean ks, STD, and Euclidean are borrowed from section 3.2.1.1. Also, mean squared error is calculated using the ranking of alternatives and the results of MSE are shown in Table 60.

Table 60: MSE for each normalization technique using ranking of alternatives.

	Max	Max-Min	Vector	Fuzzification	MSE	Rank
Max (N1)		0.666667	0.333333	0.666667	0.555556	1
Max-Min (N2)	0.666667		1	0	0.555556	1
Vector (N4)	0.333333	1		1	0.777778	4
Fuzzification (N6)	0.666667	0	1		0.555556	1

Table 61 summarizes the ordering of applied metrics (MSE is calculated above from Table 60 and the rest of the metrics are borrowed from Table 41) for each normalization technique.

Phase 4, Step 4- Plurality Voting for the SAW method

Utilizing plurality voting (PV) for the final ordering of the assessed normalization techniques to recommend the best technique for this case study produces the results shown in Table 61, which reveal that Max-Min and Fuzzification normalization techniques are both the best (highest rank), while the worst is the Vector normalization technique.

Table 61: Phase 4, Step 4- Plurality Voting ranking for SAW method

	RCI↑	Mean ks↑	STD↑	MSE↓	Euclidean↑	PV↑
Max (N1)	1	1	3	1	3	3
Max-Min (N2)	1	1	1	1	1	4
Vector (N4)	4	4	4	4	4	0
Fuzzification (N6)	1	1	1	1	1	4

It should also be noticed that although Fuzzification is one of the recommend techniques, it has the drawback of requiring definition of an appropriate membership function by analysts or experts for each criterion (Ribeiro, 1996), which is a cumbersome manual process.

3.4.1.2 **TOPSIS Method** (adapted from section 3.1.1.1)

In this section, we discuss the suitability of six normalization techniques (see Table 8) with the TOPSIS method for the case study borrowed from section 3.1.1.1.

In this case study, we implemented the assessment framework of phase 4 (Figure 11). The results of Step 3 for the related metrics (RCI, Mean ks, STD, and Euclidean) are borrowed from section 3.2.1.2 (Table 43). Also, MSE is calculated for this case study using the ranking of alternatives. The summary of Step 3 for the six chosen normalization techniques with respect to the applied metrics is shown in Table 62. After applying Step 4, plurality voting (PV), we see that Vector normalization is the best technique for TOPSIS in this case study (Table 62). Moreover, Max, Sum, and Fuzzification are the second best and Max-Min and Logarithmic are the worst techniques.

Table 62: Phase 4 (Steps 3 and 4) Summary of evaluation framework for TOPSIS method

	RCI↑	Mean ks↑	STD↑	MSE↓	Euclidean↑	PV↑
Max (N1)	2	1	3	3	1	2
Max-Min (N2)	4	5	5	5	5	0
Sum (N3)	3	1	2	2	1	2
Vector (N4)	1	1	4	4	1	3
Logarithmic (N5)	6	6	6	6	6	0
Fuzzification (N6)	5	4	1	1	4	2

3.4.1.3 AHP Method (adapted from (Vafaei et al., 2020) and (Vafaei et al., 2016a) and section 3.1.1.2)

This case study is borrowed from section 3.1.1.2 and evaluates the suitability of the chosen six normalization techniques (see Table 8) with the AHP method. Applying Steps 1 & 2 of Phase 4 and borrowing results from section 3.1.1.2, we eliminate Logarithmic and Fuzzification from the candidate techniques and continue the evaluation with only Max, Max-Min, Sum, and Vector normalization techniques. For step 3, we borrowed results from section 3.2.1.3 (Table 45) related to the RCI, Mean ks, STD, and Euclidean, and also calculated MSE for this case study (Table 63). Finally, Step4-plurality voting (PV), is calculated to recommend the best technique. As Table 63 shows, Max-Min is the more suitable technique, Max normalization is the second best, while Sum and Vector are the worst techniques for the AHP method in this case study. Although Max-Min is elected as the most suitable normalization technique, it requires an extra re-normalization because the sum of the normalized values has to be 1 in the AHP method (for more information about AHP method please see Section 2.1.4.

Table 63: Phase 4. Step3 and 4 Summary of evaluation framework for AHP method

	RCI↑	Mean ks↑	STD↑	MSE↓	Euclidean↑	PV↑
Max (N1)	**1**	**1**	**3**	**1**	**3**	**3**
Max-Min (N2)	**1**	**2**	**1**	**1**	**1**	**4**
Sum (N3)	**4**	**4**	**2**	**4**	**2**	**0**
Vector (N4)	**3**	**3**	**4**	**3**	**4**	**0**

3.4.1.4 ELECTRE Method (adapted from section 3.1.1.3)

This numerical example is adapted from section 3.1.1.2 to find the best normalization technique for the ELECTRE method using the assessment framework of phase 4 (Figure 11).

The results of Step 3 for the related metrics (RCI, Mean ks, STD, and Euclidean) are borrowed from section 3.2.1.2 (Table 47). Also, MSE is calculated for this case study using the ranking of alternatives. The results of Step 3 are summarized in Table 64. After utilizing plurality voting (PV), Step4, we observe that Max is the best technique for the ELECTRE method in this case study (Table 64). Moreover, Vector and Logarithmic are the second best, while Max-Min, Sum, and Fuzzification are the worst techniques.

Table 64: Phase 4 Step 3 and 4. Summary of evaluation framework for ELECTRE method

	RCI↑	Mean ks↑	STD↑	MSE↓	Euclidean↑	PV↑
Max	1	1	2	1	2	3
Max-Min	6	3	4	3	4	0
Sum	3	4	5	4	5	0
Vector	1	2	2	1	2	2
Logarithmic	5	6	1	6	1	2
Fuzzification	4	5	6	5	6	0

3.5 Implementation Design of Phase 4 Conceptual model

This section discusses the implementation design of the conceptual model for the final assessment framework (Figure 11). This implementation process, presented as a flowchart (Figure 12) provides relevant information for decision makers to select the best normalization technique using different case studies. The purpose of designing this model is to introduce an automatic and applicable framework to decision makers and help them select the best normalization technique for decision problems, in a user-friendly manner. For simplicity purposes, those MCDM methods and normalization techniques which depend on human/expert intervention are not shown in this model. For instance, the AHP MCDM method is removed because it needs expert's intervention to determine comparison matrices and when there are more than 7-9 criteria and/or alternatives it is too cumbersome to be used. From the normalization techniques, Fuzzification has the drawback of requiring the definition of appropriate membership functions to represent the input criteria, by analysts or experts (Ribeiro, 1996), which is a cumbersome manual process. Therefore, to have a fully automatic evaluation process using the conceptual framework, the AHP method and Fuzzification normalization technique are removed from the implemented model. Figure 12 depicts the decision-making process that the decision makers should follow.

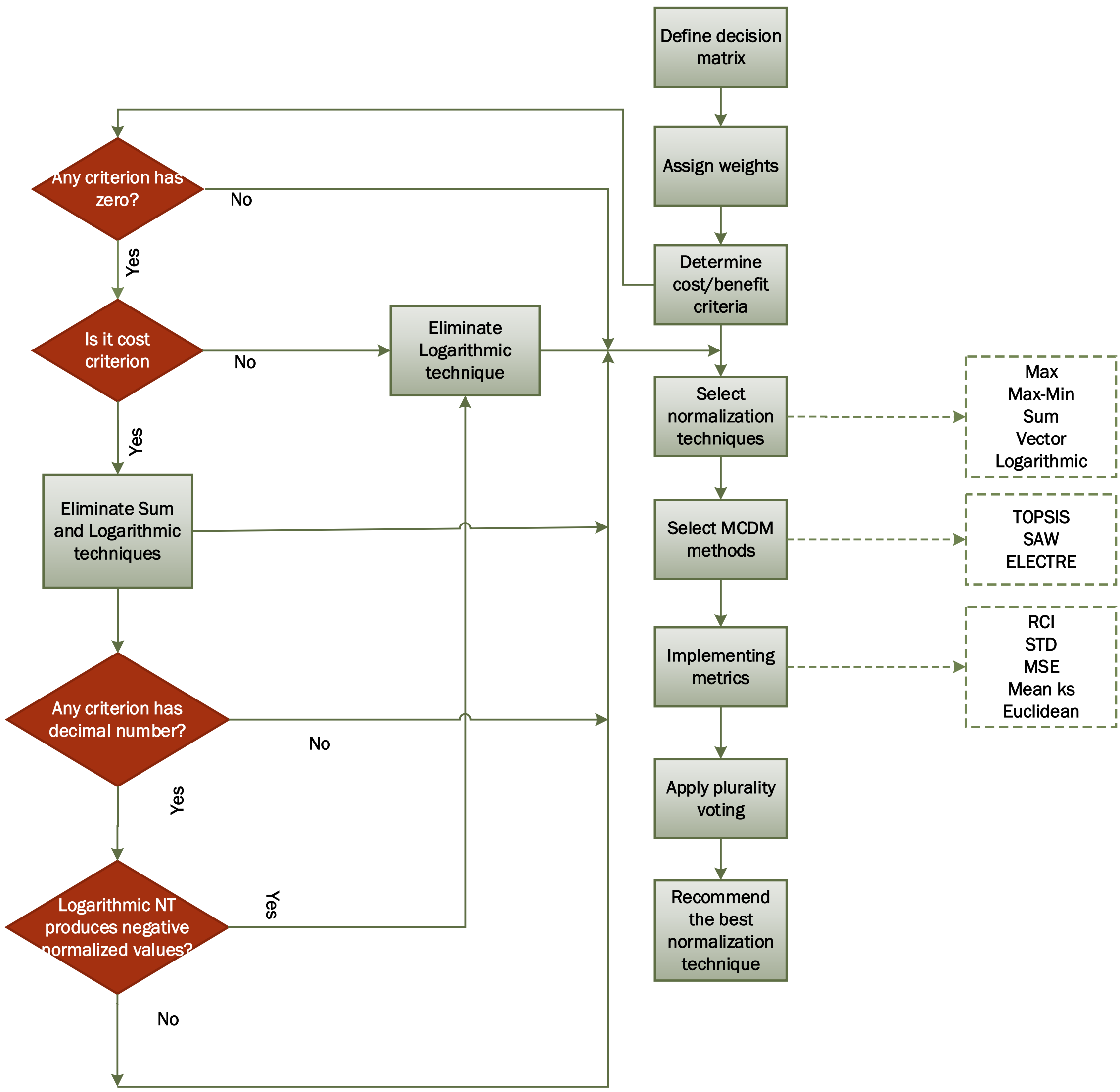


Figure 12: Design model for phase 4 of assessment framework.

Initially, the decision maker defines the decision matrix which contains the values of alternatives with respect to the desired criteria. For each criterion, he/she should assign weights and indicate cost (the lower values the better) or benefit (the higher values the better) criteria. Then it must be chosen which normalization techniques could be used in the decision problem, paying attention to the existence of zero values in the decision matrix, which causes the elimination of the Logarithmic and Sum techniques. Also, the existence of decimal numbers causes the elimination of logarithmic technique if it produces negative normalized values. Then decision maker selects which MCDM methods he/she wishes to use among TOPSIS, SAW, and ELECTRE methods for aggregation/ ranking alternatives. Finally, the design model proceeds to the metrics calculation of the assessment framework: RCI, STD, MSE, Mean ks, and Euclidean. In the end, the process will recommend the best normalization technique to the decision maker using plurality voting for the related decision problem.

MATLAB software is used for implementing this design model. The reasoning for selecting MATLAB was its ability for handling mathematical models using matrixes. The validation of the design model is discussed in the next chapter.

3.6 Summary

This chapter focused on contributions and main findings for this thesis work. It is divided into four subsections about the framework development evolution, phase 1, phase 2, phase 3 and phase 4 to select the best normalization technique in MCDM methods. Finally, another sub-section is added with the implementation design model of the framework

In the first subsection, Phase 1 of the assessment framework three steps are proposed and explained:

Phase1-Step a) Determining the Ranking Consistency Index (RCI) (from (Chakraborty and Yeh, 2009)):

Phase1-Step b) Comparative study between ranking of alternative/alternatives' values using Pearson and Spearman correlations to determine the mean ks value (Chatterjee and Chakraborty, 2014).

Phase1-Step c) Analysis and evaluation of normalization techniques consistency with three conditions (borrowed from (Celen, 2014)):

***Condition 1**:* Checking similarity of distributional properties such as means, standard deviations, minimum and maximum values.

***Condition 2**:* Checking normal distribution using Kolmogorov-Smirnov test.

***Condition 3**:* Checking the similarity ranking of alternatives by comparison of best and worst ranking of three results/alternatives.

The performance of the proposed framework for this Phase 1 is evaluated with three case studies. The contribution of this part is supported by the following publications: Vafaei et al. (2016a, 2016b, 2018a, 2018b).

Next, to improve the robustness of the assessment framework, the above steps (phase 1) were modified and some new steps were added. So, phase 2 of the evaluation framework includes the following four steps:

Phase2-Step a) Determining the Ranking Consistency Index (RCI) (Chakraborty and Yeh, 2009).

Phase2-Step b) Calculating Pearson/Spearman correlation and their mean ks values (Chatterjee and Chakraborty, 2014).

Phase2-Step c) Calculating Standard Deviation (STD) to assess the spreading out of the data set using alternatives' values (Bland and Altman, 1996; Rumsey, 2009; Yeh, 2003)

Phase2-Step d) Calculating Minkowski distances (Guo, 2004; Han *et al.*, 2012; Hassan *et al.*, 2014; Shih *et al.*, 2007) for three well-known distances as Manhattan, Euclidean, and Chebyshev.

The performance of Phase 2 evaluation framework is evaluated with three case studies. This contribution is supported by the following publications: Vafaei et al. (2016a, 2016b, 2018a, 2019).

After, to enhance the robustness of the proposed framework, Phase 3 was developed (Figure 11). The evolution from Phase 2 to Phase 3 included defining a taxonomy of metrics using some from previous phases and new ones. Further, the steps were numbered (step 1 to 4) to facilitate readability. Also, a study about outliers in data sets is included to discuss this problematic in relation to normalization techniques. A new step 4 added includes a plurality voting process to recommend the best technique. Figure 8 shows the different steps of the assessment framework in phase 3. This contribution is supported by the following publication: Vafaei et al. (2022)

Finally, to simplify the proposed framework, some improvements were performed and the final conceptual model (Figure 12) is depicted in phase 4 (Final phase). Considering that the existence of outliers in decision problems is very rare, metrics related to outliers were removed. Also, we opted by choosing only to use the Euclidean distance because results are rather similar and have the same origin as Manhattan and Chebyshev. The performance of Phase 4 framework is evaluated with three case studies. This contribution is supported by the following publications Vafaei et al. (2016a, 2016b, 2018a, 2019, 2020) Figure 13 summarizes the evolution of the assessment framework that was carried out throughout this thesis research work. It is obvious that step 3 for both Phase 3 and 4 is the only step comparable with the previous versions of the framework, i.e. step1, step2 and step 4 were not addressed in Phase 1 and 2.

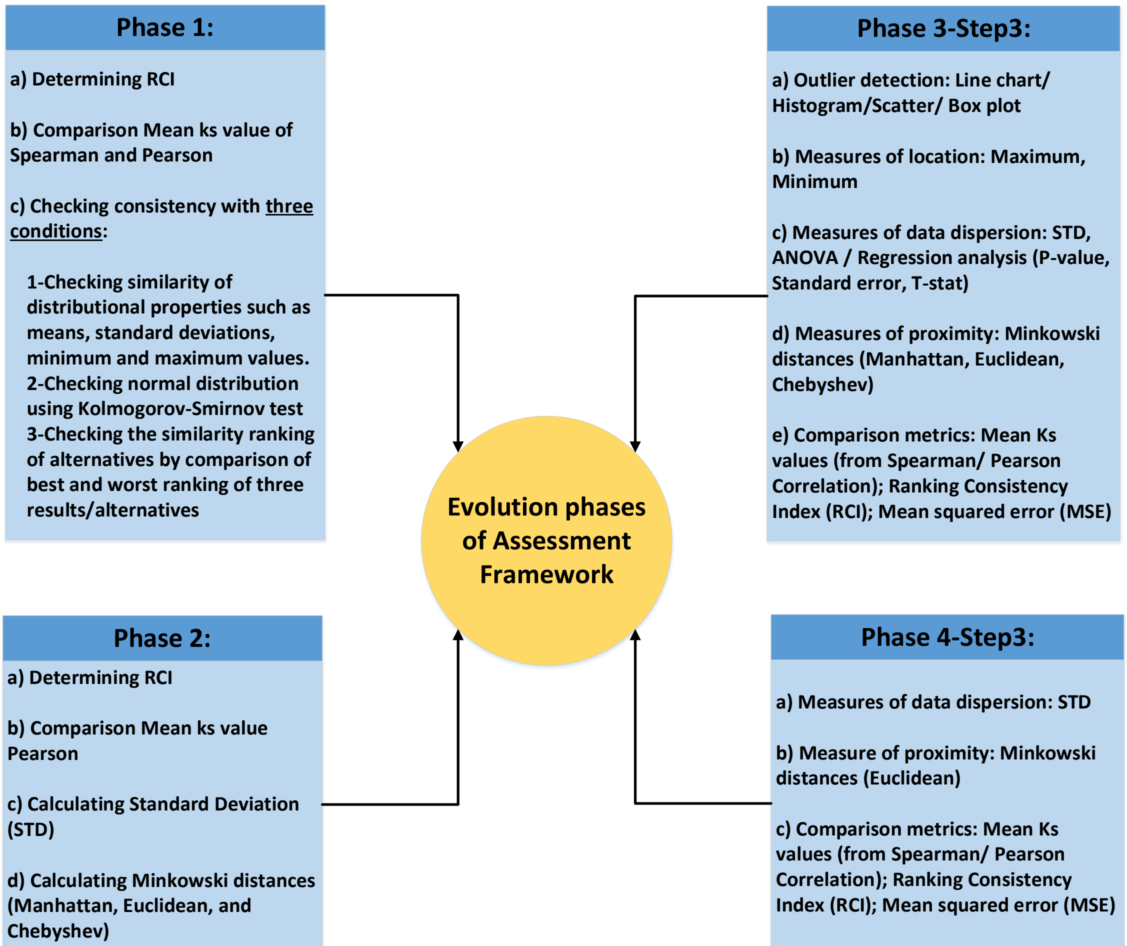


Figure 13: Evolution phases of the proposed assessment framework.

4

4 Evaluation and Validation

This chapter discusses further tests and validations performed to assess the developed evaluation framework. We tested several illustrative and representative case studies, as well as examples from the literature. The results are evaluated and validated by comparing our results with established results reported in the literature.

4.1 Validation methodology

This section presents the adopted validation process for the proposed framework to evaluate normalization techniques for usage in MCDM methods.

Many times, the validation process of any research work consists of using different methods such as case studies, prototypes, simulation, benchmarking, etc. (Camarinha-Matos, 2015; Pedersen *et al.*, 2000). Kasanen et al. (1993) proposed the Constructive Research method (CRm), which helps researchers to validate applied research, in the area of design science, by building one or more artefacts. The artefacts propose to solve a domain problem, obtaining information/knowledge on how the problem can be solved and how proposed solutions are new or better than previous ones.

In this thesis, we use the above Constructive Research method with one artefact, which is our proposed "Assessment Framework" (Table 65). Since our work is spanned over different multi criteria decision models/methods, the evaluation process needs to assess different dimensions of the generated artefact, including the applicability of the approach and its utility using case studies, benchmarks, and testimonies. Furthermore, the artefact is evaluated by the research community to ensure the robustness of the validation process. Table 65 shows the parameters and chosen items of the followed validation method, applied to our framework.

Table 65: Validation with Constructive Research method for assessment framework

Artefact (Who)	Parameter (What)	Mean (How)	Based on	Section
Assessment Framework	Applicability/Utility	Case Studies Benchmarks Testimonies	(Camarinha-Matos, 2015)	4.2
	Evaluation by the research community	Publications Presentations Panels	Evaluation by peers	4.3

4.2 Applicability and Utility Evaluation of the Framework

The parameter applicability and utility of the assessment framework is evaluated using several case studies and benchmarks from the literature. Moreover, we also collected testimonies about its usefulness from experts.

The next sub-sections describe the validations performed using the above items. The main goal is to verify if the framework's functionalities fit the research work´ objectives in terms of applicability and utility for decision makers.

4.2.1 Applicability with Case studies

We tested the proposed assessment framework with well-known MCDM methods to assess the effect of different normalization techniques on several decision problems and determine its applicability and utility. We illustrate its usability by using different scenarios that contain different input data sets.

As mentioned in section 3.5, having an automatic assessment framework for selecting normalization techniques to use with MCDM methods is the goal of this thesis. To focus the testing and validation we chose three of the most well-known MCDM methods (SAW, TOPSIS, and ELECTRE) and Max, Max-Min, Sum, Vector, and Logarithmic techniques from normalization techniques.

In the next subsections, the applicability of the proposed framework is tested using, first, 3 small scaling cases studies (Case 1, Case 2, Case 3) which have 4 criteria and 4 alternatives and, second, we performed tests on large scale decision problems using two illustrative examples: Case 4, which includes 20 alternatives and 4 criteria, and Case 5 which includes 20 criteria and 10 alternatives. The details about these cases and discussion of results obtained are in the following sub-sections.

4.2.2 MCDM problems with small scaling decision matrices: Case 1, 2 and 3

In this section, three case studies are tested. The first case (Case 1) is adapted from (Lakshmi and Venkatesan, 2014) and the other two cases (Case2 and Case3) are illustrative examples to test different types of criteria, from ordinal numbers to percentages, etc. We assessed the performance of five normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic) with three MCDM methods: TOPSIS, SAW, and ELECTRE, using three case studies including four alternatives (A1, A2, A3, A4) and four criteria (C1, C2, C3, C4), where C3 and C4 are cost criteria and C1 and C2 are benefit ones. To ensure fair comparison on the effects of different normalization techniques on MCDM methods we applied equal weights for criteria on these three case studies.

4.2.2.1 Case 1

As mentioned above, the first case study (Case 1) is adapted from (Lakshmi and Venkatesan, 2014) but instead of using a Gaussian normalization technique (proposed by the authors) we use the logarithmic normalization technique (to maintain consistency on the chosen normalization techniques for this work) together with the four other normalization techniques: Max, Max-Min, Sum, and Vector. Table 66 shows the input data for case 1 decision matrix.

Table 66: Decision matrix for case 1 [adapted from (Lakshmi and Venkatesan, 2014)].

	C1	C2	C3	C4
A1	7	9	9	8
A2	8	7	8	7
A3	9	6	8	9
A4	6	7	8	6

- **TOPSIS method**

We implemented TOPSIS method to rank the alternatives with respect to the criteria of case 1 (Table 66). Table 67 depicts alternatives' values and ranking of alternatives using tested normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 67: Alternatives' rates and ranking for Case 1 using the TOPSIS

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Logarithmic	Rank
A1	0.7401	2	0.4458	4	0.5145	2	0.5342	2	0.6208	1
A2	0.7807	1	0.6340	1	0.5294	1	0.5568	1	0.5429	2
A3	0.6667	4	0.5000	3	0.4174	4	0.4209	4	0.4853	3
A4	0.7140	3	0.5473	2	0.4780	3	0.4755	3	0.3006	4

Table 67 reveals it is impossible to select the best technique just by looking at the results because each technique ranking is quite different from the other. Thus, there is a need for using the assessment framework to recommend the best normalization technique for TOPSIS.

Table 68: Results from framework´ Step 3 & 4 for TOPSIS in Case 1

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.1655	5	0.0478	5	5.75	1	0.625	1	0.7130	1	3
Max-Min	0.2763	2	0.0798	2	3	4	2	5	0.2366	5	0
Sum	0.1727	4	0.0499	4	5.75	1	0.625	1	0.6708	3	2
Vector	0.2118	3	0.0611	3	5.75	1	0.625	1	0.7017	2	2
Logarithmic	0.4722	1	0.1363	1	0.25	5	1.625	4	0.2480	4	2

After using the framework step 3 metrics and plurality voting of step 4 we now can say (Table 73) that the best normalization technique for case 1 with TOPSIS method is the Max normalization technique, because it includes the higher counting of the first rank on the used metrics.

- **SAW method**

Another well-known MCDM method to validate the framework is SAW method. We implemented SAW for Case 1 (Table 66) and ranked alternatives. Table 69 shows the results.

Table 69: Alternatives' rates, and ranking for Case 1 using SAW

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Logarithmic	Rank
A1	0.4722	2	0.4167	4	0.2503	3	0.5008	2	0.2548	1
A2	0.5000	1	0.6667	1	0.2568	1	0.5148	1	0.2535	2
A3	0.4444	4	0.5000	3	0.2419	4	0.4813	4	0.2498	3
A4	0.4722	2	0.5833	2	0.2510	2	0.4983	3	0.2460	4

Likewise, for TOPSIS, Table 69 shows that using the assessment framework is not able to recommend the best normalization technique for the SAW method. So, applying the framework metrics of step 3 and plurality voting of step 4, the recommendation is that both Max and Max-Min normalization techniques are suitable for Case 1 with the SAW method (Table 70).

Table 70: Results of Step 3 & 4 of the framework for SAW in Case 1

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.0786	2	0.0227	2	5	1	0.875	2	0.7503	1	2
Max-Min	0.3727	1	0.1076	1	4	3	1.4375	4	0.3832	4	2
Sum	0.0212	4	0.0061	4	4.75	2	0.9375	3	0.7243	3	0
Vector	0.0477	3	0.0138	3	4	3	0.5625	1	0.7441	2	1
Logarithmic	0.0137	5	0.0040	5	0.25	5	2.1875	5	0.2206	5	0

- **ELECTRE**

We implemented ELECTRE to rank the alternatives with respect to the criteria for input data of case 1 (Table 66). Table 71 shows the results.

Table 71: Ranking of alternatives for Case 1 using ELECTRE

	Max	Max-Min	Sum	Vector	Logarithmic
A1	2	4	2	2	1
A2	1	1	1	1	2
A3	4	3	4	4	3
A4	3	2	3	3	4

Table 71 shows that it is impossible to select the best technique just by looking at the results because each technique ranking is quite different from the others. Thus, we used the assessment framework to recommend the best normalization technique for ELECTRE.

Table 72: Results of Step 3 & 4 of the framework for ELECTRE in Case 1

	Eu-clid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	5.1962	1	1.5	1	9.75	4	0.4375	3	0.8767	3	2
Max-Min	4.4721	2	1.291	2	12	1	0.3125	1	0.9175	2	2
Sum	4.3589	5	1.2583	5	11	3	0.5625	4	0.7997	4	0
Vector	4.4721	2	1.291	2	12	1	0.3125	1	0.9175	1	3
Logarith-mic	4.4721	2	1.291	2	4.25	5	0.75	5	0.7219	5	0

After using framework step 3 metrics and plurality voting of step 4 we now can say (Table 72) that the best normalization technique for case 1 with ELECTRE method is the Vector normalization technique, because it includes the higher counting of the first rank on the used metrics.

4.2.2.2 Case 2

The second case study is an adaptation of Case 1, where the input data of C1 are decimal numbers, to test the robustness of the proposed assessment when there are different types of criteria values. Input data for case 2 are shown in Table 73.

Table 73: Decision matrix Input data for case 2.

	C1	C2	C3	C4
A1	0.1	9	75	8
A2	0.325	7	42	7
A3	0.5	6	95	9
A4	0.4	2	80	6

- **TOPSIS method**

We applied the TOPSIS method to rank the alternatives with respect to the criteria for case 2 (Table 73). Table 74 depicts alternatives' values and ranking using the same five normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 74: Alternatives' values, and ranking for Case 2 using TOPSIS

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rithmic	Rank
A1	0.2617	4	0.4527	3	0.4538	3	0.4781	3	0.9686	1
A2	0.4646	1	0.7086	1	0.6850	1	0.6786	1	0.4675	2
A3	0.2904	3	0.4380	4	0.5578	2	0.5764	2	0.3371	3
A4	0.3329	2	0.5051	2	0.4136	4	0.4353	4	0.1123	4

Again Table 74 shows that it is impossible to select the best normalization technique just by looking at the results. So, we applied the assessment framework to recommend the most suitable normalization technique for Case 2.

Table 75: Results of Step 3 & 4 of the framework for TOPSIS in Case2.

	Eu-clid-ean	Ran k↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.3108	5	0.0897	5	3	4	1.75	2	0.5374	4	0
Max-Min	0.4331	2	0.1250	2	4	3	1.75	2	0.5756	3	0
Sum	0.4202	3	0.1213	3	4.75	1	1.25	1	0.6242	1	3
Vector	0.3758	4	0.1085	4	4.75	1	1.25	1	0.6101	2	2
Logarith-mic	1.2558	1	0.3625	1	0.75	5	2.25	3	-0.1081	5	2

Table 75 depicts the results obtained with steps 3 & 4 of the framework and the recommendation is that the Sum normalization technique is the best regarding plurality voting.

➢ **SAW method**

We applied the SAW method to rank the alternatives with respect to the criteria for input data of Case 2 (Table 73) and Table 76 shows the results.

Table 76: Alternatives' rates and ranking for Case 2 using SAW.

	Max	Ran k	Max-Min	Ran k	Sum	Ran k	Vector	Ran k	Loga-rithmic	Rank
A1	0.3804	3	0.4277	3	0.2253	3	0.4510	3	0.3325	1
A2	0.5520	1	0.7359	1	0.2986	1	0.5614	1	0.2680	2
A3	0.4167	2	0.3929	4	0.2515	2	0.4819	2	0.2355	3
A4	0.3784	4	0.5083	2	0.2246	4	0.4450	4	0.2080	4

Again, Table 76 is inconclusive and we need to use Step 3 and 4 of the framework to recommend the best normalization technique. Table 77 displays the results for Case 2 with the SAW method, from where it can be stated that Max is the best normalization technique for this method.

Table 77: Results of Step 3 & 4 of the framework for SAW in Case2.

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.2840	2	0.0820	2	8.75	1	0.875	1	0.7223	1	3
Max-Min	0.5343	1	0.1542	1	5	4	2.125	3	0.6138	4	2
Sum	0.1204	5	0.0347	5	8.75	1	0.875	1	0.6914	3	2
Vector	0.1855	4	0.0535	4	8.75	1	0.875	1	0.7139	2	2
Logarithmic	0.1856	3	0.0536	3	2.75	5	1.75	2	-0.0016	5	0

- **ELECTRE**

We applied the ELECTRE method to rank the alternatives with respect to the criteria of case 2 (Table 73). Table 78 depicts alternatives' values and ranking of alternatives using the tested normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 78: Ranking of alternatives for Case 2 using ELECTRE

	Max	Max-Min	Sum	Vector	Logarithmic
A1	4	3	3	3	1
A2	1	1	1	1	2
A3	3	4	2	2	3
A4	3	2	4	4	4

Table 78 shows that it is impossible to select the best technique just by looking at the results because each technique ranking is quite different from the others. Thus, there is a need for the assessment framework to recommend the best normalization technique for ELECTRE.

Table 79: Results of Step 3 & 4 of the framework for ELECTRE in Case 2

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	4.4721	1	1.2910	1	7.5	1	1.375	2	0.45	1	4
Max-Min	4.4721	1	1.2910	1	7.5	1	1.25	1	0.45	1	5
Sum	4.4721	1	1.2910	1	7.5	1	1.375	2	0.45	1	4
Vector	4.4721	1	1.2910	1	5.5	4	1.375	2	0.4	4	2
Logarithmic	4.4721	1	1.2910	1	1	5	4.875	5	1	5	2

After using the framework step 3 metrics and plurality voting of step 4 we now can say (Table 79) that the best normalization technique for case 2 with the ELECTRE method is the Max-Min normalization technique, because it includes the higher counting of the first rank on the used metrics.

4.2.2.3 Case 3

The third case study is an adaptation of Case 1 & 2, where all input data are decimal numbers, to test the robustness of the proposed assessment framework when there are decimal values for all criteria. Input data for case 3 are shown in Table 80.

Table 80: Decision Matrix input data for case 3.

	C1	C2	C3	C4
A1	0.1	0.845	0.211	0.4
A2	0.325	0.214	0.01	0.2
A3	0.5	0.1	0.699	0.6
A4	0.4	0.425	0.752	0.1

- **TOPSIS method**

We applied the TOPSIS method to rank the alternatives with respect to the criteria for input data of case 3 (Table 80). Table 81 depicts alternatives' values and ranking using the same five normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 81: Alternatives' values and ranking for Case 3 using TOPSIS.

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rithmic	Rank
A1	0.2930	3	0.5207	3	0.3181	3	0.5759	1	0.4477	3
A2	0.4563	1	0.5909	1	0.6584	1	0.5731	2	0.4544	2
A3	0.1326	4	0.3721	4	0.2125	4	0.3148	4	0.6067	1
A4	0.4352	2	0.5298	2	0.3567	2	0.5027	3	0.3839	4

Again, Table 81 shows that it is impossible to select the best normalization technique just by looking at the results. So, we applied the assessment framework to recommend a more suitable normalization technique for Case 3.

Table 82: Results of Step 3 & 4 of the framework for TOPSIS in Case3.

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.5190	2	0.1498	2	7.5	1	1.25	2	0.4094	3	1
Max-Min	0.3219	5	0.0929	5	7.5	1	0.875	1	0.4698	1	3
Sum	0.6625	1	0.1912	1	7.5	1	1.25	2	0.4635	2	3
Vector	0.4250	3	0.1227	3	3.75	4	1.625	4	0.3939	4	0
Logarith-mic	0.3275	4	0.0945	4	3	5	3.5	5	-0.7407	5	0

Table 82 presents the results obtained with steps 3 & 4 of the framework and the recommendation is that Max-Min and Sum normalization techniques are the best regarding plurality voting.

- **SAW method**

We applied the SAW method to rank the alternatives with respect to the criteria for input data of Case 3 (Table 80) and Table 83 depicts the results.

Table 83: Alternatives' values and ranking for Case 3 using SAW.

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rithmic	Rank
A1	0.5632	2	0.5323	3	0.1959	3	0.5684	2	0.2593	2
A2	0.6392	1	0.6289	1	0.3929	1	0.5983	1	0.2255	3
A3	0.2972	4	0.2679	4	0.1352	4	0.3327	4	0.3087	1
A4	0.5341	3	0.5466	2	0.2761	2	0.5344	3	0.2244	4

Again, Table 83 is inconclusive and we need to use Step 3 and 4 of the framework to recommend the best normalization technique. Table 84 displays the results, from where it can be stated that Max is the best normalization technique for the SAW method in this case.

Table 84: Results of Step 3 & 4 of the framework for SAW method in Case 3.

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.5113	2	0.1476	2	6.75	1	1.125	2	0.4782	1	2
Max-Min	0.5424	1	0.1566	1	6	3	1.25	2	0.4712	2	2
Sum	0.3858	4	0.1114	4	6	3	1.375	4	0.3931	4	0
Vector	0.4159	3	0.1201	3	6.75	1	1	1	0.4632	3	2
Logarith-mic	0.1372	5	0.0396	5	1	5	4	5	-0.8971	5	0

- **ELECTRE**

The ELECTRE method is applied to rank the alternatives with respect to the criteria of case 3 (Table 80). Table 85 depicts alternatives' values and ranking of alternatives using tested normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic) for ELECTRE method. Table 85 shows the ranking of alternatives.

Table 85: Ranking of alternatives for Case 3 using ELECTRE

	Max	Max-Min	Sum	Vector	Logarithmic
A1	1	1	2	2	3
A2	1	1	1	1	4
A3	4	4	4	4	1
A4	3	3	3	3	3

Observing Table 85it is impossible to select the best technique just by looking at the results because each technique ranking is quite different from the others. Thus, there is a need for the assessment framework to recommend the best normalization technique for ELECTRE.

Table 86: Results of Step 3 & 4 of the framework for ELECTRE in Case 3

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	5.1962	1	1.5	1	12.75	1	1.5	3	0.5136	1	4
Max-Min	5.1962	1	1.5	1	12.75	1	1.5	3	0.5136	1	4
Sum	4.4721	3	1.291	3	12.75	1	1.3125	1	0.4925	4	2
Vector	4.4721	3	1.291	3	12.75	1	1.3125	1	0.4925	3	2
Logarithmic	4.3589	5	1.2583	5	7	5	5.125	5	-0.8811	5	0

After using the framework step 3 metrics and plurality voting of step 4 we now can say (Table 86) that the best normalization techniques for case 3 with ELECTRE method are Max and Max-Min normalization technique, because they include the higher counting of the first rank on the used metrics.

4.2.3 MCDM problems with large scaling decision matrices: Case 4 and 5

In this section, we test and validate the framework with two illustrative examples (Case 4 and Case 5), which include large decision matrices, again using TOPSIS, SAW, and ELECTRE methods. The goal is to assess the behaviour of five normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic) and recommend which is the best normalization techniques for large scale MCDM problems.

Case 4 contains 20 alternatives (A1, A2, …, A20) and 4 criteria (C1, C2, C3, and C4), where C4 is a cost criterion while the others are benefit criteria. Case 5 includes 10 alternatives (A1, A2, …, A10) and 20 criteria (C1, C2, …, C20), where C19 and C20 are cost criteria. To assess the behaviour of the five normalization techniques using TOPSIS, SAW, and ELECTRE, the proposed assessment framework was applied.

To ensure fair comparison on the effects of normalization techniques on MCDM methods we applied equal weights for criteria in those two case studies.

4.2.3.1 Case 4

As mentioned above, the Case 4 example consists of 20 alternatives (A1, A2, …, A20) and 4 criteria (C1, C2, C3, and C4) in which C4 is the cost criteria. The input data for Case 4 are shown in Table 87.

Table 87: Decision Matrix input data for Case 4

	C1	C2	C3	C4
A1	138.6090	0.3349	6.4543	9
A2	154.7214	0.6395	23.4244	8
A3	158.3081	0.3441	15.4244	7
A4	157.3082	0.3487	6.8542	9
A5	144.5976	0.9301	11.2616	9
A6	138.5982	0.3346	12.2616	5
A7	131.5989	0.2391	19.1988	9
A8	132.5988	0.3437	14.1988	3
A9	144.5976	0.7252	15.2616	7
A10	138.5982	0.2297	11.2616	2
A11	132.5988	0.5342	11.1988	2
A12	135.9513	0.3387	6.8974	2
A13	119.7141	0.8204	81.2616	1
A14	112.7148	0.4248	11.1988	6
A15	112.7148	0.1292	21.1988	9
A16	128.9520	0.1337	6.8974	2
A17	116.5321	0.3268	7.2695	7
A18	114.1963	0.4259	10.2846	7
A19	131.8605	0.3250	6.2997	5
A20	129.5248	0.1241	10.3148	9

➢ TOPSIS method

TOPSIS method was applied to rank the alternatives with respect to the criteria of case 4 (Table 87). Table 88 depicts alternatives' values and ranking of alternatives using the tested normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 88: Alternatives' values, and ranking for Case 4 using TOPSIS

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Logarithmic	Rank
A1	0.0884	19	0.0930	19	0.2340	13	0.0557	19	0.2817	15
A2	0.2904	4	0.2964	4	0.3373	2	0.2571	2	0.3919	9
A3	0.1674	13	0.1709	13	0.2655	7	0.1365	10	0.4103	7
A4	0.0944	18	0.0996	18	0.2354	12	0.0598	18	0.2771	17
A5	0.2921	3	0.3036	3	0.2787	6	0.2024	3	0.1670	20
A6	0.2012	10	0.2062	10	0.2368	11	0.1192	13	0.3693	10
A7	0.1546	16	0.1536	16	0.2936	4	0.1683	7	0.5309	3
A8	0.2702	8	0.2761	8	0.2200	15	0.1576	8	0.3931	8
A9	0.2709	6	0.2806	5	0.2810	5	0.1933	4	0.2696	18
A10	0.2740	5	0.2796	6	0.1620	18	0.1403	9	0.4322	6
A11	0.3100	2	0.3185	2	0.1780	17	0.1698	6	0.2467	19
A12	0.2703	7	0.2770	7	0.1498	19	0.1306	11	0.2847	14
A13	0.9525	1	0.9490	1	0.7686	1	0.9704	1	0.5632	2
A14	0.1870	11	0.1930	11	0.2417	10	0.1147	14	0.2981	11
A15	0.1664	14	0.1639	15	0.3065	3	0.1898	5	0.6411	1
A16	0.2553	9	0.2606	9	0.1440	20	0.1195	12	0.4530	5
A17	0.1212	17	0.1256	17	0.2295	14	0.0661	17	0.2974	12
A18	0.1595	15	0.1654	14	0.2426	8	0.1014	15	0.2816	16
A19	0.1802	12	0.1855	12	0.2154	16	0.0875	16	0.2868	13
A20	0.0458	20	0.0454	20	0.2425	9	0.0515	20	0.5129	4

Again, Table 88 reveals that it is impossible to select the best technique just by looking at the results and there is a need for using the assessment framework to recommend the best normalization technique for TOPSIS.

Table 89: Results of Framework, Step 3 & 4, for TOPSIS in Case 4

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	Plurality
Max	3.6063	2	0.1850	2	13.75	2	38.175	1	0.75266123	2	1
Max-Min	3.5903	3	0.1842	3	14	1	51.225	3	0.745336228	3	1
Sum	2.5209	4	0.1293	4	3.75	5	53.125	4	0.738764223	4	0
Vector	3.8033	1	0.1951	1	10.75	3	42.075	2	0.809497074	1	3
Logarithmic	2.3912	5	0.1227	5	5.25	4	64.6	5	0.318998389	5	0

After using the framework step 3 metrics and plurality voting of step 4 we now can say (Table 89) that the best normalization technique for case 4 with the TOPSIS method is the Vector normalization technique, because it includes the highest counting of the first rank on the used metrics.

- **SAW method**

Another well-known MCDM method to validate the framework is the SAW method. We applied it to Case 4 (Table 87) and ranked alternatives. Table 90 shows their results.

Table 90: Alternatives' values, and ranking for Case 4 using SAW

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rithmic	Rank
A1	0.1339	19	0.0673	19	0.0261	20	0.2343	19	0.0435	19
A2	0.3470	3	0.3070	3	0.0646	2	0.3676	2	0.0500	10
A3	0.2466	12	0.1944	12	0.0431	11	0.2991	8	0.0525	7
A4	0.1410	18	0.0778	18	0.0272	18	0.2382	18	0.0437	18
A5	0.3152	8	0.2752	6	0.0516	5	0.3280	5	0.0381	20
A6	0.2738	9	0.2232	9	0.0399	12	0.2966	9	0.0505	9
A7	0.1920	15	0.1277	15	0.0450	9	0.2911	12	0.0587	4
A8	0.3395	5	0.2959	5	0.0493	6	0.3236	6	0.0519	8
A9	0.3407	4	0.3013	4	0.0539	4	0.3416	3	0.0441	16
A10	0.3177	6	0.2721	8	0.0482	7	0.3029	7	0.0541	6
A11	0.3936	2	0.3589	2	0.0570	3	0.3373	4	0.0447	14
A12	0.3167	7	0.2726	7	0.0440	10	0.2921	11	0.0446	15
A13	0.9446	1	0.9607	1	0.2068	1	0.7507	1	0.0611	2
A14	0.2626	11	0.2081	11	0.0391	13	0.2930	10	0.0468	11
A15	0.1762	17	0.1062	17	0.0452	8	0.2889	13	0.0666	1
A16	0.2647	10	0.2117	10	0.0380	14	0.2685	15	0.0549	5
A17	0.1871	16	0.1240	16	0.0286	17	0.2531	17	0.0451	13
A18	0.2314	14	0.1735	14	0.0366	15	0.2804	14	0.0458	12
A19	0.2324	13	0.1772	13	0.0294	16	0.2637	16	0.0438	17
A20	0.1053	20	0.0315	20	0.0265	19	0.2306	20	0.0594	3

Likewise, for TOPSIS, Table 90 shows that using the assessment framework is unavoidable to recommend the best normalization technique for the SAW method. So, applying the framework metrics of step 3 and plurality voting of step 4 (Table 91), the recommendation is that Max-Min normalization techniques are suitable for Case 4 with SAW method.

Table 91: Results of Step 3 & 4 of the framework for SAW in Case 4

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	Plural-ity
Max	3.3874	2	0.1738	2	19	2	20	3	0.7809	3	0
Max-Min	3.7531	1	0.1925	1	19.5	1	21.4	4	0.7756	4	3
Sum	0.7492	4	0.0384	4	11.25	4	18.45	1	0.8161	1	2
Vector	2.1280	3	0.1092	3	16	3	19.4	2	0.8106	2	0
Logarith-mic	0.1428	5	0.0073	5	11.25	4	61.35	5	0.2547	5	0

- **ELECTRE**

We applied ELECTRE to rank the alternatives with respect to the criteria for input data of case 4 (Table 87). Table 92 shows their results.

Table 92: Ranking of alternatives for Case 4 using ELECTRE

	Max	Max-Min	Sum	Vector	Logarith-mic
A1	20	20	20	20	19
A2	2	2	2	1	10
A3	8	8	6	7	5
A4	18	18	18	18	20
A5	6	6	6	6	18
A6	9	9	10	8	8
A7	12	12	9	10	2
A8	5	5	5	5	6
A9	2	4	3	3	12
A10	7	7	8	9	4
A11	2	3	4	4	13
A12	11	9	13	13	15
A13	1	1	1	1	3
A14	10	11	12	10	11
A15	14	14	11	12	1
A16	15	15	15	15	9
A17	17	17	16	16	14
A18	13	13	14	14	16
A19	16	16	17	17	17
A20	19	19	19	19	7

Table 92 reveals, again, that it is impossible to select the best technique just by looking at the results, thus, we used the assessment framework to recommend the best normalization technique for ELECTRE.

Table 93: Results of Step 3 & 4 of the framework for ELECTRE in Case 4

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	Plurality
Max	119.2099	1	6.1153	1	22	1	12.75	4	0.8152	2	3
Max-Min	115.4946	4	5.9247	4	22	1	12.5875	3	0.8131	3	1
Sum	116.0129	3	5.9513	3	22	1	10.525	1	0.6208	4	2
Vector	116.8589	2	5.9947	2	21.5	4	11.125	2	0.8369	1	1
Logarithmic	115.3256	5	5.9161	5	1.5	5	42.4375	5	0.1486	5	0

After using the framework step 3 metrics and plurality voting of step 4, we now can say (Table 93) that the best normalization technique for case 4 with the ELECTRE method is the Max normalization technique, because it includes the highest counting of the first rank on the used metrics.

4.2.3.2 Case 5

The fifth case study consists of 10 alternatives (A1, A2, …, A10) and 20 criteria (C1, C2, …, C20), where C19 and C20 are cost criteria and all others are benefit ones. The decision matrix input data for case 5 is shown in Table 94.

Table 94: Decision matrix input data for case 5.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	2	60	6.35	6.8	10	6.35	4.5	60	2.5	6.8
C2	0.192	0.4	0.15	0.1	0.1	0.15	0.08	0.4	0.1	0.2
C3	436	2540	1016	1727.2	1000	560	1016	2540	1016	1727
C4	95	500	3000	1500	2000	500	350	1500	3000	500
C5	102	990	1041	1676	965	915	1041	990	508	1676
C6	72	65	40	32	75	64	25	64	10.35	10.8
C7	82.5	95	80	50	55	36.25	77	75	64	25
C8	7	8	9	6	4	7	1	9	5	7
C9	9	7	6	7	2	1	9	8	6	7
C10	138.609	154.7214	158.3081	157.3082	144.5976	124.5988	135.9513	129	154	207
C11	0.3349	0.3395	0.3441	0.3487	0.3301	0.3279	0.3025	0.2925	0.2787	0.2649
C12	6.4543	11.4244	11.4244	6.8542	11.2616	19.1988	14.7153	18.5122	20.2391	21.9659
C13	5760	8840	5760	6150	13200	2529	5241	969	4948.5	8928
C14	15	22	12	25	32	54	71	95	8	7
C15	3.9516	5.5274	5.5274	3.9516	5.5274	2	12	7.13	2.249	3.463
C16	4	8	7	9	5	6.5	4.76	2.565	5.001	2.617
C17	7130	2249	3463	3694	5857	5149	7630	7985	8871.5	9758
C18	3841	6088	1406	4502	7312	7720	10673	5684	7586	2772
C19	6468	3338	2719	6159	5411	9631	1874	500	3000	1500
C20	2565	5001	2617	2612	3106	8542	9125	7270	7691	7212

- **TOPSIS method**

We applied the TOPSIS method to rank the alternatives with respect to input data of case 5 (Table 94). Table 95 depicts alternatives' ratings and ranking using the same five normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 95: Alternatives' values, and ranking for Case 5 using TOPSIS

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rith-mic	Rank
A1	0.2161	10	0.2363	10	0.2556	9	0.1701	10	0.1713	10
A2	0.6461	2	0.6432	2	0.7243	1	0.6652	2	0.8265	2
A3	0.3642	3	0.3674	3	0.3169	4	0.3244	3	0.3829	5
A4	0.2824	7	0.2937	7	0.2988	7	0.2427	8	0.4062	4
A5	0.3489	4	0.3566	4	0.3395	3	0.3129	4	0.4982	3
A6	0.2409	9	0.2524	9	0.3046	5	0.2202	9	0.3782	6
A7	0.3217	6	0.3318	5	0.2909	8	0.3081	5	0.3447	8
A8	0.7659	1	0.7582	1	0.7190	2	0.7930	1	0.8705	1
A9	0.3319	5	0.3318	6	0.3005	6	0.2971	6	0.2052	9
A10	0.2762	8	0.2856	8	0.2348	10	0.2460	7	0.3673	7

Once again, Table 95 shows it is impossible to select the best normalization technique just by looking at the results. So, we applied the assessment framework to recommend the more suitable normalization technique for Case 5.

Table 96: Results of Step 3 & 4 of the framework for TOPSIS in Case 5.

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	1.7123	4	0.1805	4	13.25	1	2.0000	1	0.9715	3	2
Max-Min	1.6451	5	0.1734	5	13.25	1	2.8000	2	0.9728	2	1
Sum	1.7382	3	0.1832	3	2	5	3.4000	4	0.9601	4	0
Vector	1.9317	2	0.2036	2	12.25	3	3.3429	3	0.9736	1	1
Loga-rithmic	2.2084	1	0.2328	1	9.75	4	4.3429	5	0.9268	5	2

Table 96 depicts the results obtained with steps 3 & 4 of the framework and the recommendation is that Max and Logarithmic normalization techniques are the best regarding plurality voting.

➢ SAW method

Here we applied SAW method to rank the alternatives for input data of Case 5 (Table 94) and Table 97 shows the results.

Table 97: Alternatives' values and ranking for Case 2 using SAW.

	Max	Rank	Max-Min	Rank	Sum	Rank	Vector	Rank	Loga-rithmic	Rank
A1	0.3196	10	0.2677	9	0.0582	10	0.2094	10	0.0794	10
A2	0.6684	2	0.6436	2	0.1593	2	0.4351	2	0.1206	2
A3	0.4613	4	0.4150	4	0.0900	4	0.2951	4	0.0989	6
A4	0.4097	6	0.3605	6	0.0812	6	0.2654	6	0.1012	4
A5	0.4647	3	0.4182	3	0.0952	3	0.3023	3	0.1050	3
A6	0.3235	9	0.2636	10	0.0701	9	0.2210	9	0.0943	7
A7	0.4319	5	0.3825	5	0.0881	5	0.2897	5	0.0990	5
A8	0.7802	1	0.7592	1	0.2097	1	0.5077	1	0.1261	1
A9	0.3909	7	0.3335	8	0.0737	8	0.2563	7	0.0843	9
A10	0.3866	8	0.3338	7	0.0745	7	0.2537	8	0.0912	8

Again, Table 97 is inconclusive and we need to use Step 3 and 4 of the framework to recommend the best normalization technique. Table 98 displays the results for Case 5 with the SAW method, from where it can be stated that Max-Min and Sum are the best normalization technique for SAW in this case.

Table 98: Results of Step 3 & 4 of the framework for SAW in Case 5.

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	1.4059	2	0.1482	2	33.25	2	0.5500	3	0.9786	2	0
Max-Min	1.5250	1	0.1607	1	30	4	0.7000	4	0.9784	3	2
Sum	0.4491	4	0.0473	4	33.75	1	0.5000	1	0.9725	4	2
Vector	0.9006	3	0.0949	3	33.25	2	0.5286	2	0.9800	1	0
Loga-rithmic	0.1385	5	0.0146	5	26.75	5	1.6786	5	0.9271	5	0

➢ ELECTRE

Now, we applied ELECTRE method to rank the alternatives with respect to the criteria of case 5 (Table 94). Table 99 depicts alternatives' ratings and ranking of alternatives using the tested normalization techniques (Max, Max-Min, Sum, Vector, and Logarithmic).

Table 99: Ranking of alternatives for Case 5 using ELECTRE

	Max	Max-Min	Sum	Vector	Logarith-mic
A1	10	10	10	10	10
A2	1	2	2	2	2
A3	4	4	3	3	5
A4	5	4	5	5	3
A5	1	2	3	3	3
A6	8	9	8	9	8
A7	6	6	6	5	6
A8	1	1	1	1	1
A9	8	8	8	8	9
A10	7	7	7	7	7

Similarly, to the above cases, Table 99 reveals it is impossible to select the best technique just by looking at the results because each technique ranking is quite different from the others. Thus, we used the assessment framework to recommend the best normalization technique for ELECTRE.

Table 100: Results of Step 3 & 4 of the framework for ELECTRE in Case 2

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	31.1288	1	3.2813	1	27.75	5	0.7250	4	0.9652	4	2
Max-Min	30.0167	2	3.1640	2	29.25	2	0.4250	1	0.9774	1	2
Sum	28.3019	5	2.9833	5	31.5	1	0.5250	2	0.9717	2	1
Vector	29.3428	4	3.0930	4	28.75	3	0.7071	3	0.9658	3	0
Logarith-mic	29.3939	3	3.0984	3	28	4	0.9821	5	0.9507	5	0

After using the framework step 3 metrics and plurality voting of step 4 we now can say (Table 100) that the best normalization techniques for case 5 with the ELECTRE method are Max, and Max-Min normalization techniques, because they include the highest counting of the first rank on the used metrics.

4.2.3.3 Final Comments on normalization techniques on scaling problems

We observed some interesting results about the relation between the number of criteria and alternatives in decision problems using different normalization techniques in MCDM (TOPSIS, SAW, and ELECTRE) methods. We tested different number of alternatives when there are 20 criteria Table 101 shows the decision matrix which contains 20 criteria and 10 alternatives. C19 and C20 are cost criteria and the other criteria are benefit ones.

Table 101: Illustrative example with 20 Criteria and 10 alternatives

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	2	60	6.35	6.8	10	6.35	4.5	60	2.5	6.8
C2	0.192	0.4	0.15	0.1	0.1	0.15	0.08	0.4	0.1	0.2
C3	436	2540	1016	1727.2	1000	560	1016	2540	1016	1727
C4	95	500	3000	1500	2000	500	350	1500	3000	500
C5	102	990	1041	1676	965	915	1041	990	508	1676
C6	72	65	40	32	75	64	25	64	10.35	10.8
C7	82.5	95	80	50	55	36.25	77	75	64	25
C8	7	8	9	6	4	7	1	9	5	7
C9	9	7	6	7	2	1	9	8	6	7
C10	138.609	154.7214	158.3081	157.3082	144.5976	124.5988	135.9513	129	154	207
C11	0.3349	0.3395	0.3441	0.3487	0.3301	0.3279	0.3025	0.292567	0.278767	0.264967
C12	6.4543	11.4244	11.4244	6.8542	11.2616	19.1988	14.7153	18.51227	20.23912	21.96597
C13	5760	8840	5760	6150	13200	2529	5241	969	4948.5	8928
C14	15	22	12	25	32	54	71	95	8	7
C15	3.9516	5.5274	5.5274	3.9516	5.5274	2	12	7.13	2.249	3.463
C16	4	8	7	9	5	6.5	4.76	2.565	5.001	2.617
C17	7130	2249	3463	3694	5857	5149	7630	7985	8871.5	9758
C18	3841	6088	1406	4502	7312	7720	10673	5684	7586	2772
C19	6468	3338	2719	6159	5411	9631	1874	500	3000	1500
C20	2565	5001	2617	2612	3106	8542	9125	7270	7691	7212

Using the data from Table 101 , first we analysed the behaviour of the chosen normalization techniques with MCDM methods for the first 3 alternatives (A1, A2, and A3). Then, we analysed the results with 4 alternatives (A1, A2, …, A4) and then continued with 5, 7, 8, 9, and 10 alternatives, respectively. The ranking of alternatives using five normalization techniques in TOPSIS, SAW, and ELECTRE methods for the cases with 3, 4, 5, and 10 alternatives are shown in Table 102, Table 103, Table 104, and Table 105. To avoid displaying too many results, the ranking of alternative for cases with 6, 7, 8, and 9 alternatives are not shown.

Table 102: Ranking for 3 alternatives with TOPSIS, SAW, and ELECTRE

		Max	Max-Min	Sum	Vector	Logarithmic
TOP-SIS	A1	3	3	3	3	3
	A2	1	1	1	1	1
	A3	2	2	2	2	2
SAW	A1	3	3	3	3	3
	A2	1	1	1	1	1
	A3	2	2	2	2	2
ELEC-TRE	A1	3	3	3	3	3
	A2	1	1	1	1	1
	A3	2	2	2	2	2

Table 103: Ranking for 4 alternatives with TOPSIS, SAW, and ELECTRE

		Max	Max-Min	Sum	Vector	Logarith-mic
TOPSIS	A1	4	4	4	4	4
	A2	1	1	1	1	1
	A3	2	2	2	2	2
	A4	3	3	3	3	3
SAW	A1	4	4	4	4	4
	A2	1	1	1	1	1
	A3	2	2	2	2	2
	A4	3	3	3	3	3
ELECTRE	A1	4	4	4	4	4
	A2	1	1	1	1	1
	A3	3	3	3	3	3
	A4	2	2	2	2	2

Table 104: Ranking for 5 alternatives with TOPSIS, SAW, and ELECTRE

		Max	Max-Min	Sum	Vector	Logarith-mic
TOPSIS	A1	2	2	2	2	2
	A2	3	3	3	3	4
	A3	4	4	4	4	3
	A4	5	5	5	5	5
	A5	1	1	1	1	1
SAW	A1	2	2	2	2	2
	A2	3	3	3	3	4
	A3	4	4	4	4	3
	A4	5	5	5	5	5
	A5	1	1	1	1	1
ELECTRE	A1	5	5	5	5	5
	A2	1	1	1	1	1
	A3	3	3	3	3	3
	A4	2	2	2	2	2
	A5	4	4	4	4	4

Table 105: Ranking for 10 alternatives with TOPSIS, SAW, and ELECTRE

		Max	Max-Min	Sum	Vector	Logarith-mic
TOPSIS	A1	8	8	2	8	8
	A2	2	2	8	2	2
	A3	3	3	5	3	5
	A4	5	5	3	5	4
	A5	9	7	6	7	3
	A6	7	9	9	9	6
	A7	4	4	4	10	10
	A8	10	10	7	4	7
	A9	6	6	1	6	9
	A10	1	1	10	1	1
SAW	A1	8	8	8	8	8
	A2	2	2	2	2	2
	A3	5	5	5	5	5
	A4	3	3	3	3	4
	A5	7	7	7	7	7
	A6	4	4	4	4	3
	A7	9	10	10	9	6
	A8	10	9	9	10	10
	A9	6	1	6	6	9
	A10	1	6	1	1	1
ELECTRE	A1	7	7	7	7	2
	A2	2	1	2	2	5
	A3	9	8	9	9	8
	A4	8	9	8	8	10
	A5	3	4	3	3	7
	A6	5	6	5	5	3
	A7	10	10	10	10	6
	A8	6	5	6	6	1
	A9	4	3	4	4	9
	A10	1	2	1	1	4

Table 102, Table 103, and Table 104 show that the TOPSIS , SAW, and ELECTRE methods produced the same ranking for alternatives using different normalization techniques i.e., there were no differences in the ranking of alternatives using different normalization techniques when the number of alternatives is less than the number of criteria. Until 9 alternatives, the results are similar, i.e. the ranking remains the same for each normalization technique. However, when A≥10 alternatives the ranking changes and produces different results. This study led to a novel conclusion about the relation of ranking results in TOPSIS, SAW, and ELECTRE methods, using different normalization techniques. It showed that to obtain consistent rankings for alternatives, the number of alternatives should be at least half the number of criteria in decision matrices, e.g. for decision problem with 20 criteria we need to have at least 10 alternatives to rank, when using TOPSIS, SAW, and ELECTRE methods.

This test was done with several decision matrixes and different input data to ensure the achieved conclusion is valid, i.e. there are no consistent rankings in MCDM methods for comparing normalization methods when the number of alternatives is half of the number of criteria.

4.2.4 Benchmarking

Within benchmarking we should compare the results of our proposed assessment framework with results from the literature. However, since this is a novel topic of research, there are only few papers about selecting the best normalization techniques using MCDM methods in the literature. So, we could only find four case studies for the SAW, TOPSIS, and WASPAS (Weighted Aggregated Sum Product Assessment Method) methods (Celen, 2014; Chakraborty and Yeh, 2012; Lakshmi and Venkatesan, 2014; Mathew *et al.*, 2017) to validate the proposed framework.

4.2.4.1 Case 6

This case study is borrowed from Celen (2014) who recommended normalization techniques using the TOPSIS method for a deposit banks´ ranking decision problem. The case study consists of 29 sub-attributes under 6 main attributes (Table 106) and 13 alternatives (A1,…, A13) to assess the financial performances of 13 Turkish deposit banks and rank them.

Table 106: Fuzzy pair-wise comparison matrix [adapted from (Celen, 2014)].

	C1	C2	C3	C4	C5	C6
C1	(1, 1, 1)	(7, 8.33, 9)	(0.11, 2.7, 7)	(0.11, 5.37, 9)	(0.11, 2.41, 7)	(7, 8.33, 9)
C2	(0.11, 0.12, 0.14)	(1, 1, 1)	(0.11, 0.41, 1)	(0.11, 0.41, 1)	(0.11, 0.41,1)	(0.11, 0.41, 1)
C3	(0.14, 3.38, 9)	(1, 6.33, 9)	(1, 1, 1)	(0.11, 4.7, 9)	(0.11, 4.7, 9)	(1, 5, 9)
C4	(0.11, 3.08, 9)	(1, 6.33, 9)	(0.11, 3.1, 9)	(1, 1, 1)	(0.11, 3.37, 9)	(0.11, 3.37, 9)
C5	(0.14, 6.05, 9)	(1, 6.33, 9)	(0.11, 3.1, 9)	(0.11, 3.37, 9)	(1, 1, 1)	(1, 6.33, 9)
C6	(0.11, 0.12, 0.14)	(1, 6.33, 9)	(0.11, 0.44, 1)	(0.11, 3.37, 9)	(0.11, 0.41,1)	(1, 1, 1)

The author first used Fuzzy Analytical Hierarchy Process (FAHP) to calculate the weights using pairwise comparison matrices. Then the TOPSIS method was applied for ranking alternatives. Further, the author tested four different normalization techniques (Vector, Max-Min, Max, and Sum techniques) to prepare dimensionless data from heterogeneous input data sets. The final rank and relative closeness of 13 alternatives are depicted in Table 107 (borrowed from Celen (2014)).

Table 107: Relative closeness (RC) and Ranking of alternatives (R) using four different normalization techniques [borrowed from (Celen, 2014)]

Banks	**Vector**		**Max-Min**		**Max**		**Sum**	
	RC	*R*	*RC*	*R*	*RC*	*R*	*RC*	*R*
A1	0.682	2	0.63	3	0.732	4	0.446	9
A2	0.492	5	0.476	7	0.595	8	0.696	3
A3	0.458	7	0.409	8	0.669	5	0.379	11
A4	0.796	1	0.805	1	0.786	2	0.545	8
A5	0.312	12	0.302	12	0.506	9	0.681	4
A6	0.442	8	0.397	9	0.603	7	0.417	10
A7	0.644	3	0.692	2	0.825	1	0.572	6
A8	0.64	4	0.555	4	0.75	3	0.257	12
A9	0.423	10	0.394	10	0.618	6	0.632	5
A10	0.427	9	0.357	11	0.23	12	0.065	13
A11	0.459	6	0.539	5	0.363	10	0.858	1
A12	0.417	11	0.496	6	0.275	11	0.776	2
A13	0.212	13	0.218	13	0.149	13	0.547	7

As Table 107 shows, ranking of alternatives varies using the four chosen normalization techniques. So, the author used four consistency conditions that analyzed the effects of normalization techniques on the mentioned case study and suggested that the Vector normalization is the best technique and Max and Max-Min are the second best normalization techniques (Celen, 2014).

We applied our proposed assessment framework for this case study to validate its accuracy when compared with Celen´s results. The obtained results of the framework (3.4) are depicted in Table 108.

Table 108: Results from framework, (Step 3 & 4) for TOPSIS case study (example from Celen (2014)).

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	Plural-ity
Vector	1.9869	4	0.1591	4	3	1	14.2051	2	0.5168	2	1
Max-Min	2.0305	3	0.1626	3	2.6667	2	12.4103	1	0.5721	1	2
Max	2.8012	1	0.2243	1	1.6667	3	16.1538	3	0.4520	3	2
Sum	1.9869	4	0.2169	2	0	4	32.2051	4	-0.0597	4	0

As Table 108 shows, when using our framework: Max-Min and Max normalization techniques are the best techniques, while the Vector normalization is the second best technique.

The comparison between our assessment framework and Celen´s work (Celen, 2014) reveals that our results are more robust and accurate because we use several metrics, from different categories (STD from measures of data dispersion; Euclidean distance from the measure of proximity; Mean Ks, RCI, and MSE from comparison metrics), while the author (Celen, 2014) only used four consistency conditions - requiring decision maker's intervention - to determine the selection of the normalization technique. Further, (Celen, 2014) work does not provide a numerical ranking of the normalization techniques while our framework provides numerical ranking with several known metrics.

4.2.4.2 Case 7

The second case study used for benchmarking is borrowed from Mathew et al. (2017) for the selection of an industrial robot using Weighted Aggregated Sum Product Assessment Method (WASPAS).

The authors analysed the effect of the six different normalization techniques in WASPAS method (Mathew *et al.*, 2017). The case study consists of 5 criteria (C1,…, C5) and 7 alternatives (A1,…, A7) with the assigned weights (Table 109). In this case study, C2 is the cost criteria (the lower values the better) and the others are benefit criteria (the higher values the better).

Table 109: Decision matrix input data and assigned weights for the case study borrowed from Mathew et al. (2017).

	C1	C2	C3	C4	C5
Weights	0.036	0.192	0.326	0.326	0.12
A1	60	0.4	2540	500	990
A2	6.35	0.15	1016	3000	1041
A3	6.8	0.1	1727.2	1500	1676
A4	10	0.2	1000	2000	965
A5	2.5	0.1	560	500	915
A6	4.5	0.08	1016	350	508
A7	3	0.1	177	1000	920

The authors selected six normalization techniques as Vector, Max, Max-Min, Sum, Logarithmic, and Enhanced accuracy for the related case study (Mathew *et al.*, 2017) to analyse the effect of the mentioned techniques on the ranking of alternatives.

Hence the Enhanced accuracy normalization technique does not belong to our chosen set of normalization techniques, however, its formulas for cost and bene-fit criteria are shown in Table 3. This technique is classified as a linear technique and applicable to add to the selected normalization techniques in the assessment framework. This status shows the user-friendly procedure of the proposed assessment framework to add/remove normalization techniques in Step 2.

Table 110 shows the alternative values and ranking of alternatives for WASPAS method using five selected normalization techniques (Vector, Max, Max-Min, Sum, Logarithmic, and Enhanced accuracy).

Table 110: Results with WASPAS method (adapted from Mathew et al. (2017)).

	Vector		**Max**		**Max-Min**		**Sum**		**Logarithmic**		**Enhanced accuracy**	
	Alt. Value	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*
A1	0.3423	4	0.2436	7	0.2150	4	0.1426	4	0.1525	1	0.7924	7
A2	0.4864	2	0.6226	2	0.6054	2	0.1830	2	0.1514	2	0.9021	2
A3	0.4898	1	0.6341	1	0.6277	1	0.1885	1	0.1509	3	0.9194	1
A4	0.4023	3	0.5067	3	0.4746	3	0.1470	3	0.1503	4	0.8570	3
A5	0.2456	6	0.3023	5	0.1466	7	0.0863	6	0.1311	6	0.8331	6
A6	0.2599	5	0.3198	4	0.1545	5	0.0970	5	0.1326	5	0.8401	4
A7	0.2347	7	0.2875	6	0.1513	6	0.0825	7	0.1282	7	0.8335	5

As Table 110 shows different normalization techniques produced different rankings of alternatives. The authors used Spearman correlation and calculated Mean ks values for each normalization technique. In the end they recommend the Max-Min normalization technique because of having the highest Mean ks value among the selected techniques (Mathew *et al.*, 2017).

Table 111: Results of Mean ks for WASPAS method (borrowed from Mathew et al. (2017)).

	Mean ks	Rank↑
Max	0.7209	4
Max-Min	0.8370	1
Sum	0.8236	2
Vector	0.8236	2
Logarithmic	0.5424	6
Enhanced accuracy	0.6808	5

We now apply our assessment framework (Section 3.4) to this case study and the obtained results are depicted in Table 112.

Table 112: Framework (Step 3 & 4) results for case study 7 (borrowed from Mathew et al. (2017)).

	Euclid-ean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	Plural-ity
Vector	0.7160	3	0.1105	3	33.8	1	1.2000	1	0.9013	2	2
Max	1.0809	2	0.1668	2	28.4	5	2.4000	4	0.8534	4	0
Max-Min	1.4355	1	0.2215	1	29.6	4	1.3143	3	0.9014	1	3
Sum	0.2885	4	0.0445	4	33.2	2	1.2000	1	0.8723	3	1
Logarith-mic	0.0720	6	0.0111	6	20	6	3.8286	6	0.7033	6	0
Enhanced accuracy	0.2831	5	0.0437	5	30.8	3	2.5143	5	0.7244	5	0

Observing results from Table 112, we conclude that our framework also selects the Max-Min normalization as the best technique because it has the highest PV. However, our framework provides more confidence and consistency because it uses metrics from different categories (STD from measures of data dispersion; Euclidean distance from the measure of proximity; Mean Ks, RCI, and MSE from comparison metrics) and could guarantee the robustness of the comparison between different normalization techniques. On the other hand, implementing PV worked as an aggregation process which enabled us obtain a single result from different used metrics. Concluding, results from our framework Table 112 prove, with more certainty and consistency, the results from Mathew et al. (2014), where they just applied Spearman correlation and calculated Mean ks values for the six chosen normalization techniques.

4.2.4.3 Case 8

This case study is borrowed from Lakshmi and Venkatesan (2014) that analyzed and recommended normalization techniques for the TOPSIS method in a car selection problem. The case study consists of 4 alternatives (A1, …, A4) regarding different car brands (Civic Coupe, Saturn Coupe, Ford Escort, and Mazda Miata) and 4 criteria (C1, …, C4) related to the car characteristics (style, reliability, fuel-eco, and cost). The decision matrix with the input data is depicted in Table 113.

Table 113: Decision matrix input data [borrowed from (Lakshmi and Venkatesan, 2014)].

	C1	C2	C3	C4
A1	7	9	9	8
A2	8	7	8	7
A3	9	6	8	9
A4	6	7	8	6

The authors used five normalization techniques (Max, Max-Min, Sum, Vector, and Fuzzification (using Gaussian membership function)) to analyze the effects of using different normalization techniques on the TOPSIS method. They calculated the relative closeness and ranking of alternatives, with TOPSIS method, using five selected normalization techniques for this case study (Table 114).

Table 114: Relative closeness (RC) and Ranking of alternatives (R) for Case Study 8 from Lakshmi and Venkatesan (2014).

	Vector		Max-Min		Sum		Max		Fuzzification (Gaussian)	
	RC	*R*	*RC*	*R*	*RC*	*R*	*RC*	*R*	*RC*	*R*
A1	0.74	1	0.88	1	0.38	1	0.26	4	0.75	1
A2	0.41	3	0.28	4	0.26	3	0.31	1	0.45	3
A3	0.17	4	0.31	3	0.009	4	0.29	3	0.02	4
A4	0.44	2	0.45	2	0.32	2	0.30	2	0.62	2

As Table 114 depicts that there is no consensus between the five selected normalization techniques for ranking alternatives. The authors (Lakshmi and

Venkatesan, 2014) calculated time complexity and space complexity for each normalization technique with the help of MATLAB and recommend the Sum normalization technique as the best technique for the case study, using the TOPSIS method.

Also, we applied metrics from Step 3 and 4 of the proposed assessment framework (Section 3.4.) and the results are presented in Table 115.

Table 115: Framework (Step 3 & 4) results for Case Study 8 using TOPSIS (adapted from Lakshmi and Venkatesan (2014)).

	Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
Max	0.8094	3	0.2337	3	10.75	1	1	2	0.5211	3	1
Max-Min	0.9587	2	0.2768	2	9	4	0.5	1	0.3364	4	1
Sum	0.5648	4	0.1630	4	10.75	1	1	2	0.5690	2	1
Vector	0.0748	5	0.0216	5	6.25	5	2.75	5	-0.5452	5	0
Fuzzification (Gaussian)	1.1016	1	0.3180	1	10.75	1	1	2	0.5694	1	4

From the obtained results (Table 115) using the proposed assessment framework, Fuzzification (Gaussian) normalization technique is the best technique, while the approach by (Lakshmi and Venkatesan (2014)), recommended the Sum normalization technique.

Comparing both approaches results, we believe that our framework provides more robust and reliable results than the ones obtained by the authors (Lakshmi and Venkatesan, 2014), because we cover a wide range of metrics from different categories (STD from measures of data dispersion; Euclidean distance from the measure of proximity; Mean Ks, RCI, and MSE from comparison metrics).In Case 8, the authors (Lakshmi and Venkatesan, 2014) just calculated time and space complexity with MATLAB, and these results are highly dependent on the style of MATLAB users/programmers. For instance, someone can code the Sum normalization technique in a manner that obtains time and space complexity twice higher than someone else. Therefore, our proposed framework ensures more accurate and reliable results to support decision makers.

4.2.4.4 Case 9

This case study is borrowed from Chakraborty and Yeh (2012) and they compared the results of using different normalization techniques for SAW and TOPSIS methods and compared the their results with Weighted Product (WP) method's results and find more proper normalization technique. They implemented WP method without using normalization techniques and produced comparable data by multiplying the weight to the input data of each criterion, then compared its results with the SAW and TOPISIS method while using different normalization techniques in these two MCDM methods. Here we do not consider the WP method because it does not belong to our chosen MCDM methods.

The case study includes 5 criteria (C1, …, C5) and 6 alternatives (A1, …, A6). The decision matrix with the input data is depicted in Table 116.

Table 116: Decision matrix input data and assigned weights for the case study 9 (borrowed from Chakraborty and Yeh (2012)

	C1	C2	C3	C4	C5
Weights	0.03	0.1	0.3	0.15	0.15
A1	690	3.1	9	7	4
A2	590	3.9	7	6	10
A3	600	3.6	8	8	7
A4	620	3.8	7	10	6
A5	700	2.8	10	4	6
A6	650	4	6	9	8

The authors (Chakraborty and Yeh, 2012) compared 4 normalization techniques namely Max, Max-Min, Sum, and Vector for the SAW and TOPSIS methods to analyze the effect of the mentioned techniques on the ranking of alternatives. Table 117 shows the alternatives´ rates and rankings.

Table 117: Results with SAW and TOPSIS method (borrowed from Chakraborty and Yeh (2012)).

		Max		Max-Min		Sum		Vector	
		Alt. Value	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*	*Alt. Value*	*Rank*
SAW	A1	0.5421	5	0.3527	2	0.1159	6	0.3987	6
	A2	0.5404	6	0.2664	6	0.1179	5	0.4041	5
	A3	0.5541	3	0.3000	5	0.1209	2	0.4190	1
	A4	0.5528	4	0.3082	4	0.1195	4	0.4070	4
	A5	0.5682	1	0.4050	1	0.1202	3	0.4106	3
	A6	0.5575	2	0.3523	3	0.1232	1	0.4137	2
TOPSIS	A1	0.4896	3	0.5651	2	0.4645	5	0.4994	4
	A2	0.4789	5	0.3450	6	0.5035	3	0.5183	3
	A3	0.5337	2	0.4137	4	0.5437	1	0.5320	2
	A4	0.4790	4	0.4148	5	0.4843	4	0.4817	5
	A5	0.5428	1	0.6290	1	0.5138	2	0.5485	1
	A6	0.4353	6	0.5244	3	0.4562	6	0.4566	6

As Table 117 shows, different normalization techniques produced different rankings of alternatives. The authors used Spearman correlation and calculated Mean ks values for each normalization techniques for both SAW and TOPSIS methods. Table 118 depicts Mean ks values for each normalization technique for the related case study.

Table 118: Results of Mean ks for SAW and TOPSIS method (borrowed from Chakraborty and Yeh (2012)).

	SAW		TOPSIS	
	Mean ks	Rank↑	Mean ks	Rank↑
Max	0.464	1	0.336	1
Max-Min	0.15	4	0.264	2
Sum	0.321	3	0.25	4
Vector	0.393	2	0.264	2

In the end (Chakraborty and Yeh, 2012) recommended tMax normalization technique for both SAW and TOPSIS methods because of having the highest Mean ks values among the selected techniques. Also, they conclude that to reach more accurate results using MCDM methods several details such as selecting normalization techniques and aggregation method have to be considered by decision makers.

We now apply our assessment framework (Section 3.4) to this case study and the obtained results are depicted in Table 119: Framework (Step 3 & 4) results for case study 9 (example from Chakraborty and Yeh (2012)).Table 119.

Table 119: Framework (Step 3 & 4) results for case study 9 (example from Chakraborty and Yeh (2012)).

		Euclidean	Rank↑	STD	Rank↑	RCI	Rank↑	MSE	Rank↓	Mean ks	Rank↑	PV
SAW	Max	0.2172	2	0.0397	2	3	3	1.7778	1	0.4640	1	2
	Max-Min	0.5912	1	0.1079	1	2.6667	4	4.7778	4	0.1500	4	2
	Sum	0.1790	4	0.0327	4	4	1	2.4444	2	0.3210	3	1
	Vector	0.1850	3	0.0338	3	3.6667	2	2.7778	3	0.393	2	0
TOPSIS	Max	0.2172	2	0.0397	2	4.3333	1	1.7778	1	0.3360	1	3
	Max-Min	0.5912	1	0.1079	1	2.6667	4	4.4444	4	0.2640	2	2
	Sum	0.1790	4	0.0327	4	3	3	2.8889	3	0.2500	4	0
	Vector	0.1850	3	0.0338	3	3.3333	2	2.0000	2	0.264	2	0

From the obtained results (Table 119) using the proposed assessment framework, for SAW method, the Max normalization technique is the best technique and for TOPSIS, both Max and Max-Min techniques are best techniques (highest PV), while the approach by Chakraborty and Yeh (2012), recommended the Max normalization technique for both SAW and TOPSIS.

As mentioned before, we believe our framework is more consistent and robust regarding recommendations because it uses metrics from different categories (STD from measures of data dispersion; Euclidean distance from the measure of proximity; Mean Ks, RCI, and MSE from comparison metrics) and could guarantee the robustness of the comparison between different normalization techniques. Further, using PV (step 4) worked as a good aggregation process to summarize results from different metrics. Concluding, results from our framework (Table 119) prove with more certainty the accuracy of results from Chakraborty and Yeh (2012), where they just applied Spearman correlation and calculated Mean ks values.

4.2.5 Testimonies

Another validation performed for this research work was to collect testimonies from experts about the proposed assessment framework, regarding its novelty and utility in recommending the best normalization techniques for MCDM methods. These testimonies represent an important peer support that this research topic is of importance for the advancement of science, regarding normalization techniques applied to MCDM methods. Below, we present a selection of testimonies, provided by unknown reviewing peers about articles published during this PhD work:

- ✓ Expert 1 (Vafaei et al., 2022):

" *The introduction is a really important section of the article, where the researchers have the opportunity to show the research gap who are acting and the contribution of the paper. Therefore, the authors may present a clearer justification based on points such as research importance, originality, and viability. Literature review provides an excellent overview of normalization techniques and assessment framework, which demonstrates that researchers have done a good literature review on the topic.*"

- ✓ Expert 2 (Vafaei et al., 2022):

" *This paper faces the normalization of data including outliners in multi-criteria decision making issues. The authors compare seven normalization techniques. The topic of the paper is very interesting, and the paper is well written and well structured.*"

- ✓ Expert 3 (Vafaei et al., 2020):

" *Very interesting and very well organized paper. The research question presented is very well formulated: it is clear and objective.* "

- ✓ Expert 4 (Vafaei et al., 2020):

" *Interesting work, well-structured and written. Clear comparisons are made. State-of-the-art is well done.* "

- ✓ Expert 5 (Vafaei et al., 2019):

" *This paper tackles a generally important problem and fits the conference.*"

- ✓ Expert 6 (Vafaei et al., 2019):

" This papers presents a recommendation framework for supporting users to select data normalization techniques that better fit the requirements in different application scenarios, based on multi-criteria decision methods. This is a well-written paper and the developed framework will be of value for decision makers."

✓ Expert 7 (Vafaei et al., 2018b):

" *The paper re-analyses normalization techniques within MCDM problems. The interesting part is the relation that the normalizations technique may have with the decision problem. Therefore, it may condition the final decision.*"

✓ Expert 8 (Vafaei et al., 2018c):

" *The paper deals with a relevant problem in our research field (DSS), from a practical point of view. In my opinion, there are three main ideas in the paper: (i) the normalizations techniques considered in the study and their expressions; (ii) the incorporation of the dynamic character and, (iii) the selection of the most suitable normalizations technique.*"

✓ Expert 9 (Vafaei et al., 2018c):

" *The paper is about a very interesting topic in the MCDM domain. Indeed, the normalization technique choice can greatly affect the final outcome. In future versions it would be interesting to see these methods applied in TOPSIS or VIKOR, MCDM methods which are versatile and allow much experimentation. The comparison of the results could be of value.*"

4.3 Evaluation in the Research Community

The author of this thesis became a member of the EURO Working Group on Decision Support Systems (EWG-DSS) [https://www.euro-online.org/websites/dss/] in 2015. This society provided a vast networking and interaction with various experts/members regarding research on related topics. This continuous interaction contributed to improvements on this PhD research work from their positive feedback and encouragement about contributions of the normalization process on decision problems and methods.

Furthermore, new contributions and results of this thesis were published in several international peer-reviewed conference proceedings, scientific journals, and book chapters.

Figure 14 depicts the published work, by type of publication, relation with thesis, and if it won prizes. The complete published research along the PhD included: six publications in international journals; five publications in proceedings of international conferences with peer reviewing (book chapter); five publications in abstract proceeding of international conferences with peer review; and one work still in progress (aiming a journal publication). Furthermore, other academic contributions include: book editor, poster, and panel participation, and awards as shown in Figure 15.

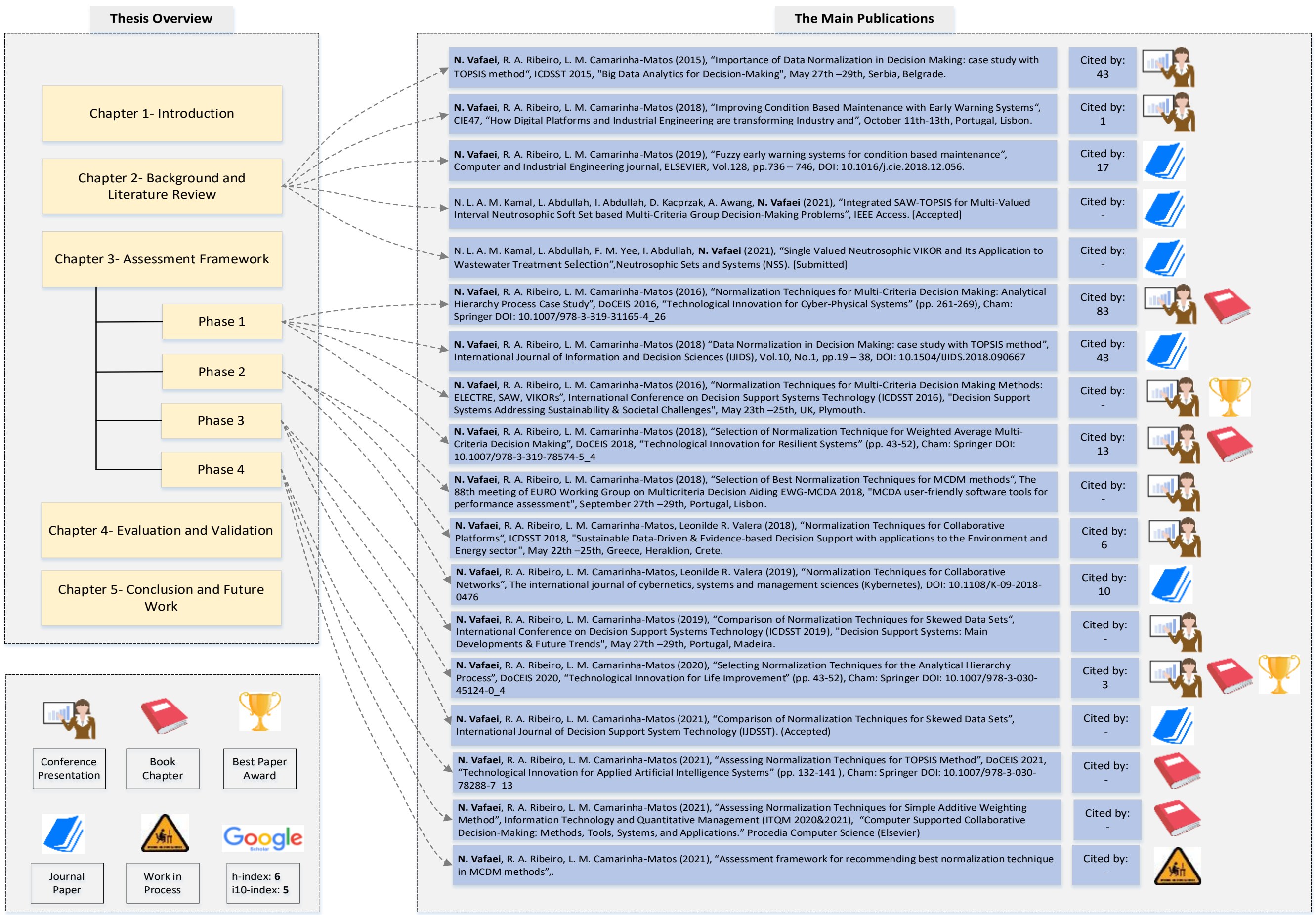


Figure 14: Thesis Outline with related main publications.

BOOK EDITOR	Camarinha-Matos, Luis M.**, Vafaei, N.**, Falcao, A., Najdi, S., Technological Innovation for Cyber-Physical Systems: 7th IFIP WG 5.5/SOCOLNET Advanced Doctoral Conference on Computing, Electrical and Industrial Systems, DoCEIS 2016, Costa de Caparica, Portugal, April 11-13, 2016, Proceedings. (DOI: 10.1007/978-3-319-31165-4)
Poster Presentation	**N. Vafaei,** R. A. Ribeiro, L. M. Camarinha-Matos (2017), “Evaluation Assessment to Select Best Normalization Techniques for Multi-Criteria Decision Making Methods: ELECTRE, TOPSIS, SAW, VIKOR”, International Conference on Decision Support Systems Technology (ICDSST 2017), "Data, Information and Knowledge Visualisation in Decision Making", May 29th –31th, Belgium, Namur.
	N. Vafaei, R. A. Ribeiro, L. M. Camarinha-Matos (2020), “Selecting Best Normalization Technique for MCDM Methods“, International Conference on Decision Support Systems Technology (ICDSST 2020), "Cognitive Decision Support Systems & Technologies", May 27th –29th, Spain, Zaragoza.
Panel Participation	**N. Vafaei,** The 10th Advanced Doctoral Conference on Computing, Electrical and Industrial Systems (DoCEIS 2019), “Technological Innovation for Industrial and Service Systems” Portugal, Lisbon, May 8-10, 2019
Awards	EWG-DSS Young Researcher Award (EURO Working Group on Decision Support Systems) award for research about Selecting Best Normalization Technique for MCDM Methods, 2020

Figure 15: Other Academic Contributions/activities.

4.4 Summary

This chapter presented and discussed the validation of the proposed assessment framework, using a Constructive Research method. We performed validations with case studies and illustrative examples, as well as benchmarks and testimonies, to show the framework usefulness in recommending normalization techniques for MCDM methods. Due to the lack of research work and papers in the literature, we could only find three case studies related with our work to perform the benchmarking. The performed validations with case studies, benchmarks as well as testimonies and published papers, demonstrate the effectiveness and applicability of the developed assessment framework. Moreover, the testimonies from experts reveal the novelty of the topic and also the importance and utility of the proposed framework in decision problems and MCDM methods. Furthermore, the published papers also acted as another validation method and showed the strong acceptance of the assessment framework, in the research community, and its positive impact on academic activities.

5

5 Conclusion and Future Works

This chapter summarizes the main findings and new contributions of this research work. Finally, it discusses limitations through different perspectives and lists open issues for future research.

5.1 Summary of the Work

This thesis research work addressed the main research question " What are the characteristics and different steps of an evaluation framework to assess and recommend the more appropriate normalization techniques to use with well-known MCDM methods (SAW, TOPSIS, AHP, ELECTRE)?". The related hypothesis was developed as " If we build a strong assessment framework to identify the best normalization technique for decision problems using well-known MCDM methods then we can ensure more robust results for ranking alternatives in the related decision problems. In other words, this assessment framework should support decision makers by recommending which normalization technique is more appropriate to solve their decision problems".

To answer the above research question and considering the formulated hypothesis, a new assessment framework was developed. This innovative assessment framework evolved during four phases by modifying and refining steps and adding more evaluation metrics. In the final phase of evolution (Phase 4), the following four steps were defined for the proposed framework (Figure 11):

Step 1: Data types: Determine the types of data in the decision problem:

- Benefit or cost criteria
- Criteria values are:
    - Ordinal number
    - Natural number
    - Real number
    - Float numbers in the unit Interval [0-1]

Step 2: Selection of normalization techniques: Choose candidates from the three main categories (linear, semi-linear, non-linear):

- Linear: Max; Max-Min; Sum
- Semi Linear: Vector
- Non Linear: Logarithmic, Fuzzification

Step 3: Evaluation of the techniques: In this step, candidate normalization techniques are assessed with the following chosen metrics:

a) Measures of data dispersion: STD
b) Measure of proximity: Minkowski distances (Euclidean)
c) Comparison metrics: Mean Ks values (from Pearson Correlation); Ranking Consistency Index (RCI); Mean squared error (MSE)

Step 4: Selection of the best techniques: Selection and recommendation of the best technique is done with plurality voting:

- Plurality voting: Selection of the best normalization technique with the large number of first order/rank, in the different used metrics.

The proposed framework helps decision makers to select the best normalization techniques for the decision problem at hand. This framework is flexible enough to add any other normalization techniques and MCDM methods. Furthermore, to develop an automatic decision process with less human intervention, the conceptual model (Figure 12) for the related assessment framework is designed.

To summarize, this thesis was divided into 5 chapters. In the first chapter, the problem statements and motivations for this research and the research question and hypothesis were defined. In the second chapter, the literature background about MCDM methods, normalization techniques and taxonomy for both were presented. also, some insights about dynamic systems and collaborative networks were introduced. The third chapter discussed the proposed framework and used case studies and illustrative examples to explain the well-known assessment tools and metrics. Moreover, the framework's conceptual model for recommending the best normalization technique in MCDM methods and its respective automatic decision process are addressed. The fourth chapter discussed the validation and testing process of the proposed assessment framework to recommend the best normalization technique in MCDM method using case studies, benchmarking, and expert testimonies; as well as accepted research work by the research community. The fifth chapter focused on the main findings and analysis of the obtained results using the proposed assessment framework and mentioned open issues for future research.

5.2 Evaluation

To validate the proposed framework, four different aspects were considered (Camarinha-Matos, 2015):

- ✓ Case studies:
  Five different case studies were implemented, three of them with small scales (4 criteria and 4 alternatives) and two of them with larger scales (one of them 4 criteria with 20 alternatives and the other one 20 criteria with 10 alternatives). From this validation it was demonstrated that the developed assessment framework is applicable for both small and large scales and reaches its mission of recommending the best normalization technique for well-known MCDM methods. Meanwhile, there was an interesting issue about ranking alternatives. using different normalization techniques regarding the number of criteria and alternative in the MCDM decision problems. The preliminary results show that when the numbers of alternatives are less than half of the numbers of criteria, different normalization techniques produce the same rank for final ordering/ranking of alternatives. This is an open issue for further research work and will be listed in the last part of the thesis as future work.

- ✓ Benchmarking:

  Due to the existence of only a few research papers about the related topic of this thesis research, three case studies that are more similar to our work were selected for benchmarking. The obtained results showed up the consistency and robustness of our assessment framework because it concurs with most other author´ results while being tested and validated with several metrics for assessing normalization techniques and providing a final ranking based on a plurality voting model. Furthermore, by using different metrics from different categories ensured more accurate results when compared with results from the literature.

- ✓ Testimonies:

  We presented interesting testimonies from 9 experts, that revised published papers during this thesis research work, about the applicability and utility of the proposed assessment framework.

- ✓ Peers' evaluation:

  The dissemination of the various stages of this research about the topics of this thesis was done through publications in peer-review as conference proceedings, book chapters, journals, and posters' presentation, and panel participation.

5.3 Novel Contributions

This thesis gathered novel contributions from different points of view. First, an assessment framework was developed during the thesis work with usage of several metrics, collected from a broad variety of categories. Also, in this evolutionary research, a new classification for normalization techniques is introduced: linear, semi-linear, and non-linear, which improves the classification of Jahan and Edwards (2015) by adding the semi-linear class , like Target-based and Vector which neither belong to the linear nor non-linear classes. Moreover, it was successfully proposed using plurality voting for recommending the best normalization technique. This is a novel usage of this social sciences method and made the developed framework more robust.

Second, designing a conceptual framework enables decision makers to rank alternatives besides recommending the best normalization technique with an automatic decision process with minimum human intervention. All these new contributions from the developed framework will provide more accurate results for decision problems.

Figure 16 represents the arisen novel contributions regarding the performed research works in this thesis.

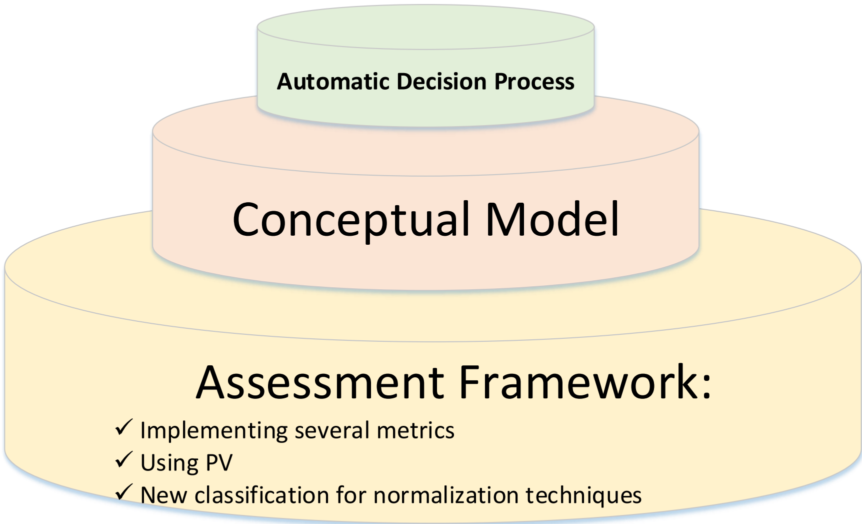


Figure 16: Novel contributions

5.4 Future Works

In this thesis, the evaluation of normalization techniques was done with differenced decision problems and there was no time for generalizing and recommending always the same normalization technique for each MCDM method (SAW, TOPSIS, etc.). The specific characteristics of MCDM methods imposed some limitations for the generalization of the results of this research work. For example, some elements should be changed for simulation (which is one the generalization methods) (i) adding/removing alternatives which may cause rank reversal and led to changes on the study concept; (ii) changing values of criteria/alternatives can cause changes on the decision matrix of the case study, which is not considered in the MCDM methods; (iii) changing weights of criteria may influence effects on ranking alternatives, here we simplified to equal weights for all criteria. All these limitations are interesting topics requiring future research.

Checking for normal distributions in section 3.1.1.1 was another limitation of this thesis work. For confirming normal distribution, using Kolmogorov-Smirnov test, the result of statistical test should be less than 1 and the result of significant level test (Sig) should be more than 0.05 (sig > 0.05) (Field, 2000; Trochim and Donnelly, 2006). In the applied case study for TOPSIS (section 3.1.1.1), we were faced with a normalization technique with statistic<1 and Sig<0.05. In this kind of cases, we cannot judge the normal distributions of normalized values. In addition, it remained some doubts about the requirement for normal distribution in the normalized values, which will be addressed in future work.

Furthermore, there were some challenges in using some statistical analysis, namely ANOVA and Regression analysis (P-value, T-stat, and Standard Error), on aggregation of data sets for comparing them with statistical analysis of input data sets. In other words, this comparison (between two data sets) would be available if these statistical analytics were applied for a single criterion of both data set. Hence, the final focus of this thesis was to recommend the best normalization technique for aggregated data sets (alternatives rating), therefore, using more research on this topic will be needed.

In addition, considering the limitations and challenges from different aspects of this thesis, some open issues for new research and future works are listed below:

- Add more metrics to the proposed assessment framework to improve the consistency of results. Namely, metrics from statistics that could be useful for input data and aggregated data sets using MCDM methods.
- Apply the proposed assessment framework to other relatively known MCDM methods such as COPRAS, MOORA, PROMETHEE, etc. besides SAW, TOPSIS, ELECTRE, and AHP.
- Analyze the effects of other normalization techniques (ex. z-transformation, etc.) on MCDM methods.

- Classify other normalization techniques in the new categories proposed by the authors (Linear, Semi-linear, and Non-linear).
- Explore a suitable model to generalize results for each MCDM method.
- Extend the application of the proposed assessment framework for the real projects, especially with the presence of big data, and analyze the results.
- Study different data distributions for input data and normalized values in MCDM decision models.
- Analyze and assess the effects of different input data (e.g. decimal numbers, zero, complex numbers, outlier, etc.) on normalization techniques and determine limitations for each normalization technique considering the input data.
- Explore a suitable model to generalize results for each MCDM method
- Implement automatic calculations for the whole assessment framework (e.g. for RCI index) to make it more user-friendly.
- Explore the relation between the number of alternatives and criteria on the ranking/ordering of alternatives using different normalization techniques.

Also, extending the application of the assessment framework to real projects, especially with big data could improve the validation of this research work. Finally, tool implementation (software) for the proposed framework will bring the opportunity of easy usage by all users from different disciplines.

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