

A Monte Carlo Simulation-Based Approach to Solve Dynamic Sectorization Problem

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Abstract

In this study, two novel stochastic models are introduced to solve the dynamic sectorization problem, in which sectors are created by assigning points to service centres. The objective function of the first model is defined based on the equilibration of the distance in the sectors, while in the second one, it is based on the equilibration of the demands of the sectors. Both models impose constraints on assignments and compactness of sectors. In the problem, the coordinates of the points and their demand change over time, hence it is called a dynamic problem. A new solution method is used to solve the models, in which expected values of the coordinates of the points and their demand are assessed by using the Monte Carlo simulation. Thus, the problem is converted into a deterministic one. The linear and deterministic type of the model, which is originally non-linear is implemented in Python's Pulp library and in this way the generated benchmarks are solved. Information about how benchmarks are derived and the obtained solutions are presented.

1. Introduction

In sectorization problems (SPs), based on administrative purposes a vast region is divided into small districts, which are called sectors. There would be a balance between sectors in terms of variables such as distance and demand. Moreover, it is intended that the formed sectors be compact. Usually, based on these two requirements, the objective functions and constraints of SPs are determined [1-5]. Since SPs have applications in diverse fields, it has been extensively studied by researchers [6-15].

In this study, unlike the previous ones in the literature, a dynamic SP (DSP) containing special points is described. In the problem, special points are service centres. The dynamism of the problem is due to the demand of the points and their coordinates change over time. As a contribution to the literature, two stochastic models are offered for this problem, where the objective function is defined based on the equilibrium of the distances and also the equilibrium of demands in the sectors. Both models contain constraints related to compactness and equilibrium. Benchmarks are derived for the problem and to solve them, the expected value of the random variables is calculated using Monte Carlo simulation. In this way, they are converted to deterministic ones. Furthermore, models, which are basically non-linear are linearized. In the experimental results section, the results for the benchmarks derived for the models are presented. Conclusion and future works form the last section of the study.

The subjects of the next sections of the study are summarized in Figure 1.

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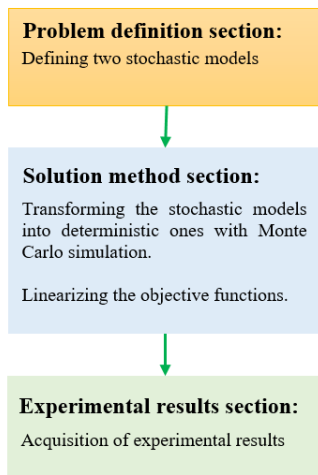


Figure 1. General scheme of the work

2. Problem Definition

In the problem, sectors are designed by assigning points in a region to service centres. Each service centre and the assigned points form a sector. Every point has a demand, which is met from the service centres. The number of points and service centres is previously known and fixed. The number of sectors that appear is equal to the number of service centres. Unlike those in the literature, in the problem of this study, the demand and coordinates of the points change over time according to statistical distributions that are known beforehand. The coordinates of the service centres are fixed. Two single-objective models are suggested for the mentioned problem. The objective function of one is defined based on the equilibrium of distances in the formed sectors, and the other is on the basis of equilibrium of demands. Compactness is provided with constraints in both models.

The used notations are summarized in Table 1. Index t is employed for random variables.

Table 1. Notations

Notation	Description
n	Total number of points
k	Total number of sectors and service centres
x_{ij}	Decision variable about assignment of point i to sector j
c_j	Service centre of sector j
$d_{ic_j}^t$	Euclidean distance between point i and the centre of sector j at moment t
D_j^t	Total distance of points from the center of sector j at moment t
\bar{D}^t	Average distance of points from the center of sectors at moment t
de_i^t	Demand of point i at moment t
De_j^t	Total demand of points in sector j at moment t
\bar{De}^t	Average demand of points in sectors at moment t
$pr_1\%$	Percentage of points whose coordinates change in time period $t = 1, \dots, T$
$pr_2\%$	Percentage of points whose demand changes in time period $t = 1, \dots, T$
τ_{eqde}	Tolerance for the equilibrium of demands
τ_{eqcm}	Tolerance for the equilibrium of compactness
f_1	Objective function related to equilibrium of compactness in sectors
f_2	Objective function related to equilibrium of demands in sectors

In both models, the decision variable is as follows:

$$x_{ij} = f(x) = \begin{cases} 1, & \text{if point } i \text{ is assigned to sector } j \\ 0, & \text{otherwise} \end{cases} \quad i = 1, \dots, n, j = 1, \dots, k \quad (1)$$

The formulation of the problem is described in the following two subsections. It should be noted that both introduced models are set for time period $t = 1, \dots, T$, but the problem is solved only once and for all time intervals. Consequently, the decision variable is independent of t .

2.1. First single-objective model (SO₁)

In this model, the equilibrium of the distances in the sectors is provided by the objective function, which is defined as in Equation (2).

$$f_1 = \text{Min} \sum_{j=1}^k |E(D_j^t) - E(\bar{D}^t)| \quad (2)$$

where $E(D_j^t) = \sum_{i=1}^n E(de_{ic_j}^t) \times x_{ij}$ and $\bar{D}^t = \sum_{j=1}^k D_j^t/k$.

Subject to

$$\sum_{j=1}^k x_{ij} = 1, \forall i = 1, \dots, n \quad (3)$$

$$\sum_{i=1}^n x_{ij} \geq 1, \forall j = 1, \dots, k \quad (4)$$

$$|E(D_j^t) - E(\bar{D}^t)| \leq E(\bar{D}^t)(1 - \tau_{eqcm}), \forall j = 1, \dots, k, 0 \leq \tau_{eqcm} \leq 1 \quad (5)$$

$$|E(De_j^t) - E(\bar{De}^t)| \leq E(\bar{De}^t)(1 - \tau_{eqde}), \forall j = 1, \dots, k, 0 \leq \tau_{eqde} \leq 1 \quad (6)$$

where $De_j^t = \sum_{i=1}^n de_j^t \times x_{ij}$, $\bar{De}^t = \sum_{j=1}^k De_j^t/k$.

Constraint (3) affords that each point is assigned to a service centre and Constraint (4) provides that at least one point is assigned to each service centre. The left-hand side of Constraints (5) shows the deviation of the total distance from the centre in each sector from the average of all sectors. In this way, Constraint (5) specifies an upper limit for it. Constraint (6) fits the same for demand.

2.2. Second single-objective model (SO₂)

In this model, the equilibrium of the demand of the sectors is yield by the objective function, which is established as in Equation (7).

$$f_2 = \text{Min} \sum_{j=1}^k |E(De_j^t) - E(\bar{De}^t)| \quad (7)$$

Also, Constraints (2) – (6) are valid.

3. Solution Method

As seen in Figure 1, in the solution method, firstly, the expected values of the random variables are obtained by a Monte Carlo simulation, and consequently, the problem is transformed to a deterministic one. Additionally, the model is linearized for easier handling. The next two subsections describe these stages.

3.1. Transforming the stochastic model into a deterministic one with the Monte Carlo simulation

To deal with the stochasticity of the model, variations in time periods $t = 1, \dots, T$ are simulated. This is done separately for each benchmark, and in fact, in this way, a numerical estimation of expected values is performed, to transform the stochastic benchmark into a deterministic one. It should be noted that actually, this is a part of the solution method and also the experimental results but not the model. The Monte Carlo simulation is done numerically on benchmarks.

Thus, if the parts of the first model that are based on the expected value, which are the objective function (2) and Constraints (5)-(6), are written deterministically, they become Equation (8) and Inequalities (9)-(10), respectively.

$$f_1 = \text{Min} \sum_{j=1}^k |D_j - \bar{D}| \tag{8}$$

$$|D_j - \bar{D}| \leq \bar{D}(1 - \tau_{eqcm}), \forall j = 1, \dots, k, 0 \leq \tau_{eqcm} \leq 1 \tag{9}$$

$$|De_j - \bar{D}e| \leq \bar{D}e(1 - \tau_{eqde}), \forall j = 1, \dots, k, 0 \leq \tau_{eqde} \leq 1 \tag{10}$$

Also, Constraints (3)-(4) are included.

Likewise, the second model is transformed into a deterministic one.

$$f_2 = \text{Min} \sum_{j=1}^k |De_j - \bar{D}e| \tag{11}$$

Constraints (3)-(4) and also (9)-(10) are applied.

Although in the literature this approach has been adopted formerly to solve other problems [15], we use it for the first time for a DSP.

3.2. Linearizing the objective functions f_1 and f_2

To linearize the objective functions, as defined in Equation (12), non-negative variables V_j^1 and V_j^2 , $\forall j = 1, \dots, k$, are added to objective function f_1 .

$$\text{Min } f_1 = \sum_{j=1}^k V_j^1 + \sum_{j=1}^k V_j^2 \tag{12}$$

Furthermore, the following k constraints are added to the model:

$$D_j - \bar{D} - V_j^1 + V_j^2 = 0, \forall j = 1, \dots, k \tag{13}$$

$$V_j^1 \text{ and } V_j^2 \geq 0, \forall j = 1, \dots, k \tag{14}$$

In a similar way and using non-negative variables V_j^3 , and V_j^4 , $\forall j = 1, \dots, k$, objective function, f_2 is also linearized.

$$\text{Min } f_2 = \sum_{j=1}^k V_j^3 + \sum_{j=1}^k V_j^4 \tag{15}$$

$$De_j - \bar{D}e - V_j^3 + V_j^4 = 0, \forall j = 1, \dots, k \tag{16}$$

$$V_j^3 \text{ and } V_j^4 \geq 0, \forall j = 1, \dots, k \tag{17}$$

4. Experimental Results

12 benchmarks are generated for the problem, which are indicated as the *Number of points* \times *Number of service centres*. Those with 1000, 500, and 50 points are classified as large, medium, and small ones, respectively. For each number of points, two different service centre numbers are appropriated. For each benchmark, values of the initial period, $t=1$, are initially derived. Till this stage, the coordinates and demands of the points in the first period are determined. The coordinates of the service centres are also arranged at this stage and remain constant for all periods. Unlike this, the coordinates and demands of the points change in each period. Two-dimensional coordinates of points and centres and also points' demands are generated based on $N(50;10)$ and $U(100;10)$, which are normal and discrete uniform distributions, respectively. We consider 10 periods for each benchmark. Therefore, the coordinates and demands of some of the points change, which are also produced according to the $N(50;10)$ and $U(100;10)$ distributions, respectively. The number of points whose coordinates and demands change in each period is defined by pr parameter. Values of 20 and 10 are chosen for pr . Following, the expected values of the demands and coordinates of the points are found. This process is done in Excel, which is a Monte Carlo simulation and as a result, the benchmark is transformed into a deterministic one.

Table 2. Obtained results for the model SO_1

Size	τ_{eqde}	$pr_1\%$	f_1
1000×300	0.99	20	5184472.12
1000×300	0.99	10	5161742.47
1000×100	0.99	20	1707890.27
1000×100	0.99	10	1700991.80
500×150	0.99	20	1284027.97
500×150	0.99	10	1281570.24
500×50	0.99	20	414728.58
500×50	0.98	10	413923.94
50×15	0.99	20	11361.37
50×15	0.95	10	11225.82
50×5	0.77	20	3502.57
50×5	0.78	10	3464.98

Table 3. Acquired results for the model SO_2

Size	τ_{eqcm}	$pr_2\%$	f_1
1000×300	0.7	20	54167.83
1000×300	0.7	10	54039.86
1000×100	0.7	20	53805.51
1000×100	0.06	10	53678.39
500×150	0	20	26762.98
500×150	0	10	26809.87
500×50	0.02	20	26403.75
500×50	0.01	10	26450.00
50×15	0.08	20	2644.69
50×15	0.09	10	2676.24
50×5	0.28	20	2266.88
50×5	0.29	10	2293.92

To solve the deterministic version of Benchmarks, the models described in Section 3 are implemented in Python's Pulp library, which is an open-source library that solves linear models. We utilize a system with an Intel Core i7 processor, 1.8 GHz with 16 GB of RAM for the implementation.

The earned results are displayed in Tables 2 and 3. The τ values in the tables are determined in the following way: after finding the range that gives the feasible result for each benchmark, the value that performs the tightest result is used.

All of the used benchmarks and implemented codes are accessible via the email address of the corresponding author.

5. Conclusions

In this work, two new stochastic models were suggested to solve a DSP. The objective function of one of them was settled on the basis of the equilibrium of distances in sectors and the second one was based on the equilibrium of demands. A novel solution method was proposed for the models that was tested on benchmarks of different sizes. In the

method, a Monte Carlo simulation was applied to each benchmark and in this way the expected values of random variables were obtained. Thus, the benchmark turns into a deterministic one. Also, during the solution method, the model which was originally non-linear was linearized, which eases the solution. The deterministic and linear versions of the models were implemented in the Pulp library that is available in Python. The implemented model solved the deterministic benchmarks. Since the proposed models and the implemented solution method was easy to handle, they can be used to solve many real-life problems.

The models of this study were single-objective ones. Real SPs, like many other problems, are generally multi-objective [16, 17]. In future studies, a more comprehensive multi-objective model will be proposed, which also will cover both single-objective models of this study.

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