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Intermittent chaotic flows in the weakly magnetised spherical Couette system

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Experiments on the magnetised spherical Couette system are presently being carried out at Helmholtz-Zentrum Dresden-Rossendorf (HZDR). A liquid metal (GaInSn) is confined within two differentially rotating spheres and exposed to a magnetic field parallel to the axis of rotation. Intermittent chaotic flows, corresponding to the radial jet instability, are described. The relation of these chaotic flows with unstable regular (periodic and quasiperiodic) solutions obtained at the same range of parameters is investigated.

1 Introduction

The magnetised spherical Couette system (MSC) system is of relevance in the study of the dynamics in the interior of stars and/or planets since it models two essential ingredients occurring in some celestial objects: differential rotation and magnetic fields. Because of its relevance, the MSC problem has motivated several experimental (e. g. [1]) and numerical (e. g. [2]) studies. Aside its astrophysical importance, the MSC problem is of interest in the study of large scale dissipative dynamical systems with symmetry [3,4] as it is $\mathbf{SO}(2)\times\mathbf{Z}_2$ equivariant, i. e., invariant by azimuthal rotations and reflections with respect to the equatorial plane.

2 Model and numerical method

A fluid of constant density ρ, kinematic viscosity ν, magnetic diffusivity $\eta = 1/(\sigma\mu_0)$ (where μ_0 is the magnetic permeability of the free-space and σ is the electrical conductivity) is filling the space within two concentric spheres (of radius r_i and r_o). The outer sphere is at rest and the inner sphere is rotating at angular velocity Ω around the vertical axis $\hat{\mathbf{e}}_\mathbf{z}$. In addition, a uniform axial magnetic field of amplitude B_0 is imposed. This is the setup of the HEDGEHOG experiment [1]. The system is governed by the non-dimensional Navier-Stokes and induction equations:

$$\begin{aligned}&\partial_t \mathbf{v} + \mathrm{Re}\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla p + \nabla^2\mathbf{v} + \mathrm{Ha}^2(\nabla\times\mathbf{b})\times\hat{\mathbf{e}}_\mathbf{z}, \\ &0 = \nabla\times(\mathbf{v}\times\hat{\mathbf{e}}_\mathbf{z}) + \nabla^2\mathbf{b}, \quad \nabla\cdot\mathbf{v}=0, \quad \nabla\cdot\mathbf{b}=0,\end{aligned} \tag{1}$$

which are obtained using $d = r_\mathrm{o} - r_\mathrm{i}$, d^2/ν, $r_\mathrm{i}\Omega$ and B_0, as scales of length, time, velocity and magnetic field, respectively. The Reynolds number is $\mathrm{Re} = \Omega r_\mathrm{i} d/\nu$ and the Hartmann number is $\mathrm{Ha} = B_0 d(\sigma/(\rho\nu))^{1/2}$ ($\mathbf{v}$ and $\mathbf{b}$ are the velocity field and the magnetic field perturbation, respectively). The inductionless approximation –valid in the limit of small magnetic Reynolds number $\mathrm{Rm} = \Omega r_\mathrm{i} d/\eta \ll 1$– is employed. This approximation makes sense in the case of the liquid metal GaInSn (with magnetic Prandtl number $\mathrm{Pm} = \nu/\eta \sim O(10^{-6})$ [5]) at moderate $\mathrm{Re} = 10^3$, since $\mathrm{Rm} = \mathrm{Pm}\,\mathrm{Re} \sim 10^{-3}$. We choose an aspect ratio $\chi = r_\mathrm{i}/r_\mathrm{o} = 0.5$ and no-slip ($v_r = v_\theta = v_\varphi = 0$) at $r = r_\mathrm{o}$ and constant rotation ($v_r = v_\theta = 0,\ v_\varphi = \sin\theta$, θ being colatitude) at $r = r_\mathrm{i}$ boundary conditions for the velocity field. For the magnetic field insulating boundary conditions are considered. The equations are solved by means of a pseudo-spectral method –spherical harmonics in the angular coordinates and a collocation method in the radial direction– with a high order implicit-explicit backward-differentiation (IMEX–BDF) time integration.

3 Preliminary results and discussion

The solutions of Eq. (1) are obtained by means of direct numerical simulations (DNS) with $n_r = 40$ radial collocation points and a spherical harmonic truncation parameter of $L_\mathrm{max} = 84$. Each solution has then a dimension of $n = (2L_\mathrm{max}^2 + 4L_\mathrm{max})(n_r - 1) = 563472$. Here we describe intermittent chaotic long transients corresponding to the commonly known "radial jet instability" (e. g. [6]), described as an equatorial radial jet directed to the outer sphere with a posterior meridional circulation moving back the fluid to the inner sphere. The intermittent transients are found in a regime exhibiting a rich variety of periodic and quasiperiodic unstable states [3, 4]. For a spherical shell of aspect ration $\chi = 0.5$ and $\mathrm{Re} = 10^3$, this regime occurs for $\mathrm{Ha} < 12.2$; these are the paramters considered in this paper. The time and volume-averaged kinetic energy density

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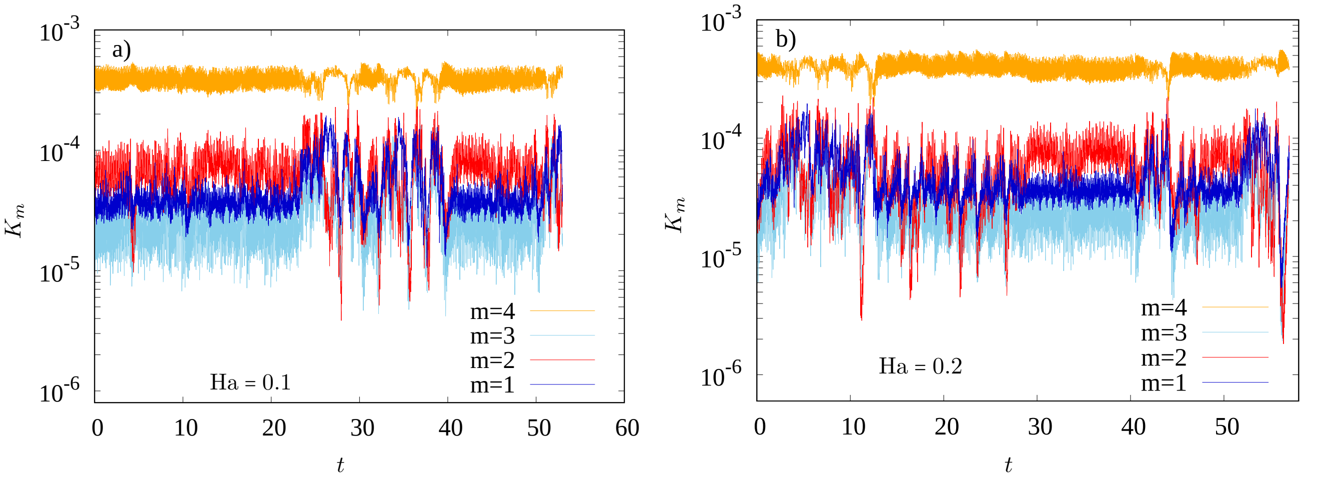

Fig. 1: Kinetic energy K_m of the modes with azimuthal wave numbers $m = 1, 2, 3, 4$ versus time at two different Hartmann numbers. (a) $\mathrm{Ha} = 0.1$ (b) $\mathrm{Ha} = 0.2$.

K_m corresponding to the azimuthal wave number m is employed as a proxy of the spatio-temporal dependence of the flows. Only the multiples of m_0 are nonzero in the spherical harmonics expansion of a m_0-fold azimuthally symmetric flow.

Figure 1 displays $K_m(t)$ for the azimuthal wave numbers $m \in \{1, 2, 3, 4\}$ and two different solutions at $\mathrm{Ha} = 0.1$ and $\mathrm{Ha} = 0.2$. For these solutions the energy is mainly contained in the $m = 4$ azimuthal wave number, with K_4 being roughly an order of magnitude larger. There are however time intervals for which the energies of the modes $m \in \{1, 2, 3, 4\}$ are of similar magnitude and time intervals for which the energies K_1, K_2 and K_3 are even smaller than K_4, roughly by two orders of magnitude. In the latter case the flow azimuthal symmetry is basically $m = 4$ and K_4 displays a nearly quasiperiodic behaviour. Notice that these events occur randomly, as it is characteristic of intermittent flows (e. g. [7]). In some cases, intermittent behaviour is driven by the existence of unstable invariant objects in the phase space (on-off intermittency of [8]) which may be the case of the solutions analysed in Fig. 1. This is feasible since unstable quasiperiodic solutions with azimuthal symmetry $m = 4$ are present in the system for $\mathrm{Ha} < 2.4$ (see [4]).

Our preliminary investigations point to the existence of on-off intermittent behaviour for MSC flows, when rotation effects dominate over the applied magnetic field. On-off intermittency has been also described in [9] for purely spherical Couette dynamos close to the onset of dynamo action. Our intermittent flows may be related to unstable symmetric quasiperiodic states previously described in [3, 4]. The phase-space trajectory of the chaotic intermittent flow may approach these quasiperiodic states, which organise the dynamics around it, but eventually could be repelled since the quasiperiodic states are unstable. The latter have certain spatio-temporal symmetries which may be identified from the time series of K_m. In addition, on-off intermittent flows can be also characterised from time series since their power spectral properties are well known (e. g. [10]).

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