
FISCAL POLICY, CONGESTION, AND ENDOGENOUS GROWTH

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Abstract

We devise an endogenous growth model with private and public physical capital, and human capital, which allows for relative and absolute congestion. According to empirical evidence, long-run growth is invariant to fiscal policy. Despite its complexity, the dynamics of the market economy and the centralized economy are analyzed in detail. We show that an increase in absolute congestion reduces the long-run growth rate of output. In contrast, relative congestion does not affect long-run growth. In the absence of congestion, it is optimal to use lump-sum taxation, and with congestion it is optimal to also tax income.

1. Introduction

Since the stimulating work of Aschauer (1989a,b), the effect of public investment has been the subject of active research in the literature on endogenous growth. Following the seminal work of Barro (1990), the bulk of this literature has regarded the current flow of public investment as a productive input in private production (e.g., Barro and Sala-i-Martin 1992; Jones, Manuelli, and Rossi 1993; Turnovsky 1996a, 1996b, 2000; Eicher and Turnovsky 2000). However, as long as productive government expenditures are intended to represent public infrastructure, it is the accumulated stock, rather than the current flow, that should be considered as the source of contribution to productive capacity. Futagami, Morita, and Shibata (1993) modify the Barro (1990) model by introducing the stock of public capital as a purely public good affecting

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the productivity of firms. They analyze a decentralized economy and show that it features transitional dynamics, in contrast to models in which public expenditure enters production as a flow, when the economy is always on its balanced growth path. Glomm and Ravikumar (1994, 1997) take account of the congestion associated with public goods. In their model, private and public capital fully depreciates each period, and so, the model behaves like the Barro's (1990) model. In particular, it does not feature transitional dynamics. Turnovsky (1997) extends the model of Futagami et al. (1993) to consider congestion and a more complete array of fiscal instruments. Gómez (2004) devises a fiscal policy that allows the optimal growth path to be decentralized when investment is irreversible.

The models of Futagami et al. (1993) and Turnovsky (1997) are limited in some respects. First, they use the simplest one-sector Ak endogenous growth model as a framework to analyze the effect of public investment. Therefore, it would be interesting to examine whether the results obtained carry over to a more general setting. Recent endogenous growth theories have attributed human capital an important role in the process of economic growth (e.g., Lucas 1988; Caballe and Santos 1993) and, although the empirical evidence is somewhat mixed, it seems to be ultimately encouraging that human capital does have a substantial impact on productivity (e.g., Krueger and Lindahl 2002). Thus, we shall analyze the effect of public capital and congestion in a model of endogenous growth that features human capital accumulation. Second, the knife-edge assumption of constant returns-to-scale in the accumulated factors of the Ak model allowed Turnovsky (1997) to introduce relative congestion but not absolute congestion in the analysis. With relative congestion, the service of public capital to the agent depends on the usage of her capital stock relative to the aggregate capital stock; with absolute congestion, the services derived by the agent from the provision of a public good depend on the aggregate capital stock. This issue is important because it has been shown (e.g., Barro and Sala-i-Martin 1992; Eicher and Turnovsky 2000) that relative and absolute congestion can have rather different effects on many aspects of the growth process. Third, the empirical evidence reported, e.g., by Levine and Renelt (1992), Stokey and Rebelo (1995), Jones (1995a), Mendoza, Milesi-Ferretti, and Asea (1997), and Tanzi and Zee (1997), supports the invariance of long-run growth to fiscal policy. This result differs from the theoretical implications of the models of Futagami et al. (1993) and Turnovsky (1997).

This paper extends the Turnovsky (1997) model to overcome the shortcomings pointed out above. We develop a model with human capital as well as public and private physical capital as productive inputs in the economy, in which public capital is allowed to be subject to absolute congestion as well as relative congestion. The theoretical implications of our model will be shown to be consistent with the invariance of long-run growth to fiscal policy, which is supported by the empirical evidence cited previously.

First, we analyze the equilibrium dynamics of the market economy. The transitional dynamics of the model is represented by a two-dimensional stable saddle-path. As Eicher and Turnovsky (2001) point out, this provides a much richer dynamics for the transition paths relative to the standard endogenous growth model (e.g., Bond, Wang, and Yip 1996), and also to the models of Turnovsky (1997) and Eicher and Turnovsky (2000), which feature a single stable root and a one-dimensional stable manifold. Stability of the steady state is independent of the level of relative congestion, and also of the level of absolute congestion in the (plausible) case that the elasticity of intertemporal substitution is lower than or equal to one. In accordance with the empirical evidence cited above, changes in public investment and income taxation do not affect long-run growth. We also examine the effects of both types of congestion on long-run growth. An increase in absolute congestion reduces the long-run growth rate of output and physical capital, while the positive or negative effect on the long-run growth rate of human capital depends on the size of the elasticity of intertemporal substitution. In contrast, relative congestion does not affect long-run growth if population is constant.

Next, we analyze how the optimal growth path attainable by a central planner can be decentralized. We show that an income tax, combined with lump-sum taxation, can correct for the negative external effects caused by both absolute and relative congestion. The optimal income tax is constant both along the transition and in the steady state. It is positive as long as there is congestion because the agent ignores the negative externality caused by congestion and overaccumulates physical capital relative to the optimum. In the absence of relative and absolute congestion, the optimal income tax is zero so public investment should be financed with lump-sum taxation.

The remainder of this paper is organized as follows. Section 2 analyzes the equilibrium dynamics of the decentralized economy. Section 3 analyzes the optimal growth path attainable by a central planner. Section 4 determines an optimal fiscal policy capable of decentralizing the optimal growth path. Finally, some concluding remarks are provided in Section 5.

2. The Decentralized Economy

The economy is inhabited by a large but fixed number N of identical infinitely lived agents, each of whom has an infinite planning horizon and possesses perfect foresight. The agent is endowed with one unit of time per period which can be allocated to work, u , or learning, $1 - u$.

2.1. Goods Production

Each individual firm produces output, y , in accordance with the Cobb–Douglas technology

$$y = Ak^\alpha (uh)^\beta K_s^\phi \quad 0 < \alpha < 1, \quad 0 < \beta < 1, \quad \alpha + \beta \leq 1, \quad 0 \leq \phi, \quad (1)$$

where k is the agent's stock of physical capital, h is the agent's stock of human capital, and K_s denotes the services derived by the agent from the stock of public physical capital. The term uh is referred to as effective labor. Following Eicher and Turnovsky (2000), the services derived by the individual firm from the aggregate public capital stock, K_g , are represented by

$$K_s = K_g (k/K)^{\theta_R} K^{-\theta_A}, \quad \theta_R \geq 0, \quad \theta_A \geq 0, \quad (2)$$

where $K = Nk$ denotes the aggregate private physical capital stock.

As Eicher and Turnovsky (2000) argue, the specification of public services (2) comprises three categories. The first one is the no-congestion case, $\theta_A = \theta_R = 0$, so that $K_s = K_g$. This case corresponds to a nonrival and nonexcludable public capital good that is available equally to each agent, independent of the usage of others. As Barro and Sala-i-Martin (1992) argue, almost all public services are subject to some degree of congestion, so that the pure public good should be viewed as only a benchmark.

The second category, $\theta_R > 0$ and $\theta_A = 0$, is referred to as pure relative congestion.¹ With relative congestion, a fixed level of public capital stock, K_g , would provide a constant level of public services, K_s , to the agent if and only if the usage of her individual capital stock increases in proportion to the usage of the aggregate capital stock. Congestion increases if aggregate usage increases relative to individual usage. The specification (2) entails that the agent can maintain a constant level of public services, given her private capital stock k , if and only if public capital grows in proportion to the aggregate private capital in accordance with $K_g/K_g = \theta_R \dot{K}/K$. Therefore, the parameter θ_R measures the degree of relative congestion. As Eicher and Turnovsky (2000) argue, an example of public good subject to relative congestion might be highways. In the special case of proportional relative congestion, $\theta_R = 1$, $\theta_A = 0$, congestion increases in direct proportion to the size of the economy, and the public good is like a private good in that the individual receives her proportional share of public capital, $K_s = K_g/N$.

The third category, $\theta_A > 0$ and $\theta_R = 0$, corresponds to pure absolute congestion, in which congestion is directly proportional to the aggregate level of private capital.² An example of public good subject to absolute congestion might be local policy services or fire protection services. The case $\theta_A > 0$ and $\theta_R > 0$ corresponds to a combination of both absolute and relative congestion. Although other specifications of congestion have been

¹This specification of relative congestion has also been used by Turnovsky (1996b, 1997, 1998), who points out that it is the standard formulation of the median voter model of congestion (e.g., Edwards 1990).

²Barro and Sala-i-Martin (1992) and Glomm and Ravikumar (1994, 1997) also consider absolute congestion.

considered in the literature,³ the specification (2) is relatively standard and general.⁴

Combining (1) and (2), output of the individual firm can be expressed as

$$y = Ak^{\alpha+\phi\theta_R}(uh)^\beta K_g^\phi K^{-\phi(\theta_R+\theta_A)}. \quad (3a)$$

Thus, the productivity of individual physical capital depends upon the usual elasticity of private physical capital, α , and a component, $\phi\theta_R$, which reflects the fact that, from the perspective of the individual agent, increasing her stock of physical capital will increase the level of government services she derives in the presence of relative congestion. Aggregate output, $Y = Ny$, is given by

$$Y = AN^{1-\alpha-\beta-\phi\theta_R} K^{\alpha-\phi\theta_A} (uH)^\beta K_g^\phi = \bar{A}K^{\alpha-\phi\theta_A} (uH)^\beta K_g^\phi, \quad (3b)$$

where $H = Nh$ is the aggregate stock of human capital. We assume that the production function of the individual firm (3a) exhibits nonincreasing returns-to-scale to private inputs,

$$\alpha + \beta + \phi\theta_R \leq 1. \quad (4a)$$

Furthermore, to ensure that private physical capital is productive in the aggregate economy, we impose the following condition:

$$\alpha - \phi\theta_A > 0. \quad (4b)$$

Note that the presence of congestion introduces a distortion because the individual firm takes aggregate capital K as given in her production function (3a). However, the central planner takes into account that $K = Nk$, and thus she considers the aggregate production function (3b).

2.2. The Government

We shall assume that the government claims a fraction g of output for investment. The evolution of public capital is given by

$$\dot{K}_g = gY, \quad 0 \leq g < 1. \quad (5)$$

The government finances its investment by using either income taxation at a rate τ , or lump-sum taxation, s , in accordance with its flow budget constraint:

$$\tau y + s = gy. \quad (6)$$

³For example, public capital might be congested by aggregate output (Turnovsky 1996a) or employment (Glomm and Ravikumar 1994, 1997).

⁴As Eicher and Turnovsky (2000) show, it encompasses as particular cases the three specifications adopted by Barro and Sala-i-Martin (1992).

2.3. Individual Optimization

The agent derives utility from the consumption of a private consumption good, c , according to the isoelastic utility function

$$\int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt \quad \rho > 0, \sigma > 0. \quad (7)$$

Here, ρ is the rate of time preference and $1/\sigma$ is the elasticity of intertemporal substitution.

Human capital, h , accumulates according to the Uzawa (1965)–Lucas (1988) technology, so that effective time is the only input in human capital accumulation,

$$\dot{h} = \delta(1 - u)h \quad \delta > 0, \quad (8)$$

where δ is the productivity parameter in the educational sector.⁵ The agent maximizes her utility function (7) subject to the constraint on private physical capital accumulation,

$$\dot{k} = i_k, \quad (9)$$

where i_k denotes private investment on physical capital, the constraint on human capital accumulation (8), and the agent's budget constraint

$$(1 - \tau)y = i_k + c + s, \quad (10)$$

taking as given the aggregate stocks of private physical capital K and public physical capital K_g in (3a), and the initial conditions $k(0) > 0$ and $h(0) > 0$.⁶

2.4. Equilibrium Dynamics

A competitive equilibrium for this economy is as a set of paths $\{c(t), i_k(t), u(t), k(t), h(t)\}$ that solves the agent's utility maximization problem and such that the government obeys its budget constraint. The current value Lagrangian of the agent's problem is

$$L = (c^{1-\sigma} - 1)/(1 - \sigma) + \lambda i_k + \mu \delta(1 - u)h + \eta [(1 - \tau)y - i_k - c - s],$$

where λ and μ are the shadow prices of private physical capital and human capital, respectively, and η is the multiplier associated with the agent's budget constraint. The first-order conditions are

⁵Using such technology in the educational sector allows endogenous growth to be compatible with any arbitrary degree of returns to scale to the reproducible factors in the goods producing technology (see, e.g., Mulligan and Sala-i-Martin 1993). Therefore, we can consider the case in which public capital is subject to absolute congestion as well as relative congestion.

⁶For simplicity, we have abstracted from the depreciation of the stocks of physical and human capital.

$$\partial L/\partial c = c^{-\sigma} - \eta = 0, \quad (11a)$$

$$\partial L/\partial i_k = \lambda - \eta \leq 0, \quad i_k \geq 0, \quad (\lambda - \eta)i_k = 0, \quad (11b)$$

$$\partial L/\partial u = \eta(1 - \tau)\beta y/u - \mu\delta h = 0, \quad (11c)$$

$$\partial L/\partial k = \eta(1 - \tau)(\alpha + \phi\theta_R)y/k = \rho\lambda - \dot{\lambda}, \quad (11d)$$

$$\partial L/\partial h = \eta(1 - \tau)\beta y/h + \mu\delta(1 - u) = \rho\mu - \dot{\mu}, \quad (11e)$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = \lim_{t \rightarrow \infty} \mu h e^{-\rho t} = 0. \quad (11f)$$

Suppose that the constraint on nonnegative gross investment is not binding, so that (11b) entails that $\lambda = \eta$, and let $w = \mu/\lambda$ denote the value of human capital measured in terms of private physical capital. Equation (11a) equates marginal utility to the shadow value of an additional unit of private physical capital. Equation (11c) can be expressed as

$$(1 - \tau)\beta \frac{y}{uh} = w\delta, \quad (12a)$$

which states that the after-tax rate of return on effective time, valued in terms of private physical capital as numeraire, must be the same in both sectors. Equation (11d), which can be expressed as

$$(1 - \tau)(\alpha + \phi\theta_R) \frac{y}{k} = \rho - \frac{\dot{\lambda}}{\lambda}, \quad (12b)$$

equates the after-tax rate of return on private physical capital to the rate of return on consumption. This expression takes into consideration that a higher relative congestion raises the productivity of private physical capital since increasing the agent's private capital entails an increase in the quantity of productive services derived from public capital. Equation (11e), which can be expressed as

$$\frac{1}{w}(1 - \tau)\beta \frac{y}{h} + \delta(1 - u) + \frac{\dot{w}}{w} = \rho - \frac{\dot{\lambda}}{\lambda}, \quad (12c)$$

equates the rate of return on human capital, both in the goods and the educational sectors, to the rate of return on consumption. Now, the return to investing in human capital includes the rate of capital gains, \dot{w}/w .

In what follows, the aggregation conditions $Ny = Y$, $Nk = K$, $Nh = H$, and $Nc = C$ will be imposed. Hereafter, let $\gamma_p = \dot{p}/p$ denote the growth rate of a variable p . Log-differentiating (11a), (11c), and (3b) with respect to time we obtain, respectively,

$$\gamma_c = -\gamma_\eta/\sigma, \quad (13)$$

$$\gamma_Y - \gamma_u + \gamma_\eta - \dot{\tau}/(1 - \tau) = \gamma_\mu + \gamma_H, \quad (14)$$

$$\gamma_Y = (\alpha - \phi\theta_A)\gamma_K + \beta\gamma_u + \beta\gamma_H + \phi\gamma_{K_g}. \quad (15)$$

From (11c) and (11e), we get

$$\gamma_\mu = \rho - \delta. \quad (16)$$

When the constraint on nonnegative gross investment, $i_k \geq 0$, is not binding, (11b) entails that $\lambda = \eta$. Combining the individual budget constraint (10) with the government budget constraint (6), multiplying by N , and substituting the private investment in physical capital from the resulting expression into (9), aggregate private physical capital in the economy is accumulated in accordance with the product market equilibrium condition:

$$\gamma_K = (1 - g)Y/K - C/K. \quad (17)$$

Since $\gamma_\lambda = \gamma_\eta$, from (13) and (11d) we derive the Keynes–Ramsey rule of optimal consumption as

$$\gamma_C = \frac{1}{\sigma}[(1 - \tau)(\alpha + \phi\theta_R)Y/K - \rho]. \quad (18)$$

The system that characterizes the dynamics of the decentralized economy in terms of the variables $q = C/K$, $z = K/K_g$, u , and $r = Y/K$, that are constant in the steady state, is

$$\gamma_q = \gamma_C - \gamma_K = \frac{1}{\sigma}(1 - \tau)(\alpha + \phi\theta_R)r - (1 - g)r + q - \frac{\rho}{\sigma}, \quad (19a)$$

$$\gamma_z = \gamma_K - \gamma_{K_g} = (1 - g)r - q - grz, \quad (19b)$$

$$\begin{aligned} \gamma_u = \delta u - & \frac{[\phi(\theta_A + \theta_R) + g(\alpha - \phi\theta_A) - \tau(\alpha + \phi\theta_R)]r}{1 - \beta} \\ & - \frac{(\alpha - \phi\theta_A)q}{1 - \beta} + \frac{g\phi zr}{1 - \beta} + \frac{\delta\beta}{1 - \beta} - \frac{\dot{\tau}}{(1 - \tau)(1 - \beta)}, \end{aligned} \quad (19c)$$

$$\begin{aligned} \gamma_r = \gamma_Y - \gamma_K = & - \frac{[(1 - g)(1 - \alpha - \beta + \phi\theta_A) + \beta(1 - \tau)(\alpha + \phi\theta_R)]r}{1 - \beta} \\ & + \frac{(1 - \alpha - \beta + \phi\theta_A)q}{1 - \beta} + \frac{g\phi zr}{1 - \beta} + \frac{\delta\beta}{1 - \beta} - \frac{\beta\dot{\tau}}{(1 - \tau)(1 - \beta)}. \end{aligned} \quad (19d)$$

Equation (19a) is obtained from (17) and (18). Equation (19b) results from (17) and (5). Solving the system (14)–(15) for γ_u and γ_Y , and substituting $\gamma_\eta = \gamma_\lambda$ from (11d), γ_μ from (16), γ_K from (17), γ_H from (8), and γ_{K_g} from (5), after some algebra, we get (19c) and (19d).

When the constraint on nonnegative private investment, $i_k \geq 0$, is binding, log-differentiating the overall resources constraint, $(1 - g)Y = C$, with respect to time yields

$$\gamma_C = \gamma_Y - \dot{g}/(1 - g). \quad (20)$$

The system that drives the dynamics of the decentralized economy is $q = (1 - g)r$, and

$$\gamma_z = -grz, \tag{21a}$$

$$\begin{aligned} \gamma_u = & \delta u + \frac{(1 - \sigma)\phi}{1 - \beta + \beta\sigma} grz + \frac{\delta\beta(1 - \sigma) - \rho}{1 - \beta + \beta\sigma} \\ & + \frac{\sigma \dot{g}}{(1 - g)(1 - \beta + \beta\sigma)} - \frac{\dot{\tau}}{(1 - \tau)(1 - \beta + \beta\sigma)}, \end{aligned} \tag{21b}$$

$$\begin{aligned} \gamma_r = & \frac{\phi grz}{1 - \beta + \beta\sigma} + \frac{\beta(\delta - \rho)}{1 - \beta + \beta\sigma} + \frac{\beta\sigma \dot{g}}{(1 - g)(1 - \beta + \beta\sigma)} \\ & - \frac{\beta \dot{\tau}}{(1 - \tau)(1 - \beta + \beta\sigma)}. \end{aligned} \tag{21c}$$

Equation (21a) results from (5) and $\gamma_K = 0$. Substituting γ_C from (20) into (13), we get $\gamma_\eta = -\sigma\gamma_Y + \sigma \dot{g}/(1 - g)$. Substituting γ_η from this equation into (14), solving the system (14)–(15) for γ_u and γ_Y , and substituting γ_μ from (16), γ_H from (8), γ_{K_g} from (5), and $\gamma_K = 0$, into the resulting expressions, we obtain (21b) and (21c), using that $\gamma_r = \gamma_Y$.

2.5. Balanced Growth Path

Now, we focus on an interior balanced growth path (or steady state) in which all variables grow at constant but possibly different rates, the shares of labor in its different uses are constant, and the fiscal policy parameters, g and τ , are stationary, i.e., $\dot{\tau} = 0$ and $\dot{g} = 0$. A hat “ $\hat{\cdot}$ ” over a variable will denote its steady-state value. The following proposition examines the existence of an interior steady state with positive long-run growth in which, therefore, the nonnegativity constraint on gross investment must be not binding.

PROPOSITION 1: *Let $1 - \alpha - \phi + \phi\theta_A > 0$. The decentralized economy has a unique steady-state with positive long-run growth,*

$$\hat{r} = \frac{\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + (\delta - \rho)\beta}{(1 - \tau)(\alpha + \phi\theta_R)(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma)}, \tag{22a}$$

$$\hat{q} = \frac{(1 - g)[\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + \beta(\delta - \rho)] - (1 - \tau)\beta(\delta - \rho)(\alpha + \phi\theta_R)}{(1 - \tau)(\alpha + \phi\theta_R)(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma)}, \tag{22b}$$

$$\hat{z} = \frac{(1 - \tau)\beta(\delta - \rho)(\alpha + \phi\theta_R)}{g[\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + (\delta - \rho)\beta]}. \tag{22c}$$

$$\hat{u} = \frac{\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A)}{\delta(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma)}, \tag{22d}$$

where the long-run growth rate of human capital equals

$$\hat{\gamma}_H = \delta(1 - \hat{u}) = \frac{(\delta - \rho)(1 - \alpha - \phi + \phi\theta_A)}{1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma}, \quad (23a)$$

and the long-run growth rate of income, consumption, private and public physical capital is

$$\hat{\gamma}_K = \frac{\beta}{1 - \alpha - \phi + \phi\theta_A} \hat{\gamma}_H = \frac{\beta(\delta - \rho)}{1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma} = \hat{\gamma}_C = \hat{\gamma}_Y = \hat{\gamma}_{K_g}, \quad (23b)$$

if and only if

$$\begin{aligned} \text{sign}(\delta - \rho) &= \text{sign}(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma) \\ &= \text{sign}[\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A)] \\ &= \text{sign}\{(1 - g)[\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + \beta(\delta - \rho)] \\ &\quad - (1 - \tau)\beta(\delta - \rho)(\alpha + \phi\theta_R)\}. \end{aligned} \quad (24)$$

Proof: See Appendix.

When there are constant returns to scale to private inputs in goods production and no externalities, the long-run growth rate in the Uzawa-Lucas model is independent of technological parameters in the goods sector (see, e.g., Barro and Sala-i-Martin 2004, section 5.2.2). However, when the Uzawa-Lucas model is extended to include public capital, Equations (23a) and (23b) show that the long-run growth rates depend on technological parameters of the production function in the goods sector even if there are constant returns to scale in private inputs both at the private level, $\alpha + \phi\theta_R + \beta = 1$, or at the social level, $\alpha - \phi\theta_A + \beta = 1$. In this latter case, the long-run growth rate of output would be $\hat{\gamma}_K = \beta(\delta - \rho)/(\beta\sigma - \phi)$. Only if there are constant returns to scale in the reproducible inputs, K , H , and K_g , in the aggregate, $\alpha - \phi\theta_A + \beta + \phi = 1$, long-run growth would be independent on the technological parameters of the goods sector, and the long-run growth rate would be simply given, as in the Uzawa-Lucas model, by $\hat{\gamma}_K = \hat{\gamma}_H = (\delta - \rho)/\sigma$.

It should be noted that the long-run growth rates of physical and human capital, along with the after-tax marginal return on private physical capital, $\hat{i} = (1 - \tau)(\alpha + \phi\theta_R)\hat{r}$, can be obtained by solving the following system:

$$\hat{\gamma}_K = (\hat{i} - \rho)/\sigma, \quad (25a)$$

$$\hat{\gamma}_K = \beta\hat{\gamma}_H/(1 - \alpha - \phi + \phi\theta_A), \quad (25b)$$

$$\hat{\gamma}_K - \hat{\gamma}_H = \hat{i} - \delta, \quad (25c)$$

where (25a) comes from (18) using that at the steady state $\hat{\gamma}_K = \hat{\gamma}_C$; (25b) results from evaluating (15) at the steady state using that $\hat{\gamma}_K = \hat{\gamma}_Y = \hat{\gamma}_{K_g}$ and

$\hat{\gamma}_u = 0$, and (25c) results from evaluating (14) at the steady state, using (16) and (11d) (with $\eta = \lambda$).

Equations (23a) and (23b) show that changes in income taxation and public investment do not affect the long-run growth rates. The invariance of long-run growth to fiscal policy follows from the assumption that the technology in the educational sector is linear in effective time. The intuition behind this result can be more easily ascertained by focusing on the case that the goods producing technology exhibit constant returns to scale to the reproducible factors K , H , and K_g , in the aggregate, $\alpha - \phi\theta_A + \beta + \phi = 1$. In this case, Equation (25b) shows that the long-run growth rates of physical and human capital coincide, $\hat{\gamma}_K = \hat{\gamma}_H$. Equation (25c) states then that in the steady state the after-tax marginal return on private physical capital in goods production is equal to the marginal return on effective time in the education sector which, given the linearity assumption, is the constant δ . Therefore, the common long-run growth rate of income, capital and consumption would be given simply by the Keynes–Ramsey rule of optimal consumption as $\hat{\gamma}_K = (\delta - \rho)/\sigma$, and would not depend upon fiscal policy parameters. If goods production does not exhibit constant returns to scale to the reproducible factors, a similar reasoning applies, as the simultaneous determination of $\hat{\gamma}_K$, $\hat{\gamma}_H$, and \hat{i} via (25a)–(25c) only involves technology and preference parameters (and not fiscal policy parameters). It should be stressed that the invariance of long-run growth to fiscal policy does not mean that it has no impact on the economy. Actually, fiscal policy affects the transitional dynamics of the economy, as system (19) clearly shows. Furthermore, Equations (22a)–(22c) show that fiscal policy also affects the steady state ratios of output to private physical capital, \hat{r} , consumption to private physical capital, \hat{q} , and private to public physical capital, \hat{z} .

From (23a) and (23b), we can derive the effect of absolute congestion on long-run growth,

$$\partial\hat{\gamma}_K/\partial\theta_A = -\phi\hat{\gamma}_K/(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma) < 0, \quad (26a)$$

$$\partial\hat{\gamma}_H/\partial\theta_A = (1 - \sigma)(\partial\hat{\gamma}_K/\partial\theta_A), \quad (26b)$$

where the sign in (26a) has been derived by assuming that the condition (28) for local saddle-path stability derived later is fulfilled. Hence, an increase in absolute congestion adversely affects the long-run growth rate of physical capital (and income) since it lowers the aggregate productivity in the economy, as in the model of Eicher and Turnovsky (2000). However, its effect on the growth rate of human capital depends on the elasticity of intertemporal substitution.⁷ As the value of σ increases the agents are less inclined to substitute intertemporally and are less responsive to changes in the interest rate. Since the net marginal return on private physical capital is given

⁷Note that Eicher and Turnovsky (2000) do not consider human capital in their model.

by $\hat{i} = (1 - \tau)(\alpha + \phi\theta_R)\hat{r}$, we can get that $\partial\hat{i}/\partial\theta_A = \sigma(\partial\hat{\gamma}_K/\partial\theta_A)$. Therefore, after an increase in absolute congestion, the reduction of $\hat{\gamma}_K$ will be lower (greater) than that of \hat{i} if $\sigma > 1$ ($\sigma < 1$). Given that $\hat{\gamma}_K - \hat{\gamma}_H = \hat{i} - \delta$ from (25c), an increase in absolute congestion will reduce $\hat{\gamma}_H$ if $\sigma < 1$, and will increase $\hat{\gamma}_H$ in the more plausible case that $\sigma > 1$. In the logarithmic utility case, $\sigma = 1$, absolute congestion does not affect $\hat{\gamma}_H$.

An increase in relative congestion does not affect the aggregate productivity in the economy (see Equation (3b)) and, therefore, does not affect long-run growth,⁸

$$\partial\hat{\gamma}_K/\partial\theta_R = \partial\hat{\gamma}_H/\partial\theta_R = 0. \quad (27)$$

Eicher and Turnovsky (2000) find instead that relative congestion adversely affects the long-run growth rate of (physical) capital and income, but this would also be the case in our model if population grows at a constant rate (see footnote 8). In contrast, relative congestion positively affects long-run growth in the model of Turnovsky (1997), but this result depends on the assumption that population is constant and normalized to unity (see Turnovsky 1997, footnote 18).

2.6. Stability Analysis

As usual, we assume that the capital stocks move sluggishly, so that $K(0)$, $H(0)$, and $K_g(0)$ are given by their historical values. Local saddle-path stability of the steady state can be ensured if the coefficient matrix of the linearization of the system (19) around its steady state $(\hat{r}, \hat{q}, \hat{u}, \hat{z})$ has two stable eigenvalues. We can state the following proposition.

PROPOSITION 2: *The steady state of the decentralized economy is locally saddle-path stable if and only if*

$$\text{sign}(\delta - \rho) = \text{sign}[\delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A)] = +1. \quad (28)$$

Proof: See Appendix.

Propositions 1 and 2 demonstrate that there exists a unique locally saddle-path stable steady-state equilibrium with positive long-run growth under mild assumptions. The transitional dynamics of the model are then represented

⁸This result relies on the assumption that population is constant. If population grows at a constant rate, it can be easily shown that $\partial\hat{\gamma}_K/\partial\theta_R < 0$, so that relative congestion would adversely affect the long-run growth rate of aggregate physical capital, and $\partial\hat{\gamma}_H/\partial\theta_R = (1 - \sigma)\partial\hat{\gamma}_K/\partial\theta_R$, so that its effect on the long-run growth rate of H would depend upon the elasticity of intertemporal substitution. The effect of absolute congestion on the long-run growth rates would be described again by (26).

by a two-dimensional stable saddle-path. This provides a much richer dynamics for the transition paths relative to the models of Turnovsky (1997) and Eicher and Turnovsky (2000), which feature a single stable root and a one-dimensional stable manifold. Proposition 2 also shows that stability is independent of relative congestion, as in the models of Turnovsky (1997) and Eicher and Turnovsky (2000), and is also independent of absolute congestion in the (empirically more plausible) case that $\sigma \geq 1$, given the assumption that $\alpha - \phi\theta_A + \phi < 1$ (see Proposition 1).⁹ If $\sigma < 1$, for given parameter values, stability is more likely the greater the absolute congestion parameter is.

3. The Centrally Planned Economy

The central planner possesses complete information and chooses all quantities directly, taking all the relevant information into account. In particular, the social planner takes into account that $K = Nk$, and thus she considers the aggregate production function (3b). The central planner maximizes the utility of the representative agent

$$\int_0^{\infty} e^{-\rho t} \frac{(C/N)^{1-\sigma} - 1}{1-\sigma} dt, \quad (29a)$$

where C denotes aggregate consumption, subject to the constraints on human capital, private and public physical capital accumulation, and the overall resources constraint,

$$\dot{H} = \delta(1 - u)H, \quad (29b)$$

$$\dot{K} = I_K, \quad (29c)$$

$$\dot{K}_g = G, \quad (29d)$$

$$Y = I_K + G + C, \quad (29e)$$

and the irreversibility constraints $I_K \geq 0$ and $G \geq 0$, where I_K and G denote private and public investment in physical capital. An optimal growth path is as a set of paths $\{C(t), I_K(t), G(t), u(t), K(t), K_g(t), H(t)\}$ that solves the planner's utility maximization problem. The central planner sets public investment, G , in an optimal manner, which is equivalent to setting optimally the ratio of public investment to output, $g = G/Y$. Hence, Equation (29d) can be expressed equivalently as $\dot{K}_g = gY$, and the resource constraint Equation (29e) as $(1 - g)Y = I_K + C$. The constraint on nonnegative public investment is then $G = gY \geq 0$.

⁹This is precisely the stability condition in the model of Eicher and Turnovsky (2000).

Let L be the current value Lagrangian of the planner's problem,

$$L = [(C/N)^{1-\sigma} - 1]/(1 - \sigma) + \lambda I_K + \mu \delta(1 - u)H \\ + \psi gY + \eta[(1 - g)Y - I_K - C],$$

where λ , μ , and ψ are the shadow prices of private physical capital, human capital and public physical capital, respectively, and η is the multiplier associated with the economy's resources constraint. The first-order conditions are

$$\partial L/\partial C = N^{\sigma-1}C^{-\sigma} - \eta = 0, \quad (30a)$$

$$\partial L/\partial I_K = \lambda - \eta \leq 0, \quad I_K \geq 0, \quad (\lambda - \eta)I_K = 0, \quad (30b)$$

$$\partial L/\partial g = (\psi - \eta)Y \leq 0, \quad g \geq 0, \quad (\psi - \eta)gY = 0, \quad (30c)$$

$$\partial L/\partial u = [\eta(1 - g) + \psi g]\beta Y/u - \mu \delta H = 0, \quad (30d)$$

$$\partial L/\partial K = [\eta(1 - g) + \psi g](\alpha - \phi\theta_A)Y/K = \rho\lambda - \dot{\lambda}, \quad (30e)$$

$$\partial L/\partial H = [\eta(1 - g) + \psi g]\beta Y/H + \mu \delta(1 - u) = \rho\mu - \dot{\mu}, \quad (30f)$$

$$\partial L/\partial K_g = [\eta(1 - g) + \psi g]\phi Y/K_g = \rho\psi - \dot{\psi}, \quad (30g)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \lambda K e^{-\rho t} = \lim_{t \rightarrow \infty} \mu H e^{-\rho t} = \lim_{t \rightarrow \infty} \psi K_g e^{-\rho t} = 0. \quad (30h)$$

As in the case of the market economy, suppose that the nonnegativity constraint on private investment in physical capital is not binding, so that Equation (30b) entails that $\eta = \lambda$. Let $v = \psi/\lambda$ and $w = \mu/\lambda$ denote the value of public physical capital and human capital, respectively, measured in terms of units of private physical capital. Comparing (30) with (11), it can be observed that the main difference is that the relevant rates of return in the market economy are the private ones, while in the centralized economy they are the social ones. Equation (30a) equates the marginal utility of consumption to the (social) shadow value of an additional unit of private physical capital. Equation (30d) can be expressed as

$$(1 - g + vg)\beta \frac{Y}{uH} = w\delta, \quad (31a)$$

which states that the (social) rate of return on effective time, valued in terms of private physical capital as numeraire, must be the same in both sectors. The left-hand side of (31a) comprises three components. The first is the marginal productivity of human capital employed in the production of goods, $\beta Y/(uH)$. As government expenditure is tied to output according to $G = gY$,

an increase in human capital employed in the goods sector also induces an increase in public physical capital, $g\beta Y/(uH)$, which is valued at its imputed real price v . The third term, $-g\beta Y/(uH)$, accounts for the resource costs embodied in the public physical capital valued at its price of unity. In equilibrium, the sum of these components must equal the return on human capital employed in the educational sector, δ , measured in terms of private physical capital as numeraire. Equation (30e), which can be expressed as

$$(1 - g + vg)(\alpha - \phi\theta_A) \frac{Y}{K} = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{31b}$$

equates the rate of return on private physical capital to the rate of return on consumption. The former relationship takes again into account that private investment also induces an increase in public physical capital. Equations (30f) and (30g) can be expressed as

$$\frac{1}{w}(1 - g + vg)\beta \frac{Y}{H} + \delta(1 - u) + \frac{\dot{w}}{w} = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{31c}$$

$$\frac{1}{v}(1 - g + vg)\phi \frac{Y}{K_g} + \frac{\dot{v}}{v} = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{31d}$$

and describe the analogous relationships for human capital and public physical capital, respectively, with the only difference that the returns to investing in human capital and public physical capital include the rate of capital gains \dot{w}/w and \dot{v}/v , respectively.

3.1. Transitional Dynamics

We shall find it convenient to express the dynamics of the centrally planned economy in terms of variables that are constant in the steady state: the ratio of consumption to private physical capital, $q = C/K$, the ratio of output to private physical capital, $r = Y/K$, the ratio of private to public physical capital, $z = K/K_g$, and the value of public physical capital measured in terms of units of private physical capital, $v = \psi/\lambda$.

The interior solution, when the nonnegativity constraints on investment in private and public physical capital are not binding ($I_K > 0$, $G > 0$), can be easily obtained. Now, Equations (30b) and (30c) entail that $\lambda = \psi = \eta$, and thus the value of public physical capital measured in terms of units of private physical capital, $v = \psi/\lambda$, must be constant and equal to

$$v = 1 = v^*, \tag{32a}$$

and (30e) and (30g) imply that the net returns on each type of physical capital are equalized. Hence, $z = K/K_g$ must be constant and equal to the ratio of the elasticity of private physical capital to the elasticity of public physical capital:

$$z = (\alpha - \phi\theta_A)/\phi = z^*. \tag{32b}$$

Since z is constant, we can derive the (time-varying) optimal public investment in physical capital from $\gamma_K = \gamma_{K_g}$ as $G = \phi(Y - C)/(\alpha - \phi\theta_A + \phi)$. Since $Y - C = I_K + G$, the former expression simply states that the optimal expenditure policy for the government consists on investing in physical capital a fraction of total investment equal to the ratio of the elasticity of public physical capital in production to the sum of the elasticities of private and public physical capital. The optimal ratio of public investment to output would then be given by

$$g = \frac{\phi}{\alpha - \phi\theta_A + \phi} (1 - q/r), \quad (33)$$

which obviously satisfies that $0 < g < 1$, and is therefore not constant but time-varying.

Using that $\lambda = \psi = \eta$, the consumption growth rate can be derived from (30a) and (30e) as

$$\gamma_C = \frac{1}{\sigma} [(\alpha - \phi\theta_A)Y/K - \rho] = \frac{1}{\sigma} [(\alpha - \phi\theta_A)r - \rho]. \quad (34)$$

From (29c) and (29e), we obtain

$$\gamma_K = (1 - g)\frac{Y}{K} - \frac{C}{K} = (1 - g)r - q. \quad (35)$$

Using (30d) and (30f) the growth rate of μ is given by

$$\gamma_\mu = \rho - \delta. \quad (36)$$

Log-differentiating (3b) with respect to time yields

$$\gamma_Y = (\alpha - \phi\theta_A)\gamma_K + \beta\gamma_u + \beta\gamma_H + \phi\gamma_{K_g} \quad (37)$$

Log-differentiating (30d), using that $\lambda = \psi = \eta$, we get

$$\gamma_Y - \gamma_u + \gamma_\lambda = \gamma_\mu + \gamma_H. \quad (38)$$

The system that characterizes the dynamics of the centrally planned economy in terms of the variables $q = C/K$, $r = Y/K$, and u that are constant in the steady state is

$$\begin{aligned} \gamma_r = & -\frac{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \phi)}{\alpha - \phi\theta_A + \phi} r \\ & + \frac{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \beta - \phi)}{(1 - \beta)(\alpha - \phi\theta_A + \phi)} q + \frac{\delta\beta}{1 - \beta}, \end{aligned} \quad (39a)$$

$$\gamma_u = \delta u - \frac{(\alpha - \phi\theta_A)q}{1 - \beta} + \frac{\delta\beta}{1 - \beta}, \quad (39b)$$

$$\gamma_q = \gamma_C - \gamma_K = \frac{(\alpha - \phi\theta_A)(\alpha - \phi\theta_A + \phi - \sigma)r}{\sigma(\alpha - \phi\theta_A + \phi)} + \frac{(\alpha - \phi\theta_A)q}{\alpha - \phi\theta_A + \phi} - \frac{\rho}{\sigma}. \quad (39c)$$

From (34) and (35), after substituting g for (33), we obtain (39c). Solving (37) and (38) for γ_Y and γ_u , substituting γ_λ from (30e), γ_μ from (36), γ_K from (35), γ_H from (29b), γ_{K_g} from (29d), and g from (33), we get (39b) and (39a), using that $\gamma_r = \gamma_Y - \gamma_K$.

Hence, if the initial ratio $z(0)$ equals its steady state, $z(0) = z^*$, the ratio of private to public physical capital, z , remains constant at its stationary value z^* , and the economy evolves along the transitional path of system (39). Alternatively, if private and public investments in physical capital are assumed to be reversible and the government can choose the ratio of private to public physical capital, z jumps immediately to its optimal value z^* , in which the net returns on each type of physical capital are equalized. After that, the economy moves along the transitional path of system (39).

Suppose that $z(0) > z^*$, so that K is initially abundant relative to K_g . Without nonnegativity constraints on investment, the adjustment entails increasing K_g and decreasing K by discrete amounts so that the ratio of private to public physical capital jumps at the initial time to its steady state, z^* , and the economy evolves along the transitional path of system (39) after that. This solution requires negative private investment at an infinite rate. Thus, when the nonnegativity constraints are considered, the one corresponding to private physical capital would be violated. The desire to lower K entails that the inequality $I_K \geq 0$ will be binding in an interval $[0, T]$, whereas $G > 0$. Now, (30c) entails that $\psi = \eta$. Hence, from (30a) and (30g), we obtain

$$\gamma_C = \frac{1}{\sigma} (\phi Y/K_g - \rho) = \frac{1}{\sigma} (\phi r z - \rho).$$

Log-differentiating (30d) with respect to time yields

$$\gamma_Y - \gamma_u + \gamma_\psi = \gamma_\mu + \gamma_H. \tag{40}$$

Solving the system (37) and (40) for γ_Y and γ_u , substituting γ_ψ from (30g), γ_μ from (36), γ_H from (29b), γ_{K_g} from (29d) and $\gamma_K = 0$, we can obtain the growth rates of $r = Y/K$ and u as

$$\gamma_r = \gamma_Y = \phi r z - \frac{\phi q z}{1 - \beta} + \frac{\delta \beta}{1 - \beta}, \tag{41a}$$

$$\gamma_u = \delta u - \frac{\phi q z}{1 - \beta} + \frac{\delta \beta}{1 - \beta}. \tag{41b}$$

The growth rates of $q = C/K$ and $z = K/K_g$ can be easily obtained as

$$\gamma_q = \gamma_C = \frac{1}{\sigma} (\phi r z - \rho), \tag{41c}$$

$$\gamma_z = -\gamma_{K_g} = (q - r)z, \tag{41d}$$

where it has been used that the overall resource constraint, $(1 - g)Y = I_K + C$, entails that $g = 1 - q/r$. As the economy evolves, the ratio of private to

public physical capital, z , decreases. At time T , it reaches its steady state value, that is, $z(T) = z^*$, and the nonnegativity constraint on private investment, $I_K \geq 0$, becomes nonbinding. From $t = T$ on, the solution is given by $z(t) = z^*$ and $v(t) = v^*$, and the economy evolves along the stable saddle-path of (39), where g is given by (33).

Suppose now that $z(0) < z^*$, so that K_g is initially abundant relative to K . Without nonnegativity constraints on investment, the adjustment entails increasing K and decreasing K_g by discrete amounts so that the ratio of private to public physical capital jumps at the initial time to its steady state value z^* , and the economy evolves along the transitional path of (39) after that. This solution requires negative public investment at an infinite rate. Thus, with irreversibility constraints, the one corresponding to public capital would be violated. The desire to lower K_g entails that the inequality $G \geq 0$ will be binding in an interval $[0, T]$, whereas $I_K > 0$. Now, Equation (30b) entails that $\lambda = \eta$. Hence, from (30a) and (30e), we obtain

$$\gamma_C = \frac{1}{\sigma} [(\alpha - \phi\theta_A)Y/K - \rho] = \frac{1}{\sigma} [(\alpha - \phi\theta_A)r - \rho].$$

Log-differentiating (30d), using that $\lambda = \eta$, yields (38). Solving the system (37) and (38) for γ_Y and γ_u , substituting γ_λ from (30e), γ_μ from (36), γ_K from (35), γ_H from (29b), and $\gamma_{K_g} = 0$, we can obtain the growth rates of r and u as

$$\gamma_r = \gamma_Y - \gamma_K = -(1 - \alpha + \phi\theta_A)r + \frac{(1 - \alpha + \phi\theta_A - \beta)q}{1 - \beta} + \frac{\delta\beta}{1 - \beta}, \tag{42a}$$

$$\gamma_u = \delta u - \frac{(\alpha - \phi\theta_A)q}{1 - \beta} + \frac{\delta\beta}{1 - \beta}. \tag{42b}$$

The growth rates of $q = C/K$ and $z = K/K_g$ can be easily obtained as:

$$\gamma_q = q + \frac{1}{\sigma}(\alpha - \phi\theta_A - \sigma)r - \frac{\rho}{\sigma}, \tag{42c}$$

$$\gamma_z = \gamma_K = r - q, \tag{42d}$$

whereas $g = 0$. As the economy evolves, z increases until, at time T , it reaches its steady state value, z^* , i.e., $z(T) = z^*$, and the nonnegativity constraint on public investment, $G \geq 0$, becomes nonbinding. From $t = T$ on, the solution is given by $z(t) = z^*$ and $v(t) = v^*$, and the economy evolves along the saddle-path of (39), with g given by (33). A similar reasoning to that in Gómez (2004) may be used to show that the continuity of the shadow prices involves the continuity of the consumption and work time paths on the optimal solution.

3.2. Balanced Growth Path

Now, we focus on an interior balanced growth path (or steady state) in which all variables grow at constant but possibly different rates and the shares of

labor in its different uses are constant. An asterisk ‘*’ will denote the steady state of a variable in the centrally planned economy. We have shown above that once z reaches its steady state value, z^* , the solution is given by $z(t) = z^*$, the nonnegativity constraints on investment in physical capital are not binding, and the economy evolves along the stable saddle-path of (39), where g is given by (33). This would also be the case if private and public investments in physical capital are assumed to be reversible and the government can choose the ratio of private to public physical capital. The following proposition examines the existence of an interior steady state with positive long-run growth in which, therefore, the nonnegativity constraints on private and public investment must be not binding.

PROPOSITION 3: *Let $1 - \alpha + \phi\theta_A - \phi > 0$. The centrally planned economy has a unique positive steady state with positive long-run growth,*

$$v^* = 1, \tag{43a}$$

$$z^* = (\alpha - \phi\theta_A)/\phi, \tag{43b}$$

$$r^* = \frac{\delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi) + \beta(\delta - \rho)}{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma)}, \tag{43c}$$

$$q^* = \frac{\delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi) + \beta(\delta - \rho)(1 - \alpha + \phi\theta_A - \phi)}{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma)}, \tag{43d}$$

$$u^* = \frac{\delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi)}{\delta(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma)}, \tag{43e}$$

$$\begin{aligned} g^* &= \frac{\phi}{\alpha - \phi\theta_A + \phi} (1 - q^*/r^*) \\ &= \frac{\phi\beta(\delta - \rho)}{\delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi) + \beta(\delta - \rho)}, \end{aligned} \tag{43f}$$

where the long-run growth rate of human capital is

$$\gamma_H^* = \delta(1 - u^*) = \frac{(\delta - \rho)(1 - \alpha + \phi\theta_A - \phi)}{1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma}, \tag{44a}$$

and the long-run growth rate of income, consumption, private and public physical capital is

$$\gamma_K^* = \frac{\beta}{1 - \alpha + \phi\theta_A - \phi} \gamma_H^* = \frac{\beta(\delta - \rho)}{1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma} = \gamma_C^* = \gamma_Y^* = \gamma_{K_s}^*, \tag{44b}$$

if and only if

$$\begin{aligned} \text{sign}(\delta - \rho) &= \text{sign}(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma) \\ &= \text{sign}[\delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi)]. \end{aligned} \tag{45}$$

Proof: See Appendix.

As in the case of the market economy, it can be easily shown that the long-run growth rates of physical and human capital along with the (social) rate of return on private physical capital, $i^* = (\alpha - \phi\theta_A)r^*$, can be obtained by solving the system (similar to (25)):

$$\gamma_K^* = (i^* - \rho)/\sigma, \quad (46a)$$

$$\gamma_K^* = \beta\gamma_H^*/(1 - \alpha - \phi + \phi\theta_A), \quad (46b)$$

$$\gamma_K^* - \gamma_H^* = i^* - \delta. \quad (46c)$$

The assumption that the technology in the educational sector is linear in effective time explains why the long-run growth rates in the market economy (23a)–(23b) coincide with those of the centrally planned economy (44a)–(44b), because the private and the social return on effective time in the educational sector, which is the constant δ , coincide.

Local saddle-path stability of the steady state can be ensured if the coefficient matrix of the linearization of (39) at its steady state (r^*, q^*, u^*) has one unstable root. We can state the following Proposition.

PROPOSITION 4: *The steady state of the centrally planned economy is locally saddle-path stable if and only if (28) is satisfied.*

Proof: See Appendix.

4. Decentralization of the Optimal Growth Path

This section devises a fiscal policy by means of which the optimal growth path attainable by a central planner can be decentralized.

Suppose first that $z(0) = z^*$, so that the nonnegativity constraints on private and public investment in physical capital are not binding. As shown above, the solution is given by $z(t) = z^*$, and the economy evolves along the transitional path of system (39), where g is given by (33). Since the nonnegativity constraint on private investment in physical capital is not binding, the dynamics of the market economy is driven by system (19). Comparing (19a) with (39c), using that the optimal ratio of public investment to output, g , is given by (33), we observe that the evolution of the ratio of consumption to private physical capital, q , will be the same for both the market economy and the centrally planned economy if and only if the tax rate on income is chosen as¹⁰

¹⁰This result can also be obtained by comparing the growth rate of consumption in the market economy given by (18) with the corresponding one in the centrally planned economy given by (34).

$$(1 - \tau^*)(\alpha + \phi\theta_R) = \alpha - \phi\theta_A. \quad (47)$$

Hence, the optimal income tax can be derived as

$$\tau^* = \frac{\phi(\theta_R + \theta_A)}{\alpha + \phi\theta_R}. \quad (48)$$

It can be readily shown that substituting g for its optimal value given by (33), z for $z^* = (\alpha - \phi\theta_A)/\phi$, and τ for its optimal value given by (48) (and so, $\dot{\tau} = 0$) into (19c), (19d), and (19b), the expressions for γ_u and γ_r in the market economy coincide with their counterparts (39b) and (39a), respectively, in the centrally planned economy, and $\gamma_z = 0$ as in the centralized economy, since $z = z^*$ (a constant). It should be noted that the optimal income tax rate is constant, and $0 \leq \tau^* < 1$, where the second inequality follows from $\alpha - \phi\theta_A > 0$. Having set the income tax rate in an optimal manner, lump-sum taxes (or transfers) should be set so as to balance the government budget.

The optimal income tax (48) corrects for the negative external effects caused by both absolute and relative congestion. The intuition is straightforward: It ensures that the social marginal return to private physical capital, as viewed by the central planner, $(\alpha - \phi\theta_A)r$, coincides with the private after-tax return, as viewed by the individual agent, $(1 - \tau)(\alpha + \phi\theta_R)r$. It is constant since the degree of congestion is constant as well, and is positive as long as there is congestion. This is because the individual agent ignores the negative externality caused by congestion and overaccumulates physical capital relative to the optimum. Therefore, the accumulation of physical capital should be discouraged through an income tax.¹¹ The optimal income tax is zero as long as there is no congestion. In this case, there is no externality, so capital income should be untaxed. The first-best optimum requires resorting solely to lump-sum taxation to finance public investment.

Both types of congestion raise the optimal income tax as

$$\frac{\partial \tau^*}{\partial \theta_A} = \frac{\phi}{\alpha + \phi\theta_R} > 0, \quad (49a)$$

$$\frac{\partial \tau^*}{\partial \theta_R} = \frac{(\alpha - \phi\theta_A)}{(\alpha + \phi\theta_R)} \frac{\partial \tau^*}{\partial \theta_A} > 0. \quad (49b)$$

Intuitively, an increase in either form of congestion raises the corresponding externality, and therefore, raises the optimal income tax needed for its correction. Equation (47) shows that an increase in relative congestion increases the (before-tax) private rate of return on private physical capital, but does not affect its social rate of return. In contrast, an increase in absolute

¹¹As pointed out in Section 2, income taxation affects the transitional dynamics and the steady state of the economy, and thus may affect welfare and restore efficiency even if it does not affect long-run growth.

congestion does not affect the (before-tax) private rate of return on private physical capital, but it decreases its social rate of return. Therefore, an increase in congestion entails that a higher income tax is required to restore the equality between the private and the social rates of return. Furthermore, the effect of absolute congestion is greater than that of relative congestion, $\partial\tau^*/\partial\theta_A > \partial\tau^*/\partial\theta_R > 0$. This is because an increase in absolute congestion drives a larger wedge between social and private marginal rates of return to physical capital than an increase in relative congestion does and, therefore, requires a larger tax to correct.

So far we have considered the case in which $z(0) = z^*$. Suppose now that $z(0) > z^*$, so that $I_K = 0$ and the optimal growth path is described by system (41) up to the point in which $z = z^*$. The corresponding system in the decentralized economy is (21). First note that (21a) and (41d), which describe the dynamics of the ratio of private to public capital, z , in the decentralized and centralized economies, respectively, coincide. Substituting the optimal value of $g = 1 - q/r$ into (21b) and (21c), it can be easily shown that they coincide with (41b) and (41a), respectively, if the income tax is constant, i.e., $\dot{\tau} = 0$. It should be emphasized that \dot{g} in (21b) and (21c) should be replaced with $\dot{g} = q(\gamma_r - \gamma_q)/r$, with γ_r and γ_q given by (41a) and (41c), respectively. Thus, the optimal growth path can be achieved by setting the optimal share of government to output equal to $g = 1 - q/r$, with γ_q given by (41c), and can be sustained by any combination of (constant) income taxation and lump-sum taxation that satisfy the government budget constraint, up to the point in which $z = z^*$. After that, the economy evolves according to the system (39), with $z = z^*$, the income tax rate should be set according to (48), and the public investment share of output, g , is given by (33).

If $z(0) < z^*$, then $G = 0$ and the optimal growth path is described by (42) up to the point at which $z = z^*$. The corresponding system in the decentralized economy is (19). First note that Equations (19b) and (42d), that describe the dynamics of z in the decentralized and centralized economy, respectively, coincide after substituting the optimal value of $g = 0$. Comparing (19a) and (42c), with $g = 0$, we see that the market economy will fully replicate the dynamic path of q in the optimal solution if and only if the tax rate on income is set according to (48). It can be easily shown that setting the income tax rate in this way, also ensures that the growth rate of u and r in the decentralized economy, Equations (19c) and (19d), coincide with their counterparts in the centrally planned economy, Equations (42b) and (42a), respectively, noting that $\dot{\tau} = \dot{g} = 0$. Thus, tax revenue must be rebated as lump-sum transfers to consumers in order to the government budget constraint be met, up to the point at which $z = z^*$. After that, the economy evolves according to (39), with $z = z^*$, the tax rate on income must be kept at its value given in (48), and the public investment share of output, g , is given by (33).

The previous discussion has assumed that private and public investments in physical capital are irreversible; (i.e., nonnegative). Alternatively, if investments are assumed to be reversible and the government can choose the ratio of private to public physical capital z , it jumps immediately to its optimal value $z^* = (\alpha - \phi\theta_A)/\phi$. After that, the economy evolves along the saddle path of system (39). As shown above, in this case the optimal growth path can be decentralized by setting the income tax rate according to (48), and lump-sum taxes (or transfers) so as to balance the government budget.

5. Concluding Remarks

This paper analyzes the effect of public investment in an endogenous growth model with private and public physical capital, and human capital. Productive government expenditures are intended to represent public infrastructure, and so, the accumulated stock of public capital, rather than the current flow of public investment, is regarded as a productive input in private production. Differently to previous literature, public capital may be subject to absolute congestion as well as relative congestion. In spite of the complexity of the dynamical systems involved, the paper provides a detailed analysis of the equilibrium dynamics of the market economy, and also of the optimal growth path attainable by a central planner. In this last case, it is shown that the irreversibility constraints on private and public investment in physical capital play a crucial role when deriving the optimal growth path. According to empirical evidence, long-run growth is found to be invariant to fiscal policy. The paper also analyzes the effect of congestion on long-run growth, and derives an optimal tax policy capable of decentralizing the first-best solution attainable by a central planner. An interesting extension would be to analyze whether the results obtained carry over to other settings; in particular, to endogenous growth models with R&D (e.g., Aghion and Howitt 1992; Grossman and Helpman 1991; Jones 1995b).

Appendix

Proof of Proposition 1: From $\hat{\gamma}_C = \hat{\gamma}_K = \hat{\gamma}_Y$ it follows that $\hat{\gamma}_r = 0$ and $\hat{\gamma}_q = 0$. Rewriting (5) as $\gamma_{K_g} = grz$, we get $\hat{\gamma}_z = 0$; i.e., $\hat{\gamma}_K = \hat{\gamma}_{K_g}$. The steady state of (19) is (22). From $\hat{\gamma}_H = \delta(1 - \hat{u})$ and $\hat{\gamma}_K = \beta\hat{\gamma}_H/(1 - \alpha - \phi + \phi\theta_A)$, which results from (15), we obtain (23). If $1 - \hat{u} > 0$ then $\hat{\gamma}_H > 0$, and to ensure the positivity of $\hat{\gamma}_K$, we must have that $1 - \alpha - \phi + \phi\theta_A > 0$. The condition $0 < \hat{u} < 1$ holds if and only if the first two equalities in (24) are satisfied, which also ensure that $\hat{z} > 0$ and $\hat{r} > 0$. However, $\hat{q} > 0$ entails that the latter equality in (24) must be met. It can be easily shown that transversality conditions (11f) are satisfied if $0 < \hat{u} < 1$. ■

Proof of Proposition 2: Linearizing (19) around its steady state $(\hat{r}, \hat{q}, \hat{u}, \hat{z})$, we obtain

$$\begin{pmatrix} \dot{r} \\ \dot{q} \\ \dot{u} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} J_{11} & (1 - \alpha - \beta + \phi\theta_A)\hat{r}/(1 - \beta) & 0 & g\phi\hat{r}^2/(1 - \beta) \\ J_{21} & \hat{q} & 0 & 0 \\ J_{31} & -(\alpha - \phi\theta_A)\hat{u}/(1 - \beta) & \delta\hat{u} & g\phi\hat{r}\hat{u}/(1 - \beta) \\ \hat{q}\hat{z}/\hat{r} & -\hat{z} & 0 & -g\hat{r}\hat{z} \end{pmatrix} \times \begin{pmatrix} r - \hat{r} \\ q - \hat{q} \\ u - \hat{u} \\ z - \hat{z} \end{pmatrix},$$

where $J = (J_{ij})$ denotes the coefficient matrix of the former system,

$$J_{11} = -((1 - \alpha - \beta + \phi\theta_A)\hat{q} + \beta\delta)/(1 - \beta), \quad J_{21} = (\rho/\sigma - \hat{q})\hat{q}/\hat{r},$$

and J_{31} is unnecessary for the subsequent analysis. The structure of the third column of J entails that $\delta\hat{u}$ is an eigenvalue. The remaining eigenvalues of J are those of the matrix \bar{J} resultant from deleting the third row and column. After simplification we get

$$\det(\bar{J}) = g[\beta\delta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + \beta(\delta - \rho)]\hat{q}\hat{z}\hat{r}/[(1 - \beta)\sigma].$$

Stability requires that there be two eigenvalues with negative real parts and, therefore, $\det(\bar{J}) > 0$; i.e., $\beta\delta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) + \beta(\delta - \rho) > 0$ which, combined with (24), entails that (28) be met. In turn, (28) entails that $1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma > 0$ (which comes from (24)) as

$$\begin{aligned} 0 &< \delta\beta(\sigma - 1) + \rho(1 - \alpha - \phi + \phi\theta_A) < \delta[\beta(\sigma - 1) + 1 - \alpha - \phi + \phi\theta] \\ &= \delta(1 - \alpha - \beta - \phi + \phi\theta_A + \beta\sigma). \end{aligned}$$

Since $\det(\bar{J}) > 0$, there are 0 or 2 eigenvalues with negative real parts. The characteristic equation for the matrix \bar{J} is $p(\lambda) = -\lambda^3 + \pi_2\lambda^2 + \pi_1\lambda + \pi_0 = 0$, where $\pi_2 = \text{tr}(\bar{J})$, π_1 is the opposite of the sum of all the leading principal minors of order 2 of \bar{J} , and $\pi_0 = \det(\bar{J}) > 0$. After simplification, we get

$$\begin{aligned} \pi_2 = \text{tr}(\bar{J}) &= \hat{q} - g\hat{r}\hat{z} - \frac{(1 - \alpha - \beta + \phi\theta_A)\hat{q} + \beta\delta}{1 - \beta}, \\ \pi_1 &= \frac{g(\alpha - \phi\theta_A + \phi)\hat{q}\hat{r}\hat{z}}{1 - \beta} + \frac{(1 - \alpha - \beta + \phi\theta_A)\rho\hat{q}}{(1 - \beta)\sigma} + \frac{\beta\delta(\hat{q} - g\hat{r}\hat{z})}{1 - \beta}. \end{aligned}$$

The first two terms on the right-hand side of π_1 are positive. If $\pi_2 \geq 0$, it must be that $\hat{q} > g\hat{r}\hat{z}$, so that the third term on the right-hand side of π_1 is also positive, and so, $\pi_1 > 0$.

The number of roots of the characteristic equation for the matrix \bar{J} with negative real parts is equal to the number of the roots of the polynomial $p(-\lambda) = \lambda^3 + \pi_2\lambda^2 - \pi_1\lambda + \pi_0$ with positive real parts. Using the Routh-Hurwitz theorem (e.g., Gantmacher 1959), the number of stable roots of the matrix \bar{J} is then equal to the number of variations of sign in the scheme

$$1 \quad \pi_2 \quad \psi_1 \quad \pi_0,$$

where $\psi_1 = -(\pi_2\pi_1 + \pi_0)/\pi_2$. Now, if $\pi_2 > 0$ then $\pi_1 > 0$, and so, $\psi_1 < 0$. Hence, we have the configuration

$$+ \quad + \quad - \quad +$$

so that there are two variations in sign. If $\pi_2 < 0$, we have the configuration

$$+ \quad - \quad ? \quad +,$$

where a question mark represents the (unknown) sign of ψ_1 , which could be even zero. Irrespective of the unknown sign, there are two variations in sign. Finally, if $\pi_2 = 0$, which entails that $\pi_1 > 0$, we substitute π_2 for a positive constant ε than tends to zero from above, and we obtain the following configuration

$$+ \quad 0 \quad - \quad +$$

so that there are two variations in sign. Hence, irrespective of the sign of π_2 , there are two variations in sign in the preceding scheme, and so, the matrix \bar{J} has one unstable and two stable roots. Thus, J has also two stable roots and, therefore, the steady state is locally saddle-path stable. ■

Proof of Proposition 3: Equations (43a)–(43b) have been obtained in the text (see (32a) and (32b)). The steady state of (39a)–(39c) is given by (43c)–(43e), and g^* follows then from (33), which satisfies $0 \leq g^* < 1$ if (45) holds. From $\gamma_H^* = \delta(1 - u^*)$ and $\gamma_K^* = \beta\gamma_H^*/(1 - \alpha + \phi\theta_A - \phi)$, which results from (37), we get (44). If $1 - u^* > 0$ then $\gamma_H^* > 0$, and to ensure $\gamma_K^* > 0$, it must be that $1 - \alpha + \phi\theta_A - \phi > 0$. The condition $0 < u^* < 1$ holds if and only if (45) is satisfied. If $1 - \alpha + \phi\theta_A - \phi > 0$ and (45) are satisfied, it can be shown that $r^* > 0$ and $q^* > 0$, and thus the steady state is feasible. It can be readily shown that the transversality conditions (30h) are satisfied if $0 < u^* < 1$. ■

Proof of Proposition 4: Linearizing (39) around its steady state (r^*, q^*, u^*) , we get

$$\begin{pmatrix} \dot{r} \\ \dot{q} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} -\frac{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \phi)}{\alpha - \phi\theta_A + \phi} r^* & \frac{(\alpha - \phi\theta_A)(1 - \alpha + \phi\theta_A - \beta - \phi)}{(1 - \beta)(\alpha - \phi\theta_A + \phi)} r^* & 0 \\ \frac{(\alpha - \phi\theta_A)(\alpha - \phi\theta_A + \phi - \sigma)}{\sigma(\alpha - \phi\theta_A + \phi)} q^* & \frac{\alpha - \phi\theta_A}{\alpha - \phi\theta_A + \phi} q^* & 0 \\ 0 & -\frac{\alpha - \phi\theta_A}{1 - \beta} u^* & \delta u^* \end{pmatrix} \times \begin{pmatrix} r - r^* \\ q - q^* \\ u - u^* \end{pmatrix}.$$

Let J denote the coefficient matrix of the former system. The structure of the third column of J entails that δu^* is an eigenvalue. The remaining eigenvalues of J are those of the matrix \bar{J} resultant from deleting the third row and column of J . After simplification, we get

$$\det(\bar{J}) = -(\alpha - \phi\theta_A)^2(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma)q^*r^* / [(1 - \beta)\sigma(\alpha - \phi\theta_A + \phi)].$$

Stability requires that there must be one eigenvalue with negative real part, and so, $\det(\bar{J}) < 0$; i.e., $1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma > 0$, which combined with (45), entails that (28) is met. In turn, (28) entails that $1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma > 0$ as

$$\begin{aligned} 0 &< \delta\beta(\sigma - 1) + \rho(1 - \alpha + \phi\theta_A - \phi) < \delta(\beta(\sigma - 1) + \delta(1 - \alpha + \phi\theta_A - \phi)) \\ &= \delta(1 - \alpha + \phi\theta_A - \beta - \phi + \beta\sigma). \end{aligned}$$

Thus, the steady state is locally saddle-path stable if and only if (28) holds. ■

References

AGHION, P., and P. HOWITT (1992) A model of growth through creative destruction, *Econometrica* **60**, 323–351.
 ASCHAUER, D. A. (1989a) Is public expenditure productive? *Journal of Monetary Economics* **23**, 177–200.
 ASCHAUER, D. A. (1989b) Does public capital crowd out private capital? *Journal of Monetary Economics* **24**, 171–188.
 BARRO, R. J. (1990) Government spending in a simple model of endogenous growth, *Journal of Political Economy* **98**, S103–S125.

- BARRO, R. J., and X. SALA-I-MARTIN (1992) Public finance in models of economic growth, *Review of Economic Studies* **59**, 645–661.
- BARRO, R. J., and X. SALA-I-MARTIN (2004) *Economic Growth*, 2nd ed. Cambridge, MA: MIT Press.
- BOND, R. W., P. WANG, and C. K. YIP (1996) A general two-sector model of endogenous growth with human and physical capital: Balanced growth and transitional dynamics, *Journal of Economic Theory* **68**, 149–173.
- CABALLE, J., and M. S. SANTOS (1993) On endogenous growth with physical and human capital, *Journal of Political Economy* **101**, 1042–1067.
- EDWARDS, J. H. Y. (1990) Congestion function specification and the ‘publicness’ of local public goods, *Journal of Urban Economics* **27**, 80–96.
- EICHER, T., and S. TURNOVSKY (2000) Scale, congestion and growth, *Economica* **67**, 325–346.
- EICHER, T., and S. TURNOVSKY (2001) Transitional dynamics in a two-sector non-scale growth model, *Journal of Economic Dynamics and Control* **25**, 85–113.
- FUTAGAMI, K., Y. MORITA, and A. SHIBATA (1993) Dynamic analysis of an endogenous growth model with public capital, *Scandinavian Journal of Economics* **95**, 607–625.
- GANTMACHER, F. R. (1959) *The Theory of Matrices*. New York: Chelsea Publishing Company.
- GLOMM, G., and B. RAVIKUMAR (1994) Public investment in infrastructure in a simple growth model, *Journal of Economic Dynamics and Control* **18**, 1173–1187.
- GLOMM, G., and B. RAVIKUMAR (1997) Productive government expenditures and long-run growth, *Journal of Economic Dynamics and Control* **21**, 183–204.
- GÓMEZ, M. A. (2004) Optimal fiscal policy in a growing economy with public capital, *Macroeconomic Dynamics* **8**, 419–435.
- GROSSMAN, G. M., and E. HELPMAN (1991) *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.
- JONES, C. I. (1995a) Time series tests of endogenous growth models, *Quarterly Journal of Economics* **110**, 495–527.
- JONES, C. I. (1995b) R&D-based models of economic growth, *Journal of Political Economy* **103**, 759–784.
- JONES, L. E., R. E. MANUELLI, and P. E. ROSSI (1993) Optimal taxation in models of endogenous growth, *Journal of Political Economy* **101**, 485–517.
- KRUEGER, A. B., and M. LINDAHL (2002) Education and growth: Why and for whom? *Journal of Economic Literature* **39**, 1101–1136.
- LEVINE, R., and D. RENELT (1992) A sensitivity analysis of cross-country growth regressions, *American Economic Review* **82**, 942–963.
- LUCAS, R. E., Jr. (1988) On the mechanics of economic development, *Journal of Monetary Economics* **22**, 3–42.
- MENDOZA, E. G., G. M. MILESI-FERRETTI, and P. ASEA (1997) On the ineffectiveness of tax policy in altering long-run growth: Harberger’s superneutrality conjecture, *Journal of Public Economics* **66**, 99–126.
- MULLIGAN, C. B., and X. SALA-I-MARTIN (1993) Transitional dynamics in two-sector models of endogenous growth, *Quarterly Journal of Economics* **108**, 739–773.
- STOKEY, N. L., and S. REBELO (1995) Growth effects of flat-rate taxes, *Journal of Political Economy* **103**, 419–450.
- TANZI, V., and H. H. ZEE (1997) Fiscal policy and long run growth, *IMF Staff Papers* **44**, 179–209.

- TURNOVSKY, S. J. (1996a) Optimal tax, debt, and expenditure policies in a growing economy, *Journal of Public Economics* **60**, 21–44.
- TURNOVSKY, S. J. (1996b) Fiscal policy, adjustment costs, and endogenous growth, *Oxford Economic Papers* **48**, 361–381.
- TURNOVSKY, S. J. (1997) Fiscal policy in a growing economy with public capital, *Macroeconomic Dynamics* **1**, 615–639.
- TURNOVSKY, S. J. (2000) Fiscal policy, elastic labor supply, and endogenous growth, *Journal of Monetary Economics* **45**, 185–210.
- UZAWA, H. (1965) Optimum technical change in an aggregative model of economic growth, *International Economic Review* **6**, 18–31.