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Numerical methods to predict human induced vibrations on low frequency stairs. Part 1: Literature review, modelling

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ABSTRACT

Recent trends towards slender construction with prominent and exigent architectural requirements often result in low frequency staircases that are significantly flexible and susceptible to unacceptable vibrations, which may promote safety concerns for their users. For structural engineers, however, there is still a lack of understanding, available information and specific design guides for predicting the dynamic behaviour of staircases due to human induced vibrations.

To address this problem, this work reviews and applies the main existing numerical methods for predicting vibrations, to evaluate their precision and provide practical guidance when designing flexible staircases.

The work developed is presented in a two-part paper. In Part 1, the actual paper, several numerical methods are introduced and a detailed description is given of how these can be employed in a design stage. The distinction between low and high frequency staircases is explained, since it directly influences the structure's behaviour and, subsequently, the selected method. A description is given of how to simulate walking dynamic loads, which forms the basis of all methods. The group effect is also discussed because it tends to considerably amplify the staircase response. Finally, the different numerical procedures are applied to a practical case and compared.

It was observed that, although the four numerical methods were employed with the same staircase, their results were different. The reasons for the higher results of Fourier series walking models are explained. In Part 2, the follow-up paper, the numerical methods are employed on a real staircase, comparing the estimated and experimental results.

1. Introduction

In recent years it has become increasingly popular in contemporary architecture to design monumental, lightweight and slender steel staircases. This usually results in flexible staircases that, when exposed to different types of human movement patterns (walking, running, jumping, etc.), tend to generate undesirable vibrations. Steel staircases subject to significant vibration levels often cause a feeling of discomfort in the individuals who experience it, and the perception that the structure is not safe.

In the designing of steel staircases with prominent architectural features, generally the effects of pedestrians are treated as equivalent static loads. From the ultimate limit state (ULS) or serviceability limit state of deflection (SLSD) point of view, that poses no problem; however, in vibration serviceability, considering only static loads may not be sufficient to avoid unsatisfactory dynamic behaviour.

When dealing with vibration serviceability, it is essential for designers to take into account characteristics such as i) pedestrians' step frequencies and variable loads induced over time, ii) the staircase's dynamic properties (mass, stiffness, natural frequencies and their mode shapes), iii) high frequency and low frequency structural response, and iv) acceptable limits of human comfort.

Pedestrian induced forces are usually obtained through force plate measurements, being designated as ground reaction forces (GRFs). Some examples of researchers who performed force plates tests on staircases are Kerr and Bishop [1–3], González [4,5] and Kasperski and Czwikla [6]. Kerr and Bishop [1–3] verified that the dynamic forces applied to stairs are higher than the forces applied to horizontal surfaces, and that it is also possible to walk stairs with higher step frequencies, which furthermore raises more concern for this type of structure. Bougard [7]

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also evaluated the loads applied on stairs by pedestrians, although only measuring maximum loads and not the GRFs time histories. This researcher reported that the results were in accordance with the British design codes, as they integrate a multiplication factor of at least 1.6 times the specified pedestrian static loads.

Assuming that the GRFs traces are identical, regardless of the foot performing the footfall, and replicable during the time it takes for the pedestrian to walk the staircase, it is possible for human walking loads also to be described by means of Fourier series. This is the main work developed by the researchers previously mentioned [1,2,4–6].

Although there is a general consensus that walking loads can be reproduced by Fourier series, according to Gaile [8,9], conducting discrete Fourier analysis on GRFs traces measurements encompasses various constraints. The measurement of a footfall force assuming that those that follow are identical may not be plausible due to the non-periodicity of the forces applied by each footfall. Also, measuring a footfall where the individual is focused on performing it may not be the most suitable situation because of its unnatural character. Therefore, this researcher presents a different methodology to obtain force time functions in terms of Fourier series based on inverse dynamics, which allows the measurements of consecutive footfalls.

Characterising the dynamic properties of staircases is a vital part of effectively reducing vibration serviceability issues, this being the main focus of diverse studies [10–12]. Setareh [10] addressed how monumental stairs should be modelled in a structural analysis software to reliably predict their natural frequencies. The study concluded that the stringer stiffness, weight of the structure, and the inclusion of non-structural elements such as stair cladding are important factors that need to be accurately included in finite element (FE) staircase models. Belver et al. [11] present a case study related to FE modelling, modal testing and FE model updating of a lively staircase. Despite recent advances using modelling techniques, it was observed that there is no guarantee that the initial FE model can estimate the modal properties of the staircase adequately, even when it is very detailed. Cappellini et al. [12] investigated the influence of moving people on changes of a staircase's modal properties and damping. For a small number of pedestrians (equal to or less than 5), the difference compared to an empty staircase, especially for natural frequencies, was not substantial.

The research carried out by Cappellini et al. [12] is amongst recently found studies that comprise a different class of models to predict dynamic responses, which take into account the interaction between the pedestrians and the vibrating structure [13–21]. According to this researcher [12], considering pedestrians interacting with a structure solely as a source of force (classical approach) can lead to inaccurate and overestimated vibrations levels, therefore, human-structure interaction (HSI) must also be included when estimating responses. HSI can be described as the influence of human bodies on the dynamic properties (i.e. modal mass, stiffness and damping) of the structures they occupy [15,18]. Despite the arising relevance of this topic, researchers are still attempting to propose suitable models to predict HSI effects and consequent changes in the dynamic behaviour of the occupied structures [20].

Depending on their natural frequency, structures can be separated into two types of dynamic behaviour. Structures with low natural frequencies can develop a resonant response, while structures with high natural frequencies display an impulsive response. Since designers tend to consider only a high stiffness in order to avoid the occurrence of resonant effects, neglecting the mass contribution can also give rise to high frequency stairs with significant serviceability problems. In this context, Santos et al. [22] proposed a simplified expression to be used on the pre-design of stair steps, avoiding the occurrence of excessive vibrations and implying that projected steps will have a natural frequency higher than 16 Hz, also avoiding the development of resonance responses. Kim et al. [23] further developed the effective impulse model from SCI P354 [24] to predict high floors responses, suggesting a new formula to be directly applied to high frequency stairs. The proposed effective impulse formula led to more accurate predictions when compared to the SCI P354 [24] model for floors, with about 10% average error. Kraincanic and Sparkes [25] measured the modal properties and human induced vibrations of a high frequency helical staircase using a calibrated smartphone accelerometer, then compared these with the numerical results obtained in an FE staircase model. The footfall analysis showed that, despite a high fundamental frequency, the responses exceeded the prescribed design guidelines.

Various design guides are available that specifically aim at assessing human comfort against vibrations [24,26–28]. However, as far as staircases are concerned, only SCI P354 [24] directly refers to acceptable vibration limits for humans, although these are the values given by Bishop et al. [3]. Due to the lack of information, some researchers have proposed their own acceptable limits [29–31].

Although the work developed in this field has increased and there is more awareness of the importance of the vibration phenomena associated with structures, there are still few studies related to the existing numerical procedures for predicting vibrations in the design stage and validation of the same with experimental programs in staircases. Consequently, to overcome the diverse limitations found in the literature, this research paper intends to review and apply the main existing numerical methods, to assess their accuracy and feasibility when designing flexible staircases.

To accomplish this, a two-part paper is presented. Part 1, the current paper, consists of a literature review, with the definition of relevant concepts and the description of the different numerical methods. Part 2, the follow-up paper, includes the employment of the different procedures presented in Part 1 to a real staircase, comparing the predicted numerical results with the measured results collected during an experimental campaign.

In this paper, the cut-off boundary between low and high frequency stairs is first presented, this being one of the most important notions for defining the structural behaviour and which numerical procedure should be employed. Then, a description is given of how to simulate the dynamic forces induced during walking, which forms the basis of all numerical methods. After outlining these concepts, the different methods are presented, with further detail of how each one can be applied in the design of staircases. Lastly, the application process for each method is summarised and they are then employed in a real case, to compare their results.

2. Cut-off frequency between low and high frequency stairs

When designing structures subjected to human activities such as walking, jumping or running, one of the most important parameters to take into account is the pedestrian's step frequency. The step frequency is of particular relevance because, if it coincides with the structure's natural frequency, a resonance can occur, which greatly amplifies its response. However, this phenomenon does not occur exclusively when the step frequency equals the structure's natural frequency; if the step frequency is one sub-multiple of the structure's natural frequency, a resonance build-up can also occur, although at a lower level of vibration.

Depending on the type of dynamic response, structures can be divided into two categories, low frequency structures (LFS) and high frequency structures (HFS). LFS respond in resonance, with accelerations increasing with consecutive steps, while HFS respond impulsively, with accelerations corresponding to one step decreasing significantly before the next step occurs. Although, the division between LFS and HFS is widely accepted in the civil engineering community, there is still much discussion regarding the limit or boundary beyond which a resonant build-up is no longer possible and the structure begins to act impulsively [32].

In the case of floors, the frequency of the 4th harmonic amplitude of the walking force is normally taken as the boundary between low and high frequency. Since on flat surfaces the maximum step frequency is usually 2.5 Hz [1,2,33], the cut-off frequency between LFF and HFF will be equal to 10 Hz. This cut-off frequency is commonly used by the majority of researchers for floors [34]. However, for staircases the boundary between low frequency and high frequency is more arguable and there is not much information in the bibliography related to which should be considered. The only design guide found in the literature that refers to the cut-off frequency for staircases is SCI P354 [24], which indicates that staircases up to 12 Hz should be treated as LFS. According to Santos et al. [22], this boundary may not be the most appropriate, because, even assuming that from the 4th harmonic amplitude the higher harmonics do not present enough energy to cause resonant effects, it is possible to walk staircases with step frequencies around 4 Hz [1,2], which makes the 4th harmonic frequency equal to 16 Hz. This could mean that staircases with natural frequencies below this value should be treated as low frequency with the possibility of a resonance build-up.

Additionally, a new study showed that there is no apparent sign that beyond 10 Hz (the 4th harmonic) the Fourier amplitudes of the remaining harmonics cannot produce a resonance build-up response in the case of floors [35]. The improved model presented suggests that the cut-off frequency between low and high frequency should shift from 10 Hz to 14 Hz, to account for higher force harmonics that can still induce significant amplification due to resonant effects. This furthermore corroborates that the aforementioned 16 Hz boundary for staircases is not overly conservative and that it is reliable, given the conditioning characteristics of this type of structure.

3. Simulation of human walking forces

In the design when dealing with topics such as human–structure dynamic interaction, the assessment of susceptibility to vibrations is based essentially on a three-fold problem, i) quantification of the walking dynamic forces, ii) obtaining the accelerations numerically, and iii) comparing the expected accelerations with an acceptance criteria. For its relevance on vibration analysis, this section presents the quantification of dynamic forces induced by a human as it moves.

3.1. Direct force measurements

The walking dynamic forces are in general known as ground reaction forces (GRFs) and their measurement is mainly performed by multi-component force plates or instrumented force measuring treadmills (IFMT). The main difference between force plates and instrumented treadmills is that the former only measure the force applied during a footfall, while the second can measure the force applied by several consecutive footfalls [32,36].

Force plates are the most commonly used instruments for conducting studies on loads generated during walking. In the case of stairs, this seems more logical, since treadmills present very different geometric properties from a staircase and cannot realistically simulate the load functions obtained during walking activities on this type of structure.

The study of GRFs obtained through force plates measurements on horizontal surfaces is widely developed in the literature. However, unlike footbridges and floors, few researchers have performed tests on

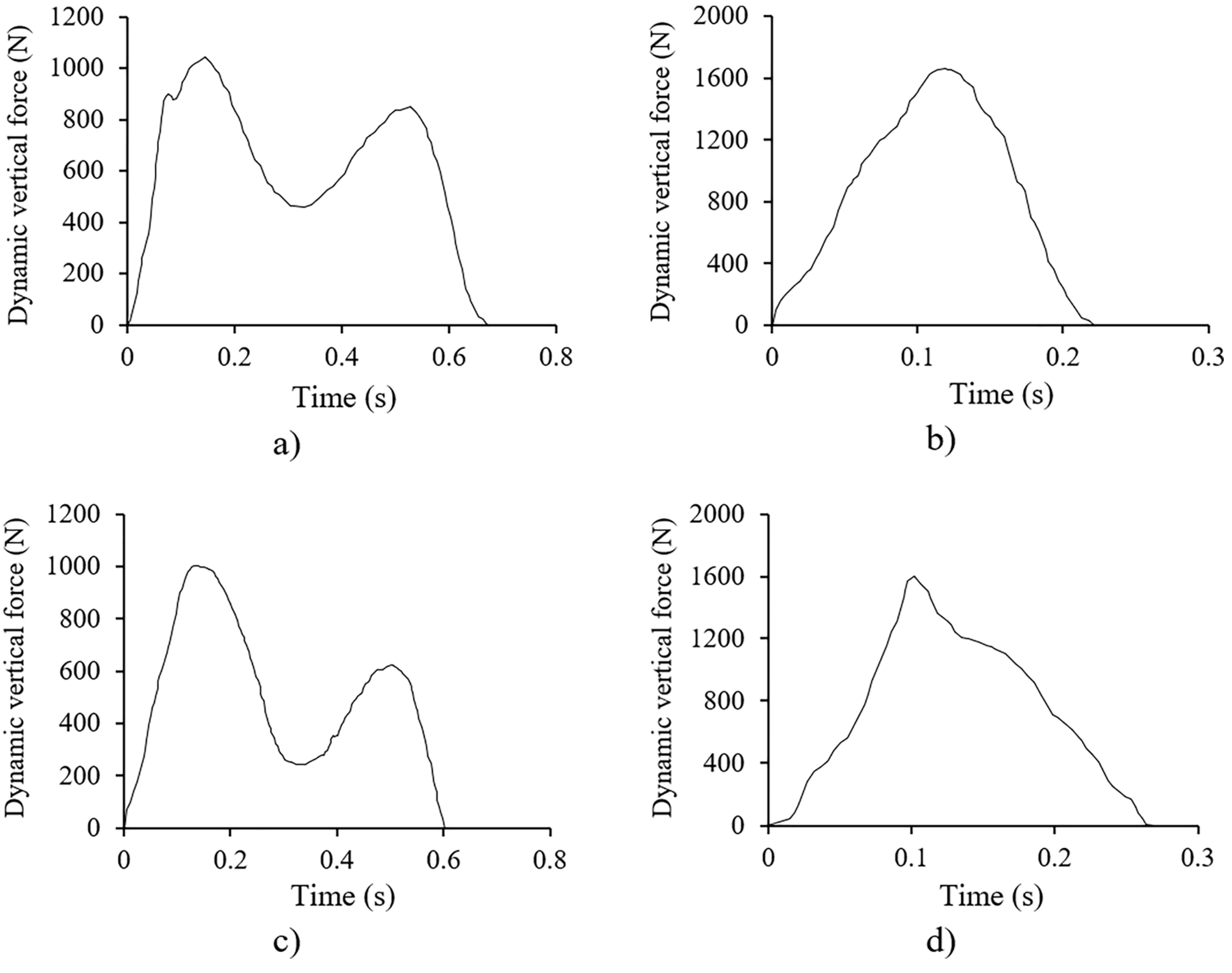

Fig. 1. Characteristic GRF traces: a) design at 1.90 Hz (Davis), b) ascent at 3.80 Hz (Kerr), c) descent at 2.60 Hz (González), and d) descent at 3.50 Hz (Kerr).

force plates to determine characteristic GRFs for staircases. Some examples are Kerr and Bishop [1,2], González [4,5] and Kasperski and Czwikla [6]. To date, the most extensive study conducted to obtain characteristic GRFs on stairs has been accomplished by Kerr [2], who obtained more than 600 footfall force traces from 25 individuals ascending and descending a staircase at different step frequencies. The elaborated work demonstrates that the GRFs traces obtained for staircases differ from those for a horizontal surface. This is related to the sequence of events at foot placement and step frequencies being different for staircases. Fig. 1a) to d) show some examples of GRFs traces measured on staircases for different step frequencies. According to various researchers [3,6,30], it is possible to walk on staircases with step frequencies varying approximately between 2.0 Hz and 4.5 Hz. For normal walking at lower step frequencies, the motion pattern of GRFs has the same characteristics as for horizontal surfaces; the first maximum occurs as the heel strikes the plate and the body weight is transferred to the stepping leg, while the second maximum occurs when the subject pushes off from the force plate with the toes, ending entirely the contact (Fig. 1a) and c)). As the step frequency increases on staircases, the GRFs traces only present one peak because, unlike for low step frequencies, the subject uses his toes to initiate contact with the plate (toe strike) and to push off (toe off) without having full contact with the foot on the force plate (Fig. 1b) and d)).

GRFs are the fundamental basis of one of the existing numerical methods to predict accelerations, and are widely applied throughout the work developed. The GRFs displayed were selected from the bibliography, the footfall forces obtained by the previously mentioned researchers being chosen [1,2,4,5], since these were directly measured on stairs. Although the GRF from Davis [37] (Fig. 1a)) was obtained on a horizontal surface, it was specifically established for designing purposes, as will be explained in Subsection 4.2.1, thus being particularly worthy of mention.

3.2. Fourier analysis

A GRF trace from a footfall can be transformed into a periodic continuous load function corresponding to the overlapping of several consecutive footfalls, which means that GRFs from force plate measurements can be described using a Fourier series. Hence, it is also possible to quantify and simulate human walking loads through Fourier series.

According to Kerr [2], in order to perform a Fourier analysis from force plate tests, two factors must be taken into account: first, Fourier analysis can only be performed on periodic functions, so it is necessary to assume that the GRFs are identical, regardless of the foot that performs the footfall (left foot or right foot) and, secondly, GRFs should be overlapped exactly after the step period $1/f_{step}$.

The force employed by pedestrians on floors, footbridges, stairs, etc. can be described in terms of a Fourier series as follows:

$$F(t) = P + \sum_{i=1}^{n} P\alpha_i \sin\left(2\pi i f_{step} t + \varnothing_i\right) \qquad (1)$$

where P represents the pedestrian self-weight, f_{step} represents the pedestrian step frequency, $\varnothing_i$ the phase angle of the harmonic i and α_i represents the amplitude of the harmonic i. α is also often referred to as the Fourier coefficient or dynamic load factor (DLF).

The studies carried out for the determination of GRFs from force plates generally have the main aim of obtaining the harmonic amplitudes for subsequent reproduction of human load walking in terms of a Fourier series, as verified in the work elaborated by the researchers referred to in Subsection 3.1, Kerr and Bishop [1–3], González [4,5] and Kasperski and Czwikla [6].

There is a considerable difference between the values obtained by these researchers, but there appears to be a consensus that, at least up to the 4th harmonic, the amplitudes present magnitudes that should be considered. Since the 4th harmonic occurs for 4 times the step frequency, this apparently demonstrates that resonance can occur at least up to the 4th submultiple of a structure's fundamental frequency, as referred to in Section 2.

The information available in the various design guides regarding the quantification of human loads in stairs through Fourier series is scarce. Only the design guide SCI P354 [24] and the international standard ISO 10137 [28] refer to how to describe the dynamic forces induced by pedestrians in this type of structure, the existing information being identical in both. It is assumed in both that, unlike the aforementioned researchers, only the first two harmonics in the case of staircases should be taken into account, and that their amplitudes values do not depend on the step frequency and should be considered constant, as shown in Table 1.

The commonly used method to obtain Fourier series describing footfalls force time histories is by performing force plate measurements. However, there is another method designated as 'inverse dynamic', which consists of measuring accelerations from an individual as he moves and then replacing these in the equilibrium equation described by Newton's 2nd Law, obtaining the footfalls forces in terms of a Fourier series. One of the researchers who developed this method further within the context of stairs was Gaile [8,9]. Table 2 shows the values of harmonic amplitudes (DLFs – dynamic load factors) proposed by Gaile [8,9].

Another important parameter for defining a Fourier series given by Eq. (1) is the phase angle $\varnothing_i$. As can be seen, Gaile [8,9] presents the phase angle values obtained from her experiments, but the other researchers Kerr and Bishop [1,2], González [4,5] and Kasperski and Czwikla [6] do not mention any information about this parameter, and in fact information found in the bibliography regarding this parameter was limited. The standard ISO 10137 [28] indicates that a conservative approach is to employ a phase angle of 90° for all harmonics except the first, which it states should be considered equal to 0°. Bachmann and Ammann [33] also report that, when it comes to numerical simulations, the most unfavourable case is to consider the phase angle of the first harmonic $\varnothing_1 = 0^\circ$ and the second and third harmonic $\varnothing_2 = \varnothing_3 = 90^\circ$. These researchers only present information concerning the first three harmonics, hence only displaying phase angles values for the same.

Table 1
Kerr and Bishop [1,2] and SCI P354 [24] harmonic amplitudes values.

	Description	Step Frequency (Hz)	DLF average value	DLF 75% of non-exceedance
SCI P354	1st Harmonic	1,2 to 4,5	1,1	–
	2nd Harmonic	2,4 to 9	0,22	–
Kerr and Bishop	Ascending, 2nd Harmonic	–	0,13	0,16
	Ascending, 3rd harmonic	–	0,06	0,08
	Ascending, 4th harmonic	–	0,03	0,04
	Descending, 2nd harmonic	–	0,2	0,25
	Descending, 3rd harmonic	–	0,09	0,11
	Descending, 4th harmonic	–	0,06	0,08

Table 2
Gaile's [8,9] proposed harmonic amplitudes values.

	Harmonic Number	DLF,α_i	Phase angle,$\varnothing_i$	Proposed DLF's values for step frequencies up to 2,3Hz
Ascending 2 Hz	1st	0,37	9,66°	$DLF(2Hz).(0,94f-0,88);$
	2nd	0,21	2,15°	$1\leqslant f(Hz)<1,95\,for\,i=1\ldots3$
	3rd	0,10	−142°	$DLF(2Hz)$
	4th	0,03	84,5°	$\left\{\begin{array}{l}1\leqslant f(Hz)\leqslant1,95\,for\,i=4,5\\1,95\leqslant f(Hz)\leqslant2,3\,for\,i=1\ldots5\end{array}\right.$
	5th	0,01	18,5°	
Descending 2,15 Hz	1st	0,60	20°	$DLF(2,15Hz).(0,99f-1,13);$
	2nd	0,13	−60,3°	$1\leqslant f(Hz)\leqslant1,85\,for\,i=1\ldots3$
	3rd	0,05	−84,5°	$DLF(2,15Hz)$
	4th	0,03	−125°	$\left\{\begin{array}{l}1\leqslant f(Hz)\leqslant1,85\,for\,i=4,5\\1,85\leqslant f(Hz)\leqslant2,3\,for\,i=1\ldots5\end{array}\right.$
	5th	0,02	93,4°	

The Fourier series representing walking loads are the basis of the second numerical method for accelerations prediction. Fig. 2a) to d) represent some examples of Fourier series for ascents and descents with different step frequencies. Fourier series were defined with the harmonics presented in Tables 1 and 2 for the different step frequencies, respectively, given by the researcher Gaile [8,9] and the design guide SCI P354 [24].

Also used were the harmonics obtained by Kerr and Bishop [1,2], presented in Table 1, since this was one of the most extensive works accomplished to date in staircases. Due to the scatter verified for the i$^{\text{th}}$ harmonic amplitude as the step frequency varies, it becomes difficult to distinguish which values should be considered. For the 1st harmonic, the researchers were somewhat able to define three distinct zones according to the step frequency, "*walking*", "*mixture*" and "*running*", as the amplitudes used in the analysis based on these zones. However, for the 2nd, 3rd and 4th harmonics, the amplitudes already present too much scattering, making it impossible to distinguish zones for the different step frequencies, so it was decided to use their mean value, shown in Table 1.

In the Fourier series defined with the harmonics given by Kerr and Bishop [1,2] and SCI P354 [24], the phase angles from the researchers Bachmann and Ammann [33] and the Standard ISO 10137 [28] were considered for the reasons previously mentioned. In the case of the Fourier series defined with the harmonics proposed by Gaile [8,9] the phase angles from Table 2 were used.

4. Numerical methods

In order to evaluate the structure's susceptibility to human induced vibrations, as mentioned in Section 3, after the characterisation of walking dynamic forces, the next step is to obtain the accelerations

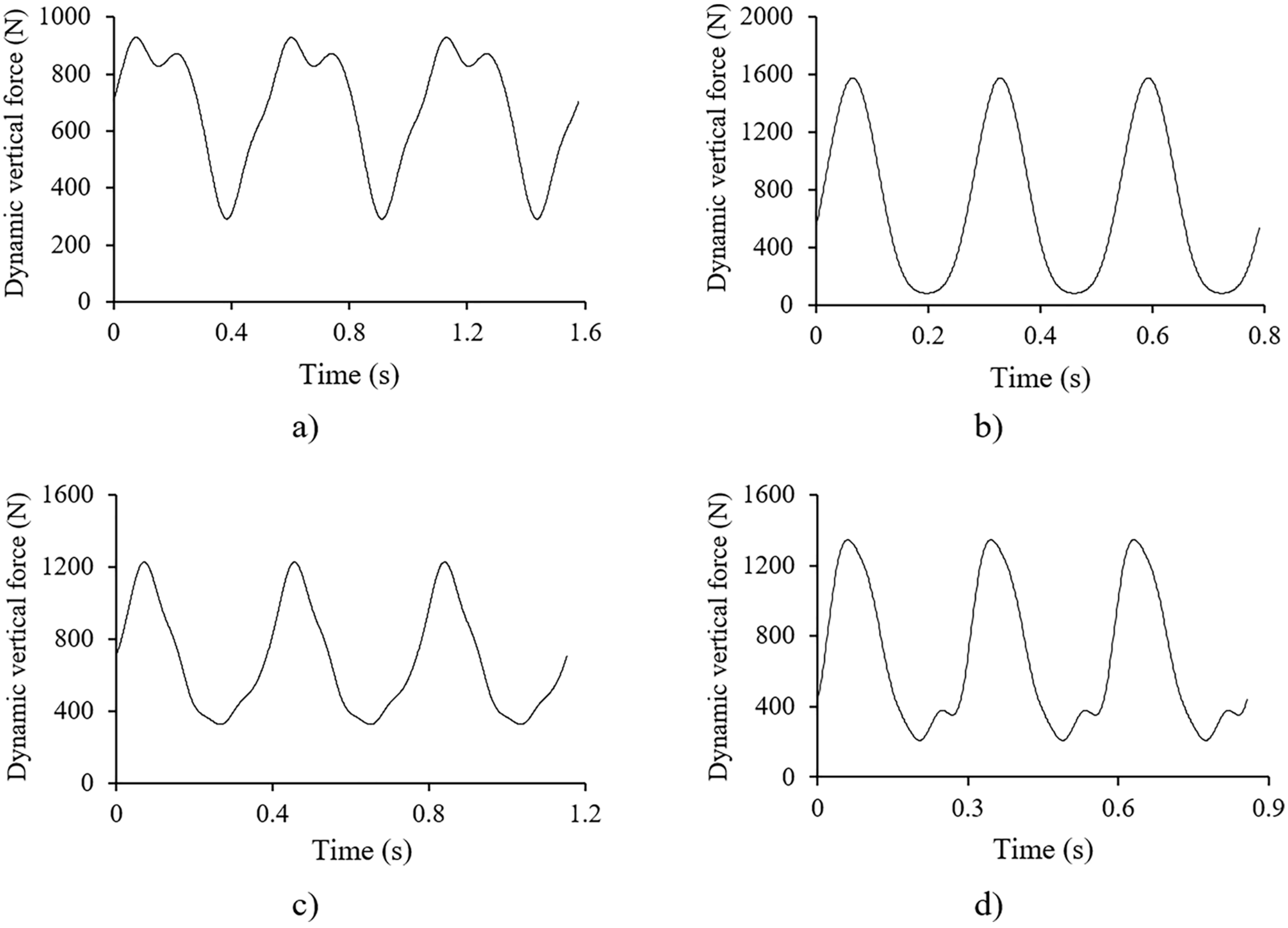

Fig. 2. Fourier series walking models: a) ascent at 1.90 Hz (Kerr), b) ascent at 3.80 Hz (SCI P354), c) descent at 2.60 Hz (Gaile) and d) descent at 3.50 Hz (Kerr).

numerically, this being one of the main shortcomings when designing steel staircases, or slender staircases in general.

As a result, this section presents the four main existing walking force based models to predict accelerations: i) footfall force time histories (GRFs), ii) Fourier series walking models, iii) steady-state analysis and iv) simplified vibration evaluation. Additionally, taking into account the arising of human-structure interaction (HSI) based models, it is also briefly described the basic principles of this approach, the work performed by different researchers, while discussing their results and exposing the current limitations for practical design purposes, and, therefore the classical approach based on walking excitation models is followed throughout this two-part paper.

4.1. *Human-structure interaction models*

Classical models of vertical pedestrian excitation on structures are based solely on external moving loads with GRF's as generated on rigid surfaces, not considering the interaction between pedestrians and the supporting structure. An approach which accounts to the fact that pedestrians are mechanical systems and, therefore, can affect the dynamic properties of the structures they occupy, has been developed in different recent works, i.e. human-structure interaction (HSI) based models. The basic principle of these models is to decompose the contact force with the supporting structure into the well-known GRF exerted by a pedestrian on a rigid surface and a mechanical interaction term which depends on the dynamic properties of both pedestrians and the structure [21].

In HSI models, pedestrians can be schematized as equivalent single-degree-of-freedom (SDOF) or multi-degree-of-freedom (MDOF) systems, as a simple inverted pendulum that oscillates in the vertical plane while moving along the structure, or as a bipedal walking model with damped compliant legs [15,16].

4.1.1. *HSI effects on footbridges*

In the case of footbridges, the system “pedestrians” is in general mathematically described by a mass-spring-damper SDOF model for each individual, a crowd dynamics model (i.e. pedestrian traffic) and a force model of individual GRF's, while the system “structure” is usually described by a mass-spring-damper SDOF model [15,17–19,21]. The researchers [15,18,19,21] presented and applied a modelling framework that comprise a detailed crowd-structure system with each pedestrian represented by an SDOF model, although with slightly variations describing the people “intelligent” behaviour between the proposed models, i.e. time varying positions, walking speed, posture, level of vibration experienced by each pedestrian, etc., to assess the impact of HSI on the structural response for a wide range of footbridge parameters and pedestrian densities. The results predominantly showing higher damping ratios of the coupled crowd-structure than the inherent damping of a real structure and a reduction on the structural response when compared with traditional approaches that not feature HSI effects, mainly for lower natural frequencies. However, this becoming more pronounced for pedestrian crowds, which might not be the case of staircases.

Due to the lack of a reliable statistical model available in literature for pedestrian's equivalent dynamic properties, F. Tubino [16] and E. Shahapbpoor et al. [17], respectively conducted, an extensive Monte Carlo simulation and a detailed experimental campaign to identify the SDOF walking human model parameters. E. Shahapbpoor et al. [17] observed that HSI effects yielded to ranges of 2.8–3.0 Hz for the natural frequency and 27.5–30.0% for the damping ratio of the SDOF walking model, while F. Tubino [16] observed that if an average pedestrian frequency is assumed lower than about 3.0 Hz, the coupled footbridge-pedestrian system will predict a significant increase in the damping ratio, the HSI being negligible assuming frequencies higher than 3.0 Hz. F. Tubino [16] stated that the results obtained in a non-dimensional form could be potentially used for the study of HSI effects on a wide range of structures, when a reliable characterization of pedestrian's dynamic parameters is available, however, this not being the case yet.

4.1.2. *HSI effects on staircases*

The studies found directly to staircases are scarce [12–14,20]. In these, to account for the HSI, the passive ground reaction forces (PGRF's) are used to properly define an equivalent dynamic model of a structure occupied by moving pedestrians and then active ground forces (AGRF's) are applied to the modified model to estimate the structural response [12–14,20]. A common assumption and shortcoming of the HSI approach for footbridges is the modelling of the structure and the pedestrian as an SDOF model, whereas the proposed models for staircases, no restrictions on the number of DOF's is required, which can lead to a more accurate response assessment, especially in the case of a high modal density.

G. Busca et al. [14] studied the influence of passive pedestrians on the dynamic properties of a real staircase by adding PGRF's to the modal model of the empty structure in terms of experimentally measured and average values of apparent masses available in literature [38] for different postures. It was observed a high increase of damping ratios and a slight decrease of natural frequencies when considering PGRF's. Also, the proposed model was able to predict changes in the modal parameters due to the presence of people, even when employing the average values of apparent mass obtained by Matsumoto and Giffin [38]. As an extension of this research, A. Cappellini et al. [12] and M. Berardengo et al. [20] evaluated the vibration response of steel staircases by adding AGRF's to the joint H-S system containing the PGRF's. A statistical approach was used to compare RMS accelerations experimentally measured and numerically obtained by different models: empty structure, joint H-S (G_H), H-S model 1, H-S model 1b and H-S model 1c. These models evolving between them by including changes in position, one or two feet in contact and different apparent mass curves for each posture within a step. Only employing AGRF's (classical approach) to the empty structure led to overestimated responses, while simultaneously including AGRF's and PGRF's, it was possible to predict numerical and experimental results closely spaced. Besides improvements and different complexity of the different proposed models, all correlated approximately well with measurements.

Despite showing promising results and the arising importance of HSI based models, according to several researchers, their employment remains a challenge since reliable calibration based on experimental evidence it is still scarce, therefore difficult to introduce general considerations that can be applied to any structural example [14–16,20]. A question can also be raised, it is possible to predict HSI effects in a design phase using average values of apparent masses instead of measured values after construction, although no information about people who will occupy the structure is previously known [14]. Considering the existing limitations, is not surprising that all relevant design guides still suggest walking force based models [24,26–28], the UK recommendations for the design of permanent grandstands [39] being the only guideline that specifically require taking into account both passive and active contributions [18].

The HSI models are complex and require a high number of parameters and level of characterization, as exemplified in Table 3, which are not broadly known and available for design purposes, also not being yet integrated in commonly used commercial structural analysis software and suitable for hand-based calculations. Hence, deemed outside the scope of this work, considering the urgent need of guidance on how to design slender, lightweight and long span staircases on day-to-day routines. Furthermore, as seen in the previous subsection, the HSI effects are more pronounced for crowd situations and for lower natural frequencies, which may not be the case of the majority of staircases, since large groups are not commonly expected and higher natural frequencies can also be excited, as explained in Section 2.

Table 3
HSI models parameters and characterization.

Structure	Model	Parameters	Loading		
Footbridge	Structure SDOF	Mass (m_b)			
		Damping (c_b)			
		Stiffness (k_b)			
	Pedestrian SDOF	Mass ($m_{p,i}$)			
		Damping ($c_{p,i}$)			
		Stiffness ($k_{p,i}$)			
	Crowd Model	Position Variation ($x_{p,i(t)}$)			
		Velocity ($v_{p,i(t)}$)			
		Frequency ($f_{p,i(t)}$)			
		Time Variation (Δt)			
	Walking GRF's ($F_{p,i(t)}$)	Classical Description			
Staircase	Structure MDOF	Mass ($\mathrm{m}_{\mathrm{b,i}}$)			
		Damping ($\mathrm{c}_{\mathrm{b,i}}$)			
		Stiffness ($\mathrm{k}_{\mathrm{b,i}}$)			
	PGRF's (f^{p})	Apparent Masses ($\mathrm{M}^*_{\mathrm{a,i}}$)	Ascending and Descending	Right foot	Posture 1a and 1d [20]
					Posture 2a and 2d [20]
					Posture 3a and 3d [20]
				Left foot	Posture 1a and 1d [20]
					Posture 2a and 2d [20]
					Posture 3a and 3d [20]
				Both foot	One Posture [20]
		Position Variation ($\mathrm{x}_{\mathrm{p,i}}$)			
	AGRF's (f^{a})	Classical Description			

4.2. *Walking force based models*

4.2.1. *Footfall force time histories*

This is the most rigorous method of predicting accelerations numerically among the four aforementioned main classical procedures, since it consists of consecutively applying GRFs directly obtained from force plate's tests on stairs, simulating in a more realistic manner the application of consecutive footfalls during the pedestrian's movement.

When performing the analysis, first, the GRFs traces for ascents and descents need to be selected from the literature, as shown in the examples of Fig. 1, where step frequencies corresponding to submultiples of the staircase's fundamental frequency are chosen, in order to predict the conditioning response due to resonance amplification. If the first four submultiples are outside the range of commonly used step frequencies in daily routines, i.e. 1.90 Hz to 2.80 Hz, GRFs traces within this interval should also be chosen. Then, the selected GRFs traces are applied to a finite element staircase model to reproduce the pedestrian's walking at the desired step frequency. Fig. 3 demonstrates the simulation of the first two steps at instants t_1 and t_2, the remaining steps necessary for the pedestrian to transverse the flight of stairs being simulated following the same reasoning.

Once GRFs traces have been applied to the FE model, numerical accelerations can be calculated. Finite element software's feature a time domain analysis that allows to obtain the structure's numerical response, designated as time history analysis.

An essential parameter to perform a time history analysis is the definition of a damping ratio value. Based on experimental data, for staircases mainly made of steel with few or without non-structural elements, damping ratios of approximately 1% of critical are generally estimated. Another relevant parameter that could raise some uncertainty about a straightforward definition is the time step size. According to current practice, the time step size can be equal to one tenth of the period of the last mode considered. However, Davis [37] states that 0.005 s is a resolution sufficient to avoid the loss of considerable acceleration peaks. The number of output time steps is selected taking into account the duration required for an individual to walk the staircase. Modal superposition with the inclusion of 10 vibration modes can be selected for a feasible computation of the numerical responses. Another option is to use direct integration; however, as detailed in Part 2 of this paper, it leads to excessively overestimated responses and provides no advantage over modal superposition.

For each GRF trace at a different step frequency, a time history analysis is required. This can be seen as a method constraint, since it makes the acceleration calculation process slow for routine designs. Another drawback is due to the scarcity of existing footfall force measurements in the literature, where it is not always possible to find GRFs traces with the desired step frequencies. To overcome this shortcoming, some GRFs for stairs found in the literature need to be multiplied in the horizontal axis (time) by a scaling factor in order to obtain the desired step frequency to employ in the design stage. One of the only researchers to obtain GRFs footfall traces for design purposes was Davis [37], who presented load functions with step frequencies ranging from 1.6 Hz to 2.2 Hz, spaced from 0.083 Hz to 0.083 Hz. The GRFs traces given by this researcher are indicated for design, since their frequency content presents harmonic amplitudes close to the harmonic amplitudes values recommended by Young and Willford [40], with a 75% probability of not being exceeded. However, only GRFs footfall traces for floors were obtained, and it is questionable if the same can be representative of GRFs for stairs.

4.2.2. *Fourier series*

The second method employed to calculate accelerations numerically is through footfall forces defined by Fourier series. As mentioned in Subsection 3.2, in order to perform a Fourier analysis it is necessary to transform the GRF from a footfall into a periodic continuous force function of several consecutive footfalls. This assumption allows the GRFs from consecutive footfalls to be described using a Fourier series given by Eq. (1). The importance of Fourier series in the simulation of human walking cannot be denied. Due to the fact that all scientific papers and design guides dedicated to human induced vibrations provide harmonic values (although this have more veracity for floors than for stairs) and are easy to apply, the Fourier series is a simplified approach in the design phase, to predict the accelerations to which a given staircase will be subjected.

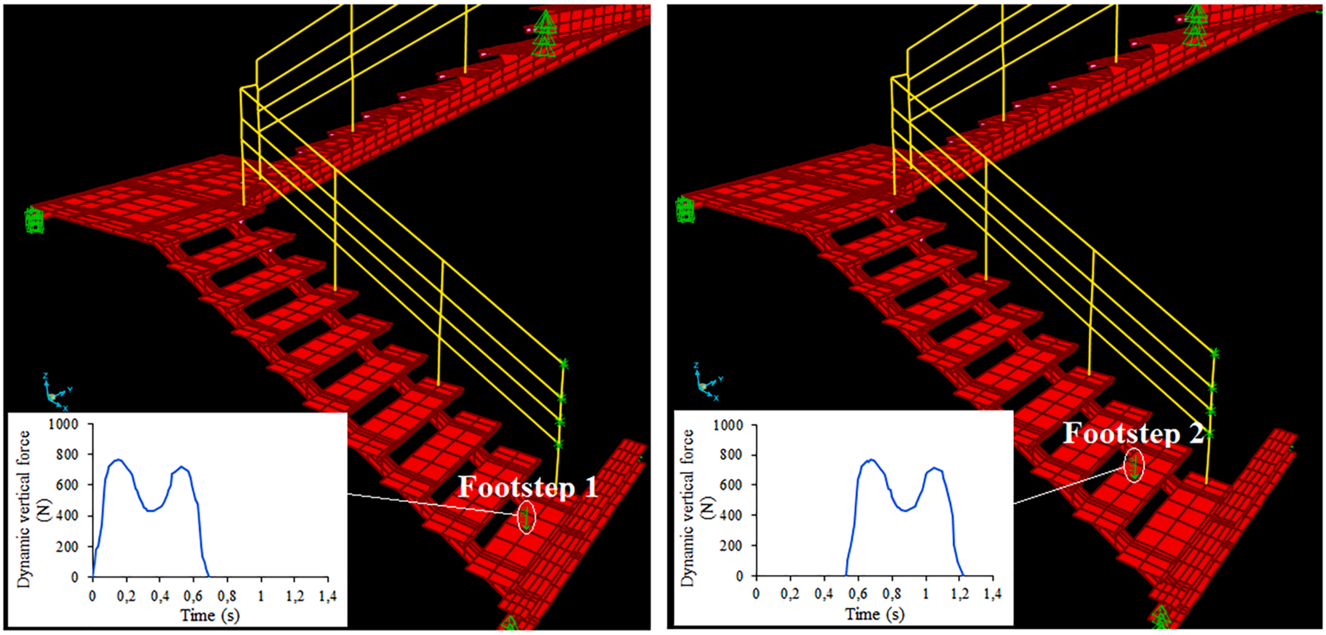

Fig. 3. Pedestrian walking simulation with typical GRFs, footsteps 1 and 2 at instants t_1 and t_2.

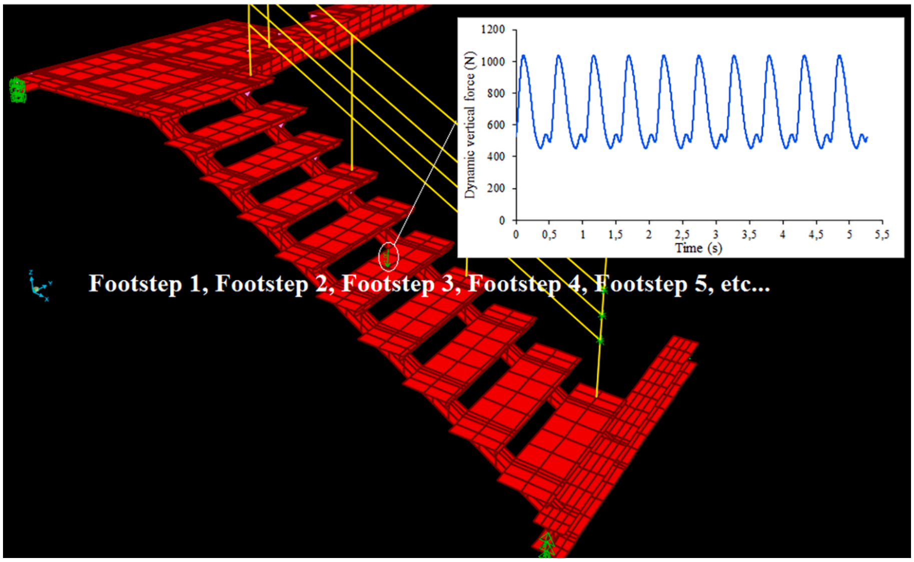

Fig. 4. Fourier series application at midspan for consecutive footsteps at different instants.

Similarly to the first method, before performing a numerical analysis, the Fourier series footfall traces with the desired step frequencies (first four submultiples and/or most common step frequencies) need to be calculated, as seen in the examples of Fig. 2. Following this, it is necessary to verify how these should be applied to the numerical model. Applying the Fourier series traces along the flight of stairs, in the same manner as the GRFs traces described in Subsection 4.2.1, may not be the most accurate. Undoubtedly, forces employed by several consecutive steps can be simulated using a Fourier series, but it should be noted that when, in a real situation, the two feet overlap in two consecutive steps (right and left foot), the forces generated are applied at two different points and not at the same point, as it is assumed for defining Fourier series traces. This is of particular relevance, since applying two GRFs traces on two separate steps over a simultaneous period does not yield the same result as applying two GRFs traces on the same step over a simultaneous period. Fourier series can only simulate periodic continuous functions in an extremely realistic way if they always act at the same point over time.

For this reason, it seems more coherent to apply the Fourier series traces to the numerical model always at the same point during the time required for the individual to walk the flight of stairs, i.e. at midspan, since this is where highest accelerations are expected. As only acting at midspan, the results obtained are expected to be conservative. According to Davis [37], when it is intended to perform human walking simulations using Fourier series, these should always be applied to the numerical model at the same point, in accordance with this paper. Fig. 4 outlines how Fourier series traces are applied to the numerical model.

As in the first method (see Subsection 4.2.1), after applying the Fourier series traces, the accelerations are computed, performing a time history analysis with an identical procedure.

4.2.3. Steady-state analysis

The determination of the vibration modes and, respective, shapes and natural frequencies is highly important in the dynamic analysis of a structure. However, it becomes difficult or even impossible to understand which modes are most likely to be excited by pedestrian walking if only their frequencies and shapes are known. One method to predict which modes are most likely to be excited is to perform a steady-state analysis. This numerical analysis is described in detail by Barrett [41]. Briefly, steady- state analysis computationally assesses the magnitude of the dynamic response of the various vibrations modes as a function of their frequency by applying a series of harmonic loads (loads described by a sine or cosine function) to specific frequency increments. The aforementioned magnitude of the dynamic response is designated as the frequency response function (FRF).

Based on steady-state analysis and the harmonics amplitude, Davis proposed a method, originally for floors [37] and, more recently, for staircases [30], to calculate peak accelerations numerically. This method takes advantage of the fact that the majority of a structure's response due to walking occurs when the frequency of a harmonic equals the frequency of one of the dominant vibration modes. It is important to recognise that this is a simplified method that can only be employed on staircases with low natural frequencies, i.e. lower than 16 Hz.

The first step in applying this method is to perform a steady-state analysis using a structural software, to determine which mode is most susceptible to be excited, as well as its magnitude of acceleration.

Steady-state analysis requires the use of hysteretic damping instead of viscous damping. However, according to Chopra [42], hysteretic damping can be considered the double of viscous damping. Considering the damping ratio recommendation in Subsection 4.2.1 of 1%, then the hysteretic damping should be equal to 2% for steel staircases. The second and last step is the multiplication of the maximum acceleration magnitude by the amplitude of the harmonics that equal the staircase's fundamental frequency and by the pedestrian self-weight, thus obtaining the peak accelerations due to walking. It should be assumed that only the amplitude of the first four harmonics is relevant for resonant effects to occur.

Fig. 5 represents an example of an FRF graph performing a steady-state analysis, with acceleration magnitudes obtained at midspan of an FE model created from a real staircase. The FRF was calculated for a frequency range between 10 and 20 Hz since, for this particular case, no vibration mode was obtained for frequencies below 10 Hz, and for frequencies above 20 Hz vibration modes are unlikely to be excited by the commonly used step frequencies. As can be observed, the first vibration mode with a frequency of 13.90 Hz is clearly the most susceptible to generating the largest acceleration response. In accordance with the aforementioned, this means that the frequency of the 4th load harmonic when walking at 3.50 Hz is the only one capable of matching the structure's fundamental frequency (13.90 Hz) and, therefore, the maximum acceleration magnitude (1.04%g/N) must be multiplied by its amplitude.

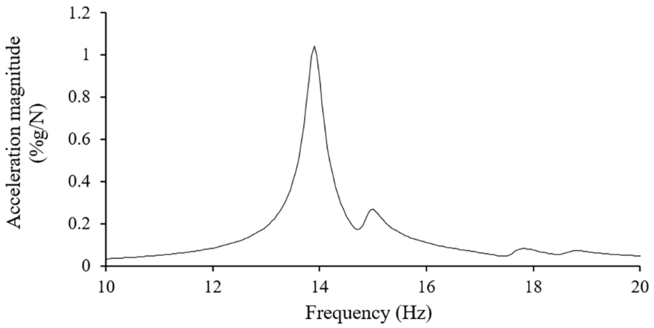

Fig. 5. Magnitudes of acceleration FRF for frequencies between 10 Hz and 20 Hz.

In the majority of design cases, the maximum value of the acceleration magnitude can be multiplied by the average harmonics amplitudes obtained by Kerr [2], presented in Table 1, since overestimated responses are expected due to steady-state resonance build-up at the midspan. However, if a higher margin of safety is required, with narrower restrictions against potentially disturbed users, the harmonics amplitudes defined by Kerr [2] with a 75% probability of not being exceeded, also presented in Table 1, should be used.

4.2.4. Simplification vibration evaluation

Davis and Avci [31,43] in recent studies proposed a new simplified procedure to evaluate vibration serviceability, which is based on manual calculations without the need to use an FE software. The proposed simplified procedure accounts for the fact that current evaluation methods rely on FE analysis-based response predictions which are not fast or easy enough for routine structural design usage. Similarly to the steady-state analysis, employing this simplified method obtains the peak accelerations of the n$^{\text{th}}$ harmonic frequency that matches the staircase's fundamental frequency. Usually, stairs consist of a pair of parallel beams connected by closely spaced transverse bending elements. Thus, the first vibration mode resembles that of a simply supported beam with uniform mass, i.e. a half-sine wave. According to the researcher's experience with FE numerical analysis on slender staircases, the second vibration mode frequency is, in general, at least double that of the fundamental frequency, which places it outside the pedestrian's excitable frequencies, making it possible to neglect the second and higher modes without losing much accuracy. Therefore, the staircase's response is mostly conditioned by its fundamental frequency mode and can be treated as an SDOF simply supported beam.

The first step for applying this simplified method is to obtain the staircase's fundamental frequency, in order to verify which n$^{\text{th}}$ harmonic in the range of the pedestrian's step frequencies could match it. The staircase's fundamental frequency can be estimated using Eq. (2):

$$f_n = \frac{\pi}{2}\sqrt{\frac{gEI}{wL^4}} \tag{2}$$

where g represents the gravitional acceleration, EI the stringer flexural stiffness, w the uniform weight (force per length) and L the stringer length. However, Davis and Avci [31,43] suggest that if more refined estimates are required, the staircase's fundamental frequency should be obtained by creating a detailed FE model.

After estimating the staircase's fundamental frequency, the peak acceleration at midspan (the point of highest accelerations) can be obtained by the following equation:

$$a_{sMidspan} = \frac{\alpha P cos^2\theta}{2\beta M} \tag{3}$$

where M represents the staircase modal mass, β the viscous damping ratio, θ the staircase angle from the horizontal and α and P were already defined for Eq. (1) (see Subsection 3.2). The staircase's modal mass M should be considered approximately equal to half of its total mass, which is the modal mass of a simply supported beam. The researchers do not refer to values for the damping ratio β, stating that it must be set using engineering judgment based on experimental data. However, the damping was experimentally measured on two different staircases. In the first staircase, with no non-structural components, the damping ratio was 1.1% and in the second staircase, with non-structural components,

Table 4
Calibration factors$R$ for different n$^{\text{th}}$ harmonics.

Harmonic Number	R_{50}	R_{25}	R_{10}
2	0,5	0,65	0,8
3	0,7	0,9	1,1
4	0,7	0,85	1,0

the damping was 3.8%. The harmonic amplitudes α should be taken from Table 1, considering the average values in the same manner as for steady-state analysis.

According to Davis and Avci [31,43], it is often necessary to calculate accelerations at locations away from the midspan. For example, considering a long staircase, users could potentially stand at one of the intermediate landings and perceive uncomfortable vibrations, while others are walking up or down the flight of stairs. Therefore, to compute peak accelerations at any location of interest, the previously given equation can be rewritten as follows:

$$a_s = \frac{\alpha P cos^2\theta}{2\beta M}\phi_r\phi_e \tag{4}$$

where ϕ_r represents the mode shape amplitude at the distance from the end of the stringer to the response location (observer) and ϕ_e the mode shape amplitude at the distance from the end of the stringer to the pedestrian excitation force location, both measured on the diagonal. The mode shape amplitudes can be manually calculated by $\phi_i = sin\frac{\pi x_i}{L}$, or directly obtained from an FE staircase model. As reported by the researchers, employing Eq. (4) leads to overestimated peak accelerations, because resonant build-ups do not last long enough to achieve a steady-state response and footsteps are not perfectly periodic. The former two effects are taken into account by multiplying the peak accelerations, respectively, by a resonant build-up envelope function and empirical calibration factors, obtaining the final recommended equation for design purposes as follows:

$$a_s = R\frac{\alpha P cos^2\theta}{2\beta M}\phi_r\phi_e\left(1 - e^{-100\beta}\right) \tag{5}$$

The calibration factors R were established by calculating the ratios between the measured and predicted peak accelerations. Table 4 shows the final calibrations factors R for the different nth harmonics. R_{50}, R_{25} and R_{10} result in predictions that are exceeded by measurements 50%, 25% and 10% of the time, respectively. Davis and Avci [31,43] suggest that R_{50} should be used for design and R_{25} and R_{10} for unusual situations where the owner requires a wide margin of safety against complaints.

5. Group effect

As reported by Bishop et al. [3], also one of the main difficulties related to staircase vibrations is the lack of quantification of the group enhancement effect. According to the same researchers, a group of pedestrians can significantly amplify the vibrations on staircases. Bishop et al. [3], in an attempt to demonstrate the group effect on stairs, simulated the dynamic forces generated by groups of 9, 18 and 27 pedestrians. The amplification factors with a higher probability to occur were 2, 2.4, and 3.1 for groups of 9, 18, and 27 pedestrians, respectively, although for a large group of pedestrians Bishop et al. [3] verified that in a real situation the damping ratio of staircases considerably increased. So, according to the same, for a group of 27 pedestrians a lower value for the amplification factor seems to be more plausible.

Cappellini et al. [12] also studied the influence of a group of pedestrians in the staircase's damping ratio, mainly for a relatively high number of people (more than 9). However, based on their measurements, the damping ratio for steel staircases does not significantly differ from 1%, regardless of the number of people considered, being consistent with the value referred to in Subsection 4.2.1. Concerning the staircase's natural frequencies, their values were not substantially affected by increasing the number of pedestrians. Comparing measured and numerical RMS accelerations, these researchers concluded that the consideration of an empty staircase model leads to an overestimation of predicted structural vibrations referring to a group of pedestrians. A key point to reliably predict the vibrations amplitudes of moving people is the correct identification of the staircase's modal parameters based on the human–structure (H–S) joint system. Still, in this work, there is no reference to the amplification effect between an individual and a group of pedestrians.

Kerr [2], to further develop the gap reported by Bishop et al. [3], also quantified the forces generated by a group of pedestrians through amplification factors. This researcher verified that enhancement factors varying between 2.0 and 3.0 for a group of 4 pedestrians and of 6 for a group of 9 pedestrians are more likely to occur when the individuals' walking is synchronised, this being the most conditioning scenario and with a higher probability of taking place.

Regarding the four numerical procedures presented in this paper, in the first two methods, described in Subsections 4.2.1 and 4.2.2 respectively, it is possible to directly predict the amplified accelerations generated by a group of pedestrians. The procedure to calculate group accelerations with footfall force time histories and Fourier series walking models is identical to that for a single pedestrian, except for the number of GRFs and Fourier series traces to be employed, because in this case it is intended to simulate the walking of various individuals, and, for the definition of the arrival time in time history analysis, since it is necessary to take into account the different intervals of time at which the individuals walk the staircase. The GRFs and Fourier series traces are applied along and at midspan of the flight of steps, respectively, during

Table 5
Comparison of step-by-step procedure application of each numerical method.

Footfall Force Time Histories	Fourier Series	Steady State Analysis	Simplified Vibration Evaluation
Build a F.E. numerical model to obtain the staircase's fundamental frequency			Estimate fund. frequency
Employ on HFS and LFS		Only employ on LFS	
Verify the step frequencies that match the fundamental frequency submultiples, considering descents will be the governing case			
Choose GRF's traces from literature	Define the Fourier Series traces from literature	Select the nth harmonics that match the fundamental frequency from literature	
Apply along the staircase	Apply at staircase midspan	Apply a unit load at staircase midspan	–
Perform a Time History Analysis, using a damping ratio of ξ (usually 1%)		Do a Steady State Analysis, hysteric damping of 2ξ	Employ Eq. (4), using a damping ratio of ξ
If staircase's submultiples are high, also compute accelerations due to common used step frequencies (1,90 to 2,80 Hz)		–	–
–	–	Multiply the max. acc. mag. by the nth harmonic amplitude	–
Simulate group responses, increasing the number of force walking traces		Amplify the isolated peak accelerations by a factor of 3	
Compare predicted accelerations with acceptable limits of the different researchers and design guides (specified in Part 2 of this paper)			
Modelling time (with shell and bar elements): 6 to 8 h			–
Time for each analysis: 10 min Total time: 14 h		Time for each analysis: 5 min Total time: 10 h	Time for each calculation: 3 min Total time: 10 min

the time required for the number of considered individuals to walk the staircase, analogously with the individual analysis.

With the procedures detailed in Subsections 4.2.3 and 4.2.4, as for the majority of simplified numerical methods, the peak accelerations due to a group of pedestrians cannot be directly obtained. Consequently, Davis et al. [30,31,43] suggest that a simple approach to obtain the group effect performing a steady-state analysis or employing the simplified vibration evaluation is to amplify the peak accelerations caused by a single pedestrian by a factor of 3, in accordance with the work elaborated by Kerr [2].

6. Comparison of numerical methods

The previous sections defined the most important concepts, explained how to characterise the walking dynamic forces on staircases and comprehensively detailed and described the several steps involved in applying the different numerical methods when predicting accelerations, also referring to how estimate the amplification response due to the group effect. In this section, the different steps required to apply each numerical method are compared, and the modelling and calculation time for each procedure is estimated. After that, the various methods are applied to a real staircase with known liveness to assess and compare their results.

6.1. Application procedures

With the aim of providing a more straightforward and practical approach on how to employ the different numerical procedures to predict accelerations in a design phase, this subsection elaborates and presents a table encompassing a summary of the necessary application process for each method described in the previous section. Table 5 gives a simplified step-by-step procedure to apply the four design methods, including all the series of actions needed to estimate walking induced accelerations when designing flexible staircases, from the construction of the FE staircase model and force simulation to the prediction of accelerations, group amplification, and comparison with other researchers' and design guides' acceptance criteria. For routine design usage, Table 5 also includes the estimated time required to model an FE staircase, with shell and frame elements, and to perform each numerical analysis referring to the different methods.

6.2. Numerical results

Next, to evaluate the main differences between the distinct responses generated by an individual and a group of pedestrians, and by the four numerical methods, these procedures were employed on a real staircase example.

The staircase used in the analysis is predominantly made of steel and is described in Part 2 of this paper. For the scope of Part 1, it is relevant to mention that from the modal tests and the FE model a fundamental frequency equal to 13.9 Hz was estimated. Therefore, the sample staircase can be treated as an LFS with the possibility of applying the four numerical methods, since its fundamental frequency is within the range referred to in Section 2, i.e. less than or equal to 16 Hz, thus having a real and high probability of a resonant phenomenon occurring during the lifespan of the structure when walking at a step frequency of 3.50 Hz (the 4th submultiple of the fundamental frequency).

Fig. 6a) and b) give two examples of numerical acceleration graphs obtained for a single pedestrian at the FE model midspan (the location with the highest accelerations), respectively, for a normal ascent at 1.90 Hz and a fast descent at 3.50 Hz. The two acceleration graphs were estimated using the first numerical method, the GRFs footfall traces

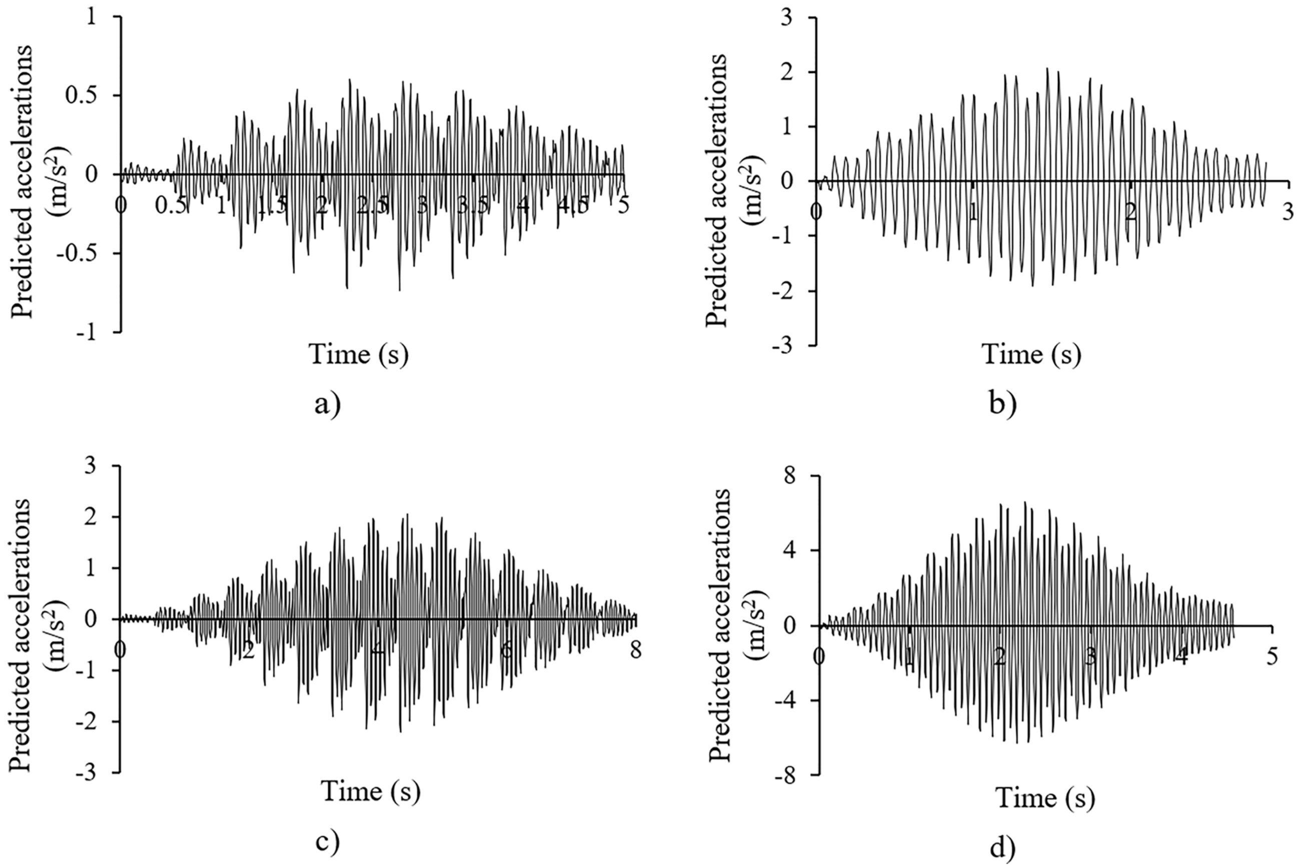

Fig. 6. Numerical accelerations examples due to GRFs: a) individual ascent at 1.90 Hz (González), b) individual descent at 3.50 Hz (Kerr), c) group ascent at 1.90 Hz (González) and d) group descent at 3.50 Hz (Kerr).

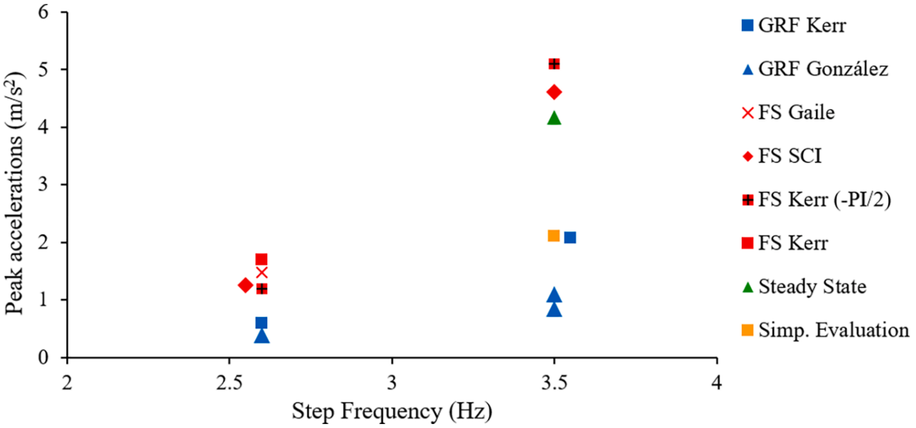

Fig. 7. Comparison between peak acceleration employing the different numerical methods.

employed in the simulations being selected from the researchers González [4,5] and Kerr [2]. It is important to highlight that, for the fast descent at 3.50 Hz, a resonant response can be clearly observed, this being possible because the GRF trace was applied to the numerical model with 0.2857 s increments (1/3, 50 Hz). For the normal ascent at 1.90 Hz an impulsive response can be seen, with accelerations decaying significantly after each step being applied, since this step frequency is outside the range of the staircase's fundamental frequency submultiples.

An attempt was also made to obtain the numerical accelerations referring to a group of four pedestrians. To calculate the group accelerations with GRFs footfall traces, the procedure defined in Section 5 and indicated in Table 5 was followed. For the group analysis, the same step frequencies and GRFs traces as for the individual analysis were used, the accelerations being also obtained at the midspan of the FE model, so the amplification effect can be directly verified. Fig. 6c) and d) represent the two numerical accelerations graphs obtained for a group of pedestrians. As predicted, for the ascent at 1.90 Hz and the descent at 3.50 Hz respectively, an impulsive and resonant response was obtained, similar to individual accelerations. The acceleration values are approximately 2 to 3 times higher than the acceleration values obtained for a single pedestrian, as seen in Fig. 6a) and b), being close to the amplification factors obtained by Kerr [2] for a group of four pedestrians.

The four numerical methods were applied to the analysed staircase following the series of steps detailed in Subsections 4.2.1 to 4.2.4 Fig. 7 presents the peak accelerations generated by a single pedestrian at midspan for normal descents at 2.60 Hz (descents in general) and fast descents at 3.50 Hz (plausible resonant scenario). The values in Fig. 7 represent the peak accelerations for an individual pedestrian, so the responses of each numerical method can be directly compared, without applying any intermediate amplification factor.

Employing GRFs footfall traces gives rise to lower predicted accelerations among the four numerical procedures, except when comparing the values estimated with the GRF Kerr and the simplified vibration evaluation for the descent at 3.50 Hz. This was as expected, since it is the least conservative method, where the footfall force functions are applied along the staircase steps, realistically simulating pedestrian walking. Among the different GRFs traces employed, the GRF Kerr and GRF González generated close peak accelerations for the normal descents (2.60 Hz) and relatively distinct peak accelerations for the fast descents (3.50 Hz). As also predicted, the peak accelerations are higher for the descents at 3.50 Hz, due to resonance amplification.

Despite the advantages of using Fourier series to simulate the dynamic forces applied by pedestrians, as can be seen from Fig. 7, when applying the Fourier series traces and the GRFs traces the results obtained are substantially different. The Fourier series, as explained in Subsection 4.2.2, are originated from the overlapping of two consecutive footfalls, and when this load function is applied only in one step of the FE staircase model, the force generated by the right foot and the left foot is applied at the same point; this is not the case of the GRFs load function since, as it represents only one footfall, during the period when the right foot and the left foot are acting simultaneously, the two functions are being applied at different points, i.e. at different steps. This could be one of the reasons why Fourier series load functions result in higher acceleration values, especially when dealing with step frequencies where there is a possibility of a resonance build-up. This is a relevant observation, as it may mean that Fourier series only simulate periodic forces accurately when always applied at the same point and not in these particular cases, i.e. human walking forces. For the different Fourier series traces used in the normal descents at 2.60 Hz and the fast descents at 3.50 Hz, approximate peak accelerations were obtained.

For the aforementioned reasons given in Section 4.2.3, the steady-state analysis was only performed for the descent at 3.50 Hz. For this step frequency, the predicted peak acceleration was higher than the peak accelerations obtained with the GRFs traces and the simplified vibration evaluation method, being closer to the values estimated with Fourier series traces. Steady-state analysis predicts peak accelerations due to a unit sinusoidal load applied at midspan multiplied by the n^{th} harmonic amplitude that matches the staircase's fundamental frequency, so it is also expected to generate conservative results, being highly dependent on the considered values of harmonic amplitudes.

Similar to the steady-state analysis, the simplified vibration evaluation method was only employed for the descent at 3.50 Hz. The peak acceleration was computed using Eq. (4) for the staircase midspan, to be directly compared with the predicted peak accelerations of the remaining methods. Although not considering any response reduction, the predicted peak acceleration with this procedure is not as high as those predicted with the Fourier series and steady-state analysis, almost coinciding with the estimated value of the GRF Kerr trace.

It must be emphasised that the accelerations shown in Fig. 7, obtained by all methods, were not affected by any adjustment factor, as proposed by various researchers [30,31,37,44,45], since it was intended to compare the predicted results of the four numerical methods on the same basis, taking into account only their fundamental theory. The different values of adjustment factors proposed in the literature are discussed and presented in Part 2 of this paper.

7. Summary and conclusions

The vibration serviceability of lightweight and slender monumental steel staircases is currently one of the major concerns and challenges facing structural engineers. To overcome this difficulty, this paper aimed to provide insights and practical guidance, from a design point of view, on how to predict vibrations of flexible staircases to avoid the occurrence of inadequate dynamic behaviour and, subsequently, any potential feeling of unsafety when pedestrians use them. For this to be achieved, the four main existing numerical methods for estimating vibrations in the design phase were presented, described and applied to a real staircase to compare the differences in the results obtained by each procedure.

From the work developed, various relevant conclusions can be drawn:

- A structure's dynamic behaviour depends on its fundamental frequency. Low frequency structures (LFS) respond with resonance and high frequency structures (HFS) respond impulsively. In the case of staircases, based on the reasons set out, a cut-off frequency between LFS and HFS equal to 16 Hz seems plausible.
- The selected numerical method depends on whether it is HFS or LFS. The numerical methods using GRFs and Fourier series footfall traces can be employed on LFS and HFS, while steady-state analysis and the simplified vibration evaluation can only be applied on LFS.
- There is a scarcity of GRFs footfall traces for staircases in the literature, with limited step frequencies and none specifically obtained for design purposes. The GRFs traces vary significantly with the step frequency; hence the relevance of using GRFs with the exact intended frequency content, or close, to accurately employ this numerical method.
- Footfall force time histories and Fourier series walking models are rigorous methods to simulate pedestrians' walking loads and predict acceleration intervals, while steady-state analysis and simplified vibration evaluation are simplified methods to predict peak accelerations due to walking harmonic amplitudes. As a consequence, the selected method directly influences the modelling, analysis and calculation time and hence the overall work time for the process.
- It was verified that the amplification generated by a group of pedestrians is, in general, 2 to 3 times higher than for an individual.
- For fast descents at 3.50 Hz (the 4th submultiple of the fundamental frequency in the staircase analysed) higher numerical accelerations were obtained due to resonance amplification, also demonstrating the importance of step frequencies matching the fundamental frequency submultiples on the staircase response.
- Fourier series walking models and steady-state analysis resulted in higher acceleration values than footfall force time histories and the simplified vibration evaluation.
- For the first time, the reasons why Fourier series walking models tend to generate much higher results are explained. The Fourier series footfall traces are derived from the overlapping of the force generated by the right and left foot (two GRFs traces) and are simultaneously applied at the same step (located at midspan), and can therefore be expected to originate overestimated responses.

The four numerical methods analysed gave rise to different results, although applied on the same staircase. In Part 2 of this work, it will be seen which method most successfully compares with reality.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Kerr SC, Bishop NMW. Human induced loading on flexible staircases. doi: 101016/jengstruct200710006; Vol. 23; 2001, p. 37–45.

[2] Kerr SC. Human induced loading on staircases. Ph.D. thesis. London (UK): University College of London, Mechanical Engineering Department; 1998.

[3] Bishop NWM, Willford M, Pumphrey R. Human induced loading of flexible staircases. Saf Sci 1995;18(4):261–76.

[4] González H. Numerical simulation of human induced vibrations of stairs. MSc. thesis. Weimar, Germany: Bauhaus-Universität Weimar; 2013.

[5] Meinhardt C, Zabel V, Gonzalez H. An in-situ case study induced vibrations on slender staircases. In: Proceedings of the 9th International Conference on Structural Dynamics, EURODYN 2014. Porto, Portugal; 2014.

[6] Kasperski M, Czwikla B. A refined model for human induced loads on stairs. Proceedings of the 30th IMAC, A Conference on Structural Dynamics. Jacksonville, Florida, USA; 2012.

[7] Bougard A. Human loading on staircases Structures and Buildings 2002; 152: 371–80. doi.org/10.1680/stbu.2002.152.4.371.

[8] Gaile L. Footfall Induced Forces on Stairs. 4th International Conference CIVIL ENGINEERING'13. Jelgava, Latvia; 2013.

[9] Gaile L. Dynamic Response of Light-Weight Lattice Towers to Human Induced Loads. Ph.D. thesis. Riga, Latvia: Riga Technical University; 2014.

[10] Setareh M. Vibration serviceability of monumental stairs. In: Proceedings of The National Conference On Undergaduate Research (NCUR). Asheville, North Carolina, USA; 2016.

[11] Belver AV, Stana Z, Dang HV, Istrate M, Iban AL. Modal testing and FE model updating of a lively staircase structure. In: Proceedings of the SEM IMAC XXX Conference. Jacksonville, FL USA; 2012.

[12] Cappellini A, Manzoni S, Vanali M, Cigada A. Evaluation of the dynamic behaviour of steel staircases damped by the presence of people. Eng Struct 2016;115:165–78.

[13] Vanali M, Berardengo M, Manzoni S. Numerical model for human induced vibrations. International Modal Analysis Conference, IMAC XXXV. Garden Grove (CA, USA) January 30–February 2; 2017, p. 179–86.

[14] Busca G, Cappellini A, Manzoni S, Tarabini M, Vanali M. Quantification of changes in modal parameters due to the presence of passive people on a slender structure. J Sound Vib 2014;333(21):5641–52.

[15] Venuti F, Racic V, Corbetta A. Modelling framework for dynamic interaction between multiple pedestrians and vertical vibrations of footbridges. J Sound Vib 2016;379:245–63.

[16] Tubino F. Probabilistic assessment of the dynamic interaction between multiple pedestrians and vertical vibrations of footbridges. J Sound Vib 2018;417:80–96.

[17] Shahabpoor E, Pavic A, Racic V. Identification of mass–spring–damper model of walking humans. Structures 2016;5:233–46.

[18] Shahabpoor E, Pavic A, Racic V. Structural vibration serviceability: New design framework featuring human-structure interaction. Eng Struct 2017;136:295–311.

[19] Caprani CC, Ahmadi E. Formulation of human–structure interaction system models for vertical vibration. J Sound Vib 2016;377:346–67.

[20] Berardengo M, Drago L, Manzoni S, Vanali M. An approach to predict human–structure interaction in the case of staircases. Arch Appl Mech 2019;89(10):2167–91.

[21] Van Nimmen K, Lombaert G, De Roeck G, Van den Broeck P. The impact of vertical human-structure interaction on the response of footbridges to pedestrian excitation. J Sound Vib 2017;402:104–21.

[22] Santos J, Andrade P, Escórcio P. Pre-design of laterally supported stair steps. Eng Struct 2019;182:51–61. https://doi.org/10.1016/j.engstruct.2018.12.018.

[23] Kim S-Y, Lee C-H, Kim N-E. Effective impulse model for prediction of vibration response of high-frequency steel staircases. J Constr Steel Res 2016;126:129–38.

[24] Smith AJ, S.J. H, P.J. D. SCI P354 - Design of steel floors for vibration: a new approach. Steel Construction Institute: Ascot, Berkshire (UK); 2009.

[25] Kraincanic I, Sparkes A. Human induced vibrations of staircases: Measurements and analysis. In: Zingoni A, editor. Proceedings of the SEMC International Conference on Structural Engineering, Mechanics ann Computation. Cape Town, South Africa; 2019.

[26] Murray TM, Allen DE. AISC - Steel design guide series 11: floor vibrations due to human activity. American Institute of Steel Construction 1997.

[27] Institution BSBS. BS 6472 - Evaluation of human exposure vibration in buildings (1Hz to 80Hz). BSi; 1992.

[28] Standardization IOf. ISO 10137 - Bases for design of structures, serviceability of buildings and walkways against vibrations. International Organization for Standardization; 2007.

[29] Zhou Bin, Ren Xiao Song, Lu Xi Lin. Vibration analysis and evaluation of the indoor spiral steel stair. Adv Mater Res 2011;163-167:36–43.

[30] Davis Brad, Murray Thomas M. Slender monumental stair vibration serviceability. J Archit Eng 2009;15(4):111–21.

[31] Davis Brad, Avci Onur. Simplified vibration serviceability evaluation of slender monumental stairs. J Struct Eng 2015;141(11):04015017. https://doi.org/10.1061/(ASCE)ST.1943-541X.0001256.

[32] Middleton CJ, Brownjohn JMW. Procedures for vibration serviceability assessment of high-frequency floors. doi: 101016/jengstruct200710006; Vol. 30; 2007, p. 1548–59.

[33] Bachmann H, Ammann W. Vibrations In Structures, Induced by Man and Machines. IABSE (International Association for Bridge and Structural Engineering): Zurique, Switzerland; 1987.

[34] Middleton CJ, Brownjohn JMW. Response of high frequency floors: a literature review. doi: 101016/jengstruct200710006 2009; 32:337-52.

[35] Mohammed AS, Pavic A, Racic V. Improved model for human induced vibrations of high-frequency floors 2018;168:950–66. https://doi.org/10.1016/j.engstruct.2018.04.093.

[36] Muhammad Zandy O, Reynolds Paul. Probabilistic Multiple Pedestrian Walking Force Model including Pedestrian Inter- and Intrasubject Variabilities. Adv Civil Eng 2020;2020:1–14.

[37] Davis DB. Finite element modeling for prediction of low frequency floors vibrations due to walking. Ph.D. thesis. Virginia, USA: Virginia Polytechnic Institute; 2008.

[38] Matsumoto Y, Griffin MJ. Mathematical models for the apparent masses of standing subjects exposed to vertical whole-body vibration. J Sound Vib 2003;260(3):431–51.

[39] IStructE. Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Action: recommendations for the design of stadium grandstands for vibrations. London; UK2008.

[40] Young P, Willford MR. A Design Guide for Footfall Induced Vibration of Structures. Concr Soc 2006.

[41] Barret AR. Dynamic Testing of In-Situ Composite Floors and Evaluation of Vibration Serviceability Using Element Method. Ph.D. thesis. Virginia, USA: Virginia Polytechnic Institue and State University; 2006.

[42] Chopra AK. Dynamics of structures, theory and applications to earthquake engineering. Prentice Hall, International Edition: New Jersey, USA; 1995.

[43] Davis B, Avci O. Simplified vibration response predicition for slender monumental stairs. In: Structures Congress, ASCE. Boston, Massachusetts, USA 2014.

[44] Setareh M. Vibration analysis and design of a monumental stair. In: Springer, editor. Proceedings of the 2013 Annual Conference on Experimental and Applied Mechanics. Lombard, Illinois, USA; 2013.

[45] Eid R, Seica M, Stevenson D, Howe B. Staircase vibrations due to human activity. Proceedings of Structures Congress, ASCE. Las vegas, Nevada, USA; 2011.