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## **PROBLEMS WITH VARIATION: AN EDUCATIONAL EXPERIENCE OF CULTURAL TRANSPOSITION WITH PROSPECTIVE PRIMARY TEACHERS**

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*The paper presents some theoretical reflections and some methodological notes about a Professional Development (PD) path worked out during the last two years by Italian researchers for prospective Primary teachers. The theoretical construct of Cultural Transposition defines the framework of the PD path's activities and the related research. It was used to define an interesting cultural lens to delineate possible new approaches for effective pre-service teacher education programs, in particular for the Primary level. The defined methodology was based on the possibility to reflect about the decentralization of didactic practices based on a specific cultural context through one or more contacts with other "realities" coming out from different selected cultural contexts. In the last section of the paper we argue that the contact with a different didactic perspective coming from the Chinese "problems with variation" (one of the stimuli proposed in the PD' path), encouraged some reflections by the Italian pre-service teachers on the use of "variation" in Math word-problems for an early approach to Algebra in Primary level.*

### **INTRODUCTION**

Looking to new scenarios that School is living in recent years, the classroom realities that teachers and students observe are nowadays changing, enriched by new modes, new stimuli, new routines, new didactical processes that comes from inside and outside social and cultural classroom contexts. These scenarios are in many cases very complex and difficult to study for the Mathematics Education Communities. One pioneering works (Bishop, 1988) highlighted the importance of recognizing Math practices as social phenomena that are embedded in those cultures and those societies that generated them. D'Ambrosio (2006) putted in evidence that taking care about cultural and social issues into mathematical practices contributes to the understanding of cultures and the Mathematics itself. Nowadays the awareness of taking into account cultural and historical contextualization in Math classroom teaching practices and the crucial assumption that culture permeates Mathematics education practices is well known by the all the Math Education Communities. According to Bartolini Bussi and Martignone (2013) what is rather new is the awareness about the effect and the possible benefits of cultural diversity in Math teacher education research. According to this view some works, through qualitative and/or quantitative approaches (Bartolini Bussi et al., 2014; Bartolini Bussi et al. 2017; Di Paola 2016; Mellone and Ramploud 2015), pointed out how Math Education researchers, coming into contact with educational practices adopted in other cultural contexts, are able to *deconstruct* (Derrida, 1967) them, reconsidering the themes of educational intentionality defined as background of their educational practices (Mellone et al., 2019). In recent years, considering the important of these aspects for the classroom teaching

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practice, some Professional Development (PD) paths for teachers were designed and realized in Italy (e.g. Bartolini Bussi and Martignone, 2013) and in other countries (Chen, 2017). The paper presents some theoretical reflections and some methodological notes about the PD's path implemented during the last two years in Palermo and in Enna for prospective Primary teachers (around 250 selected on a voluntary base). The PD activities permitted them to "get in touch" with many selected didactic classroom realities, coming from different cultural contexts (e.g. Italian, Chinese, Singaporean, Arabic, African). In this paper, for space reason, we argue about one of the proposed stimuli related to maybe the most significant Mathematics tools for the Chinese Primary classroom practices: the "*problems with variation*" (Bartolini Bussi et al. 2014; Sun, 2011). The choice to propose this content to PD's pre-service teachers was due to the awareness that this particular approach could be useful to stimulate a possible reflection about an early approach to Algebra in Primary level, not usual in Italian classroom. In the last section of the paper we discuss about it.

### **THE CULTURAL TRANSPOSITION**

The *Cultural Transposition* construct (Mellone and Ramploud, 2015) focused the idea to the "*condition for decentralizing the didactic practice of a specific cultural context through contact with the didactic practices of different cultural contexts*" (Mellone et al., 2019, p. 199). This construct is inspired by the Skovsmose's (1994) approach according to which Mathematics is seen as an imperceptible "construction" that plays an important role in the subtended connected societies and in general in the Human condition. In this perspective, offering the possibility to prospective Primary teachers to "get in touch" with different educational practices, coming out from different cultural contexts can help them not only to become more aware of their social and cultural paradigm as regards classroom teaching practices but also to deconstruct their thought. Of course this "changing process" is very complex and needs more and more opportunities of reflection and *contaminations* (Bartolini Bussi et al., 2014). With this awareness, the PD path was designed with the aim to favour in all involved prospective teachers a passage, a transition, from a previous own condition to another one, more complex but also more rich, critic, conscious and stable. We are also convinced that use of the *Cultural Transposition* construct in a teachers training is useful to promote future more effective educational choices related to classroom practices.

### **THE PD'S TEACHING PATH: METHODOLOGICAL NOTES**

According to the declared aim and what literature discusses on PD path, based on the same research subject (e.g. Bartolini Bussi e al., 2017, Mellone et. al., 2019) the PD' training activities was designed following this frame:

1. Pre/Post-questionnaire: teachers' consciousness about classroom practices and cultural resources,
  2. Theoretical input on the *Cultural Transposition* paradigm, analysis on different school systems and typical classroom practices (e.g. Italian, Chinese, Singaporean, Arabic, African),
  3. Focus group: *Why/When/Where/What/How culturally transpone in your classroom?*,
  4. Observation (by video of classroom activities, Textbooks analysis, etc.) and critical discussion about teaching classroom practices coming from different cultural context,
  5. Teachers implementation of classroom activities designed according the *Cultural Transposition*,
  6. Written teaching report and group discussion: *What happen in your classroom?*
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The author of this paper was involved in all phases of the PD' path, as plenary speakers and group leader of the focus group; two teacher/researcher in Math Education supported his work and participated also to analyse the collected data. The pre-service teachers were engaged for 220 hours. During all PD's phases the data were collected by video and audio recording. These were opportunely analysed by the author of this paper and the two teachers/researchers and were used to redefine step by step the designed PD design. Pre-service teachers were exiting to participate and to analyse for the first time Math classroom practices, coming out from different cultural contexts. They never did it before during their teacher training. According to what we proposed they noticed and actively discussed analogies and differences in classroom setting, teachers and students behaviours, classroom practices, tasks design and use with students, etc. In this paper, for space reason, we discuss, even if briefly, one of the example proposed during the fourth PD path's phase. We proceed to next papers the qualitative and quantitative analysis of the data related to the questionnaires, the focus group and the written report that we collected.

### **DISCOVERING THE PROBLEMS WITH VARIATION: PRELIMINARY RESULTS**

As we said before referring to "*problems with variation*" are one of the most significant didactic approach for the Chinese Primary Math classroom practices. In our PD's path we proposed it, given special attention to one of the many interesting aspects that the "*problem with variation*" approach offers: the shift from arithmetic aspect to a structural vision of a mathematical word-problem. The choice to reflect on this aspect with pre-service Primary teachers was due to the awareness that in general in Italian Primary school no care is often given to Early Algebra, except for some activities regarding patterns or regularities (Spagnolo and Di Paola, 2010). In the Italian curriculum and the related textbook (we studied them with the PD' pre-service teachers), the study of Algebra starts, in fact, from grade 7 as the study of the formal algebraic language, consisting of activities of transforming algebraic expressions or solving equations. On the contrary, Chinese educational curriculum explicitly states that the main goal in teaching arithmetic is to develop students' understanding of quantitative relationships and their use of mathematical signs to represent them. In this perspective, the algebraic approach observed in Chinese and Italian Primary classroom practices are strongly different (Spagnolo and Di Paola, 2010). We discussed a lot with the PD's prospective teachers about it and we gave them the possibility to read part of these curricula (in Italian and English version) and to analyse some video of Chinese classroom practices based on the "*problems with variation*". We did it with the aim to understand the subtended cultural aspects behind the educational choices actuated by teachers in classroom. Through laboratory activities PD's teachers compared these approaches to those used in Italian ones, traceable also in many Italian Primary School textbooks. They observed that in the latter there are only isolated problems, structured mainly as word-problems with addition, subtraction, multiplication and division in a progressive but strongly partitioned and split development perspective. Also often no representation or of mathematical signs is linked to the text (Mellone et al., 2019). In such way they discovered that use of word-problems in Chinese and Italian Mathematics classroom practices are different! Classically, the "*problems with variation*" word-problems concerning the same reality contexts and the same arithmetical structure presented on the same page one after the other. The students' task is not only to solve the problems but (the most important aspect for Chinese teachers) also to recognize what all

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situations have in common. It is clear that what is essential is the “fluctuating” from a purely arithmetical aspect (typical for the Italian textbook context) to a more relational “ground”, oriented towards an early Algebra (Mellone et al., 2019). According to the construct of *Cultural Transposition*, the offered *contamination* gave to PD’s prospective Primary teachers, the opportunity to discover something new, “a-typical” for the Italian cultural context and very powerful for the Algebraic thought at Primary level. During the fifth PD’ phase they designed ad hoc some problem with variation testing these at school, following what they “learn by Chinese teachers”. The results were impressive; to approach “variation” permitted them to rethink the typical Italian classroom practices to word-problems in a effective way. As one of prospective teacher wrote in the post-questionnaire: “Thanks to Cultural transposition approach, used for this interesting course, I rethought my knowledge, I rethought me, as future Math teacher. The problems with variation are really powerful to find Math relation in a word-problem and can help students to approach Algebra in a informal way. I will treasure this in my future teaching”.

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