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Spatial analysis of public transportation infrastructure in Santiago, Chile, using the continuous approximation method

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Abstract

Santiago, the capital city of Chile, has seven million inhabitants in an area of 850 km^2. This city has a metro network with seven lines extending 140 kilometers and transports approximately 2.6 million people daily. The bus system has undergone significant transformations over the last three decades. The most relevant change having been Transantiago, the public transportation system implemented in 2007 for Santiago, Child, which combines the use of Metro and buses (BRT). Metropolitan Mobility Network (called Red) is the latest version of the public transportation plan. This paper aims to analyze the current subway infrastructure using the continuous approximation method for Santiago, Chile. We previously proposed a macroscopic methodology to identify the needs for an adequate level of service in urban mobility and transportation, and we applied it to Santiago's Metro network. Our work focuses on functionality and demand distribution. Santiago's demand varies spatially in volume and extension throughout the city. Using the latest origin-destination survey from 2012, we deduct the critical components in this current network structure. It is worth mentioning that the Metro design bases its network on a ring-radial structure. With our macroscopic model applied to Santiago, Chile, we have detected infrastructure needs in the current transit network. The supply of infrastructure should increase for two reasons: first, to achieve balanced cost levels between users and the agency and second, to reduce subway occupations. The optimal model outcomes for Santiago define the optimal network in which the system requires five rings and ten end-to-end longitudinal lines (20 radial routes), including lower levels of occupation. The obtained results are a good preliminary solution, considering the subway infrastructure supply could be sub-estimated in the public transportation plan.

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Keywords: Network design; Subway; Continuous approximation; Concentric city; Santiago, Chile.

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1. Introduction

Santiago, the Chilean capital city, is an extensive city of 850 km^2, with a population of 7 million inhabitants. The city's road network tends to have a concentric structure with the city's original principal roads crossing the city from East to West. However, only one of the rings proposed with the urban planning tools is in operation. Regarding public transportation, Transantiago is the urban public transportation system that operates in the Santiago metropolitan area. Santiago's Metro is an articulated mode of transportation for Transantiago. The Metro transports more than 2.6 million people daily, on its seven lines extending 138 kilometers and 136 stations. The Metro will continue expanding in the next decade and is expected to reach 220 kilometers of extension with three new projected lines, in addition to the extension of existing lines. Several scientific works analytically study the operation of a transit system, e.g., classic articles such as Vuchic and Newell (1968), Wirasinghe and Ghoneim (1981), Chua (1984), etc. Some of these works analyze concentric cities using a structure of polar coordinates for radial networks, such as Haight (1964) and Smeed (1968, 1965). In this paper, we analyze the current transit infrastructure in Santiago, Chile. The objective is to analyze the Metro network using a proposed macroscopic methodology to identify infrastructure needs to reach an adequate service level in urban mobility and transportation. The continuous approximation method uses analytical formulations and transit information from the latest origin-destination survey in 2012 to deduct the critical components in this current network structure. Our work focuses on functionality and demand. We assume that demand varies spatially in volume and extension over the city. A balance between user and agency costs provides the most efficient network configurations. The city of Santiago has a heterogeneous distribution considering its demand and network distribution.

The next section presents the city of Santiago and its structure, delving into the subway network. After that, we explain the methodology, which we apply to the Santiago case. Finally, we present the outcomes and conclusions.

2. General features of Santiago, Chile

The city of Santiago belongs to the Metropolitan Region of Santiago, which has six provinces. The province of Santiago is the central province, which has 32 city councils called Communes. Fig. 1 shows Gran Santiago including these 32 zones plus 2 more zones: San Bernardo and Puente Alto, which belong to other provinces; however, these zones are considered as part of the city. The two latter Santiago, founded in the 16th century around the Mapocho River. Santiago has three modes of public transportation: subway (Metro), buses (BRTs and traditional buses), and a commuter train (MetroTren). The Metro is a current system with six lines. The opening of the last one (line 3) was in 2017. The future network plan incorporates three new lines and three extensions for old routes. Fig. 1 presents the current subway network and the projected lines, including line extensions.

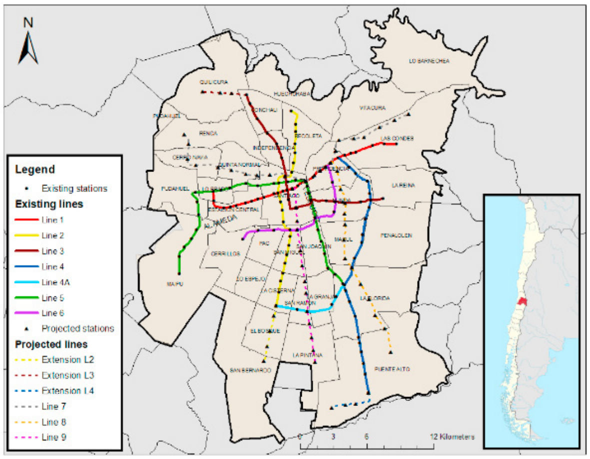


Fig. 1. Metro network map: existing lines and future projects.

3. Methodology

The mathematical model considers the continuous approximation (CA) method proposed by G.F. Newell. The method has applications for transit problems (e.g., Medina-Tapia et al., 2021, 2020, 2013), private transportation issues (e.g., Estrada et al., 2021; Medina-Tapia and Robusté, 2019, 2018), and logistics problems (e.g., Pulido et al., 2015). The region of the modeling is a concentric city of radius R [km]. Santiago is not a perfectly circular city; thus, we adapted the urban shape to the modeling but using the real parameters from surveys. The concentric city has ring and radial routes considering the rush hour of the city (P_m) as the period of analysis. We assume non-homogeneous continuous distribution over the city in which each point (r,θ) in polar coordinates has a different density value $D(r_f,\theta_f,r_t,\theta_t)$ in [user/km^4·h] represents the trip density distribution from a point (r_f,θ_f) to (r_t,θ_t).

3.1. Cost functions

The model formulation contains two components: user (T_T^u in [user·h/P_m]) and agency (C_T^a in [\$/P_m]) costs as show in Equation 1, where can also see that the travel time value (μ in [\$/user·h]) multiplies the user cost function.

$$TC_T = \mu \cdot T_T^u + C_T^a \tag{1}$$

Nomenclature

T	duration of the peak period or rush hour [h/P_m]
$d^c(r)$	distance between ring transit routes with a radius r [km/route]
$\Phi^r(\theta)$	distance as an angle between radial routes with an angle θ [radian/route]
$h^c(r)$	headway between vehicles at a ring route on r [h/veh]
$h^r(\theta)$	headway between vehicles at a radial route on θ [h/veh]
K^v	vehicle capacity [user/veh]
K^h	minimum time between consecutive vehicles [h/veh]
$K^{d(c)}$	minimum separation between two transit routes [km/route]
$K^{d(r)}$	minimum separation between two transit routes [rad/route]
μ	travel time value by average user [\$/user·h]
φ^k	unitary cost per vehicle [\$/veh·P_m]
φ^g	driver's wage per hour [\$/shift·h]
φ^o	operating cost per kilometer traveled on a cruising speed [\$/veh·km·route]
φ^p	linear infrastructure cost [\$/km·route·P_m]
φ^s	nodal (stop or station) infrastructure cost [\$/station·route·P_m]

3.1.1. User costs

The total time of users ($T_T^u = T_A + T_W + T_V + T_T$ in [user·h/P_m]) contains four functions: access (T_A), waiting (T_W), trip (T_V), and transfer time (T_T), presented in Medina-Tapia et al. (2021). The time functions in Equation 2 represent the total time for each trip stage of a transit system. The calculation of these functions comes from the integration of the local time function over a circular region:

- Access time: Users lose time to get to the closest station or the destination from the origin. First, demand in rush hour ($f^A(r,\theta)\cdot T$ in [user/km^2·P_m]) is the user density that board/alight at a station/stop during the period. Second, the average accessibility time per user, $t^A(r,\theta)$ in [h], depends on the time perception and the average access time.
- Waiting time: The passenger density boarding a vehicle is $f_l^W(r,\theta)\cdot T$ in direction $l \in L$ during rush hour ([user/km^2·P_m]). The average waiting time per passenger at a station is $t_l^W(r,\theta)$ in [h], which depends on the time perception factor and time headway of a service (Medina-Tapia et al., 2013).

- In-vehicle travel time: The total travel time depends on two components: the user load density in rush hour ($f_l^V(r,\theta)\cdot T$ in direction $l\in L$ in [user/km·P_m]), and the travel time per kilometer ([h/km]).
- Transfer time: Two factors comprise this local time function. The user density that transfers at a point (r,θ) to direction $l\in L$ ($f_l^T(r,\theta)\cdot T$ in [user/km^2]) and the average transfer time function.

3.1.2. Agency costs

The agency cost has three components (C_T^a in [\$/P_m]): capital (C_K), operational (C_O), and infrastructure cost (C_I). The last two costs have sub-components. The operational cost ($C_O=C_G+C_V$) includes the on-vehicle crew cost (C_G), and in-operation vehicle cost (C_V). The infrastructure cost ($C_I=C_P+C_S$) includes the linear (C_P) and nodal infrastructure (C_S). The explanations of Equations 2 are at Medina-Tapia et al. (2021).

- Capital cost: The cost value ($C_K=\sum_{l\in L}F_l\cdot\varphi^k$ in [\$/P_m]) depends on the fleet in direction $l\in L$ (F_l in [veh]) and the cost per vehicle (φ^k in [\$/veh·P_m]).
- Operational cost: First, the total salary ($C_G=F\cdot\eta^d\cdot T\cdot\varphi^g$ in [\$]) is in proportion to three components: the fleet (F_l, $l\in L$), the number of work shifts on a vehicle (η^d), and the salary in rush hour ($T\cdot\varphi^g$). Second, the total operating cost (C_V in [\$]) comprises two components: the number of vehicles that run on a corridor ($2T/h^c(r)$ or $2T/h^r(\theta)$ respectively), the operation cost per unit of distance traveled on a cruising speed (φ^o) considering the width of a transit corridor $d^c(r)$ or $\Phi^r(\theta)$.
- Infrastructure cost: The linear infrastructure cost (C_P) depends on the number of routes and its length of rings and radial routes ($1/d^c(r)$ or $1/\Phi^r(\theta)\cdot r$ in [km·route] in direction $l\in L$), and the unitary cost (φ^p in [\$/km·route·P_m]). The unitary cost per km in the rush hour has a fixed component and another variable part ($\varphi^p=\varphi^{p(f)}+\varphi^{p(v)}\cdot T$ in [\$/km·route·P_m]. The nodal infrastructure (C_S) depends on the number of intersections $1/\Phi^r(\theta)\cdot 1/d^c(r)$ and the unitary cost φ^s in [\$/station·route·P_m], considering each intersection has four stations or stops.

3.2. Problem formulation and optimization

The TNDFSP (transit network design and frequency setting problem) minimizes the system's total cost, taking a heterogenous demand distribution into account. The formulation of the total cost of a transit system (Equation 2) in monetary units ([\$/P_m] contains two components (Estrada et al., 2011): the user cost component (T_T^u in [user·h/P_m]), which is multiplied by the travel time value (μ in [\$/user·h]), and the second component of agency costs (C_T^a in [\$/P_m]).

$$Min\ TC_T=\mu\cdot(T_A+T_W+T_V+T_T)+(C_K+C_V+C_I) \quad \text{(a)} \qquad (2)$$

s.t.

$$\max_{\theta,l\in\{c_c,c_a\}}\left(f_l^V(r,\theta)\cdot d^c(r)\cdot h^c(r)\right)\le K^v \quad \forall\, r \quad \text{(b)}$$

$$\max_{r,l\in\{r_i,r_o\}}\left(f_l^V(r,\theta)\cdot\Phi^r(\theta)\cdot\frac{R+r}{2}\cdot h^r(\theta)\right)\le K^v \quad \forall\,\theta \quad \text{(c)}$$

$$d^c(r)\ge K^{d(c)} \quad \forall\, r \quad \text{(d)}$$

$$\Phi^r(\theta)\ge K^{d(r)} \quad \forall\,\theta \quad \text{(e)}$$

$$h^c(r)\ge K^h \quad \forall\, r \quad \text{(f)}$$

$$h^r(\theta)\ge K^h \quad \forall\,\theta \quad \text{(g)}$$

The problem is a fixed spatial transit system whose mathematical problem is a nonlinear system that includes inequality constraints. First, the problem has four decision variables ($d^c(r),\Phi^r(\theta),h^c(r),h^r(\theta)$). The variables are spatial and temporal, and variables do not change along a corridor. Second, the problem has three sets of constraints. The first of these (Equations 2(b) and (c)) ensures that occupancy does not exceed the capacity of each vehicle (K^v in [user/veh]). Second, the minimum distance between stations ensures that transit vehicles reach the cruising speed before arriving at the next station and can correctly brake (Equations 2(d) and (e)). In the case of radial routes (Equation 2(e)), $K^{d(r)}=K^{d(c)}/r_{min}$, where r_{min} is a minimum radius in which this constraint applies in [km/route] or [rad/route], respectively. Finally, the operator requires a minimum separation (time) between consecutive vehicles (TRB, 2013). Equations 2(f) and (g) ensure that the optimum frequency is feasible (K^h in [h/veh]).

4. Results

4.1. Modeling inputs

The modeling considers Santiago's urban shape approaching a concentric city, explained by Medina-Tapia et al. (2021). The modeled city has a radius of 15 km (R), and the rush hour lasts 1.5 hours (T). Table 1 shows the parameters in each stage of a trip, using the time perception from TRB (2013).

Table 1 – Parameters used for user and agency cost functions.

Demand parameters							
T [h]	α [dimensionless]	β [dimensionless]	γ [dimensionless]	δ [dimensionless]	v^w [km/h]	χ^T [m]	$v^a(r)$ [km/h]
1.5	2.2	2.1	1.0	2.5	3.0	40	$v^a(r) = 3.0 + 1.4\bar{6} \cdot r$

Operational parameters													
v^t	a^a	a^d	τ	τ'	τ''	τ'''	τ^s	t^f	η^d	K^v	K^d	K^h
[km/h]	[m/s²]		[s/station]					[min]	[shift/veh]	[user/veh]	[km/route]	[s/veh]
80	1.3	1.3	19.2	5	35	0	42.1	6	1	1,494	0.481	123

Economic parameters							
μ	φ^k	φ^g	φ^o	$\varphi^{p(f)}$	φ^p	$\varphi^{s(f)}$	φ^s
[\$/user·h]	[\$/veh]	[\$/shift·h]	[\$/km·veh]	[\$/km·route·P_m]		[\$/station·route·P_m]	
1.48	135.6	27	3.7	245.1	248.8	169.9	172.4

The proposed model considers a concentric city, which has a public transportation system with two types of transit services: ring and radial routes. Santiago is a concentric city but does not have a perfect circular form; however, the modeling assimilates that the city has this urban form. The last Origin-Destination survey (MTT, 2012) sets over 700 zones for Gran Santiago city; however, the modeling simplifies zoning by dividing the city into 9 OD macro-zones: a central zone, four inner zones, and four outer zones (Fig. 2).

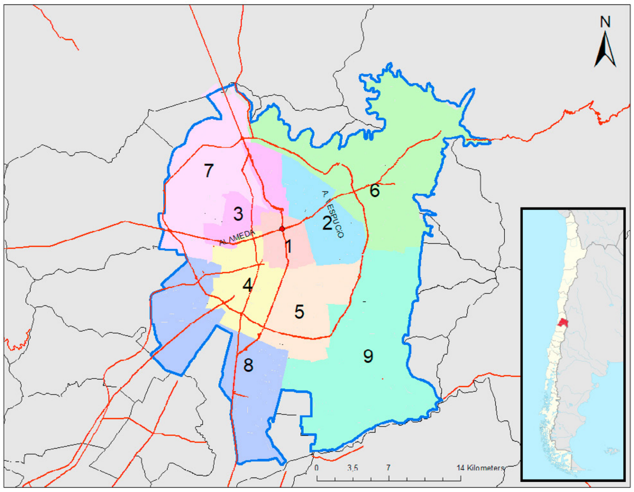


Fig. 2. OD macro-zones used for the Santiago of Chile modeling.

The method calibrates a continuous function in which the total trips of each origin-destination is the value in the OD trip matrix (T_{ij} where i is the origin zone, and j is the destination zone), as shown in Equation 3. The coordinates $\theta^i_{f(2)}$-$\theta^i_{f(1)}$ and $r^i_{f(2)}$-$r^i_{f(1)}$ define the origin zone. Meanwhile, the coordinates $\theta^i_{t(2)}$-$\theta^i_{t(1)}$ and $r^i_{t(2)}$-$r^i_{t(1)}$ define the destination zone. Using the function of Equation 3, we obtain the function of generated demand $\lambda(r,\theta)$ at a point (r,θ) in [user/km^2·h] (Equation 4), and attracted demand $\rho(r,\theta)$ at a point (r,θ) in [user/km^2·h] (Equation 5). Fig. 3 shows the functions of trip generation (Fig. 3(a)) and attraction (Fig. 3(b)) obtained from Equations 4 and 5. Fig. 3(c) represents the total trip density function, i.e., the sum of generated and attracted demand functions.

$$T_{ij} = \int_{\theta_{f(1)}^{i}}^{\theta_{f(2)}^{i}} \int_{r_{f(1)}^{i}}^{r_{f(2)}^{i}} \int_{\theta_{t(1)}^{i}}^{\theta_{t(2)}^{i}} \int_{r_{t(1)}^{i}}^{r_{t(2)}^{i}} D(r_f, \theta_f, r_t, \theta_t)\, r_t\, dr_t\, d\theta_t\, r_f\, dr_f\, d\theta_f \quad (3)$$

$$\lambda(r,\theta) = \int_0^{2\pi} \int_0^{R} D(r,\theta,r_t,\theta_t)\, r_t\, dr_t\, d\theta_t \quad (4)$$

$$\rho(r,\theta) = \int_0^{2\pi} \int_0^{R} D(r_f,\theta_f,r,\theta)\, r_f\, dr_f\, d\theta_f \quad (5)$$

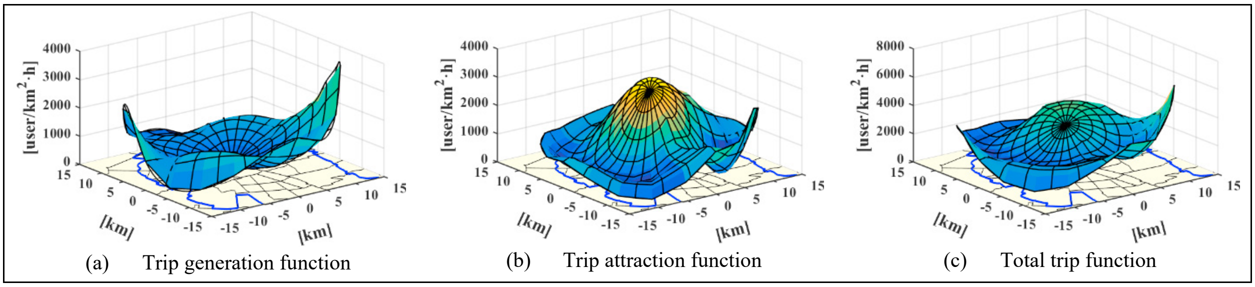


Fig. 3. Functions of trip generation and attraction estimated from Santiago's OD matrix (MTT, 2012).

4.2. Optimal solutions obtained from the model

The problem formulation is optimized using KKT conditions. Each point has an optimal solution of density and headway for ring and radial routes. Fig. 4 presents the optimal transit density profiles for Santiago, Chile, obtained from the model. The blue line in Fig. 4(a) represents the optimal ring route density, and Fig. 4(b) represents the optimal solution for radial routes. The red points represent the optimal location of a route obtained from the discretization process.

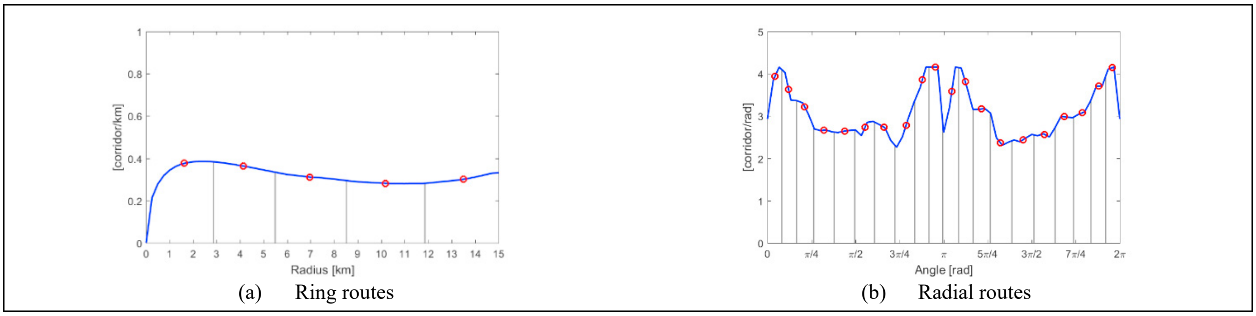
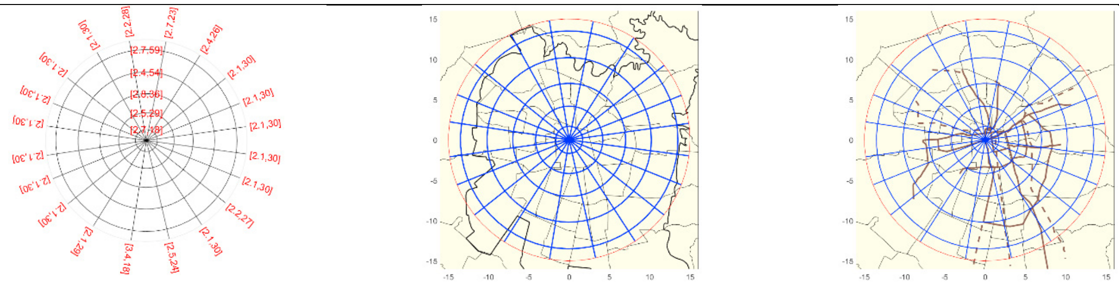


Fig. 4. Optimal density profiles for Santiago, Chile.

From optimal transit densities, Fig. 5(a) contains the optimal network structure in which a label on each route shows the optimal headway in minutes and the fleet size in trains for rings and radial routes. Regarding ring routes in Fig. 5(a), the model proposes five transit corridors, whose distribution is slightly higher around the city center. Regarding radial routes in Fig. 5(a), the model proposes 20 corridors with non-homogeneous distribution with higher density in three zones: East zone (above 0 radians, including Providencia and Las Condes communes), West zone (above and below π radians, including Estación Central, Maipú, Pudahuel communes), and Southeast (below 0 radians, including La Florida and Puente Alto communes). It is worth noting that 20 radial lines could represent ten lines from one side of the city to the opposite. Regarding ring routes in Fig. 5(a), headways take on similar values between 2.4 and 2.8 minutes. However, the cycle times on each route are different considering the vehicle-kilometers traveled increases as the route approached the city periphery. Thus, the fleet size that needs a route increases from 18 to 59 trains. Regarding radial routes in Fig. 5(a), all routes have the same length, but the fleet size depends on the headway and the optimal ring density giving fleet needs from 18 to 30 trains. In a city whose transit vehicles travel long distances, slight headways modify the fleet size significantly. Fig. 5(b) shows blue lines representing the optimal transit structure, the red circle represents the analyzed area, and the map in the background shows the communes of Santiago, Chile. Fig. 5(c) also includes the current infrastructure and the future subway projects as brown lines for Santiago, Chile, where continuous lines are existing infrastructure, and dashed lines are the future projects. Currently, Santiago's Metro has seven lines; however, it represents nine existing radial routes, including their extensions; meanwhile, the three new projected lines represent four more radial routes. Globally, the subway mobility plan proposes 13 radial routes, which should increase to 20 radial routes, according to the model. Regarding rings, the transit system only has two incomplete rings, which should increase to 5 ring routes.

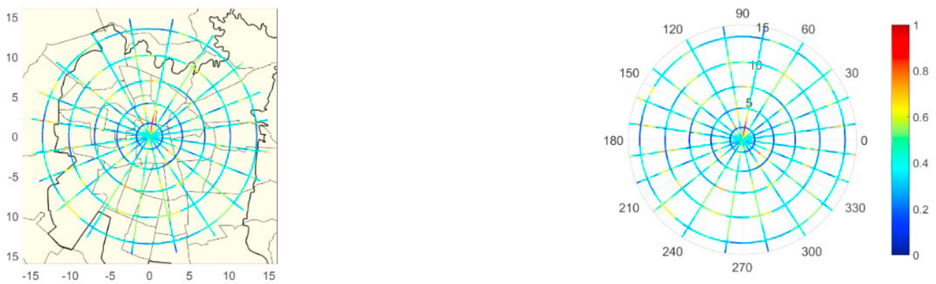


(a) Optimal structure and headways (b) Optimal transit structure (c) Proposal and subway infrastructure

Note: In Fig. 5(a), [X1, X2] in red represents the optimal operation attributes in which X1 is the time headway in minutes, and X2 is the fleet size in trains for each route defined optimally.

Fig. 5. Optimal transit network scheme for Santiago, Chile.

From optimal transit densities, Fig. 6(a) and (b) show occupation levels of transit routes for Santiago, Chile. The former overlays the optimal transit network with the map of communes of Santiago, Chile. Regarding occupation levels, rings have a maximum occupation of up 75%; however, the average occupation is around 25%. On the other hand, radial routes reach an occupation of up 92% with an average occupation around 37%. Therefore, the global transportation system is not stressed by high occupancy levels.



(a) Santiago and occupation levels (b) Occupation levels

Fig. 6. Map of occupation levels obtained from the optimal transit network.

5. Conclusions

Santiago, Chile, has a consolidated subway network of more than 50 years with seven lines currently operating. The investment plan considers the extension of some of these lines and the construction of three future projects. The modeling in the paper uses standard values of operating data. However, some parameters were adapted to the Chilean case, such as the value of time, the train capacity, and the transit demand. In this line, the continuous travel distribution function obtained from the travel matrix origin-destination between macro-zones allowed to represent the generation and attraction of trips. A territory does not have a staggered structure with breaks between two OD-zones boundaries. On the contrary, demand generally varies smoothly from one side of an OD zone boundary to the other side. The model obtained a macroscopic proposal for subway infrastructure needs, i.e., the modeling applied a theoretical mathematical model previously presented, defining a relevant approach to determine the infrastructure needs for the city of Santiago. The proposed model for Santiago considers 5 rings and 20 radial routes. The comparison between this proposal and the current network plus the planned defines Santiago's infrastructure needs: 13 new radial routes (equivalent to 7 more lines) and four more rings. Future studies could study complementing these central services with tram services, particularly in Santiago's central commune. It is worth noting that slight headway changes modify the fleet size significantly. The fleet size depends on the headway and the opposite optimal route density or transit stations for the modeling. The latter relates to the time spent due to users boarding, alighting, and others, increasing the cycle time. The proposal network proposes lower occupation vehicle levels: the maximum occupation of rings is 75%, and radial lines have a maximum occupation of 92%. However, the average occupation of rings is around 25%, and the average radial occupation is 37%. Therefore, Santiago, Chile, should increase infrastructure supply for the subway network in future strategic planning. The results are considered an interesting preliminary proposal; however, the research approach should be refined through complementary studies. These studies should consider information from all available origin-destination zones, periods, transport purposes, and others. Future studies should also analyze current services with complementary modes, and personal mobility modes within a sustainability framework.

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