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Incorporating Real World Geometry Problems Into Geometry Classrooms

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Incorporating Real World Geometry Problems into Geometry Classrooms

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Heather Renee Postier

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Background

Mathematics has been a constant love in my life. Throughout my public school years, I always enjoyed going to math class. As I was finishing up high school, some of my favorite math teachers asked me about what major I planned to pursue in college. My teachers exposed me to the life of a math educator and explained why I would be a good candidate for the field. Thus, my journey of becoming a middle/secondary math teacher began.

Before I first started attending classes at Butler University, I had a pre-conceived idea that I would be picking one math topic and would be teaching that topic for the rest of my career. Once I actually started attending my education and math classes at Butler, I soon realized that I would be learning mathematics as a whole, which would prepare me for any class I would potentially teach in the future. As I have taken a variety of math classes throughout college, I have always leaned toward and loved geometry. It was one of my favorite classes in high school, and I thoroughly enjoyed learning more about the subject in college. However, as I went through the motions in my classes, I noticed that my Butler educators were pushing me away from the typical black and white thinking of mathematics, towards a bigger picture. I was pushed to think about math being applied to the real world through real world problems.

In my first year at Butler University, I took a math education class with Dr. Ryan Flessner. Dr. Flessner really pushed us as students to think more like an educator and less like a college student. We began the semester with basic elementary mathematics topics. While this seemed simplistic, we were constantly asked to explain why we were solving the problems with specific methods. I remember particularly struggling with understanding why problems were solved the way that they were. One of the first problems we did together as a class was a simple division problem with fractions. Most of the students in the class, including myself, used a method that we had learned in school called “keep, change, flip.” When dividing two fractions, we learned that you keep the first fraction, change the division sign to a multiplication sign, and then flip the other fraction. You then proceed with multiplying the fractions and obtaining an answer.

When we presented Dr. Flessner with our case for solving the problem, he asked us, “Why did you use this method?” This is the moment I realized that I had never thought about why I was implementing certain methods when solving math problems. I did not know why I was using the “keep, change, flip” method. I just knew how and when to use it. I wanted to figure out why I was using certain “shortcuts” to solve problems and what these “shortcuts” were actually doing. When you implement the “keep, change, flip” method, you are actually multiplying the first fraction by the reciprocal of the second fraction in order to make the denominator equal to one. Thus, you are essentially dividing by one, which equates to whatever the numerator originally was multiplied by the reciprocal. Unfortunately, teachers do not typically explain this to students. Teachers tend to struggle to answer the dreaded questions that students ask of, “Why are we doing this?” and “When are we going to use this?” The typical answer given by teachers is, “You will use this in future math classes.” I had never thought twice about my teachers’ answers to these questions until Dr. Flessner’s class. I began to think about why students had to learn about huge topics in high school like geometry, pre-calculus, and calculus. Math courses are put in place in high school to develop a student’s critical thinking and problem-solving skills. However, this is not the only reasoning for math classes in high school. There are several reasons as to why students should undergo twelve years of mathematical education. There are several jobs, even non-stereotypical mathematical jobs, that use skills learned in a high-school math classroom.

Teachers should be using more real-world topics and real-world based questions in math classrooms (Casa, Firmender, Gavin, & Carroll, 2016). Real-world problems can help students understand why they are learning such a fascinating school topic. This practice will help students expand their learning "experiences beyond the classroom" and find solutions to real problems (Ubiz, & Aydinyer, 2019). If teachers were to incorporate more real-world topics into math classes, such as geometry, students would understand why they are learning such rigorous topics in school (Moss, Hawes, Naqvi, & Caswell, 2015).

Standards for Mathematical Learning

Just like all high school subjects, math has a set of standards set by each individual state. Forty out of 50 states have agreed to use the Common-Core State Standards (Indiana Department of Education, 2019). These standards are set in place to guide students' learning and understanding of the specified content. By looking through the state standards for Indiana, Georgia, and the Common Core[1], I have found several standards that align with the argument for implementing real-world topics into mathematics.

The introductory section for mathematics on the Common Core website introduces real world application of mathematics by stating that the applications of triangles are "fundamental in many real-world and theoretical situations" (Indiana Department of Education, 2019) The Indiana state standards backs this introduction up with several state standards on how triangles should be used to solve real-world and mathematical problems. An example of this would be Indiana Standard G.T.5, "Use properties of congruent and similar triangles to solve real-world and mathematical problems involving sides, perimeters, and areas of triangles" (Indiana Department of Education, 2019). The states that do implement common core clearly have an

[1] As a college student in Indiana I need to look at Indiana. I grew up in Georgia and plan to return there to teach. I am also examining the Common Core since that is the most commonly used set of standards in the United States.

agenda to integrate real-world problems into learning and understanding mathematics; however, it is only evident with a few select topics, such as triangles.

Research Question and Process

There are several real-world problems that involve several different types of mathematics, not just the few specified topics in the state standards. Research shows that there are several articles addressing the urgency of integrating real-world problems into mathematics (e.g., Bently, T. 2012; Gainsbury, 2008; Prawvichen, Siripun, & Yuenyon, 2018; Tharp & Uprichard, 1992; Whitin, 1993). These works discuss many different topics of mathematics that can be learned by teaching with real world applications. Because of my passion for real world applications of mathematical content and my professional obligations of meeting the standards, I intend to answer the following question:

How do teachers incorporate real world problems into geometry classrooms?

In order to complete this research, I plan to conduct a thorough literature review. This will include looking at sources that contain research related to real world problems in high school geometry classrooms. Through this literature review I plan to look at themes in the research conducted to impact what I do while student teaching at Butler University. By funneling the research down to common themes, I will get a better understanding of how to incorporate real world problems into geometry classrooms.

Conducting the Search for Literature

Butler University provides several online resources for their students to use. As a part of Butler University's Irwin Library, they have a Database section on their webpage. I used this Database section to look for and use the search engines to find sources that relate to my topic. The first Database used in my research Web of Science. In order to find sources that related to my thesis, I used specific words in the search bar on Web of Science to narrow down my findings. This search on Web of Science resulted in six sources that pertain to integrating real world geometry in secondary education classrooms. I started this research on Web of Science in early January of 2020. Butler University has since canceled their subscription with Web of Science; thus, I needed to find a new Database. I contacted Ryan Flessner, my thesis advisor, and he recommended I use EBSCO from now on. In August of 2020 I began my research on EBSCO and used all databases.

When I started searching for sources on EBSCO, I started with the same three key words I used on Web of Science: Real World Geometry. This table outlines the words I searched, the number of sources derived from the searches, and the number of sources I read and used for research.

Search Words	Number of Sources	Number of Sources Read
Real World Geometry	6,807	N/A
Real World Geometry Education	502	N/A
Real World Geometry Teaching	340	N/A
Teaching Real world geometry in high school	53	7
Integrating Real World Geometry	35	10

Problem Based Learning in Mathematics	29,237	N/A
Problem Based Learning in Geometry	2,858	N/A
Problem Based Teaching in Geometry	1, 198	N/A
Problem Based Real World Teaching in Geometry	58	10
Inquiry Based Learning Geometry	86	7
National Council of Teachers of Mathematics	10, 608	N/A
National Council of Teachers of Mathematics Inquiry	243	N/A
National Council of Teachers of Mathematics Inquiry Learning	163	8

Reviewing the Literature

As the data from the table indicates, several articles came up in many of my searches. Sometimes when searching I had to type in more broad terms or more specific terms to get a collection of articles that was manageable to sift through. As much as I had hoped every article would be relatable to the topic, that was simply not true. There were plenty of articles that were not chosen to be read during the research process. When reading the abstract of sources, I tried to look for articles and books that specifically related to high school mathematics, geometry, and real-world applications. If an article did not exactly correlate with these topics, I did not pick it for this research.

Some examples of topics that came up were integrating technology into the classroom, incorporating inquiry in science classrooms, and full-on science related articles. None of these types of topics would help me address the question posed in this thesis, thus those articles were not chosen.

In some cases, an article may have been related to this research, but it was not written in English. My first language is English; thus, I was conducting all research in English. These articles very easily could have related to the topic and been good to use but I could not read them without heavy translation. Any article found in the search that was not written in English was not used for research purposes.

I also tried to avoid articles related to elementary education. I plan to teach secondary mathematics, and this research is focused primarily on geometry and high school mathematics. I picked very few articles that discussed elementary education. If I choose to read those articles, it was because they had very good tools or examples of real-world mathematics and application. For example, I decided to pick *A Geometric Scavenger Hunt,* because it had great examples for engagement and real-world usage, even though it focused on a 3rd grade classroom. (Smart & Marshall, 2007)

Many articles fit under the categories that I was looking for and were very useful. Of these sources I noticed that many of them came from the National Council of Teachers of

Mathematics (NCTM). Quite a few of my sources actually came from specific articles in the journals *The Mathematics Teacher* or *Mathematics Teaching in Middle School* which are both published by NCTM. Because I noticed that many of the sources I was picking were from NCTM, I decided to include the terms NCTM in my search for sources so I could find additional related sources. I believe this abundance of NCTM sources is a good thing as one of the best ways to get teaching advice and research is from a council of teachers. According to NCTM's (2000) *Principles and Standards for School Mathematics,* (the council) advocates the application of geometry in real-world situations. (NCTM, 2009). Thus, their standards and the sources published by them correlate directly with my research question.

Findings

After conducting my research, I realized that my research question could be answered. Interestingly, I feel like my research answers more than just my original question. It also addresses questions related to how and why teachers should incorporate real world applications and inquiry learning into mathematics classrooms in general. Each article had several categories it could fall under, but I decided to narrow these categories for organizational purposes. These categories help to answer the question of not only how to apply mathematics in real world situations, but they also describe why teachers should incorporate real world problems into the mathematical concepts they teach. Thus, my findings are organized in the following categories:

- Inquiry
- Student Engagement
- Mathematical Understanding

Inquiry

It only takes one student's inquiring question to launch a lesson with real world applications. If the students seem excited about something, roll with it. I know this idea seems crazy as teachers take so much time to plan lessons and do not have that much time in the school year to stray from certain topics. So many teachers choose a traditional approach when teaching mathematics. Many may ask the question, "If it works, why change it?". In response, Hannifin and Scott (2001) note, "Most traditional instructional programs are grounded in an objectivist learning theory, which holds that knowledge exists outside of the minds and must be transmitted to the learner by some means, primarily by a teacher, for learning to occur" (p.122). Through an objectivist learning mindset, teachers are planning instruction around themselves, not around the students. Based on the Indiana State Standards for mathematics, PS.2, students should be able to "Reason abstractly and quantitatively" (Indiana Department of Education, 2019). How can a student do this without the chance to inquire and problem solve in the classroom on their own?

One teaching method to incorporating inquiry into the classroom is Problem-Based Learning (PBL). PBL is a teaching method in which students work in collaborative groups and learn through facilitated problem solving (Hmelo-Silver, 2004). Teachers guide students in solving a problem, most likely a real-world problem, which could have been presented by a student. Teachers can incorporate both PBL and inquiry together in the mathematics classroom as one way to optimize student learning.

There are numerous possibilities for learning opportunities that can occur through inquiry in the classroom. Many students are interested in social justice. In the year 2020 alone, there were several social justice issues that came to light (Khan, 2021). Students are highly aware of the events and issues in their environment and lives (Bender & Schreiber, 1980). Teachers can incorporate such events and interests into the classroom to help students' understandings and inquiries. "Students observe the power of mathematics, understand why it is important, and are empowered to view the world through mathematics" (Johnson, 2011, pg.175) when teachers incorporate their interests and their lives in the classroom.

Crafting inquiry-based lessons can be difficult. But if the inquiry relates to the content standards, why not integrate the question into the lesson? A teacher's importance "in facilitating student inquiry…is unquestioned; [a] teacher's ability or willingness to adapt their teaching style [to] facilitate such inquiry is, however, another matter." (Hannifin & Scott, 2001, p.123). Teachers are encouraged to continually engage in professional development and self-reflection. Research now states that inquiry and real-world applications are the way of learning for the future (McGregor, 2016). Thus, teachers must make an effort to try new things and not be afraid to experiment with inquiry and real world applications in their lessons. Afterall students of mathematics are asked to do this every day. "Fostering a spirit of inquiry is an essential habit of mind that builds a classroom of thinking, conjecturing students" which is what all schools and teachers want (Whitin & Whitin, 2009, p.465). When students are engaged in inquiry, teachers can connect mathematics to the real world more easily and students' mathematical understanding and motivation can grow exponentially (Sheppard, 2009).

When starting lessons about discovery and inquiry, it can help to start small. Lessons planned for the week can be spiced up with inquiry by adding short, open-ended questions into guided questions (Chissick, 2004). When starting a lesson as students to consider certain things about an image or a situation to help lead them to the discovery. For example, to help students recognize similar triangles, you can ask them to draw two proportional triangles and allow them to come to conclusions about the triangle's similarity. There are several ways to do this in lessons and this can be a first step in incorporating inquiry into the classroom.

An example of inquiry-based learning is shown in the article *A Geometric Scavenger Hunt* (Smart & Marshall, 2007). A teacher was asked if there were geometric shapes in the woods outside. The teacher created a four-day lesson on real-world applications through students' questions, and it applied to the standards and students' interest. "When a student feels that he or she is more vested in actual learning" and it is catered to them, "retention of the material tends to rise significantly" (Wolf, Linderman, Wolf, & Dunnerstick, 2011, p.557). Additionally, when the students are more vested in the learning process, their attitude towards learning changes. This attitude toward learning consists of "curiosity, flexibility, critical reflection, and [honesty]" (Salim, & Tiawa, 2015). When students have good attitudes towards learning, they will have an even higher motivation to excel and be more invested in the lesson. Thus, the teacher in *A Geometric Scavenger Hunt*, was able to get students more motivated and invested in geometric shapes by creating this lesson around this one student's inquiry.

When teachers integrate a student's question of inquiry into lessons, the students will benefit in more ways than one (Copes, 2000). Not only will they be more invested in the lesson and have the motivation to excel in the content area, but their understanding of the content area will increase (Mudaly, n.d.). In their study, *Implementation of Structured Inquiry Based Model Learning Toward Students' Understanding of Geometry* (Salim, & Tiawa, 2015), the researchers found that the students understanding of geometry concepts was significantly higher with the structured inquiry model than with the conventional learning style. The research showed that students can learn effectively in this way and will have a slight positive shift in the way they view mathematical concepts. Thus, another reason why the traditional teaching style alone can limit positive effect on student learning.

When teachers are planning for an inquiry lesson, they need to be prepared for the lesson to not go as planned. Inquiry-based learning allows students to think outside the box which means that some students will most likely approach the problem from a different angle than the teacher. It is very natural for students to emerge with unexpected mathematical results and strategies when in an inquiry-based setting (Manouchechri,2006). Thus, in order to be prepared for such things, the teacher should plan questions to help guide students to mathematical understandings. If the teacher has not planned these questions, they will need to adapt and think quickly to help students understand why their approaches are mathematically correct or incorrect (Cook, Hartman, Pierce & Seaders, 2017).

If high school students are still in the stage of concrete operations, based on the theory of Piaget (Theerasan & Yuenyong, 2018), they will have a difficult time learning and seeing particular characteristics of three-dimensional shapes. Therefore, if you change the approach and learning strategy from concrete to self-constructed or inquiry, the students can understand the concepts and characteristics on their own (Theerasan & Yuenyong, 2018). This investigation could be with guided instruction so that later in life the students may be better able to rediscover, investigate, and search for the meaning in everyday life. The process of inquiry-based learning can assist students in becoming lifelong learners. The mathematical goal is for the students to understand the mathematical concepts they are learning at a specific time, but the ultimate goal of allowing students to discover their love of learning should be the focus of the educational process and lesson plan.

Student Engagement

Most students have a favorite subject in school, and often that - subject is not math. Thus, math teachers struggle with student engagement almost every day. However, it is a teacher’s job to teach the content and make sure the students are able to stay engaged so that they learn the material. Wolf, Linderman, Wolf, and Dunnerstick (2011) note “Lessons should incorporate varied activities that engage all students. If lesson planning accounts for different learning styles, students will be exposed to more than one type of learning, which increases the possibility of success and the desired outcome” (p.557). One way increase student engagement is to incorporate various real-world scenarios into the classroom while allowing students to engage in group work. This way students can also enjoy what they are learning. Students should feel the joy of learning when in the classroom and not dread the content. Teachers can help them experience this joy through inquiry and student engagement (Yeh, 2021).

A study done in a government school in Calabar municipality showed that “in general, the students’ gave positive responses” to inquiry-based learning and real-world topics (Ekwueme, Ekon,, & Ezenwa-Nebife, 2015, p.50). This article concluded that 87.5% of students enjoyed this type of learning, 91.7% of students prefer this method to typical pen/paper lecture, and 85.8% students said that they could arrive at mathematical solutions with more confidence after learning this way. These students were interested in how math can be applied to their lives and the world around them. This increased their interest in the subject and the type of learning, thus making it possible to increase student engagement through inquiry and real-world applications.

Incorporating inquiry and real world applications into lessons takes time and planning. Yet, when teachers plan assignments that incorporate inquiry and their students' interests, the student's engagement increases. Even students who have traditionally struggled with mathematics enjoy math more when teachers incorporate their interest in the lesson (Johnson, 2011). Zimmer and Finn (2012), define engagement into four main categories. These categories are that engagement is essential for learning, engagement is multifaceted with behavioral and psychological components, engagement and disengagement are developmentally appropriate, and student engagement modifies developmentally throughout the years. Thus, all students at some point or another will struggle with engagement due to their development. Therefore, teachers can help all students see why engagement is so essential for learning through real world application.

A key to student engagement relates to the problems being authentic. With many problem-solving approaches, students are given artificially contextualized problems where the teacher has a specific answer and method of approach in mind. Instead the problems need to be authentic and relatable. Otherwise the students have no real sense of wonder, engagement, or understanding (Fielding-Wells, O'Brien, Makar, 2017). As presented in the previous inquiry section, inquiry learning is best derived from a question or state of wonder. The same applies for student engagement. The students are best engaged when there is a sense of wonder in the mathematical concept (Fielding-Wells, O'Brien, Makar, 2017). Thus, the problems should be relatable and authentic to drive a challenge and goal of understanding.

Students will also appreciate the lesson more and engage more deeply in the mathematics when learning opportunities include representations of all peoples (Frankenstein, 1990). This includes incorporating different races, genders, and cultures in the classroom. (Banks, 1991). This can occur through daily problems, data sets, or exploring different mathematicians. If students can see themselves in the data and relate to mathematicians, they are more likely to ask questions, wonder, and connect the content to themselves. Teachers can also link math to different disciplines such as math and history through art and architecture. Students can embrace a multicultural understanding of mathematics by learning about how several cultures have contributed to mathematics throughout history (Desai & Safi, 2020). The teacher could take this further by showing how the same cultures used math to contribute to art and design. This allows students to feel welcome in the classroom environment (Eichorn, DiMauro, Lacson, & Dennie, 2019) and the chance to experience the diverse history of mathematics.

Teachers can also increase student engagement through real world application is through the use of technology and software programs. In a study about using technology in the geometry classroom (Erbas & Yenmez, 2011), students were asked to explore geometric congruency and similarity through multiple software programs. There was a control group and an experimental group. The experimental group used software environments such as Logo-based Turtle geometry, Cabri-Geometre, and Geometer's Sketchpad. Not only did the experimental group participate more in the learning of the content, but the article also stated that the students in this group showed a greater interest and motivation than the control group. Technology and software programs can make the classroom and learning environment more fun and thus increase student engagement. Through a fun learning environment, the technology programs can enhance mathematical understanding through use of visual representations and moveable representations (Quinn, 2020). These software programs can allow students to explore mathematical concepts through visual and kinesthetic learning while increasing engagement.

Using technology does not just have to be through software programs, it can be through games as well. When students engage with mathematical concepts through the lenses of games, it activates reasoning, strategy, and logic skills (Soldano, Luz, Arzarello, & Yerushalmy 2018). Using games in the classroom makes learning fun for students while keeping them engaged in the classroom. Afterall, students remember most about the experience they have in the mathematics classroom rather than the actual mathematics concepts (Suranto, 2018). If a teacher can incorporate a game or some sort of technology into the classroom, student engagement and motivation increase (Hwa, 2018). Students love playing games in and out of the classroom. Students gain higher mathematical understanding when teachers incorporate games in the classroom with additional bits of real-world application.

Mathematical Understanding

Not only are inquiry and student engagement significantly addressed when using real world application in mathematical classrooms, but mathematical understandings are also significantly improved. An average student understands mathematical concepts in a traditional classroom; however, a student's understanding of a mathematical concept, such as geometry, is significantly higher with a structured inquiry-model than that of students who receive conventional learning (Lavy & Shriki, 2010). As noted in previous sections, a structured inquiry model allows students to investigate mathematical concepts through real world application. Without proper planning, implementing structured inquiry lessons can be daunting for both the teacher and the students.

The Common Core State Standards for Mathematics (CCSSM) calls for students to create mathematical representations and identify important information and relevant variables from problems (Hendrickson and Kisner, 2015). CCSSM Standard 8.G.9. states that students need to

"Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems" (CCSSM, Standard 8.G.9). In the article *Aunt Bertha's Ice Cream Shoppe* (Hendrickson and Kisner, 2015), teachers applied formulas of cones, cylinders, and spheres to ice cream. The problem asks students to find the best dimensions for ice cream cones for Aunt Bertha to make a profit in her shoppe. This problem combines that State Standards and real-world application to allow mathematical understanding for students. This situation also uses problem-based learning and allows students to gather data, draw conclusions, and report their findings on the most profitable dimensions for an ice cream cone. Determining dimensions of objects to obtain a profit is a key skill in many job types, thus this problem is also relatable to the students.

Students tend to learn best in their zone of proximal development (ZPD), which is the "gap, or distance, between what a learner can achieve independently and what [they] can achieve with the help of a more knowledgeable other (MKO)" (Hannifin & Scott, 2001, p.123). In the case of a classroom, the teacher would be the MKO. When students are asked to take on a new task with a new mathematical concept, they are being asked to explore the unknown. Teachers should not expect students to be successful without a MKO and instructional scaffolding. When teachers are incorporating real world application, "teachers can provide instructional scaffolds that assists students in going from the unknown to the known", so that they may grasp the mathematical understanding (Hannifin & Scott, 2001, p.123). After all, teachers cannot just expect students to explore real world problems and magically understand the mathematical contexts. It is with the support of the teacher (MKO), the ZPD, and instructional scaffolding that students can connect these real-world applications with specific mathematical concepts presented in state standards (Goos, 2004).

To help guide students' learning, the teacher should "help students make connections among various solutions, tie student ideas to important mathematical structures, and extend student inquiry by posing questions and tasks that challenge their initial interpretations of problems or their false generalizations" (Manouchechri, 2006, p.290). When planning for inquiry-based lessons, teachers cannot plan for everything the students will ask and come up with. However teachers should be prepared to answer commonly asked questions. Even better, teachers should have questions ready to guide the students in discovering the correct mathematical reasoning. Because some students will inevitably come up with incorrect or incomplete solutions, teachers must be prepared to guide students back to the task at hand or use their new questions and new solutions to teach certain mathematical concepts (McDonald, 1988). Importantly, teachers must become comfortable with students developing solutions that are different from their own. It is essential that teachers remember that "Problem solving skills help [students] realize that there is often more than one way to solve a mathematical problem" (Obara, 2010, p.128). Teachers must guide students to mathematical understandings and the realization that there can be different solutions to mathematical problems.

In an inquiry based investigation, pre-service teachers were provided with a meterstick, TI-84 calculator, graph paper, and eleven PVC view tubes of different lengths. (Obara, 2010). The view tubes were all constructed from household items (e.g., toilet paper rolls, wrapping paper rolls, paper towel rolls). This helped the students relate mathematics to real world contexts. There were three students in each group. One student was tasked with looking through the tube, another with measuring what the first student was seeing through the tube, and the third was to record measures. Before the investigation, students were asked to predict if the relationship would be linear, power, or exponential. The students were asked to collect, organize, and graph data. When attempting to graph the data, all student groups agreed that the relationship was not linear, power, or exponential, but hyperbolic instead. The relationship being the height of the tube and the amount visible through the hole. Through this they created graphical representations, algebraic representations, and geometric representations to decide which would best fit the data for the relationship.

This may seem like a massive undertaking, but it started with household items. Through the inquiry project, students were able to build several skills at once. They practiced problem-solving skills, communication with others, investigation, mathematical modeling, and learning by doing all through one experiment. If students would have learned these skills with a traditional teaching approach, they probably would have been given the data and asked to graph the hyperbolic relationship. Through the context of inquiry learning with real world objects, students were able to gain so much more, identify that the relationship was hyperbolic on their own, and learn mathematical skills.

There are several examples of objects from the real world that can be applied in the classroom to guide students in mathematical understanding. Teachers can use something as simple as a soup can to help students understand volume and surface area. Certain packaging types can help with tangents and area. Honeycombs can be used to help students understand similarity, angles, and tessellations. Cracking can help with angles and perpendicular lines. Flowerpots and handbasins can be used to understand functionality of the truncated cone. Teachers can use the golden ratio to help students physically see mathematics in art and nature. To discuss a coordinate plane, a teacher can use dining-tables, floors, and pavements. Science teachers and mathematics teachers can teach an interdisciplinary lesson through DNA and its shape of the Helix (e.g., Bender & Schreiber, 1980; Mallik, n.d; Salim & Tiawa, 2015; Whitin & Whitin, 2009). Mathematics is everywhere! If teachers can get students to see this realization, they can get students to understand incredible mathematical concepts and help them love the subject.

Conclusion

The question posed to conduct this research was:

How do teachers incorporate real world problems into geometry classrooms?

There is no single answer to this question, but there are many answers. The research done in this literature review not only answers this question but also the question of why. The research reviewed was organized into the sections of Inquiry, Student Engagement, and Mathematical Understanding. Teachers can incorporate the real world into mathematics classrooms through inquiry, problem-based learning, and every day real world objects. Teachers should do this because it increases student engagement and leads students to higher mathematical understandings. Not only does incorporating the real world into the classroom do all of these things for students, it does so much more. It can allow students to see mathematical connections to themselves, their interests, and the world around them. It can allow students to grow in skills such as communication with others, investigation, mathematical modeling, and the love of learning. There is not doubt that incorporating real world problems into any mathematical classroom will increase students' engagement and understanding. It is time to use inquiry based teaching methods in addition to traditional methods for the benefit of the future generations.

Appendix A

The purpose of this Appendix is to show a completed lesson using the research found in this literature review. This lesson can be used in any geometry classroom and is here as an example for the readers.

Lesson for Integrating Real World Application into a Geometry Classroom

Standards:

G.CI.1 Define, identify and use relationships among the following: radius, diameter, arc, measure of an arc, chord, secant, tangent, congruent circles, and concentric circles.

G.CI.4 Solve real-world and other mathematical problems that involve finding measures of circumference, areas of circles and sectors, and arc lengths and related angles (central, inscribed, and intersections of secants and tangents).

Process Standards:

PS.1: Make sense of problems and persevere in solving them.

PS.3: Construct viable arguments and critique the reasoning of others.

PS.6: Attend to precision.

PS.8: Look for and express regularity in repeated reasoning.

Essential Question: How can we use geometric concepts to solve real world problems of profit and design?

Enduring Understanding: Students will use geometric concepts to solve real world problems of profit and design.

SMART Objective: Students can understand geometric concepts by working in groups to solve the following production and manufacturing questions.

Warm up: Each day for a warmup, the teacher can present questions on diameter, circumference, and surface area so students are brushing up on these concepts before digging into the problem solving.

Day 1:

Students will be presented with the following problem:

You and your friends are trying to open up a business for energy drinks. You are offered two different dimensions for cans that you can make your energy drinks out of. Company A is selling two dozens of a can with a height of 5.2 in and a radius of 1.5 inches for the price of $52.75. Company B is selling 30 cans with a height of 4.3 in and a diameter of 2.8 inches for the price of $45.63. Your job is to find the circumference and surface area of each companies can and to determine which company's offer is cheaper by the can.

Students will work In groups to solve the problem to determine which company has the cheapest cans.

Day 2:

Students will be presented the following problem:

Now that you have determined which company has the better deal, you can choose to manufacture your energy drink with that company or on your own. If you decide to manufacture cans on your own, the price of production per 28 cans will be $50.99. Also, if you decide to manufacture on your own, you will be able to decide height and radius of the can, thus having more product if you wish. The only catch is, the taller the height, the smaller the radius. And the heigh cannot exceed 10 in, and the radius cannot be less than 1 in. You will determine if it is cheaper to manufacture on your own or go with the company of your choice. You will also determine the dimensions, circumference, and surface area of the can that you and your group would manufacture.

Students will work in groups to solve the problem presented to them. Students will also explain their choice and why in a short paragraph.

Day 3:

Students will have time to finish up both problems and write their paragraph for reasoning. Students will draw and design their can and logo. Then each group will present to the class which option they chose and why, and their can design.

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