Universidade do Porto
FEUP Faculdade de Engenharia

Optimization of Mechanical Systems
(Otimização de Sistemas Mecânicos)
M.EM005

Optimal Design Applications

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Preface

Optimization is a branch of Mathematics and Numerical Methods that has been the subject of intensive theoretical and applied research over the last decades. In particular, Engineering is one of the areas where optimization has found propitious ground for its application and theoretical and practical development. The development of numerical methods in engineering and science in general, as well as the parallel development of computational capacities, has made available to engineers and scientists powerful tools for the analysis and resolution of the most varied and complex problems in their respective areas. Once equipped with the ability to analyse and solve complex problems, the natural path of the engineer and scientist is to try to obtain the best solution for these problems, the optimal solution.

Design optimization can be defined as the rational establishment of a design that is the best within all possible designs according to one or more predefined objectives and obeying a prescribed set of geometric constraints and/or constraints related to the behaviour and integrity of systems in engineering, technological constraints, etc.

The correct formulation of the design optimization problem is critical because the quality of the optimal solution is dependent on the formulation of the problem. For example, if a critical constraint is omitted in the formulation, the optimal solution will probably not satisfy the constraint in question. Moreover, if too many constraints are considered or if they are inconsistent, there may be no solution to the problem. However, once the problem is formulated properly, there is usually an algorithm to solve it.

To apply mathematical concepts to optimal design research, the underlying mathematical problem must be formulated and the respective model built. To do so, it is necessary to define the design variables, the objectives and the constraints of the problem. The nature of these mathematical entities depends on the available information about the design problem, which can be deterministic, probabilistic/stochastic or fuzzy. On the other hand, optimization algorithms can be classified in three major classes: mathematical programming, optimality criteria and bio-inspired algorithms.

This document presents a set of applications that represent a challenge for students in learning the curricular unit of Optimization of Mechanical Systems. Initially are suggested examples of unconstrained problems involving the calculation of the extreme values of polynomial functions. It is also proposed the minimization/maximization of polynomial functions subject to constraints. Finally, real scenario problems involving the optimal design of mechanical systems in two areas are proposed: i) structures and mechanical components; ii) thermal and fluid mechanics systems.

Students are invited to use optimization methods with different approaches in the search for extreme values, comparing the results obtained and the associated costs.

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Carlos Conceição António, Prof. Catedrático

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1. Polynomial problems (PP)

1.1 Unconstrained problems

Exercise PP 1:

$$\text{Minimize } f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2 \tag{1.1}$$

from the starting point $\mathbf{X_1}(x_1, x_2) = (0{,}0)$ using the steepest descent (Cauchy) method.

Exercise PP 2:

Solve the following problem using the steepest descent (Cauchy) method:

$$\text{Minimize } f(x_1, x_2) = x_1^2 + x_2^2 + x_1x_2 - 1 \tag{1.2}$$

Exercise PP 3:

Consider the following problem:

$$\text{Maximize } f(x_1, x_2) = x_1 - x_2 - 2x_1x_2 - x_1^2 - 2x_2^2 \tag{1.3}$$

a) Write the necessary optimality conditions.
b) Determine the optimum using the Fletcher and Reeves conjugate gradient method.
c) Determine the optimum using the Polack and Ribière conjugate gradient method.

Exercise PP 4:

$$\text{Minimize } f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2 \tag{1.4}$$

from the starting point $\mathbf{X_1}(x_1, x_2) = (0{,}0)$ using the DFP and BFGS methods [3].

Exercise PP 5:

Use the steepest descent method to find the minimum of the unconstrained objective function $U(x, y)$ given by:

$$U(x, y)2x + \frac{20}{xy} + \frac{y}{3} \tag{1.5}$$

Solve the problem analytically and compare with the numerical results. Analyse the performance of the method.

Exercise PP 6:

Minimize the function:

$$f(x, y, z) = xy + \frac{1}{xz} - 16y^2 + z \tag{1.6}$$

As the initial point consider the point with coordinates $(1.0, 0.5, 0.5)$, in the search space. Choose two different unconstrained search methods and two different univariate search methods, to determine the optimum search-step. Combine the methods and compare their performance.

1.2 Test functions

Exercise PP 7:

The use of test functions aims to show how well a specific algorithm works compared to other algorithms. Each test function is minimized from a standard starting point. The total number of function evaluations required to find the optimum solution is taken as a measure of the efficiency of the algorithm. Some of the commonly used test functions are given below. The initial solution is denoted by $\mathbf{X_1}$ and the optimal solution is represented by $\mathbf{X}^*$.

1. Rosenbrock's parabolic valley [1]:

$$f(x_1, x_2) = 100\,(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{1.7}$$

$$\mathbf{X_1}(x_1, x_2) = (-1.2, 1.0)\,, \quad f(\mathbf{X_1}) = 24.0$$

$$\mathbf{X}^* = (1.0, 1.0)\,, \quad f(\mathbf{X}^*) = 0.0$$

2. A quadratic function [1]:

$$f(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2 \tag{1.8}$$

$$\mathbf{X_1}(x_1, x_2) = (0, 0)\ ,\ \ f(\mathbf{X_1}) = 74.0$$

$$\mathbf{X}^* = (1, 3)\ ,\ \ f(\mathbf{X}^*) = 0.0$$

3. Powell's quartic function [1]:

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5\,(x_3 - x_4)^2 \\ +(x_2 - 2x_3)^4 + 10\,(x_1 - x_4)^4 \end{aligned} \tag{1.9}$$

$$\mathbf{X_1}(x_1, x_2, x_3, x_4) = (3, -1, 0, 1)\ ,\ \ f(\mathbf{X_1}) = 215.0$$

$$\mathbf{X}^* = (0, 0, 0, 0)\ ,\ \ f(\mathbf{X}^*) = 0.0$$

4. Fletcher and Powell's helical valley [1]:

$$f(x_1, x_2, x_3) = 100\,\left\{[x_3 - 10\,\theta(x_1, x_2)]^2 + \left[\sqrt{x_1^2 + x_2^2} - 1\right]^2\right\} + x_3^2 \tag{1.10}$$

where

$$2\pi\theta(x_1, x_2) = \begin{cases} arctan\left(\dfrac{x_2}{x_1}\right) & if\ \ x_1 > 0 \\ \pi + arctan\left(\dfrac{x_2}{x_1}\right) & if\ \ x_1 < 0 \end{cases}$$

$$\mathbf{X_1}(x_1, x_2, x_3) = (-1, 0, 0)\ ,\ \ f(\mathbf{X_1}) = 10000.0$$

$$\mathbf{X}^* = (1, 0, 0)\ ,\ \ f(\mathbf{X}^*) = 0.0$$

5. A nonlinear function of three variables [1]:

$$f(x_1, x_2, x_3) = \frac{1}{1+(x_1 - x_2)^2} + sin\left(\frac{1}{2}\pi\, x_2\, x_3\right) + exp\left[-\left(\frac{x_1 + x_3}{x_2} - 2\right)^2\right] \tag{1.11}$$

$$\mathbf{X_1}(x_1, x_2, x_3) = (0, 1, 2)\ ,\ \ f(\mathbf{X_1}) = 1.5$$

$$\mathbf{X}^* = (1, 1, 1)\ ,\ \ f(\mathbf{X}^*) = 3.0\ \ \ \text{(maximum)}$$

6. Freudenstein and Roth function [1]:

$$f(x_1, x_2) = \{-13 + x_1 + [(5 - x_2)\, x_2 - 2]\, x_2\}^2 + \{-29 + x_1 + [(x_2 + 1)\, x_2 - 14]\, x_2\}^2 \tag{1.12}$$

$$\mathbf{X_1}(x_1, x_2) = (0.5, -2)\ , \ \ f(\mathbf{X_1}) = 400.5$$

$$\mathbf{X}^* = (5, 4)\ , \ \ f(\mathbf{X}^*) = 0.0$$

$$\mathbf{X}^{**} = (11.41, -0.8968)\ , \ \ f(\mathbf{X}^{**}) = 48.9842$$

7. Beale's function [1]:

$$f(x_1, x_2) = [1.5 - x_1\,(1 - x_2)\,]^2 + [2.25 - x_1\,(1 - x_2^2)\,]^2 + [2.625 - x_1\,(1 - x_2^3)\,]^2 \tag{1.13}$$

$$\mathbf{X_1}(x_1, x_2) = (1, 1)\ , \ \ f(\mathbf{X_1}) = 14.203125$$

$$\mathbf{X}^* = (3, 0.5)\ , \ \ f(\mathbf{X}^*) = 0.0$$

8. Wood's function [1]:

$$f(x_1, x_2, x_3, x_4) = [10\,(x_2 - x_1^2)]^2 + (1 - x_1)^2 + 90\,(x_4 - x_3^2)^2 + (1 - x_3)^2 + 10\,(x_2 + x_4 - 2)^2 + 0.1\,(x_2 - x_4)$$

$$\mathbf{X_1}(x_1, x_2, x_3, x_4) = (-3, -1, -3, -1)\ , \ \ f(\mathbf{X_1}) = 19192.0$$

$$\mathbf{X}^* = (1, 1, 1, 1)\ , \ \ f(\mathbf{X}^*) = 0.0 \tag{1.14}$$

1.3 Constrained optimization problems

Exercise PP 8:

Consider the following problem:

$$\text{Minimize } f(x_1, x_2) = 2(x_1 - 2)^2 + 4(x_2 - 1)^2$$

subject to

$$2x_1 + 8x_2 \leq 6$$

$$2x_1 \geq 2x_2 \tag{1.15}$$