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From Galileo to Navier and Clapeyron. The Intuition of a Genius versus Engineering Rigour

De Galileo a Navier y Clapeyron. De la intuición del genio al rigor ingenieril

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Abstract

Galileo (1564-1642), in his well-known *Discorsi* (Galileo, 1638), briefly turning his attention to the fracture of a beam, starts an interesting discussion on the beam's breakage as well as its location. Could the section and breaking point of a beam have been determined beforehand? Furthermore, is it specific to the material? What Galileo did was not merely challenge a physics problem, but the prevailing knowledge of his time: namely, Aristotelianism on one hand, and Nominalism on the other. As a matter of fact, must the breakage of an element be treated as a universal or is it particular to a given material?

The present essay aims to prove how Galileo, confronting the structural problem and bringing it into the realm of science, was not just raising a problem but, using Salviati's words, he also established what actually takes place. Many years later, with the progress of physics, strength of materials and theory of structures, figures such as Claude Navier (1785-1836) and Benoît Clapeyron (1799-1864) confirmed once again that the Pisan turned out to be right.

This article intends to combine technical fields such as strength of materials and theory of structures with others like the history of science and philosophy proper. A cooperative approach to these disciplines can be doubtlessly helpful to improve the knowledge, learning and teaching of their different curricula, giving the reader a global, holistic perspective.

Keywords: Aristotelian legacy; scientific revolution; Pisan; Discorsi; beam.

Resumen

Galileo (1564-1642), en su conocido *Discorsi* (Galileo, 1638), introduciendo una breve consideración sobre la rotura de una viga, establece una interesante discusión sobre su fallo a la vez que la localización del mismo. Porque, ¿fuere posible establecer a priori la sección y el punto de rotura de una viga? Es más, ¿hay una relación con el material? Galileo se enfrentaba, a parte del problema físico en sí, al saber imperante del momento, esto es, por un lado el aristotelismo y, por otro, el nominalismo. De hecho, ¿la rotura de un elemento puede ser tratado como un universal o bien es particular de cada material?

El objetivo del presente estudio fuere mostrar como Galileo plantea no sólo el problema sino que, en boca de Salviati, afirma lo que realmente sucede. Años más tarde, con el avance de la física, la resistencia de materiales y la teoría de estructuras, figuras como Claude Navier (1785-1836) y Benoît Clapeyron (1799-1864) confirmaron nuevamente que el pisano tenía razón.

Este artículo pretende combinar campos técnicos como la resistencia de materiales y la teoría de las estructuras, con otros como la historia de la ciencia y la filosofía propiamente dicha. Un enfoque cooperativo de estas disciplinas puede ser sin duda útil para mejorar el conocimiento, el aprendizaje y la enseñanza en los diferentes planes de estudio, dando al lector una perspectiva global y holística.

Palabras clave: legado aristotélico; revolución científica; Pisano; Discorsi; viga.

1. Introduction

There is no doubt that the structural problem has been inextricably linked with human beings (Jaramillo, 2011, Castro Villalba, 1996). The search for a dwelling, a place that can be used as a shelter and even for the storage of food is reflected in the vast amount of works which have survived to our day or have been referenced elsewhere. During the earlier constructive ages, characterised by low work tension, defining a correct form of structure prevailed (Pons, 2017). In this way, the so-called structural problem becomes a problem of geometry. The experience of masters, which had been cemented throughout the centuries, held the status of building rules which dominated the *modus operandi* carried through. This geometric notion of construction prevailed and, therefore, it seemed possible to apply the adage according to which *if a building works, it will also work if built twice its side* (Heyman, 2004, 7) ensuring the functioning of construction.

> The rules of experience have ensured the survival of Greek and Roman temples. (Heyman, 2004, 6)

This world view changed radically from the 16th century preceding the Scientific Revolution. The advent of the Modern Age resulted in the observation of reality as well as the development of theories and the indictment against certain dogmas of the Middle Ages which had been sacrosanct until then. Galileo, perhaps more than any other single person, was responsible for the birth of modern science (Hawking, 2009).

As is usually said:

> After "God is the truth" and theology as an instrument to apprehend it of the Middle Ages, there came "truth is science" and the scientific method as the instrument to apprehend it. (González de Posada, 1994)

Galileo posited that with the increase of required stresses in constructions, along with increasing forms and volumes, structural problems will arise, to the extent that in some cases it may cause them to collapse into rubble. However, the Pisan did not exert any influence on construction (Aroca, 1999) since geometric elements were still prevalent. In spite of all these considerations, some witnesses refer to the need for theoretical studies; "Ars Sine Scientia Nihil Est" (Ackerman, 1949)*;* practice amounts to nothing without theory. In Galileo, precisely, approaches which run against this initial (geometrical) conception will be tackled.

Already nearing the end of his life, secluded in his home at Arcetri (Florence), Galileo finishes his book Discourses and Mathematical Demonstrations Relating To Two New Sciences (Galileo, 1638), finally printed in Holland. The title itself is a real declaration of intent for what is considered his most important work. The book, in which fundamentals of mechanics are exposed, is structured in four dialogues where each lasts one day. In it we found three interlocutors. *The three interlocutors are Salviati, who speaks for Galileo; Sagredo, who represents Galileo as a younger man, and puts forward views on occasion that the older Galileo has rejected; and Simplicio, who might represent a very young Galileo, and who acts as a foil to the other two more learned scientists* (Heyman, 1998, 3). This work will be fundamental for Isaac Newton to be able to publish the Principia (Philosophiæ Naturalis Principia Mathematica, 1687) one of the most important works in the history of science. The year is 1638.

No long after the first day, we came across the excerpt we intend to discuss. Actually, speakers in the dialogue want to address more general issues. They aim to find out whether resistance and strength multiply in connection to matter increase. Discussing the previous relationships, Salviati mentions the case of a thick, marble column which fractured precisely at the point that needed reinforcement. There begins an interesting dissertation on the causes to counter the perplexity they caused. "They are the subject of our discussions today" (Drake 1974).

Our intention here is to examine Galileo's description of the resistive case of the beams: "a field full of beautiful and useful considerations" (Drake, 1974). Once again, Galileo shows how exacting he can be. He longs to find an essence that can be mathematically expressed. Could it be possible to express in mathematical terms the fracture of a beam? In order to reach this lofty goal, it is necessary to examine all the stresses which act on it, some of which are unknown a priori, and to predict the beam's behaviour when facing those stresses. What is more, will all beams behave the same or is it necessary to know their substance to predict their behaviour? Can we generalise a theory for the study of a beam or is the behaviour specific to each element? Galileo undertakes the study of why and how the beam breaks, achieving — as we will see — striking results.

Galileo is rigorous with problems that we will subsequently examine through the prism of the strength of materials and structures theory. Among other cases, the Pisan raises the problem of a cantilever's breakage, a beam's resistance under the action of its own weight, and here, the supports' influence in the fracture. In spite of it all, it would not be systematised until a few centuries later, with figures such as Claude Navier and Benoit Clapeyron. Although going by the name of strength of materials (sometimes also known as structures theory), the discipline tackles the problem by way of tensile analysis and resistive capacity as a final step before design or verification. In the case of Navier, focusing on tension (s) — deformation (ε) relationships (Navier 1883), became the first to grapple with the statically indeterminate problem which we encounter in the analysis of continuous beams (Timoshenko, 1953). Clapeyron, on the other side, will render the calculation of hyperstatic structures more agile (as in the case of Galileo's beam), which makes it possible to obtain the stresses generated in the reactions to external loading (Clapeyron, 1857). These contributions reveal that the reflections adduced in the dialogue between the three characters were the embryo of subsequent laws which have eventually formulated the resistive problem in a more thorough way.

2. The formulation of the problem. Galileo (1564-1642)

The excerpt from the text.

> A very large column of marble was laid down, and its two ends were rested on sections of a beam. After some time had elapsed, it occurred to a mechanic that in order to ensure against the breaking of its own weight in the middle, it would be wise to place a third similar reinforcement there as well. This suggestion seemed opportune to most people, but the result showed quite the contrary. Not many months passed before the column was found cracked and broken, directly over the new support at the centre. (Drake, 1974)

Let's tackle the problem, then, along with Galileo's considerations:

> A very large column of marble was laid down, and its two ends were rested on sections of a beam. (Drake, 1974)

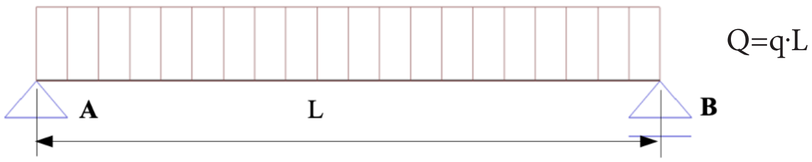


Figure 1. Articulated — supported at ends with a uniformly distributed load.

In this initial case, it would seem logical that the critical section should be the central one being it too at its maximum deflection. The load carried by each support (R_A and R_B) would amount to half of the whole. Therefore if the load value per length unit is *q* (kN/m) the corresponding value of reactions will have the value:

$$R_A = \frac{ql}{2} = \frac{Q}{2}, R_B = \frac{ql}{2} = \frac{Q}{2} \qquad (1)$$

Even though when the physical sense is assumed correct, should we want to analytically validate it, we would have to posit the equilibrium of the (rigid) solid, thereby reducing the problem to the formulation of statics equations. Since it is an isostatic problem, that is, one in which the number of unknowns to be determined (three) equals the number of equations, the problem will be possible to tackle directly with the same, which obviously allows the same final result.

$$\sum F_x = 0; \sum F_y = 0; \sum M_z(x) = 0; \qquad (2)$$

We can read:

> After some time had elapsed, it occurred to a mechanic that in order to ensure against the breaking of its own weight in the middle, it would be wise to place a third similar reinforcement there as well. (Drake, 1974)

Galileo, who had a vast knowledge of classical culture, had seen in his hometown Pisa how temples which featured great spans between pillars had the corresponding columns in intermediate points in order to downplay the deformation which occurred in the upper lintel. It is fair to mention that in Galileo's time the *Treatise*[1] (Gonzalez, 1993) are well known, especially in a land as wealthy in the culture of construction as Italy, even though his literature mostly reflects upon the aforementioned geometrical relations and upon the expertise of master craftsmen throughout the centuries.

[1] Marcus Vitruvius, Jacopo Vignola, Vincenzo Scamozzi, Andrea Palladio, Leon Battista Alberti.

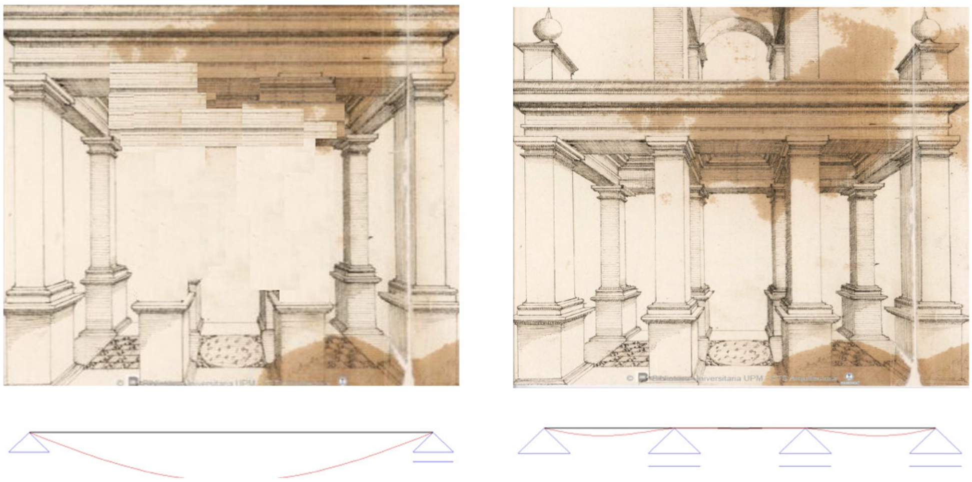


Figure 2. Effect of supports upon deflections.

There is no doubt this premise made him consider that incorporating the midway C support would help minimise the effect of deflection at said point.

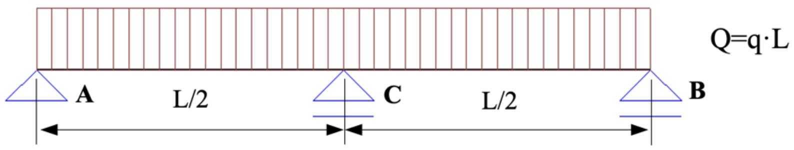


Figure 3. Beam with three supports and uniformly supported load.

In this new situation, we would be encountering the first problem. What value would each support have to withstand? As a matter of principle, *logic* would seem to indicate it is the same value, that is, the load's value distributed between the supports.

$$R_A = \frac{ql}{3} = \frac{Q}{3}, R_B = \frac{ql}{3} = \frac{Q}{3}, R_C = \frac{ql}{3} = \frac{Q}{3} \quad (3)$$

However, keeping in mind that the central support must withstand twice as much load as the previous ones (owing to a larger tributary area)[2], it would appear that the best answer is:

$$R_A = \frac{ql}{4} = \frac{Q}{4}, R_B = \frac{ql}{2} = \frac{Q}{4}, R_C = \frac{ql}{4} = \frac{Q}{2} \quad (4)$$

> Since the value for reactions A and B decreased [...] *this suggestion seemed opportune to most people*. (Drake, 1974)

[2] Section of a structure contributing to the load on a structural element.

How must we check the validity of the previous result from an analytic point of view? In this case, statics equations lead us to an indeterminate result, which is the reason we are not able to establish anything accurately in this regard. Therefore, the initial question on the calculation of value in each support remains indeterminate, entering the "dark world of hyperstatic structures" (Cardellach, 1910) which would not be implemented generically for many years to come.

> [...] the laws of Statics retreat impotently, not having the means to uncover those reactions; the structure comes under the non-isostatic category and, indeed, the problem remains a murky one indeed. (Cardellach, 1910)

The craftsman skillfully relies on intuition. That said, if the physical sense was to fail, how would the correct result be attained? That is to say, what can we do with *all events which are beyond expectation, especially when some precaution taken to prevent problems turns out to be a powerful cause of them* (Drake, 1974).

Surely, in this last case statics equations prove insufficient (we have four unknowns: A_x, A_y, C, B), and therefore, how can we determine analytically the value of reactions? In point of fact, one of the great problems which structure calculators have faced has been their determination in situations where statics equations are not enough. In other words, the so-called hyperstatic structures —sometimes known as 'statically indeterminate'.

Salviati moves on:

> […] but the result showed quite the contrary. Not many months passed before the column was found cracked and broken, directly over the new support at the centre. (Drake, 1974)

Galileo comes up with a new problem which, worded by the sceptical Simplicio seems "praeter spem" (Galileo, 1638); unexpected. That is because according to logic we would suppose that laying a new reinforcement, their work would be better: in no case would it collapse. Let's see what is, in this case, Salviati's consideration.

> For the two pieces of the column being placed flat on the ground, it was seen that the beam-section on which one end had been supported had rotted and settled over a long period of time, while the support at the middle remained solid and strong. This had caused one half of the column to remain suspended in the air; and, abandoned by the support at the other end, its excessive weight made it do what it would not have done had it been supported only on the two original [beams], for if one of them had settled, the column would simply have gone along with it. (Drake, 1974)

This unfortunate accident led to the study of cantilever beams' structural typology, which can be perfectly approached using statics equations, and that Galileo himself develops in his *Discourses* (days one and two mainly).

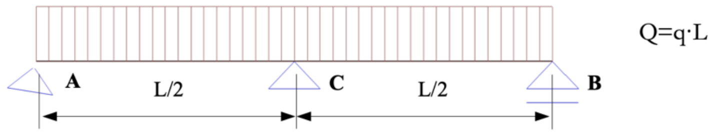


Figure 4. Case of loss of contact between beam and support A.

In this case, static equations lead us to:

$$R_C = ql = Q_{Total}, \quad R_B = 0 \ (5)$$

That is, the full value of the load becomes completely subsumed by the central support, thereby greatly increasing its value and turning into a very likely point for a fracture to start… as it took place in fact. Galileo had guessed what would happen to the piece, highlighting the central support's increase in value; he predicted which part would be the source of the fracture.

In spite of all that, the resolution of the more general problem, the hyperstatic case, was still unsolvable. Its resolution would allow us to predict the exact value in the reinforcements. We were saying that this means entering the world of indeterminate structure, those in which the numerous amount of unknown reactions makes the calculation more complicated (there are not enough equations) and their physical sense of work becomes more complex.

3. Solving the hyperstatic problem. Benoit Clapeyron (1799-1864)

It was necessary to discover — comparatively agile — analytic methods before achieving the structural resolution of the element without the need to resort to geometric postulates which are based on the premise of relatively low working stresses and, consequently, the material was not correctly used to the full. In fact, it would not be until the mid-19th century — that is, two centuries after the publication of the *Dialogues* — that the Frenchman Benoit Clapeyron would introduce a theorem allowing the resolution of the hyperstatic typology in a rigorous and agile way. Clapeyron presented in 1857 his article *Calcul d'une poutre* élastique *reposant librement sur des appuis inégalement espacés* (Clapeyron, 1857). There the resolution of the hyperstatic problem is clearly and orderly presented, drawing the following conclusion.

Soient l_0 et l_1 les ouvertures de deux travées consécutives, soient pour chacune d'elles p_0 et p_1 les charges par mètre courant, soient Q_0, Q_1 et Q_2 les moments correspondants à chacun des trois appuis consécutifs, on aura la relation

$$l_0 Q_0 + 2(l_0 + l_1) Q_1 + l_1 Q_2 = \frac{1}{4}(p_0 l_0^3 + p_1 l_1^3).$$

Figure 5. Theorem of three moments or Clapeyron's Theorem[3].

Although sometimes Henri Bertot is credited for it, since he published two years before a similar article in the same journal, most experts (Heyman, 2004, 126) attribute it to Clapeyron, calling it *Theorem of three moments* or *Clapeyron's Theorem*, in the Frenchman's honour. With the said formulation, the science of structures saw a radical shift since it now allowed, among other things, the resolution of the cumbersome hyperstatic problem which had caused so much difficulty up to that moment. Clapeyron justifies thus the aim of his work:

> I examine in the first place the case of a straight beam resting on two supports at both ends, the section is constant, it supports a uniform load; we give, moreover, the moment of the strengths acting at the ends of supports. The elastic curve equation affecting the beam's axis, the mechanical conditions to which all its points are submitted, and part of the total weight sustained by each support are deduced.
>
> The solution to the general problem is thus reduced, to the determination of moments of strengths tending to produce the breakage of the beam in each one of the supports on which it rests. This is to be achieved expressing that both elastic curves corresponding to two successive stretches are tangent to each other in the intermediate support, and that moments are equal. (Clapeyron, 1857, personal translation)

Now, applying this new formulation we are able to find the value for the central C support in an accurate way (figure 3). Using the preceding theorem, we are able now to find the moment's value on the left side of the hinge joint C which will allow us to find the correct value of the full reaction at said support.

[3] l_0 and l_1, be the openings of two successive segments, be for each one of them p_0 y p_1 loads per metre, be Q_0, Q_1 and Q_2 the corresponding moment to each one of the three successive supports, the relation will be achieved.

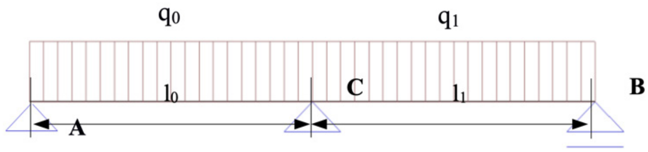


Figure 6. Theorem of three moments or Clapeyron's Theorem.

$$l_0 M_A + 2(l_0 + l_1) M_C + l_1 M_B = \frac{1}{4}(q_0 l_0^3 + q_1 l_1^3) \qquad (6)$$

Obtaining thereby the corresponding value of the moment at the central section (in our case): and.

$$2\left(\frac{l}{2} + \frac{l}{2}\right) M_C = \frac{1}{4}\left[q\left(\frac{l}{2}\right)^3 + q\left(\frac{l}{2}\right)^3\right]; \; |M_C| = \left|\frac{ql^2}{32}\right| \qquad (7)$$

Once this is known, conveniently, the solid's equilibrium is stated — having now become isostatic — in order to obtain the value of the reaction at central point.

$$R_C = \frac{10ql}{16} \quad (8)$$

An overall and detailed examination of the three different situations Galileo suggested allows us to observe that the beam's central point is subject to higher bending moment values in the former case (a beam with two supports at both ends) and in the latter case (a cantilever beam).

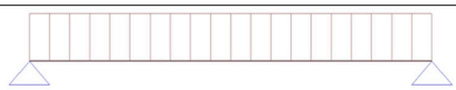
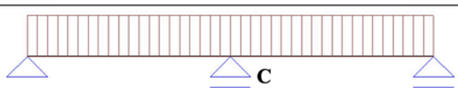
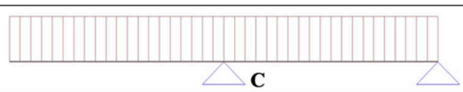
Case raised	Moment value Left	Moment value Center	Moment value Right
	0	$\frac{ql^2}{8}$	0
	0	$\left\|\frac{ql^2}{32}\right\|$	0
	-	$\left\|\frac{ql^2}{8}\right\|$	0

Table 1. Different raised situations with the values of bending moments at the supports.

If shear stress is what is being compared now, we can clearly see how the third case, corresponding to the cantilever beam, is the one suffering from higher effects due to this stress.

These relationships allow us to conclude that the C central point of the beam has been subjected, in the third case, to the most detrimental bending and shear stress, and therefore it is the most likely candidate for a breakage, just like Galileo predicted.

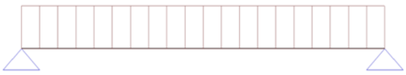
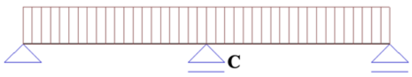
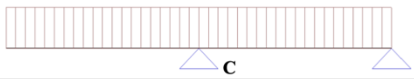
Case raised	Reaction value Left (R_A)	Reaction value Central (R_C)	Reaction value Right (R_B)
	$\frac{ql}{2}$	-	$\frac{ql}{2}$
	$\frac{3ql}{16}$	$\frac{10ql}{16}$	$\frac{3ql}{16}$
	-	ql	0

Table 2. Different situations posited with the values of reactions on supports.

4. Studying the distribution of tensions s. Claude Navier (1785-1836)

Having reached this point, we will continue with the second of our initial intentions in this article; the location of the fracture in the critical section. Closely reading the proposal made by Galileo-Salviati we stumble upon a consideration that would be interesting to point out. From the previous excerpt of the *Discorsi* we underscore: $l_0 = l_1 = \frac{l}{2}$ and $q_0 = q_1 = q$.

> […] the column was found cracked and broken, directly over the new support at the centre. (Drake, 1974)

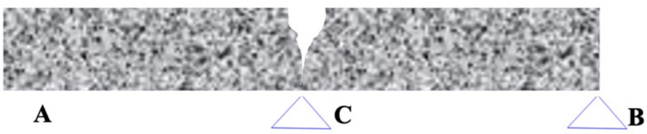


Figure 7. Overview of the material's fracture.

After indicating the critical section where the material would break, Galileo sharpens his rhetoric justifying in which part of the section the breakage would take place. Generalizing the proposed section would bring us to study the distribution of stresses in the material, as well as its mechanical properties. The actual problem consists in determining the study of the beam in its elastic behaviour. Knowing its deformation and its parameterisation, the distribution of stress and therefore its tensile and compressive areas. Actually, with Galileo, we find a group of scientists whose generalist problem consisted in the location of the neutral axis[4]. Professor Heyman's studies exhibit several theories on its location prior to Claude Navier's formulation (Heyman, 2004, 22).

Figure 8. Stress distribution of a cantilever beam according to Galileo, Mariotte and Navier. (Heyman, 2004, 22).

Claude Navier's proposed distribution is accepted nowadays, and we are consequently able to state that the elongation and shortening of every fibre in a section will be directly proportional to the distance between the fibre in question's corresponding point and the neutral line of the section analyzed.

Galileo does not speak in terms of mechanical properties, but, stating where the breakage takes places he is indirectly assuming that resistance in stress in marble (above area) is smaller than its compressive resistance (below). There Pisan undoubtedly did not have this quantified data. Mechanical properties of rocks were first studied scientifically in the mid-18th century. Until then, builders had resorted to the observation of built examples: if that stone or fabric has subsisted throughout several generations, that proves it is good (Huerta, 2004). However, subsequent studies verified the Galilean reasoning once again. The following chart shows how the compressive strength of stone materials is much higher than their tensile strength.

Material/values	$\sigma_{compression}$	$\sigma_{tension}$
Chalk	2-12	0.1-1.5
Limestone	7-40	0.5-5
Compact	40-100	4-15
Schist	15-70	1-10
Granite	60-180	6-15
Quartzite	80-300	7-20

Table 3. Resistive values (N/mm^2) of a number of materials. (Huerta, 2004, Delbecq, 1983).

[4] Line dividing section between part in tension and part in compression. Along it tension will be zero.

Consequently, Galileo was right once more. The section would break from above because that is the material's area which is subject to tension, and since tensile strength is smaller than compressive strength, the fracture will start from that specific part.

Almost 400 years have passed since the publication of the *Discorsi*. Today, there is still ongoing research in academia on continuous beam theory, as well as dimensioning with different materials. Students, mostly using numerical methods, work with the case of the beam proposed by Galileo and observe that the critical section is in the central support with a stress distribution in its upper part with fibres in tension. They observe that this upper section needs to be strengthened by means of working with particular materials, such as the concrete that must be reinforced in this area since it's susceptible of becoming a fracture start point.

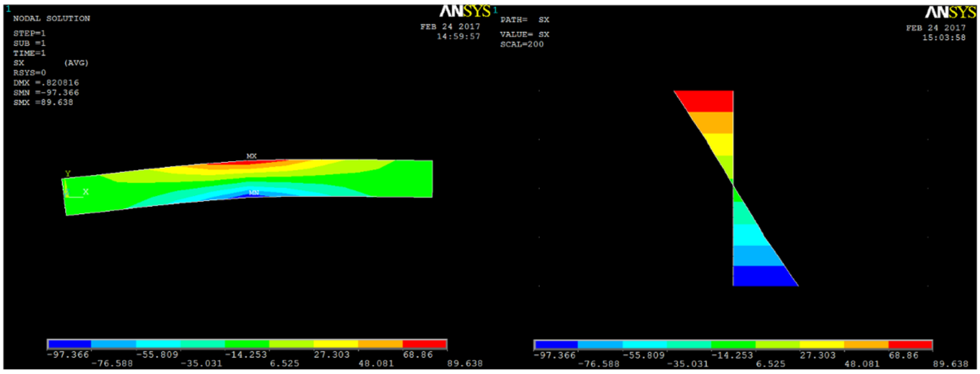


Figure 9. (a) Stress level curve. (b) Normal stress distribution curve in the section of support C. The top side is in tension being the lower part compressed.

Both implicitly and explicitly they validate what Galileo had already predicted years before in Italy.

5. Conclusion

On the basis of a text written 400 years ago, its contextualisation and the reasonings that the author offered allowed us to formulate a number of case studies which nowadays fall under the fields of strength of materials and/or structures theory, as well as introducing contributions by different scientists into our study.

For that purpose, we have used a brief fragment of Galileo's *Discorsi* where the Pisan postulates about the fracture of a beam modifying its external support elements. Today we know that this breakage depends not just on the connection with the environment and the load it must carry, but also on the material itself as well as the geometry of the resistive element expressed in a mathematical formulation of universal character but with specific parameters.

Years have passed and new theories have confirmed a great deal of the claims Salviati-Galileo made almost 400 years ago. There is no doubt that the Pisan established the foundations of a new physics that broke with the prevailing Aristotelian legacy and would come to shape reality — one which called upon the scientific revolution — finding a way out of perplexity.

In words of philosopher Ortega y Gasset[5]:

> Merely considering our respect for Galileo should be enough to realise that he is even more deserving of our fervour. Located in a particular quadrant, lodged in a great slice of the past which has a most precise shape: the beginning of the Modern Age, the system of ideas, assessments and drives which has come to dominate and nurture the historical soil which extends precisely from Galileo until our very own days. Our interest in Galileo is therefore not so selfless and generous as we could initially guess. (Ortega y Gasset, 1995)

The study of a historical text has allowed us to introduce into teaching an interesting debate that encompasses philosophical and historical approaches on the one hand, and scientific and technical approaches on the other.

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[5] José Ortega y Gasset (1883-1955), Spanish philosopher and essayist.

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