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Perspective

Wigner's friends, tunnelling times and Feynman's "only mystery of quantum mechanics"

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Abstract – Recent developments in elementary quantum mechanics have seen a number of extraordinary claims regarding quantum behaviour, and even questioning internal consistency of the theory. These are, we argue, different disguises of what Feynman described as quantum theory's "only mystery".

Real mystics don't hide mysteries, they reveal them. They set a thing in broad daylight, and when you've seen it it's still a mystery. But the mystagogues hide a thing in darkness and secrecy, and when you find it, it's a platitude.

G. K. Chesterton in *The arrow of heaven*

I will take just this one experiment, which has been designed to contain all of the mysteries of quantum mechanics.... Any other situation in quantum mechanics, it turns out, can always be explained by saying "You remember the case of the experiment with the two holes?"

R. P. Feynman in *Character of Physical Law*

Introduction. – We would like to begin by assuring the reader that the first item in the epigraph was not chosen in order to be offensive. However, the two quotes suit the purpose of this article so well that it would be a pity not to include them. Since Wigner's surprising suggestion [1] that the laws of quantum mechanics may need to be modified to accommodate human consciousness, there have been many other eye-catching claims, associated with quantum behaviour. The list includes, to name a few, particles being in several places at the same time [2], electrons "disembodied of their charge" [3], quantum "Cheshire cats" [4], photons with "discontinuous trajectories" [5], observer-dependent facts [6,7], doubts about the internal consistency of elementary quantum theory [8] and the conflict of "faster-than-light tunnelling" with special relativity [9]. A recent claim that it takes a finite amount of time to tunnel across a potential barrier can be found in [10].

Feynman, for his part, maintained that all rarities of quantum mechanics can be traced back to the double slit experiment. So are these quantum "paradoxes" just different form of the familiar double-slit conundrum? We show that most, if not all of them, are.

The double-slit experiment. Feynman's rules. – It is worth recalling the description, given by Feynman in his undergraduate text [11]. An electron, emitted by a source (s), can reach a point on a screen, x, by passing via two slits, labelled 1 and 2. If it is impossible *in principle* to establish which of the two paths was taken, the probability of arriving at x is given by the absolute square of the sum of the path amplitudes, $P(x) = |A(x \leftarrow 1 \leftarrow s) + A(x \leftarrow 2 \leftarrow s)|^2$. If it is possible, even *in principle*, the path probabilities are added, $P(x) = |A(x \leftarrow 1 \leftarrow s)|^2 + |A(x \leftarrow 2 \leftarrow s)|^2$. The fact that these two situations are mutually exclusive constitutes the Uncertainty Principle (UP).

A useful illustration can be provided by considering a two-level system (S) (a spin-1/2) and two probes $\overline{D}$ and D, used to measure the spin's condition at t_1 and $t_2 > t_1$, respectively. The joint initial state at some $t_0 < t_1$ is

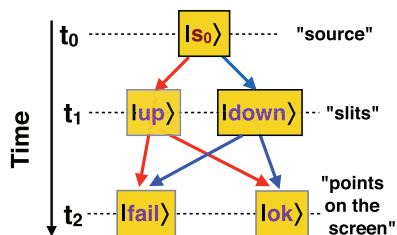─────────
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Fig. 1: A primitive double slit experiment. A spin can reach its final states via four virtual paths.

$|\Psi(0)\rangle = |D(0)\rangle \otimes |\overline{D}(0)\rangle \otimes |s_0\rangle$, and just after t_1 we have

$$|\Psi(t_1)\rangle = |D(0)\rangle \otimes [\langle up|s_0\rangle|\overline{D}(up)\rangle \otimes |up\rangle$$
$$+\langle down|s_0\rangle|\overline{D}(down)\rangle \otimes |down\rangle], \quad (1)$$

where $|up\rangle$ and $|down\rangle$ form a measurement basis in the spin's Hilbert space, and we assumed, for simplicity, only the spin has its own dynamics.

If the second measurement engages only the spin, by using a measurement basis,

$$|ok\rangle = \alpha|up\rangle + \beta|down\rangle, \quad |fail\rangle = \gamma|up\rangle + \delta|down\rangle, \quad (2)$$

the probability for each of the four possible outcomes is easily found to be

$$P(i,j) = |A^S(j \leftarrow i \leftarrow s_0)|^2, \quad (3)$$
$$i = up, down, \quad j = ok, fail,$$

where

$$A^S(j \leftarrow i \leftarrow s_0) \equiv \langle j|\hat{U}^S(t_2,t_1)|i\rangle\langle i|\hat{U}^S(t_1,t_0)|s_0\rangle, \quad (4)$$

and $\hat{U}^S(t',t)$ is the spin's evolution operator. If the second measurement engages both the spin and the probe $\overline{D}$, by using a basis containing the states

$$|ok\rangle = \alpha|\overline{D}(up)\rangle \otimes |up\rangle + \beta|\overline{D}(down)\rangle \otimes |down\rangle,$$
$$|fail\rangle = \gamma|\overline{D}(up)\rangle \otimes |up\rangle + \delta|\overline{D}(down)\rangle \otimes |down\rangle, \quad (5)$$

the probabilities for the two corresponding outcomes are given by

$$P(j) = |A^S(j \leftarrow up \leftarrow s_0) + A^S(j \leftarrow down \leftarrow s_0)|^2, \quad (6)$$
$$j = ok, fail.$$

In the first case, system's past can be determined by inspecting the state of the probe $\overline{D}$ after the experiment is finished. Accordingly, the four scenarios in fig. 1 can be distinguished, and endowed with the probabilities in eq. (4). In the second case, the record carried by $\overline{D}$ has been erased and, according to the UP, it should be impossible to say whether the spin's condition at t_1 was up or $down$. A reader, worried about the collapse of the wave function, may find some comfort in noting that the calculation of probabilities is reduced to evaluation of matrix elements of unitary operators in the system's Hilbert space, and the collapse problem is not mentioned at all.

The UP rule does not rely on human experience. Mere existence of a photon, scattered near one of the slits, and carrying a record of the electron's past, is enough to destroy the interference pattern, even if the photon is never observed.

Feynman's advice was to accept the above rules as the definitive content of the quantum theory, and avoid in this way the "blind alley", reserved for those who ask "how can it be like that?" [12]. We follow the advice, and ask instead a simpler question, are the recently discovered quantum "paradoxes" mere illustrations of Feynman's rules? Some of them, such as those in [2–5], certainly appear to be just that. With one of the two holes closed, an electron is able to arrive where it could not arrive with both of them open. For this reason, the amplitudes should not be used to prove that the electron was at a particular place in the past[1]. This explains the "quantum paradoxes" constructed in [2–5] (for more details see [13]). Would it also be true for the cases discussed in [6–10]?

The question of consciousness. – Recent work on various versions of the Wigner's friend scenario [6–8] rarely mentions the consciousness of the participants explicitly. However, the original Wigner's analysis [1] was motivated by his concern about the role played by consciousness in quantum measurements, and we must attend to the question before we proceed. According to Wigner [1], eq. (1) is valid when it describes an inanimate object, but is unacceptable if $|\overline{D}(up/down)\rangle$ is understood as a state of conscious Wigner's friend, (F), who has just seen an outcome up or $down$. Would the Feynman's rules of the previous section apply in the presence of intelligent Observers?

There are at least three ways to look at an Observer's $(e.g., F', W')$, consciousness, while maintaining that quantum mechanics should apply to all physical objects, regardless of their size and complexity.

(a) Condition of consciousness can be deemed to be fully determined by that of the inanimate "physico-chemical substrate" [1] of an Observer's organism. Then Wigner's objection can be dismissed by noting that in eqs. (1) the states $|\overline{D}(up)\rangle$ and $|\overline{D}(down)\rangle$ are used for calculating the odds on F seeing up or $down$, but never both up and $down$ at the same time. However, the difficulty, resolved in this manner, returns when one considers eqs. (5), (7). Then an outcome ok, seen by Wigner (W), who performs the second measurement, would leave the joint system, which now includes the friend F, in an undesirable state of "suspended animation" [1], $\alpha|\overline{D}(up)\rangle \otimes |up\rangle + \beta|\overline{D}(down)\rangle \otimes |down\rangle$.

(b) A different problem arises if one were to try placing Observer's "extra-observational intellectual inner life" [14] outside the scope of quantum theory. Now, after having seen an up, F becomes permanently aware of his/her outcome and this information resides in a place obscure to quantum reasoning. Then if W duly entangles all material

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[1]Another Feynman's quote [12] comes to mind: "*But when you have no apparatus to determine through which hole the thing goes, then you cannot say that it either goes through one hole or the other. You can always say it - provided you stop thinking immediately and make no deductions from it. Physicists prefer not to say it, rather than stop thinking at the moment*".

records produced by F, his probability, $P(ok)$ in eq. (7) should contain an interference term $2\mathrm{Re}[A^{S*}(ok \leftarrow up \leftarrow s_0)A^S(ok \leftarrow down \leftarrow s_0)]$. But if F were able to declare that his/her outcome was up, the Uncertainty Principle would be violated, since W's interference picture would have to co-exist with F's "which way?" information.

(c) The third possibility [15,16] is a compromise between the first two, and is consistent with Feynman's analysis of the double slit experiment. As in (b), F's consciousness should not be analysed by means of quantum theory. However, unlike in (b), an Observer is not permanently aware of his/her outcome but needs, when necessary, to consult a material record such as the one kept by his/her memory or in a note, accessible to human senses. A mere act of perception on the part of an Observer adds nothing to quantum analysis, but a new record produced as a result (recall Feynman's photon destroying the double slit interference), needs to be taken into account. As in (a) only material objects need to be subjects of a quantum analysis.

Accepting (c) has further implications. In particular, at any given time, the facts about the outcomes of past experiments are contained in the set of material records. The facts are objective in the sense of being equally valid to all intelligent agents. We note that Feynman [11] goes even further in suggesting that facts do not need to be verified, in order to make the presence of records felt. For example, the mere existence of an additional spin, prepared by F in a state $|up\rangle$ after seeing an up, would remove the interference term from W's probabilities in eq. (7) even if this second spin were to be sent to Alpha Centauri, and never seen by anyone again. A record can be created, as well as destroyed, in which case the information it contains may be irretrievably lost. Observers are free to decide which experiments to make, and can sometimes use the arrangements already provided by Nature, as would happen, for example, with Wheeler's "cosmic interferometer" [17]. Finally, as suggested by Wigner, Observer's consciousnesses "never seem to interact with each other directly, but only via the physical world" [1], $i.e.$, via the world of meaningful records.

Throughout the rest of the paper, we will continue to assume that quantum theory is valid for all material objects, and leave the Observers all but outside our discussion in accordance with (c). Next we turn to the extended Wigner's friend scenario, recently proposed in [8] in order to expose alleged inconsistencies of the elementary quantum mechanics.

Extended Wigner's friend scenarios and quantum postmodernism. – With what has just been said, the original Wigner friend scenario fits the description of the double-slit experiment (1), where the states $|\overline{D}(up/down)\rangle$ refer to all material records produced by F. One can, however, talk himself into a logical contradiction with the help of the wave function in eq. (1). Indeed, measuring the spin, F obtains a definite outcome up with the probability (to shorten the notations, we put $\hat{U}^S(t',t) = 1$) $\langle\Psi(t_1)|up\rangle\langle up|\Psi(t_1)\rangle = |\langle up|s_0\rangle|^2$. On the other hand, the probability of W's outcome ok in eq. (7), $\langle\Psi(t_1)|ok\rangle\langle ok|\Psi(t_1)\rangle$, contains an interference term $2\mathrm{Re}[\langle s_0|down\rangle\langle down|ok\rangle\langle ok|up\rangle\langle up|s_0\rangle]$. This should leave the question "up or $down$?" without an answer, yet F appears to know that the answer was up. The reader may be able to see through this apparent contradiction. We will give a full explanation after considering the case where a similar "contradiction" appears in a yet more dramatic form.

In what has become known as the extended Wigner's friend scenario (EWFS) [8], two two-level systems, called "the coin" (C), and "the spin" (S), respectively, are prepared in a product state at $t = t_0$. At t_1 an Observer $\overline{F}$ measures the coin in a basis $|heads/tails\rangle$, and at $\tau > t_1$ the coin is entangled with the spin. At $t_2 > \tau$, F measures the spin in a basis $|up/down\rangle$. The experiment is finished when at $t_3 > t_2$ external Observers, $\overline{W}$ and W, measure the entire material content of $\overline{F}$'s and F's labs using bases

$$
\begin{aligned}
&|\overline{Fail}/\overline{Ok}\rangle = \\
&[|\overline{D}(heads)\rangle \otimes |heads\rangle \pm |\overline{D}(tails)\rangle \otimes |tails\rangle]/\sqrt{2}, \\
&|Fail/Ok\rangle = \\
&[|D(up)\rangle \otimes |up\rangle \pm |D(down)\rangle \otimes |down\rangle]/\sqrt{2}.
\end{aligned} \tag{7}
$$

The initial state and the interaction at $t = \tau$ are chosen so that the state of $\overline{F}$'s and F's labs just before $\overline{W}$ and W make their measurements is given by

$$
\begin{aligned}
|\Phi\rangle = [&|\overline{D}(tails)\rangle \otimes |tails\rangle \otimes |D(up)\rangle \otimes |up\rangle \\
+&|\overline{D}(heads)\rangle \otimes |heads\rangle \otimes |D(down)\rangle \otimes |down\rangle \\
+&|\overline{D}(tails)\rangle \otimes |tails\rangle \otimes |D(down)\rangle \otimes |down\rangle]/\sqrt{3},
\end{aligned} \tag{8}
$$

and can be used to evaluate four different probabilities. The likelihood of $\overline{F}$ and F seeng the outcomes $heads$ and up is zero,

$$
P(heads, up) = \langle\Phi|heads\rangle\langle heads| \otimes |up\rangle\langle up|\Phi\rangle = 0. \tag{9}
$$

A simple calculation shows that also the probabilities of $\overline{F}$ and W seeing $tails$ and Ok, and of F and $\overline{W}$ seeing $down$ and $\overline{Ok}$ vanish,

$$
P(tails, Ok) = \langle\Phi|tails\rangle\langle tails| \otimes |Ok\rangle\langle Ok|\Phi\rangle = 0, \tag{10}
$$

$$
P(down, \overline{Ok}) = \langle\Phi|down\rangle\langle down| \otimes |\overline{Ok}\rangle\langle\overline{Ok}|\Phi\rangle = 0. \tag{11}
$$

Equations (9)–(11) suggest that $\overline{W}$ and W will never see $\overline{Ok}$ and Ok, at the same time. Yet, a similar calculation shows that the outcomes $\overline{Ok}$ and Ok will occur together in about $1/12$ of all trials,

$$
P(\overline{Ok}, Ok) = \langle\Phi|Ok\rangle\langle Ok| \otimes |\overline{Ok}\rangle\langle\overline{Ok}|\Phi\rangle = 1/12. \tag{12}
$$

All four probabilities in eqs. (9)–(12) are legitimate results, and the apparent contradiction needs to be resolved in one way or another.

Frauchiger and Renner [8] were quick to tell their readers that this is where quantum mechanics looses the plot, by making too many conflicting predictions where only one is required.

Alternatively, one may assume that all "facts", as expressed by eqs. (9)–(12) are indeed valid at the same time, but not to everyone. In other words, they refer to private perceptions of the participants, held in "sealed" laboratories. With the labs isolated from each other, there is no danger of comparing the conflicting outcomes, and all perceived results are equally valid or, if one prefers, equally invalid. The view that quantum theory may only describe such "observer-dependent" facts was proposed by Brukner [6] and found further support, e.g., in [7].

There is, however, no need for a radical departure from the standard textbook rules [11]. The "contradiction", discussed in the first paragraph of this section, is a spurious one. The probabilities in eqs. (4) and (7) refer to two mutually exclusive scenarios, in which W either erases all records produced by F, or preserves them. Like the proverbial cake, a record cannot be both present and destroyed, and the results (4) and (7) should never be played against each other (we would like to avoid using an over-used term "contextual paradox"). The wave function (1) just before W's measurement contains no information about the course of action W is about to take, and contains the answers for each of the W's arrangements. It remains one's own responsibility to decide which one to use.

Precisely the same happens when the number of participants is increased to four. With the choice of the initial state, and the coupling between the coin and the spin, the system maps onto a three-slit setup, where each of the four "points on the screen" can be reached via three virtual paths (see fig. 2 and ref. [18] for more details). Now $\overline{W}$'s and W's may either erase or preserve $\overline{F}$'s and F records, respectively. There are four exclusive scenarios, and one easily finds [18] that i) eq. (9) holds true if both records are preserved, ii) eq. (10) is valid, provided only $\overline{F}$'s record survives, iii) eq. (11) is valid, provided only F's record survives. Finally, iv) eq. (12) applies when both records are erased, and all information about the past is irretrievably lost. Again, there is no contradiction if one follows Feynman's rules of ref. [11]. Next we consider a different case where Feynman's analysis of the double-slit conundrum also plays a crucial role.

The search for the "tunnelling time". – The amount of time it takes a *classical* particle to cross a given region of space is a useful quantity. In quantum tunnelling, a particle enters and leaves the barrier region, so it is only natural to assume that it spends there some duration τ. In a recent *Nature* publication [10] the authors put a number on the duration spent by a tunnelling atom in the barrier, and considered the issue "resolved".

Well, not quite, as we will show next. There are at least two approaches measuring the duration spent by a quan-
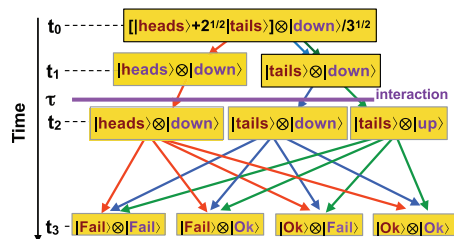

Fig. 2: Twelve virtual paths of a joint system $\{coin + spin\}$ in extended Wigner's friend scenario.

tum particle in a given region of space Ω. One, originally proposed by Baz' [19], and recently used in [10], consists of equipping the particle with a magnetic moment (spin) which rotates in a small magnetic field, confined to the Ω. For a classical particle, a ratio of the rotation angle φ to the Larmor frequency ω_L yields the desired result $\tau_{cl} = \varphi/\omega_L$.

For a quantum particle, making transition between initial and final states $|\psi\rangle$ and $|\phi\rangle$ over a time T, one finds many possible durations, and is able to define the corresponding probability amplitudes $A(\phi \leftarrow \tau \leftarrow \psi)$ [20] by summing the amplitudes $\exp\{iS[x(t)]\}$ [21] over the Feynman paths, spending in Ω precisely τ seconds. With this, a non-relativistic transition amplitude can be seen to result from interference between all allowed durations,

$$\langle\phi|\hat{U}(T)|\psi\rangle = \int_0^T A(\phi \leftarrow \tau \leftarrow \psi)\mathrm{d}\tau. \qquad (13)$$

With many virtual durations in place, the final state of the spin, travelling with the particle, will be a superposition $\int_0^T |\varphi = \omega_L\tau\rangle A(\tau \leftarrow \psi)\mathrm{d}\tau$, where $|\varphi\rangle$ stands for the initial spin's state, rotated by an angle φ around the direction of the field. Notably, a *sum* of such rotations is not a new rotation (there is no single angle to divide by ω_L), and the Larmor clock method meets with its main difficulty (see fig. 3(b)).

Equation (13) describes a multi-slit problem where a particle can reach the "point on the screen", $|\phi\rangle$, by passing through a continuum of "slits", labelled by τ, as shown schematically in fig. 3(a). As in the original double-slit case, one has a choice between keeping the transition intact, and not knowing the duration τ, or measuring τ at the cost of destroying the interference, and with it the studied transition [20,22,23]. The only exception is the classical case, where $A(\phi \leftarrow \tau \leftarrow \psi)$ rapidly oscillates everywhere but in the vicinity of the classical value $\tau = \tau_{cl}$, and $\omega_L\tau_{cl}$ defines a unique angle by which the spin rotates. In a classically forbidden tunnelling transition no unique duration can be selected. To make the matter worse, the $A(\phi \leftarrow \tau \leftarrow \psi)$ is not itself small, and the very small tunnelling amplitude in the l.h.s. of eq. (13) is a result of a very precise cancellation. Any attempt to perturb this delicate balance is, therefore, likely to destroy tunnelling.

The second method to measure the delay, experienced by a particle in a potential, is to compare the particles's

Fig. 4: The distance between the centre of mass of a tunnelling wave packet and that of its freely propagating counterpart. An attempt to deduce from it the duration spent in the barrier meets with the same difficulty as the application of a Larmor clock shown in fig. 3(b).
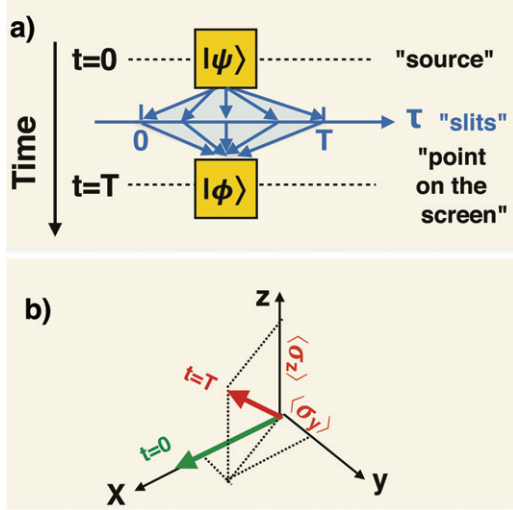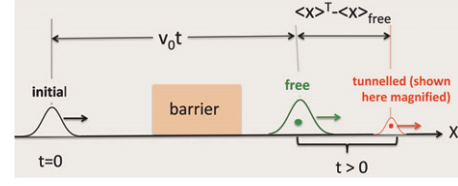

Fig. 3: (a) A non-relativistic quantum particle can reach its final state $|\phi\rangle$ by spending any virtual duration $0 \leq \tau \leq T$ in a specified region of space Ω. (b) In each virtual scenario, a spin-$1/2$ initially polarised along the x-axis rotates in the xy-plane by $\varphi = \tau\omega_L$. However, a superposition of such rotations is not a new rotation in the xy-plane, and at $t = T$ the spin acquires also a non-zero z-component. For a weakly perturbing Larmor clock one finds $\langle\sigma_y(T)\rangle \approx \mathrm{Re}[\tau_w]$ and $\langle\sigma_z(T)\rangle \approx \mathrm{Im}[\tau_w]$, with τ_w defined in eq. (22).

final position with that of its free moving counterpart [24] (see fig. 4). For a classically forbidden transition, it meets with the same difficulty. The state of a particle with a momentum p and energy E, transmitted across a finite range barrier, or well, is given by $T(p)\exp(ipx)$, where $T(p)$ is the barrier's transmission amplitude. Using a Fourier transform $\xi(x') = (2\pi)^{-1}\int_{-\infty}^{\infty} T(p')\exp(ip'x')\mathrm{d}p'$, one can write the transmitted state as a superposition of plane waves, each displaced in space by a distance x' [25,26],

$$T(p)\exp(ipx) = \int \mathrm{d}x'\xi(x')\exp[ip(x-x')]. \quad (14)$$

A similar expression exists for a transmitted wave packet with a mean momentum p_0,

$$\psi^T(x,t) = \exp[ip_0x - iE(p_0)t]$$
$$\times \int G_0(x-x',t)\eta(x',p_0)\mathrm{d}x', \quad (15)$$

where $\eta(x',p_0) \equiv \exp(-ip_0x')\xi(x')$, and $G_0(x,t)$ is the envelope of the same initial wave packet, as it would be in the absence of the potential,

$$\psi_0(x,t) \equiv \exp[ip_0x - iE(p_0)t]G_0(x,t). \quad (16)$$

In general, a sum of spatial shifts is not a new shift, and no single shape is selected from the collection of the envelopes $G(x-x',t)$ in eq. (15). Again, the only exception is the classical limit, where an oscillatory $\eta(x',p_0)$ develops a stationary region around a classical value $x' = x'_{cl}(p_0)$, and

one recovers the classical result $\psi^T(x,t) \approx \psi_0(x-x'_{cl},t)$. For a barrier not supporting bound states, $\xi(x') \equiv 0$ for $x' > 0$ [25], so that none of the envelopes in (15) are *advanced*, relative to the freely propagating $G_0(x,t)$. In a tunnelling regime, the spatial delay experienced by the particle is lost to interference [26], just like the duration τ of the previous example. The reader may reasonably ask if, according to the Uncertainty Principle, a tunnelling time cannot exist, how could McColl conclude [24] that tunnelling is a delay-free process? And what was measured in the experiment reported in [10]?

Complex times and weak "measurements". – To answer these questions, we consider again the double-slit case (1)–(4), but this time with the probe $\overline{D}$ replaced by a von Neumann pointer [14] with position x, prepared in a state $|G_0\rangle$, $\langle x|G_0\rangle \sim \exp(-x^2/\Delta x^2)$. The pointer is set up to measure an operator $\hat{B} = |up\rangle B_1\langle up| + |down\rangle B_2\langle down|$ with eigenvalues B_1 and B_2. Now the state of the pointer, provided the spin is found in a state $|ok\rangle$, is given by

$$G(x,t_2) = G_0(x-B_1)A_1 + G_0(x-B_2)A_2, \quad (17)$$

where we use a shorthand A_1 and A_2 for $A^S(ok \leftarrow up \leftarrow s_0)$ and $A^S(ok \leftarrow down \leftarrow s_0)$, respectively. The probability for the observed system to arrive in $|ok\rangle$ is

$$P(ok) = \int |G(x,t_2)|^2 dx = |A_1|^2 + |A_2|^2$$
$$+2\mathrm{Re}[A_2^*A_1] \times \int G(x-B_1)G(x-B_2)\mathrm{d}x. \quad (18)$$

The pointer carries a record of the spin's condition at t_1, and in the accurate limit $\Delta x \to 0$ it is always possible to find out whether it was *up* or *down*. In the opposite limit, $\Delta x \to \infty$, a pointer's reading x cannot be used to distinguish between the paths, and $P(ok) = |A_1 + A_2|^2$. The possible pointer readings are distributed between $-\infty$ and ∞, and the value of $\hat{B}$, measured in this way, is well and truly *indeterminate*.

One can also use eq. (18) to calculate the average pointer reading $\langle x\rangle$, conditional on the system arriving at $|ok\rangle$. For $\Delta x \to \infty$ one obtains [27]

$$\langle x\rangle \xrightarrow[\Delta x \to \infty]{} \mathrm{Re}\left[\frac{B_1A_1 + B_2A_2}{A_1 + A_2}\right]. \quad (19)$$

At first glance, eq. (19) appears to contradict the Uncertainty Principle, because a definite value, $\langle x \rangle$, has been obtained in a situation where everything was meant to be indeterminate. This is, however, not so, since the initial and final states $|s_0\rangle$ and $|ok\rangle$ can always be chosen so as to give the r.h.s. of eq. (19) *any* desired value —large, small, positive, negative, or zero [27]. The Uncertainty Principle still applies, albeit at a different level. Clearly, one thing that can be learnt from eq. (19) is that adding the system's amplitudes, multiplied by B_1 and B_2, dividing the result by their sum, and taking the real part of the fraction, gives a particular number. It is far less clear, since Feynman's rules [11] give no clue in this regard, whether eq. (19) can have any other significance.

The non-perturbing "weak" limit $\Delta x \to \infty$ was first studied in [28], where the quantity in brackets in eq. (19) was called the "weak value of an operator $\hat{B}$", equally written as

$$B_w \equiv \frac{\langle ok|\hat{B}|s_0\rangle}{\langle ok|s_0\rangle}. \qquad (20)$$

The authors of [28] made two claims which, while no doubt helping subsequent popularity of the subject [29], have led to a fair amount of confusion, including that surrounding the tunnelling time problem discussed here. Firstly, it was claimed that eq. (20) "defined a new kind of quantum variable", whereas, as we have seen, they were describing a particular combination of the familiar probability amplitudes. Secondly, they found it surprising that a weak value of a spin-$1/2$ component could take a value of 100. According to the Uncertainty Principle, it would be surprising if it could not.

It is easy to see what all this means for the quest to find and measure "the tunnelling time". Neglecting the spreading of the wave packet, $G_0(x,t) \approx G(x - v_0 t)$, and comparing eq. (15) with eq. (17), we note that we are dealing with an inaccurate measurement of the quantity which we earlier described as the spatial delay, or shift, x', with which the transmitted particle with a momentum p_0 leaves the barrier. The particle's own position x plays the role of the pointer, and by sending $\Delta x \to \infty$ we can make the measurement weak. In this limit, the distance between the centre of mass of the transmitted wave packet and its freely propagating counterpart is given by an analogue of eq. (19) [26],

$$\langle x \rangle^T - \langle x \rangle_{free} \xrightarrow[\Delta x \to \infty]{} \text{Re}[x'_w] \equiv$$
$$\text{Re}\left[\frac{\int x' \eta(x',p_0)\mathrm{d}x'}{\int \eta(x',p_0)\mathrm{d}x'}\right] = -\partial_p \Phi(p_0), \qquad (21)$$

where $\Phi(p)$ is the phase of the transmission amplitude, $T(p) = |T(p)|\exp[i\Phi(p)]$. This can be verified experimentally [30], but what can be learnt from such an experiment? The answer is the same as before: a barrier can be characterised by distribution of virtual shifts it imposes upon the transmitted particle. The measured

distance (21) is a weighted sum of the corresponding amplitudes. An attempt to give this quantity a deeper meaning immediately meets with difficulties[2]. For a broad rectangular barrier of height V, width d, and a non-relativistic particle of mass μ, one finds $T(p,V) \sim \exp(-ipd)\exp(-d\sqrt{2\mu V - p^2})$. According to eq. (21), the (small) tunnelled pulse is *advanced* by roughly the barrier's width d, no matter how broad the barrier is. This Hartmann effect [32] has more dramatic consequences if one tries to deduce from eq. (21) the "time $\mathcal{T}$ the particle has spent in the barrier." The result $\mathcal{T} \approx 0$ seems to point towards a conflict between quantum mechanics and special relativity [9]. The problem, however, has no relativistic implications. In eq. (15) the envelopes $G(x - x', t)$, from whose front tails the transmitted wave packet is built, are all delayed even relative to free propagation, and certainly so relative to the motion at the speed of light.

Time measurements by means of a weakly perturbing Larmor clock [10,19] suffer from the same deficiency [20, 22,23]. They inevitably involve a "complex time", τ_w [23],

$$\tau_w \equiv \frac{\int_0^T \tau A(\phi \leftarrow \tau \leftarrow \psi)\mathrm{d}\tau}{\int_0^T A(\phi \leftarrow \tau \leftarrow \psi)\mathrm{d}\tau} = \text{Re}[\tau_w] + i\text{Im}[\tau_w], \quad (22)$$

another "mean value", calculated with an alternating complex value distribution, which cannot be interpreted as a meaningful duration spent in a given region of space. Indeed, by choosing $|\phi\rangle$ so that $\langle\phi|\hat{U}(T)|\psi\rangle \to 0$, one can always ensure that $\text{Re}[\tau_w]$, $\text{Im}[\tau_w]$, and $|\tau_w|$ exceed the duration of motion, T, which rather proves the point. A measured value of $\text{Re}[\tau_w]$ represents a relation between the amplitudes $A(\phi \leftarrow \tau \leftarrow \psi)$ in eq. (13), while the Uncertainty Principle ensures that the duration spent in the region where the spin rotates remains indeterminate, just like the slit chosen in the double-slit experiment. We are back to Feynman's "only mystery of quantum mechanics".

Conclusions and discussion. – In summary, one finds Feynman Uncertainty Principle at the centre of many recent developments in elementary quantum mechanics. The principle, we recall, implies that in an interference phenomenon the system's past remains indeterminate, and an inquiry inevitably destroys the phenomenon. In [12] Feynman wrote "*They can give you a wider class of experiments than just the two slit interference experiment. But that is just repeating the same thing to drive it in*". Well, they did, and often without mentioning the "thing" in question. Today "Feynman's blind alley" [12] comes adorned with many extraordinary, yet not particularly useful claims. Most of them arise either from mixing incompatible scenarios (we promised not to use the words "contextual paradox"), or from giving probability

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[2]One can object by saying "but tunnelling particles *do* arrive early at a fixed detector [30]". Yes, but it this is just a different way of measuring the distance in l.h.s. of eq. (21). The delay of interest x' in its r.h.s. remains scrambled by averaging with an alternating distribution [31].

amplitudes a meaning they were not suppose to have in the first place (see footnote [1]).

Thus, accepting the general principles of quantum mechanics, as given in [11], one notes that the seeming conflict between the statements (9)–(12) does not prove that quantum theory is "inconsistent" but only that an interference pattern cannot co-exist with the knowledge of the slit used by an electron. Neither does it imply that each Observer is entitled to his or her own facts. The probabilities (9)–(12) refer to mutually exclusive situations, when some of the virtual scenarios can be distinguished, and some others cannot.

The so-called "weak measurements", used in [2–4,28], quantify particular relations between Feynman's amplitudes [27], but provide no insight into the amplitudes' meaning or physical significance beyond what was said in [11]. A quantum particle is not in two places at the same time —it is either in one place, or it is impossible to say where it is (see footnote [1]). Similarly, using the "weak values" as the evidence of the particle's presence does not prove that photons can be found in a place they never entered [5], but only that two non-zero amplitudes may sum up to zero [33].

For the same reason, tunnelling which results from interference between different delays, whichever way one looks at it, cannot be described by a single meaningful duration, and poses no threat to special relativity. Finding such a delay would invalidate the UP, and uproot quantum mechanics "protected" by the principle [11]. This is why the claim to have measured the traversal time and "*resolved the controversy regarding how long a tunnelling particle spends in the barrier region* made in [10] by Ramos *et al.* is grossly misleading, even if the experiment itself is technically perfect.

In summary, it may not be a bad idea to check whether a proposed experiment, no matter how spectacular, could be just another illustration of Feynman's "only mystery" [11]. If it is, quantum mechanics could only be accused of being quantum mechanics as we know it. If it is not, a new theory would be needed. Yet by ignoring the Uncertainty Principle altogether one risks arriving at a conclusion, which is either wrong, or after scrutiny, will prove to be a "repetition of the same thing" [12], in other words, a platitude.

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