Escola de Camins
Escola Tècnica Superior d'Enginyeria de Camins, Canals i Ports
UPC BARCELONATECH

Microsimulation of Connected Automated Vehicles in Platooning Conditions in Highways

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Màster en:
Enginyeria de Camins, Canals i Ports

Barcelona, 12/09/2021

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TREBALL FINAL DE MÀSTER

ABSTRACT

The perspective of autonomous vehicles running on our roads has come closer and closer to reality over the last decades, and this technology might be implemented during the upcoming years. However, this will only be achievable through constant investigation and efforts in all the fields of science covered by the Connected and Automated Vehicles (CAV). The present study focuses on the behaviour of CAV platoons in highways, and more precisely experiments an algorithm to get rid of propagating perturbations, such as the bullwhip effect, that affects the proper functioning of the platoon. This algorithm manages to simulate a sequential acceleration where each vehicle is given an individual pattern to follow before it even performs its manoeuvre, thanks to the current and objective parameters that its disposes of. These individual patterns are coordinated and prevent therefore any bullwhip effect. Nevertheless, a misconception in the theoretical model elaborated prevents the vehicles to reach their objective Desired Space Gap (DSG), and modifications must still be made to obtain an operational algorithm in regular driving conditions. Some ideas are mentioned to improve the model in this sense, as well as an alternative method to look into, based on dynamic equations.

ACKNOWLEGEMENT

This work could not have been possible without my first tutor, Professor Francesc Soriguera, who trusted me to contribute to this investigation process he started many years ago. His precious support and guidance, from the beginning to the end, helped me to focus and invest myself as efficiently as possible on my research, and I do hope to have been up to the task.

I would also like to thank Marcel Sala, my second tutor, who provided me many relevant pieces of advice and ideas that nourished my reflexions and allowed me to have a better insight of the topic I was to study. His way of seeing concepts and explaining them inspired my own thinking process for this TFM.

Lastly, I would like to address to my father, as well as my girlfriend Kelly, a huge thank-you for their encouragements and their recommendations, which always resulted to be of much help and pertinent, and also for their unconditional moral support throughout this study.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION AND OBJECTIVES:

1.1 Objectives

This report is the result of one semester of research at the Polytechnical University of Catalonia (UPC) of Barcelona, and constitutes the Final Master Work to be delivered at the end of the second year of the master's course at the Civil Engineering School, ETSICCPB. As an investigation work, it consists in a prolongation of a years-long research process, an additional step in the field of Connected and Automated Vehicles platooning that I hope brings valuable information, and will be profitable to the future investigators on this topic.

Boukhellouf (2019) managed to develop and algorithm that allows the formation of stable platoons of connected and automated vehicles in fluid traffic conditions, thanks to the elaboration of an acceleration pattern for the members of a platoon. However, it was brought to light through investigation and analysis that some phenomena could perturbate the process during an acceleration phase, and generate instabilities as well as discomfort for the passengers. The objective of this work is to determine strategies, based on the precedent investigations or on other propositions, that could eradicate or at least smooth up satisfactorily these perturbations. By doing so, it is hoped to make a step further in the elaboration of an algorithm of control of connected and automated platoons under regular conditions, that is to be extended then to critical conditions in the future.

The idea developed in this work appeared after the study achieved by Boukhellouf (2019). As a matter of fact, while taking some aspects of his work, it is in reality quite distant from the vision that had been adopted so far: it consists in letting aside the dynamic approach, and determine if a viable acceleration phase of the platoon could be established, using the initial and objective states, the mechanical capacities of the vehicles, and the safety and comfort regulations. The process that was followed to elaborate, formalize, test, and analyse this strategy constitutes the main topic of this report.

Before entering the core of the study, it appears fundamental to clarify as much as possible the items that constitute the problem, and to provide a state-of-the-art that can stand as a solid frame to accompany the reader and facilitate its interest in the approach that was carried out.

1.2 Definitions

1.2.1 Automated Vehicle

An Automated Vehicle (AV) is a vehicle disposing of computers systems and mechanical equipment allowing it to manoeuvre itself, completely or partially, without any human interference. This equipment are sensors that can be of different types (cameras, radars, ultrasounds, lidars), and whose utility is to provide information to the computing system about the physical environment of the vehicle. With such data, the algorithm can determine the actions and the behaviours to follow, and put them into application so that no human intervention is necessary at any moment.

Different illustrations of the automation of vehicles have been already implemented to some current vehicles, such as emergency braking, Cruise Control and Adaptive Cruise Control (ACC) - that allows the vehicle to stay below the speed limit and conserve a safe following distance - or also the Advance Driver Assistance Systems (ADAS), that among other things allows ACC, adaptive light control, and automatic parking. However, not all technologies are considered equivalent in terms of automation, and the Society of Automotive Engineers (SAE) implemented a scale to classify the different technologies and systems according to their degree of automation, as can be observed in the figure below.

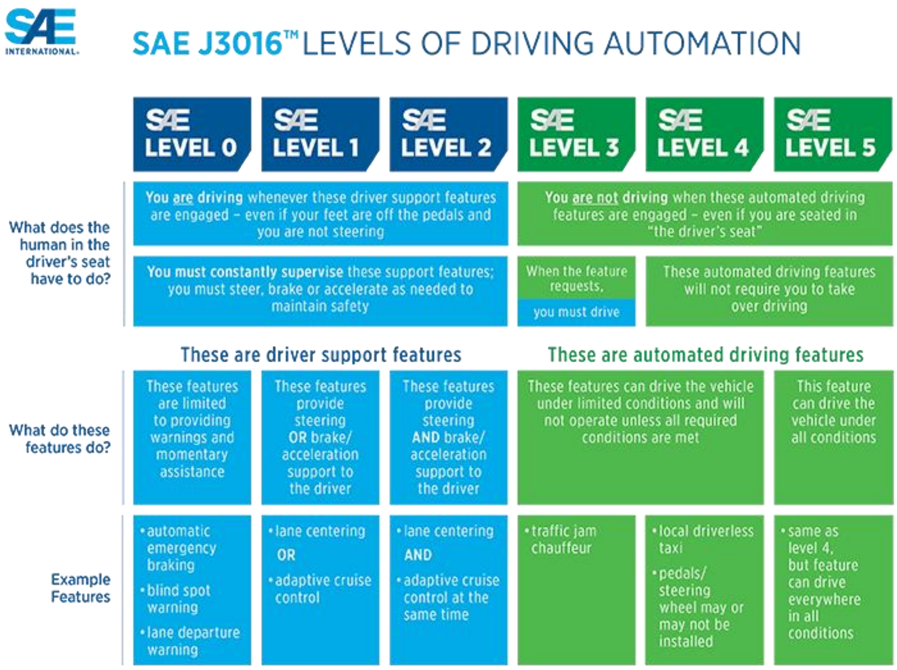

***Figure 1 :** The 6 levels of driving automation.* Source: SAE, Society of Automotive Engineers.

The automation of the vehicles that humans drive is not a caprice of consumers. On the contrary, it responds to very concrete and sensible issues that need to be dealt with. Firstly, the safety of the users and of the other people sharing their environment: a tremendous majority of car accidents are caused by human errors, 90% according to Smith (2013). This order of magnitude indicates that human error is by far the main cause of car accidents in the world. Thus, the automation of vehicles could significantly reduce the number of incidents, since drivers would not be humans anymore, but algorithms with deeper analysis capacities and faster decision making and reactions. With automated vehicles, many lives could be saved: according to the WHO Global Status Report on Road Safety of 2018, 1.35 million people die in car crashes every year, and tens of millions suffer severe injuries.

Secondly, the performances of the transportation system could be improved. Even if studies disagree on this topic, it is believed that the implementation of automated vehicles could generate less accidents, and also favour rational behaviours, deeper and faster decision making, which would lower the time spent on the road at both microscopic and macroscopic scales. At the microscopic scale on the one hand, improving the performances of the vehicle would allow it to find and adopt the best solution more quickly and with more flexibility than a human driver. On the other hand, at the macroscopic scale, avoiding accidents, irrational behaviours and unpredictable manoeuvres would favour a smoother circulation and reduce the apparition of perturbations that are often local and temporary, but tend to propagate themselves to larger scales in terms of time and space. Yet, these implications are not unanimous, and studies about the performances of AVs must still be carried out.

Thirdly, the social impact that automated vehicle could have if implemented at large scale is not to be disregarded: indeed, elderly people or individuals suffering from incapacities and unable to drive could use a car and therefore gain mobility. Furthermore, the reduction of the travel time as a consequence of better performances and flowing traffic will allow the users to dedicate their time on more valuable activities, that could be either professional or personal. Besides, since the users are free from holding the wheel and focusing their attention on controlling the vehicle, the irremovable time spent within the vehicle could also be used at their convenience.

Fourthly, the democratization of automated vehicles could have a positive impact on the environment and on ecology in general, as it could significantly reduce the emissions of carbon dioxide: shorter travel time at optimized velocities (to take into account the pollution generated), less congestions (smoother traffic) are among the environmentally friendly consequences of the vehicle’s automation.

1.2.2 Connected Vehicle

A connected vehicle (CV) is a vehicle able to receive and transmit in real time information to some operator or connected item. This information can be relative to its own state (position, speed, acceleration…), to other connected items in its close environment, or to general information about the traffic. Necessarily, it means that the vehicle needs an entity to transmit information to or receive data from. For this reason, different families of connectivity have been distinguished, depending on which entity the vehicle is able to communicate with. For a vehicle to be qualified as connected, it must enter in at least one of the following groups:

- **Vehicle-to-vehicle (V2V)**: the vehicle is able to share wirelessly data with one or multiple vehicles, such as their positions, speeds, dimensions, or mechanical limits (braking capacities, precision of the sensors…) for instance.
- **Vehicle-to-infrastructure (V2I)**: wireless and bi-directional exchange of data between the vehicle and the road infrastructure. Much information, like the colour of the lights, the speed limits, the detours, and the traffic state for instance could be shared between the organism in charge of the gestion of the road infrastructure, and the vehicles. As a matter of fact, information such as whether a light is red or green would be automatically given to the

vehicles, but there would still be room for human analyse and decision: should an operator of the traffic administration conclude that a specific measure has to be taken, they would be able to apply it and share it to the vehicles concerned (for example, a temporary reduction of the speed limit to avoid congestions in a specific aera).

- **Vehicle-to-everything (V2X)**: the vehicle is able to share data with any connected item, even a smartphone.

In the same way as automated vehicles, the primal objective of connected vehicles is to ensure a higher level of safety to all the users of the road: motorized vehicle users of course, but also cyclists and pedestrians. By providing extra information to the drivers, that they could not have perceived themselves, their decisions are more informed. In particular, the drivers could have access to crucial information that was simply out of their reach – a car hidden behind the corner of a building, a motorcycle in the blind spot – and with the help of which, accidents and casualties could be avoided.

As an objection to connected vehicles, one may argue that by providing more information to the driver, a feeling of safety is created, which can lower their attention and even lead them to adopt more dangerous habits (DeLucia *et al.*, 2018). It is perfectly correct that the information received are only useful if the receptor considers them in its decision-making process. However, because the right solution to a particular situation can be complex to find or to perform, or because the stream of data can result in more than what a human mind is able to handle, simply connected vehicles may not be as efficient as expected.

1.2.3 Connected and Automated Vehicle

A connected and automated vehicle (CAV) is a vehicle disposing of the capacities described previously: it is able to receive and transmit data from and to its environment, whether the infrastructure or another vehicle, and thanks to its algorithm, it can make use of the data collected to perform manoeuvres without needing any control or surveillance from a human.

Such vehicles do not only take advantage of both equipment, but they actually make the most of what the two systems have to offer: the stream of data, collected by the connected equipment and transmitted to the algorithm of automation, can be much more easily treated by the system than by a human brain, thanks to higher memory and processing capacities. Besides, the automation increases the range of action of the vehicle, compared to the case where a human driver would be in control: some manoeuvres that were not or hardly achievable by a human will be feasible for a computational system.

With artificial intelligence (AI) learning, the vehicle is able to identify the most relevant data and execute the best manoeuvre depending on the situation it is facing, which also improves its performances. In addition, the coordination and the manoeuvrability reached by CAVs allow them to generate a traffic as smooth as possible, considering that they constitute a sufficient proportion of vehicles in the traffic to generate more than a local effect.

Connected and automated vehicles are the ones that are considered in this study. There will be no focus on how the vehicles collect and transmit information, it will just be assumed that they manage to do so, except maybe in specific conditions which represent an abnormality in the operation of the process. Furthermore, the study is centred on a specific formation of vehicles called a platoon.

1.2.4 Platoons

A platoon of CAVs is a group of vehicles that moves in a coordinated way, where the behaviour of each vehicle depends on the leader's one. The coordination is ensured by capacities of communication between vehicles and a computer system able to use this information (position, speed, acceleration, jerk…) to identify and perform the adequate manoeuvres.

Figure 2 : *CACC 6-car platoon, as part of the Connect and Drive Project.* Source: Federal Highway Administration Research and Technology.

Platoons can be classified in typologies according to their characteristics, among which the formation adopted and the communication strategy. The formation refers to the global scheme adopted by the group of vehicles: forming a single queue, two queues next to each other, or in a "stairs" shape, for example. As for the communication strategy, it is relative to how the vehicles share the data with each other, or in other words, it refers to which vehicle is a transmitter and/or a receptor with respect to the others. The figure below (Hobert, 2012) illustrates some different communication strategies in the "queue" formation:

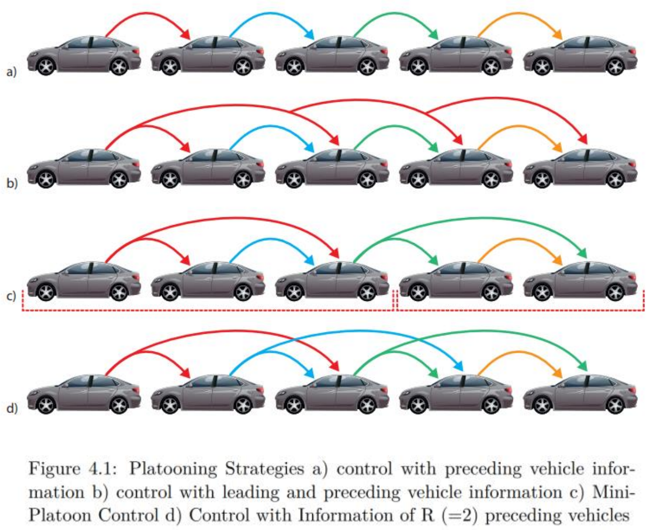

Figure 3 : *Examples of communication strategies in platoons.* Source: Hobert (2012)

As illustrated above, there are many different options regarding the communication strategy. The typology that was considered in the study is the following: the leader transmits information to all its followers, and each vehicle receives and transmits information to its direct neighbours. Consequently, unlike the examples showed in figure 3, the information can also go from a vehicle to its direct leader (the information can run up through the platoon). Keeping the same style of presentation as the one of figure 3, the communication strategy used in this study can be illustrated as follows:

Figure 4 : *Communication strategy considered in this study.* Source: Hobert (2012)

Platooning, the strategy consisting in using platoons, regroups all the phases of the life of a platoon, from their creation to their dismantling, passing though their regular state.

- The creation of the platoon, usually regrouping vehicles sharing a common destination, or having at least a common path.
- The regular state, that includes different situations: the incorporation of a new vehicle to the already existing platoon, the platoon's adaptation to the variations of the leader's behaviour, and the stable state. Notice that the total length of a platoon is limited by its range of communication, for the leader needs to be able to communicate with all the platoon's members. In the configuration used in this study for instance, since the leader has to provide information to all its followers, its range of communication is the limiting factor, whereas the configuration "a", presented in figure 3, would not have this issue.
- The dismantling, where one or various vehicles leave the platoon and therefore are not coordinated in the same way as when they were part of it. This phase can appear under different circumstances, such as a simple necessity for a vehicle to follow its own path, unfavourable conditions for the platoon to work safely, or even an emergency braking situation.

The platooning strategy with CAV has proven to be of interest, as it manages to increase the flow capacity of the existing infrastructures, among other advantageous consequences: creating a safer environment for all road users, improving the travel time of vehicles, and also providing benefits in terms of economic, social, and environmental considerations, as presented in the state of the art of this report.

2 STATE OF THE ART:

Autonomous vehicles have been studied since the beginning of the XXth century, in the 1920s, when the first tests were run with the "American Wonder", a radio-controlled 1926 Chandler driving through the traffic jams of Broadway and Fifth Avenue. As technology went improving through time, each decade saw its share of experiments, like in the 1960s with the driverless Citroen DS of the United-Kingdom's TRRL (Transport and Road Research Laboratory), that would follow the road with a speed of 130km/h far better than any human driver would, thanks to its ability to interact with magnetic cables installed below the surface (Reynolds, 2001); or for example, Mercedes-Benz's vision-guided van in the 1980's (Oagana, 2013), able to reach a speed of 63 km/h on streets without traffic.

Nowadays, more elaborated tools have been developed, and the best lead to achieve autonomous vehicles seems to be connected automated vehicles (CAVs). Based on real-time wireless exchange of data, CAVs have not entirely solved the question of autonomous vehicle yet. Various issues are still currently studied, many propositions are being discussed and a variety of ideas argued not only about the technical approach and the feasibility of such technology, but also about other aspects such as the reliability of the systems, the data safety, the social acceptancy, and the environmental repercussions.

As fully automated vehicles are considered a plausible technology to be achieved by the half of the century, public authorities from many countries already implemented regulations and test areas, with the objective of acquiring more practical knowledge on the CAV and familiarize the populations to the presence of this breakthrough. In 2020, the company Waymo launched in the city of Phoenix, Arizona (USA) a service of driverless taxis, and keeps running experiments in 25 other cities around the country (Piquard, 2021). In May 2021, Germany voted a law creating a legal framework so that CAV can go on open roads all over the territory starting from 2022 (Cazenave, 2021).

Currently, one of the main issues that States are confronted to regarding CAVs, is the acceptancy of this new technology by the populations. Indeed, the integration of CAVs into the market is to be gradual since it would be impossible to change the worldwide stock of cars within a short period of time. The question of the interaction between connected automated vehicles and regular vehicles (RV) becomes fundamental, and it is the subject of many studies such as the ones of Orosz (2019), and Brunett *et al.* (2013). The first study came to the conclusion that even in human-driver dominated conditions, the presence of CAVs could improve the traffic flow and reduce the perturbation propagation resulting in human drivers' behaviours, either with or without any cruise control device inside the regular vehicles.

The second reveals that having platoons of CAVs with short time headways tended to influence human drivers nearby, as they would themselves adapt their behaviours and reduce their own headways. The capacity of the infrastructure was therefore increased, but it was observed in the experimentations that many human drivers spent more time than usual with a time headway inferior to 1 second, which represents a safety threshold. As a consequence, further studies must still be run in order to measure the impact of CAVs on human drivers, which may not be only beneficial. With this idea, some computer simulations have also been produced at bigger scale, like the one of Covas *et al.* (2020), that takes place in the city of São Paulo and shows an improvement of the average travel time of all vehicles.

The protection of data is also a main issue in the field of CAVs, and was as a matter of fact an important part of the law about autonomous vehicles voted in Germany in May 2021 (modification of the law 19/27439). Besides, apart from the legal range of use that companies have regarding the collected data, another really specific problem to any digital connected system is the protection against hacking and other cyber-attacks. When using a CAV, users – and people around them – actually rely on informatic and electronic devices to ensure their safety, when any perturbation affecting these devices could have grave consequences. The studies led by Dong *et al.* (2018) and Chen *et al.* (2020), focused on the impact on traffic flow and safety distance changes caused by cyber-attacks on CAVs. It was showed that cyber-attacks consisting in a small deformation of the data relative to position and speed (that is, incorrect values still within the respective ranges of error accepted by the CAV system) could generate negative impacts on the road capacity, higher risks of rear-end collisions and an increase of fuel consumption and air pollution. In addition to that, attacks relative to position were more harmful than the ones relative to speed, the damage caused by slight attacks was higher on deceleration phases than on acceleration ones, and many slightly attacked CAVs were more dangerous than fewer severely affected CAVs. These different works provide useful insight for the elaboration of strategies to counter the effects of cyber-attacks that could endanger the road users' lives.

The strategy of platooning is also of much interest in the field of CAVs, due to the improvements it implies in terms of road infrastructure capacity, and the social and environmental benefits that could result from its implementation. Both works of Dennis et *al.* (2017), as well as Sun & Zhao (2013), propose a formal verification of CAVs platooning, consisting in the design of a global behaviour algorithm, able to decide under which conditions a vehicle could join or leave a platoon. Indeed, the feasibility of CAVs platooning is known to be within reach, but research still need to be lead in order to achieve this goal. Some investigations on truck platooning (Tsugawa, 2013) managed to show that the platooning strategy could save up to 15% of the overall consumed fuel at high-speed driving, along with a reduction of around 2-5% of CO_2 emissions, and better road capacity due to a more limited spatial occupation.

Boukhellouf (2019) also studied this particular aspect of CAVs, as it proposes an explicit algorithm to control the manoeuvres of the vehicles being part of a platoon, and more particularly the longitudinal behaviour during steady states, acceleration phases and deceleration phases in highway conditions. This approach is quite similar to the one carried out by General Motors (Mishra, 2014), who began to develop in the 1950's and over several decades, a model of longitudinal control of human-driven vehicle. Indeed, it was aimed to describe the behaviour of a driver with the expression of the acceleration of the vehicle, since this parameter is directly controlled by the driver thanks to the pedals.

It started with the Pipes model (Mishra, 2014), based on the Californian legislation that imposes the minimal distance between two following vehicles to be equal to the length of a car times its speed in mph and divided by 10. According to this assumption, the minimal distance increases linearly with speed, while the minimum headway decreases proportionally to the inverse of the speed, with a minimal value of 1.36s. Next came Forbes' theory (Mishra, 2014), another linear model based on the minimal reaction time to perceive the need to decelerate and use the brakes (equal to the reaction time plus the time needed to travel a distance equal to the length of the vehicle), which gave very similar results to Pipes' model.

The GM theory evolved through the years, but the main concept remained unchanged. The change of acceleration to adopt, depends on the perceived difference of speed between the two following vehicles (called "stimulus") and the ability for the follower driver to react in consequence (called "sensitivity"). The driver's acceleration response is therefore strictly proportional to the difference of speed between the two vehicles. The coefficient of proportionality was at first a constant value, but experience showed that this parameter could vary along quite an important range. Therefore, the models tended to incorporate more factors in this term, in order to represent reality more accurately.

Since the distance between the vehicles was one major cause of variability, a first idea was to introduce two factors of proportionality: one when the vehicles are close, and another one when they are far apart. However, the difficulty in choosing a value for each of these parameters led to a simplification, resulting in the use of a single value that would depend on the inverse of the space gap.

Lastly, a fourth model included the current speed of the vehicle in the expression, as it had been noticed that the higher the speed, the more sensitive the driver was to the stimulus. The fifth and final model added power coefficients "m" and "l" to the vehicle's speed and space gap terms, such that the expression of the acceleration of the follower vehicle results as:

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha_{l,m}[\dot{x}_{n+1}(t+\Delta t)]^{m}}{[x_n(t) - x_{n+1}(t)]^{l}}\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right] \quad (1)$$

Equation 1 as presented in (Mishra, 2014).

The study of Boukhellouf (2019) also proposes an acceleration formula, see equation (2), but in the context of platoons of CAVs, it depends on the leader's acceleration, the difference of position between the two vehicles minus the desired space gap (DSG), and the difference of speed between them:

$$a_n(t+\delta) = a_0(t) + k_1\big(x_{n-1}(t) - x_n(t) - DSG_n(t)\big) + k_2\big(v_{n-1}(t) - v_n(t)\big) \quad (2)$$

The parameters k_1 and k_2 had to be determined in the study, to calibrate the formula and obtain an acceptable behaviour. Indeed, in the original equation, these two parameters were linked, but the tests performed during the investigations in these conditions were not satisfying, and it was therefore decided to make them independent. Thanks to this formula and an algorithm based on it, the calibration could be done and thus a functional behaviour was found. Nevertheless, it presents undesirable instability phenomena in some manoeuvres, such as the bullwhip effect, that will be explained more in details in the methodological part, and whose resolution would definitely improve this model.

As was showed, the utility and the complexity of CAVs are such that it justifies the many studies performed about them, in all the fields of application related: technicity, efficiency, cyber protection, social and environmental impact. Even knowing the goal achievable, many questions left must be answered, and investigation must still go on in order to make this technology achievable in the coming years.

3 METHODOLOGY:

3.1 The Bullwhip effect

The objective of this study is to get rid of the instability phenomenon that appear along a platoon of CAVs during their acceleration and deceleration phases, or at least to significantly reduce its magnitude. This phenomenon, called the **bullwhip effect**, is composed of two different perturbations that alter the desired behaviour of the platoon.

The first consequence of the bullwhip effect is the amplified response that a vehicle gives as it is further in queue. Indeed, when the leader vehicle performs a manoeuvre, the information is transmitted with some delay to its followers, allowing them to adapt their own behaviour to the new piece of information received. However, the n-th follower needs to adapt its behaviour with respect to the previous vehicle, and thus the further it is with respect to the leader, the more its response will be amplified. For example, a platoon that has to decelerate will see each consecutive vehicle decelerate more and more than the previous vehicle, until a point where the safety and comfort of the passengers is not guaranteed anymore.

The other perturbation generated to the bullwhip effect is related to the minimum safety distance with the previous vehicle, also called Desired Space Gap (DSG). The following illustrative example considers the case of a positive acceleration, but the reasoning is also perfectly valid with deceleration: as the platoon accelerates, the desired space gap needs to increase, since higher velocities require higher safety distances, to allow the vehicles to perform adequate manoeuvres and in the worst case, to perform an emergency braking without colliding with the previous vehicle. Consequently, since all the vehicles composing the platoon accelerate at the same time, each of them has to accelerate less than the previous one, such that the DSG between them increases correspondingly to the gain of velocity. Nevertheless, it may happen that a vehicle far in the platoon has to “accelerate” so little, that it has in fact to decelerate, and the followers behind it will also decelerate more and more intensely. Therefore, we are now facing an acceleration phase where the vehicles down the queue are decelerating. This contradictory behaviour is typical of the bullwhip effect, and must definitely be avoided to ensure safety, efficiency, and comfort to the passengers.

3.2 Sequential Acceleration

The instability that is the bullwhip effect is due to the fact that the considered model determines the acceleration of a vehicle with respect to the previous vehicle, causing any error to propagate and amplify along the platoon. In this study, the idea of a behaviour not depending on the followed vehicle was conceived: in order to avoid the bullwhip effect, each vehicle must know its manoeuvre before even starting the positive acceleration phase, such that any information received by a vehicle is only used to check and potentially correct the behaviour determined beforehand. More precisely, the acceleration patterns that must follow the vehicles of the platoon are determined by the initial and objective conditions (velocities and related desired space gaps), the mechanical limitations of the vehicle (maximal acceleration allowed) and the safety and comfort regulations (maximal jerk).

To begin with, the length of the platoon, that is, the number of vehicles composing the platoon, constitutes an important issue. It has been explained that the bullwhip effect tended to be more present and observable in longer platoons. However, long platoons are fundamental in the platooning strategy, since it permits a diminution of the space occupancy and therefore, an increase of the infrastructure capacity. Reducing the length of platoons may not be an interesting solution considering this point, as it could reveal counterproductive. To conserve the possibility of long platoons without having to deal with propagating perturbations, it is proposed to implement sequential acceleration phases: while a certain part of the platoon will perform the acceleration manoeuvre, the rest of the vehicles will remain the corresponding stable state regime (whether at the initial velocity or at the objective one), until the accelerating vehicles reach their objective, and another group can now perform the manoeuvre. This process repeats until all vehicles are in the objective stable state.

By doing so, the platoon is temporarily split up into two or three smaller groups, each one having its own regime, and whose members will change over time: the "leading group", composed of the vehicles who achieved the objective velocity and stable state, the "acceleration group" which is performing the acceleration process (the vehicles part of this group could be considered as "active"), and the "following group" that has maintained its velocity and is waiting for its moment to proceed to the acceleration. Notice that during the first acceleration phase, the "leading group" does not exist yet, and the platoon is thus split up in only two groups, as well as during the last acceleration phase, where the "following group" does not exist anymore.

With this process, long platoons are allowed during most of their trips, as they only split up into smaller platoons temporarily, that is when they perform a change of speed. This allows to benefit from the increase of the infrastructure's capacity, and prevent intense amplifications of perturbations along the platoons. But how exactly would this sequential acceleration work? Let us start answering this question by determining the number of vehicles that would proceed to the acceleration during one phase of a sequential acceleration.

3.3 Number of active vehicles

3.3.1 Space gap time and acceleration time

In order to determine the number of active vehicles during a sequential acceleration, let us consider first two vehicles with the same initial velocity v_0 and an initial space gap d_0 between them. There are two questions that need to be answered: assuming they each dispose of an acceleration $a_1(t)$ and $a_2(t)$, how long would it take them to reach an objective velocity v_f, starting from v_0? And how long would it take them to increase the space gap between them from d_0 to d_f, starting with the same initial conditions?

Regardless of the vehicle considered to answer the first question, the time T_v to reach v_f from v_0 is:

$$T_v = \frac{v_f - v_0}{\bar{a}} \tag{3}$$

Where $\bar{a}$ **is the average acceleration of the vehicle over time**.

As for the second question, the time T_d for two vehicles 1 and 2 to reach a final space gap d_f starting from an initial gap d_0 is computed as follows:

$$d_f = d_0 + \int_0^{Td} \left(v_1(t) - v_2(t)\right) * dt \tag{4}$$

Let us have a closer look at the term inside the integral. The difference of velocities of the two vehicles can be expressed in the following way:

$$v_1(t) - v_2(t) = v_0 + \int_0^t a_1(x) * dx - \left(v_0 + \int_0^t a_2(x) * dx\right) \tag{5}$$

Thus:

$$v_1(t) - v_2(t) = \int_0^t (a_1(x) - a_2(x)) * dx \tag{6}$$

By introducing the **average difference of acceleration between the two vehicles $\overline{\Delta a}$**, we obtain:

$$v_1(t) - v_2(t) = \overline{\Delta a} * t \tag{7}$$

Implementing this result to the original equation gives:

$$d_f = d_0 + \int_0^{Td} \overline{\Delta a} * t * dt \tag{8}$$

The integral can now be computed, and it gives the following formula:

$$d_f = d_0 + \overline{\Delta a} * \frac{T_d^2}{2} \tag{9}$$

Finally, the time T_d can be expressed as:

$$T_d = \sqrt{\frac{2 * \left(d_f - d_0\right)}{\overline{\Delta a}}} \tag{10}$$

3.3.2 Safety requirements

At this step of the reasoning, safety must be considered. Indeed, the desired space gap must be met at any moment, in particular d_f must be reached before the objective velocity v_f: this condition ensures that, in case of emergency braking, the distance between each vehicle is sufficient to avoid collision. Therefore, the time T_d must be inferior or equal to T_v:

$$T_d \leq T_v \tag{11}$$

Which using the results obtained previously is equivalent to the expression:

$$\sqrt{\frac{2*\left(d_f-d_0\right)}{\overline{\Delta a}}} \leq \frac{v_f-v_0}{\bar{a}} \tag{12}$$

The behaviour that is implemented now consists in having a certain number k of vehicles accelerating such that the difference of average acceleration between each consecutive vehicle is uniform, so that all the space gaps increase at the same rate. In order to do so, the average difference of acceleration between the two vehicles $\overline{\Delta a}$ is imposed to be equal to a fraction of the average acceleration of the leader with respect to time $\overline{a_{leader}}$:

$$\overline{\Delta a} = \frac{\overline{a_{leader}}}{k} \tag{13}$$

This would have two important consequences: each vehicle further in the queue will have a smaller acceleration than the previous one, but always strictly positive, and secondly all the active vehicles would reach the objective desired space gap simultaneously.

Besides, with this new piece of information, it has to be noticed that since the leading vehicle has the greatest acceleration, the rest of the vehicles in the platoon will satisfy the inequality if the leader does so. More precisely, if the first follower verifies it, this remains true, but it can be assumed that considering the leading vehicle is a more conservative approach, and may even make more sense in the case where there is a queue of platoons.

If we now insert the expression into the inequality relative to the leader, it becomes:

$$\sqrt{\frac{2*\left(d_f-d_0\right)*k}{\overline{a_{leader}}}} \leq \frac{v_f-v_0}{\overline{a_{leader}}} \tag{14}$$

The isolation of the variable k can be processed:

$$\sqrt{k} \leq \frac{v_f-v_0}{\sqrt{2*\left(d_f-d_0\right)*\overline{a_{leader}}}} \tag{15}$$

And because k represents a number of vehicles, it is a positive integer superior or equal to 1, which allows to write:

$$k \leq \frac{\left(v_f-v_0\right)^2}{2*\left(d_f-d_0\right)*\overline{a_{leader}}} \tag{16}$$

By applying safety requirements and proposing conditions on the accelerations of the vehicles, an expression of the maximum number of active vehicles was obtained. This expression depends on the initial and final conditions of space gap and velocities, as well as the average acceleration of the leader. The next step of the process is to determine this unknown variable.

3.4 Average acceleration of the leader

For general knowledge, an average car can go from 0 km/h to 100 km/h in 9 seconds, which represents an acceleration a_{car} of around 3.08 m/s².

Regarding the model that is being established, let us consider that the acceleration is trapezoidal and symmetric during the transition phase T: it reaches a maximal value a_{max} at $t=t_1$ and due to the symmetry, starts to decrease at $t= t_2$ such that $T-t_2=t_1$. The trapeze shape allows to have an acceleration pattern close enough to reality, since the maximal acceleration cannot be reached instantly, and the symmetric property is taken to simply some further calculations, even if it is admitted that it is a strong hypothesis that could be discussed in future investigations.

The parameter a_{max} is not the absolute maximal acceleration that can be reached by a vehicle. As a matter of fact, it refers to the maximal acceleration allowed in the considered acceleration phase. If the change of speed to proceed is low, for example (in other words, in v_f and v_0 are relatively close), then deploying the maximal acceleration potential of the vehicle may be overreacting. Therefore, the parameter a_{max} does not only depend on the mechanical limits of the vehicle (or even, of the entire platoon), it must also be adapted to the manoeuvre that is aimed.

Going back to the general equation seen previously:

$$\int_0^T a(t) * dt = \bar{a} * T \tag{17}$$

It can be developed into the following formulation, thanks to the hypothesis on the shape of the acceleration along time:

$$\bar{a} = a_{max} * \left(1 - \frac{t_1}{T}\right) \tag{18}$$

However, in order to ensure a minimum comfort to the passengers of the vehicle, the jerk has to be limited, and therefore there exist a maximal acceptable jerk called j_{max} that verifies the equation:

$$t_1 = \frac{a_{max}}{j_{max}} \tag{19}$$

Which combined with the previous equation allows to write:

$$\bar{a} = a_{max} * \left(1 - \frac{a_{max}}{j_{max} * T}\right) \tag{20}$$

Besides, the acceleration phase must allow a vehicle to reach a velocity v_f from v_0, in consequence the acceleration time T must be equal to T_v:

$$T = T_v = \frac{v_f - v_0}{\bar{a}} \tag{21}$$

Adding this information to the previous equation, we obtain the average acceleration:

$$\bar{a} = \frac{a_{max}}{1 + \frac{a_{max}^2}{j_{max} * (v_f - v_0)}} \tag{22}$$

By introducing mechanical limitations, safety regulations and comfort considerations, we obtained an expression of the average acceleration of the leading vehicle, that depends on the maximal allowed acceleration, the maximal jerk allowed and the initial and objective velocities. The next step consists in expressing the acceleration of all the vehicles taking part in the acceleration phase.

3.5 Acceleration pattern for any vehicle

3.5.1 General equations

As previously seen, the average acceleration of the leader from a stable state with a velocity v_0 to another one with a velocity v_f is:

$$\overline{a_{leader}} = \frac{a_{max}}{1 + \frac{a_{max}^2}{j_{max} * (v_f - v_0)}} \tag{23}$$

Where a_{max} is the highest allowed acceleration for the cars forming the platoon, and j_{max} an imposed level of comfort for the passengers. The acceleration lasts T_v and has a symmetrical trapezoidal shape where a_{max} is reached after t_1 such that:

$$t_1 = \frac{a_{max}}{j_{max}} \tag{24}$$

We intend to give to all the vehicle a symmetrical trapezoidal shape, similarly to the leading vehicle. It was stated that the difference of average acceleration of two consecutive vehicle would be constant and equal to a fraction of the leader's average acceleration. Thus, considering the i-th follower, its average acceleration is equal to the leader's one minus a fraction (i-1)/k of this value:

$$\bar{a_i} = \overline{a_{leader}} * \left(1 - \frac{i-1}{k}\right) = \frac{a_{max}}{1 + \frac{a_{max}^2}{j_{max} * (v_f - v_0)}} * \left(1 - \frac{i-1}{k}\right) \tag{25}$$

Apart from that, and as was showed previously, the raw definition of the average value of a trapezoidal-shaped acceleration is:

$$\bar{a_i} = a_{max,i} * \left(1 - \frac{t_{1,i}}{T_{v,i}}\right) \tag{26}$$

Where:

$$t_{1,i} = \frac{a_{max,i}}{j_i} \tag{27}$$

and

$$T_{v,i} = \frac{v_f - v_0}{\bar{a_\iota}} \tag{28}$$

As can be noticed in the previous equations, the two parameters $a_{max,i}$ and j_i must be determined in order to obtain an explicit formulation of the acceleration of the i-th vehicle. Different approaches have been explored with this objective, but only the one presented in part 3.5.4 is functional.

3.5.2 Uniform jerk: $j_i = j_{max}$

In this first case, the jerk is the same for every vehicle. Even before doing any calculation, this configuration was immediately discarded: indeed, if all the vehicles have the same jerk, the for a certain duration, consecutive vehicles will see their accelerations and velocities increase at the same rate. This causes the space gap between them to remain constant while their speeds increase, meaning that the acceleration is not realized in safe conditions: the space gap becomes too small with respect to the velocities that are reached. Since the safety conditions are not met, this option cannot be retained.

3.5.3 Uniform maximal acceleration: $a_{max,i} = a_{max}$

In this scenario, the maximum acceleration reached is uniform, in other words, the height of the acceleration trapeze is the same for all the vehicles of the acceleration phase.

Hence:

$$\bar{a_\iota} = \frac{a_{max,i}}{1 + \frac{a_{max,i}^2}{j_i * (v_f - v_0)}} = \frac{a_{max}}{1 + \frac{a_{max}^2}{j_i * (v_f - v_0)}} \tag{29}$$

Which means that the jerk of the i-th vehicle j_i is (with i being an integer between 1 and k):

$$j_i = \frac{a_{max}^2}{\left(\frac{a_{max}}{\overline{a_{leader}} * \left(1 - \frac{i-1}{k}\right)} - 1 \right) * (v_f - v_0)} \tag{30}$$

Using the expression of $\overline{a_{leader}}$ found previously, it can be written that:

$$j_i = \frac{k - i + 1}{\frac{(i-1) * (v_f - v_0)}{a_{max}^2} + \frac{k}{j_{max}}} \tag{31}$$

Consequently, the time $t_{1,i}$ to reach a_{max} for the vehicle i is:

$$t_{1,i} = \frac{a_{max}}{j_i} = \frac{1}{k - i + 1} * \left(\frac{(i-1) * (v_f - v_0)}{a_{max}} + \frac{k * a_{max}}{j_{max}} \right) \tag{32}$$

We have the satisfaction to observe that, for i=1 (leading vehicle), $j_1 = j_{max}$ and $t_{1,1}=t_1$, and in addition, for the last vehicle (i=k), $j_k = 0$ (which is normal since the vehicle does not accelerate) and $t_{1,k}$ tends to infinity (since there is no acceleration, it is also coherent). Besides, j_i is decreasing when i increases and $t_{1,i}$ increases, which means the maximum jerk is never overcome (comfort ensured).

Despite the encouraging results obtained so far with this hypothesis, it was found that **this method does not work in practice.** The reason is that for model to be functional, the trapezoidal shape of the acceleration must be ensured (or at the limit, the triangular shape), and it was found that the existence of such trapeze (or triangle) is not guaranteed for all the active vehicles of an acceleration phase. Indeed, except for the first active vehicle, there is not guarantee that in any conditions, the trapeze of acceleration will be achievable.

To obtain this result, the relationship between $t_{1,i}$, the time during which the i-th vehicle's jerk is increasing, and T_v,i, the duration of the entire acceleration phase of the i-th vehicle , was studied. The condition to ensure the existence of a trapezoidal acceleration is that the maximal acceleration a_{max} must be reached, at least punctually. If that is the case, there is indeed a trapezoidal acceleration, or in the limit case, a triangular one. For this to happen, due to the symmetry of the trapeze, $t_{1,i}$ must be inferior or equal to half of $T_{v,i}$:

$$t_{1,i} \leq \frac{T_{v,i}}{2} \tag{33}$$

After developing each term with its corresponding formula, a division must be performed to isolate the variable k. Nonetheless, the sign of the term used for the division is unknown, and thus a hypothesis must be taken to proceed to the final calculations: the term is either strictly positive, or it is strictly negative.

As a result, for any value of i except 1, it occurs that whichever hypothesis concerning the sign of the term is made, both options result in a contradiction: in both cases, the inequality obtained at the end of the process happens to be the opposite of the hypothesis previously decided to perform the division. In other words, making the hypothesis that the term to perform the division is strictly positive leads to a result where this term has to be negative, and vice-versa.

At the light the results of these two cases, where one of the two parameters j_i and $a_{max,i}$ is constant, it appeared that the only other option left was to consider that this pair of variables is specific to every single vehicle.

3.5.4 Specific individual jerk and maximal acceleration

In this last case, the jerk and the maximal allowed acceleration are different for each active vehicle. Let us remember the original formula of the average value of a trapezoidal acceleration of the i-th vehicle:

$$\overline{a_i} = a_{max,i} * \left(1 - \frac{t_{1,i}}{T_{v,i}}\right) \tag{34}$$

Which can also be written in the following way:

$$\overline{a_i} = a_{max,i} * \left(1 - \frac{a_{max,i} * \overline{a_i}}{j_i * (v_f - v_0)}\right) \tag{35}$$

This equation can be reorganized into a second order polynomial equation of unknown $a_{max,i}$:

$$-\frac{\overline{a_i}}{j_i * (v_f - v_0)} * a_{max,i}{}^2 + a_{max,i} - \overline{a_i} = 0 \quad (36)$$

The discriminant of this equation has the following expression:

$$\Delta = 1 - \frac{4 * \overline{a_i}^2}{j_i * (v_f - v_0)} \quad (37)$$

For this equation to have a real solution, the discriminant must be positive or null, which implies that the jerk j_i verifies:

$$j_i \geq \frac{4 * \overline{a_i}^2}{(v_f - v_0)} \quad (38)$$

Let us now introduce the coefficient $\alpha \geq 1$ such that:

$$j_i = \alpha * \frac{4 * \overline{a_i}^2}{(v_f - v_0)} \quad (39)$$

Resolving the equation and keeping the greater value, we obtain:

$$a_{max,i} = 2 * \alpha * \overline{a_i} * \left(1 - \sqrt{\frac{\alpha - 1}{\alpha}}\right) \quad (40)$$

Despite starting with two apparently independent parameters j_i and $a_{max,i}$, it was possible to obtain an explicit equation for both of them thanks to the use of a coefficient α, that will be studied further on. Besides, the existence of the trapeze is always possible with these new expressions of j_i and $a_{max,I}$: the equation (33) is indeed equivalent to the following one, which stands true for any value of α greater or equal to 1:

$$t_{1,i} \leq \frac{T_{v,i}}{2} \Leftrightarrow \sqrt{\frac{\alpha-1}{\alpha}} \geq 0 \Leftrightarrow \alpha \geq 1$$

Two things can be noticed concerning the new formulas of jerk and maximal acceleration. First, it can be shown that this latter is a positive and decreasing function for α superior of equal to 1, which means that the maximal acceleration allowed cannot exceed twice the average acceleration:

$$a_{max,i} \leq 2 * \overline{a_i} \quad (41)$$

Secondly, the jerk of a vehicle must be strictly smaller than the one of the followed vehicles, and the absolute maximal jerk admissible being the limit of comfort, 0.9m/s^3:

$$j_i = \alpha * \frac{4 * \overline{a_i}^2}{(v_f - v_0)} \leq j_{i-1} = \alpha * \frac{4 * \overline{a_{i-1}}^2}{(v_f - v_0)} \quad (42)$$

Since the average acceleration decreases as a vehicle is further in the queue according to the established model, this inequality holds true for any active vehicle. Let us have a look at the second restriction:

$$j_i = \alpha * \frac{4 * \bar{a_i}^2}{(v_f - v_0)} \leq 0.9 \quad (43)$$

Which can be rewritten:

$$\alpha \leq \frac{0.9 * (v_f - v_0)}{4 * \bar{a_i}^2} \quad (44)$$

With the objective of choosing a value for α in a simple way, it is proposed to test the following method:

$$\alpha = \max\left(1; \frac{0.9 * (v_f - v_0)}{4 * \overline{a_{i-1}}^2}\right) \quad (45)$$

By using the average acceleration of the previous vehicle, the inequality is still verified, and we have now explicit equations for both parameters, the jerk, and the maximal acceleration:

$$\alpha = \max\left(1; \frac{0.9 * (v_f - v_0)}{4 * \overline{a_{i-1}}^2}\right) \quad (46)$$

$$j_i = \alpha * \frac{4 * \bar{a_i}^2}{(v_f - v_0)} = 3.6 * \left(\frac{\bar{a_i}}{\overline{a_{i-1}}}\right)^2 \quad (47)$$

$$a_{max,i} = 2 * \alpha * \bar{a_i} * \left(1 - \sqrt{\frac{\alpha - 1}{\alpha}}\right) \quad (48)$$

The following step consists in putting this theory into practice, as shown in the next part.

4 NUMERICAL EXPERIMENTS

To perform the numerical experiments, the set of equations obtained through the established model has been implemented into an algorithm, written in MatLab. Three functions, jerk, speed, and acceleration were coded, as well as a main file that performs the behaviour as designed in the theory. These codes are fully available in the Appendix part, at the end of this document.

The experiments are focused on the behaviour of CAV platoons on highways, in their "regular states", that is when no vehicle joins or leaves the platoon. More precisely, it consists in starting with a platoon formed by N vehicles in a stable initial state with a velocity v_0 and the related desired space gap d_0, and to make it accelerate according to the model elaborated in the theoretical part until an objective stable state with a velocity v_f and its corresponding DSG, d_f.

Several configurations with different velocities or number of active vehicles are run, and the graphs along time of several parameters are provided: the space gap, the individual position, velocity, acceleration, and jerk for each vehicle of the platoon.

4.1 Mechanical limit and comfort parameters

As previously explained, some restrictions have been taken in order to meet the minimum imposed comfort level and mechanical limits of the vehicles. Therefore, the maximal jerk acceptable retained for the experiments described is **j_{max} = 0.9 m/s³**, value taken from the work of Boukhellouf (2019). Besides, the maximal reachable acceleration is taken equal to **a_{max} = 2.5 m/s²**. Other values could naturally be used, but this experiment does not focus on the optimal or more accurate values of these parameters. Consequently, the results obtained through this process are to be considered as references, and not strictly exact values.

4.2 Desired Space Gap

The desired space gap (DSG) associated to both velocities are computed thanks to the formula developed by Boukhellouf (2019) that he expressed in the following way:

$$DSG_n(t) = minSG + v_0(t)\delta + \frac{v_0(t)^2}{2b} * \frac{\beta}{1-\beta} + l_{n-1} \tag{49}$$

This formula states that the desired space gap for a vehicle n at time t is the sum of the minimal space gap (arbitrary value of gap when the vehicles are stationary), the latency, which is the product of the leader's velocity by the delay of transmission delta, the gap resulting in the difference of deceleration capacity between the vehicle and the previous vehicle, and finally the length of the latter. The parameter β of this equation is taken equal to 20%, as recommended by Boukhellouf (2019).

4.3 From 110 km/h to 130 km/h

In this configuration, a platoon of N=20 vehicles, has to perform and acceleration from their initial velocity of 110 km/h to the objective one, that is 130 km/h (maximal velocity allowed in the French highways). The parameters of this trial are summarized in the next table:

Parameter	Value	Units
Maximal acceleration a_{max}	2.5	m/s²
Maximal jerk j_{max}	0.9	m/s³
Length of the platoon N	20	veh
Initial velocity v_0	110	km/h
Objective velocity v_f	130	km/h
Initial DSG d_0	15.23	m
Objective DSG d_f	20.41	m

***Table 1 :** Platoon parameters, first configuration (from 110 to 130 km/h)*

Thanks to this data, the maximal number of active vehicles during an acceleration phase k can be determined. It this case, k is found to be equal to 2, which means that the whole acceleration sequence of the platoon will be in 10 phases of 2 vehicles. Besides, the average acceleration of the leader is equal to 1.111 m/s². The next figures present the different graphs obtained with this configuration, and can also be observed at bigger scale in the Appendix section:

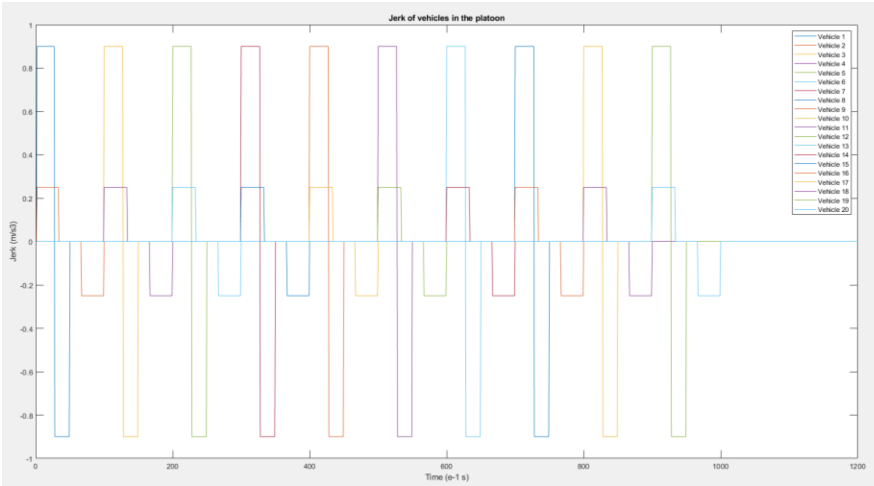

Figure 5 : *Jerk versus time, 110 to 130 km/h configuration*

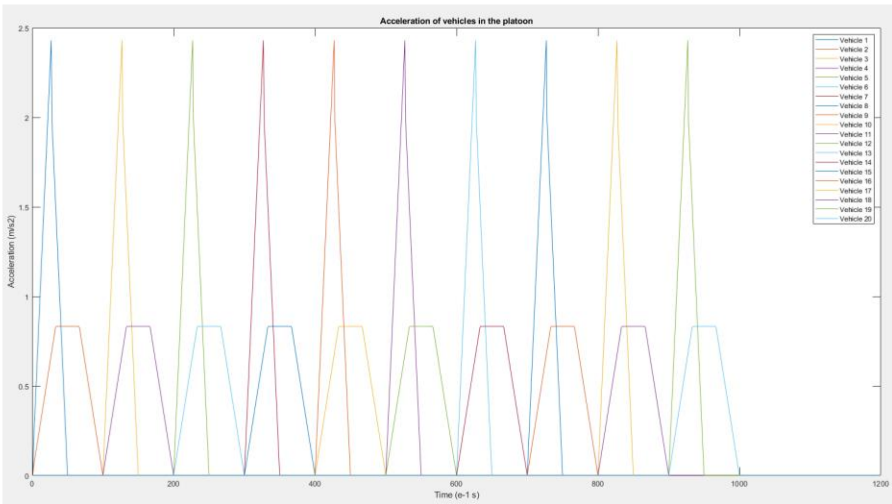

Figure 6 : *Acceleration versus time, 110 to 130 km/h configuration*

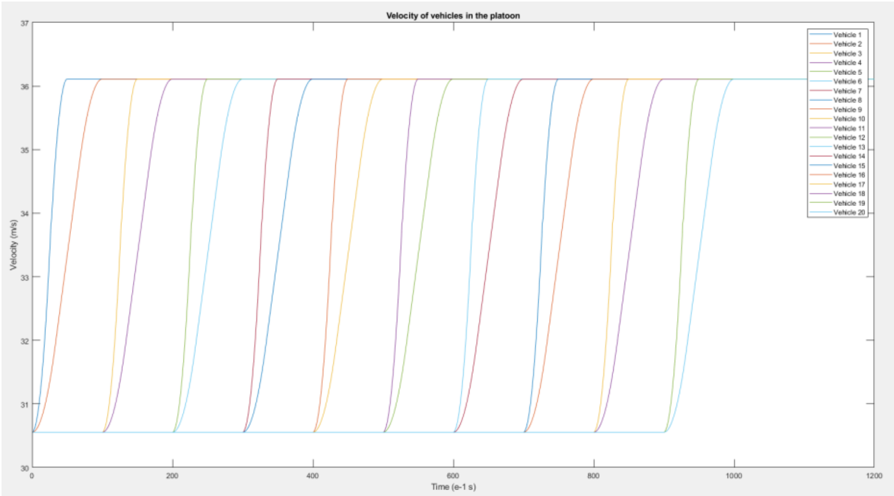

Figure 7 : *Velocity versus time, 110 to 130 km/h configuration*

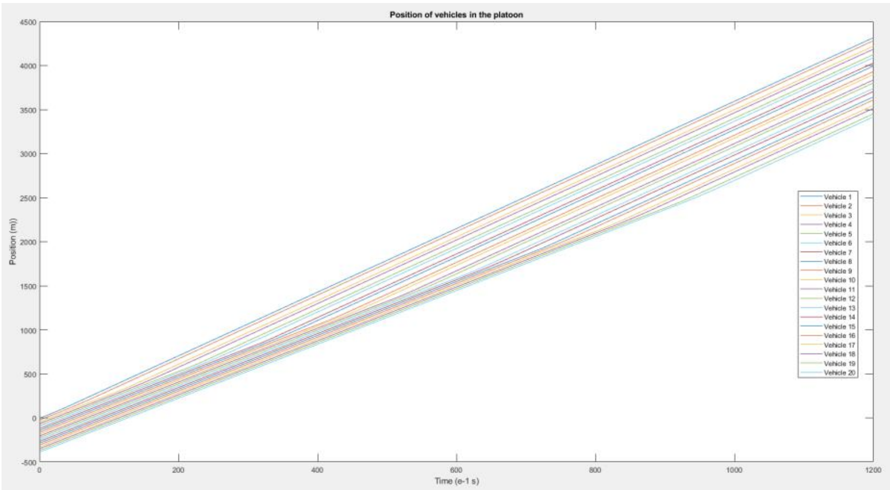

Figure 8 : *Position versus time, 110 to 130 km/h configuration*

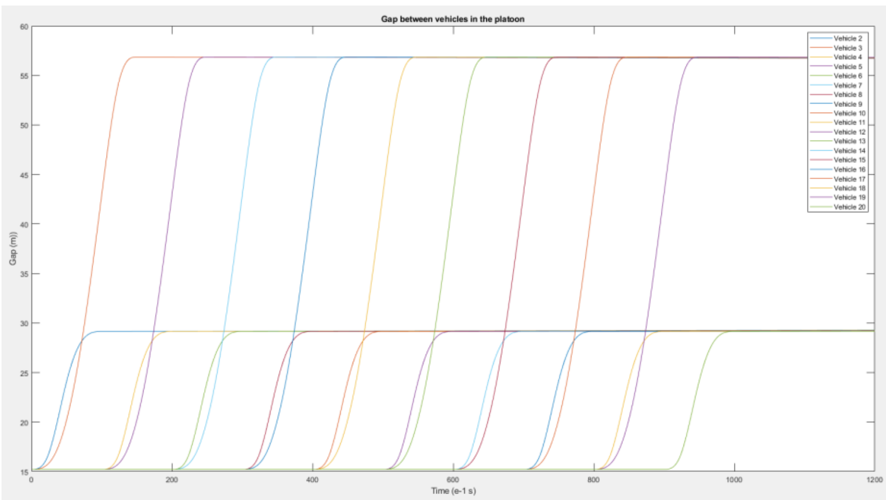

Figure 9 : *Space gap versus time, 110 to 130 km/h configuration*

As a first and general comment, it can be noticed that the sequential aspect of the model is clearly observable, as the curves repeat the same patterns in a cyclic way, and the total duration of the platoon's acceleration is around 100 seconds, 1 minute and 40 seconds.

Concerning the jerk graph (Figure 8), the maximal jerk of the second active vehicle is significantly inferior to the leading active vehicle, and this can also be observed in the acceleration graph (Figure 9): the maximal acceleration is more than half smaller, and it lasts twice as much. About the acceleration, the trapeze shape is observable, even though the active leader's acceleration pattern seems to tend towards the triangular limit case.

As for the velocity graph, it seems that all vehicles manage to reach the objective speed in the desired order, going from the head of the queue until the last follower (Figure 10).

Clear problems are visible in the position and space gap graphs, figure 11 and 12: indeed, despite the fact that the vehicle reach the objective velocity, the desired space gap is far from being reached. It can be observed in the position graph, that present parallel lines with different spacings between them, and it is even clearer in the space gap graph, that displays space gaps of 29.16 m for the "second active vehicles" and 56.84 m for the "leading active vehicles". Two issues are to be highlighted: instead of having a single value of space gap equal to the objective DSG, there is a specific space gap for each active vehicle depending on its position during its acceleration phase, and all of these space gaps turn out to be higher than the DSG (respectively 29.17 m and 56.83 m). The reasons of these results are discussed more in detail later in the report, along with the ones obtained in the next configurations.

4.4 From 90 km/h to 130 km/h

This other configuration only changes the initial velocity and DSG, with the idea of observing the behaviour of the model in the case of a wider speed change. The initial velocity is set to be 90 km/h, resulting in a change of speed of 40 km/h instead of 20 km/h previously.

Parameter	Value	Units
Maximal acceleration a_{max}	2.5	m/s²
Maximal jerk j_{max}	0.9	m/s³
Length of the platoon N	20	veh
Initial velocity v_0	90	km/h
Objective velocity v_f	130	km/h
Initial DSG d_0	10.81	m
Objective DSG d_f	20.41	m

Table 2 : *Platoon parameters, second configuration (from 90 to 130 km/h)*

As can be noticed in the following figures, the increase of the speed difference between the initial one and the objective one causes the number of active vehicles to increase as well: in this case, k happens to be 4, which means that the whole acceleration sequence of the platoon will be in 5 phases of 4 vehicles. Besides, the average acceleration of the leader is equal to 1.5385 m/s², higher than the 1.111 m/s² of the previous case with the initial speed of 110 km/h.

The cyclic behaviour is once again perfectly identifiable: the active vehicles start accelerating at the same time, with smaller jerk and acceleration than their direct leader, and end up reaching the objective velocity. Besides, the total operation is longer than in the previous case, about 144 seconds, which is 2 minutes and 24 seconds. This is quite evident: since the range of speed is more important, the acceleration phase mechanically lasts longer.

However, a look at figure 15 shows that the last active vehicle accelerates significantly longer than the other active vehicles: indeed, it takes it 29 seconds to reach the objective velocity, against respectively 7.3, 9.7 and 14.5 seconds for the others.

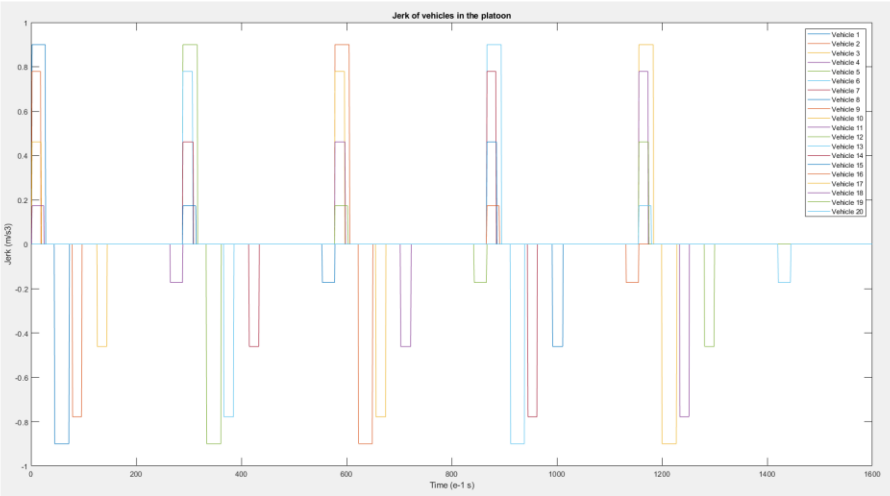

Figure 10 : *Jerk versus time, 90 to 130 km/h configuration*

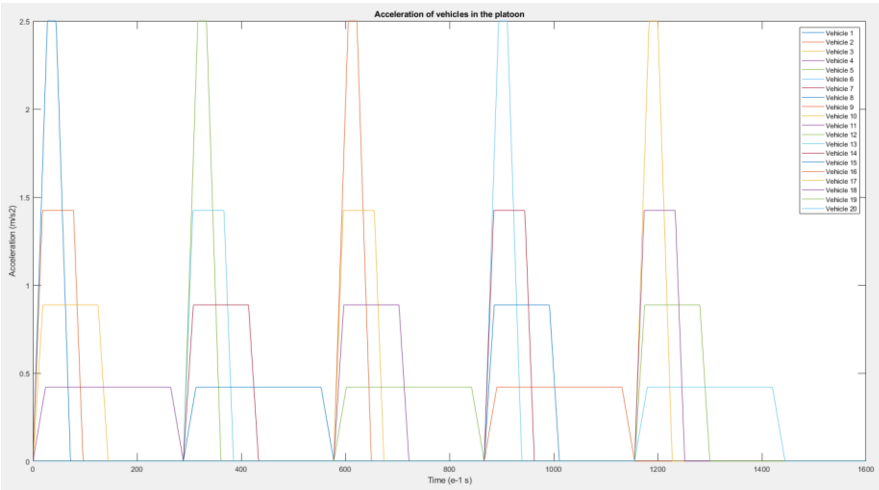

Figure 11 : *Acceleration versus time, 90 to 130 km/h configuration*

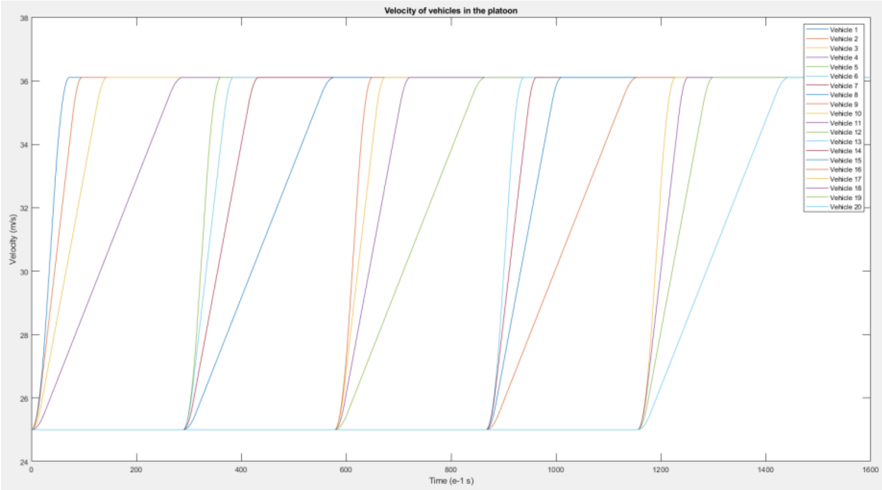

Figure 12 : *Velocity versus time, 90 to 130 km/h configuration*

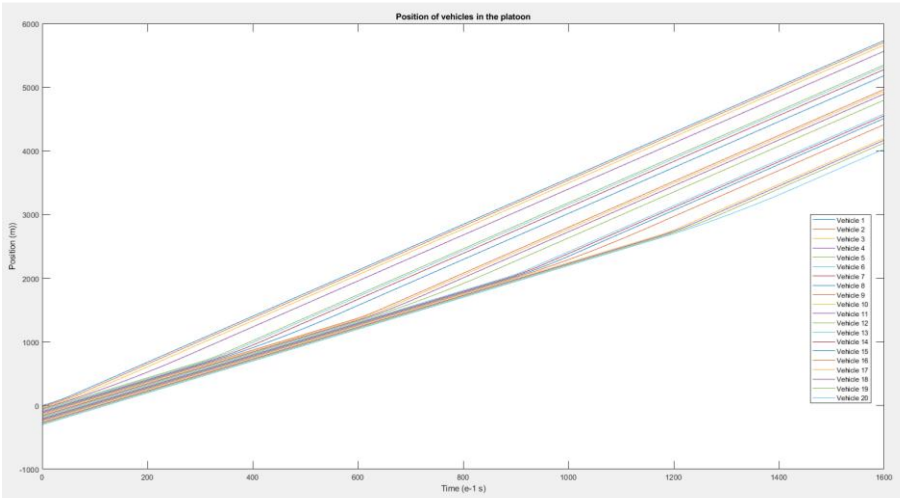

Figure 13 : *Position versus time, 90 to 130 km/h configuration*

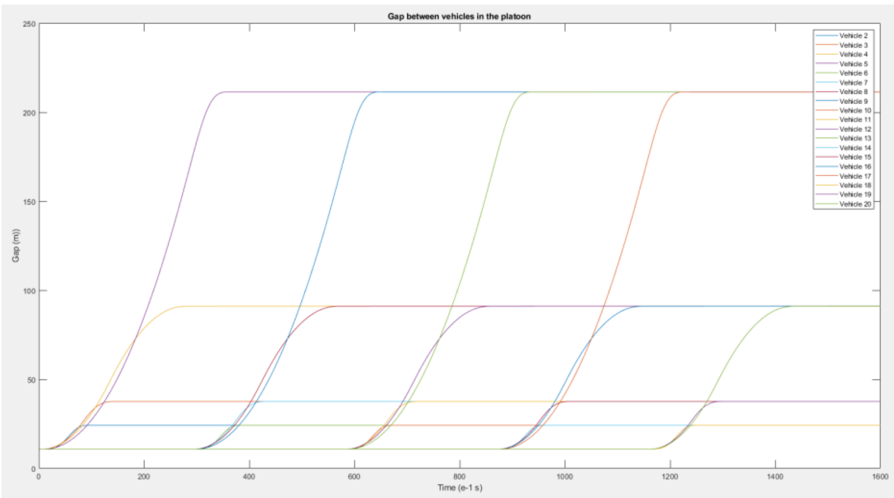

Figure 14 : *Space gap versus time, 90 to 130 km/h configuration*

Concerning the space gap, the same problem happens in this configuration: each active vehicle has its specific space gap, and none of them correspond to the objective DSG: the leading active vehicles even reach a space gap of 211.46 m, more than ten times the DSG. As mentioned in the previous test, this result is explained in the analysis section of the report, but after these two configurations where the maximal number of active vehicles is used, it seems interesting to consider a case with a number of active vehicle different from the maximal value allowed by the theory.

4.5 From 90 km/h to 110 km/h, with k-1

The idea of this configuration is to determine which changes occur when the number of active vehicles is reduced. This trial has the same parameters than the previous one, as can been seen in the table 3, but the code was modified to use k-1 as number of active vehicles instead of k. Therefore, there are no more 4 vehicles performing the acceleration during each phase, but 3, causing the scheme of phases to become 6 phases of 3 vehicles and a seventh one with the remaining 2 vehicles.

Parameter	Value	Units
Maximal acceleration a_{max}	2.5	m/s^2
Maximal jerk j_{max}	0.9	m/s^3
Length of the platoon N	20	veh
Initial velocity v_0	90	km/h
Objective velocity v_f	130	km/h
Initial DSG d_0	10.81	m
Objective DSG d_f	20.41	m

Table 3 : *Platoon parameters, third configuration (from 90 to 130 km/h, with k-1)*

The graphs regarding this configuration are available here below, as well as in the Appendix:

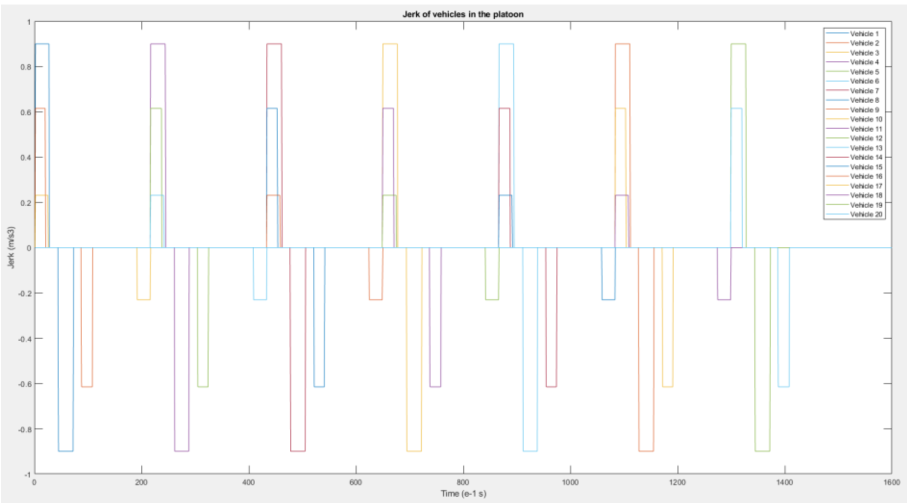

Figure 15 : *Jerk versus time, k-1 configuration*

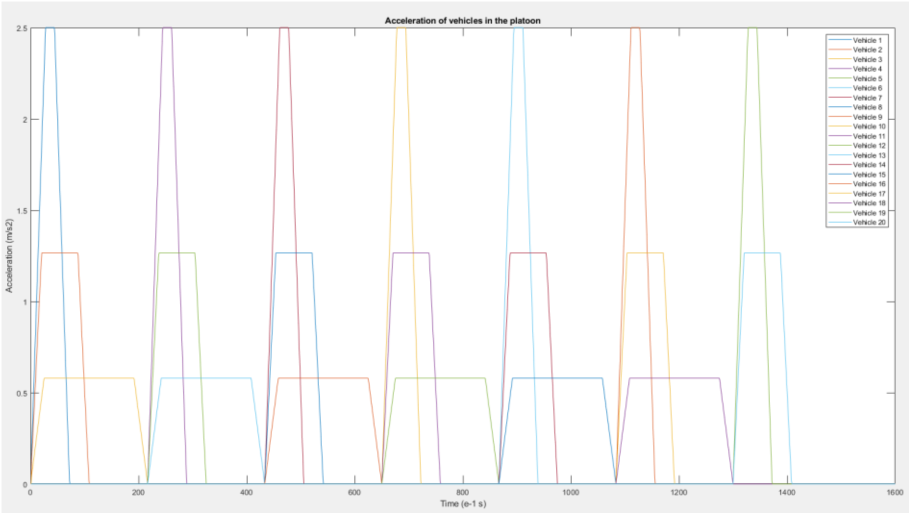

Figure 16 : *Acceleration versus time, k-1 configuration*

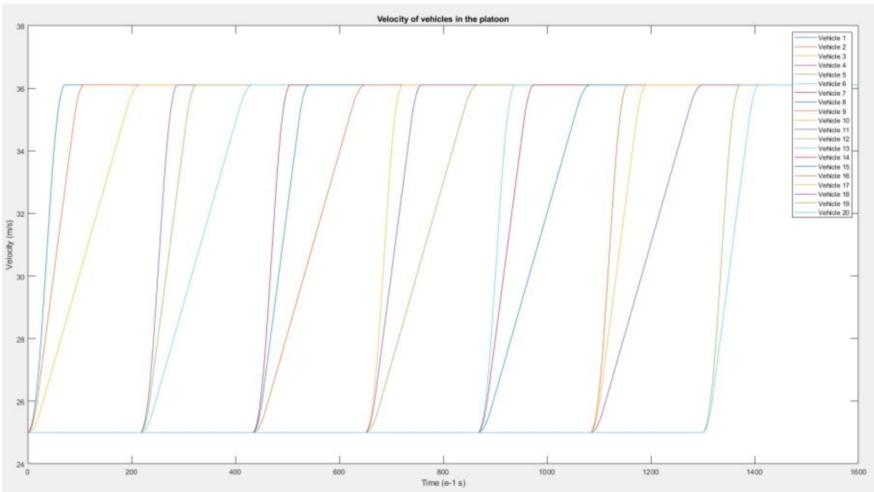

Figure 17 : *Velocity versus time, k-1 configuration*

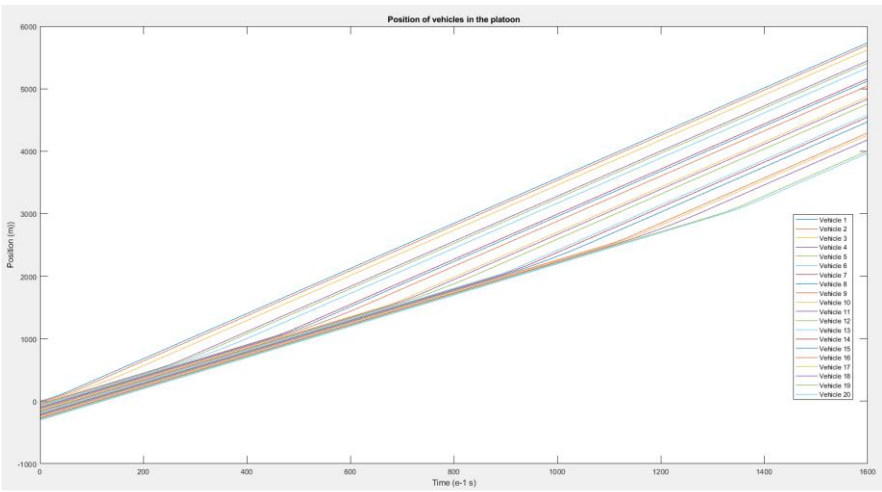

Figure 18 : *Position versus time, k-1 configuration*

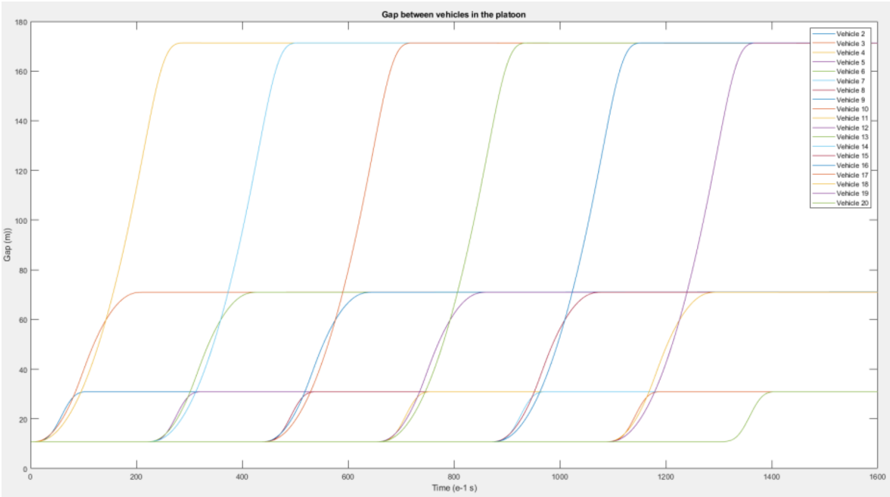

Figure 19 : *Space gap versus time, k-1 configuration*

Apart from the previously made observations, that still hold true in this case, it can be added in this particular case that the duration of the acceleration process is very similar to the one before, when the number of active vehicle k was maximal: T_{total} = 141 s, compared to 144 s.

Furthermore, the maximum space gap reached is 171,3 m, 19% less than when k is maximal. Reducing the number of active vehicles seems thus to lower the problem of "over-gapping" identified in the other tests run. It is now legitimate to ask what would happen in the extreme case, that is when k is as small as possible.

4.6 From 90 km/h to 130 km/h, with k = 1

As far as this last configuration is concerned, it consists in the limit case of the previous experiment, where the number of active vehicles was not the maximal one: indeed, here the acceleration is individual, and it can be argued that this configuration does not correspond to a platoon anymore. However, it is run to see if this limit case, or maybe even off-limit case, confirms the observations made in the case with k-1. As before, the parameters remain unchanged, expect for k that is imposed to be 1.

Parameter	Value	Units
Maximal acceleration a_{max}	2.5	m/s²
Maximal jerk j_{max}	0.9	m/s³
Length of the platoon N	20	veh
Initial velocity v_0	90	km/h
Objective velocity v_f	130	km/h
Initial DSG d_0	10.81	m
Objective DSG d_f	20.41	m

Table 4 : *Platoon parameters, fourth configuration (from 90 to 130 km/h, with k=1)*

The different graphs of this particular configuration are showed in the following figures, and can also be found with bigger scale in the Appendix section:

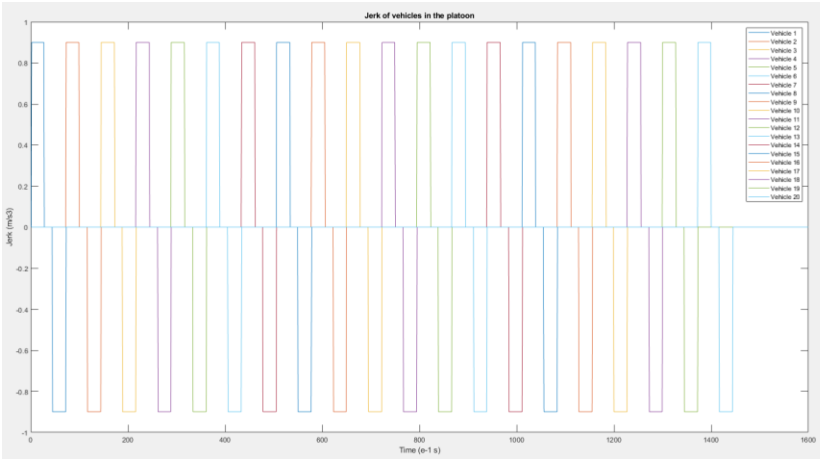

Figure 20 : *Jerk versus time, k=1 configuration*

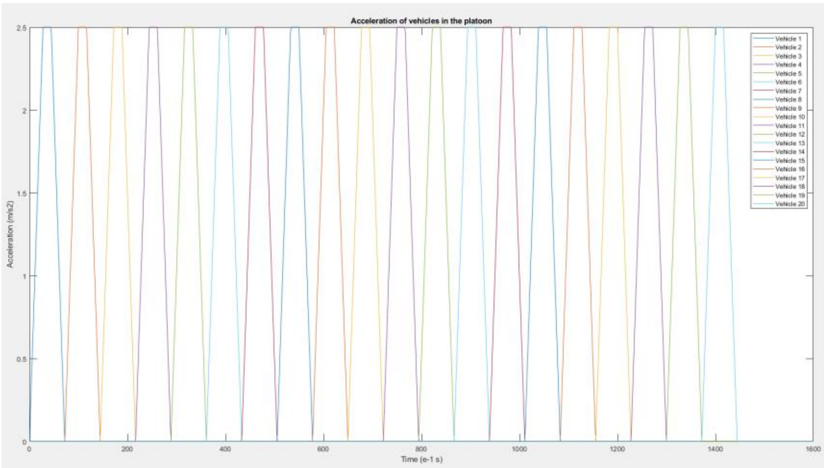

Figure 21 : *Acceleration versus time, k=1 configuration*

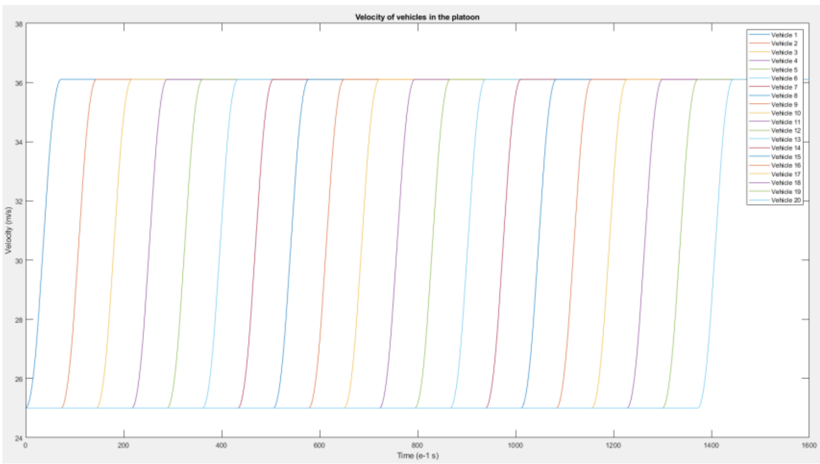

Figure 22 : *Velocity versus time, k=1 configuration*

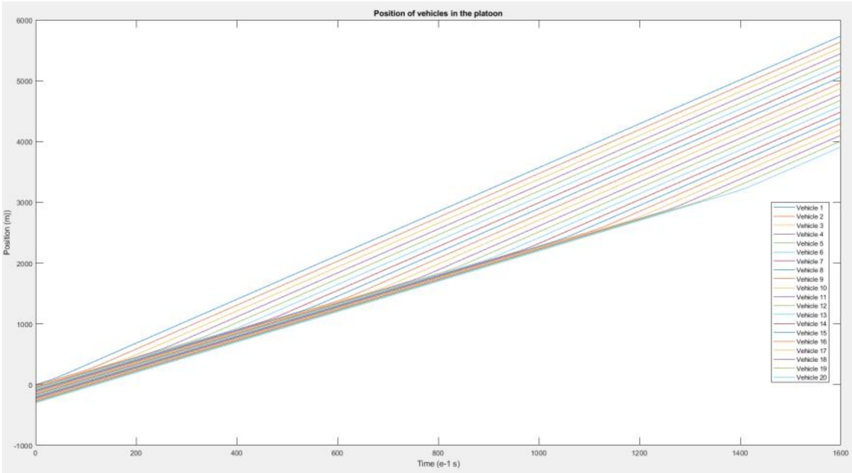

Figure 23 : *Position versus time, k=1 configuration*

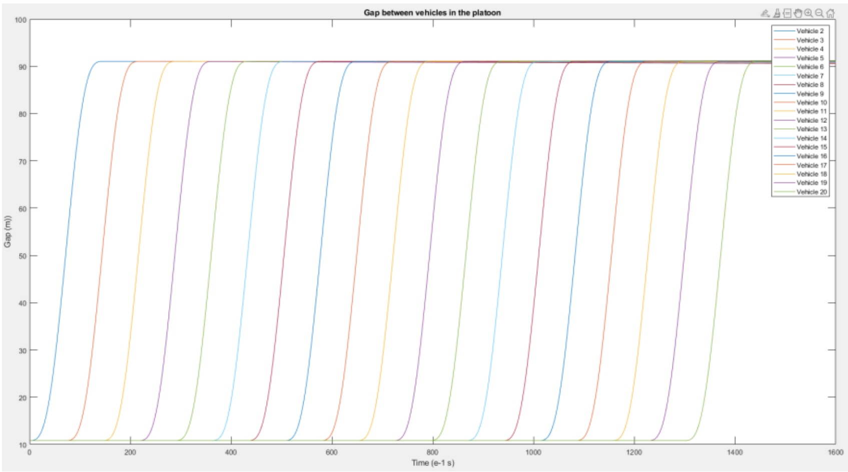

Figure 24 : *Space gap versus time, k=1 configuration*

As could have been expected, with k equal to 1, since every active vehicle is a leading active vehicle, they all follow the same behaviour, with just an interval between each sequence. As a consequence, the space gap between them is always the same (Figure 26 and 27), and is equal in this case to 90.98 m, which is 57% smaller than in the case where k is maximal, but still 4.46 times bigger than the objective desired space gap. Concerning the total duration of the manoeuvre, it remains very similar to the last results, with a duration of 144 seconds.

The overall results of this experimental phase are interesting, as they reveal what seems to be misconception in the elaboration of the theory. Nevertheless, it also showed that the conceptual idea used in the very same theory can be a lead worth following to resolve the problem of bullwhip effect.

5 ANALYSIS OF THE RESULTS

At the light of the results of the experimental process, it is possible to have more insight about the model that was established and to identify its performances and its limits. The most visible result is the final space gap at the end of the acceleration process. Indeed, while it was expected to be equal to the objective desired space gap, all the configurations that were tested showed that this goal was never achieved, and more precisely, the result was most of the time far from the aimed value. The origin of this unsatisfying result is to be found in the theoretical approach, and it has two origins.

On the one hand, when two consecutive vehicles accelerate, the increase of spacing between them is duly considered by the model, but only until the leading vehicle reaches the objective velocity: from this moment, the model does not take into account the space generated by the difference of velocity of both vehicles, due to a formulation of $\Delta\bar{a}$ that should be improved, the average acceleration difference between two consecutive vehicles. Indeed, a better expression would integrate the fact that the gap grows until both vehicles reach the objective velocity, not when only one of them does so.

The second cause of these tremendous gaps between consecutive acceleration sequences is due to the fact that, when a new acceleration sequence begins, the space gap between the new leading active vehicle is not the initial space gap d_0 anymore, since the last vehicle of the previous sequence has already reached another stable state (even if it is different from the objective stable state). Hence, with initial conditions actually different from the ones used in the theory, it is naturally impossible that the space gap that had been created during the previous sequence gets resorbed. An option to get rid of these large gaps between sequences could be to modify the leading active vehicle's behaviour such that it covers the increment of distance caused by the previous acceleration sequence: the average acceleration of the leader would be computed with its actual initial space gap instead of d_0, while the rest of the vehicles would follow the "normal behaviour" elaborated.

As for more satisfying results, it can be observed that the constraint relative to the maximal jerk is verified for all the vehicles of every single configuration. Since the jerk keeps getting lower as the vehicle is further in the acceleration sequence, the limit is never overcome and thus the comfort criterion is respected.

In addition, the graphs of position obtained through the different experiments do not show any bullwhip effect, whether in a form of an amplification of the acceleration response or a contradictory behaviour. This result comes from the non-dynamic approach used to determine the acceleration pattern: since this latter is determined before the actual acceleration phase, and that a vehicle's behaviour does not depend on its direct leader's one, each vehicle disposes of an individual, previously elaborated pattern to follow, and can therefore disregard the other vehicles behaviours under normal traffic conditions.

To sum up, the developed theory and its computational application reveal and interesting path to achieve short DSG without the bullwhip effect and propagation errors. However, due to the previous discussed limitations of the model, this remains to be proven yet and will be part of future research.

6 CONCLUSIONS AND FUTURE INVESTIGATIONS

The main objective is partially achieved, as the proposed model does prevent from the bullwhip effects during the acceleration phase, but is still incomplete and presents limitations that must be overcome in order to stand as a satisfactory solution.

The non-dynamic approach of the model, different from the previous approach used by Boukhellouf (2019), has the advantage to provide a coordinated and individual pattern to each vehicle before the acceleration phase even begins (as observed thanks to the algorithm coded during this work), which makes impossible any propagation of error along the platoon and allows a vehicle to consider the other vehicles' behaviours only to check and possibly slightly adapt its own, or to prevent any extraordinary event, such as an emergency braking.

However, the theory focuses on positive accelerations, whereas of course decelerations phases shall appear in any trip. This situation, non-covered by the current theory, is necessary to design a more complete model, and even if it seems that most of the theory could be transposed straight forwardly to the case of negative acceleration, a particular attention should be given to the specificities of deceleration, such as the moment where the process should start: indeed, in the case of a reduction of speed, all the vehicles of the platoon should reach the objective velocity before entering the zone of lower speed, whereas in the case of an acceleration, the vehicles can maintain their initial velocity in the higher-speed-limit portion of road.

Besides, the theoretical part of the model has weaknesses that diminish its efficiency. The space gaps that are generated by this behaviour are much too large, and thus incompatible with the idea of optimization of space and higher infrastructure capacity. Some ideas have been emitted in the analysis part concerning this issue, and it is clear that its resolution would drastically improve the model and get it closer to an efficient and useful tool for connected and automated platoons.

Nevertheless, the approach of this work is not the only one that can be used to solve the problem of bullwhip effect. Although it is a lead worth following and studying more deeply, an adaptation of the dynamic approach used in previous works, which would consist in adapting a vehicle's acceleration with either to its direct leader or to the platoon's leader depending on its DSG relative error, could also be promising, if not to get rid of the bullwhip effect, at least to reduce it satisfactorily.

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8 APPENDIX

Function jerk:

```
function j=jerk(v0,vf,a_max,j_max,k,m,t0,t,i)

a_avleader = a_max/(1+(a_max^2/(j_max*(vf-v0)))) ; %average acceleration of the leader of the sequence
Tm = t0 + (m-1)*k*(vf-v0)/a_avleader ; % moment when the m-th sequence begins

a_av = a_avleader*(1-(i-1)/k) ; %average acceleration of vehicle i
T=(vf-v0)/a_av ; % duration of the acceleration for vehicle i
gamma = max(1,0.225*(vf-v0)/(a_avleader*(1-(i-2)/k))) ; %coefficient of existence of the acceleration trapeze

if i ~= 1 %for all active vehicles except the leading one
    amax_i = 2*gamma*a_av*(1-(1-1/gamma)^0.5) ;
    jer = 4*gamma*a_av^2/(vf-v0) ;
else % for the leading vehicle
    amax_i = a_max ;
    jer = j_max ;
end

t1 = amax_i/jer ; % t1 is equal to amax_i/j

if t < Tm+t1 % increasing phase of the trapeze from T-t1 to T
    j = jer ;
elseif (t>=Tm+t1) && (t<Tm+T-t1) %constant maximal acceleration a_max
    j = 0 ;
elseif (t>=Tm+T-t1) && (t<Tm+T) % decreasing phase of the acceleration trapeze from T-t1 to T
    j = -jer ;
else
    j = 0 ;
end
end
```

Function acceleration:

```
function a = acceleration(v0,vf,a_max,j_max,k,m,t0,t,i)

a_avleader = a_max/(1+(a_max^2/(j_max*(vf-v0)))) ; %average acceleration of the leader of the sequence
Tm = t0 + (m-1)*k*(vf-v0)/a_avleader ; % moment when the m-th sequence begins

a_av = a_avleader*(1-(i-1)/k) ; %average acceleration of vehicle i
T=(vf-v0)/a_av ; % duration of the acceleration for vehicle i;
gamma = max(1,0.225*(vf-v0)/(a_avleader*(1-(i-2)/k))) ; %coefficient of existence of the acceleration trapeze

if i ~= 1 %for all active vehicles except the leading one
    amax_i = 2*gamma*a_av*(1-(1-1/gamma)^0.5) ;
    j = 4*gamma*a_av^2/(vf-v0) ;
else % for the leading vehicle
    amax_i = a_max ;
    j = j_max ;
end

t1 = amax_i/j ; % t1 is equal to amax_i/j

if t < Tm+t1 % increasing phase of the trapeze until reaching a_max (j*t1)
    a = j*(t-Tm) ;
elseif (t>=Tm+t1) && (t<=Tm+T-t1) % constant maximal acceleration a_max
    a = amax_i ;
elseif (t>Tm+T-t1) && (t<Tm+T) % decreasing phase of the acceleration trapeze from T-t1 to T
    a = j*(T+Tm-t) ;
else
    a = 0 ;
end
end
```

Function speed:

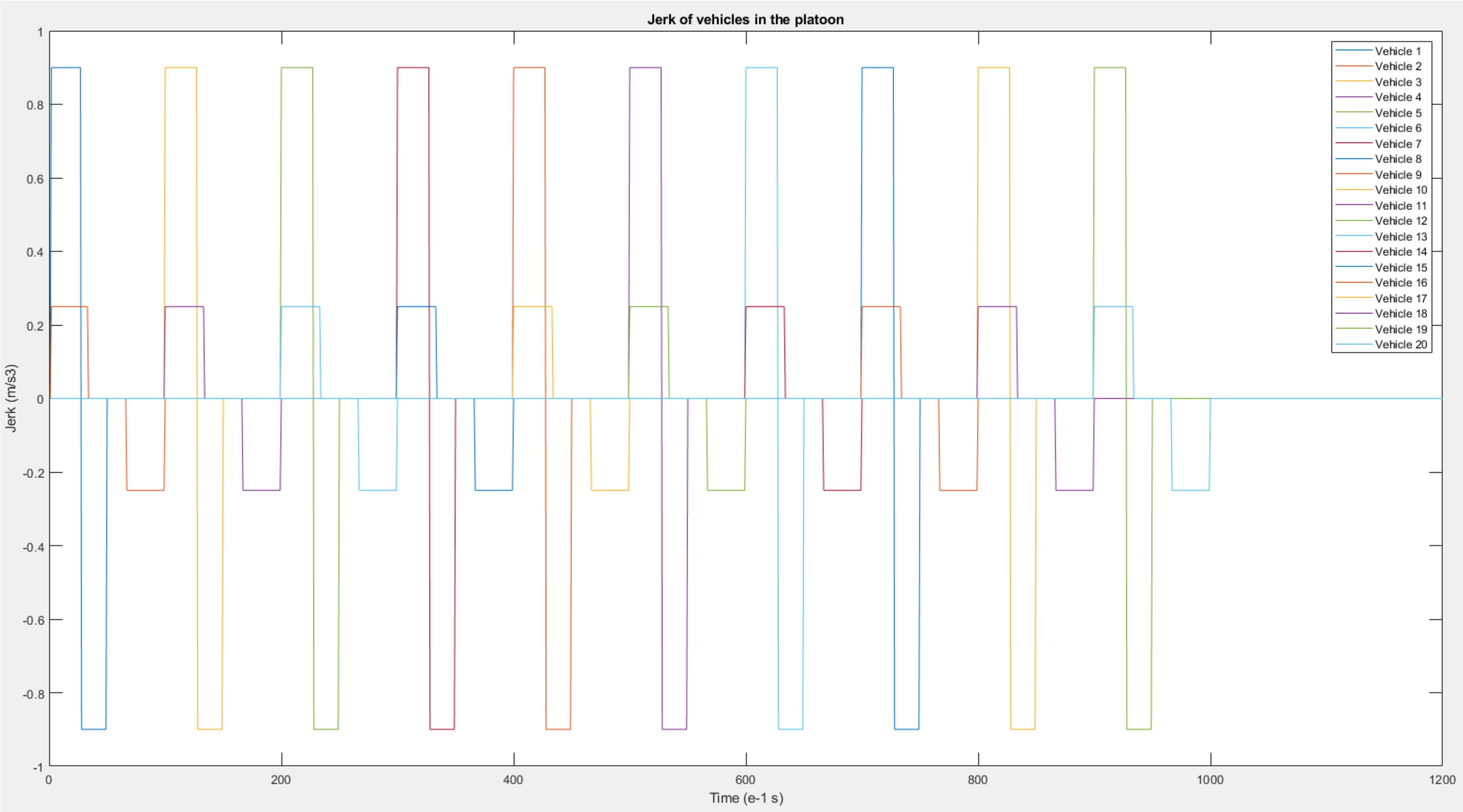
```
function v=speed(v0,vf,a_max,j_max,k,m,t0,t,i)

a_avleader = a_max/(1+(a_max^2/(j_max*(vf-v0)))) ; %average acceleration of the leader of the sequence
Tm = t0 + (m-1)*k*(vf-v0)/a_avleader ; % moment when the m-th sequence begins

a_av = a_avleader*(1-(i-1)/k) ; %average acceleration of vehicle i
T=(vf-v0)/a_av ; % duration of the acceleration for vehicle i
gamma = max(1,0.225*(vf-v0)/(a_avleader*(1-(i-2)/k))) ; %coefficient of existence of the acceleration trapeze

if i ~= 1 %for all active vehicles except the leading one
    amax_i = 2*gamma*a_av*(1-(1-1/gamma)^0.5) ;
    j = 4*gamma*a_av^2/(vf-v0) ;
else % for the leading vehicle
    amax_i = a_max ;
    j = j_max ;
end

t1 = amax_i/j ; % t1 is equal to a_max/j

if t < Tm+t1 % increasing phase of the trapeze until reaching a_max (j*t1)
    v = j*(t-Tm)^2/2 + v0 ;
elseif (t>=Tm+t1) && (t<=Tm+T-t1) % constant maximal acceleration a_max
    v = amax_i*(t-(Tm+t1)) + v0 + j*t1^2/2 ;
elseif (t>Tm+T-t1) && (t<=Tm+T) % decreasing phase of the acceleration trapeze from T-t1 to T
    v = -j*t^2/2 + j*t*(T+Tm) + v0 + j*t1^2/2 + amax_i*(T-2*t1) + j*(Tm+T-t1)^2/2 - j*(Tm+T)*(Tm+T-t1) ;
else
    v = vf;
end
end
```

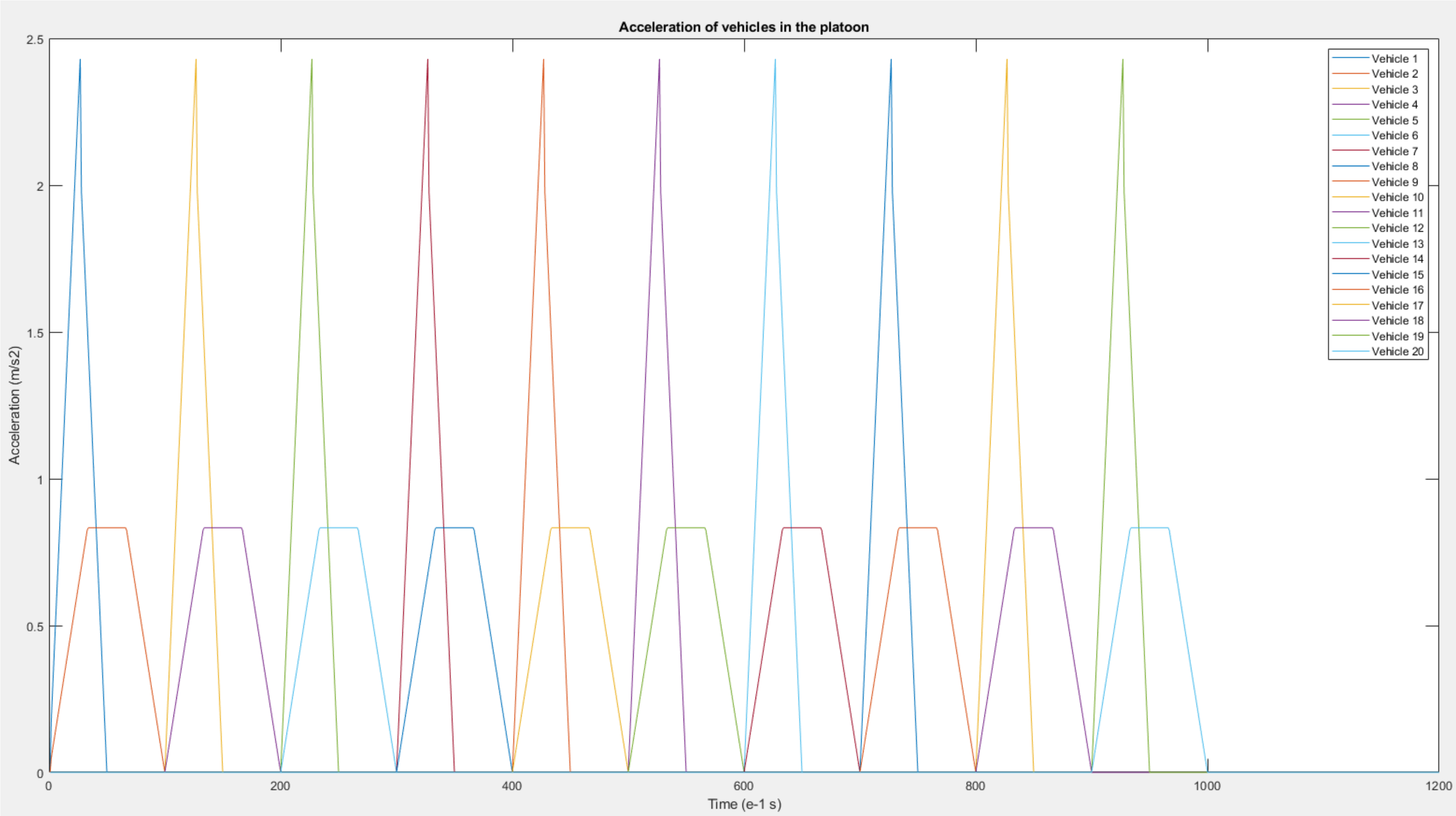
Main code:

```
%% Time parameters of the simulation
t0 = 0 ;
time_param = 1600 ; % parameter to determine SimTime
delta = 0.1 ;%increment of time [s]
SimTime=time_param*delta ; % duration of the simulation [s]


%% Parameters of the platoon
N = 20 ; %number of vehicles of the platoon
v0 = 110/3.6 ; % velocity of the initial stable state [m/s]
vf = 130/3.6 ; % velocity of the final stable state [m/s]
d0 = 0.5 + v0*delta + v0^2/(2*10)*(0.2/(1-0.2)) ; % space gap of the initial stable state (DSFG formula of Mathis, with b=10m/s2 and alpha = 0.2) [m]
df = 0.5 + vf*delta + vf^2/(2*10)*(0.2/(1-0.2))  ; % space gap of the final stable state [m]
j_max = 0.9 ; % maximal jerk ensuring comfort [m/s3]
a_max = 2.5 ;%min(2.5, alpha*sqrt(j_max*(vf-v0))) ; % maximal acceleration permited by the platoon AND allowing the trapeze to exist [m/s2]

a_avleader = a_max/(1+(a_max^2/(j_max*(vf-v0)))) ; % average acceleration of the leader [m/s2]
k = floor((vf-v0)^2/(2*(df-d0)*a_avleader)) ; % maximum number of vehicle in the same acceleration sequence
s = floor(N/k) ; % number of "entire" sequences during the UGSA
r = mod(N,s) ; % number of vehicles forming the "uncomplete" sequence

if r == 0 %determination of the number of sequences
    M = s ;
else
    M = s+1 ;
end

%% Matrices and initial conditions

Positions = zeros(time_param, N) ;
Speeds = ones(time_param, N)*v0 ;
Accelerations = zeros(time_param, N) ;
Jerks = zeros(time_param, N) ;
Gap = zeros(time_param, N-1) ;

for i = 1:N
    Positions(1,i) = -(i-1)*(5+d0) ; % the origin for position is the front of the leader vehicle
end

for i = 1:N-1
    Gap(1,i) = d0 ;
end

%% Functions of jerk, acceleration and speed

jer = @jerk ; %naming the function of the jerk, defined in another m file
acc = @acceleration ; %naming the function of the acceleration, defined in another m file
spe = @speed ; %naming the function of the velocity, defined in another m file
```

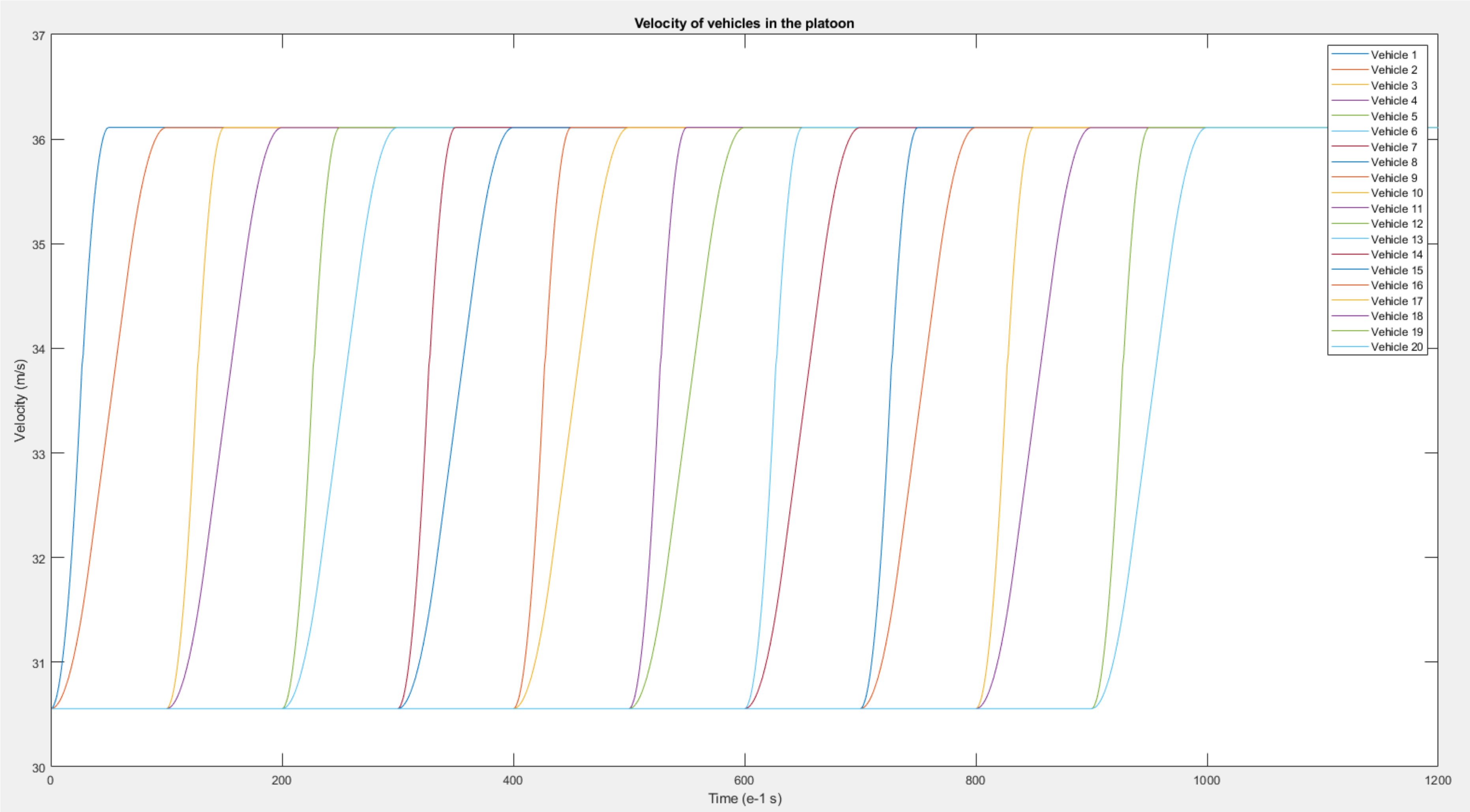
```
%% Algorithm of the UGSA strategy

m = 1 ; % iteration variable of a sequence
while m <= M %while there is still an acceleration sequence to realize for some vehicles of the platoon
    for t=linspace(2,time_param,time_param-1) % after every step of time of delta
         if t*delta >= t0 + m*k*(vf-v0)/a_avleader % when the last vehicle of a sequence finishes its acceleration phase
             m = m+1 ; % the iteration index of the sequences is incremented by 1
         end
        for i=1:N % for all the vehicles of the platoon
            if (i < (m-1)*k+1) || (i > min(m*k,N)) % if the vehicle is not concerned by the sequence m
                Jerks(t,i) = 0 ; %the jerk is null
                Accelerations(t,i) = 0 ; %the acceleration is null
                Speeds(t,i) = Speeds(t-1,i) ; %the velocity is constant
                Positions(t,i) = Positions(t-1,i) + Speeds(t-1,i)*delta + Accelerations(t-1,i)*delta^2/2 + Jerks(t-1,i)*delta^3/6 ; % the position is determined by the func
            elseif (i >=(m-1)*k+1) && (i<=min(m*k,N)) % if the vehicle is concerned by the sequence m
                Jerks(t,i) = jer(v0,vf,a_max,j_max,k,m,t0,t*delta,i-k*(m-1)) ; % the jerk is determined by the function jerk
                Accelerations(t,i) = acc(v0,vf,a_max,j_max,k,m,t0,t*delta,i-k*(m-1)) ; % the acceleration is determined by the function acceleration
                Speeds(t,i) = spe(v0,vf,a_max,j_max,k,m,t0,t*delta,i-k*(m-1)) ; % the speed is determined by the function speed
                Positions(t,i) = Positions(t-1,i) + Speeds(t-1,i)*delta + Accelerations(t-1,i)*delta^2/2 + Jerks(t-1,i)*delta^3/6 ; % the position is determined by the func
            end
%          if t*delta >= t0 + m*k*(vf-v0)/a_avleader % when the last vehicle of a sequence finishes its acceleration phase
%              m = m+1 ; % the iteration index of the sequences is incremented by 1
%          end
        end
    end
end

for t=linspace(2,time_param,time_param-1)
    for i = 1:N-1
        Gap(t,i) = Positions(t,i) - Positions(t,i+1) -5 ;
    end
end

%% Graphs

T = linspace(1,time_param,time_param) ;

% Color = cool(N) ;
% Color(1,:) = [1 0 0] ;
% figure

for n=1:N
    txt = ['Vehicle ',num2str(n)];
    plot(T,Jerks(:,n),'DisplayName',txt) ;
    hold on ;
end
legend(gca,'show')
title('Jerk of vehicles in the platoon')
xlabel('Time (e-1 s)');
ylabel('Jerk (m/s3)');

figure
for n=1:N
    txt = ['Vehicle ',num2str(n)];
    plot(T,Accelerations(:,n),'DisplayName',txt) ;
    hold on ;
end
legend(gca,'show')
title('Acceleration of vehicles in the platoon')
xlabel('Time (e-1 s)');
ylabel('Acceleration (m/s2)');

figure
for n=1:N
    txt = ['Vehicle ',num2str(n)];
    plot(T,Speeds(:,n),'DisplayName',txt) ;
    hold on ;
end
legend(gca,'show')
title('Velocity of vehicles in the platoon')
xlabel('Time (e-1 s)');
ylabel('Velocity (m/s)');

figure
for n=1:N
    txt = ['Vehicle ',num2str(n)];
    plot(T,Positions(:,n),'DisplayName',txt) ;
    hold on ;
end
legend(gca,'show')
title('Position of vehicles in the platoon')
xlabel('Time (e-1 s)');
ylabel('Position (m))');

figure
for n=1:N-1
    txt = ['Vehicle ',num2str(n+1)];
    plot(T,Gap(:,n),'DisplayName',txt) ;
    hold on ;
end
legend(gca,'show')
title('Gap between vehicles in the platoon')
xlabel('Time (e-1 s)');
ylabel('Gap (m))');
```

First configuration: from 110 to 130 km/h

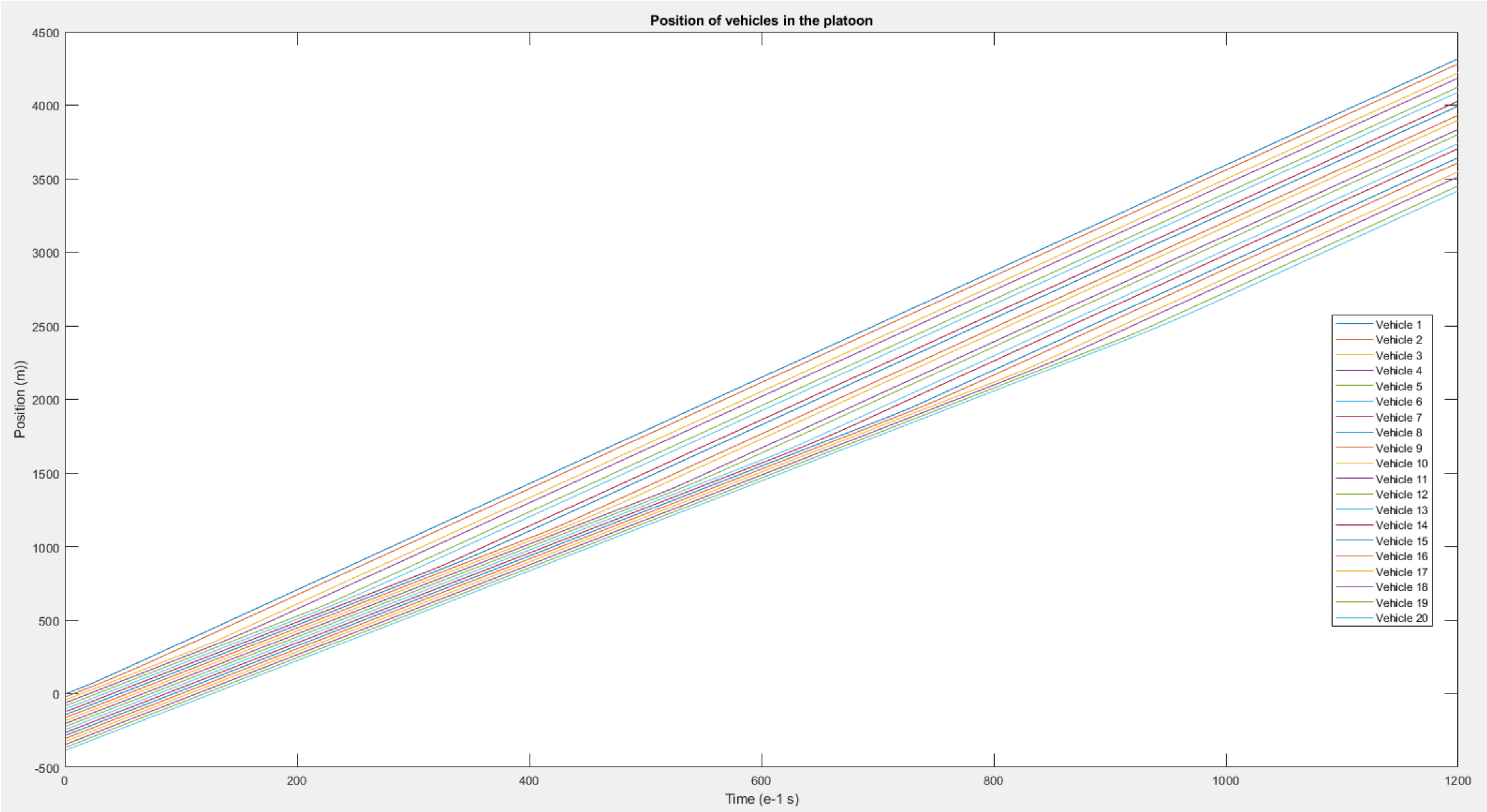




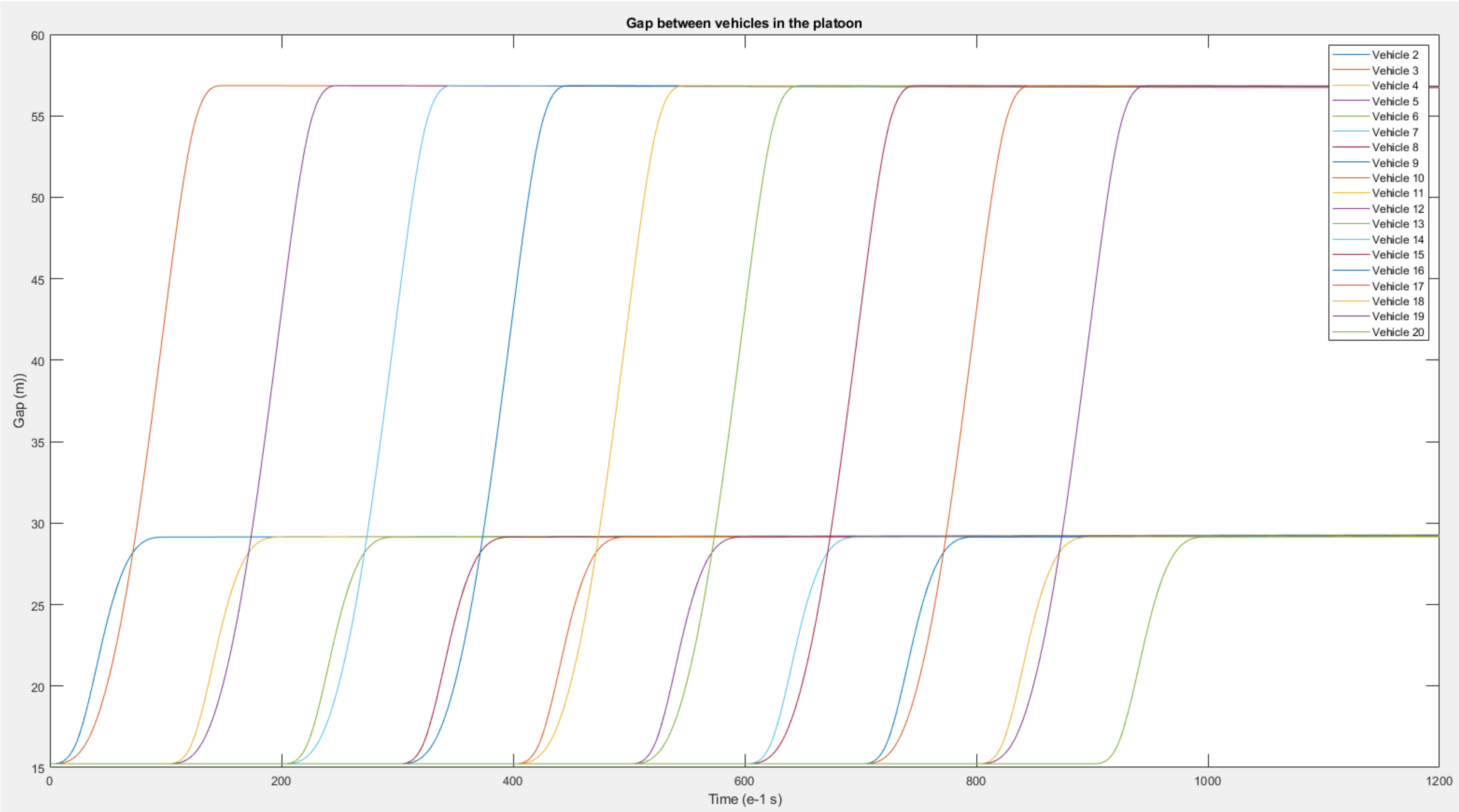
Acceleration of vehicles in the platoon
Acceleration (m/s2)
Time (e-1 s)
0
0.5
1
1.5
2
2.5
0
200
400
600
800
1000
1200
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20



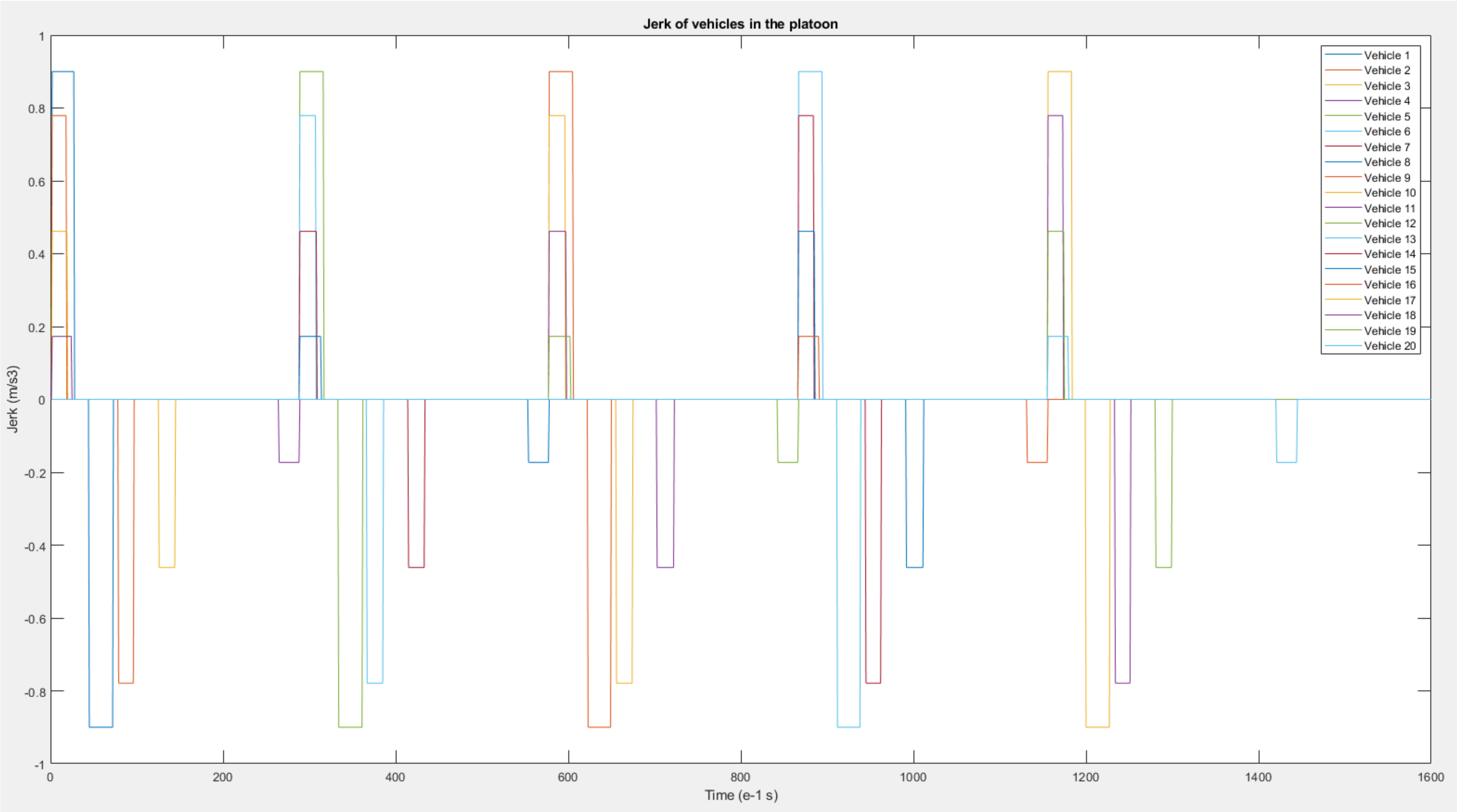
Velocity of vehicles in the platoon
Velocity (m/s)
Time (e-1 s)
30
31
32
33
34
35
36
37
0
200
400
600
800
1000
1200
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

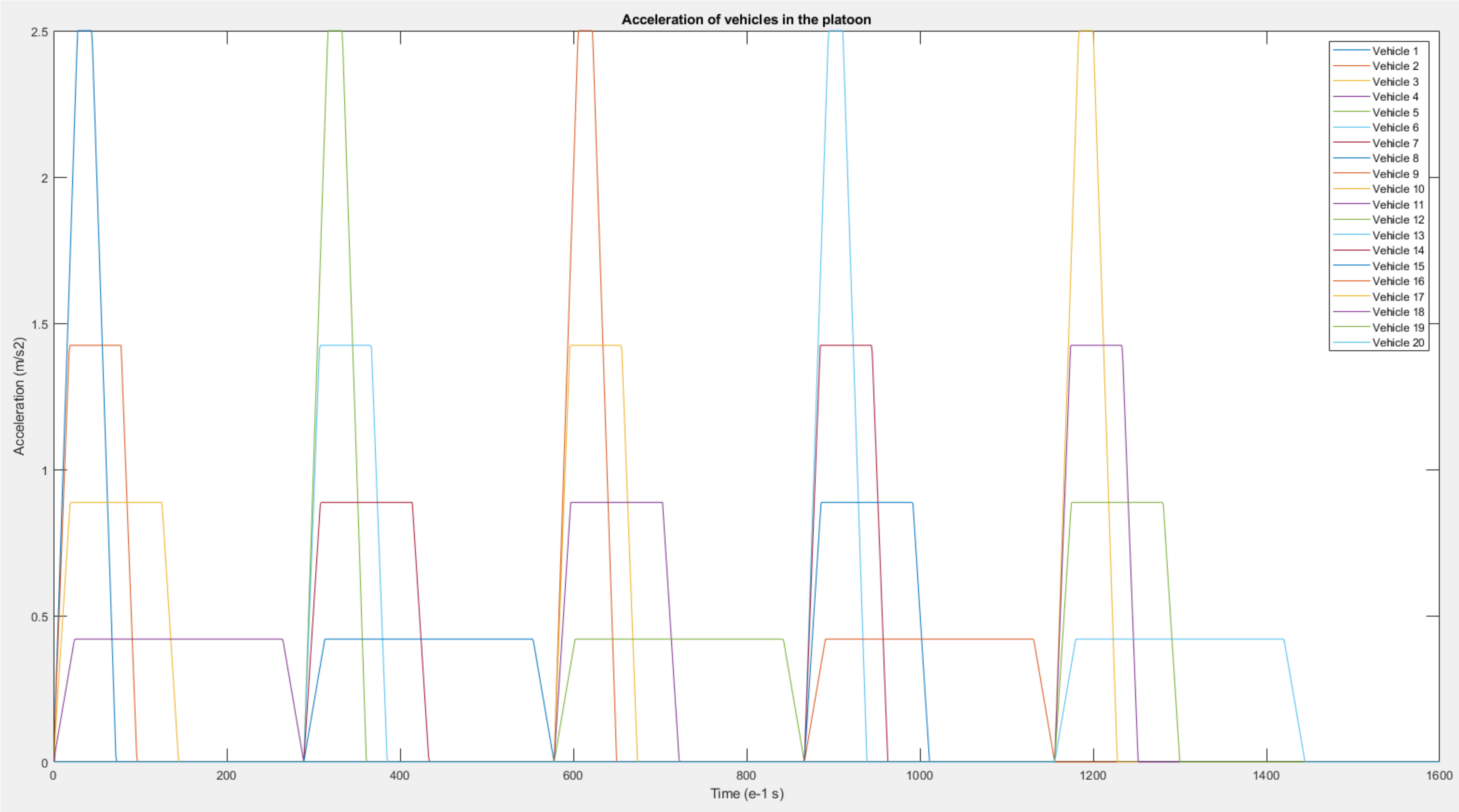


Position of vehicles in the platoon
Position (m))
Time (e-1 s)
4500
4000
3500
3000
2500
2000
1500
1000
500
0
-500
0
200
400
600
800
1000
1200
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

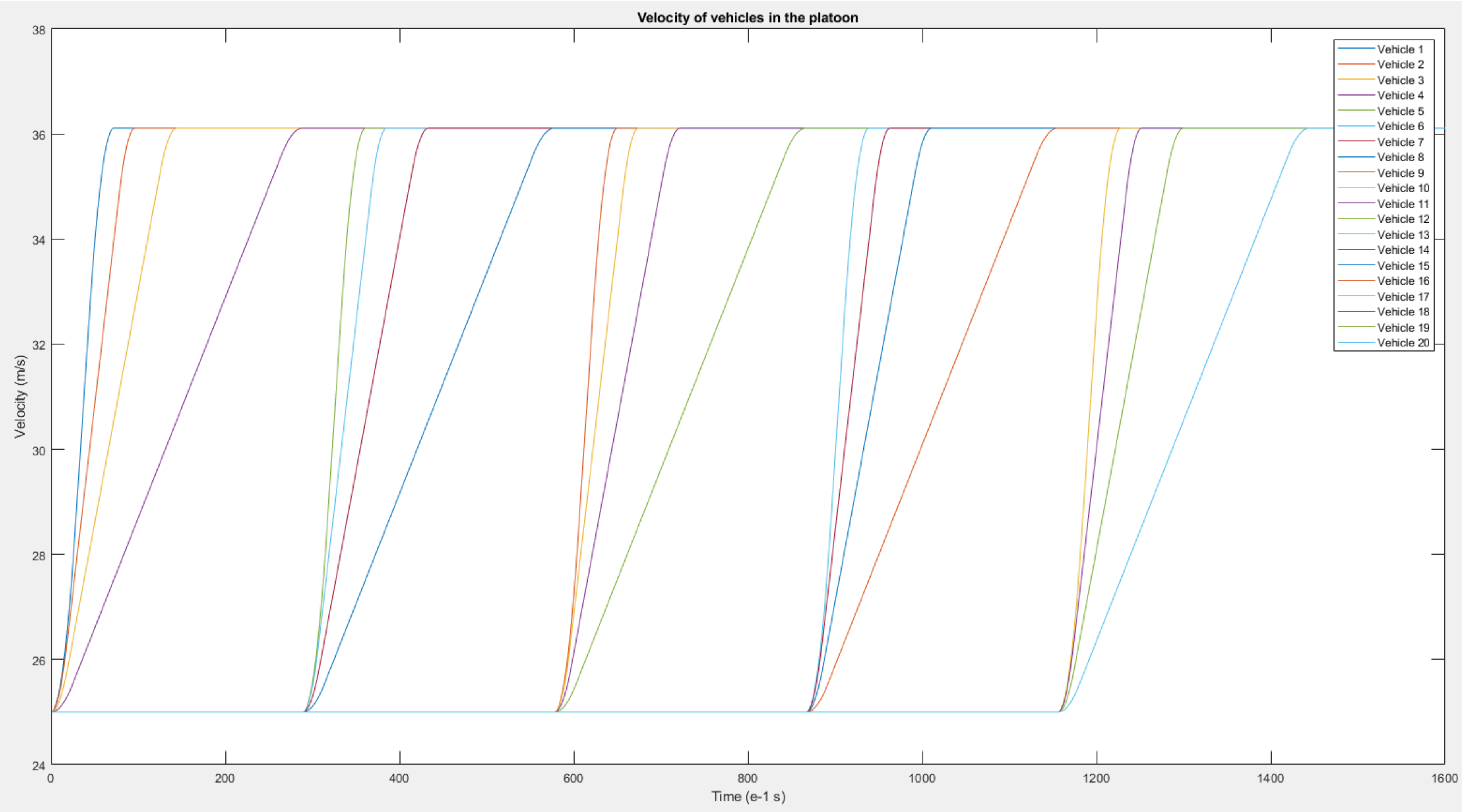

Gap between vehicles in the platoon
Gap (m))
Time (e-1 s)
60
55
50
45
40
35
30
25
20
15
0
200
400
600
800
1000
1200
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

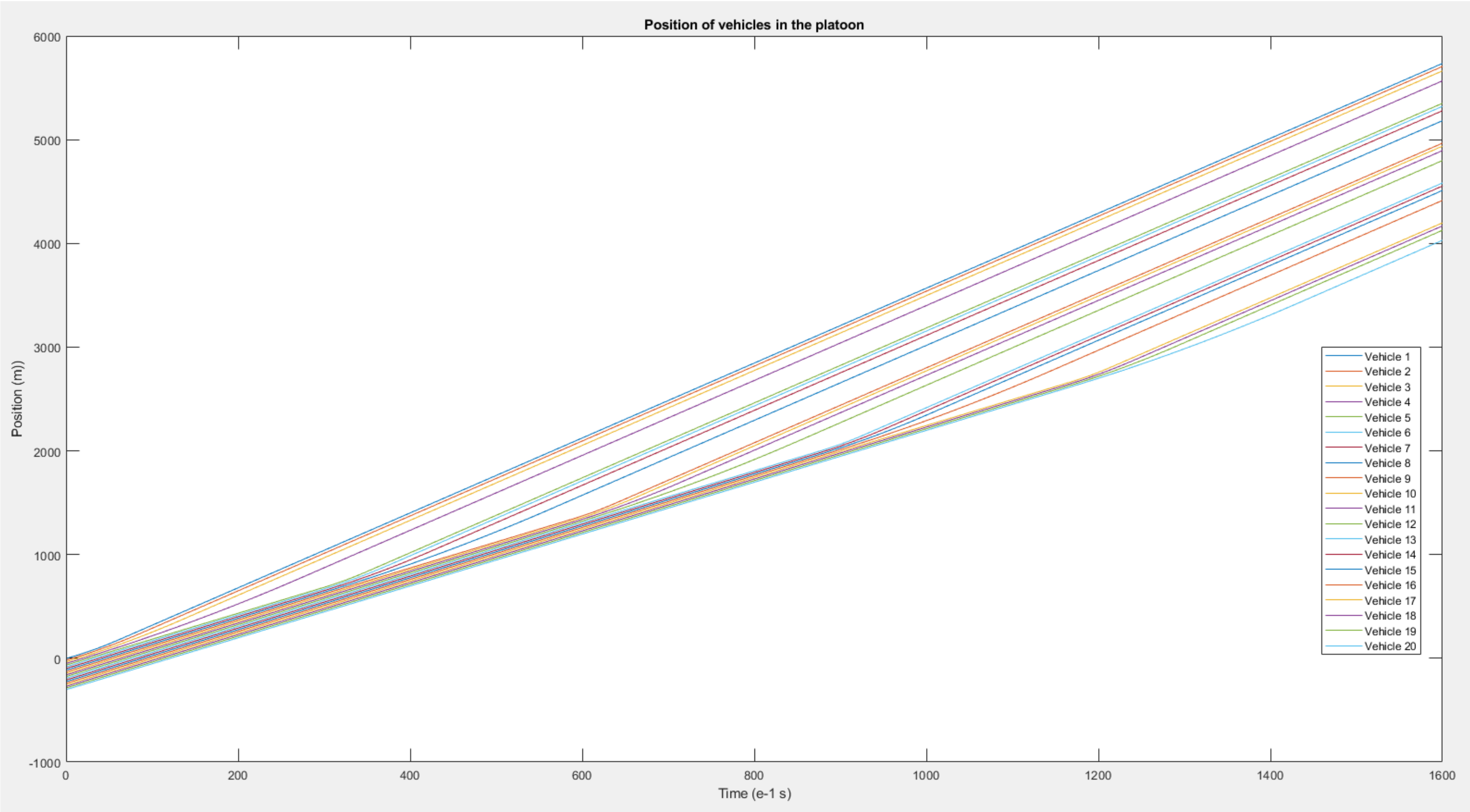
Second configuration: from 90 to 130 km/h

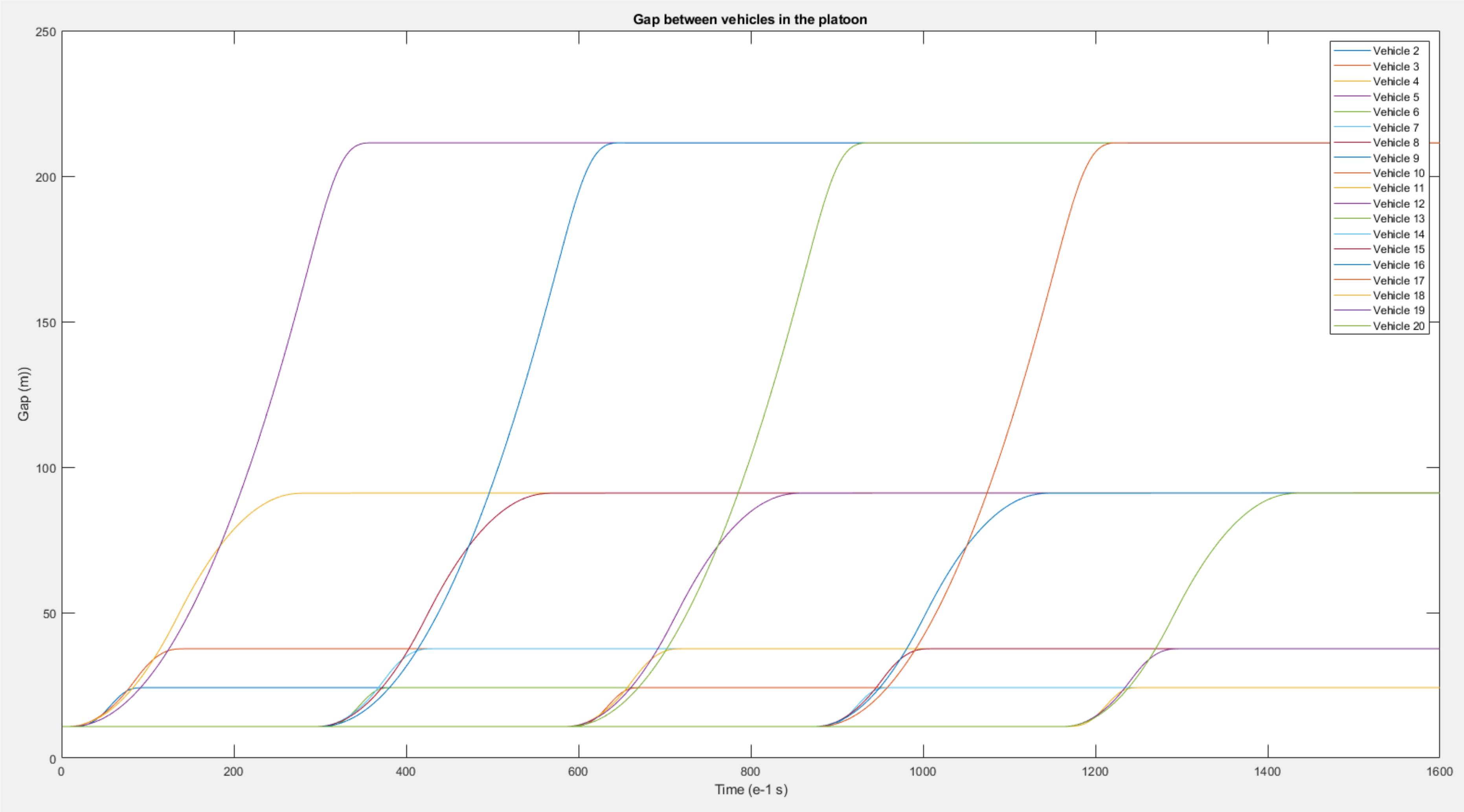




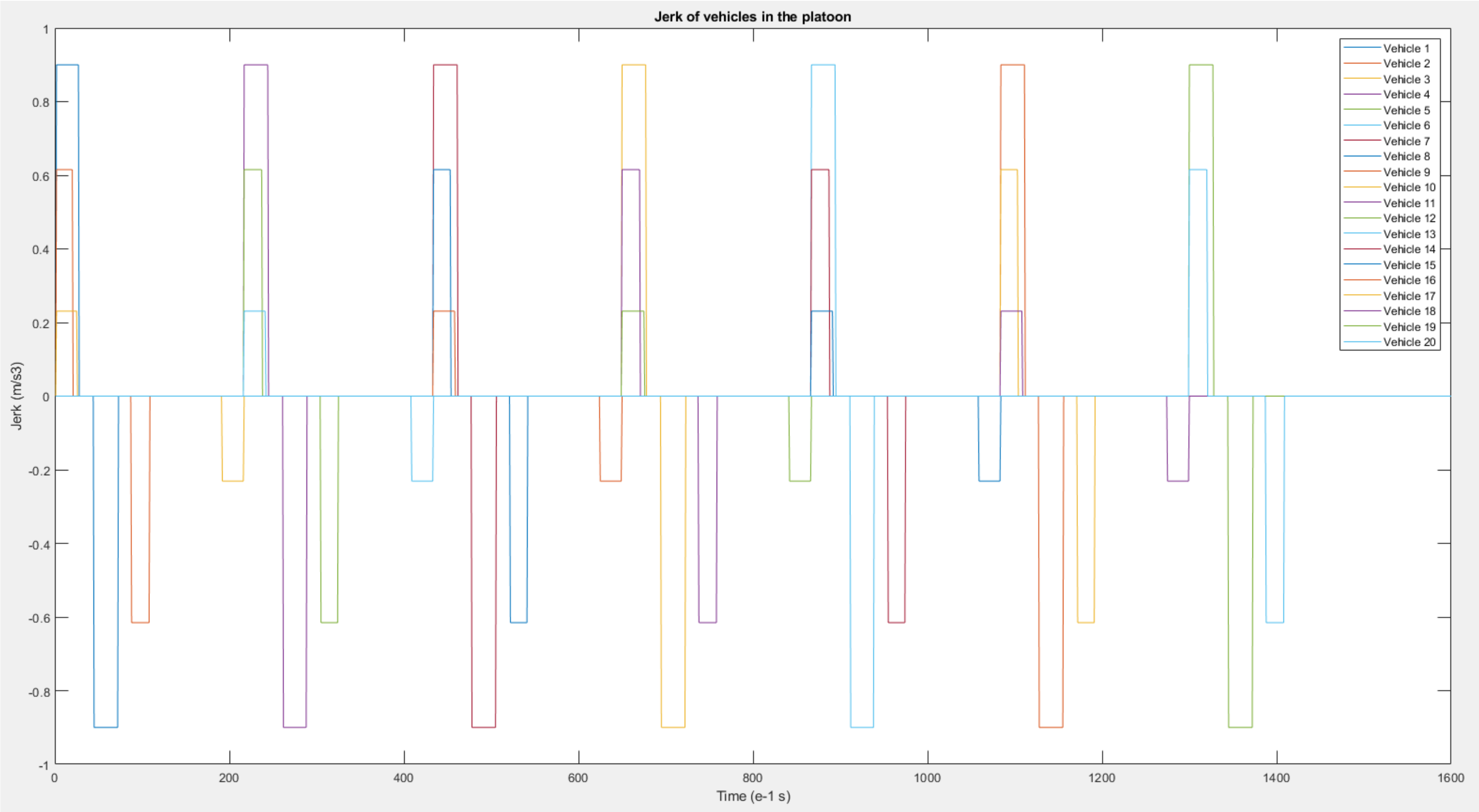
Acceleration of vehicles in the platoon
Acceleration (m/s2)
Time (e-1 s)
0
0.5
1
1.5
2
2.5
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

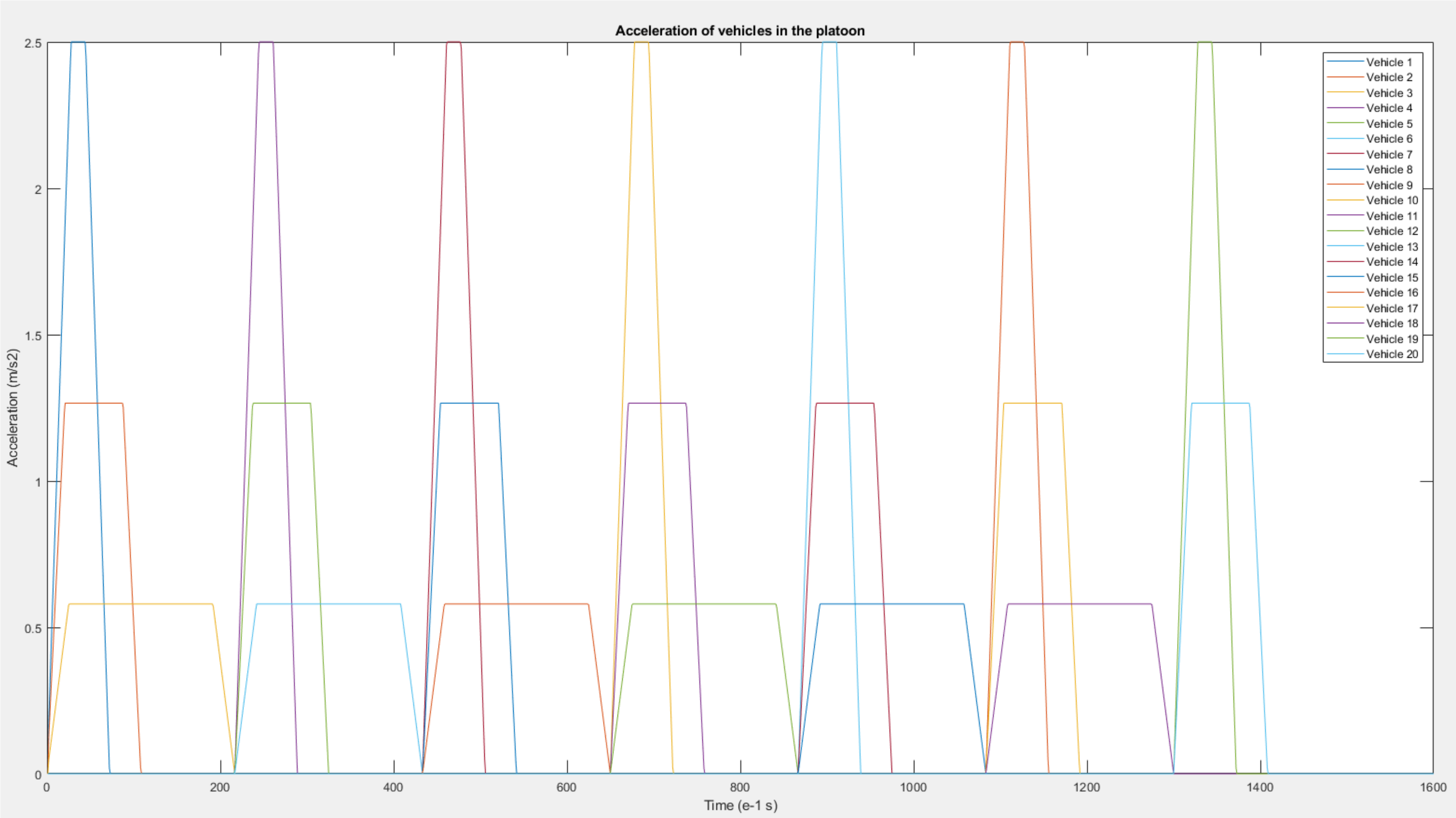

Velocity of vehicles in the platoon
Velocity (m/s)
Time (e-1 s)
38
36
34
32
30
28
26
24
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20


Position of vehicles in the platoon
Position (m))
Time (e-1 s)
6000
5000
4000
3000
2000
1000
0
-1000
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

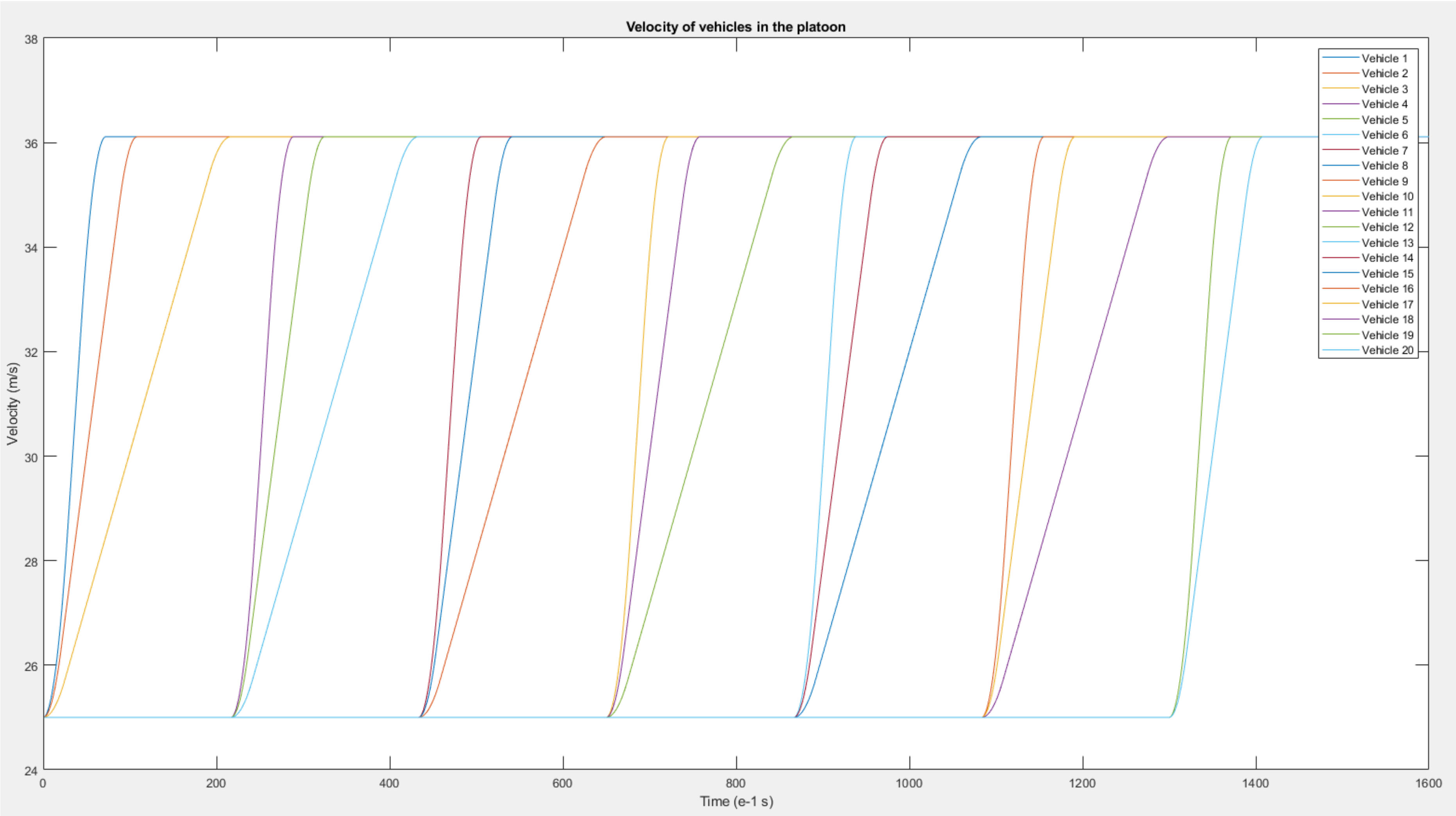

Gap between vehicles in the platoon
Gap (m))
Time (e-1 s)
0
50
100
150
200
250
0
200
400
600
800
1000
1200
1400
1600
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

Third configuration: from 90 to 130 km/h, k-1

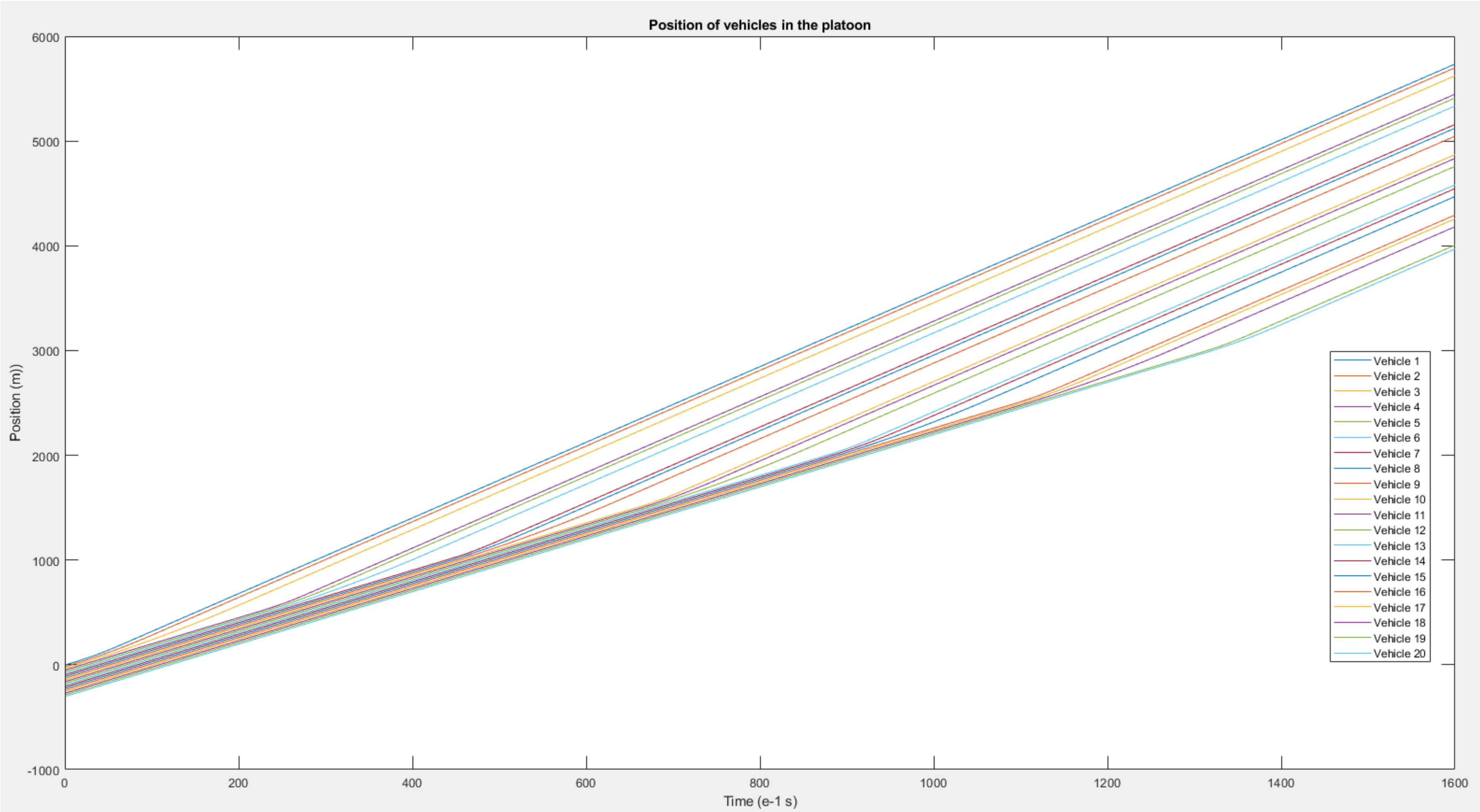


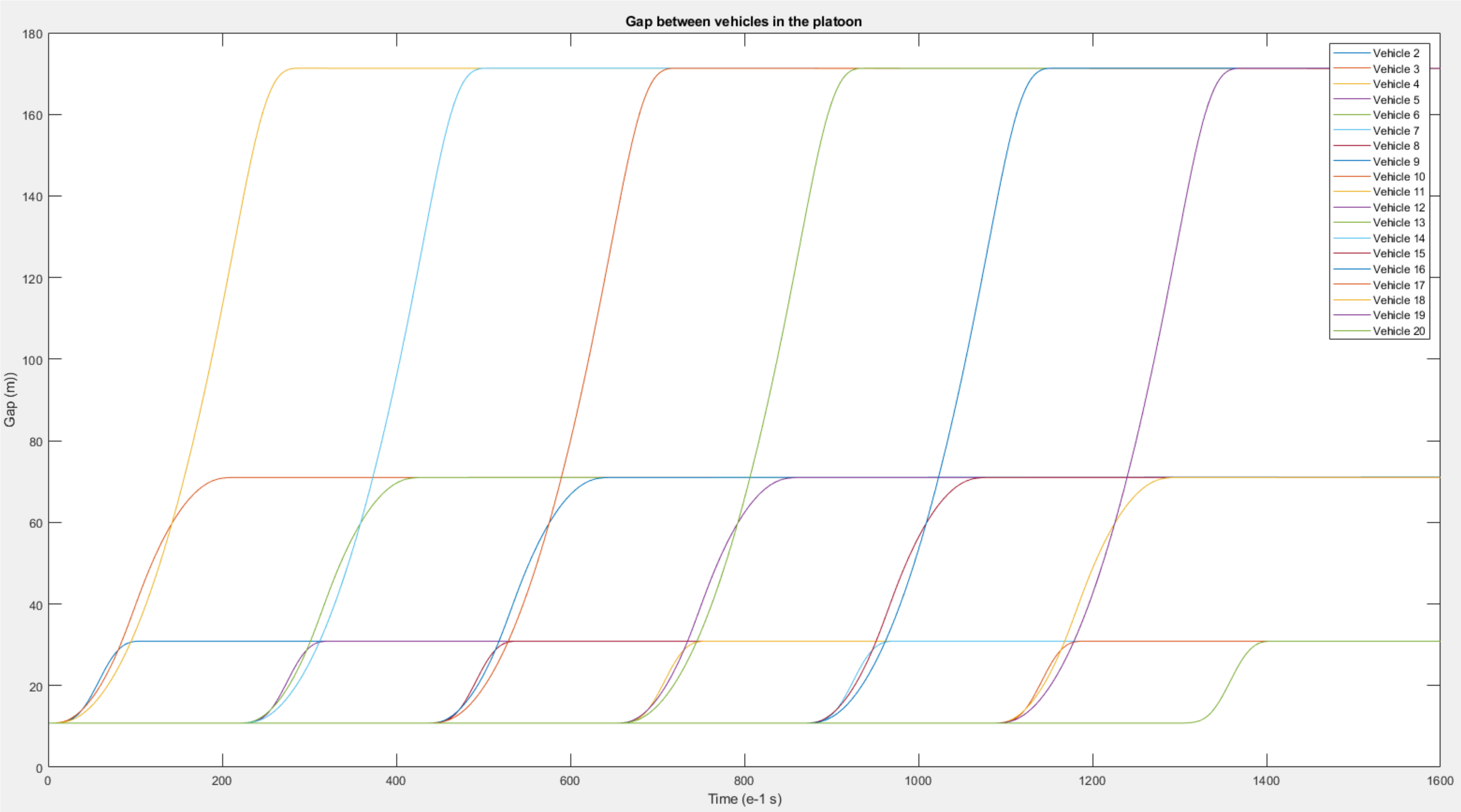


Acceleration of vehicles in the platoon
Acceleration (m/s2)
Time (e-1 s)
0
0.5
1
1.5
2
2.5
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

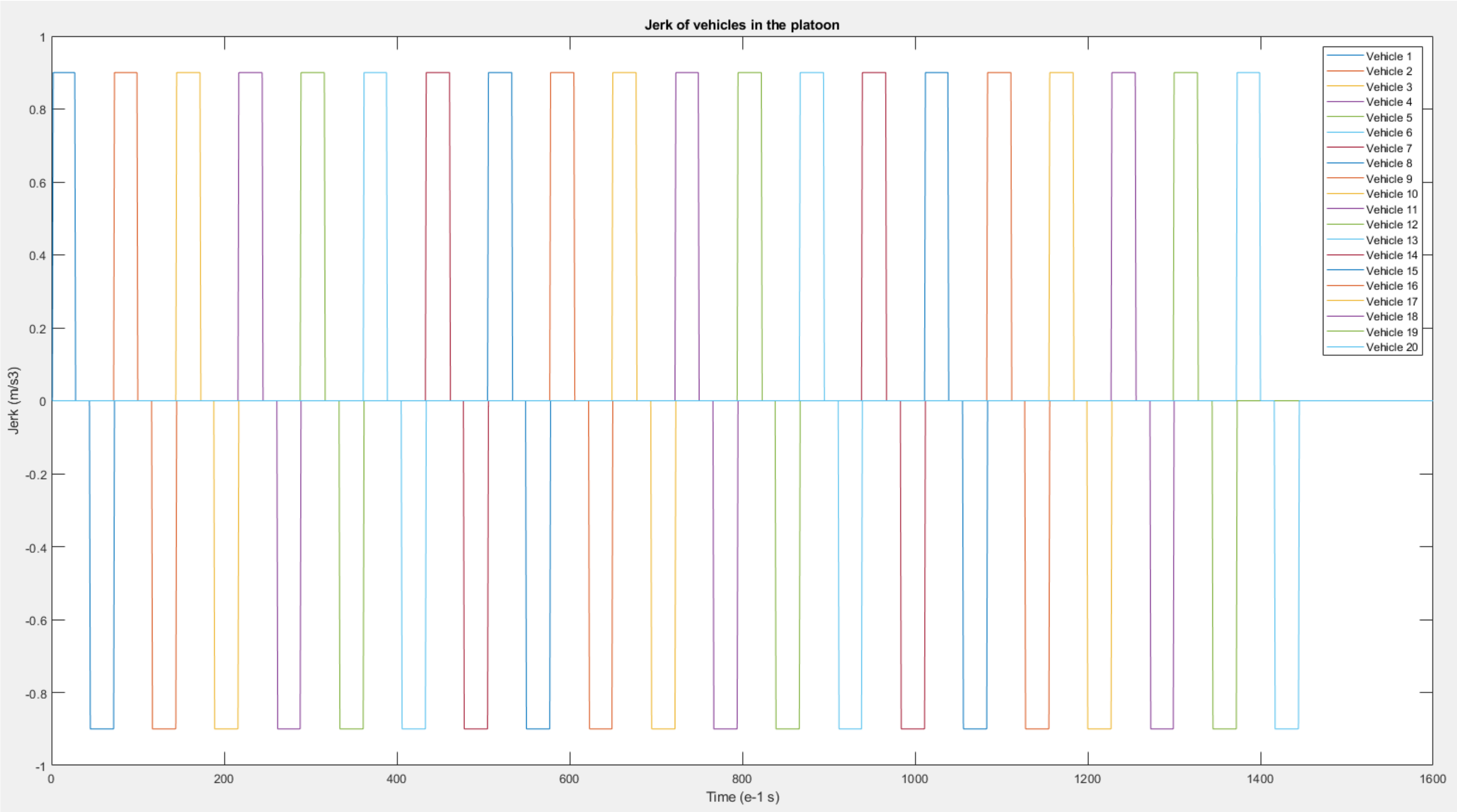


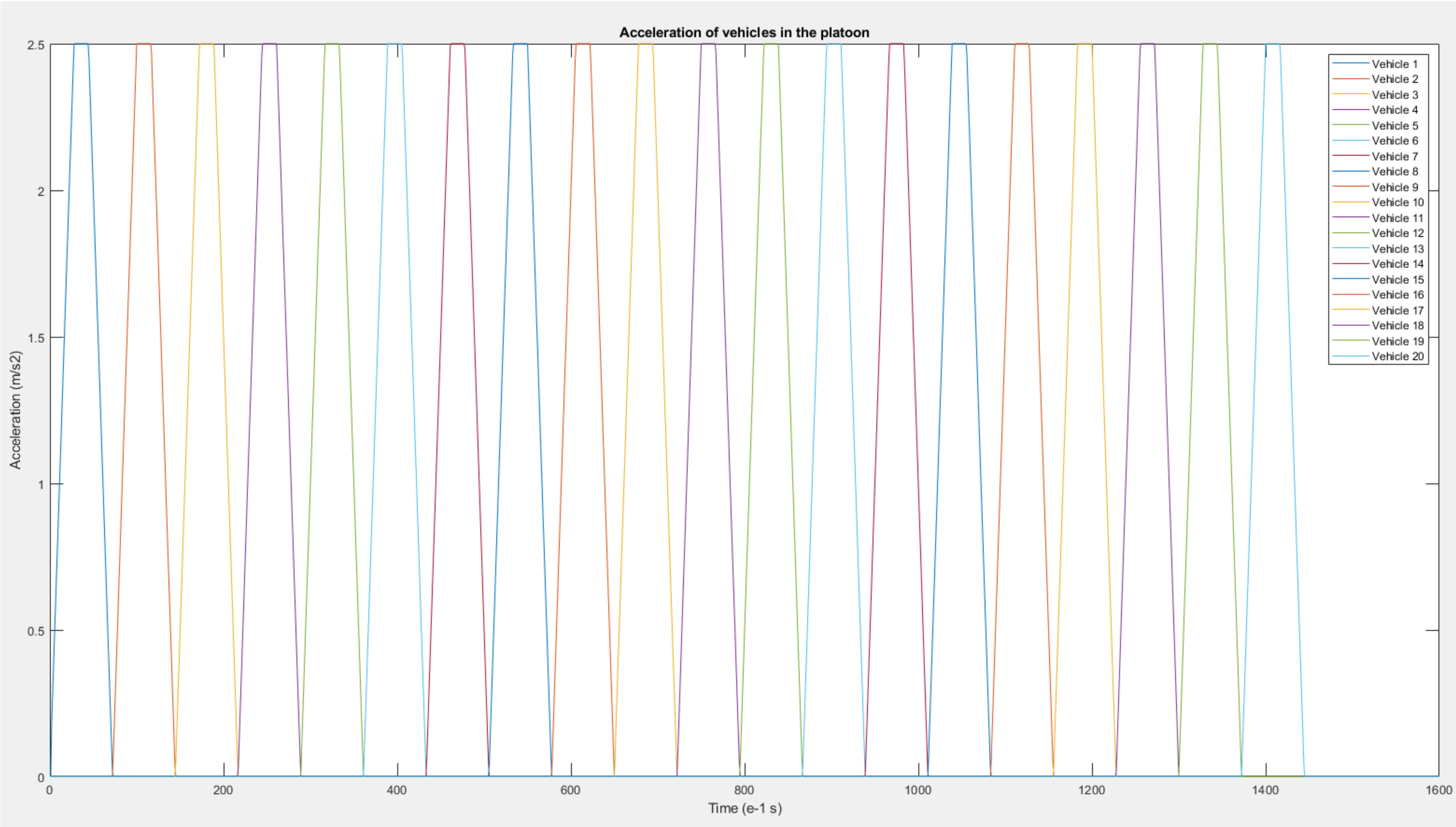
Velocity of vehicles in the platoon
Velocity (m/s)
Time (e-1 s)
38
36
34
32
30
28
26
24
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

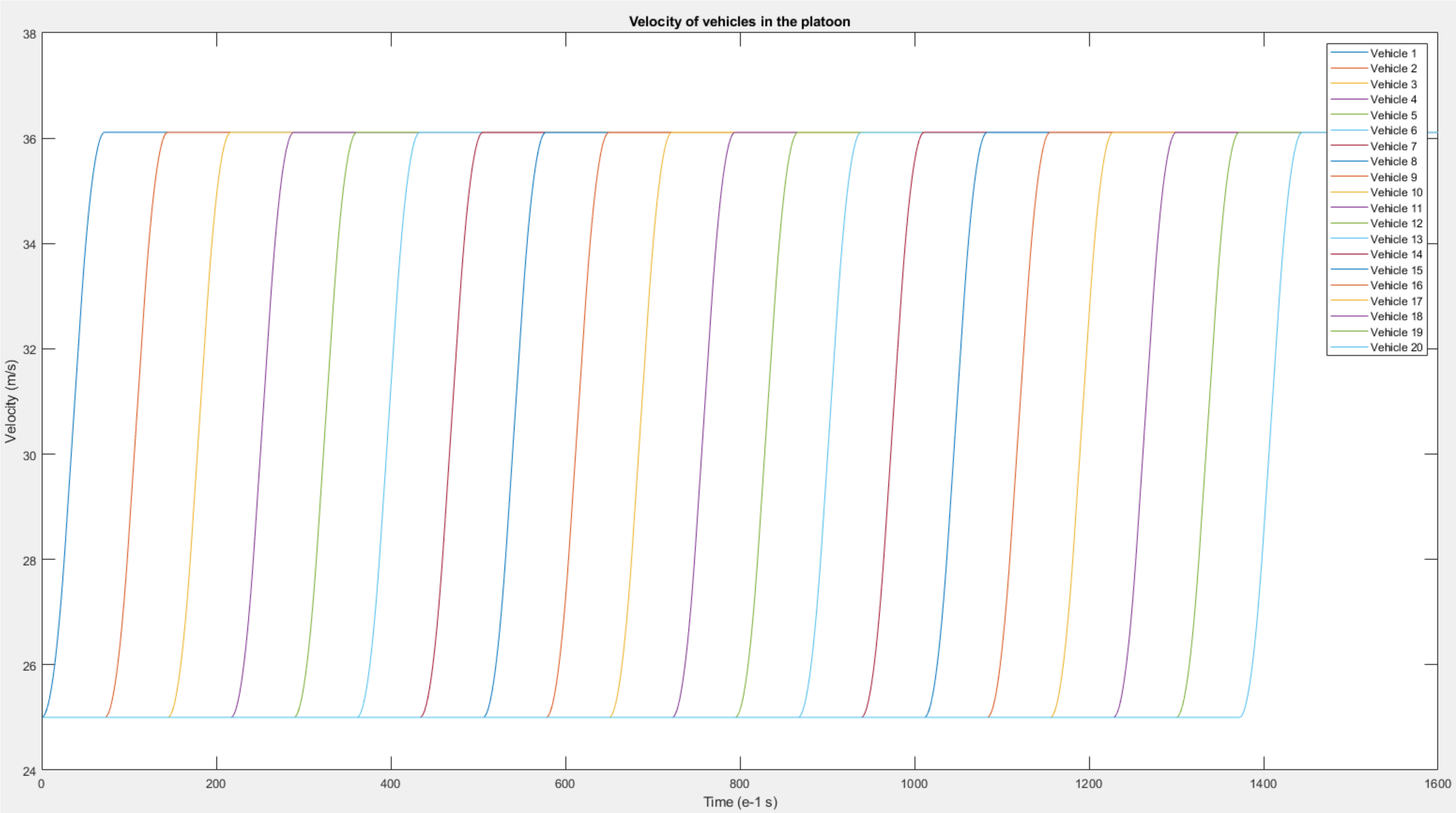

Position of vehicles in the platoon
Position (m))
Time (e-1 s)
6000
5000
4000
3000
2000
1000
0
-1000
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20


Gap between vehicles in the platoon
Gap (m))
Time (e-1 s)
0
20
40
60
80
100
120
140
160
180
0
200
400
600
800
1000
1200
1400
1600
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20

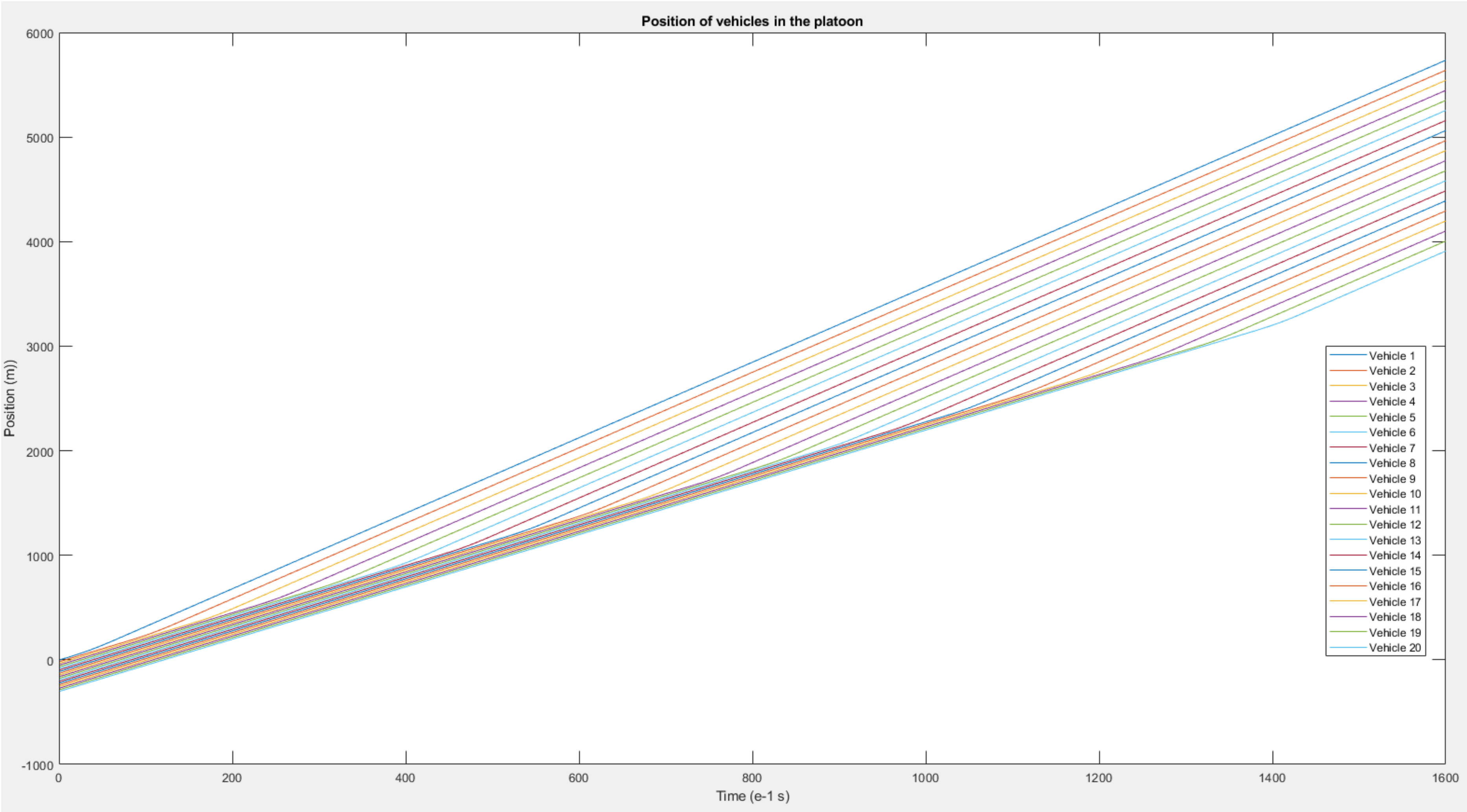
Fourth configuration: from 90 to 130 km/h, k=1

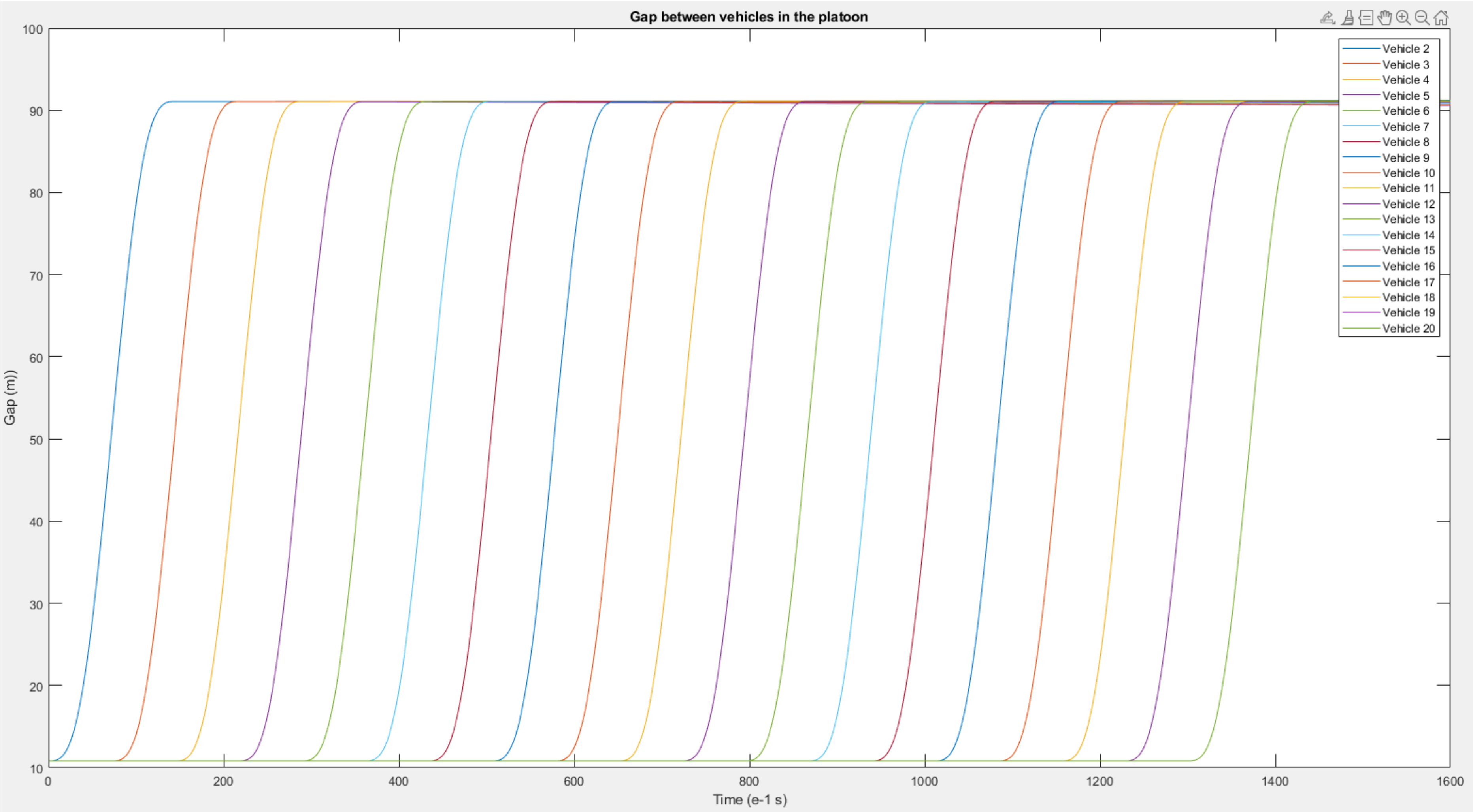



Acceleration of vehicles in the platoon
Acceleration (m/s2)
Time (e-1 s)
0
0.5
1
1.5
2
2.5
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20



Velocity of vehicles in the platoon
Velocity (m/s)
Time (e-1 s)
38
36
34
32
30
28
26
24
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20


Position of vehicles in the platoon
Position (m))
Time (e-1 s)
6000
5000
4000
3000
2000
1000
0
-1000
0
200
400
600
800
1000
1200
1400
1600
Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20


Gap between vehicles in the platoon
Gap (m))
Time (e-1 s)
100
90
80
70
60
50
40
30
20
10
0
200
400
600
800
1000
1200
1400
1600
Vehicle 2
Vehicle 3
Vehicle 4
Vehicle 5
Vehicle 6
Vehicle 7
Vehicle 8
Vehicle 9
Vehicle 10
Vehicle 11
Vehicle 12
Vehicle 13
Vehicle 14
Vehicle 15
Vehicle 16
Vehicle 17
Vehicle 18
Vehicle 19
Vehicle 20