Teaching and Teacher Education 62 (2017) 60–70

Contents lists available at ScienceDirect

Teaching and Teacher Education

journal homepage: www.elsevier.com/locate/tate

Research paper

Mathematics teachers' motivations for, conceptions of, and experiences with flipped instruction

Zandra de Araujo*, Samuel Otten, Salih Birisci [1]

University of Missouri, 121 Townsend Hall, Columbia, MO, 65211, USA

HIGHLIGHTS

- Teachers flip because of colleagues' recommendations and potential student benefits.
- Teachers describe student collaboration as the most significant benefit of flipping.
- Teachers attend more to creating instructional videos than to planning class time.
- Flipped instruction shifts instructional interactions in mathematics classrooms.

ARTICLE INFO

Article history:
Received 20 June 2016
Received in revised form
22 October 2016
Accepted 29 November 2016
Available online 12 December 2016

Keywords:
Secondary school mathematics
Teaching models
Teaching practices
Flipped instruction

ABSTRACT

Flipped instruction in school mathematics has been occurring more frequently. This study investigated two teachers' motivations for, conceptions of, and experiences with flipped mathematics instruction. We found that the teachers were motivated to flip based on colleagues' recommendations and potential benefits for students. The teachers discussed changes to their instruction as a result of transitioning to flipped instruction and we interpreted those changes in terms of how teachers perceived the interactions among students, mathematics content, and themselves. Although the teachers viewed many of these changes as beneficial, some presented new challenges as they navigated their changing role in the classroom.

Recently, with the ever-increasing supply of online resources and technological tools, flipped instruction has gained popularity but empirical research has not kept pace with implementation. Thus far, research examining flipped classrooms has primarily featured examinations of student outcomes in flipped versus non-flipped classes (e.g., Clark, 2015; Coufal, 2014; DeSantis, Van Curen, Putsch, & Metzger, 2015; Zack, Fuselier, Squire, Lamb, & O'Hara, 2015). One limitation of these studies is that they have not adequately accounted for the variability that is possible *within* flipped classes (de Araujo, Otten, & Barisci, in press). Many studies refer to flipped instruction as a single approach to instruction, but understanding the variability in how teachers conceive and enact flipped instruction is crucial in order to understand how to support teachers in effectively enacting flipped mathematics instruction.

A second limitation of prior studies of flipped instruction is that they have focused on students' experiences and outcomes, not accounting for the teacher as the most central force in flipped instruction. Furthermore, many studies that examined teachers' changes in practice involved educational initiatives introduced by school districts, teacher educators, or instructional designers. Few studies have looked at teachers' decisions to change practice aside from external forces. Interestingly, flipped instruction is one area in which teachers often initiate the change themselves. Because of the personal initiative teachers take in flipping their classes, examining teachers' motivations for doing so is a worthy endeavor (Gorozidis & Papaioannou, 2014). Teachers are not only instrumental in enacting flipped instruction (Bergmann & Sams, 2012; Enfield, 2013) but they are often the initial proponent of the innovation, in contrast to many other initiatives that are driven by changes to policy or standards (e.g., Common Core State Standards). In other words, it is largely individual teachers who are deciding in

* Corresponding author.
E-mail addresses: dearaujoz@missouri.edu (Z. de Araujo), ottensa@missouri.edu (S. Otten), birisci@gmail.com (S. Birisci).
[1] Present Address: Artvin Coruh University, Faculty of Education, Department of Primary Education, 08000, Artvin, Turkey.

http://dx.doi.org/10.1016/j.tate.2016.11.006

increasing numbers to flip their classes, and thus it is important to know why they are making this decision.

The purpose of this study is to present two cases of mathematics teachers who have flipped their instruction. We explore their decisions to implement flipped instruction and their perceptions of its benefits and challenges with regard to student engagement, planning and preparation, and teacher-student relationships. We also examine the ways in which teachers perceive the interactions among themselves, students, and mathematical content to differ in flipped classrooms.

1. Literature review

1.1. A theoretical frame for instruction

Teachers are uniquely positioned as decision-makers with regard to the shift to flipped instruction while simultaneously being involved in implementing the instruction. To conceptualize instruction, we draw on Cohen, Raudenbush, and Ball (2003) and the instructional triangle (see Fig. 1) wherein teaching is defined through the interactions that occur among and between teachers, mathematical content, and students. Furthermore, each of these aspects and their interactions occur within various environments (e.g., classroom, school). Cohen and colleagues argued that these interactions occur regardless of the classroom format: “The interactions occur in distance learning, small groups in classrooms, informal groups, tutorials, and large lectures” (p. 122). Thus, although this particular conceptualization of instruction was created before the widespread implementation of flipped instruction, we argue that it is appropriate to extend its use to that format.

Students not only interact with one another, but also with the teacher. These interactions may be student or teacher led, but are conducted with the purpose of facilitating students in understanding the content. The students interact with the content through the tasks and experiences the teacher selects or designs. In a non-flipped class, the teacher may lecture, and witnessing the lecture is the experience through which the students interact with the content. Similarly, in a flipped class, lecture videos may serve as a proxy for the teacher's live lectures. Beyond lectures, there may also be investigations or problem sets, selected or designed by the teacher, through which the students have opportunities to learn mathematical content.

1.2. Considerations for teachers flipping their classes

With regard to instruction in flipped classes specifically, various authors have described different instructional principles (e.g., Chen, Wang, Kinshuk, & Chen, 2014; Hamdan, McKnight, McKnight, & Arfstrom, 2013; Strayer, Hart, & Bleiler, 2015). Overall, the approach to instruction in flipped classrooms typically involves shifting content delivery from in class to outside of class. Thus we can hypothesize that teachers' adoption of flipped instruction might be related to the affordances of individualized, technology-based content delivery or to the new possibilities for the use of class time. Indeed, some advocates of flipped instruction have positioned it as an exemplary use of instructional technology (Flipped Learning Network, 2012) and others have pointed to the potential for student collaboration in class (Bergmann & Sams, 2012; Hamdan et al., 2013).

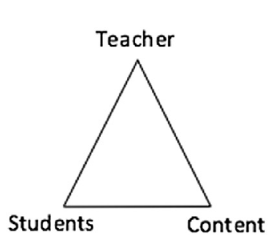

Fig. 1. Instructional triangle adapted from Cohen et al. (2003).

Empirical research on flipped instruction, however, is still in its infancy, with a preponderance of anecdotal work and self-studies (e.g., Bergmann & Sams, 2012; Southmayd, 2014; Tucker, 2012). A common theme among the accounts of flipped mathematics instruction is that teachers flip in the hopes that students develop deeper understandings of mathematics (Bergmann & Sams, 2009; Ford, 2015; Fulton, 2012; Hamdan et al., 2013; Lage, Platt, & Treglia, 2000; McGivney-Burelle & Xue, 2013; Sickle, 2015). For example, Strayer et al. (2015) decided to flip their college precalculus courses in order to foster a deeper understanding of the mathematics content with students. In analyzing their implementation of flipped instruction, the instructors did perceive students as developing a greater understanding of content and also the instructors developed greater insight into students' thinking. Teachers also perceived that flipped instruction can help maintain consistency among multiple sections of a course in terms of pacing and content because teachers of the various sections can send the same video home for students to watch (Strayer et al., 2015).

Teachers are also choosing to flip their classrooms in the hopes that students gain more positive attitudes towards mathematics and are more engaged (Chen et al., 2014; Moore, Gillett, & Steele, 2014). This increased engagement is possible because students are together as they work on mathematics, allowing for collaboration, and flipped instruction increases the amount of class time available for work. Proponents of flipped instruction even suggest that “different subgroups might benefit from the student-centered support from both the teacher and fellow classmates” (Hamdan et al., 2013, p. 8).

Although first-hand accounts of flipping note several benefits, these studies have also acknowledged potential challenges for teachers. A common challenge is ensuring students view the videos prior to class (Chen et al., 2014; Ford, 2015; Palmer, 2015). Teachers have also reported that the flipped instructional model increased their work load (Enfield, 2013; Talbert, 2014) because of needing to develop the videos (Ford, 2015; Wanner & Palmer, 2015). Although teachers could use pre-made videos, many choose to make their own videos and some teachers have reported that their students prefer teacher-made videos (Palmer, 2015).

Other concerns raised suggest that many potential benefits, such as active student collaboration, are possible in classrooms regardless of whether or not they are flipped (Hamdan et al., 2013). This suggests that flipped instruction may not be innovative or new, but rather an unneeded repackaging for how teachers might deliver effective, research-based instructional strategies. Others assert that flipped classrooms are not addressing broader problems such as an overloaded school curriculum (Southern California Public Radio, 2013) Moreover, issues of access and equity may be exacerbated because not all students have access to the needed technologies (e.g., internet) at home. Finally, there are fears that mass adoption of flipped instruction may lead to less skilled teachers being hired to provide students with generic lecture videos and then serve more as classroom aids than skilled professionals (Bergmann & Sams, 2012). Despite these concerns, teachers continue to adopt flipped instruction, which leads to the need for research to understand the ways in which flipped instruction is adopted and teachers' perceived outcomes from this instructional model. By understanding teachers' motivations, teacher educators and researchers might be better able to leverage these motivations in supporting effective instructional practices.

2. Method

In this study we were interested in understanding how teachers came to flip their classrooms. We also wanted to know the ways in which the teachers conceptualized flipped instruction in terms of the resources they drew upon and the interactions among themselves, students, and the mathematics. We examined the following questions:

1. What motivated the teachers to flip their mathematics instruction?
2. How do the teachers describe their enactments of flipped mathematics instruction?
3. What challenges and benefits do teachers perceive from their enactments of flipped instruction?
4. In what ways do teachers conceive of interactions among teachers, students, and content in flipped instruction and how does this align with (or differ from) non-flipped instruction?

These questions were examined in the context of secondary and post-secondary mathematics because, not only is this our particular area of interest, but it is also one of the most common domains in which flipped instruction is occurring (Flipped Learning Network, 2012).

2.1. Participants and context

Once we received ethical approval from our university's institutional review board, we recruited teachers for our study. Two teachers participated in this study, Ms. Temple[2] and Ms. Schaefer. We invited Ms. Temple to participate in our study because we learned through mutual acquaintances that she had been implementing flipped instruction for a few years. Ms. Temple has a Bachelor's degree in mathematics with an emphasis in education and a Master's degree in secondary school administration. She had been teaching for 18 years, 5 of which involved flipped instruction. She stated that she had always tried to incorporate technology into her teaching and had been active for many years presenting at regional conferences. Each teacher and her participating students provided informed consent in adherence with our institution's ethical standards.

We focused on one section of Ms. Temple's 8th grade mathematics course (20 students aged 12–14 years) that she taught in a public middle school. She was flipping this course for the fifth time. Ms. Temple partially used the Holt *Algebra 1* textbook as a class resource in addition to her personally-developed iBook (For details on Ms. Temple's iBook design, see de Araujo et al., in press.).

Ms. Schaefer contacted us and offered to participate in our study when she learned of our intent to study flipped instruction. Ms. Schaefer has Bachelor's and Master's degrees in mathematics education and had been teaching mathematics for 14 years (5 years at the secondary level, 9 years at the post-secondary level). She also had experiences working on revisions to a middle school textbook series.

For this study, we focused on Ms. Schaefer's college algebra course that she taught in a computer lab at a community college. The course was an 8-week, hybrid course. This meant that the duration was half that of a traditional semester-long course and the course was a blend of traditional face-to-face meetings and online instruction, including MyMathLab online homework (a resource from Pearson Education Inc.). Although she had taught the course in a non-flipped manner several times, the data described below came from her first semester flipping. There were 24 students in Ms. Schaefer's flipped college algebra course, about half of whom had been in her intermediate algebra course during the prior 8-week term. The students ranged in age (18 years and older) and career goals, however Ms. Schaefer pointed out that for many of her students this was their terminal mathematics course. Both teachers were in the Midwest region of the United States.

2.2. Data sources and analysis

This study is part of an ongoing project examining teachers' conceptions and enactments of flipped instruction. The larger project sought to provide more nuanced understandings of teachers' *enactments* of flipped instruction and included teacher and student surveys, teacher interviews, instructional artifacts such as tasks and videos, and lesson observations of multiple teachers. For the present study we were particularly interested in teachers' conceptions of flipped instruction and we chose to focus on two particular teachers who taught content that was relevant to secondary mathematics (as opposed to elementary mathematics or undergraduate mathematics) and for whom we had complete data with regard to their motivation for and conceptualization of flipped instruction (as opposed to simply data about how they implemented it on particular days). In these ways, Ms. Temple and Ms. Schaefer were similar but it was also interesting that they had different backgrounds and experiences flipping. Furthermore, because the present study addresses the teachers' perspectives on flipped instruction, we did not examine the lesson observations and instead focused our analysis on the teacher surveys and interviews.

The surveys consisted of both open-ended and Likert-type items. We designed the surveys to collect the teachers' background information, particularly with regard to flipped instruction (e.g., How long have you taught using flipped instruction? Discuss what led to your use of flipped instruction), their views on what constitutes flipped instruction (e.g., Describe a typical day in your flipped class), and also their thoughts regarding their experiences with various technologies (e.g., What is your comfort level with video-editing technology?) and the potential benefits of flipped instruction (e.g., In general, discuss the ways, if any, that flipped instruction supports your struggling learners).

With Ms. Schaefer we administered the survey and conducted an initial interview about flipped instruction in general and then conducted three cycles of observation. For each cycle we audio-recorded a pre-interview to allow Ms. Schaefer to describe the lesson we were observing. We were particularly interested in understanding the instructional decisions she made related to the flipped format. After the lesson observations we conducted post-interviews to allow Ms. Schaefer to discuss her thoughts on the lesson. We also collected copies of Ms. Schaefer's instructional videos and any tasks she used in class. With Ms. Temple we also collected the survey data and conducted an extended interview about flipped instruction in general, but because Ms. Temple's lesson observation data was collected from a distance, we did not conduct pre- and post-lesson interviews. We did, however, collect copies of her instructional materials, including her instructional videos and tasks. In both instances, the teachers' instructional materials were used to confirm their description of the materials during the interviews and on the survey.

We conducted a thematic analysis of the data. First, two members of the research team identified sections of the interview and survey data in which the teachers discussed flipped instruction, including instances where they mentioned non-flipped instruction as a way to compare or contrast with flipped instruction. We first broadly separated all the interview excerpts and survey data into three categories: teachers' motivations for flipping their

[2] All names are pseudonyms.

instruction, teachers' vision for flipped instruction, and outcomes from flipped instruction. For example, in post-interview 1, Ms. Schaefer said,

> Until you start trying to do them on your own and that's when you struggle. And that's the purpose of flip in my opinion. [It] is—I'm there when you struggle. I want to be there to help you through this and maybe some guided questions with this worksheet of things that I know you're going to struggle with and then that will maybe make sense, I don't know.

Because she was describing what she perceived to be the purpose of flipped instruction, we coded this as teachers' vision for flipped instruction.

Following this initial round, the research team open coded data in each of these three sets using analytic memos with regard to the particular aspects of instruction the teachers discussed. For example, teachers' motivations for flipping their instruction was separated into outside influences and internal motivations. Teachers' vision was separate into flipped at-home work, flipped in-class work, video use in class, resources for flipped, and flipped benefits. For the outcomes from flipped instruction, we distinguished among benefits and struggles. These analytic memos were compiled to answer research questions 1–3. Finally, we interpreted all the themes through the instructional triangle. This meant that for each of the three categories of data, we distinguished among the teachers' thoughts related to their roles and interactions with students and content and their interactions with students and students' interactions with the mathematical content in flipped classrooms. We also attended to the ways in which they described differences in these interactions in flipped classrooms as opposed to non-flipped classrooms. In these ways, we addressed research question 4. To structure the presentation of findings, we address research questions 1–3 in sequence and within each sub-section we make connections to the instructional triangle that is entailed in research question 4.

3. Findings

3.1. Motivations for flipping instruction

Ms. Temple and Ms. Schaefer's initial forays into flipped instruction were catalyzed by similar events. Ms. Temple began flipping her class after learning about several high school teachers' experiences flipping their classes. She discussed a particular encounter with a fellow mathematics teacher as an impetus for flipping.

> He just said, “Hey, I think you should try this. It's really cool how much time you get with your kids in class. I think you would love it.” And so, after he bugged me for a couple months, I said, “Alright, what the heck! I'll try it.” So I tried it with my advanced class and I was like, ”Oh my gosh! This is so cool!” Because, me talking in the front of class and then you bore yourself so many days that you know your kiddos are poking their eyeballs out. … And, I did [flip]. And, was like, “Oh my gosh. I can talk to these kids and physically see what they are doing.” (Ms. Temple, Initial Interview)

Similar to Ms. Temple's experience described above, Ms. Schaefer began flipping her class after hearing about the benefits of flipping from other teachers.

> I have heard about it [flipped classrooms] for a while and attended several conferences where there were speakers on flipped classrooms. It seemed like there was a huge range of ideas on what flipped classrooms really are, but I liked the idea of having time to really deepen the understanding of math and make the real world connections that you typically do not get to do. (Ms. Schaefer, Survey)

Whereas Ms. Temple discussed her desire to increase her interaction and engagement with students as a main source of motivation for flipping her classroom, Ms. Schaefer found appeal in the prospect of helping students develop deeper mathematical understandings. It is interesting to note that both teachers had attended professional conferences in which teachers reported on the benefits of flipped instruction.

With regard to the instructional triangle (Fig. 1), the teachers' motivations for flipping centered on a change in the teacher-student interactions. In particular, they described potential benefits that could arise from increased class time engaging with students in a manner different from the traditional speaker-receiver roles common in mathematics classrooms. By moving lectures outside of the classroom, Ms. Temple and Ms. Schaefer anticipated more flexible time in class for teachers to take on a new instructional role. As in prior studies on flipped instruction (e.g., Bergmann & Sams, 2009; Ford, 2015; Fulton, 2012), the teachers' motivations for flipping their instruction arose mainly from perceived benefits to students and they expressed those potential benefits in light of limitations to non-flipped instruction. The teachers also referred to a desire to change relationships to content. In other words, part of the appeal of flipped instruction was an opportunity to redress some drawbacks of traditional non-flipped teaching with regard to the richness of content.

3.2. Teachers' conceptualizations of flipped instruction

Although the teachers had similar reasons for flipping their classes, their descriptions differed with regard to how they would enact flipped instruction and what this instructional model would afford them and their students. In the following sub-sections, we present an overview of each teacher's conceptualization of her flipped as well as the reasons for these decisions.

Ms. Temple. Ms. Temple described flipped instruction as “a method to deliver the basic content outside of class (through video, reading, explorations) and then work together and dig deeper in class together” (Survey). Initially, Ms. Temple flipped her class without assigning videos for students to view at home. Instead, she would send students home with guided notes to complete while reading their textbook. Several years ago she started also assigning videos and guided notes to further support students' understanding of mathematical concepts.

Ms. Temple produced her homework resources in iBook format. Because her school had a one-to-one iPad program, this allowed all of her students to download the iBook at school and access it at home, even if they did not have internet access.

> So I started in [20]11–12 with no video, and then, the fall of 2012, I had a student teacher. And we were just kind of doing the reading thing, and she went to a conference and really wanted to add video, and I said, “Okay, my only want is that we make it so that no internet is required.” And that's how I found out about iBooks. (Ms. Temple, Initial Interview)

Ms. Temple's iBooks included written content regarding particular concepts, worked examples, quizzes, and videos in a single file. Ms. Temple's decision to include videos as part of students' homework was contingent upon all of her students having access to the videos at home. She found that iBooks satisfied this criterion and they also allowed her to include interactive experiences for students (Fig. 2). "When they open that [iBook] up it's got videos, it's got explanations, it's got other little widgets in it for them to use" (Initial Interview). Ms. Temple thought it important to allow students to read about mathematical concepts and have opportunities to solve problems that further reinforced those concepts. The video embedded within the iBooks provided both explanations and worked examples. When asked to describe her videos, which she created in Explain Everything (2011), she stated, "My videos explain the basic math concepts. They are not trying to get at the deeper parts of the math but the surface level stuff" (Survey). She chose to create the videos herself because she thought it "important that my students hear me" (Survey). She thought the iBook homework freed up in-class time that could best be used to allow students to develop deeper understanding of the concepts introduced in the videos.

Ms. Temple envisioned her flipped mathematics lessons beginning at home. The students were to read through the iBook at home and work through the accompanying notes. Ms. Temple had created a typical format for this process as explained below.

> So the typical day starts out with what they do outside of class to prepare, and that would be they either watch a video, read in their textbook and kind of draw some conclusions, or find an example, or go through an exploration … And then, at some point in time in the notes it will flip to "play your video." … that video is more the road to mathematics, more the basic skills. It's me talking, doing examples that they have right in their guided notes and them doing it with me. And then after I do an example or two or explain something, I will ask them to try it on their own. Then I will say pause the video…then play the video again when you are ready to check … At the end of those notes, there is a spot where they have for the summary and to write a question. The summary is just one or two sentences, "What did you learn? What do you remember? What do you recall?" … Or if they don't have a question, they are supposed to write a question to ask their group the next day to challenge them. (Initial Interview)

Ms. Temple created this format because she thought it was important to show students ideas and processes but thought it was also important for students to practice these processes on their own. She also thought that coupling student activity with the iBook would provide a solid foundation upon which the subsequent class time could build.

When students arrived to class, Ms. Temple expected them to work independently on an opening problem. The opening problem might serve as a review of a prior topic or reference a specific example from the prior night's homework. Next, the students, who were in desks grouped by four, begin work on what Ms. Temple referred to as "team discussions." These discussions served as the main portion of in-class activity and included tasks at a variety of levels of difficulty related to the prior night's homework. The students were encouraged to work as a team throughout this portion of class time. Students would then work on "old school math practice" typically consisting of exercises from their textbook. Throughout the discussions and practice, Ms. Temple checked in with students and answered questions. The class typically ended with a brief quiz covering the day's topics. Although this format was relatively standard from lesson to lesson, Ms. Temple stated that the practice may occasionally be replaced with applications or games.

In summary, the at-home portion of Ms. Temple's flipped class involved most prominently the interaction between students and content as students received explanations and tried examples based on what they had been shown. The teacher was also present at home vicariously through the videos embedded in the iBook. Throughout the in-class portion of her lessons, there were conventional segments where students worked individually on content as directed by the teacher but Ms. Temple also regularly planned collaborative activities. In these cases, Ms. Temple

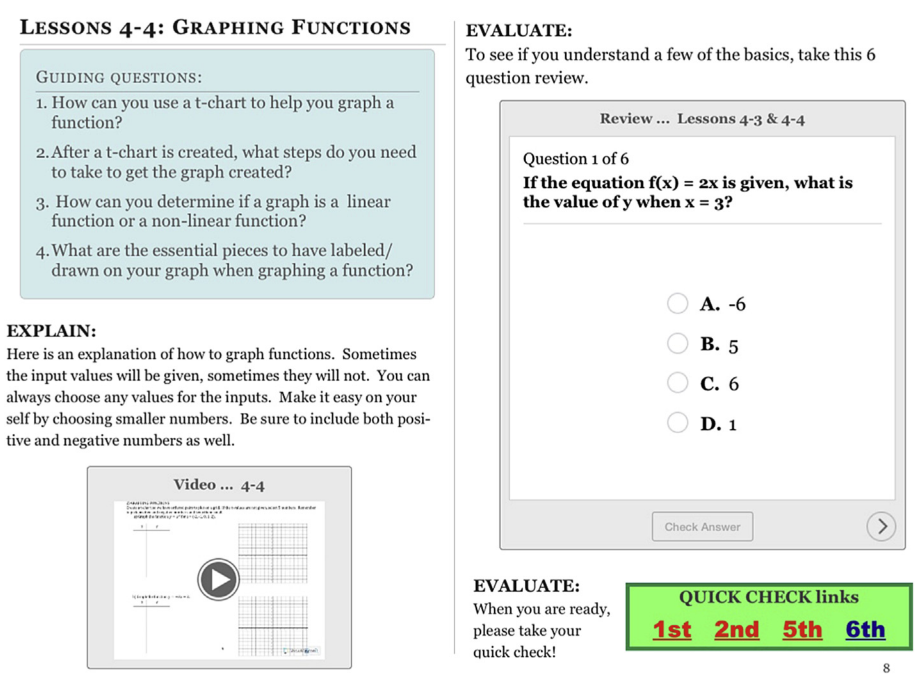

Fig. 2. A screenshot from one of Ms. Temple's iBook homework assignments with an embedded video in the lower-left corner.

described her role as that of a facilitator. She described flipped instruction as allowing her to give her students more freedom because she doesn't "mandate the order necessarily in class." She explained that it also allowed for more student-to-student interaction rather than relying on her.

> Before I flipped, it was me delivering, saying, "This is what this word means. This is how these pieces go together. This is how you solve an equation. Solve these steps. Copy down these notes. Try this problem." And so I was more, "Here it is." … Now [with flipped instruction] I feel like the whole hour I get to have small discussions … My conversations now are so much more targeted than they were last time. I would just think I was hitting the right things that they didn't understand by just guessing. But now my help with them and my conversations with them is way more accurate, because they come in with a little bit. (Initial Interview)

Discussions such as this suggest that although Ms. Temple described her role in flipped instruction as more supporting than leading, she also thought that the flipped format allowed her to have more meaningful interactions with students.

Ms. Schaefer. Ms. Schaefer provided her definition of flipped instruction in the survey:

> Students watch prepared videos or reading assignments prior to coming to class. Then we use class time to really dive into the material and get a better understanding of the concepts. We get a chance to use critical thinking skills to understand and make connections with the mathematics. (Survey)

Thus, Ms. Schaefer conceptualized her flipped lessons occurring in two connected phases: at-home work and in-class work. Ms. Schaefer's homework consisted of approximately 2–4 instructional videos that the students would watch before each class. Although she sometimes used videos provided by the textbook publisher, Ms. Schaefer preferred to create her own videos because she thought the publisher videos lacked explanations. She also made her own videos to include her voice and personalize the delivery for her students.

Unlike Ms. Temple, Ms. Schaefer did not present expository text or student problems in conjunction with her videos, but like Ms. Temple, she created the videos using Explain Everything (2011). She then posted them to YouTube. The videos typically consisted of Ms. Schaefer discussing a key process or procedure and talking through worked examples. Initially, Ms. Schaefer had hoped to keep the video duration to 10 min. However, the videos we collected from her were often closer to 20 min. She thought that part of the reason for the longer length was because of the commentary that she added in.

> It's [her commentary] like what I would say in class …"Hey, I know you are going to have a pitfall here. Hey, make sure you pay attention to this." And I think I'm saying too much of that in my videos and they are longer than what I really want them to be. (Pre-Interview 1)

Ms. Schaefer expected students to watch the videos at home or in the computer lab prior to class, taking notes or recording questions as they saw fit. She described her directions to students in an interview.

> I told them it's basically as though I were talking to you. If you were in class, what would you be doing? Taking notes, writing things down, important things that I say. Same concept [for the flipped videos]. Instead of me talking now, I'm talking at home to you. (Post-Interview 2)

In some instances, Ms. Schaefer supplied a PDF document containing a worked example in lieu of or in conjunction with a video.

For the in-class time, Ms. Schaefer initially set a routine schedule that involved splitting the 90-min class into three 30-min sections. She explained the purpose of the segments.

> So 30 minutes of questions [about the video homework], 30 minutes of this activity that I have, this worksheet that they are going to work through, and then 30 minutes of them working on [problem sets] and asking if they have any questions. (Pre-Interview 1)

Over the course of the semester, Ms. Schaefer found the first 30 min to be a "waste of time" because the students were not consistently watching the videos. She seemed justified in this thought during our time observing her because the view counter on her videos was less than the number of students in her class. To address this issue, Ms. Schaefer replaced the first segment with a brief "video quiz" in an attempt to incentivize watching the videos, and then she divided the remaining time between the two remaining segments (worksheet and problem sets).

The worksheet portion of the class centered on tasks Ms. Schaefer created to address key ideas from the videos. Students were welcome to work with partners or groups on these worksheets but they were not grouped together nor required to work collaboratively. Finally, the problem set portion of class involved students completing online practice problems provided by the textbook publisher. These problems had to be completed individually but students could advise one another or compare and discuss answers as they worked. Many of the students completed these individually, in part because they often progressed through the problem sets at different paces.

Overall, the at-home portion of Ms. Schaefer's flipped class entailed the typical instructional triangle as students were presented with mathematical explanations and expected to record notes about the content, and the teacher was present in the form of audio commentary on the videos. The teacher and students, however, were not able to interact dynamically as they would if they were physically together during the content presentation. For the in-class portion, the students interacted primarily with the content through the worksheets and problem sets and they also had opportunities to interact with each other as they worked. Ms. Schaefer's role focused on responding to students' questions that arose as they worked on the worksheet or practice problems. In general, she did not plan to provide whole-class instruction unless a difficulty arose and she was unable to "get around to everyone" and so she drew the class's attention to one or more big ideas on a worksheet. She described her role in her flipped classroom as a substantial departure from her non-flipped classes. The teachers' changing roles and interactions with students framed much of the teachers' perceptions of the benefits and challenges of flipped instruction, as we discuss in the following section.

3.3. Benefits and challenges of flipped instruction

Both teachers described benefits and challenges stemming from flipping their instruction. Note that these perceptions were made in the midst of flipping (Ms. Temple in her 5th year, Ms. Schaefer in reflecting upon lessons she had just carried out), not beforehand. In this section we discuss both the benefits and challenges the teachers described resulting from their shift to flipped instruction.

Benefits. According to Ms. Schaefer and Ms. Temple, the most significant benefit of flipping their instruction was increased collaboration among students. The teachers specifically discussed an increase of student-to-student talk compared to their prior non-flipped teaching where much of the mathematical talk had been between themselves and some students. Ms. Temple explained,

> In my traditional class they (observers) wouldn't hear kids talk as much. They come in my flipped class, kids are talking *all* the time about math. In the traditional, my tables were more facing the front. … They'd walk in now, kids are in groups… They would notice now, kids helping each other all the time … In my traditional, they would see more hands waiting, and kids not going to each other for help as much. (Interview)

Ms. Schaefer echoed this sentiment in describing the main difference, and benefit, of her flipped classrooms as "collaboration, definitely collaboration" (Post-Interview 1). The teachers thought the increased collaboration was beneficial because students did not have to wait for the teacher to help them. Instead, their peers often served in this role. Ms. Schaefer discussed this benefit of students helping one another but also noted a related benefit that, during the expanded in-class work time of flipped instruction, she was more available to students when they struggled. In a non-flipped format, those struggles often happen when the student is outside of class and the teacher is unavailable.

The teachers largely ascribed the collaborative benefits to the change in their role from that of a lecturer to more of a facilitator. Because the flipped classroom had students working on mathematics tasks in class, the dynamics shifted from the students as listeners to the students as doers of mathematics. The teachers also thought that the students were more confident talking in class because they had a preliminary understanding of the topics due to the video homework. Although it is possible that the teachers could have increased student collaboration in their non-flipped classes, both teachers acknowledged that by moving the content delivery outside of class, they were able to foster the conditions for increased student collaboration.

Both teachers noted that increased collaboration connected with increased student engagement. Ms. Temple stated, "I think they are more engaged with flipped" (Initial Interview). This engagement, according to the teachers, related to students being more active in their work and taking on more responsibility for learning. Ms. Schaefer explained in the following excerpt:

> Ms. Schaefer: I had several of these students in the first 8 weeks in intermediate algebra [a non-flipped prior course], and they are now talking with other students where they never would before. And I would try to talk to them and they would go into their shell and so there was a couple of students that went to their shell the first 8 weeks that I'm seeing coming out of that shell a little bit the second 8 weeks [which is the flipped class] … with the flipped classroom they're coming out of their shell and getting help and working on stuff.
>
> Interviewer: And do you attribute the flip for being part of the cause of that?
>
> Ms. Schaefer: I do. (Post-interview 1)

Ms. Schaefer noted that, because her students were not what she referred to as "math people," the increased engagement she saw was particularly exciting for her to witness.

In addition to changes in classroom interactions, the teachers also described benefits stemming from the video homework. The teachers noted that because students watched videos prior to attending class they were more prepared to engage in the mathematics tasks. Ms. Schaefer explained:

> Where in a traditional classroom they come in with a blank slate and so they can't really contribute during class time, where these students watched the videos before and so they are helping other students and they're saying–, they're using math words. (Post-Interview 1)

Ms. Temple described similar benefits in her flipped classroom, "My conversation with them (students) is way more accurate because they come in with a little bit (of knowledge). So when I ask them what don't they get, they're telling me the specifics" (Initial Interview). The teachers thought that by watching the videos, the students had more prior knowledge on which to draw and this led to their increased and more substantive contributions in class. Ms. Schaefer noted that students were better able to explain their thinking after watching the videos and that students also had notes and videos to turn back to if they were unsure of an answer. Thus, the at-home portion of the flipped classes was allowing students to have some initial experiences with content that the teachers ascribed as a benefit to what they were able to accomplish when they came to class. The other main benefit they saw was the increased student-to-student interactions in class, with the teacher able to intervene strategically with students and content rather than having to deliver the entirety of the content in class.

With respect to content delivery, Ms. Schaefer stated that because the videos covered content the night before, more class time was free for students to delve more deeply into the mathematic. Ms. Temple also mentioned having more time in class which she used to focus on her "bottom 10%" (i.e., low-performing students). In general, both teachers found the additional time in class beneficial, however, the increased class time also created some challenges.

Challenges. The most significant challenge the teachers discussed with regard to flipped instruction was the time required to plan lessons and develop resources. Whereas the students had the benefit of increased time to work in class, the teachers' personal time was taxed because of the effort required to develop lesson resources, especially the videos. Ms. Schaefer frequently referenced the copious amount of time she spent creating her lecture videos. For example, in the initial interview she remarked, "it is taking me *hours*, HOURS, plural, to come up with these videos." Much of her concern about time may be because she was flipping for the first time. Ms. Temple admitted that an early barrier to flipping her instruction was time and it was not until her student teacher helped create the videos that she fully flipped her class. Following the initial time investment in creating her flipped instructional resources, Ms. Temple stated that the time needed to plan her course was similar to that of a non-flipped course. Ms. Schaefer also acknowledged that she thought her first time flipping would require significantly more planning time than subsequent implementations. Though both teachers described the process of shifting to flipped instruction as time consuming, the opportunities afforded by flipped instruction served as encouragement to continue. This was particularly evident in Ms. Temple's case because she had been flipping for five years and was adamant that she would never return to traditional instruction.

In light of this commitment, it was not surprising that Ms. Temple did not identify many challenges with flipped instruction. She and Ms. Schaefer did acknowledge, however, that one challenge was ensuring students watched the videos. Ms. Temple saw this challenge as similar to addressing students who did not complete homework in a traditional class, remarking "that kid that doesn't do anything outside of school, they still don't do anything outside of school. It's (flipped instruction), not magic pixie dust." Ms. Schaefer, however, did think that students not watching videos was a significant challenge of flipped instruction. In an attempt to address this concern, Ms. Schaefer experimented with video quizzes meant to hold students accountable for viewing the videos. Even with these quizzes, she still found that students were not consistently watching the videos before class. She acknowledged that the length of her videos might be contributing to fewer students watching them and was trying to reduce the length. Holding students accountable for watching videos continued to be a challenge for Ms. Schaefer and she was eager to keep trying new strategies to increase students' homework participation.

Although Ms. Temple repeatedly credited flipped instruction with allowing her to better know her students "both mathematically and non-mathematically," Ms. Schaefer faced challenges in this regard because she found herself less connected with students in her flipped class and described this as a significant challenge of flipped instruction.

> I feel like I am losing the connection with my students. When I was lecturing I could look around the classroom and see their faces to determine whether they were getting the concept or not. Now I just hope they get the concept or ask me if they are struggling with the material. (Survey)

Ms. Schaefer remarked on this outcome a number of times. She also acknowledged that she could probably use the additional class time more effectively saying, "I don't feel like I'm capitalizing on that in-class instruction time. I feel like I need to do a better job of that… Because I think if that was awesome, I'd be super sold on it [flipping]" (Post-interview 3). She hoped that learning to better use class time would help her regain the relationships with students she felt were lost her first time flipping.

In summary, there were some notable patterns that arose across these benefits and challenges identified by the teachers. The main benefits—student engagement and collaboration, more substantive student contributions, and additional class time available for mathematical work—were benefits to *students*. In contrast, the major challenges—needing ample planning time, trying to assure students watched the videos—were framed in terms of challenges for the *teachers*. In the following section, we further discuss these findings and the resulting implications in terms of the overall shifts in the instructional triangle.

4. Discussion

Ms. Schaefer and Ms. Temple's flipped classes shared many aspects. Both teachers began flipping their classrooms in large part because of encouragement from other educators and for the potential student benefits. They had also conceptualized their use of in-class and at-home contexts similarly. Though both teachers described comparable benefits and challenges from their enactments, they did perceive different outcomes with regard to their relationships with students. To gain a more nuanced understanding of how flipped instruction might impact instructional interactions, we discuss the differences between flipped and non-flipped classrooms in terms of the instructional triangle (Cohen et al., 2003).

In examining the classrooms through the instructional triangle we see some interesting shifts when comparing non-flipped and flipped instructional models. In the following sections we discuss the different interactions present in flipped and non-flipped classrooms. To do so, we address the instructional purposes (content delivery or practice/application) across both contexts (in class or at home).

4.1. Content delivery

In non-flipped classrooms, the content delivery occurs in class and all three components of the instructional triangle (Cohen et al., 2003) are present—the students, the teacher, and the mathematics content. Because flipped instruction is often described as simply switching the class context with the home context, one might expect that the instructional triangle would remain intact for content delivery at home. In fact, this is not the case with flipped classrooms as the teacher has been removed from the content delivery portion of the lesson, replaced with the instructional resources (lecture video, multimedia etc.) she selects or creates. Thus, for those teachers who utilize the home context for content delivery, the instructional triangle is fundamentally altered.

We represent the at-home situation in a flipped classroom as a triangle with an extended dashed segment (Fig. 3). The new node, video/media, refers to various media types such as text, images, sound, and of course video. The new dashed segment connecting teacher and video/media represents the time that teachers spend creating videos for at-home use and the teachers' attempt, through these videos, to maintain a virtual connection to their students during content delivery. Their stated desires to be a visible part of those videos is similar to what instructional design scholars have called "social presence" in online education (Garrison, Anderson, & Archer, 2000), and it has been found to be an important component of students' at-home learning experience. This academic notion of social presence connects with the personal desires of Ms. Schaefer to make her videos reflective of her personality and Ms. Temple to make her at-home resources more interactive. Nonetheless, the teacher's at-home connection is indirect, which may be part of why Ms. Schaefer, for example, felt a disconnect from her students relative to the non-flipped teaching where she could "see their faces" during lecture.

Although the teacher is one step removed during flipped content delivery, the videos and multimedia are introduced, which have different affordances and constraints. For example, videos can be paused and replayed as needed on an individual basis, and teachers can embed interactive features such as those used by Ms. Temple (de Araujo et al., in press). Incorporating such features, however, may require even more time commitment from teachers and may also draw on different skills (e.g., technological savvy, digital design) than preparing a successful lecture in the non-flipped in-class content delivery. Therefore, the flip is not merely a change in instructional context for content delivery but an important shift in the instructional triangle that implicates benefits and challenges.

Also notable is that the teacher's role in delivering content in a non-flipped class is more dominant that in a flipped classroom. We indicate this in bolding the teacher in the non-flipped, content delivery triangle. In non-flipped classrooms as described by the teachers, the teacher is setting the pace through which the information is conveyed and actively steering the students in their activity with the content. Teacher-dominant non-flipped classrooms are frequently the norm in secondary mathematics (Stigler & Hiebert, 1999). At-home in a flipped classroom, however, students are able to watch the videos at their leisure and have more control over the pace of the lesson. Thus, students are more prominent in the instructional triangle but it can also mean that students may not necessarily watch the videos, which was a key challenge for the teachers in this study.

Another aspect of note related to content delivery is that in non-flipped classes, collaboration among students is possible, though

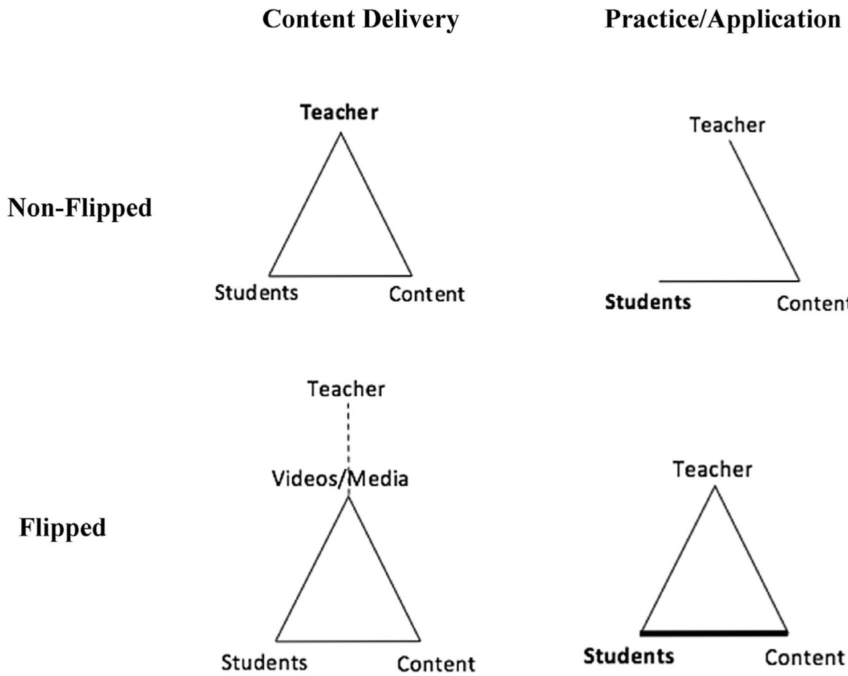

Fig. 3. Instructional interactions in flipped and traditional classes.

not always capitalized upon, as they learn new content. In a flipped classroom, the content delivery can occur for students on an individual basis. While this may allow students to set the pace of their learning, this may also make it more difficult for students to work together as they are receiving the content delivery.

4.2. Practice and applications

In flipped classrooms the practice/application occurs in class whereas in a non-flipped classroom much of this work occurs at home. Comparing these two situations (Fig. 3) reveals that the instructional triangle differs between flipped in-class work and non-flipped at-home work, though the teachers' goals for each were similar—for students to practice the procedures, deepen their understanding, create connections to applications of knowledge, and reinforce the ideas presented during the content delivery phases.

In a non-flipped, at-home context, the teacher and student do not interact directly: rather, the main interaction is between the student and content as the students commonly complete practice problems. In contrast, the teachers' use of class time for practice/applications in flipped classrooms included all aspects of the instructional triangle. In a flipped classroom, the teacher is present in real time to support and guide students as they practice or apply the knowledge and skills developed in the content delivery portion of the lesson. Another change is that students had greater access to other students when completing tasks, thus student-to-student interaction could be developed in ways not necessarily possible in a non-flipped, at-home context. One thing that notably remained consistent between both teachers' flipped and non-flipped teaching was the mathematics content, including the curriculum. The teachers did not report selecting different tasks for students in flipped; they used existing curriculum materials.

The teacher's in-class role in flipped instruction was more reactive than in a non-flipped classroom. The teachers' described being responsive to students' needs and questions rather than steering the pace and content of the classroom. Conversely, the students took on a more proactive role in terms of deciding the pace and their interaction with the materials. The teachers noted that the students in flipped classrooms come to class with prerequisite knowledge upon which they could build. The teachers perceived this as benefitting students and allowing them to be more active in class. The students' interactions were also greater with one another than with the teacher during in-class flipped instruction. We illustrate these shifts in Fig. 3 with bold emphasis. Although the teacher maintains strong connections when deciding the content and resources to use in class during flipped instruction, the teachers perceived the connection between them and their students to differ from what was typical of their non-flipped classrooms. In Ms. Temple's case, she perceived this to be a strengthening of that link between teacher and students, though still more reactive as the students were more deeply connected with the content and resources and only asked for her help as needed. In Ms. Schaefer's case, she perceived the student-teacher connection as weaker than in her traditional classroom. In both cases, the teachers now perceive the students to have much stronger connections with the content, resources, and one another, however the teacher's connection to the students is changed. Our modified instructional triangles are an amalgamation of the teachers' experiences. Thus, although the exact relationships between teachers and students differ for Ms. Temple and Ms. Schaefer, overall, our representations capture the changes we found in general interactions as teachers shifted from non-flipped to flipped instruction.

5. Implications and conclusion

Teachers' motivations and visions for flipping their classroom align with much of the literature on effective mathematics teaching (increased collaboration, discourse, deeper understanding, etc.). We note that many of these aspects would be carried out in class; yet, much of the teachers' planning time and attention in flipped classrooms is not on the in-class activity but on the at-home activity. Though the teachers' attention to the at-home resources is necessary because the home context is where the content delivery has shifted, the same attention was not given to the in-class context of their flipped classrooms. However, in-class time is arguably very important (de Araujo et al., in press). Class time is when students are present and so teachers can foster rich communication within and among students and mathematics. Furthermore, the teacher is present in class to help guide and support student activity. In not carefully considering the use of class time, teachers who flip their classes might experience what Ms. Schaefer referred to as an "emporium model" of teaching in which students are working individually at their own pace. Although some may perceive this as a benefit of flipped, this model of instruction does not align with many aspects of effective mathematics instruction as advocated for in the literature, such as communication and mathematical discourse (Herbel-Eisenmann, Steele, & Cirillo, 2013).

Teachers would benefit from knowledge and support related to the creation of high quality video and multimedia resources. The teachers spent a significant amount of their lesson planning efforts on developing the at-home resources for students. This was sensible because in flipped classrooms the teacher is replacing themselves with the videos as the source of content delivery. Thus, videos should be carefully planned, developed, and delivered. Supporting teachers in this effort could connect to other initiatives to support teachers' technological knowledge such as Technological Pedagogical Content Knowledge (Koehler & Mishra, 2005). Furthermore, though not characteristic of Ms. Schaefer and Ms. Temple's classrooms, teachers could conceptualize flipped instruction outside of the typical exposition-practice format of mathematics instruction and this might result in different types of videos. For example, teachers may create what we refer to as set-up videos (de Araujo et al., in press). These videos would feature situations that would motivate subsequent in-class activity. The use of such videos would perhaps change the interactions in the classroom, though more research is needed to understand how and why these changes might occur.

Another means for supporting teachers would be to better prepare them for making effective use of the expanded in-class time. Even though Ms. Schaefer and Ms. Temple appreciated the increased student collaboration, neither had as an explicit goal an increase in collaborative *discussions* with their classes. One such way to support such discussions is through the use of rich mathematics tasks (Smith & Stein, 2011). If teachers continue to use the same procedure-based problems that characterize traditional mathematics homework in class, the likelihood of engaging in collaborative discussion may be minimal due to the nature of the tasks. Often, time is cited by teachers as the primary barrier to enacting rich tasks and having such discussions and mathematical discourse (Henningsen & Stein, 1997; Herbel-Eisenmann & Cirillo, 2009). More collaboration could be beneficial, but teachers must consider the tasks they use to support that collaboration as well as the quality and purpose of that collaboration (Herbel-Eisenmann et al., 2013).

Although research on flipped instruction is still scarce, teachers' implementation of this instructional model is not. By examining the perspectives and experiences of teachers who have chosen to implement flipped instruction, we can identify ways to support them and can also reflect on whether their motivations for flipping and perceived benefits align with scholarly ideas about the potential of flipped instruction as an educational innovation. Furthermore, helping teachers capitalize on the various phases of instruction is important in helping them maximize the benefits that may be possible through flipped instruction, while also minimizing the challenges that arise. It is by understanding teachers' perceptions of and goals for flipped instruction that we can help to align their practice with instructional strategies that literature suggests might help them achieve those goals.

Acknowledgements

This work was supported by the ReSTEM Institute in the College of Education at the University of Missouri. We thank Abigail Heffern and Nicole Fyten for their assistance with the analysis and Dr. Milan Sherman for his comments on this paper. We also thank the teachers, without whom this work would not have been possible.

References

Bergmann, J., & Sams, A. (2009). Remixing chemistry class: Two Colorado teachers make vodcasts of their lectures to free up class time for hands-on activities. *Learning & Leading with Technology, 36*(4), 22–27.
Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. Alexandria, VA: International Society for Technology in Education.
Chen, Y., Wang, Y., Kinshuk, & Chen, N. S. (2014). Is FLIP enough? Or should we use the FLIPPED model instead? *Computers & Education, 79*, 16–27.
Clark, K. (2015). The effects of the flipped model of instruction on student engagement and performance in the secondary mathematics classroom. *Journal of Educators Online, 12*(1), 91–95.
Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 119–142.
Coufal, K. (2014). *Flipped learning instructional model: Perceptions of video delivery to support engagement in eighth grade math*. Beaumont, TX: Lamar University (Unpublished doctoral dissertation).
de Araujo, Z., Otten, S., & Barisci, S. Conceptualizing "homework" in flipped mathematics classes. *Journal of Educational Technology and Society*, in press.
DeSantis, J., Van Curen, R., Putsch, J., & Metzger, J. (2015). Do students learn more from a flip? An exploration of the efficacy of flipped and traditional lessons. *Journal of Interactive Learning Research, 26*(1), 39–63.
Enfield, J. (2013). Looking at the impact of the flipped classroom model of instruction on undergraduate multimedia students at CSUN. *TechTrends, 57*(6), 14–27.
Explain Everything. (2011). *Explain everything (version 3.21) [Mobile application software]*. Retrieved from http://itunes.apple.com.
Flipped Learning Network. (2012). *Improve student learning and teacher satisfaction in one flip of the classroom*. Retrieved on October 28, 2015 from http://flippedlearning.org/cms/lib07/VA01923112/Centricity/Domain/41/classroomwinwinfographic7-12.pdf.
Ford, P. (2015). Flipping a math content course for pre-service elementary school teachers. *PRIMUS, 25*(4), 369–380.
Fulton, K. P. (2012). 10 reasons to flip. *Phi Delta Kappan, 94*(2), 20–24.
Garrison, D. R., Anderson, T., & Archer, W. (2000). Critical inquiry in a text-based environment: Computer conferencing in higher education. *The Internet and Higher Education, 2*(2–3), 87–105.
Gorozidis, G., & Papaioannou, A. G. (2014). Teachers' motivation to participate in training and to implement innovations. *Teaching and Teacher Education, 39*, 1–11.
Hamdan, N., McKnight, P., McKnight, K., & Arfstrom, K. (2013). *A review of flipped learning*. New York, NY: Flipped Learning Network. Retrieved on June 6, 2016 from http://www.flippedlearning.org/cms/lib07/VA01923112/Centricity/Domain/41/LitReview_FlippedLearning.pdf.
Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524–549.
Herbel-Eisenmann, B. A., & Cirillo, M. (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. Reston, VA: NCTM.
Herbel-Eisenmann, B., Steele, M., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator, 1*, 181–196.
Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research, 32*(2), 131–152.
Lage, M., Platt, G., & Treglia, M. (2000). Inverting the classroom: A gateway to creating an inclusive learning environment. *Journal of Economic Education, 31*(1), 30–43.
McGivney-Burelle, J., & Xue, F. (2013). Flipping calculus. *PRIMUS, 23*(5), 477–486.
Moore, A. J., Gillett, M. R., & Steele, M. D. (2014). Fostering student engagement with the flip. *Mathematics Teacher, 107*(6), 22–27.
Palmer, K. (2015). Flipping a calculus class: One instructor's experience. *PRIMUS, 25*(9–10), 886–891.
Sickle, V. S. (2015). Adventures in flipping college algebra. *PRIMUS, 25*(8), 600–613.
Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.
Southern California Public Radio (Producer). (2013, February 20). *Can flipping the classroom fix the educational system? [Audio Podcast]*. Retrieved from http://www.scpr.org/programs/airtalk/2013/02/20/30599/can-flipping-the-classroom-fix-the-educational-sys/.
Southmayd, R. (2014, May 18). *Fort Mill teachers flip classrooms, change how they teach. Fort Mill Times*. Retrieved on June 16, 2016 from http://www.

heraldonline.com/news/local/education/article12292973.html.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Strayer, J. F., Hart, J. B., & Bleiler, S. K. (2015). Fostering instructor knowledge of student thinking using the flipped classroom. *PRIMUS, 25*(8), 724–735.

Talbert, R. (2014). Inverting the linear algebra classroom. *PRIMUS, 24*(5), 361–374.

Tucker, B. (2012). The flipped classroom. *Education Next, 12*(1), 82–83.

Wanner, T., & Palmer, E. (2015). Personalising learning: Exploring student and teacher perceptions about flexible learning and assessment in a flipped university course. *Computers & Education, 88*, 354–369.

Zack, L., Fuselier, J., Squire, G. A., Lamb, R., & O'Hara, K. (2015). Flipping freshman mathematics. *PRIMUS, 25*(9–10), 803–813.