ISSN: 2692-5079

A New Model for Location-Allocation Problem Based on Sectorization

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Abstract—Many models have been proposed for the location-allocation problem. In this study, based on sectorization concept, we propose a new single-objective model of this problem, in which, there is a set of customers to be assigned to distribution centres (DCs). In sectorization problems there are two important criteria as compactness and equilibrium, which can be defined as constraints as well as objective functions. In this study, the objective function is defined based on the equilibrium of distances in sectors. The concept of compactness is closely related to the accessibility of customers from DCs. As a new approach, instead of compactness, we define the accessibility of customers from DCs based on the covering radius concept. The interpretation of this definition in real life is explained. As another contribution, in the model, a method is used for the selection of DCs, and a comparison is made with another method from the literature, then the advantages of each are discussed. We generate benchmarks for the problem and we solve it with a solver available in Python's Pulp library. Implemented codes are presented in brief.

Index Terms—Location-Allocation Problem, Sectorization, Covering Radius, Python, Pulp, Linear Programming

I. Introduction

The aim of the location-allocation problem (LAP) is to find the optimal location of service centres as well as allocating demand zones to each of them [1]. In this study, we address this problem based on the idea of sectorization. In sectorization problems (SPs), a large region is split into smaller ones for administrative goals. SPs have several applications in territorial management of sales, water, healthcare, public transportation, internet networking, municipality, electric power, emergency service, police patrol, social facilities, etc [2]–[10]. Two important criteria for SPs are compactness and equilibrium, which can be added to mathematical models as both constraint and objective function [11]–[14].

In the problem of this study, there is a set of customers in a region whose coordinates are known in advance. They have predetermined demands so that they are assigned to distribution centres (DCs) in order to meet the demands, taking some criteria into account. A subset of the DCs is chosen, therefore, some of them may not be open, in which case customers are not assigned to them. The coordinates of them are also certain. Thus, the problem consists of the selection of DCs and the assignment of customers to them. Various solution methods have been proposed in the literature for this problem. Teymourifar et al. [14] proposed a two-stage method, in which a subset of DCs is selected in the first stage and the corresponding sectorization subproblem of the subset is solved in the second stage. In the mentioned study, at the first stage, different subsets are searched and since the model is multi-objective and Pareto optimal solutions are found [14]. Unlikely, in this study, we do the selection of DCs and the assignment of the customers in one stage. Also, instead of compactness, we manage the accessibility of the customers from DCs with a constraint defined based on the concept of covering radius [15]. Different from previous studies that used the covering radius concept for LAP, we integrate it with the sectorization approach.

In other sections of the study, experimental results are presented after problem definition. Then, implementation is briefly described. The conclusion is the last part of the study.

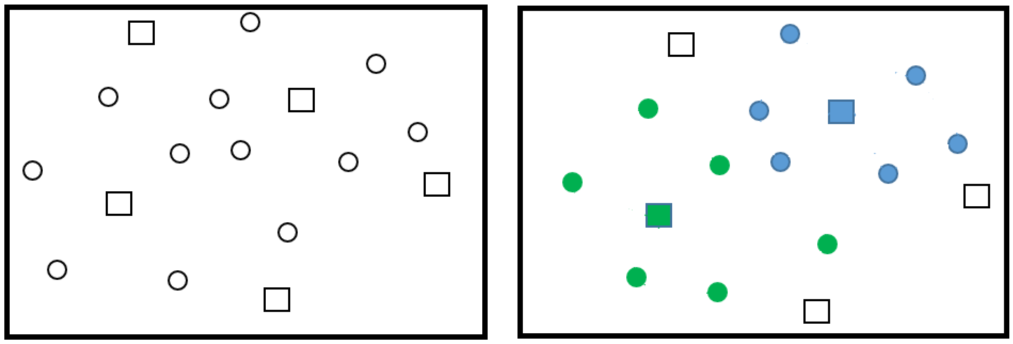
II. Problem Definition

The problem dealt with in this paper includes a set of potential DCs and customers in different locations. Considering the objective function, sectorization is done by selecting a subset of DCs to be opened and assigning a subset of customers to each. Thus, each DC and its assigned customers creates a sector.

We provide an illustrative example that helps to better understand the problem and the solution method. In Fig. 1(a), potential DCs and customers are represented as squares and circles, respectively. As shown in Fig. 1(b), two of DCs is selected to open and customers are assigned to them so that the objective function is minimized. The resulting two sectors are shown in green and blue colours.

Some of the used terminology and notations are summarized in TABLE I.

In the model, we use binary decision variables Y_j, and X_{ij}, which are defined as in Equations 1 and 2.



(a) (b)

Fig. 1. An illustrative example of the problem [14].

TABLE I
Used notations.

Objective functions:	
f	Objective function defined based on the equilibrium of distances in sectors
Sets and indexes:	
$i \in \{1, ..., \bar{I}\}$	Index of customers
$j \in \{1, ..., \bar{J}\}$	Index of DCs and sectors
Parameters:	
D_{ij}	Euclidean distance between customer i and DC j
$\bar{D}$	Average distance of customers from DCs
r_{ij}	Binary parameter about if customer i is in the covering radius of DC j or not
De_i	Demand of customer i
$\bar{De}$	Average demand of customers in sectors
τ_{equ}	Tolerance for the equilibrium criteria
τ_r	Tolerance for the covering radius of DCs
n_{max}	Upper limit for the number of opened DCs
Variables:	
D_j	Total distance of customers from DC j in sector j
De_j	Total demand of customers in sector j
Binary decision variables:	
Y_j	Decision variable about if DC j is opened or not
X_{ij}	Decision variable about if customer i belongs to sector j or not
Positive decision variables:	
A_j	Positive variable to linearize the objective function
B_j	Positive variable to linearize the objective function

$$Y_j = \begin{cases} 1, & \text{if } \textit{DC j is opened} \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$$X_{ij} = \begin{cases} 1, & \text{if } \textit{customer i is in sector j} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

There is a DC in each sector, so if customer i is in sector j, which likewise implies that it is assigned to DC j.

The objective function is defined as in Equation 3, which is related to equilibrium of distances in sectors:

$$f = Min \sum_{j=1}^{\bar{J}} |D_j - \bar{D}| \quad (3)$$

where $D_j = \sum_{i=1}^{\bar{I}} D_{ij} \times X_{ij} \quad \forall \in \bar{J}$ and $\bar{D} = \sum_{i=1}^{\bar{I}} \sum_{j=1}^{\bar{J}} \frac{D_{ij}}{\bar{J}}$. It should be noted that $\bar{D}$ is not dependent on decision variables. Hence, it is calculated as a parameter, not a variable.

As defined in Constraint 4, at least one customer is assigned to each sector.

$$\sum_{i=1}^{\bar{I}} X_{ij} \geq 1, \quad \forall \in \bar{J} \quad (4)$$

In the reference [13], two definitions of compactness are given, one of them as an objective function and the other one as a constraint. Since these definitions are based on the total distance of the customers from the assigned DC, they cannot measure the accessibility of each customer to the DC. In a different way, we use constraint 4, which is a linear Constraint, to ensure the accessibility of each customer from the related DC. The linearity of the constraint also makes it easy to manage.

Each customer is assigned to only one sector, which is supplied by Constraint 5:

$$\sum_{j=1}^{\bar{J}} X_{ij} = 1,\ \forall i \in \bar{I} \tag{5}$$

Constraint 6 is to ensure that all customers are on the covering radius of at least one of the opened DCs.

$$\sum_{j=1}^{\bar{I}} r_{ij} Y_j \geq 1,\ \forall i \in \bar{I} \tag{6}$$

r_{ij} is a binary parameter, which is equal to one if customer i is in the covering radius of DC j, and otherwise is equal to zero. r_{ij} is calculated based on τ_r. Details are given in the section of experimental results.

Customers only receive service from one of the opened DCs. However, as defined in Constraint 6, each customer is in the covering radius of more than one DC. For the customers, this can be an advantage in emergency situations, which can be a case in which the DC that the customers in the assigned sector ordinarily receive service from, is damaged.

Customer i receives service from the opened DC j only if it is in its covering radius, which is defined as in Constraint 7.

$$X_{ij} \leq r_{ij},\ \forall i \in \bar{I}\ and\ \forall j \in \bar{J} \tag{7}$$

Customer i can be assigned to DC j and receives service from it only if DC j is open, which is guaranteed by Constraint 8.

$$X_{ij} \leq Y_j,\ \forall i \in \bar{I}\ and\ \forall j \in \bar{J} \tag{8}$$

DC j is open if at least one customer is assigned to it, which is provided by Constraint 9.

$$Y_j \leq \sum_{i=1}^{\bar{I}} X_{ij},\ \forall j \in \bar{J} \tag{9}$$

Applying Constraint 10, an upper limit can be specified for the number of opened DCs.

$$\sum_{i=1}^{\bar{J}} Y_j \leq n_{max} \leq \bar{J} \tag{10}$$

It is expected that the equilibrium be provided in terms of demands between the formed sectors, which is afforded by Constraint 11.

$$|De_j - \bar{De}| \leq \bar{De}(1-\tau_{equ}),\quad \forall j = 1,...,\bar{J},\quad 0 \leq \tau_{equ} \leq 1 \tag{11}$$

where $De_j = \sum_{i=1}^{\bar{I}} De_i \times X_{ij}$, $\bar{De} = \sum_{i=1}^{\bar{I}} \sum_{j=1}^{\bar{J}} \frac{De_j}{\bar{J}}$.

III. Implementation

In this section, we present some details about implementation, which is done in the Pulp library of Python on an Intel Core i7 processor, 1.8 GHz with 16 GB of RAM.

In Python, after installing Pulp library, it can be imported using the following command:

$from\ pulp\ import\ *$

Since the Pulp library solves only linear models, using positive variables A_j and B_j the objective function, which is defined as in Equation 3, is linearized.

$$D_j - \bar{D} - A_j + B_j = 0,\ \forall j = 1,...,\bar{J} \tag{12}$$

$$A_j\ and\ B_j \geq 0,\ \forall j = 1,...,\bar{J} \tag{13}$$

$$Min\ f = \sum_{j=1}^{\bar{J}} A_j + \sum_{j=1}^{\bar{J}} B_j \tag{14}$$

More details about linearization can be found at the reference [14].

Sets of customers and DCs are defined as follows:
$setI = range(1, \bar{I})$
$setJ = range(1, \bar{J})$

Decision varaiable X_{ij} is defined as follows:
$xVars = LpVariable.dicts(name = "xVars", indexs = (setI, setJ), lowBound = 0, upBound = 1, cat = LpInteger)$

In the reference [14], although in the problem definition part more than one binary decision variable is introduced, only one of them is used in the solution process, which is X_{ij}. In this study, we use Y_j too, which is defined as in Equation 1. It is included in the model as follows:

$yvars = LpVariable.dicts(name = "yvars", indexs = setJ, lowBound = 0, upBound = 1, cat = LpInteger)$

Also, the positive variables used for linearization are defined as follows:
$A = LpVariable.dicts(name = "A", indexs = setJ, lowBound = 0, cat = LpInteger)$

$B = LpVariable.dicts(name = "B", indexs = setJ, lowBound = 0, cat = LpInteger)$

The name and type of the implemented model are defined as follows:
$prob = LpProblem("MIP\_Model", LpMinimize)$

Constraints are included to the model as follows:
Constraint 4:
$for\ j\ in\ setJ:$
$prob += lpSum(xVars[i][j]\ for\ i\ in\ setI) >= 1,""$

Constraint 5:
$for\ i\ in\ setI:$
$prob += lpSum(xVars[i][j]\ for\ j\ in\ setJ) == 1,""$

Constraint 6:
$for\ j\ in\ setJ:$
$prob += lpSum(r[i,j] * Y[j]\ for\ i\ in\ setI) >= 1,""$

Constraint 7:
$for\ i\ in\ setI:$
$for\ j\ in\ setJ:$
$prob += xVars[i][j] <= r[i,j],""$

Constraint 8:
$for\ i\ in\ setI:$
$for\ j\ in\ setJ:$
$prob += xVars[i][j] <= yVars[j],""$

Constraint 9:
$for\ j\ in\ setJ:$
$yVars[j] <= lpSum(xVars[i][j]\ for\ i\ in\ setI),""$

Constraint 10:
$lpSum(yVars[j]\ for\ j\ in\ setJ) <= nMax,""$

As the constraints defined based on the absolute value cannot be employed directly in Pulp, Constraint 11 is managed with two following constraints:

$for\ j\ in\ setJ:$
$prob += lpSum(de[i] * xVars[i][j]\ for\ i\ in\ setI) - deBar <= dej[j] * (1 - tau),""$

$for\ j\ in\ setJ:$
$prob += lpSum(-de[i] * xVars[i][j]\ for\ i\ in\ setI) + deBar <= dej[j] * (1 - tau),""$

where $deBar$ and $dej[j]$ are $\bar{De}$ and De_j in TABLE I, respectively.
Constraint 12:
$for\ j\ in\ setJ:$
$prob += lpSum(d[i,j] * xVars[i][j]\ for\ i\ in\ setI) - dBar - A[j] + B[j] == 0,""$

where $dBar$ is $\bar{D}$ in TABLE I.

The linearized objective function illustrated as in Equation 14, is appended to the model as follows:
$prob += lpSum(A[j]\ for\ j\ in\ setJ) + lpSum(B[j]\ for\ j\ in\ setJ)$

But the real value of the model's objective function, determined in Equation 3, is acquired like following:
$sum = 0$
$for\ j\ in\ setJ:$
$for\ i\ in\ setI:$
$sum += abs(d[i,j] * xVars[i][j].varValue - dBar)$

We utilize the solver $GLPK\_CMD$ in the Pulp library using the following command:
$prob.solve(GLPK\_CMD())$

IV. Experimental Results

Three benchmarks are created as 30×5, 450×75 and 900×150, which are indicated as the *Number of customers* × *Number of potential DCs*. To calculate the value of r_{ij}, we use parameter τ_r. Suppose that the maximum distance between all customers and DCs in a benchmark is d_{max}. In this case, if the distance between customer i and DC j is equal to or less than $d_{max} \times \tau_r$, r_{ij} is equal to one, otherwise, it is equal to zero. The value of one for r_{ij} means that customer i is reachable from DC j, thus is assignable to it.

In the benchmarks, two-dimensional coordinates of customers and DCs and also customers' demands, are generated according to $N(50;10)$ and $U(100;10)$, which are normal and discrete uniform distributions, respectively.

In Table II, the results are given according to the different values of τ_{equ} and benchmark size. Parameter τ_{equ} varies within the interval [0, 1]. At first, it is checked in which range of this parameter feasible solutions can be obtained, and then the tightest value is selected for each benchmark. The outcomes are given in TABLE II, which are for the case in which $n_{max} = \bar{J}$. The results in the table are for $\tau_r = 0.5$, but the same results are obtained for $\tau_r = 0.33$.

V. Conclusion and Future Works

In this study, based on the concept of sectorization, a new model is suggested for a LAP, in which the selection of DCs and the assignment of customers to them are done at the same stage. Its difference with a model

TABLE II
Obtained results for the benchmarks

Benchmark size	τ_{equ}	f
30 × 5	0.79	1.9e+03
450 × 75	0.98	5.92e+05
900 × 150	0.99	2.37e+06

from the literature that solves the same problem in two stages is discussed. The objective function of the model is defined based on the equilibrium of the distances in sectors. Different from the studies in the literature, the accessibility of customers from DCs is managed with a constraint, which is defined based on the covering radius instead of the compactness concept. The advantages of this definition are: (i) it is a simple linear constraint that can be included in solvers easily, uses a measure of accessibility for each customer, rather than the total distance within the sectors, and (iii) in real-life problems, it can be easily interpreted in terms of accessibility.

This study also has some limitations. For example, $\bar{D}$ in objective function 3 and $\bar{D}e$ in Constraint 11 are defined independently from the decision variables. Therefore, they are parameters and not variables. This is because the Pulp library could not find results for the case where they are declared as variables. It may be possible to find the result for this case with a different solver. In addition, although it is defined as $n_{max} \leq \bar{J}$ in Constraint 10, it is used as $n_{max} = \bar{J}$ in the experimental results. In future studies, more comprehensive results will be presented considering this matter.

In future studies, new models will be proposed for the multi-objective version of the problem.

Acknowledgements

This work is financed by the ERDF - European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020 Programme and by National Funds of the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia within project POCI-01-0145-FEDER-031671.

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