Symmetrical and Geometrical Cycles in Twelve-Tone Composition: Developments Toward a New Model

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ABSTRACT

The development of a pre-compositional model is proposed in this study based on two systems with two design perspectives: Schoenberg's *Serialism* and Perle's *Twelve-Tone Tonality* (from now on TTT). Schoenberg's perspective reveals a linear design where the set has functions like those of a *motive*; on the other hand, Perle's design result in harmonic simultaneities based on symmetric cycles. The authors propose a model for 12-tone composition that assumes an interactive approach between the horizontal and the vertical statements toward a new pre-compositional system based on geometrical and symmetrical issues. This model is being implemented in PWGL for Computer Aided Composition (CAC) in order to assist the extrapolation of the Motivic/Harmonic fundamental requirements of the model. Finally, the empiric outcome produced in the form of musical composition was analyzed, although not presented in its entirety in the scope of this paper.

1. INTRODUCTION

The melodic/harmonic dichotomy is generally viewed in serialistic composition context with a straight connotation with a recent past of tonal functionalism, a connection that should be avoided in light of a necessary paradigmatic breakdown. However, the question in its deeper significance (the relations between the horizontal and the vertical entities) is real, and won't disappear by simply eluding a certain morphologic heritance. To Schoenberg, the twelve-tone set has the function of a *motive* [1], and it is this linear polarity of the *motive* that ought to transmit unity to the musical work. However, the huge universe of 479 001 600 possible permutations of the 12-notes turns the selection of the right set pivotal in pre-compositional assignments.

With the exception of monodic texture, simultaneities result from three basic processing factors: from overlapping lines derived from the original series resulting in a kind of counterpoint (e.g. Schoenberg op.25 no.1, Dallapiccola Goethe Lieder no.2), from the vertical segmentation of the series (e.g. op.33a Schoenberg, op. 37), and from composer's free selection (e.g. Berg's Volinkonzert).

The available studies around composers of the so-called Second Viennese School prove that the choice of the set was never taken randomly or unsystematically. Features such as melodic contour, intervallic relationship, symmetrical properties, invariance, segmentation, and other relevant aspects were studied and experimented patiently (sometimes to exhaustion) by these composers before the final selection of the set. It is this 'unique' set that Schoenberg called the *motive*, the linear stating at the beginning of the work that proposes a reflexive approach of characteristic reference of the set that leads to the desired *unity* of the work as a whole.

In a study related with the *pluralistic analysis*, Jairo Moreno confronts two diametrically opposed ideas of music-theoretical concept of *motive*, namely that of Heinrich Schenker and Arnold Schoenberg. Moreno writes that for Schoenberg, "the motive not only represents a concrete expression of the musical Idea but also lies at the origin of all processes of thematic transformation that he identifies as the historical common denominator uniting his musical making to that of his predecessors"; Schenker considers "motives in terms of voice-leading transformations of simpler underlying event [2] and rejects "those definitions of form which take the motive as their starting point and emphasize manipulation of the motive by means of repetition, variation, extension, fragmentation, or dissolution" [3].

Perle was concern with this problem of organizing the simultaneity of sounds in musical composition with twelve notes. Although departing from a miscalculation, his theory derives also from the 12-note universe – and later would be called *Twelve-Tone Tonality.* Ernst Krenek, confronted with Perle's first serialistic attempted – a sketch for a string quartet – told him he had misinterpreted the twelve-tone system, but at the same time he had made what he called 'a discovery'. This misinterpretation is the symmetrical cycle. A set based on symmetric cycles offers consistent and predictable simultaneities to be extracted from a twelve-tone matrix.[1]

As could be observed in Figure 2 in the next chapter, Perle's system has its final pre-compositional results in the form of chord progressions, vertical entities with structural organization that suggests function *degrees* analogous as those present in functional tonality. So, it

[1] In many circumstances, the process of selection of the twelve-tone set is a constraint of these incidental findings by studying the invariance. These sets, showing similar features after suffering the usual transformations of the system (R, I, RI), minimize what Perle perceived as some weaknesses of the system.

could be easily perceived a melodic/harmonic distinction between Schoenberg and Perle's pre-compositional methods.

It is the aim of this research to develop a model that gathers the best of both worlds: a model for 12-tone composition that assumes an interactive approach between horizontal dimension (the linearity of the motive), and vertical dimension (the simultaneities in the harmony). The new pre-compositional model extrapolates the motivic organization in serialism and the TTT premises of symmetry into the universe of Geometry. In order to assist the extrapolation of the Motivic/Harmonic fundamental requirements of the model, the open GL application PWGL[2] was selected due to its characteristic powerful features in working with lists, lists of lists, and so on.

2. SCHOENBERG'S MOTIVE AND PERLE'S TTT HARMONY

2.1 Schoenberg's Motive: Serialism and Harmony

Schoenberg defined and systematized the dodecaphonic set in order to substitute some unifying and formative advantages of scale and tonality. Since the very beginning of the Twelve-Tone System many composers dealt with the persistent problem of the harmonic subject. "How does one use a linearly oriented system of musical organization with consistent and aurally logical harmony? Chords do not exist as such in Schoenberg's system, as they arise solely from the simultaneous unfolding of given aspects of the work's basic set. Even those chords, expressing portions of the set vertically are idiosyncratic to the ordering of the set." [4] Although functioning as a *motive* as pointed out by Schoenberg himself, sooner or later the compositional process will get juxtaposition of events – or notes simultaneities. But those simultaneous occurrences arise out of the contrapuntal manipulation of the set. As pointed out by Rosenhaus "Schoenberg's is a primarily linear system which deals with simultaneity solely as a result of counterpoint" [5]. "Harmony in such works tend to arise from "accidents" of counterpoint and not from any regular chordal constructs" [6].

2.2 Perle: Cycling Sets as Harmony

George Perle was an American composer, professor emeritus at the Aaron Copland School of Music and an awarded Pulitzer Prize for Music, thanks to his Twelve-tone system of composition based on cyclic intervals and symmetrical inversions. This structural method for composition started with the analytical work he has done with Alban Berg's compositions. Perle considers that the principle of "symmetry" present in his model is the key element that establishes a direct relationship with the Tonal System. He believe that "Symmetry can serve as the foundational premise of a coherent and natural twelve-tone harmonic language, just as the triad does for the harmonic language of diatonic tonality" [7].

George Perle's theory had its origin due to a misinterpretation on Schoenberg's system: In the late thirties: after realizing that from a twelve-tone set one could prepare a matrix, he assumed that any note in this matrix could be selected (note axis) and four neighbor notes (two horizontal and two vertical) set as members of the harmony. The resulting harmonies were ambiguous and inconstant. The amount of existing intervals in the set multiplies by the number of harmonic possibilities in its realization. Later Perle realizes that twelve-tone sets based on constant cycles of symmetrical intervals disclose a consistent pattern that could be used harmonically with no ambiguity or inconstancy.

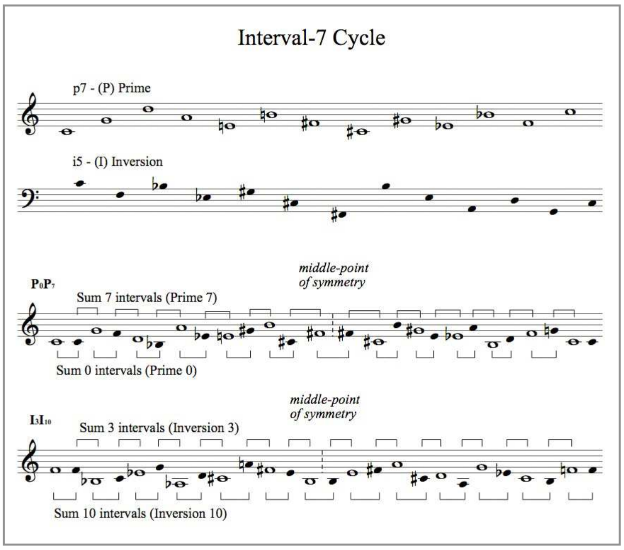

Figure 1. Perle's Interval-7 Cycle.

Figure 1 above presents a symmetrical cycle of interval seven (p7) and its inversion (i5). The terminology reflects the intervallic distance according to ascending or descending movement in the cycle. The interpolation of both cycles (the prime p7 and its inversion i5) results in a new set (in this case named p0p7). This time, Perle's terminology[3] p0p7 reflects the constant interval sums present in the cycle: sum 0 and sum 7. The last system presents a variation of the same interval-7 cycle: i3i10. Starting from a different initial point, interval sums disclose different patterns. However, the linear profile is an inversion (transposed) of p0p7.

The *middle-point of symmetry* reveals two things that emerge in any symmetrical cycle: a reverse ordering invariance, and a tritone transposition (T6) equivalence of the first half of the cycle. In effect, the distinction between retrograde-related row forms was eliminated and any note was free to move to either of its neighbors in either of two inversionally complementary forms [7].

[2] PWGL is a free cross-platform visual language based on Common-Lisp, CLOS and OpenGL, specialized in computer aided composition. It is based on many concepts and ideas that were originally developed for PatchWork. PWGL is developed at Sibelius Academy in Finland Finland by a research team consisting of Mikael Laurson, Mika Kuuskankare, Vesa Norilo, and Kilian Sprotte. (Information retrieved from http://www2.siba.fi/PWGL/) in 31st March 2014.

[3] The complete collection of dyadic cyclic sets could be accessed in Perle's Appendix where he explains the meaning of equivalent sets. 'Sets that share the same pair of tonic sums are equivalent to each other. […] Thus, the interval-1 set p0p1 is equivalent to *i1i0,* the set of the same tonic sums and the complementary interval, 11; the interval-2 set p0p2 is equivalent to the interval-10 set *p2p0*; the interval-3 set p0p3 is equivalent to the interval-9 set *i3i0*; etc (Perle 1996, 253).

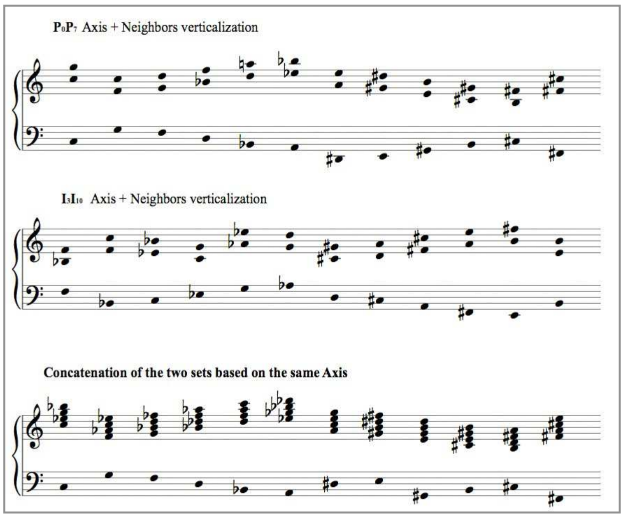

Figure 2. Cognate Cycles p0p7 and i3i10.

Figure 2 presents the harmonic result concatenation of two cognate[4] cycles. The first six chords could be related with the last six chords likewise before: they still disclose a reverse ordering invariance, and a tritone transposition (T6). Figure 3 presents an excerpt from an orchestral work commissioned by 'Guimarães European Cultural Capital 2012' named *Abertura em forma de Pena.* The phrase between bar 50 and 55 was constructed based on Perle's symmetrical cycles present on Figure 2. The music excerpt respects the ordering of the set: the axis line starts in the tuba continuing in the clarinet part, and the associated harmonies were distributed on other instruments.

Figure 3. Abertura em Forma de Pena.

[4] Cognate cycles share the same interval in terms of absolute value. E.g.: ascending interval 7 and its inversion descending interval 7 or ascending 5. This could be observed on Figure 2: in fact, the entire sequence of i3i10 is an inversion of p0p7.

Authors like Carrabré [8], Foley [9], and Rosenhaus [5] proposed new approaches and extensions to Perle's theory. James Carr [10] developed an application that he called 12-tt 2.0 capable to generate an 'encyclopedia' with lists of vectors and matrices; Gretchen Foley developed an application named 'T3RevEng' directed to Aided Analysis [11]; Headlam created a set of applications to matrix identification [12]; and Christopher Winders designed an application that can slide a cyclic set over another in way to produce every possible vertical alignments [11].

2.3 Some Preliminary Conclusions

The fundamental difference between these two systems could be denoted in advance from their theoretical foundation: in Schoenberg's serialism the harmonic implication of any counterpoint is ultimately arbitrary – harmony tend to arise from "accidents" of counterpoint and not from any regular chordal structure; in Perle's TTT verticality and harmonic relations are the solely genesis of his theory, leaving to a secondarily issue the linear approach – in fact, the latter results from the unfolding of the chordal structures.

Table 1 presents a synoptic description of both perspectives with their benefits and weaknesses. Vertical and horizontal dimensions were analyzed and described to a better comparison between each method.

	Horizontal Dimension	Vertical Dimension
Serialism	1. The set already starts from the linear premise; 2. It works as a thematic motive; 3. The Matrix is an auxiliary transformation of the set in R(etrograde), I(nversion), and RI. It does not present a grounded systematic basis for harmonic definition.	1. A pre-compositional criteria for adjacencies of the elements of the set; 2. Linear layering of the set – counterpoint; 3. Segmentation of the set. Usually in hexachords, tetrachords e trichords derived from the properties of the integer twelve.[5] 4. The total content of the set impose pre-compositional restrictions determinants in excluding some harmonic relations.
TTT	1. Symmetrical cycles result in predictable linearity; 2. Segmentation of this kind of sets have always the same interval characteristics; 3. Poor motivic interest.	1. Vertical simultaneities result from interpolation of symmetrical cycles. 2. A pre-compositional criteria for axis elements and adjacent neighbors of the set;

Table 1. Horizontal and Vertical Dimensions.

[5] In serial twelve-tone harmonic practice, "when linearly adjacent elements are simultaneously stated, the original ordering may be disregarded so long as the harmonic entity is identical in content with the segment of the set. The harmonic relevance of the linear formation is thus in inverse proportion to the number of elements that constitute this segment. Obviously, if the harmonic formation contains only two notes the vertical and the horizontal adjacencies will be identical. And if it contains twelve notes it will have no relation to a unique linear arrangement since it could function as a verticalization of any set" (Perle, 1991, p. 85).

It could be assumed that both systems, although based on the same 12-tone genesis, assume different design perspectives with implications in the compositional praxis. Serialism manipulates the set on many structural levels such as the melody and rhythm but not in terms of a regular, and predictable, harmony. The latter arises out of the contrapuntal manipulation of a selected twelve-note row and its permutations. Schoenberg´s twelve-tone system is ultimately linear, one that maintains the integrity of an ordered set, while Perle's system of twelve-tone tonality is ultimately harmonic, one in which the use of traditional influences like "tonality" and "mode" assumes the integrity of the cyclic sets involved. TTT is a "composer-selective" system allowing for regular and predictable chordal structures; Serialism is a "composer-ordered" system [6].

The idea of a new model emerges: a model capable of merging the best of both dimensions.

3. GEOMETRIC HARMONY

3.1 A New Symmetrical Approach

Symmetry seems to work as an integrational constraint that brings equilibrium in 12-tone pre-compositional organization. In order to extrapolate Perle's idea, other symmetrical approaches were studied, namely those related with geometry. Some geometrical figures present an organization of lines, edges, and vertices that could be directly applied to 12-tone segments. The following examples represent a few of those geometrical constraints. To get a normative unity (also present in Perle's system) a rule determine that a chord to be selected must reveal the same sum in their integers based on the chromatic values from C=0 to B=11. Figure 4 represents a geometrical solution to organize trichords of equal sum.

Like composers persistently search of invariances in serial music, here we find an extrapolation of those symmetrical properties in terms of geometrical proportional equivalencies. Each side of the Hexagon represents a trichord. Trichords must share all the same equal sum.

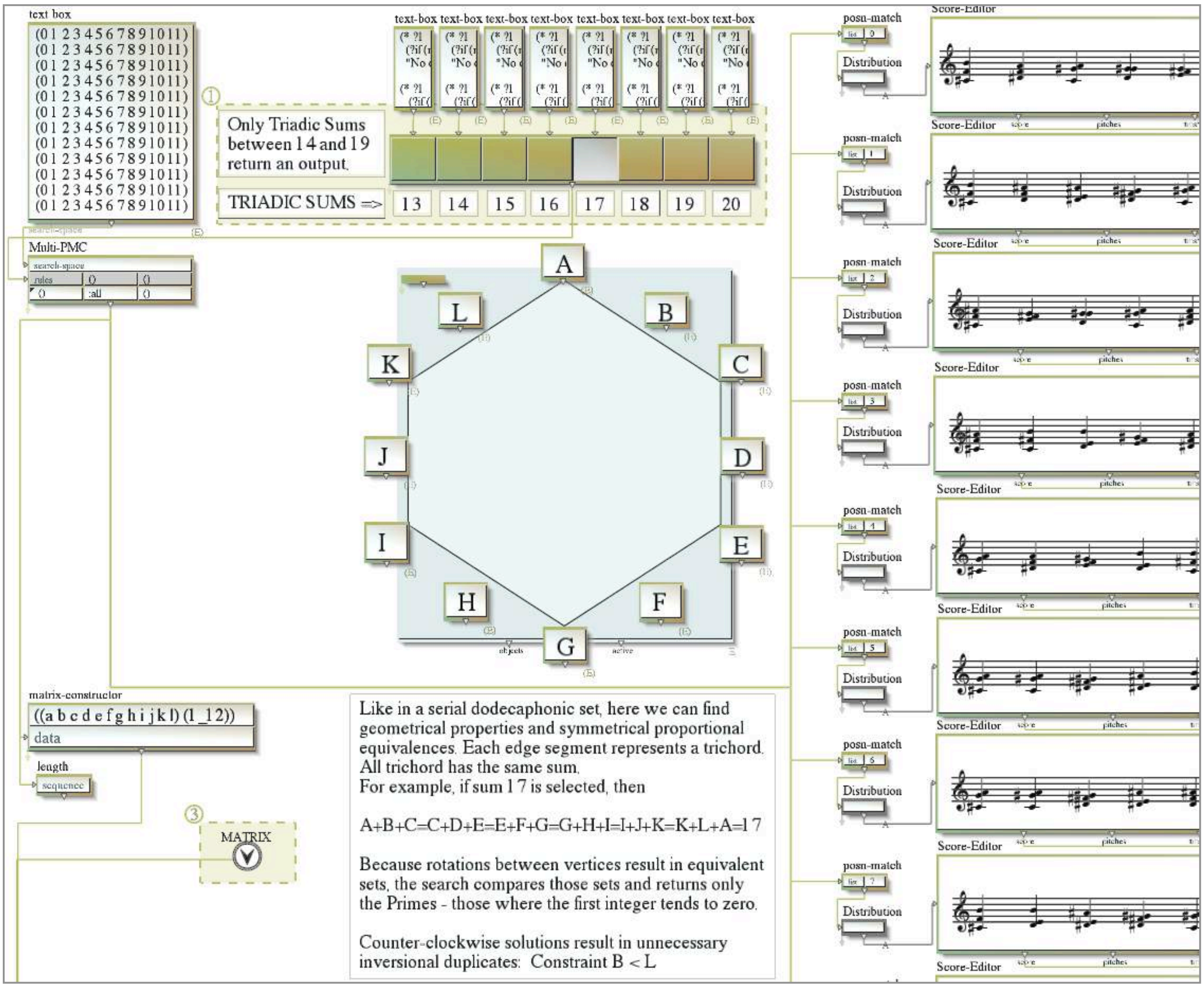

Figure 4. Geometrical Solution for Equivalent Sum Trichords Based on the Hexagon.

For this to be accomplished it is necessary that: A+B+C=C+D+E=E+F+G=G+H+I=I+J+K=K+L+A=sum

A	B	C	D	E	F	G	H	I	J	K	L
0	10	4	9	1	11	2	7	5	3	6	8

Table 2. Resultant Direct Application.

The PWGL patch found solutions to sums between 14 and 19. As the rotation of the set based on the edges result in equivalent sets, the solutions present only the *primes* – those whose first element is closer to 0. Table 2 exemplifies a solution. Each letter relates directly to a position in the set.

To define the six trichords of the set we must extract the values of each side of the hexagon:

Trichords [A, B, C] = [0, 10, 4]; [C, D, E] = [4, 9, 1]; [E, F, G] = [1, 11, 2]; [G, H, I] = [2, 7, 5]; [I, J, K] = [5, 3, 6]; [K, L, A] = [6, 8, 0].

Another invariance emerges from the solutions obtained counter-clockwise. As can be observed on the hexagon symbol, the sequence [A B C D E . . . etc.] is mirrored in its retrograde [A L K J I . . . etc.]. Lets look for example at the results for a sum of 14:

Sum 14:
0 10 4 9 1 11 2 7 5 3 6 8 (0) ←
0 10 4 8 2 11 1 7 6 3 5 9 (0) ←
0 10 4 7 3 9 2 11 1 5 8 6 (0) ←
0 9 5 3 6 7 1 11 2 8 4 10 (0)
0 8 6 3 5 7 2 11 1 9 4 10 (0)
0 6 8 5 1 11 2 9 3 7 4 10 (0)

The first three solutions are already represented by the retrograde of the last three. Thus, a condition B < L was included in the constraint rules code in order to reduce the number of equivalent solutions to their primes:

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(i2 i12
    (?if (< i2 i12)))
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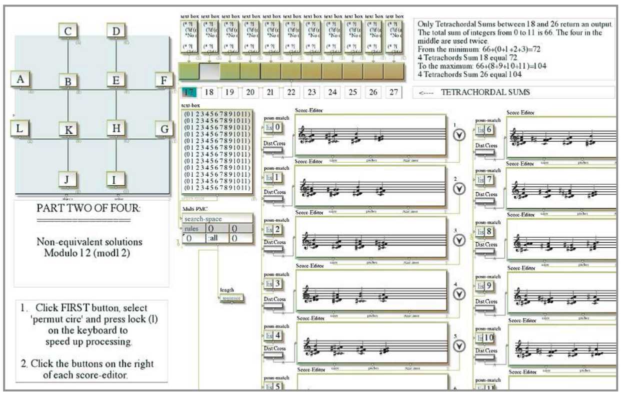

Figure 5. Geometrical Solution for equivalent Sum Tetrachords Based on Double-Cross.

Figure 5 is another representation of a geometrical solution to organize tetrachords of equal sum based on the chromatic values from C=0 to B=11. The patch found solutions to sums between 18 and 26.

Table 3 and Figure 6 illustrate the first solution for Sum=18. We can thus define the four Tetrachords according to the geometric figure as follow: [A, B, E, F] =

A	B	C	D	E	F	G	H	I	J	K	L
4	3	6	8	1	10	11	0	9	7	2	5

Table 3. Resultant Direct Application.

[4, 3, 1, 10]; [D, E, H, I] = [8, 1, 0, 9]; [G, H, K, L] = [11, 0, 2, 5]; [J, K, B, C] = [7, 2, 3, 6].

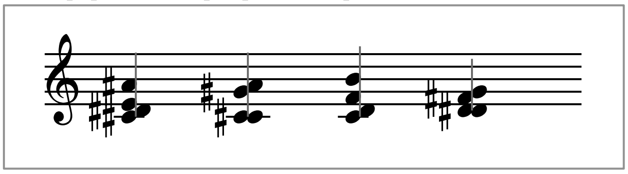

Figure 6. First Solution for Sum=18.

3.2 Pre-compositional Application

As related before in chapter 2, it is the aim of this new model to achieve balance between vertical and horizontal dimensions. The following practical application was the result from a commission for the *Filarmónica União Taveirense*. The purpose was to sign a decade of musical direction under the baton of João Paulo Fernandes. The work intitled “87.658,13H” is easily related with the amount of hours and minutes that 10 years comprise, visible also by the metrical distribution of bars during the introduction and the final of the piece with recurrent sequence of the time signatures 8/4, 7/4, 6/4, 5/4, 8/4, and 13/8. The melodic construction is based on the serial set [0, 5, 10, 9, 8, 11, 4, 6, 7, 3, 2, 1]. The two inicial motives result from splitting the set in two melodic segments as shown in Figure 7.

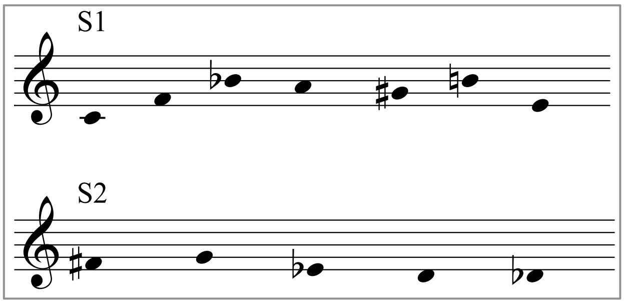

Figure 7. Set Divided in Two Melodic Segments.

The PWGL patch used for the pre-compositional unit could be seen in Figure 8 on the next page. Following the principles of the model proposed, and based on a geometric *six-point Star* representation, one of the possible solutions presented by the application is related with the geometric harmony fulfilled by the tetrachord *Sum 22*. Only harmonic sequences containing the melodic segment could be used.

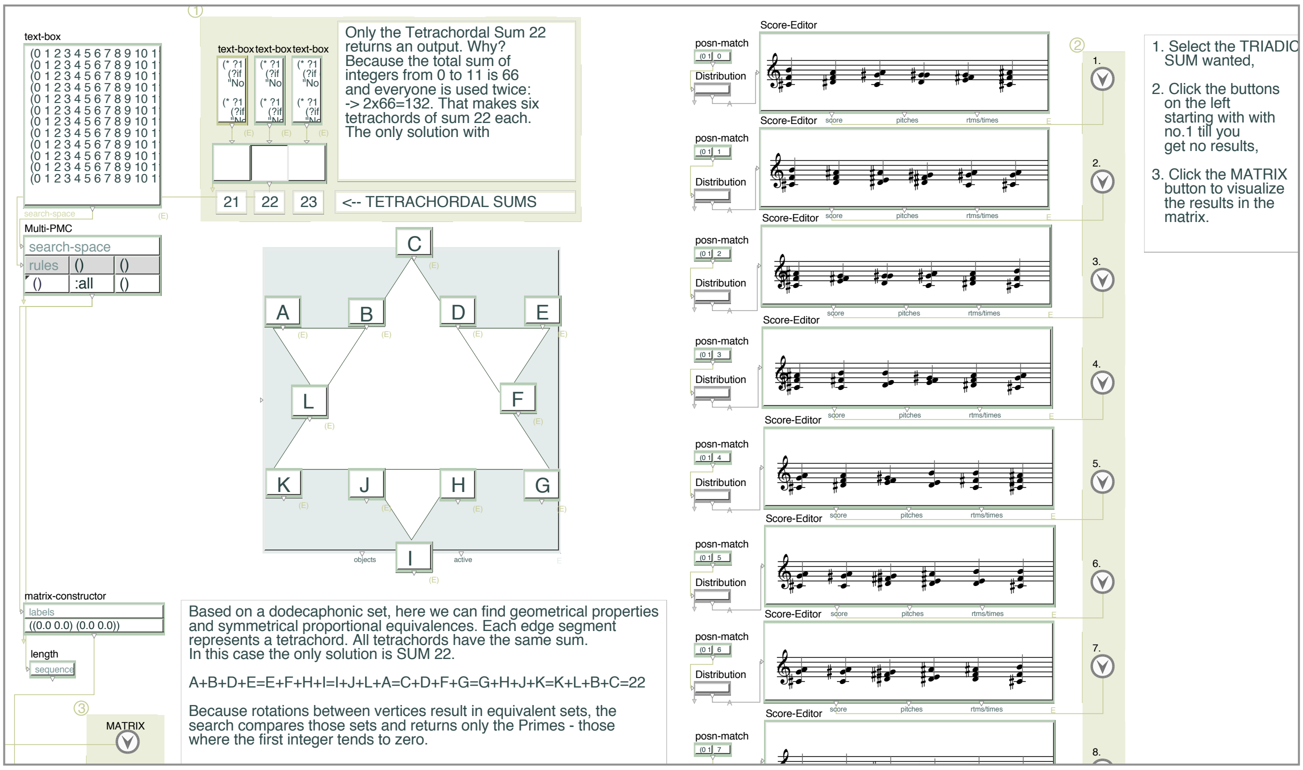

Figure 8. Geometrical Solution for equivalent Sum Tetrachords Based on a six-point Star.

The example in the Figure 9 is representative of one of the positive solutions founded compatible with the first melodic segment S1.

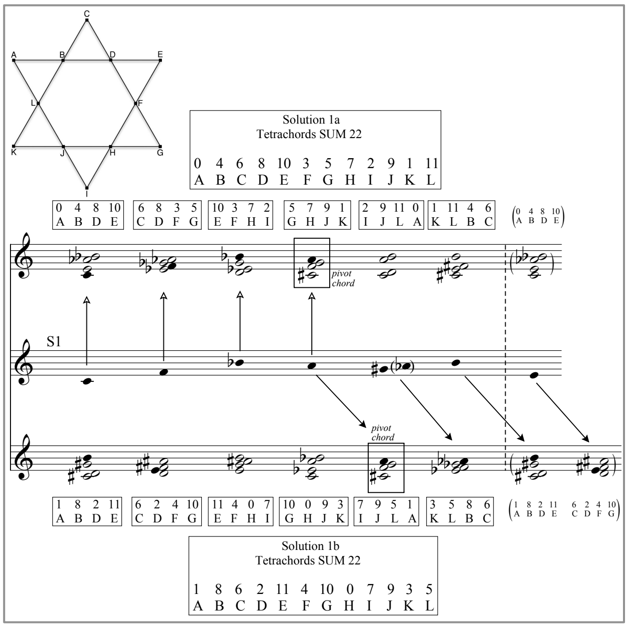

Figure 9. Melodic Segment S1.

4. CONCLUSIONS

Schoenberg and Perle's methods of composition still present a sparkling freshness nowadays due to their solutions concerning Twelve-tone music composition. The model presented in this report is being developed and clarified with theoretical and practical approaches resulting in musical works. Analytical studies of these compositions should be prepared, and offered in future articles. Still, an expansion on Perle's theoretical system towards a new pre-compositional model, suggests a comparative analysis with other relevant 12-tone structural organization theories. A PWGL specific library dedicated to this matter will be a subsequent stage of development in near future research.

Acknowledgments

We would like to thank to Dr. Patrick Carrabré for his supportive approach, especially in the very beginning of the investigation. We are also thankful to IPP – Politechnique Institute In Oporto for his support during this research.

5. REFERENCES

[1] A. Schoenberg, *Style and Idea: Selected Writings of Arnold Schoenberg*, 60th Anniversary Edition ed. Berkeley: University of California Press, 1975, p.215.

[2] J. Moreno, "Schenker's Parallelisms, Schoenberg's Motive, and Referential Motives: Notes on Pluralistic Analysis," *College Music Symposium,* vol. 41, pp. 91-111, 2001.

[3] H. Schenker, *Free Composition* vol. 3. New York: Longman, 1979, p.131.

[4] S. Rosenhaus, "Harmonic motion in George Perle's Wind Quintet No. 4," Ph.D. dissertation, New York University, 1995, pp.7-8.

[5] S. Rosenhaus, "Harmonic motion in George Perle's Wind Quintet No. 4," Ph.D. dissertation, New York University, 1995, p.164.

[6] S. Rosenhaus, "Harmonic motion in George Perle's Wind Quintet No. 4," Ph.D. dissertation, New York University, 1995, p.247.

[7] G. Perle, *Twelve-Tone Tonality*, 2nd edition ed. Berkeley: University of California Press, 1996.

[8] P. T. Carrabré, "Twelve-Tone Tonality and the Music of George Perle," PhD dissertation, City University of New York, 1993.

[9] G. Foley, "Pitch and Interval Structures in George Perle's Theory of Twelve-Tone Tonality," Ph.D. dissertation, University of Western Ontario, 1999.

[10] J. Carr, "George Perle and the Computer: An Uneasy Alliance," *International Journal of Musicology,* vol. 4, pp. 207-215, 1995.

[11] G. Foley and C. Cusack, "Computer-Assisted Analysis of Music in George Perle's System of Twelve-Tone Tonality," *Computer Music Journal,* vol. 30, pp. 53-66, 2006.

[12] P. Lansky, G. Perle, and D. Headlam, "Twelve-note Composition," in *The New Grove Dictionary of Music and Musicians*, S. S. a. J. Tyrrell, Ed., second edition ed. London: Macmillan, 2001.