Sebastian Lange, Peter Sokolowski, Eckehard Schöll, Xinghuo Yu

Using revealed-bidding in power markets: A paradigmatic model

Using revealed-bidding in power markets: A paradigmatic model

Sebastian Lange[1,2], Peter Sokolowski[2], Eckehard Schöll[1] and Xinghuo Yu[2]

[1] is with Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany
[2] is with School of Engineering, RMIT University, Melbourne, VIC 3000, Australia

Abstract— **Electricity markets underwent profound changes and are still far from static. With the decentralization and the rise of microgrids, electricity markets face new objectives, which may require new auction mechanisms. With the majority of electricity markets employing sealed bid auctions, this paper explores the use of revealed bid auctions. The proposed market system publicly announces each submitted bid and allows other sellers to change their standing bids based on the announced bid. Most importantly the achievement of power balance and of strategy equilibria is investigated. For the analysis of strategy loops, not only the standing bids of all sellers are considered, but also their capabilities to change their bids, which leads to the rise of strategy loops. A method is proposed to prevent these strategy loops. The insights gained in the dependency of marginal price and demand are of interest to auctioneers allowing rudimentary control over the marginal price using demand response schemes.**

I. INTRODUCTION

According to a report by IRENA [1], nearly 60% of the additional electricity needed to achieve universal electricity access will be supplied outside of large scale power grids by 2030. For these microgrids, efficient, transparent trading mechanisms are needed. However, as trading adds substantially to fluctuations in power grids, a better understanding of power markets is relevant for large scale grids, too [2].

Trade among generators and consumers can be arranged in two different ways [3]. On the one hand, there is bilateral trade where buyers and sellers directly negotiate trade conditions in pairs of two. On the other hand, trade can be arranged through an intermediary. In this case, all generators sell their electricity to said intermediary, which sells it on to the retailers. The sellers are "playing a game" to determine who is producing which quantity of electricity for which price. This process can be understood as a reverse auction [4].

There is no evidence that in general, neither bilateral nor mediated markets are superior over the other [3]. However, markets handling time frames of a day or less before dispatch are potentially benefiting of mediated market forms [3].

The way the bidding in an electricity auction takes place can be organized in different ways. Most commonly sealed-bid auctions are used in electricity markets [6]. Morey argues that using sealed bids is inevitable, as revealed bidding would hast a non-competitive behavior, which Klemperer quoted as "implicit collusion" [3], [5]. However, it needs to be stressed that Klemperer attests that a related collusion phenomenon can arise in sealed-bid, uniform-price auctions [5]. This is reinforced by Clauser *et al.* stating that sealed-bid auctions are susceptible to collusion [7]. Furthermore, the use of sealed bids leads to sellers suffering higher uncertainty. Thus, this work aims to investigate the basic characteristics of revealed-bidding systems in power auctions.

The paper is organized as follows: Sec. II introduces the proposed market system, which is analyzed in Sec. III. The used methodology, simulating the proposed market system is given in Sec. IV, while simulation results are provided and discussed in Sec. V. Sec. VI summarises the paper.

II. PROPOSED MARKET SYSTEM MODEL

In this model, the trade between multiple companies selling electricity and a single mediator acting on behalf of all buyers is described. The purpose of the market system is to arrange the supply of the demand D. The proposed peculiarity is the use of reveal instead of sealed bidding. Explaining the model, first, the structure of the proposed market system is described. Subsequently, the trading, taking place in form of an auction is laid out.

A. Market system structure

The proposed market system consists of N-sellers and a single buyer. The sellers only consider the cost of generation and maintenance of their generating assets, but neglect other costs such as fees for transmission. Transmission can take place unrestricted and lossless. D is variable but inelastic.

The N-sellers are the companies owning one or multiple generators which are either steam-driven, wind- or solar-powered. As state variable these companies exhibit their overall current power output $q_n \in \mathbb{N}^+$, with $n \in [1, N]$ being the identifier of each company. This q_n is bounded by a lower and upper generator capacity limit $q_{min,n}$ and $q_{max,n}$ so that $q_{min,n} \leq q_n \leq q_{max,n}$.

Furthermore, each company is characterised by a cost function $C_n(q_n)$ measuring the generation cost in Dollar ($), with $C_n(q_n) > \$0, \forall q_n \in [q_{min,n}, q_{max,n}]$. Additionally $C_n(q_n)$ needs to ensure that a q_n-increase will result in a lower price per produced MWh $\c{C}_n(q_n)$, *i.e.*:

$$\frac{d}{dq_n}\left(\frac{C_n(q_n)}{q_n}\right) = \frac{d}{dq_n}\c{C}_n(q_n) < 0\frac{\$}{\text{MWh}},$$
$$\forall\ q_n \in [q_{min,n},\ q_{max,n}],\ q_n \neq 0\ \text{MWh} \quad (1)$$

In case $[q_{min,n}, q_{max,n}]$ does not include 0 MWh, define

$$C_n(0\ \text{MWh}) = \tfrac{1}{2}C_n(q_{min,n}). \quad (2)$$

What kind of cost function is utilized and how the generator capacity limits are determined, depends on the type of generator employed and is described in the appendix.

In this work, the total price the n-th seller offers reflects solely $C_n(q_n)$. Margins are only considered indirectly: In cases where a seller would make the same profit for two different q_n-offers, the seller will offer the larger q_n. The rationale is, that for a larger quantity the seller could earn more by adding a margin to each MWh offered.

Representing all buyers, a single mediator is installed often called the Independent System Operator (ISO) [9]. As described below, in this model trade is facilitated through auctions for which the ISO will act as the auctioneer.

Each day is divided into *dispatch intervals* of equal length, which are equal to the trading intervals. This work's sole focus is the trading dynamics within one dispatch interval.

B. Reverse, revealed-bid auction

Which sellers are allowed to generate, which amount of electricity and what price each seller can charge is determined using a reverse auction. Each bid submitted in an auction is publicly announced, *i.e.* revealed. A bid consists of one overall quantity $q_n \in \mathbb{N}^+$ the n-th seller is willing and able to generate and a price $p_n \in \mathbb{R}$ given in Dollar ($) requested for each unit of q_n supplied. The employed reverse auction is conducted as shown in Fig. 1 and as follows:

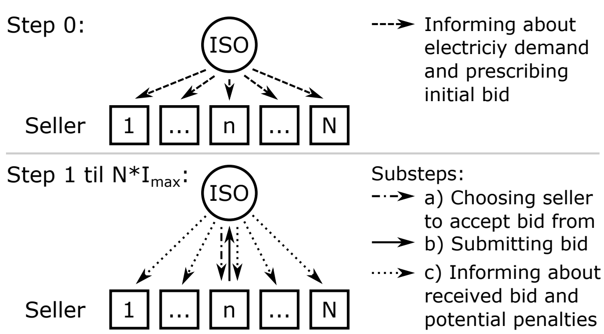

Fig. 1. Course of reverse, revealed-bid auction in proposed market system.

In an initial step prior to accepting bids, the ISO announces D and assigns an initial bid to each seller. The ISO then commences the bidding process by creating a random order of sellers to submit their bids in. The submission of bids occurs subsequently and is announced publicly right away. After one iteration, *i.e.* once every seller had the chance to submit a bid, the ISO creates a new random order of sellers and continues to accept bids. Each seller is able to hold one bid at a time but allowed to replace its standing bid.

The parameter $\tilde{\Upsilon} \in \mathbb{N}$ reflect how often bids have been placed or changed in total in this trading interval. Every time the n-th seller places or changes its bid the current value of $\tilde{\Upsilon}$ is stored in $\tilde{\Upsilon}_n$. The merit order is given by the bijective function $j : [1, N] \to [1, N],\ j(n) = \eta$ which maps n on its placement in merit order η. For each pair of sellers $n, \tilde{n} \in [1, N], n \neq \tilde{n}$ the merit order has to satisfy:

$$j(n) < j(\tilde{n}), \quad \text{if } q_n = 0 \neq q_{\tilde{n}} \tag{3}$$

$$j(n) < j(\tilde{n}), \quad \text{if } p_n < p_{\tilde{n}} \wedge (q_n = 0 = q_{\tilde{n}} \vee q_n \neq 0 \neq q_{\tilde{n}}) \tag{4}$$

$$j(n) < j(\tilde{n}), \quad \text{if } p_n = p_{\tilde{n}} \wedge \Upsilon_n < \Upsilon_{\tilde{n}} \wedge (q_n = 0 = q_{\tilde{n}} \vee q_n \neq 0 \neq q_{\tilde{n}}). \tag{5}$$

(3), (4), (5) omit the unit of q_n. The price per supplied MWh a seller n receives from an auction can then be noted as:

$$a_n = \begin{cases} p_{n_{la}} & \text{if } j(n) \leq J_{la} \wedge q_n \neq 0\text{MWh} \\ 0 & \text{else,} \end{cases} \tag{6}$$

making it a last accepted uniform price auction. Given the inverse function $j^{-1} = g(\eta) = n$, the last accept sellers n_{la} occupying the J_{la}-th position in merit order is given by:

$$n_{la} = \arg\max_{J_{la}} g(J_{la}) \quad \text{subject to} \quad \sum_{J=1}^{J_{la}} q_{g(J)} < D. \tag{7}$$

The set holding all bidders is called $\mathbb{S}$. The n-th seller's bid is accepted when $j(n) \leq J_{la}$ and $q_n \neq 0$ MWh. The set holding all accepted bidders is called $\mathbb{A} \subseteq \mathbb{S}$.

The auction is terminated by the ISO once either: A) No seller replaced its standing bid with a differing q_n-offer. Or B) The maximum number of iterations I_{max} is reached.

III. Model Adequacy Analysis

Each seller's objective is the maximisation of its profit

$$P_n^* = \max_{q_n} P_n = \max_{q_n} (a_n q_n - C_n(q_n)) \tag{8}$$

Given this objective the following seller behavior arise:

A. Participation of sellers in the proposed market system

With the sellers' sole objective being defined in (8) their motivation to participate can be mapped out as follows:

A seller $n \in \mathbb{A}$ is guaranteed $a_n \geq \c{C}_n(q_n)$. This is due to the employed uniform-price auction and as $p_n = \c{C}_n(q_n)$. Hence, these sellers do not experience a loss. On the other hand, as the cost of generation is defined to be $C_n(q_n) > \$0$ the cost term does always have a negative influence on the profit. Thus, a seller $n \in \mathbb{A}$ is raising P_n compared to not placing a bid, *i.e.* placing a bid with $q_n = 0$ MWh.

If a seller n can not place a bid so that $n \in \mathbb{A}$, the strategy leading to the maximum profit is bidding $q_n = 0$ MWh. This is due to the constraints enforced on the $C_n(q_n)$.

Recall that bids can be replaced for free and that the stopping criteria for the auction are known to all sellers. Aware of the stopping criteria, each seller n will try to place a bid so that $n \in \mathbb{A}$, as this is the only chance to avoid a loss. Should this attempt fail, n is still able to play $q_n = 0$ MWh. Given I_{max} is chosen sufficiently large every seller will at least attempt to place a bid.

B. Strategies and Bid Shading

The ISO prescribes the initial bid for each seller. Hence, when (re-)placing a bid each seller n can follow three strategies. Either n increases, keeps or decreases q_n. As issuing a new bid with the same q_n will lead to an increase in η for n no player will reissue a bid but keep its standing bid. This strategy is therefore omitted from the discussion.

Increasing the generated, and thus placed, quantity will lead to a decrease in p_n, compare (1). If $n \in \mathbb{A}, n \neq n_{la}$, this decrease in cost will not effect a_n. Hence, n increases P_n when q_n increases, as long as $n \in \mathbb{A}$, $n \neq n_{la}$.

Additionally, increasing q_n can influence $j(n)$: Let a seller n with $n = n_{la}$ increase q_n and in turn decrease p_n. The lower price offered can lead to undercutting another sellers bid so that $j(n) < J_{la}$ after the increase of q_n. In this case the seller may no longer has zero profit, but $P_n > \$0$.

An increase in q_n can push the current last accepted bid out of the accepted bid pool, which can have three consequences for seller n: 1) If n pushes itself out of $\mathbb{A}$, P_n will become negative. 2) If n pushes another seller $\tilde{n}$ out a_n might lowers causing a decrease in P_n. However, 3) if n pushes $\tilde{n}$ out of $\mathbb{A}$, $\tilde{n}$ will try to reduce $q_{\tilde{n}}$ to get reaccepted into $\mathbb{A}$. This reduction in $q_{\tilde{n}}$ will lead to an increased $p_{\tilde{n}}$. So if $\tilde{n}$ is reaccepted into $\mathbb{A}$, every seller $k \in [1, N]$ with $j(k) < j(\tilde{n})$ will earn more from the auction.

Now considers a scenario where the bidder offering the lowest p_n decides to increase q_n so that $q_n > D$. Then every seller would experience a loss, which can be prevented by:

Market rule 1: If the n-th seller's bid satisfies $q_n > D$ then the n-th seller's bid is rejected by the ISO.

Decreasing q_n will lead to a higher p_n, compare (1). Then the only way to increase P_n is to cause an increase in a_n. To offset the increase in $\c{C}_n(q_n)$ when reducing q_n, an even higher increase in a_n is needed.

To increase a_n the seller n has to lower q_n so far that a bid $\tilde{n}$ satisfying $j(\tilde{n}) > J_{la}$ before the reduction of q_n, satisfies $j(\tilde{n}) = J_{la}$ after the reduction of q_n. Adding a seller to $\mathbb{A}$ bears a non-zero probability that a_n increases. Seller n's behavior of not bidding the lowest p_n possible while satisfying $j(n) \leq J_{la}$ is known as bid shading. As it influences a_n, sellers that posses the ability to successfully shade their bid, are known as having market power.

C. On the equilibrium states of an auction

A strategy equilibrium (SE) is defined as the state of an auction, when no seller n can increase P_n. In a Nash-Equilibrium each seller will only consider the current bids of all other sellers. However, the discussed auctions are repeated frequently with the same sellers participating and outcomes being announced publicly, which can influence SE. *E.g.*, sellers are aware of their competitors generation capabilities and can therefore anticipate other sellers reactions:

A seller $\tilde{n}$, with $\tilde{n} \notin \mathbb{A}$, will bid $q_{\tilde{n}} = 0$ MWh. In a Nash-Equilibrium a seller n would then only consider $q_{\tilde{n}} = 0$ MWh for seller $\tilde{n}$, when evaluating its next move. Now consider n reducing q_n by Δq so that $\tilde{n}$ can get accepted by offering $q_{\tilde{n}} = q_{min,\tilde{n}}$. If the acceptance of $\tilde{n}$ leads to a change in a_n from a_{old} to a_{new} satisfying

$$\frac{q_{old}}{\c{C}_n(q_n))} < \frac{a_{new}}{\c{C}_n(q_n - \Delta q)} \left(1 - \frac{\Delta q}{q_n}\right), \quad (9)$$

seller n can increase P_n. Seller n is only able to increase P_n when a former rejected seller $\tilde{n}$ is able to place an accepted bid as a result of the reduction in q_n. However, if n reduced q_n by Δq and a seller $\hat{n}$, with $\hat{n} \in \mathbb{A}$ increases $q_{\hat{n}}$ by Δq, a decrease in P_n will result. This will happen when seller $\hat{n}$ is able to increase $P_{\hat{n}}$ further by increasing $q_{\hat{n}}$ by Δq than by receiving the higher a_{new}. Seller $\hat{n}$ will prefer to increase $q_{\hat{n}}$ when

$$\left(\frac{a_{new} - \c{C}_{\hat{n}}(q_{\hat{n}})}{a_{abs} - \c{C}_{\hat{n}}(q_{\hat{n}} + \Delta q)} - 1\right) q_{\hat{n}} < \Delta q$$
$$\exists\ \hat{n} \in [1, N],\ \hat{n} \neq n,\ j(\hat{n}) < J_{la}, \quad (10)$$

holds true, where a_{abs} represents $a_{\hat{n}}$ after the bid shade of seller n, but before the reaction of any other seller.

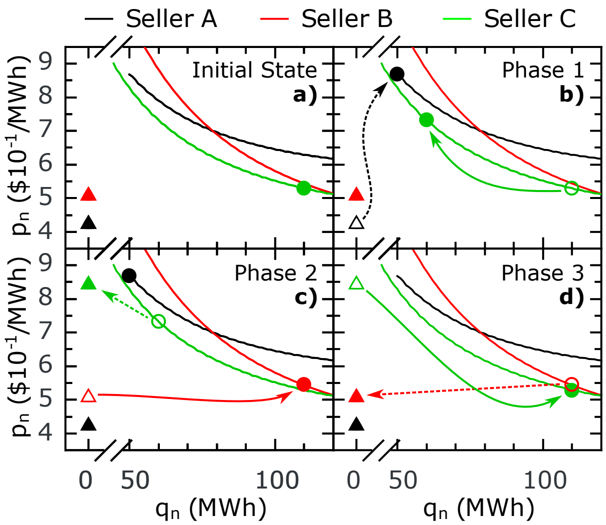

Fig. 2. Example SL. a) shows initial auction state; b) shows bid shade of Seller C and expected reaction of Seller A; c) shows Seller B taking advantage of situation; d) shows Seller C undoing bid shade, forcing Seller B back to its initial bid, reopening an opportunity for Seller C to bid shade. Filled symbols represent current bids, while empty symbols represent bid before replacement. Circles mark bids within accepted bid pool. Triangles mark bids rejected from accepted bid pool. Solid arrows show an action taken by a seller. Dashed arrows show other sellers reaction to an action.

A major disadvantage when considering the reactions of other sellers to a bid shade, additional to standing bids, is the introduction of strategy loops (SL). Let $q_{n,t}$ represent the quantity offered by the n-th seller at the t-th number in time the ISO offered n to replace its bid. A SL describes the behavior of a group of sellers, for which in each iteration at least one seller replaces its last bid $q_{n,\tilde{t}}$ with a bid $q_{n,t}$ so that $q_{n,t} \neq q_{n\tilde{t}}$, but $q_{n,t} = q_{n,\hat{t}}$ for $\hat{t} \in N^+, t > \tilde{t} > \hat{t}$.

What follows is an example SL between one seller C $\in \mathbb{A}$ who shades its bid, inducing a reaction of two initially rejected bidders A, B $\in \mathbb{S}$. For the example SL to occur the cost functions of the three involved sellers need to be defined for the following points: For Seller A $q_A^{(1)}$, for Seller B $q_B^{(1)}$ and for Seller C $q_C^{(1)}$ and $q_C^{(2)}$, which need to satisfy $q_C^{(2)} = q_C^{(1)} + q_A^{(1)}$ and $q_B^{(1)} = q_C^{(1)}$ as well as $\c{C}_C(q_C^{(2)}) < \c{C}_B(q_B^{(1)}) < \c{C}_C(q_C^{(1)}) < \c{C}_A(q_A^{(1)})$. Before the bid shade, that triggers the SL, takes place D is satisfied and the following auction state persists: $q_C = q_C^{(2)}$ and $q_A = q_B = 0$ MWh. The loop itself is described in Fig. 2.

SLs are favorable for each seller n that exhibits $n \notin \mathbb{A}$ prior to the bid shade but is able to achieve $n \in \mathbb{A}$ due to the bid shade. These sellers hope that the auction is terminated when the SL is in a phase when $n \in \mathbb{A}$. However, the occurrence of a SL is not at any point in time in the interest of all sellers. With regards to the presented SL, seller C always undoes the reduction of q_C as most reactions of the other sellers caused P_C to decrease. Thus, a seller n needs a method to consider the consequences of SLs. Should the consequence of the SL be a reduction of P_n, seller n can refrain from bid shading and thus stop the SL.

A way for seller n to consider the consequences of SLs is to keep a record of P_n and of all bids whenever a bid is placed. When n plans to place a bid with q_n, n has to check if n had placed q_n previously and if the current bids of the other sellers match their bids when n had placed q_n before. If both hold true, n can then check if P_n decreased due to another sellers reaction since then, which indicates an unfavorable SL.

D. Matching demand and supply

It can be shown, that even though

$$D = \sum_{n=1}^{N} q_n \ , \ q_n \in [q_{min,n} \ , \ q_{max,n}] \tag{11}$$

the following can hold true in a SE

$$D \neq \sum_{n}^{N} q_n \ ,$$
$$\forall q_n = \begin{cases} q_n \in [q_{min,n} \ , \ q_{max,n}] & \text{if } j(n) \leq J_{la} \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Therefore an additional mechanism is needed to ensure that in a SE supply matches demand, *i.e.* that (12) does not hold true while (11) does. Here it is proposed to solve the issue using an additional market rule:

Market rule 2: If both (11) and (12) hold true given a set $\mathbb{S}$, the seller(s) holding the highest relative quantity bid(s) will receive $a_n = \$0$.

Market rule 2 can be understood as the punishment of sellers, who block the entrance of additional sellers into $\mathbb{A}$, when (12) does hold true. The choice of which sellers are punished is made by comparing their relative quantity bid $r_n = (q_n - q_{min,n})(q_{max,n} - q_{min,n})^{-1}$. The seller(s) exhibiting the highest r_n-value is (are) then punished.

To ensure power balance within other auction models, some researchers utilize parts of the first rejected bid to fill the gap between demand and supply, for example as done in [8]. Utilizing partial bids holds two flaws: Firstly, bids have to be manipulated to a quantity which might be undesirable for the seller holding the bid. Secondly, this manipulation holds a non-zero probability to cause a violation of the sellers' generator capacity constraints, especially $q_{min,n}$.

As market rule 2 makes an adaptation of bids by the ISO unnecessary, the proposed approach is advantageous for sellers compared to approaches similar to [8].

IV. SIMULATION METHOD

To evaluate the proposed market system the sellers bidding behavior is simulated for the test system given in the appendix. The simulation is conducted as follows:

Firstly, D and $q_{min,n}, q_{max,n} \forall\ n \in [0, N]$ are determined. Subsequently, the initial bids $q_{n,0} \in [q_{min,n}, q_{max,n}]$ are drawn at random and assigned to the corresponding sellers.

Secondly, the bidding behavior is simulated. To do so, the N sellers proceed to submit their bids in the ISO prescribed order. When seller n is next in line, n will calculate P_n for every offer $q_n \in [q_{min,n}, q_{max,n}]$ subject to the other sellers current bids and if a seller $\tilde{n}$ bids 0 MWh also $q_{min,\tilde{n}}$ using (8) and the market rules. Out of the evaluated bids the seller chooses the bid which fits its strategy criteria best. Mostly the best strategy is playing the bid returning the immediate highest profit, but considerations like bid shading or the prevention of SLs may favor other bids. The ISO accepts bids until one of the two termination criteria is hit.

The above-described methodology is chosen as this brute-force-like approach is a simple way to gain an overview of the proposed market system and may be improved in future works. To allow a statistical evaluation of the computed market system each auction is conducted ι-times. Quantities depicted in Section V are averaged over these ι-datasets.

V. RESULTS

Using the test system, cost functions and generator capacity constraints given in the appendix, Table I shows the minimum possible supply q_{min}, the maximum possible supply q_{max}, the minimum price a seller can offer at best p_{min} as well as the highest price a seller might offer p_{max}.

TABLE I
SUPPLY CAPACITIES OF ALL SELLERS IN THE TEST SYSTEM.

Company	q_{min} in MWh	q_{max} in MWh	p_{min}	p_{max}
1	20	200	\$0.22	\$1.03
2	20	210	\$0.21	\$1.09
3	20	200	\$0.20	\$1.05
4	0	3	\$0.001	\$0.001
5	0	0	\$0.001	\$0.001
6	0	8	\$0.001	\$0.001
7	0	0	\$0.001	\$0.001
8	0	20	\$0.001	\$0.001
9	0	40	\$0.001	\$0.001

A. Results regarding power balance, strategy equilibria and marginal price development

To see how the test system behaves using the proposed market system a parameter sweep is conducted for D. The simulation is configured for sellers to consider bid shading and to prevent SLs. The results of the sweep are visualized in Fig. 3. One of the most important findings shall be noted before these figures are explained: For each of the 5000 auctions that were conducted for each of the 681 possible D-settings a SE was reached and the supply was matched by the demand. An analytic prove that every auction conducted under the proposed market system will reach a SE, which

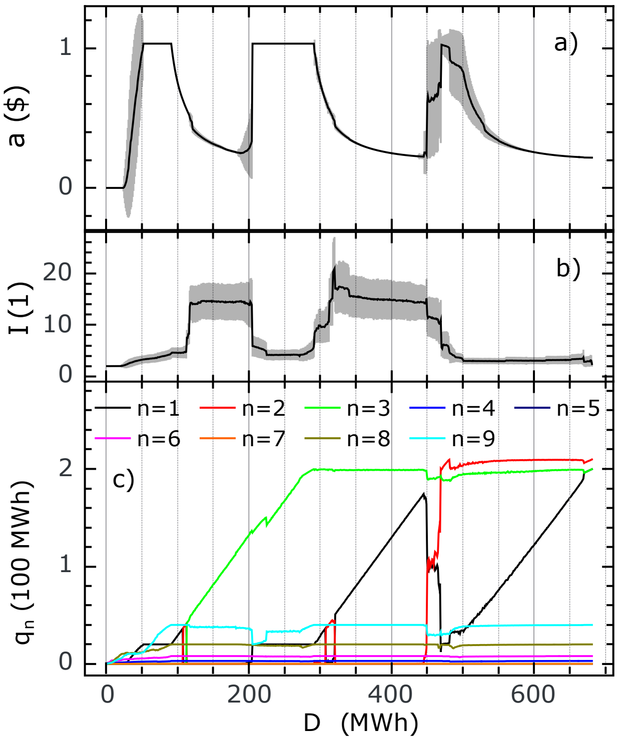

Fig. 3. Simulation of proposed market system for test system in appendix, $\iota = 5000$, $I_{max} = 100$ and SL prevention. a) shows marginal price; b) shows number of preformed iterations. Shaded areas depict corresponding standard deviation; c) shows final q_n-bid.

matches supply and demand is outstanding. However, are more than 1.7 million successfully conducted auctions a promising indicator that the mechanism works satisfactorily.

The complex behavior of the marginal price a and I is explainable using mostly the final bid quantities shown in Fig. 3c. For brevity, only the main observations are discussed.

Firstly, the differences in a for different values of D shall be pointed out. They often originate from the last accepted bid operating either close to or further away from the respective $q_{min,n}$. This observation is of importance as grid operators which are able to influence D can affect a. D can be effected using *e.g.* demand response schemes.

Secondly, the differences in the standard deviation for a shall be noted. When the standard deviation of a is not zero, while SE are reached consistently for a given D, different SE can be reached for D. Which SE is reached depends on the initial conditions and the order that bids are accepted in.

The most prominent feature of I over D are the increased I-values and standard deviation when SLs are prevented, compare Fig. 4. Pronounced changes in I often correlate to pronounced changes in q_n for at least two bidders.

B. Results regarding the impact of strategy loops

The D-sweep for the proposed market system without SL prevention shows two intervals exhibiting a notably increase in I, labelled A and B in Fig. 4. These intervals indicate the demand for which SLs can arise.

For interval A, I is almost constantly reaching I_{max}. Only for $D \in$ C $\subset$ A, I shows slight deviations from I_{max}, which

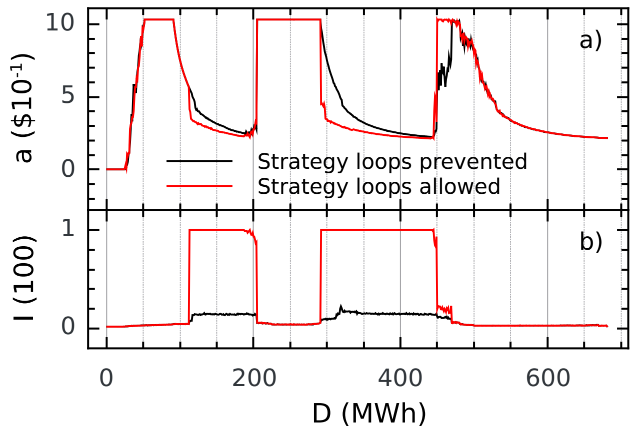

Fig. 4. Impact of SL prevention on the auction price a iterations needed to reach a SE I. Test system in the appendix, $I_{max} = 100$ and $\iota = 1000$.

increase towards larger values of D. As I is far lower for the simulations in which sellers prevent unprofitable SLs, it can be concluded that for interval A SLs can occur. As to the explanation of the deviation of I from I_{max} for $D \in$ C, the increase in the standard deviation for a is pointed out, compare Fig. 3a. The notable deviation of the a-standard deviation from zero indicates the existence of multiple SE which can be reached. In fact, in the simulation two different SE could be detected for each $D \in$ C. Only one of each SE was prone to exhibit a SL. On interval C, SE where $n_{la} = 3$ are prone to trigger SLs unless the loops are prevented. As for $D \in$ C, SE with $n_{la} = 3$ are less likely to be reached with increasing D, I drops slightly with increasing D.

For $D \in$ B, I shows a similar behaviour. When $D \in$ B $\setminus$ D, I reaches the programmed limit I_{max}, while for $D \in$ D a deviation of I from I_{max} can be observed. Why that is follows the same explanation as provided above, including that the SLs arise from auction states, that when SLs would be prevent would constitute a SE with $n_{la} = 3$.

The marginal price increases when SLs are prevented. This was to be expected as SLs are induced by bid shading. In turn, bid shading is committed by sellers trying to increase their profit by using their market power to raise a. When a is decreasing as a result of the bid shade the bid shading seller has a better strategy to play hence a SE is not reached, but a SL is created. Preventing such a SL will keep a from decreasing as a result of unsuccessful bid shading.

VI. CONCLUSIONS

This paper introduced a market framework for electricity procurement auctions using a revealed bidding approach and allowing bidders to repeatedly replace their bids.

Two market rules were proposed to ensure that supply matches demand, focusing on the phenomenon of bid shading. Combining a sellers ability to replace and to shade its bid allowed for the prevention of strategy equilibria due to the emergence of strategy loops. Numerical simulations have confirmed the effectiveness of the market rules and the approach to prevent strategy loops. An influence of the demand on the marginal price was observed, which the

TABLE II

COST FUNCTION PARAMETERS AND GENERATOR CAPACITY CONSTRAINTS OF THE GENERATORS (UNITS AS IN APPENDIX).

Generator m	Seller n	Type	a_m	b_m	c_m	e_m	f_m	$q_{min,m}$	$q_{max,m}$	η_m^{PV}	S_m^{PV}	q_m^{rat}	v_m^{ci}	v_m^{rat}	v_m^{co}
1	1	steam	2	0.1	8	0.8	0.0835	50	125	-	-	-	-	-	-
2	1	steam	1.5	0.09	9	0.4	0.114	20	75	-	-	-	-	-	-
3	2	steam	0.6	0.13	9	0.6	0.0078	20	100	-	-	-	-	-	-
4	2	steam	5	0.08	10.8	0.09	0.063	30	110	-	-	-	-	-	-
5	3	steam	3	0.12	6.5	1	0.1	20	100	-	-	-	-	-	-
6	3	steam	2	0.1	10	0.4	0.185	50	100	-	-	-	-	-	-
7	4	solar	0	0	10^{-3}	0	0	0	-	0.15	0.23	-	-	-	-
8	5	solar	0	0	10^{-3}	0	0	0	-	0.2	0.086	-	-	-	-
9	6	solar	0	0	10^{-3}	0	0	0	-	0.15	0.19	-	-	-	-
10	7	wind	0	0	10^{-3}	0	0	0	-	-	-	35	4	15	31
11	8	wind	0	0	10^{-3}	0	0	0	-	-	-	20	5	16	32
12	9	wind	0	0	10^{-3}	0	0	0	-	-	-	40	6	14	28

Independent System Operator can use to partially control prices using demand response schemes.

Further validation of the above-mentioned results is suggested due to the limitations of the model. In future works, especially an improvement of the rudimentary consideration of sellers' margins promises a better real world comparison.

VII. ACKNOWLEDGEMENT

This work was partially supported by Deutsche Forschungsgemeinschaft (DFG) in the framework of SFB 910.

APPENDIX

The appendix contains the used test system. While the following paragraphs detail the asset m owned by seller n, Tabel II contains the parameters characterising $C_m(q_m)$, $q_{min,m}$ and $q_{max,m}$, all quantities given in MWh. Each seller is eligible to own one asset unless he owns steam-driven generators of which the seller can own multiple, but no additional other types. Units are given ones, then omitted.

Steam-driven generators exhibit fixed $q_{min,m}^{SD}$ and $q_{max,m}^{SD}$. Their cost function is given by

$$C_m^{SD}(q_m) = a_m q_m^2 + b_m q_m + c_m + |e_m \sin(f_m(q_{min,m} - q_m))| \quad (13)$$

with a_m $\left(\frac{\$10^{-4}}{(\text{MWh})^2}\right)$, b_m $\left(\frac{\$}{\text{MWh}}\right)$, c_m (\$), e_m (\$) and f_m $\left(\frac{1}{\text{MWh}}\right)$ being fixed parameters. As sellers owning steam-driven generators are allowed to hold multiple assets of this type, but can hold only one bid at a time, these sellers have to solve an economic load dispatch and a unit commitment problem to obtain the cost function $C_n(q_n)$ which they use to determine p_n.

Solar-powered generators are treated as in [10]. Their marginal cost is set to zero Dollar, but constant maintenance costs are assumed: $C_m^{PV}(q_m) = a_m$, with a_m being a constant. $q_{min,m}^{PV}$ is set to 0 MWh, where as $q_{max,m}^{PV}$ is type, installation and environment dependent and determined by

$$q_{max,m}^{PV} = \eta_m S_m \Phi_m (1 - \tfrac{1}{200}\, T_m). \quad (14)$$

with photovoltaic-panel conversion coefficient η_m (1), surface area S_m (km^2), solar irradiance Φ_m (W/m^{-2}) and temperature T_m ($^\circ C$).

Wind-powered generators behave like solar-powered generators regarding $C_m^{Win}(q_m) = a_m$ and $q_{min,m}^{Win} = 0$. $q_{max,m}^{Win}$ is determined by wind speed (WS) v_m, cut in WS $v_{ci,m}$, rated power $q_{rat,m}$, cut out WS $v_{co,m}$, rated WS $v_{r,m}$; all WS in $\frac{m}{s}$ such that

$$q_{max,m}^{Win} = \begin{cases} q_{r,m} \frac{(v_m - v_{ci,m})^2}{(v_{ci,m} - v_{r,m})^2} & \text{if } v_{ci,m} \le v_m < v_{r,m} \\ q_{r,m} & \text{if } v_{r,m} \le v_m \le v_{co,m} \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

To determine $q_{max,m}$ for solar- and wind-powered assets in a simulation v_m, Φ_m and T_m are drawn at random using the following limits: $v_m \in [0, 40]$, $\Phi_m \in [0, 1000]$ and $T_m \in [0, 40]$.

REFERENCES

[1] R. Ferroukhi, J. Sawin, and F. Sveriss, REthinking Energy 2017: Accelerating the global energy transformation. Abu Dhabi: International Renewable Energy Agency, 2017.

[2] B. Schäfer, C. Beck, K. Aihara, D. Witthaut, and M. Timme, "Non-Gaussian power grid frequency fluctuations characterized by Lévy-stable laws and superstatistics," Nature Energy, vol. 3, no. 2, pp. 119–126, Feb. 2018.

[3] M. J. Morey, "Power Market Auction Design: Rules and Lessons in Market-Based Control for the New Electricity Industry," Edison Electric Institute, Washington, DC 20004-2696, Sep. 2001.

[4] N.-H. von der Fehr and D. Harbord, "Spot Market Competition in the UK Electricity Industry," Economic Journal, vol. 103, no. 418, pp. 531–46, 1993.

[5] P. Klemperer, "What Really Matters in Auction Design," The Journal of Economic Perspectives, vol. 16, no. 1, pp. 169–189, 2002.

[6] L. T. A. Maurer and L. A. Barroso, Electricity Auctions: An Overview of Efficient Practices. Washington, DC 20433: The International Bank for Reconstruction and Development/The World Bank, 2011.

[7] L. Clauser and C. R. Plott, "On the Anatomy of the 'Nonfacilitating' Features of the Double Auction Institution in Conspiratorial Markets," in the double auction market: institutions, theories, and evidence, D. J. Friedman and J. M. Rust, Eds. Reading, MA: Addison-Wesley, 1992, pp. 333–353.

[8] L. Parisio and B. Bosco, "Market Power and the Power Market: Multi-Unit Bidding and (In)Efficiency in Electricity Auctions," International Tax and Public Finance, vol. 10, no. 4, pp. 377–401, Aug. 2003.

[9] M. G. Pollitt, "Lessons from the history of independent system operators in the energy sector," Energy Policy, vol. 47, pp. 32–48, Aug. 2012.

[10] C. Li, Y. Xu, X. Yu, C. Ryan, and T. Huang, "Risk-Averse Energy Trading in Multienergy Microgrids: A Two-Stage Stochastic Game Approach," IEEE Transactions on Industrial Informatics, vol. 13, no. 5, pp. 2620–2630, Oct. 2017.