# Modified bistable modules for bias deployable structures 

M.J. Freire-Tellado *, M. Muñoz-Vidal, J. Pérez-Valcárcel<br>Grupo de Estructuras Arquitectónicas (GEA), Estructuras Singulares (GES), Dpto. Construcciones y Estructuras A.C.A. (CEACA), E.T.S. Arquitectura, Universidade da Coruña, Campus da Zapateira s/n, 15071 A Coruña, Spain

## A R T I C L E I N F O

## Keywords:

Deployable structures
Bistable structures
Scissor-hinged mechanism
Reciprocal structures
Deployment sequence


#### Abstract

Bias deployable grids are meshes with two directions of rotation on the ground plan with respect to the edges. They offer benefits such as three-dimensional resistance with supports around the entire perimeter of a rectangular layout, and consist exclusively of load-bearing scissors as opposed to the usual combinations of loadbearing scissors and bracing scissors. However, their resistance to angular distortion is limited, and they require auxiliary elements to maintain the fully deployed position. Nevertheless, they are very promising solutions for medium-span emergency buildings.

This paper proposes a bistable module adapted to bias deployable structures. The geometrical incompatibilities of several modules are analysed together with their behaviour based on the kinematic models that were built, which alternate different types of nodes and different geometries of the perimeter scissors, making it possible to calibrate the level of incompatibility introduced. The dimensions of the nodes are also taken into account. The tests are checked against the results of several series of dynamic calculations.


## 1. Introduction

In a humanitarian disaster situation, it is necessary to be able to quickly provide buildings to meet the various needs of the affected population. Deployable structures [1-4] are a good alternative in these cases, since once they are made, they can be stored in compact packages that take up little space and are relatively light; they can be transported to the location where they are needed, where they are opened (deployed) and become fully operational in a short space of time. This is especially true for medium-span buildings: while there are proposals for their use in residential modules [5], their characteristics are better suited to assembly buildings.

A very large group of deployable structures are those based on the pantograph. They are mesh-like structures organized on the basis of elements formed by two beams, generally straight, forming a blade with a pin joint located in the central part, so that the set resembles scissors, which is why they are known as Scissor Like Elements, SLEs.

The lines that join the ends of the bars of these elements can intersect each other ('polar SLEs') or be parallel ('translational SLEs'). The use of translational SLEs grants certain specific characteristics to deployable structures, in such a way that they are identified as their specific group (deployable structures consisting of translational units) [6,7].

For deployable structures to be openable, they must be geometrically
compatible throughout the entire deployment process. However, once the desired final position is reached, it is necessary to add auxiliary elements to stabilise the assembly. As time went by, a second possibility arose: the structure must be geometrically compatible in its initial and final states, while during the unfolding process it presents geometric incompatibilities that it is able to overcome by deforming its components within the elastic phase. Said incompatibilities are capable of stabilizing the structure in the deployed position under the action of reduced loads without the need to add auxiliary elements. These structures are known as bistable deployable structures.

Zeigler was who first noted the possibility of using incompatibility during folding as a way to keep the structures deployed [8]; Krishnapillai returned to the idea and patented the first module to solve and exploit it [9,10]; Clarke analyses the motion of Zeigler's mechanism [11]; Rosenfeld and Logcher [12,13] synthesise the state of the art and develop and experimentally analyse proposals for bistable modules, which they call 'clickables'. Gantes [14-18], who analysed and popularised these structures over a long period of time makes an outstanding contribution to the study of these structures. Other researchers reported the bistability of certain structural types (such as deployable domes) and devised systems for their design and calculation [19]

Bias Deployable Grids (BDGs) [20,21] are translational SLE

[^0]deployable structures with two directions of deployment arranged rotated with respect to the enclosure they and which in the deployed position have equal projections on the support plane, i.e. the deployed structure forms a mesh of rotated squares. As a result, the scissors in both directions are the same, and all the SLEs used are both load-bearing (supporting the weight) and bracing (they laterally stabilise the perpendicular SLEs), increasing the strength of the whole. The system of scissors can have a flat, curved, irregular or mixed profile, and allows for the construction of 2 and 4 pitched roofs, barrel vaults, groin vaults, and slanted vaults (with a circular or free profile), rectangular, L-shaped, Tshaped or combinations of rectangles, as well as the inclusion of openings and skylights in the structure. In addition, all of the above solutions with a constant slope can be constructed with only two types of bars, complying perfectly with the abovementioned requirements.

However, these structures are prone to some angular distortion, especially in the lengthwise direction, and require an auxiliary system to hold them in the final deployed position while the perimeter stabilization bars are fitted in place.

For this reason, the base BDG can be combined with a set of SLEs to improve the performance of the whole. In the case of a BDG in the shape of a hipped roof, one of the possibilities is the addition of horizontal SLEs adjacent to the ridge of the roof, as shown in (Fig. 1). If the BDG is made with symmetrical blades (Fig. 1a), the horizontal SLEs have to be composite to ensure compatibility (Fig. 1c). If asymmetrical scissors are used (Fig. 1b), it is possible to make the horizontal SLEs simple (Fig. 1d), but at the cost of completely conditioning the structure.

Proceeding in this way, a group of ridge modules is formed, reminiscent of the bistable diagonalised modules proposed by Krishnapillai [10], studied and refined by Gantes throughout his work [18], then by Friedman \& Ibrahimbegovi [22] and most recently by Arnouts et al, [23-27] and Zhao et al [28].

Like the previous ones, the new proposed module is made up of four equal perimeter SLEs joined at their ends by 'hub' nodes (represented as hollow circles in the drawings). Four equal diagonal scissors start from these nodes and meet at the two central nodes located on the module axis. From here the differences arise: in the new module (Fig. 2a), the perimeter elements are composite SLEs, consisting of four straight beams joined together by four other 'pivotal or revolute joints' (drawn as filled circles) as opposed to the simple SLEs formed by two beams joined by a pin of the conventional modules ((Fig. 2b) based on [17]).

The diagonal scissors of the module drawn are symmetrical oblique

SLEs (regular curved translational units [7]), (Fig. 3b), while in the existing modules they are asymmetrical oblique SLEs (irregular curved translational units), (Fig. 3c), even if the new module can also incorporate this latter type depending on the proportions of the perimeter blades, (Fig. 3a). This possibility allows adjusting the length of the short sections of the perimeter composite SLEs in case of need, but a priori it seems to be of exceptional use.

However, this change allows that, in addition to being able to vary the inclination of the diagonal SLEs as in the existing modules, that of the perimeter SLEs can be modified. Therefore, the new modules have the advantage of allowing more variants.

Thus, the new modules can be geometrically understood as a generalization of those proposed in the bibliography, and in the same way as in these, use regular polygons with a greater number of sides as a basis.

The application of these modules to the BDG entails the inversion of the priority of the SLEs - in the bibliography the main ones are the perimeter scissors; in the proposed module the main ones are the diagonal SLEs.

In previous studies, two different materials are usually used for the perimeter SLEs and for the diagonal ones of the module: aluminium and acetal [17]; aluminium and high-density polyethylene (HDPE) [24]. The indicated inversion of the bearing priority of the scissors requires that the diagonals cannot see their bearing capacity reduced by introducing a less resistant material, which leads to build all the bars with the same material or to reverse the use of the previous materials. For this reason, in this case the degree of introduced incompatibility lies predominantly in the geometry adopted for the module.

In addition, it is necessary to investigate the repercussions of the introduction of double SLEs: as will be seen later, new limitations appear derived from the width of the bars (an aspect that, in our best know, has not been taken into account by the existing bibliography [24,26]). Thus it is necessary to complete the geometric developments published including the necessary modifications for this new situation.

In Fig. 1, Fig. 2.a, Fig. 4.b and 4.c the lower diagonal bars have been drawn in a horizontal position, so that they are in extension and bending occurs. The bending of the module cannot therefore be achieved by applying horizontal forces alone (some vertical force must be included) and falls outside of the limit positions defined in [25] (Fig. 5).

Comparison of the elevations of the new module and those proposed in the bibliography shows that the inclination of the diagonal SLEs is


Fig. 1. Four-sided BDGs with horizontal SLEs added at the ridge. Resulting bistable modules.


Fig. 2. Proposed basic module (a) and existing module (b). 3D view. Perimeter SLEs in blue; diagonal SLEs in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)


Fig. 3. New module: asymmetrical (a) and symmetrical (b) diagonal SLEs. Existing module: asymmetrical diagonal SLEs (c).
(a)



Fig. 4. Side view of a bistable module (a) and two examples of a modified bistable module (b, c). Perimeter SLEs in blue; diagonal SLEs in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)


Fig. 5. Limit positions of a bistable module (side view): lower limit (a) and higher limit (b). (outer SLEs in blue, inner SLEs in red) [25]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
much smaller in the bibliographical references, raising doubts about the performance of the new module. However, no simplified method has been found to predict its behaviour: the literature review revealed a gap between the geometrical definition of a given bistable module and the detailed analysis of the mechanical conditions of the deployment, making the project adjustment very complex (e.g. Gantes $[17,18]$ requires three complete interactions to adjust the geometry of the plain model he employs to make a deployable flat slab). A simple method is needed that allows rapid geometric adjustment of the solution. The understanding of the requirements imposed by these modules is also incomplete.

There is another implicit change derived from the way in which the module has been designed: in the published solutions, the bistable deployable structure is built by the repetition or combination of bistable modules; in this case, starting from a given deployable structure, some modules are made bistable in order to improve the characteristics of the assembly by combining bistable modules with others that are not, following the line started in [27].

This text introduces the following novelties:

- A new bistable module is proposed, the bias bistable module, and the operation and bistability of the module is experimentally verified.
- A new method of geometric analysis is proposed that allows considering the geometric incompatibilities during the deployment, the real dimensions of the bars and nodes and the errors in the lengths of the bars.
- The results of the previous studies are represented in a single graph, which constitutes an effective tool for the design phase.
- Different models of the module are built with different types of nodes (including reciprocal linkages in the central nodes) and perimeter SLEs, studying the folding and deployment process. The results of this studies are plotted on the graphics resulting from the previous geometric analysis
- Two series of dynamic calculations of the base module are carried out and their results are contrasted with those obtained in the geometric studies and in the kinematic models
- The repercussion of small variations in the lengths of the bars on the folding conditions of the module is analysed


## 2. Study of folding incompatibilities of the module

The new module (Fig. 2a) forms a square whose sides are comprised of composite scissors and whose diagonals are two regular translational SLEs placed at $90^{\circ}$, the main scissors. The slopes of the bars that form these blades are either of opposite signs or one of them is null, as is the case in Fig. 4b and c.

For full compatibility, during the folding-unfolding process the horizontal projection of the module should be scaled homothetically. Therefore, the comparison of the projections on the horizontal plane of the perimeter SLEs with those of the diagonal ones at each moment of the folding process makes it possible to detect the differences that occur and their intensity, i.e. the degree of incompatibility.

In this study, these incompatibilities are analysed using two parameters whose usefulness will be shown throughout the text:

The Relative Geometric Incompatibility, RGI, understood as (1)
$R G I=\frac{d \cos 45-\frac{L}{2}}{d \cos 45}$
And the Unitary Geometric Incompatibility, UGI, defined as (2)
$U G I=\frac{d \cos 45-\frac{L}{2}}{d_{o}}$
$L$ and $d$ are the projections of the module on the horizontal plane at each instant of the folding process (Fig. 6), while $L_{0}$ and $d_{0}$ are the values of $L$ and $d$ corresponding to initial position. It is possible to express $L$ and $d$ at each instant as a function of the angle of inclination $\phi$ of the main bar of the diagonal SLE (Fig. 7): as the exterior scissors and the diagonal


Fig. 6. Schematic module ground plan.
ones share the same vertical axis - the ridge - they can be collapsed into the same vertical plane, converting the problem into one of triangle geometry. In this way the values of $L$ and $d$ during the folding process are obtained using a conventional spreadsheet.

The geometric study starts from the deployed position of the module and analyses the geometric incompatibilities that occur during its folding. To do this, the angle $\phi$ is varied from an initial value $\phi=\phi_{0}$ to the full folded value, $\phi=\pi / 2$, plotting the results obtained.

According to Fig. 7 notation, the former expressions are:
$R G I=\frac{2 b \cos \phi \cos 45-(e+f) \sin \gamma}{2 b \cos \phi \cos 45}$
$U G I=\frac{2 b \cos \phi \cos 45-(e+f) \sin \gamma}{2 b \cos \phi_{o} \cos 45}$
When approaching full folding, the RGI equation leads to an indeterminacy of the type $0 / 0$ when the theoretical lengths of the bars are introduced into it; however, it is relevant when using construction values, as will be seen later.

With the previous definitions, positive results imply that the projection of the diagonal SLEs is greater than the one of the perimeter SLEs.

The slope of the major diagonal bar has been taken as a reference because it is considered to clearly describe the folding process, facilitates the comparison of systems with translational SLEs of any type, and has advantages for image analysis.

In order to take into account the layout variants of the composite SLEs, the incompatibility has been evaluated according to their design angle $\psi$ (Fig. 8), finding that it is strongly non-linear, from a 'stress-free' situation to a situation of maximum incompatibility. This occurs when the composite scissors are horizontal $\left(\psi=0^{\circ}\right)$ and is cancelled when the bars of the perimeter SLEs (blue) have the same angles as those of the main scissors (red), $\phi=\phi^{\prime}$, which in this case corresponds to $\psi=25^{\circ}$. This result is the generalisation of the situation of flat meshes with three directions of deployment without incompatibilities.

## 3. Models with eccentric bar axis nodes

A group of kinematic roof models on a scale of $1 / 10$ was constructed (Fig. 9), having a slope in true magnitude of $25^{\circ}$. For this purpose, methacrylate bars were laser cut (8 bars for diagonal SLEs and 16 more for perimeter SLEs) ${ }^{1}$ and 10 PLA nodes were produced using a 3D printer, of the type known as eccentric bar axis nodes ([29] p. 144) measuring $10 \times 10 \times 10 \mathrm{~mm}^{3}$.

The first module was designed with the arrangement shown in Fig. 4b (double symmetrical perimeter SLEs, $\psi=0^{\circ}$, in blue in Fig. 10 and was constructed with $10 \times 4 \mathrm{~mm}^{2}$ methacrylate bars in all cases. The second one (Fig. 12) was drawn according to Fig. 4c (perimeter SLEs with simple symmetry, $\psi=12.52^{\circ}$, shown in magenta in Fig. 10), reducing the section of the perimeter blade bars to $6 \times 4 \mathrm{~mm}^{2}$ to achieve a more compact folding, since the closing of the compound scissors limits the folding of the module [21]. Finally, a third module was built by combining the previous blade types, with scissors of the same type on the parallel faces.

The width of the bar determines the spacing between the axis of the end joints of the composite blade bars (Fig. 11) according to the formula (5):
$S=2(e+f) \sin \left(\frac{1}{2} \arcsin \frac{w}{e}\right)$
With the previously indicated dimensions of the SLEs, the formula

[^1]

Fig. 7. Folding of the diagonal (red) and perimeter (blue) SLEs on the same vertical plane. The dotted line shows an intermediate deployment position corresponding to a half projection of the main blade in fully deployed position, $D=0.5 D_{0}=d_{0}$ (Geometric data: Main SLEs: $a=2.00 \mathrm{~m} ; \mathrm{b}=2.734792 \mathrm{~m} ; \mathrm{h}_{1 \mathrm{o}}=3.73046127 \mathrm{~m}$. Perimeter or bracing SLEs: $e=2.36739582 ; f=0.70987298$ ). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)


Fig. 8. Definition of the reference angle $\psi$ of the composite SLEs
shown above results in a separation of 43.21 mm for 10 mm wide bars and 26.04 mm for 6 mm wide bars (the inner angle is $9.859^{\circ}$ and $5.896^{\circ}$ respectively). This leads to maximum inclinations of the reference bars of $\varphi=86.08^{\circ}$ and $\varphi=87.65^{\circ}$ in Figs. 7 and 8 compared to the theoretical $90^{\circ}$.

The models were made with eccentric bar axis nodes as they maintain the theoretical centre-to-centre distances, (Fig. 13), and so it was assumed that their behaviour would not interfere with the
incompatibilities that occur during folding. Fig. 14 shows the variation in the theoretical geometrical incompatibility of these models.

The UGI curves show the bistability of the cases studied: both in the situation of complete folding ( $\varphi=90^{\circ}$ ) and of maximum opening, the absolute geometric incompatibility (UGI) is null. For $\Psi=25^{\circ}$, the entire curve shows a null incompatibility, so it is a 'stress-free' situation. The RGI curves reflect the relative incompatibility, which in the folding situation is asymptotic, due to although the incompatibility tends to zero, so does the opening of the module.

The positive values of the graphs imply that, during deployment, the perimeter SLEs tend to be tensioned and the diagonal ones compressed.

The deployment process displayed a clearly bistable behaviour: from the actual folded position, $\mathrm{P}_{0}$ (the theoretical maximum closed position $\varphi=90^{\circ}$ is not reached due to the wide of bars), to begin deployment, one of the perimeter nodes of the module has to be pulled horizontally. From a given point, $\mathrm{P}_{1}$, the opening process continues on its own under its own weight without the application of any horizontal force; the process stops at a point $\mathrm{P}_{2}$, as the deployment approaches the position corresponding to the greatest geometrical incompatibility; from this point onwards, the application of some external force is necessary to continue the aperture. In the first few moments, if the force is released, the deployment is stabilised; but as the maximum UGI incompatibility is approached, if the load is removed before it is exceeded, the structure closes slightly. Once the maximum incompatibility point $\mathrm{P}_{3}$ has been exceeded, the deployment continues by itself, opening suddenly: this is the well-known 'snap through' of bistable modules. Obviously the precise points at which the changes in the stop-motion situation occur depend on both the layout of the module and the friction with the support surface (the behaviour of the models was tested on glass, melamine and linoleum surfaces), so that


Fig. 9. Three models with eccentric bar axis nodes, M1 (a), M3 (b) and M2b (c).


Fig. 10. Perimeter blade design for the bistable module models studied.


Fig. 11. Maximum closing position of a composite blade


Fig. 12. Model number 2, M2.
the lower the friction, the greater the section where the process does not require external forces. Table 1 shows the openings corresponding to these positions on a glass surface.

While overcoming of the maximum incompatibility, the bars deform mainly due to bending of the horizontal axis [18], visibly sagging, from which they then recover without any problem.

In the early stages of deployment, certain perimeter composite scissors appear to be stress-free, possibly due to small construction differences arising from system clearances and tolerances and other secondary effects, such as the rotation of the nodes. It's considered that these same reasons explain the small horizontal displacements observed in Figs. 16 and 29 between the theoretical and experimental values.

The difference between the folding and unfolding processes is noteworthy: the latter can be achieved by applying exclusively horizontal forces, but this is not the case in folding, which requires the application of vertical forces to overcome the snap (two vertical and
opposite forces were applied on the central nodes), although once this phase has been overcome, folding can be achieved by only applying two opposite horizontal forces on the lower diagonal nodes. Folding can also be achieved by applying a single vertical downward force at the lower centre node, but applying a vertical upward force at the upper node lifts the model: the perimeter scissors are sufficiently effective to maintain the shape.

During folding, the intermediate equilibrium positions that were found during deployment were not achieved. Furthermore, if the application of external forces is interrupted, the snap-through effect is not sufficient to compensate for the action of the structure's own weight, and the structure would return to the deployed configuration.

Fig. 14 explains the timing and intensity of the snap through phase but not the initial phases of the opening, for which only the RGI curves shed some light. In order to specify the snap phase more precisely, the initial slope of both curves has been adjusted. For this purpose, the UGI formulation is redefined by calculating the percentage of incompatibility over the half-length of the reference bar ('b' in Fig. 7), obtaining the curve designated as MUGI (Modified Unitary Geometric Incompatibility) (6),
$M U G I=\frac{d \cos 45-\frac{L}{2}}{b}$
Fig. 15 shows the result, showing that the snap tends to coincide with the separation point of both curves.

In Model 1, opening only occurred without the application of any additional force after the snap. However, from a certain moment onwards, prior to the snap, it is necessary to apply two diagonally opposite forces, since a single horizontal force only displaces the model. It is therefore considered to be in line with the general behaviour described above. Models 2 and 3 displayed the general behaviour described above.

Initially these models were built with perimeter nodes that only allow unrestrained rotation on the axis perpendicular to each face, while


Fig. 13. Schematic plan drawing of the model using eccentric bar axis nodes and introducing a reciprocal node in the central one (only one level of bars is plotted).


Fig. 14. UGI and RGI incompatibilities of the M1 $\left(\psi=0^{\circ}\right)$ and M2 ( $\psi=$ $12,52^{\circ}$ ) models

Table 1
Individual points of the opening process.

| Model | $\mathrm{P}_{0}(\mathrm{~mm})$ | $\mathrm{P}_{1}(\mathrm{~mm})$ | $\mathrm{P}_{2}(\mathrm{~mm})$ | $\mathrm{P}_{3}(\mathrm{~mm})$ | Opening $(\mathrm{mm})$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | $42-44$ | - | - | $427-428$ | $475-477$ |
| 2 | $32-33$ | $210-212$ | $370-374$ | $407-410$ | $474-476$ |
| 3 | $34-40$ | 185 | $353-368$ | $425-430$ | $473-476$ |
| 1 M | $42-44$ | $194-210$ | $373-374$ | $407-412$ | $475-478$ |
| 2 M | $32-33$ | 195 | - | - | $473-475$ |

imposing a certain constraint on rotation with the axis of rotation contained in the plane of the node face. These nodes in Models 1 and 2 were then replaced by the nodes in Fig. 16, which for practical purposes function as spherical ball-and-socket joints. The response was greatly softened, as can be seen in Table 1:

The deployment on a glass surface of Model 1 M (M: modified) behaves according to the general pattern described above, with an opening phase only occurring under the action of its own weight prior to the 'snap' phase, which does not exist in Model 1.

During deployment on a smooth surface, Model 2 M remains stable until a horizontal distance of about $190-195 \mathrm{~mm}$ is reached between the horizontal axes of rotation of the composite SLEs. From that point onwards, the opening starts of its own accord until it is fully completed, the 'snap' being distinguished by the speed at which the movement takes place. Although the modules are different, the result seems to confirm


Fig. 15. Geometric incompatibilities related to the angle of the bar. Model 1. MUGI curve.
that in a situation with little or no friction, beyond a certain point, the folding process requires the permanent application of the force as indicated in [23].

The interpretation of the process is made clearer with the aid of the geometric incompatibility graph shown in Fig. 17, the whose X-axis represents the horizontal distance mentioned above.

The difference between the points of maximum geometric incompatibility and P3 may be due to friction effects and measurement problems.

In the graph a new variable $(\Delta \mathrm{L})$ is introduced defined as:
$A L=\Delta L=\frac{2 b \cos \phi \cos 45-(e+f) \sin \gamma}{(e+f) \sin \gamma}=\frac{d \cos 45-L / 2}{L / 2}$
During the various folding processes carried out, it became evident that the nodes influence the process in an active way, undergoing horizontal axis rotations that tend to reduce the degree of incompatibility between the different scissors (Fig. 14). In any case, the influence of this type of nodes is not particularly significant.

Given the good performance of the previous models, Model 3 was built by combining the perimeter composite SLEs of the previous models, (Fig. 18), whose side elevations correspond to Fig. 4b and 4c. In the folded position, this model has different dimensions depending on the perimeter scissors that are fitted, forming a rectangle on the ground plan. The general operation described above is maintained, with a


Fig. 16. Model 2 M. Folded Position. Top View
behaviour close to that of Model 2, perhaps because, if the nodes allow it, the least incompatibility applies: the module readjusts itself by deforming on the ground plan.

The module has also been modified by adding a reciprocal support [30] to the upper vertex, (Fig. 13), which limits the maximum opening and assists the perimeter SLEs in resisting gravitational force. Its inclusion did not produce any variations in the described deployment process.

In this way, the kinematic viability of bistable modules with reciprocal nodes is verified and a new opportunity is opened for the application of these nodes within the research on the systematic application of reciprocal nodes in deployable structural solutions that our team is carrying out.

It has already been shown that during the process of overcoming the
incompatibility, the bars mainly deform by bending of the horizontal axis within the elastic range. This deformation occurs in the bars of the diagonal SLEs according to the diagram shown in [13]. Similar deformations occur in the bars of the perimeter SLEs, but their shape depends on the blade layout in question, and are particularly visible in modules with smaller bar cross-sections.

After deployment, a transverse outward deformation of the perimeter scissors is visible, which, in the module with two types of composite SLEs (Model 3) is more evident in the scissors with smaller cross-section bars. The very design of the composite blades undoubtedly has a decisive influence on this deformation, due to the rotations allowed by the clearances.

## 4. Numerical analysis: Stresses in the bars

A dynamic calculation of this module has been carried out with the programme ARTIC [31] to verify the behaviour and fit of Model 1. For this work, a new type of graph has been developed that shows the evolution over time of the different efforts and that includes the changes in the reactions.


Fig. 18. Module with combined composite SLEs


Fig. 17. Model 2. Theoretical geometrical incompatibilities during deployment in relation to the horizontal distance between the extreme rotary axes of the perimeter SLEs. P0, P1, P2 and P3 from Table 1 Model 2 (proportional)

### 4.1. Materials and methods

The module has dimensions of $5.66 \times 5.66 \mathrm{~m}$ on the ground plan and 3.73 m in height, using aluminium tubular bars with a circular crosssection of 100.5 for the diagonal blades and 100.3 for the perimeter composite SLEs, with the characteristics shown in the attached sheet ${ }^{2}$.

The model studied is shown in Fig. 19 below:
Two types of dynamic calculations have been performed. In the first, the folding of the model is achieved by applying horizontal loads in X and Y directions on the corner nodes. The loads, 4.00 kN each, are oriented inwards and kept constant during deployment. With this, a resultant diagonal force of 5.65 kN towards the centre of the model is originated in each node.

In addition, vertical upward loads were applied to the upper nodes of the four corners. These loads were increased in successive calculations until reaching a sufficient magnitude ( 4.00 kN also) to overcome the point of maximum incompatibility.

This type of calculation would represent the behaviour of the structure in real time if the indicated loads were applied from the beginning and kept constant. It takes into account the inertial forces of accelerations and decelerations, so it will be designated as 'dynamic calculation with constant loads'.

The second type can be called quasi-dynamic calculation since the behaviour of the structure is forced by setting constant vertical displacements of the corner nodes in each interval of the calculation. For this reason, in the obtained graphs the curve of the Z displacement of the corners is a line of constant slope. In these calculations the so-called 'corner forces' appear, the name used to designate the vertical forces that must be applied at each instant in the corner upper nodes to achieve the aforementioned constant vertical displacement.

### 4.2. Results and discussion

The following figures summarise the studies carried out. Figs. 20 and 21 reflect the results of the first group of calculations, while Figs. 22 and 23 show those of the second.

The graphs show the variation over time of the axial and bending forces of the bar as well as the displacement of its upper end node, a parameter that indicates whether the module has exceeded the position of maximum incompatibility.

In these graphs, the horizontal axis indicates the time and the vertical axis the values of the displacements or internal forces as the case may be. The scale of the displacements remains constant in all of them with a maximum value of 2.8 m as the geometry is the same. However, the internal force scale is slightly modified to better visualise the results.

Fig. 22 represents the variations of the internal forces in the diagonal bars of the module, indicating the axial forces in its two stretches and the bending maximum moment. It also reflects the energy of the system and the corner forces necessary to carry out the process. Fig. 23 shows the same in the edge bars.

The general behaviour obtained is quite uniform:
During the process the diagonal bars (bars 1 and $4^{3}$ ) are under compression while the perimeter bars (bars 11 and 12) are in a tension situation (stretch 1 of bar 4 and stretch 2 of bar 11 contradict this

[^2]statement but with small values)
In these graphs we call 'point of maximum incompatibility' the point at which the internal energy of the system is maximum (therefore unstable point), tending to a state of minimum energy readapting its shape either towards folding or unfolding. This point has been marked with a vertical dotted line on the graphs to facilitate analysis.

In the dynamic calculation, this point marks the beginning of the clear ascent of the corner nodes. Simultaneously, both types of diagonal bars reach their maximum compressions (the absolute maximum axial force is achieved in stretch 2 of bar 1), and the perimeter bars, the maximum tension (absolute maximum in bar 12). This leads the structure to reach its maximum energy.

It is noteworthy that in the quasi-dynamic calculation, the 'corner force' reaches its maximum just before the previous point, subsequently reducing its value even arriving to change its sign, increasing it again in the final section, in a similar manner to [23]. This maximum coincides with a relative maximum in the bending of bar 4 , which soon after becomes zero, rebounding later.

The maximum compression of bar 4 occurs shortly after maximum incompatibility at once the maximum bending, that occurs in the bar 1 (the greatest bending and the greatest axial compression occur in this bar), a circumstance that induces think that the axial of the first (bar 4) causes the bending of the second (bar 1).

Once the incompatibility has been overcome, the efforts decrease as the folding continues. In any case, the bars subjected at each instant to the greatest internal forces are changing throughout the process.

In summary, the calculations show that the greatest efforts are produced in the diagonal bars, especially in bar 1, which support the highest axial and bending values. In the perimeter bars, despite their symmetrical layout, the bars that originate from the supports are in a worse situation, although what is perhaps surprising is that the behaviour is not too different despite the fact that one of the types does not reach the supports.

## 5. Models with offset rotating axis nodes

The influence of the looseness of the perimeter blade attachment and its influence on the performance of the models was a constant concern in previous models. For this reason, a series of new models with offset rotating axis nodes were developed [29].

A model was proposed with this type of node made of PLA using a 3D printer with a similar design to those found in the bibliography, but thicker. Stainless steel bolts and nuts were used as connecting elements, the heads of which are embedded in the fins of the nodes (Fig. 24).

To compensate for the 4 mm thickness of the node fins and to prevent transverse bending of the bars a washer was inserted between each pair of intersecting bars, made of the same material and with the same thickness as the bars.

In this case the central $x$-shaped node alters the compatibility conditions of the system, as it introduces an additional length (the extension of the distance between the axis of rotation) which the perimeter SLEs must absorb. In the bistable modules referred to in the literature, there are two ways to solve the problem [26], either by adjusting the length of the beams, using the Adapted Beam Lengths (ABL) system, or by compensating the size of the nodes, using Adapted Hub Dimensions (AHD). Both solutions are based on the fact that the single perimeter scissors define the folding properties of the module. In this case it is the other way round: these conditions are determined by the diagonals, so that the previous answers do not apply.

A way to absorb the increase in length due to the node is by inserting an extra piece. This solution behaved correctly during deployment but, of course, it collapsed when it reached the intended opening position: the component behaves like a connecting rod subjected to the compressions induced on the perimeter SLEs when exceeding the intended opening position, (Fig. 25), which indicates that the geometric condition of maintaining the projections of [7] is necessary, not sufficient.


Fig. 19. Calculation diagram.


Fig. 20. Dynamic Calculation Type 1. Diagonal Bars. Results.

To resolve the incompatibility, it was decided to prioritise compatibility in the deployed position: the increase in length of the diagonal is absorbed by extending the short sides of the composite SLEs (Fig. 26). This provides full compatibility in the deployed position, while
improving the system's packaging conditioned by the composite blade construction solution described above.

The most striking feature of the behaviour of the resulting model was the unexpected impact of the weight of the connecting elements, which


Fig. 21. Dynamic Calculation Type 1. Perimeter Bars. Results.


Fig. 22. Quasi-Dynamic Calculation. Diagonal Bars. Results.
led to an unexpected final position with the central bars exceeding the horizontal and with alternating transverse deformations of the perimeter SLEs. Once detected, it was found that this problem also occurred in earlier models, but in a barely visible form.

Also noticeable were the rotations of the nodes, of the vertical axis and opposite directions in the central nodes, and of horizontal axis in the perimeter nodes (Fig. 27). During deployment, the diagonal SLEs are subjected to compression with bending, and the perimeter scissors to traction with bending: the eccentric compression is responsible for the rotation of the central nodes, which are stabilised by the four symmetrically arranged bars.

The rotation of the edge nodes contributes to reducing the stresses in the bars, as it shortens the length of the bars, which are tensed during
deployment. An approximate measurement obtained rotations between $10^{\circ}$ and $15.26^{\circ}$ in the support nodes (in the upper nodes they are higher) during different moments of the deployment. This behaviour has not yet been taken into account in the studies included in the literature.

The linkages used showed a marked tendency to loosen, especially those connecting the perimeter SLEs, which is evidence of energy dissipation. In addition, these blades move transversely (the plane they define is prone to cylindrical deformation) and tend to separate. This behaviour is similar to that observed with the modification of the perimeter nodes in the previous models: the horizontal axis rotation of the perimeter nodes causes tilts in the edges of the base hub of the node, which induces transverse deflections on the perimeter SLEs when the nodes incorporate constraints to free rotation. This is therefore a


Fig. 23. Quasi-Dynamic Calculation. Perimeter Bars. Results.


Fig. 24. Model 4b. Folded Position. Top View
component of incompatibility due to the movement restrictions caused by the nodes themselves, which has not been reported to date.

Deployment of the model on a smooth surface reproduced the pattern shown above, with the following values (Table 2):

In the physical models constructed, the lengths of the bars have a limited precision due to the manufacturing tolerance. This adjustment is important in this case, as the expressions obtained are very sensitive to rounding (Fig. 28). In this graph, the x -axis is taken as the horizontal distance between the extreme turning axes of the composite scissors (the horizontal opening of the module) to facilitate the experimental contrast, in particular of RGI, which cancels out at both ends.

The graph starts from a situation of zero incompatibility, which corresponds to a theoretical opening of 3.11 cm . Below this value, rapidly increasing negative incompatibility values are obtained. However, the modulus cannot really be compacted any further due to the dimensions of the nodes and bars: the measured folded dimension is


Fig. 25. Increase of incompatibility when the expected equilibrium position is exceeded.


Fig. 26. Module built with correction of the lengths of the central SLEs
slightly larger than the previous value.
The nodes have an active behaviour: their operation depends on the negative or positive value of the stress (they behave like compression


Fig. 27. Rotation of external nodes

Table 2
Selected points of the deployment process.

| Model | $P_{0}(\mathrm{~mm})$ | $P_{1}(\mathrm{~mm})$ | $P_{2}(\mathrm{~mm})$ | $P_{3}(\mathrm{~mm})$ | Opening (mm) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 37 | 151 | $349-361$ | $404-409$ | $472-475$ |

rods), and the rotations they undergo modify the distances between the end joints of the SLEs. They also induce transverse deflections in the perimeter bars during deployment and transverse deformations after deployment, as they modify the theoretical opening positions. The incompatibility to be overcome has a geometric component, but also depends on the restrictions of the nodes to transverse bending: nodes that do not impose this restriction significantly reduce the incompatibility of the solution compared to those that do. This can therefore be described as a new type of incompatibility due to movement restriction in the deployment phase.

In the previous physical model, a finite-dimensional node has been introduced which, despite being plotted in compliance with the compatibility conditions of the SLEs, affects the compatibility of the structure during deployment, as is shown in Fig. 29. This also shows the curve corresponding to the theoretical situation of the module and those
corresponding to two situations with approximate bar lengths, one of which overlaps with the theoretical situation.

Fig. 29 shows that the solution developed increases the maximum incompatibility of the system and significantly affects the variation of the incompatibilities from the folded position: compared to an almost linear growth of the theoretical solution (and of the solution with appropriately adjusted approximate lengths) the incompatibility appears suddenly with small openings, and then recovers an almost linear progression.

## 6. Model with manufacturing errors

It was also decided to assess the significance of a module manufacturing error. For this, an error is assumed in the manufacture of the bracing SLEs, moving the cut-off point 2.534 mm (1.415\%) out of the theoretical compatibility point. Built as sated, the model was able to unfold: in the closed position the bars appear to be slightly deformed and the nodes rotated to increase spacing.

Fig. 30 reflects the geometric incompatibilities of the module by moving the cutting point of the perimeter SLEs the values indicated in the legend. It also includes the curves corresponding to the theoretical and construction dimensions of the module. This displacement allows the second stability position to coincide with the actual folding situation of the module, also affecting the degree of geometric incompatibility introduced, but undoubtedly the study of other factors such as looseness or the influence of nodes has to be previously clarified.

## 7. Conclusions

The new modules with perimeter composite SLEs developed from the bias deployable structures have a bistable behaviour proved with physical and computer models, and constitute a generalization of those proposed by Krishnapillai, although with their own characteristics.

The proposed modules have greater flexibility because they admit more variants (mainly by modifying the perimeter SLEs), are able to incorporate the use of bearing improvements such as reciprocal nodes, and allow various forms of introduction of incompatibilities (by modifying the layout of the perimeter SLEs, the sections and materials of bars, etc...), enabling the degree of incompatibility introduced to be modulated. The physical models carried out have demonstrated the good operation of the solution even using different SLE combinations.


Fig. 28. Incompatibilities in relation to opening. Construction dimensions


Fig. 29. UGI graph of incompatibilities considering the actual size of the central node.


Fig. 30. Graph of incompatibilities of a module with construction errors.

The study of geometric incompatibilities during the deployment of the module is a method that allows approaching the operation of the module with simple means from the determination of the maximum geometric incompatibilities. The proposed geometric incompatibility graphs allow an efficient comparison and adjustment of solutions, substantially reducing the effort to develop the final solution. They also reflect the repercussions of the types of knots used and the maximum opening positions of the module that derive from the alteration of the lengths obtained in the compatibility equation.

The dynamic calculations carried out confirm that the sign of the axial forces acting on the bars can be deduced from the geometric incompatibility graphs obtained and also the importance of the maximum incompatibility point, since this point is associated with the maximum energy of the module as well as the maximum compression, and also with other maximums that occur in its vicinity. Obviously, the dynamic analysis provides lots of other relevant information to understanding the behaviour of the module, such as that the instants in which the
maximum compression and the maximum bending occur are not coincident, that the impact of bending on the diagonal and perimeter bars is very different, or that the internal forces in the perimeter bars are hardly modified even when the incompatibility is not overcome.

The behaviour of the nodes conditions the operation of the solution, not only because it depends on the sign of the axial force, but also because the rotations they undergo modify the theoretical dimensions of the model and induce transverse bending on the bars of the perimeter SLEs.

## Funding

This research was carried out as a part of the Spanish Research Project on Deployable and Modular Constructions for Situations of Humanitarian Catastrophe, CODEMOSCH (Reference BIA2016-79459-R), funded by the Spanish Ministry of Industry, Energy, and Competitiveness (MINECO). Financing of the open access fee: Universidade da

Coruña / CISUG.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References

[1] Pérez PE. A reticular movable theatre. Archit J 1961;30(134):299.
[2] Escrig F. Expandable space structures. Int J Space Struct 1985;1(2):79-91. https:// doi.org/10.1177/026635118500100203.
[3] Hernández C, Zalewski W. Pabellón Itinerante de Exposiciones TaraTara, Estado de Falcón, Venezuela; 2005 (based on the ESTRAN 1 Protoype developped by Hernández Merchán and Zalewski, 1987-2000) [http://www.grupoestran.com/](http://www.grupoestran.com/).
[4] Escrig F, Pérez Valcárcel J, Sánchez J. Deployable Structures squared in Plan. Design and construction.. In: Spatial structures: heritage, present and future. Proceedings of the IASS international symposium 1995: June 5-9, Milano, Italia; 1995. p. 483-92.
[5] Pérez-Valcárcel J, Muñoz-Vidal M, Suárez-Riestra F, López-César I, FreireTellado M. A new system of deployable structures with reciprocal linkages for emergency buildings. J Build Eng 2021;13:101609. https://doi.org/10.1016/j. jobe.2020.101609.
[6] Sánchez Cuenca L. Geometric models for expandable structures. In: Escrig \& Brebbia, Ed. Mobile and Rapidly Assembling Structures II. Proceeding of the $2^{\text {nd }}$ MARAS 1996, Seville, Spain, June 17-20, 1996. Southampton: Computational Mechanics Publications; 1996. p. 93-102.
[7] Roovers K, De Temmerman N. Deployable scissor grids consisting of translational units. Int J Solids Struct 2017;121:45-61. https://doi.org/10.1016/j. ijsolstr.2017.05.015.
[8] Zeigler TR. Collapsible self-supporting structure. US Patent 3,968,808. USA; 1976.
[9] Krishnapillai A, Zalewski WP. The design of deployable structures - Kinematic design, Unpublished Research Report, Department of Architecture. Boston: Massachusetts Institute of Technology, MIT, USA; 1985.
[10] Krishnapillai A. Deployable Structures. US Patent US07219548. USA; 1988.
[11] Clarke RC. The kinematics of a novel deployable space structure system. In: Nooshin H, editor. The Kinematics of a Novel Deployable Space Structure System. London: Elsevier Applied Science Publishers; 1984. p. 820-2.
[12] Rosenfeld Y, Logcher RD. New concepts for deployable-collapsable structures. Int J Space Struct 1988;3(1):20-32. https://doi.org/10.1177/026635118800300103.
[13] Rosenfeld Y, Ben-Ami Y, Logcher RD. A prototype 'Clicking' scissor link deployable structure. Space Struct 1993;8(1-2):85-95.
[14] Gantes CJ, Connor JJ, Logcher RD, Rosenfeld Y. Structural analysis and design of deployable structures. Comput Struct 1989;32(3/4):661-9. https://doi.org/ 10.1016/0045-7949(89)90354-4.
[15] Gantes CJ. A design methodology for deployable structures. Ph.D. Thesis. Boston, USA: Massachusetts Institute of Technology, MIT; 1991.
[16] Gantes CJ. Geometric constraints in assembling polygonal deployable units to form multi-unit structural systems. Space Struct 1993;4:793-803.
[17] Gantes CJ, Connor JJ, Logcher RD. A systematic design methodology for deployable structures. Int J Space Struct 1994;9(2):67-86. https://doi.org/ 10.1177/026635119400900202.
[18] Gantes CJ. Deployable structures: analysis and design. Southampton, Boston: WIT Press; 2001.
[19] Valcárcel JP, Escrig F, Estévez J. Expandable triangular cylindrical vaults. In: Congreso Internacional de Métodos Numéricos en Ingeniería y Ciencias Aplicadas. Concepción, Chile; 1992. p. 327-36.
[20] Freire Tellado MJ, Muñoz Vidal M, López César I, Pérez Valcárcel JB. Estructuras desplegables de aspas para cubiertas inclinadas (Scissor-hinged deployable structures for inclined roofs). Informes de la Constr 2019;71(556):e311. https:// doi.org/10.3989/ic. 64120.
[21] Freire-Tellado MJ, Muñoz-Vidal M, Pérez-Valcárcel J. Scissor-hinged deployable structures supported perimetrally on rectangular bases scissor-hinged deployable structures supported perimetrally on rectangular bases. J Int Assoc Shell Spatial Struct 2020;61(2):158-72.
[22] Friedman N, Ibrahimbegovic A. Overview of highly flexible, deployable lattice structures used in architecture and civil engineering undergoing large displacements. YBL J Built Environ 2013;1:85-103. https://doi.org/10.2478/jbe-2013-0006.
[23] Arnouts LIW, Massart TJ, De Temmerman N, Berke PZ. Computational modelling of the transformation of bi-stable scissor structures with geometrical imperfections. Eng Struct 2018;177:409-20. https://doi.org/10.1016/j.engstruct.2018.08.108.
[24] Arnouts LIW, Massart TJ, De Temmerman N, Berke PZ. Computational design of bistable deployable scissor structures: trends and challenges. J Int Assoc Shell Spatial Struct 2019;60(199):19-34. https://doi.org/10.20898/j. iass.2019.199.031.
[25] Arnouts LIW, Massart TJ, De Temmerman N, Berke PZ. Multi-objective optimisation of deployable bistable scissor structures. Autom Constr 2020;114: 103154. https://doi.org/10.1016/j.autcon.2020.103154.
[26] Arnouts LIW, Massart TJ, De Temmerman N, Berke PZ. Geometric design of triangulated bistable scissor structures taking into account finite hub size. Int J Sol Struct 2020;206:84-100. https://doi.org/10.1016/j.ijsolstr.2020.09.009.
[27] Arnouts LIW, Massart TJ, De Temmerman N, Berke P. Coupled sizing, shape and topology optimisation of bistable deployable structures. J Int Assoc Shell Spatial Struct 2020;61(206):1-11. https://doi.org/10.20898/j.iass.2020.009.
[28] Zhao Z, Hu W, Yu L. Experimental and numerical studies on the deployment process of self-locking cuboid foldable structural units. Adv Struct Eng 2020;23 (16):3496-508. https://doi.org/10.1177/1369433220940817.
[29] Begiristain J. Sistemas Estructurales Desplegables para Infraestructuras de Intervención Urbana Autoconstruidas. Ph. D. Thesis. San Sebastián: Universidad del País Vasco; 2015.
[30] Pérez-Valcárcel JB, Suárez-Riestra F, Muñoz-Vidal M, López-César IR, FreireTellado MJ. A new reciprocal linkage for expandable emergency structures. Structures 2020;28:2023-33. https://doi.org/10.1016/j.istruc.2020.10.008.
[31] Muñoz-Vidal M. ARTIC 3 Computer program. Spain. DL: C-79-12. RPI: C-4662011. ISBN: 978-84-92794-52-2 [https://articrigid.blogspot.com/](https://articrigid.blogspot.com/).


[^0]:    * Corresponding author.

    E-mail addresses: manuel.freire.tellado@udc.es (M.J. Freire-Tellado), manuel.munoz@udc.es (M. Muñoz-Vidal), juan.pvalcarcel@udc.es (J. Pérez-Valcárcel).

[^1]:    ${ }^{1}$ Bar Lengths: Diagonal SLE ( $10 \times 4 \mathrm{~mm}^{2}$ in section): Bar 1: $225+225 \mathrm{~mm}$ Bar 2: $164.55+164.55 \mathrm{~mm}$.Perimeter bar length: Model $\mathrm{n}^{\circ} 1\left(10 \times 4 \mathrm{~mm}^{2}\right.$ in section): all $194.77+58.40 \mathrm{~mm}$.Model $\mathrm{n}^{\circ} 2\left(6 \times 4 \mathrm{~mm}^{2}\right.$ in section): Bar 1: $210.12+68.63 \mathrm{~mm}$ Bar $2: 179.43+58.61 \mathrm{~mm}$.

[^2]:    ${ }^{2}$ Calculation Data: SLE bar lengths: Diagonal ( $2.735+2.735 \mathrm{~m} ; 2.000+$ 2.000 m ) Perimetral: all $2.367+0.710 \mathrm{~m}$. Structure dimensions in plan: 5.657 x $5.657 \mathrm{~m}^{2}$. Height of the highest point: 3.730 m .Horizontal Loads on the perimetral nodes: $400 \mathrm{kp}(3.924 \mathrm{kN})$ Bars: Diagonal SLEs 100.100 .5 mm tubes (Area $=1492 \mathrm{~mm}^{2}$, Inertia $=1688000 \mathrm{~mm}^{4}$ ).Perimetral SLEs: 100.100 .3 mm tubes (Area $=914 \mathrm{~mm}^{2}$, Inertia $=1076000 \mathrm{~mm}^{4}$ ). Material: Aluminium 6060 T5 E=70 $000 \mathrm{MPa}, \mathrm{f}_{0,2}=120 \mathrm{MPa}$, Specific weight: $2700 \mathrm{kN} / \mathrm{m}^{3}$
    ${ }^{3}$ Bar 1: diagonal bars numbered as 1, 2, 5, 6; bar 4: diagonal bars, numbers 3, 4, 7 ; bar 11: perimeter bars $9,11,13,15,17,19,21,23$ and, at last, bar 12: perimeter bars $10,12,14,16,18,20,22,24$ (all references from Fig. 19).

