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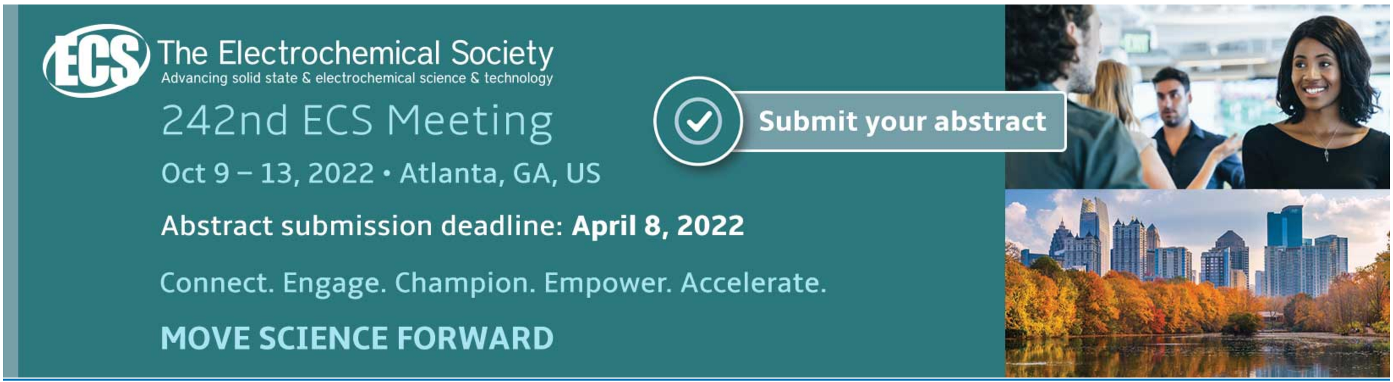

Numerical methods to predict vibration serviceability on high frequency stairs

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Abstract. With the increasingly deepen knowledge on structural dynamic behaviour, nowadays, monumental staircases and their steps are usually designed with high stiffness and low mass, obtaining high fundamental frequencies, out of the interval of step frequencies excitable by the human walking. However, neglecting the structure's mass can also result in with high levels of impulsive responses. Furthermore, often the connection between the staircase and its steps experience an almost null rotational stiffness, which can cause local vibrations in steps, where the structural behaviour of two elements is nearly independent from each other. In this scenario, steps will most likely exhibit undesirable vibrations. Hence, intending to improve future designs, this paper compares different numerical methods that can be used to predict human induced vibrations in this type of structures, applying them to a real staircase with a high local liveness. The results showed that the footfall force time histories method could realistically predict the vibrations for almost all tested step frequencies, while the effective impulse could accurately estimate the vibrations for descents at different step frequencies. The walking models defined by Fourier coefficients only generated close vibrations for descents at 3.3 Hz.

1. Introduction

Structures dynamic behaviour can be divided into two types of response, depending on their natural frequency. Low frequency structures (LFS) can be assumed to develop a resonant build-up when pedestrian's step frequencies match its natural frequency or are one of its sub-multiples [1-3], while high frequency structures (HFS) act impulsively, with a transient response [4].

It is becoming a notable and common trend to design prominent monumental staircases with increasingly slender and lightweight elements and long and unsupported spans. This is usually done by increasing the staircase's stiffness, obtaining high fundamental frequencies to avoid resonant effects. However, neglecting staircases mass can also generate unacceptable levels of impulsive responses, failing to meet the vibration serviceability requirements [4, 5].

In many cases, the connection between the staircase and its steps exhibit an extremely low rotational stiffness and the dynamic behaviour of these two is nearly independent from each other, which can develop local vibrations much more severe than a global phenomenon [6-8].

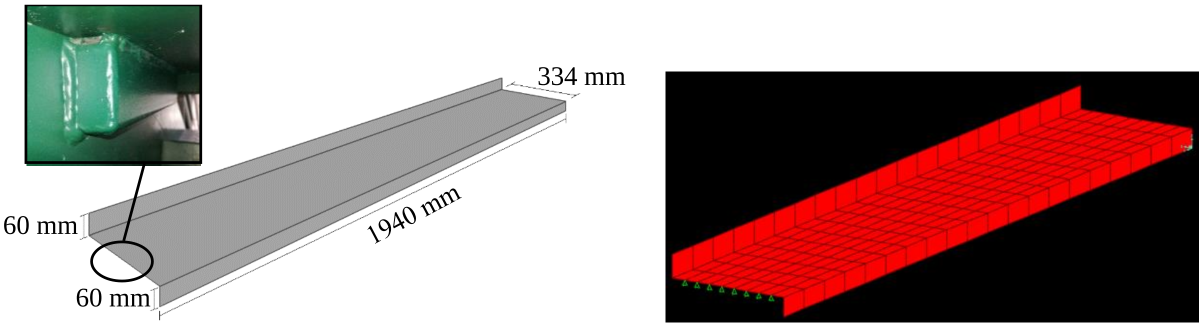
Due to the existing lack of knowledge, this paper applies and compares different numerical methods that can be used when designing this type of structures, to predict human induced vibrations. First, a real steel staircase with high frequency steps (HFS) and local vibrations is experimentally tested. Then, a Finite Element (FE) model of one of the steps is created and the different numerical methods are described and applied to estimate vibrations due to the walking forces. In the end, the vibrations experimentally measured and numerically obtained are compared, evaluating which method could more realistically predict vibrations in a local phenomenon.

2. Experimental campaign

2.1. Staircase description and dynamic properties

The steel staircase studied in this paper is located inside a public building in Funchal, Madeira, and it was conceived to allow pedestrians access between two of the floors. Due to the wear and tear by pedestrian's use over the years, the auxiliary hollow profile that connects the stair steps with the rest of the structure is weakly welded (Figure 1a), resulting in a connection with a practically null rotational stiffness. Therefore, the dynamic behaviour of the steps is independent from the rest of the structure, with its users experiencing it high levels of local vibrations.

Since the vibrations are local, it is only of interest to describe the geometrical properties of the steps. The same are each composed of a metal plate with a thickness of 6 mm covered with a thin coating of synthetic rubber sheet, being its dimensions represented in Figure 1a).



a) b)

Figure 1. Sample steel stair step: a) welding connection and geometric dimensions; b) FE model.

An ambient modal analysis has been performed to obtain the staircase steps natural frequencies, and corresponding mode shapes, as represented in Table 1. The frequencies and shapes of the two local vibrations modes experimentally measured are different from the global modes, evidencing that responses are associated with the stair steps movement and not of the entire structure. The damping ratio was estimated using the half-power bandwidth method, obtaining a value approximately equal to 0.82% of critical. This damping ratio, although estimated in a steel stair step, it is in agreement with different authors [1-3, 9], who obtained for a complete steel stair, a value close to 1% of critical.

Table 1. Experimental and numerical vibrations modes.

Modes		Experimental	Numerical
Nº	Shape	Frequency (Hz)	Frequency (Hz)
1	Vertical w/ torsion	24.1	24.0
2	Torsion	42.6	45.6

2.2. Walking tests

Several walking tests were performed at different step frequencies, ranging from normal walks to fast runs, for latter comparison with the vibrations predicted employing the numerical methods.

Experimental measurements were conducted for a single pedestrian walking the staircase, and then having a group of 4 pedestrians (2+2) traversing it. As it was observed in Table 1, the stair steps have a natural frequency close to 24 Hz, which is higher than the cut-off frequency of 16 Hz proposed by Santos et al. [6], where resonant effects can no longer occur. Additionally, since the dynamic behaviour of each step is nearly independent and only one footfall is applied, there is no possibility of a resonant build-up.

Hence, participants in the walking tests were instructed to ascent and descent the staircase with step frequencies more commonly used in their daily routines, as described in Table 2, and, not intending to reach a resonance response.

Table 2. Walking tests.

Step frequency	Number of trials	
	Single pedestrian	Group of pedestrians (2+2)
Normal descent at 2.2 Hz	4	4
Fast descent at 3.3 Hz	4	4
Normal ascent at 2.0 Hz	4	4
Fast ascent at 3.0 Hz	4	-

2.3. *Measured results and discussion*

Comparing the accelerations measured for a single pedestrian and a group of pedestrians, it was verified that the maximum values obtained for both typologies were close and, therefore, the group enhancement can be excluded.

In this subsection, the accelerations measured for a single pedestrian are compared with the acceptable limits proposed by various design guides and researchers [1, 3, 10-12], aiming to easily perceive the level of vibration which the sample staircase is subjected. Figure 2 presents the comparison between the peak accelerations measured for each individual ascent and descent at different step frequency and the proposed acceptable limits.

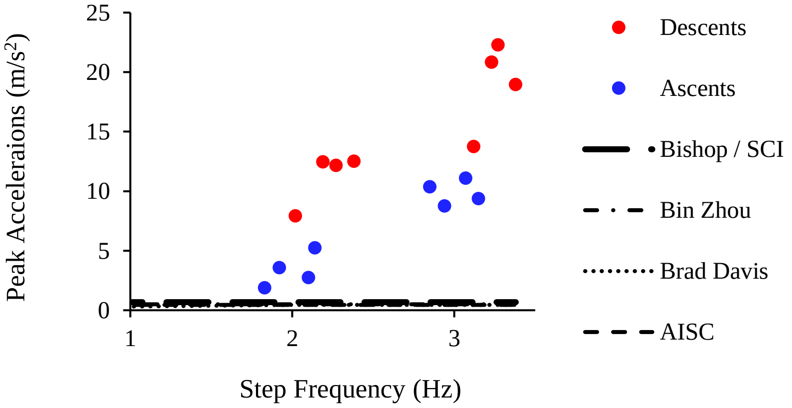


Figure 2. Comparison between experimental peak accelerations and acceptable limits of vibration.

Observing Figure 2 it is possible to notice that accelerations are extremely higher than the various limits, reaching for descents at 3.3 Hz peaks approximately twice the gravitational acceleration (≈9.81 m/s^2). This demonstrates that LFS could not always generate conditioning responses. Structures designed with a high natural frequency, but neglecting the contribution of its mass, might also give rise to significant impulsive responses, as is the case of the sample stair steps (24 Hz and 46,2 kg). Figure 2 also indicates that accelerations increase as the step frequency increases, being higher for descents, which is in accordance with results obtained by other authors [1-3, 9, 13].

3. Numerical modelling

3.1. FE model

To implement the different numerical methods, it is required to create an FE model of the analysed structure. For the reasons previously explained, it was only necessary to develop a FE model of one of the stair steps using the analysis software SAP2000. The metal plate that constitutes the step structure was modelled using shell elements with a thickness of 6 mm and the dimensions represented in Figure 1a). The contribution of the thin coating of synthetic rubber sheet can be assumed as negligible for the step stiffness, thus only its mass (6 kg/m^2) was considered when modelling. The lack of rotational stiffness given by the weakly welded connection between the stair steps and the remaining structure was simulated using pinned supports. Figure 1b) shows the FE model built.

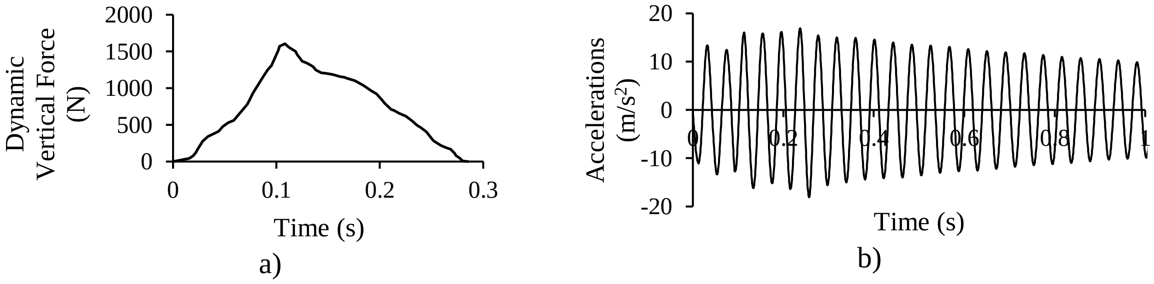
For the accelerations being effectively predicted by the different numerical methods, the FE model need to be calibrated, so the local vibration modes are close to those numerically obtained. Standard eigenvalue analysis in SAP2000 was used to estimate the FE model vibration modes and its respective frequencies, as shown in Table 1. As it can be seen from Table 1 the experimental and numerical local modes are approximated, validating the assumptions made when building the FE model.

3.2. Numerical methods

Considering the current scarcity on guidance for designing steel staircases with high local vibrations, which is an increasingly observed phenomenon, in this subsection are presented and described the three main existing numerical methods to predict human induced vibrations on this type of structure: i) footfall force time histories (GRFs), ii) Fourier coefficients walking models, and iii) effective impulse.

3.2.1. Footfall force time histories (GRFs). Walking dynamic forces directly measured on horizontal surfaces, i.e. floors and footbridges [14], and on inclined surfaces, i.e. staircases [9, 15-17], are commonly designated as ground reaction forces (GRFs). This forms the basis of the first numerical method, which consists of employing GRFs traces, directly obtained from force plate measurements on stairs, on an FE model, realistically simulating the pedestrian's movement when walking the real staircase.

In this work, a total of 13 GRF's traces from different researchers force plate tests were used [9, 15-17], 7 for ascents and 6 for descents, selecting the same step frequencies chosen for the walking tests performed, as seen in Table 2, so the accelerations could be directly compared with the experimental results. The different GRFs traces were applied to the stair step FE model, simulating the participants' behaviour during the walking tests, and, then performing for each a time history analysis in SAP2000, to compute the numerical accelerations. The damping value experimentally measured of 0.82% of critical was considered for all time domain analysis. Figure 3a) represents an example of one GRF trace measured by Kerr and Figure 3b) the numerical accelerations computed for this GRF performing a time history analysis.



a) b)

Figure 3. Footfall force time histories: a) descending at 3.3 Hz; b) numerical accelerations.

3.2.2. Fourier coefficients walking models. A GRF trace from a footfall can be transformed into a periodic continuous walking load model corresponding to the overlapping of several consecutive footfalls. This means that dynamic forces employed by pedestrians on floors, footbridges, stairs, etc. can be described in terms of a Fourier Series, as follows:

$$F(t) = P + \sum_{i=1}^{n} P\alpha_i \, sin\left(2\pi i f_{step} t + \emptyset_i\right) \tag{1}$$

where P represents the pedestrian self-weight (N), f_{step} represents the pedestrian step frequency (Hz), $\emptyset_i$ the phase angle of the harmonic i (rad) and α_i represents the amplitude of the harmonic i. α_i is also often referred to as the Fourier coefficient.

Analogously to the previous subsection, the second numerical method consists of applying Fourier series footfall traces on an FE model, simulating the pedestrian's walking forces on the actual staircase to predict the accelerations. However, the footfall's forces employed being indirectly obtain by Equation (1) and not directly measured.

Since it is intended to simulate the force induced by one footfall, and not consecutive footfalls, the second numerical method is more rigorously designated as walking models defined by Fourier coefficients (FC), and not by a Fourier series. In total, 12 walking models were defined, 6 for ascents and 6 for descents, with Fourier coefficients obtained by different researchers and proposed by a design guide for staircases [10, 15, 18]. For coherence, Fourier coefficients were chosen and Equation (1) was defined with the step frequencies employed for the walking tests and the GRFs traces. For each walking model a time history analysis was performed to compute the accelerations, using the same damping ratio value. Figure 4a) represents an example of a walking model defined with Fourier coefficients given by SCI P354 [10] and Figure 4b) the corresponding numerical accelerations.

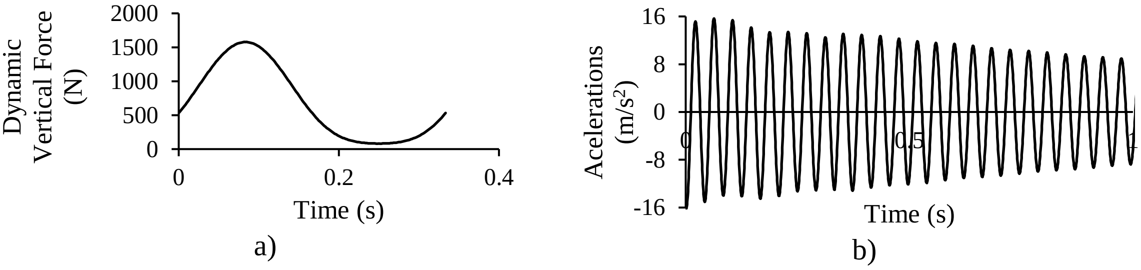


Figure 4. Fourier coefficients walking models: a) ascending at 3.0 Hz; b) numerical accelerations.

3.2.3. Effective impulse. Contrary to the other numerical methods previously presented, the effective impulse is a simplified method to predict numerical vibrations exclusively in high frequency structures. It was also developed to be used in floors, being arguable its use in stairs or stair steps, taking into account the differences on its geometric properties (inclination) and employed dynamic forces and step frequencies.

The effective impulse approach can be defined as the transient response originated by a footfall force on floors with high fundamental frequencies. This particularly method is widely explained and employed in a previous work developed by the authors [8]. Equations (2) and (3) are used to calculate the effective impulse and resulting accelerations, respectively, according to SCI P354 [10]:

$$I_{eff} = 60 f_s^{1,43} / f_i^{1,30} \left(\frac{P}{700}\right) \tag{2}$$

$$a(t) = \sum_{n=1}^{N} 2\pi f_i \sqrt{1-\xi^2} \mu_{e,i} \mu_{r,i} \frac{I_{eff}}{M_i} \sin\left(2\pi f_i \sqrt{1-\xi^2} t\right) e^{-\xi^2 \pi f_i t} W_i \tag{3}$$

where I_{eff} is the effective impulse (Ns), f_s is the step frequency (Hz), f_i is the natural frequency of the mode i of the floor (Hz), M_i is the modal mass of mode i (Kg) and W_i is a weighting factor for human perception of vibrations, $\mu_{e,i}$ is the mode shape amplitude at the point on the floor where the effective impulse I_{eff} is applied, and $\mu_{r,i}$ is the mode shape amplitude at the point where the response is to be calculated, both amplitudes obtained considering a unity or normalized mass from FE output.

The numerical accelerations were predicted employing Equation (3) for the same step frequencies, but without the possibility to distinguish between ascents and descents. Figure 5 represents an example of the numerical accelerations obtained by the effective impulse method at 2.2 Hz.

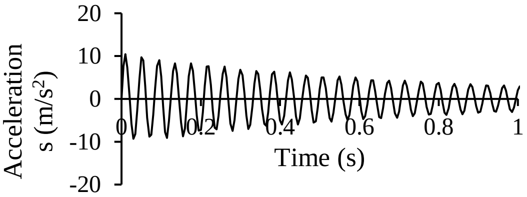
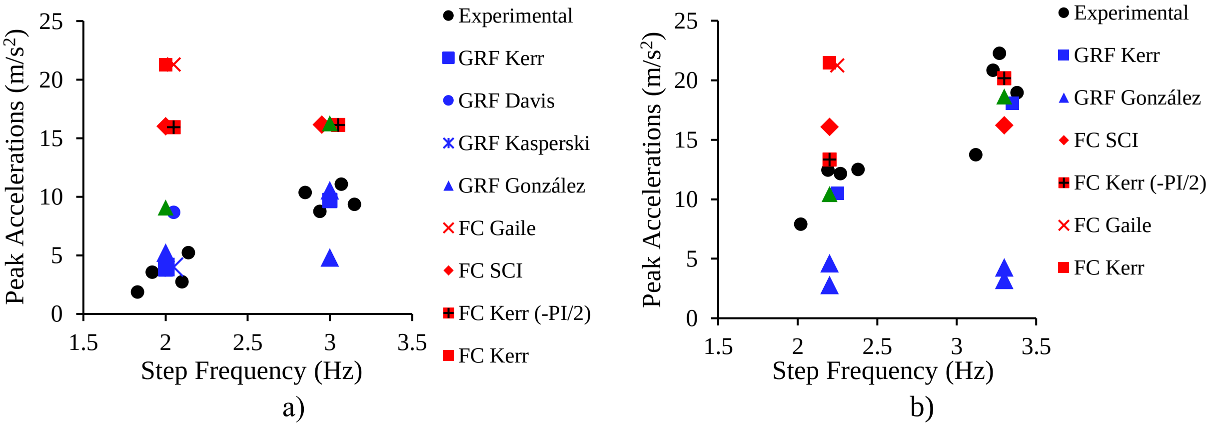


Figure 5. Effective impulse accelerations for 2.2 Hz.

4. Comparison between experimental and numerical vibrations

Figure 6 presents the peak accelerations experimentally measured and numerically obtained with the three numerical methods, for ascents and descents at different step frequencies.



a) b)

Figure 6. Comparison between experimental and numerical accelerations: a) ascents; b) descents.

Comparing peak accelerations experimentally measured and numerically obtained by GRFs traces, results are in general approximated. This evidences that, employing GRFs traces on the step FE model and, then, performing time history analysis, it was possible to accurately simulate the individuals walking on the sample staircase. However, this method presents some shortcomings, since there is a lack of GRFs traces from staircases measurements found in literature, when attempting to design this type of structure. The FC walking models originated far conservative results, except for descents at 3.3 Hz. This could be explained by the fact that Fourier coefficients are obtained by the overlap of consecutive footfalls on the same place, which does not occur in real staircases. The impulsive effective method estimated approximated results for descents at 2.2 Hz and 3.3 Hz. Since accelerations only increase with the step frequency (not differentiating between ascents and descents) and experimentally are higher for descents, the results yield to close values.

5. Summary and conclusions

The studied staircase exhibits a phenomenon of extremely high local accelerations, reaching values close to 18 m/s^2, demonstrating that impulse responses can also be conditioning, when the mass contribution is neglected. This paper aims to employ and compare three different numerical methods that can be used to predict vibrations when designing this type of structure. The application of GRFs directly measured on staircases could accurately predict the accelerations, being the most effective method, as also confirmed by other researchers [9, 13, 16]. The FC walking models gave rise to overestimated accelerations, except for descents at 3.3 Hz, while the impulse effect was able to closely predict accelerations for descents at 2.2 Hz and 3.3 Hz. Despite the differences on the obtained results, all three methods present some limitations, hence further development is still required, or other approaches should be considered when designing stair steps, i.e. based on pre-design expressions [6].

Acknowledgements

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