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Research Project Cover Sheet

Suggested citation:
Zafar, M. I. (2021). Improving Flood Estimation in Ungauged Catchments. *Undergraduate Research Report No. 2021RP080*. Department of Civil Engineering, University of Bristol, Bristol, U.K.

Declaration:
This paper entitled "Improving Flood Estimation in Ungauged Catchments" was submitted in partial fulfilment of the requirements for the unit **CENG30007 Research Project 3** to the Department of Civil Engineering, University of Bristol.

I declare that the work in this dissertation was carried out in accordance with the Regulations of the University of Bristol. The research on which this paper is based was carried out under the supervision of **Dr M. Rico-Ramirez** during the academic year **2020-21**.

The work is entirely my own work and is original, except where indicated by special reference in the text and no part of the paper has been submitted for any other academic award.

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Date: 28th May 2021

IMPROVING FLOOD ESTIMATION IN UNGAUGED CATCHMENTS

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ABSTRACT: The flood estimates are critical for studies involving flood mapping, hydraulic structure planning and design, flood risk assessments, and reservoir operations. Estimates have traditionally been obtained by regression equations. QMED is a medium sized flood obtained from the median of annual-maximum (AMAX) series – the highest flow observed in each water year. For gauged catchments, techniques involving the annual-maximum-series (AMS) and peak-over-threshold (POT) are traditionally used. However, for the ungauged sites, in order to link the index flood to catchment descriptors, statistical models such as multiple regression are most widely used. In this paper, the Flood Estimation Handbook (FEH) statistical methodologies are analysed and hence recalibrated using all data available as well as cross validation process, the availability of more flow records produced better results. Similarly, using correlation analysis, appropriate catchment descriptors are selected, and a new simple non-linear regression model is proposed. Artificial Neural Networks (ANNs) have been used for gauged catchments but rarely for ungauged catchments. The use of ANNs is investigated to estimate the flood index from the catchment descriptors and are compared to traditional non-linear regression models. The National River Flow Archive (NRFA) data has been utilized to estimate the index flood (QMED) for 337 ungauged catchments in the UK. The results showed that i) there are several catchment descriptors (e.g. catchment area, mean annual rainfall) that are directly correlated to QMED; ii) the recalibrated FEH models produce better results than the original models and so it is important to recalibrate this model as new data becomes available; iii) the proposed non-linear power-law model produces slightly better results than the FEH model; iv) the QMED estimates obtained from ANNs have shown improved performance as compared to the traditional non-linear regression models. In addition to that, given the fact that the number of catchments is not large enough to separate the data set in calibration and validation, we used cross validation (where each model is trained for n-1 data points, and then tested against the remaining 1 unseen data point demonstrating the real performance of model), to test the true performance of the models. The cross-validation results showed that the performance of the models decreases, but the ANNs still produces slightly better results compared to the non-linear regression models.

KEYWORDS: QMED, Ungauged, Flood Estimation Handbook (FEH), Artificial Neural Networks (ANN), index flood, regression.

STATEMENT OF ORIGINALITY: In this paper, the QMED estimation model given in Flood Estimation Handbook (FEH) published by Institute of Hydrology (IH 1999) and improved FEH (Kjeldsen es al. 2008) model are recalibrated with the newer and better parameters making use of all flow records available up until the end of September 2019. A new non-linear regression model for the estimation of QMED from flood data is constructed using the multiplicative-structure method. Artificial neural networks (ANNs) are used to predict QMED at ungauged locations, and the results obtained from ANNs outperformed all previous approaches.

1. INTRODUCTION

Floods are one of the most damaging natural disasters and considered a major natural hazard within the UK, a report from Environment Agency (EA, 2012), shows that there are over and above 5 million people just in England and Wales that are living in flood prone areas. In the last couple of decades, flooding has not only resulted in huge economic damage but also caused loss of life in every corner of globe (Gaume et al., 2009). Therefore, the design of extreme hydrological events such as floods is an essential element for understanding and mitigating flood risks. It is also required for the planning of variety of water resources systems to reduce the vulnerability of people and public and personal property. On the other hand, accurate estimation of flood events at ungauged catchments is very complex process and an uneasy one to understand. This paper therefore makes an attempt on investigating the reliability of the existing QMED estimation models (IH 1999 and Kjeldsen es al.2008) and explores a potential to improve the estimation of QMED using catchment descriptors.

There are different flood estimation methods available in the literature. These estimates are commonly calculated by fitting annual maximum (AMAX) series of peak flow for a catchment or regional combination of catchments to the statistical models (Stedinger et al., 1993). There is a range of regionalization techniques to choose from (Cunnane, 1988, 1989). Once the geographical region is selected using regionalization techniques implemented by Cunnane (1988) for the derivation of regional flood frequency curves. Floods for a given return period can be measured from the index flood values using the developed regional flood frequency curves and the steepness of a regional flood frequency curve can be measured fairly well using only two parameters: annual average rainfall and the region's median catchment area (Meigh et al., 1997). Dalrymple's (1960) index-flood method (IFM) is a conventional and straightforward technique, for the catchments where very less flow data is available or that are ungauged. However, in flood risk studies this methodology is still applicable because it shows considerably better results than some recent regionalization methods (Malekinezhad et al. 2011). And it is a standard procedure in Flood Estimation Handbook (FEH), the FEH is the established criteria used in the estimation of flood risks in a given region and to estimate local flood risks and, as a result, designing a flood-resistant infrastructure, in its application to the UK, it uses median of the annual maximum flood as the index-flood, QMED, this differs from the more conventional method of using the mean of the annual maxima. Because the median is a more stable indicator that is less influenced by the magnitude of a particularly large flood event, while the mean will fluctuate significantly. In a study conducted by Sun et al. (2000), it was discovered that using radar data in conjunction with rainfall data from nearby gauges, a method known as cokriging, improved the efficiency of flood estimates. The uncertainty in the input data used for modelling was also quantified in the same analysis.

For any ungauged catchment, the index flood is estimated using a combination of a multiple regression non-linear model, which links the index flood to a set of catchment descriptors. The statistical model given in the improved FEH is routinely used to obtain the flood estimates, it is assumed that the index flood or QMED, i.e., the median of the set of annual maxima (AMAX) flood data, can be explained by catchment descriptors, however, the uncertainty of the results is very high (Kjeldsen, T. R., 2015). This paper attempts to answer the research question: Can we improve flood estimates using models based on catchment descriptors? To achieve this, the objectives of this paper are: (i) to implement the current FEH method to estimate QMED based on catchment descriptors, (ii) to recalibrate the models available in the literature with the availability of more flow records, (iii) to carry out analysis of the available catchment descriptors to choose for modelling, the ones seen to be highly correlated, (iv) to develop a new non-linear regression model, and (v) to investigate the use of artificial neural networks and their efficiency compared with the traditional non-linear methodologies.

2. LITERATURE REVIEW

The Flood Studies Report (FSR) was the primary comprehensive methodology for flood frequency estimation in the UK, published by the Natural Environment Research Council (NERC, 1975). This report was derived from the Index Flood Method (IFM) developed by United States Geological Survey (Dalrymple, 1960). The IFM is based on the assumption that the flood flows in a hydrologically similar region, these regions are standardized by index flood and are identically distributed. For the estimation of the parameters for a flood frequency curve the data is gathered from the stations within a defined region and then this dimensionless curve is scaled by the index flood of the catchment of interest (Grover et al., 2002). Conventionally, these regions are defined by political or geographic boundaries. Thomas and Benson (1970) predicted flood quantiles for four different regions in the United States using multiple regression. Similarly, Tasker et al. (1996) observed more accurate results found that subdivision into smaller geographically based subregions. In the same study, they observed that using the “region of influence” method to generate a unique area for each ungauged catchment gave the smallest Root Mean Square error in the 50-year flood estimate for the uncalibrated catchments in Arkansas.

In the same way the Flood Studies Report (FSR) divided the UK into 10 geographical regions. For each region, flood frequency curve specific to each catchment can be obtained as the product of a regional dimensionless growth factor and index flood, estimate of index flood can be derived from a regression model established from the catchment descriptors such as catchment area, annual average rainfall, soil type, etc or could be directly obtained from the observation as the mean annual maximum flood. Whereas flood frequency curve is a probabilistic model linking a flood magnitude to flood rarity, the inverse of return period. Residual mapping done using geostatistics is an alternate approach to the development of separate models for each region. The residual map attempts to eliminate the bias caused by geographic differences that are not considered in the model.

Following the breakthroughs achieved in the estimation of regional flood frequency, and the “region of influence” (ROI) method (Burn, 1990) and as the method of L moments was introduced (Hosking and Wallis, 1993), the Institute of Hydrology (IH) published the Flood Estimation Handbook (FEH) with improved procedures for flood estimation. The key developments in flood frequency estimation include: (i) use of hydrological similarity for the formation catchment specific growth curve instead of regional values given in FSR, (ii) in the ungauged catchments for the estimation of index flood standardised a methodology of data transfer from nearby hydrological analogous (donor) sites (iii) the FEH adapted the median annual maximum flood, QMED, as the measure of index flood because for the shorter data series median is found less sensitive to the irregularities than mean, and (iv) the catchment descriptors are derived automatically from gridded electronic data and it is no longer limited by the need to be able to compute descriptors manually from 1:50000 paper maps. The review of relevant literature showed that the Index Flood Model and procedures similar to the FEH have also been developed for other parts of the world, it was found that the Meigh et al. (1997) method, the Görgens (2007) Joint Peak-Volume (JPV) method and the Haile (2011) method are available for application such as for: Africa (Mkhandi et al., 2000), and Europe (Castellarin et al., 2012).

The creation of much improved database of systematically recorded flood data done by HiFlows-UK recommended the changes in the FEH methodology. The improved FEH procedures were developed by the Environment Agency (2008), it has kept the use of hydrological similarity method and the Index Flood Method (IFM). However, the statistical models for the estimation of index flood and dimensionless growth curve were improved and these are implemented in the WINFAP-FEH v3 software (WHS, 2009). Due to the flexible structure of statistical models, they have better descriptive ability than the physically based models (Brath et al., 2009). For the calculation of design floods, various procedures are provided in the improved FEH, nevertheless, dependent of the availability of data. Although hardly any guidance is provided on how to assess the uncertainty of these estimates (Kjeldsen et al., 2008). For example, the 95% or 68% confidence intervals are generally used and that are mainly valid for the transfer of data from nearby gauged catchments in order to get accurate estimates of QMED, index flood, at the ungauged catchments. While the sites where no flood data is available, regression models are used to directly estimate index flood from the catchment descriptors. The studies by Pappenberger and Beven (2006); and Hall, (2011) showed that in flood management there is urgent need to give crucial importance to risk and uncertainty. For the assessment of uncertainty in the statistical models for design flood estimates new methods have been developed by Kjeldsen et al., (2008). It also assessed the accuracy of index flood (QMED) estimates at the ungauged sites, used in the improved FEH.

2.1 The Flood Estimation Handbook method (1999)

The median annual maximum flood (QMED) model described in the Institute of Hydrology's (IH 1999) Flood Estimation Handbook (FEH) is a well-known model of this kind in the UK. This model has been widely used in the United Kingdom, for example, in flood defence planning, flood risk analysis, new construction planning, and determining the rarity of significant rainfalls or floods.

The recommended method in FEH for the estimation of QMED at the sites where no flood peak data is available, or are ungauged, is to transfer data from nearby donor sites or from distant analogue ones. A precondition for such transfer is that the donor sites must be hydrologically similar in terms of catchment area, soil type and rainfall. However, using basic statistical regression techniques, it is possible to estimate index floods based on catchment descriptors such as area, base flow index, and wetness. Distinct algorithms are provided in FEH for the QMED estimation in urban sites and rural sites and even for the individual catchments. On the other hand, flood estimates obtained solely on the basis of catchment descriptors, according to Reed and Robson (1999), are poorer than the ones obtained directly from the flood peak values. Categorization of the sites into similar groups can be difficult and hence establishing a sufficient collection of donor sites is not always feasible. FEH warns that basic discrepancies between sites could lead to not only the transmission of wrong information, but also the establishment of incorrect flood projections. Even though artificial neural networks have been used for the classifications of catchments (Thandaveswara and Sajikumar, 2000),

2.2 The improved FEH equation (2008)

The proposed improvements are prompted in part by the HiFlows-UK (Environmental Agency 2012) initiative, which resulted in the construction of a much better database of routinely collected flood data (Kjeldsen et al., 2008). Not only are the information records significantly longer than before, but the HiFlows-UK project also invested a lot of effort on standardizing and assessing the entire dataset. This means that the amount of data available for analysis has significantly increased. Feedback from FEH users, both informal and official, has also had an impact on the revised procedures. Most technical features of the approach can be changed without significantly altering the approach's structure. Most technical specifics of the strategy are revised to improve the procedure's performance without significantly modifying the methodology's structure. The theoretical statistical underpinning that underpins the technique has been significantly improved as a result of the upgrades. In addition, certain novel descriptors of catchment topography and native climate that have been proposed since the FEH study have been considered. A replacement descriptor for measuring floodplain extent, in particular, has been developed and is now incorporated in the updated processes. Following Kjeldsen et al. (2008)'s study, the following important changes have been made: a model for estimating the median using a replacement regression model.

2.3 Flood estimation using Artificial Neural Networks (ANNs)

ANNs have been used to perform flood estimation in the past (Dawson, C., et al., 2006). Since the effective training techniques for ANNs were developed (Rumelhart and McClelland, 1986), these models have been used to solve a variety of hydrological problems, including rainfall-runoff modeling and river discharge forecasts (Abrahart et al., 2004). Dastorani et al. (2010) also evaluated the applicability of ANN in the forecasting of precipitation amount before its occurrence and reported it to be a reliable model for estimation. A study conducted by (Liong et al, 2000) on the river stage prediction and produced results with a high degree of accuracy and a short computational time, making ANNs a desirable forecasting tool, a sensitivity research was also undertaken, which recommended reducing the number of input neurons (in that case from eight to five), despite the estimated accuracy level not being considerably affected. Dawson and Wilby (2001) done research on the application of ANNs in hydrological modeling, describing it as an emerging field of research with a wide range of methodologies. Two sets of studies conducted by Govindaraju (2000) investigate the function of artificial neural networks (ANNs) in hydrology and provide some fundamental criteria for their use, as well as their strengths and limitations, and compare them to other hydrology modeling approaches. The ANN model is better at detecting non-linear relationships between observed and anticipated data sets (Hsu, K., L., et al, 1995 and El-Shafie, A., et al, 2011). Lapedes, A., et al. (1987) used ANN models to study non-linear data series and discovered that ANN models have superior generalization capabilities than regression-based models. However, there are relatively few studies involving the application of ANNs to flood estimation at ungauged sites. For example, a regionally trained neural network was equivalent to earlier Q2 regression estimates in the Gulf of Texas (Muttiah et al., 1997), another research was conducted for the US river basin to assess the peak storm discharge during a two-year period. Hall and Minns (1998) reported that the classification of hydrologically similar zones is an important component of regionalization techniques and the use of these techniques on flood data from the southwest of England and Wales has shown that groups can be formed by Representative Regional Catchments (RRCs), which have hydrologically more sensible qualities than those provided solely by geographical closeness. Using data from sites in Sumatra and Java, Hall et al. (2000) used four to twelve input catchment descriptors to predict the scale and location parameter of Gumbel distribution for annual floods, however Dastorani and Wright (2001) reported that for the index flood estimation, QMED, seven catchment descriptors were sufficient for sites in the United Kingdom. This paper discusses the application of ANNs to predict the index flood for a much larger sample of selected catchments in the UK. Reason why this research was considered essential is because in the most recent study carried out by Dawson, C., et al., (2006) only trained the ANNs with the data for the urban regions, in addition to that, the study was also restricted only to the locations that had at least ten years of flood data, and the index flood predictions were inevitably over or under-estimated. Since ANNs rely heavily on data, new flow records have the potential to improve flood estimates.

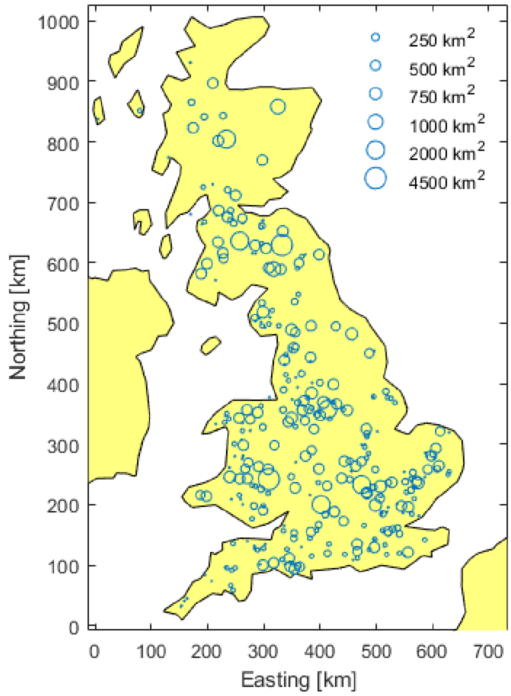
3. DATASETS

The dataset used in this paper is obtained from the National River Flow Archive (NRFA). It consists of data for 935 gauging stations containing annual maximum (AMAX) series, peak over threshold (POT) and catchment descriptors (CD). The AMAX series contains the largest observed flow (in cubic metres per second, abbreviated to m^3s^{-1} and sometimes also referred to as 'cumecs') in each water year. The Peaks Over Threshold (POT) series contains all peak flows that are greater than a given threshold flow, the threshold is generally set to include an average of 5 events per year. The Catchment Descriptor (CD) data is a set of properties that determine the hydrological characteristics of the catchment. A total of 544 stations are recommended for use in pooling groups, 337 stations are suitable for QMED, if QMED is likely to be within 30% of its true value, and 53 stations are suitable for neither. A group of 337 stations is chosen for the analysis. Figure 1a shows the location of all these catchments and their catchment area whereas Figure 1b shows their standard average annual rainfall.

3.1 Data quality

Out of already very small number of available catchments, data has to be screened for discordancy. For the recalibration of parameters of the model a careful selection of data points is very crucial. To avoid the estimates to be biased the chosen data points should be considerably consistent and in spite of that there should be enough data to appropriately define the model parameters (Grover et al., 2002). Therefore, the data for 337 stations is subjected to screening to look for any irregularities and independence. It was observed that three stations, 25808, 25809 and 25810, have noticeably small catchment areas of 0.75 km^2, 0.05 km^2, 0.04 km^2, respectively. Consequently, these stations are discarded from the recalibration and estimation of new parameters.

The chosen dataset, suitable for QMED estimation, from National River Flow Archive (NRFA) has 22 catchment descriptors (CDs) available for each catchment, the summary of CDs is shown in the Table 1. First of all, a MATLAB script was run to extract the catchment descriptors for each site from the dataset and then all of them were compiled in a single file alongside of catchment numbers and their coordinated for the ease of analysis. Although, some of these descriptors upon analysis proved to be more crucial in terms of the sensitivity of the models than the others. The annual maximum series (AMS) data from the NRFA folder for each catchment was then extracted and compiled into a single file using another MATLAB script. Finally, the median of AMS for each catchment was calculated to produce the final value of QMED measured.



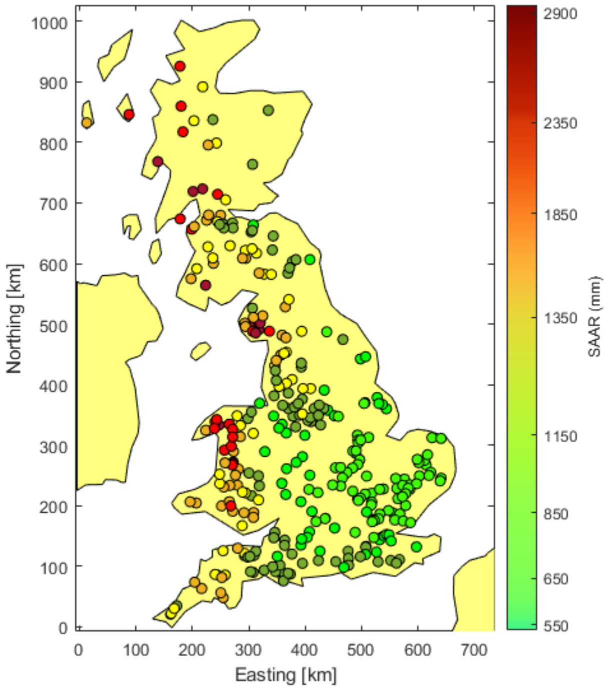


Figure 1: Locations of the catchments used in this study displaying (a) their catchment area (AREA) and (b) standard average annual rainfall (SAAR)

Table 1: All the available catchment descriptors (CDs)

Descriptors	**Unit**	**Summary**
ALTBAR	m	Mean catchment altitude above sea level
AREA	km^2	Catchment drainage area
ASPBAR		Mean direction of all the inter-nodal slopes in the catchment – represents dominant aspect of catchment slope
ASPVAR		Invariability of slope direction
BFIHOST		Base Flow Index - soil drainage type
DPLBAR	km	Mean drainage path length
DPSBAR	m/km	Mean catchment slopes
FARL		Flood attenuation due to rivers and lakes
LDP	km	Longest drainage path
PROPWET	mm	Proportion of time when soil moisture deficit ≤ 6mm
RMED1D	mm	Median annual max 1Day rainfall
RMED1H	mm	Median annual max 1Hour rainfall
RMED2D	mm	Median annual max 2Day rainfall
SAAR	mm	Standard average annual rainfall 1961-90
SAAR4170	mm	Standard average annual rainfall 1941-70
SPRHOST		Standard Percentage Runoff - soil drainage type
URBCONC1990		Concentration of sub/urban land cover 1990
URBCONC2000		Concentration of sub/urban land cover 2000
URBEXT1990		Extent of urban and suburban cover 1990
URBEXT2000		Extent of urban and suburban cover 2000
URBLOC1990		Location of urban and suburban cover 1990
URBLOC2000		Location of urban and suburban cover 2000

4. METHODS

Due to its flexible structure, the statistical models have a stronger descriptive capacity than the rigidly constructed physical models. It is also evident from the literature that the statistical indirect methods, such as QMED regression model, are more precise than the conceptual indirect models for the prediction of QMED at the ungauged sites (Brath et al., 2009). The rigid structure of the conceptual approach decreases its dependence on the specific knowledge of the individual stations and, consequently, strengthens its stability. Consequently, statistical models may not be able to provide physical interpretation or insight into other inter-related factors in flooding, leading to possible changes in flooding.

4.1 The Flood Estimation Handbook method (1999)

Various procedures to estimate QMED were provided in volume 3 of the FEH (Reed and Robson, 1999), some of the common ones are; from annual maxima (AM), peaks over threshold (POT) and catchment descriptors (CD). Annual maxima method is only used if the flood data is available for more than 14 years. The recommended method in FEH for the estimation of QMED at the sites where no flood peak data is available, or are ungauged, is to transfer data from nearby donor sites or from distant analogue ones. A precondition for such transfer is that the donor sites must be hydrologically similar in terms of catchment area, soil type and rainfall.

The statistical QMED model given in the FEH (1999) - catchment descriptor method, is advised to be used with the transfer of data method from the nearby sites. This way the obtained values for QMED will be refined and it also uses a longer flood record. The regression equation can also be used with the catchment descriptors only if transfer of data is not possible because of unavailability of suitable sites nearby, and the site record is less than two years long. This regression model was applicable to all the ungauged catchments in the UK with the area greater than 0.5 km^2. The equation 1 is based on the analysis of over 1000 stations, their QMED values and catchment descriptors obtained from the FEH CD-ROM 1. These are essentially the catchments with $URBEXT < 0.025$.

$$QMED_{rural} = 1.172\ AREA^{AE} \left(\frac{SAAR}{1000}\right)^{1.560} FARL^{2.642} \left(\frac{SPRHOST}{100}\right)^{1.211} 0.0198^{RESHOST} \qquad 1.$$

where

$$AE = \text{area exponent} = 1 - 0.015 ln\left(\frac{AREA}{0.5}\right) \qquad 2.$$

with r^2 (coefficient of determination) = 0.916 (on log scale), 0.905 (GLS-scale, Generalised Least Square) and the fse (factorial standard error) = 1.549.

The urban adjustment factor, which is calculated using the following formula, describes how an urban catchment differs from its rural counterpart.

$$UAF = (1 + URBEXT)^{0.83} PRUAF \qquad 3.$$

where

$$PRUAF = 1 + 0.615\ URBEXT\left(\frac{70}{SPRHOST} - 1\right) \qquad 4.$$

The QMED obtained from equation 1 is then corrected for the unsuccessfully mitigated effect of urbanisation using the following formula:

$$QMED = UAF\ QMED_{rural} \qquad 5.$$

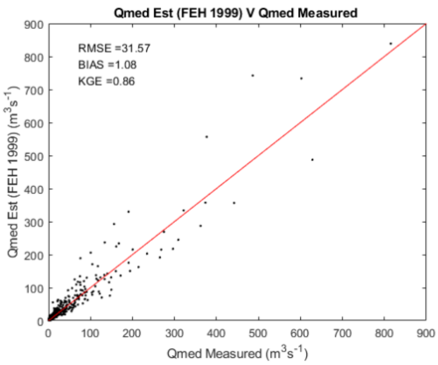
The original FEH model (Equation 1), is also evaluated for RMSE, KGE and BIAS in order to ensure the fair comparison of all the models in this paper. In the Figure 5(a) the estimated QMED from this model is plotted against the measured QMED from the AMS.

4.2 The improved FEH equation (2008)

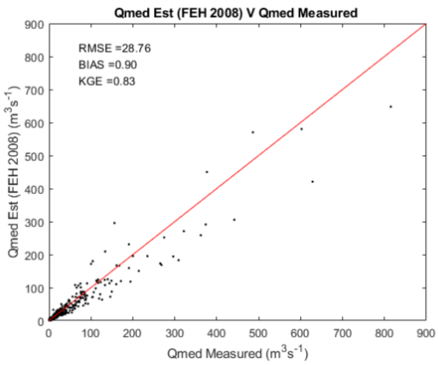
The Centre for Ecology and Hydrology modified the older model released in Flood estimation manual (1999) in 2008. (CEH), The work of Kjeldsen et al. (2008) on the calculation of floods from minor catchments is not without flaws. Although the 602 flood datasets used to generate the revised QMED equation appear to represent a sizable sample size, only 46 of the 602 flood datasets drain an area of less than 25 km2. In addition, removing repeated entries for three of those catchments decreases the number of minor catchments in the sample to just 41. Perhaps because the project team was able to quickly get the yearly maximum flood level. Kjeldsen et al. (2008) did not attempt to create an equation specially targeted at small catchments to replace the QBAR regression equation presented in IH 124, possibly because the research team was able to gather yearly maximum flood peak values from just a small sample. Kjeldsen (2010), on the other hand, offers new recommendations for altering QMEDrural to account for urbanization.

$$QMED_{rural} = 8.306\ AREA^{0.851} 0.154^{\left(\frac{1000}{SAAR}\right)} FARL^{3.445} 0.046^{(BFIHOST)^2} \qquad 6.$$

The results from the modified FEH (Kjeldsen et al. 2008) approach showed considerably smaller errors than the previous FEH approach (IH 1999), as shown in Figures 5a and 5b.



(a)



(b)

Figure 5: The plots of estimated QMED against QMED measured (a) displaying results of (FEH 1999) model (b) displaying results of (FEH 2008) model

4.3 New non-linear model

A new multiplicative non-linear power-law model (equation 7) is proposed to estimate QMED. The changes in catchment descriptors (CDs) have a scaling effect on the QMED and the degree of this effect is influenced by the exponent terms *b*, *c*, *d*, …. This form of the equation yields a linear structure that can be used with normal multivariate statistical procedures.

$$QMED = A\, X_1^b\, X_2^c\, X_3^d \ldots \qquad 7.$$

Where *A*, *b*, *c* … represent the model parameters that have to be estimated, X_i represent a given catchment descriptors. Writing the equation in this form gives a linear structure that allows standard multivariate statistical procedures to be applied. The substantial research on multiple regression analysis conducted by Thomas and Benson (1970) resulted in the mathematical equation of the similar form.

4.4 Artificial Neural Networks (ANNs)

A neural network's computational capacity is evidently derived from two factors: first, its massively parallel distributed structure, and second, its capacity to learn and hence generalize. And it is designed to use a computational approach to simulate natural neural networks (Hayek, S., 2009). A neural network consists of a number of interconnected nodes called neurons that are linked together by weights between layers. They are divided into three fundamental layers: an input layer, does not perform any calculations and just feeds the network with information, an intermediate hidden layer, and an output layer that produces results. The architecture of a typical ANN is shown in Figure 2, though it varies. The most extensively used among several varieties of ANNs is the feed-forward neural network, it gets this name because data goes through the network from the input layer to the hidden layer, and then to the output layer.

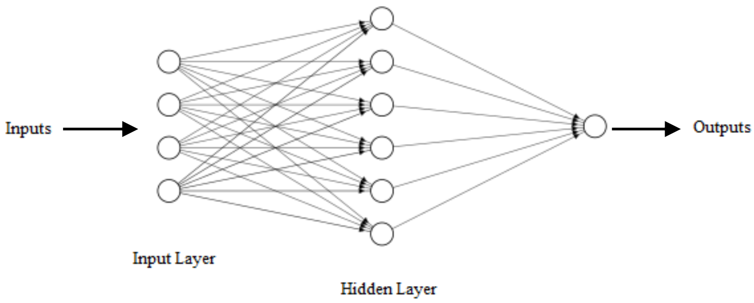


Figure 2: Typical two layered feed-forward neural network

4.5 Performance Indicators

For an objective evaluation of model performance, goodness-of-fit measures are critical. It is essential to select objective performance indicators in order to evaluate the performance of prediction approaches using various estimation methods and model parameters. To compare the estimated QMEDs with the actual QMED values, the following objective functions were used.

4.5.1 Kling-Gupta Efficiency (KGE)

The fundamental assumptions that the data are linear and normal in nature, and that outliers are not present in the datasets are inherent in the calculation of *KGE*. The criterion of *KGE* is based on a weighted average of the three component metrics i.e., correlation, bias, and variability.

To compute γ and β, the bias between estimated and observed mean QMEDs and the bias between estimated and observed standard deviation of QMED was used, respectively.

$$KGE = 1 - \sqrt{(r-1)^2 + (\gamma-1)^2 + (\beta-1)^2}; \qquad \gamma = \frac{cd}{rd}; \qquad \beta = \frac{cm}{rm} \qquad 8.$$

where r = is the linear Pearson correlation coefficient between estimated and observed values, *rm* is the average of observed values, *cm* is the average of estimated values, *rd* is the standard deviation of observed values and *cd* is the standard deviation of estimated values (Gupta, et al. 2009).

4.5.2 Root Mean Square (RMSE)

The standard deviation of estimation errors is called Root Mean Square Error (RMSE). RMSE is a measure of how far the data points are from the regression line. In other words, it indicates how tightly the data is clustered around the line of best fit. calculating the average of squared errors Taking the result's square root.

In order to calculate the RMSE, first of all the error is calculated by subtracting the estimated values from observed values. Then the estimation errors are squared, after calculating the mean of squared errors finally the square root is computed of the obtained value, that is:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{N}} \qquad 9.$$

where N is the total number of data points, $\hat{x}_i$ represents the estimated values and x_i the observed values.

4.5.3 Bias

The Bias indicates whether the model has a tendency to over-or under-predict estimated values. It is simply calculated by adding all the observed and estimated values separately and then by dividing estimated values to the observed ones.

$$BIAS = \sum_{i=1}^{n} \frac{\hat{x}_i}{x_i} \qquad 10.$$

where $\hat{x}_i$ are the estimated values and x_i are the observed values.

5. RESULTS AND DISCUSSION

Correlation analysis was carried out assess the correlation between QMED and catchment descriptors. Figures 3 and 4 show the top ranked descriptors based on their correlation with QMED after an extensive review. Another possible benefit of doing this analysis is that any outliers and non-linear interactions can also be detected by plotting descriptors against each other in the form of a matrix. The pattern observed here tends to differ slightly from the one reported in Flood Estimation Handbook (IH 1999) which is due to the availability of new flow records. Figure 3 shows only the descriptors that were implemented in FEH model (IH 1999) i.e., *AREA*, *SAAR*, *FARL*, *SPRHOST* and *BFIHOST*. For example, the descriptors that shows high correlation of at least 0.6 or high with QMED are *AREA*, *LDP* (Longest Drainage Path) and *DPLBAR* (mean drainage path length). *ALTBAR*, *SPRHOST*, *DPSBAR*, and *SAAR*, on the other hand, exhibit a correlation of 0.3 with QMED.

Some cross-correlation can also be seen in the Figures 3 and 4, but it is not desirable. Only variables with low cross-correlation would be included in an ideal model. There are two possible reasons, first, it leads to a great number of alternative model choices which all have comparable fits and many of which have poorly stated parameters. Secondly, it means that a model has favoured a variable rather than another variable and so confuses the interpretation (Reed and Robson, 1999). And that is why in the original FEH model certain strongly cross-correlated variables were redefined. For example, *SPRHOST* and *BFIHOST* had relatively high cross-correlation of 0.93 and were both very significant for the model, thus a new variable *RESHOST* was established in place of *BFIHOST*, and it was reconstructed to have a low correlation with *SPRHOST* while retaining the information from *BFIHOST*.

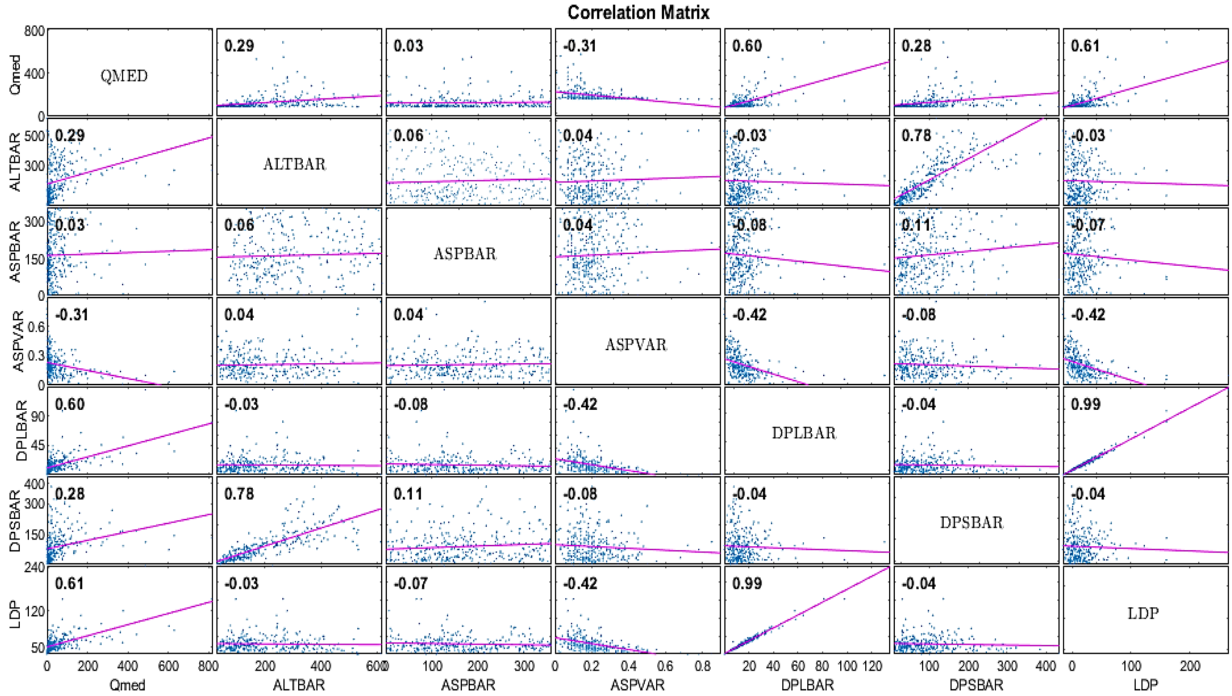


Figure 3: A correlation matrix of 6 highly correlated variables

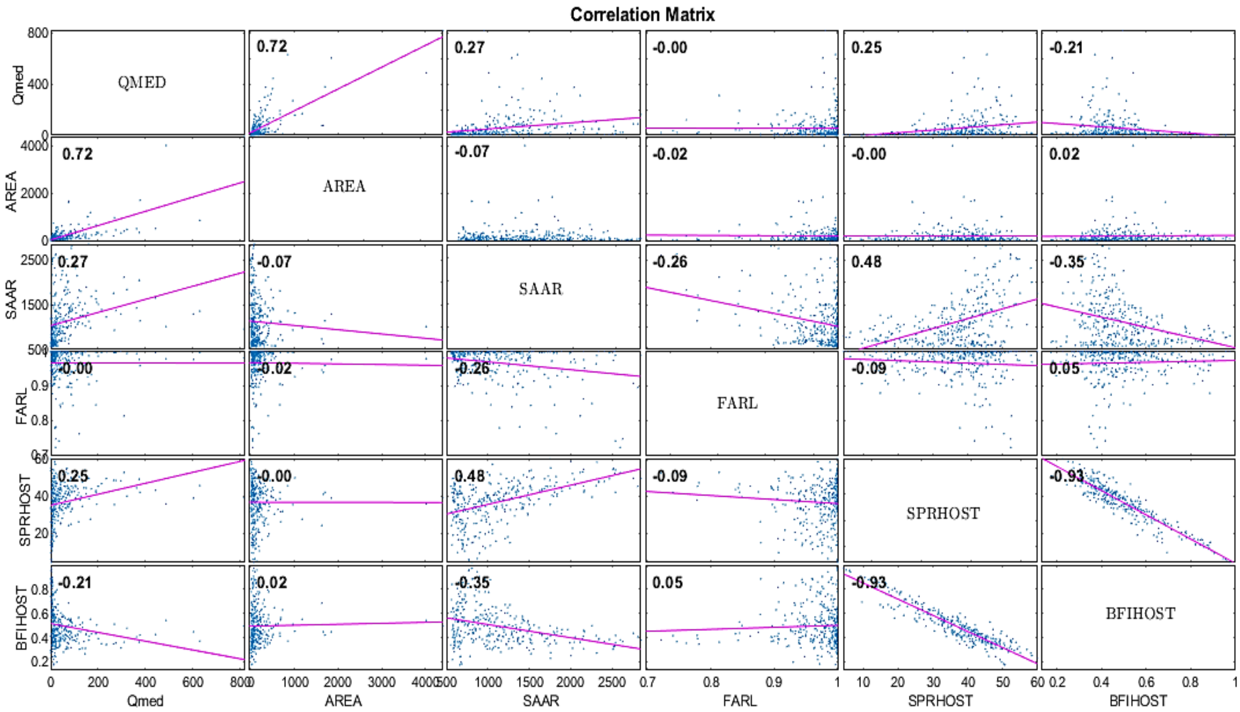


Figure 4: A correlation matrix of other 6 correlated variables

5.1 Recalibration of non-linear models from the literature using 100% of data.

All the non-linear regression models mentioned above have been re-calibrated using 100% of the data. This means that all accessible datasets were utilized to calibrate the models, and that the same dataset was used for the validation of models, resulting in the best possible results for each model.

5.1.1 Recalibrated FEH QMED equation

The non-linear model provided in Flood Estimation Handbook (Reed and Robson, 1999) was tested with the latest available data in the similar manner as it was published in the FEH. The statistical model for the estimation of Index flood, QMED, published in the literature (equation 1), was not performing as expected. The QMED estimates were considerably off the mark in terms of the expected results. The error was noted in equation (2), the calculation of *AE*, area exponent, in the literature the given expression was with the natural log (base 2) however when the model was tested the by replacing the natural log the common logarithm (base 10) the expected behavior of model was observed.

Once the normal behavior of model was reestablished, the original FEH model was recalibrated using 100% with new flow data. The parameters obtained after the re-calibration process are than plugged into the same equation (1), so the new equation is given below.

$$QMED_{R99} = 3.4410\ AREA^{AE} \left(\frac{SAAR}{1000}\right)^{1.7427} FARL^{4.4710} \left(\frac{SPRHOST}{100}\right)^{0.4760} 0.0120^{RESHOST} \quad 11.$$

$$AE = 1 - (7.94e-3)\ LOG_{10}\left(\frac{AREA}{3.84e-9}\right) \quad 12.$$

$$RESHOST = BFIHOST + 0.5619\left(\frac{SPRHOST}{100}\right) - 0.3332 \quad 13.$$

Where $RESHOST$ is the relative responsiveness of a catchment, residual from linear regression of two primary variables that summarise soil characteristics: $SPRHOST$ and $BFIHOST$.After the evaluation of the performance of model the results are displayed on the Figure 6a, where the updated QMED estimates from the recalibrated model are plotted against the measured QMED. The RMSE has improved by over 22% than the original FEH, and the results are completely unbiased, as it can be observed. In addition, the KGE indicator demonstrates a 10% improvement in performance over the original FEH model (IH 1999).

5.1.2 Recalibrated Improved FEH QMED equation

Similar to the previous approach, the model published in the literature is tested with the new dataset. The parameters obtained after the re-calibration process are than plugged into the same equation (6), so the new equation is given below. The results are plotted against the QMED from the median of annual maximum series to get the performance of model and the errors are also plotted on the graph.

$$QMED_{R08} = 12.7471\ AREA^{0.8571}(0.1291)^{\frac{1000}{SAAR}}\ FARL^{5.0919}\ (0.0182)^{BFIHOST^2} \quad 14.$$

The RMSE of the re-calibrated model is about 12% lower than the revised FEH model (Kjeldsen et al. 2008), and the estimates are 10% less biased, similarly the KGE has improved 10%.

Another significant behavior that can be noticed following the re-calibration of both of these models is that the original FEH model (IH 1999) outscored the revised FEH model (Kjeldsen et al. 2008) in every performance evaluation metric. It can be concluded from this that the original FEH model should be recalibrated as the new flow data becomes available.

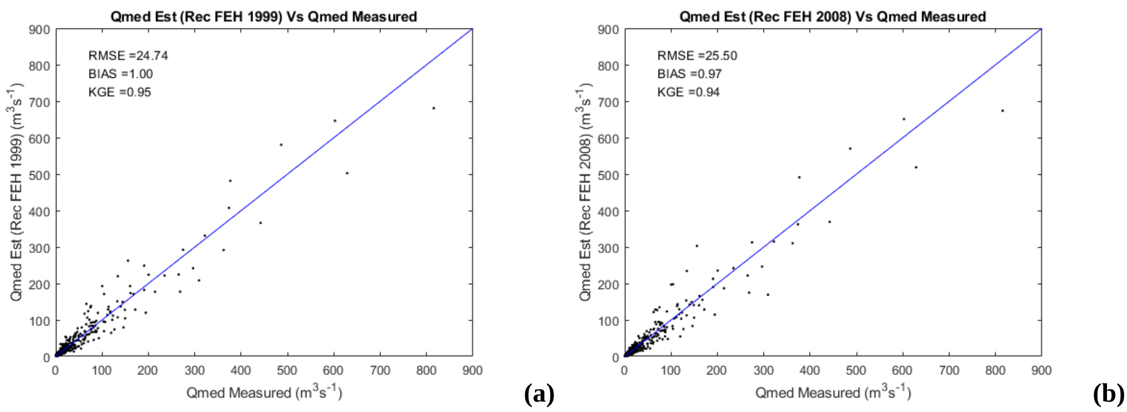


Figure 6: The plots of recalibrated estimated QMED against QMED measured (a) displaying results of recalibrated (FEH 1999) model (b) displaying results of recalibrated (FEH 2008) model

5.1.3 New non-linear model

In an attempt to improve the QMED estimates a new regression model is proposed. Regression has long been used to link a desired flood quantile to physiographic, geomorphologic, and climatic aspects of the catchment.

Typically, the analysis is carried out with the power-form equation (15). The selection of the catchment descriptors for the model is made on the basis of correlation analysis (Figure 3 and 4). Analysis suggested that 7-variable model is preferred containing $AREA$, $FARL$, $SAAR$, $SPRHOST$, $PROPWET$, $ALTBAR$ and $BFIHOST$.

$$QMED = A\, X_1^b\, X_2^c\, X_3^d \ldots \qquad 15.$$

where X_i represent a given catchment descriptor. Using nonlinear regression equation to directly estimate the parameters in power-low model yielded better results than log-linear models for estimates of the 10- and 100-year floods in Quebec (Pandey and Nguyen 1999). After the re-calibration of equation (15), the parameters that are obtained are plugged in resulting in equation (16).

$$QMED = A * AREA^{0.899} FARL^{5.3503} SAAR^{1.6567} 0.003^{BFIHOST^2} SPRHOST^{-0.9337} PROPWET^{0.0806} ALTBAR^{0.1469} \qquad 16.$$

where $A = 2.8384e - 4$.
After incorporating more variables with the correlations value ranging from 0.26 to 0.94, the results demonstrated an improvement over earlier non-linear regression models. The RMSE figure shows that the estimates from this model have a 23 percent lower error rate than the original FEH and a 16 percent lower error rate than the revised FEH model. The KGE and bias factor of the estimations show a similar trend. Both exhibited a ten percent improvement.

All of the re-calibrated models performed better in terms of estimating greater values of QMED, indicating that they are suitable for estimating high QMED values.

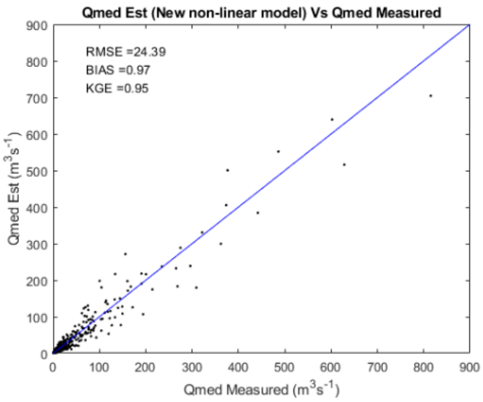


Figure 7: The results of new non-linear model obtained using multiplicative structure approach ploted estimated QMED agains measured QMED

5.2 Recalibration of non-linear models from the literature using Cross-Validation.

In addition to that, since the number of catchments is insufficient to separate the data set in calibration and validation, we employed cross validation to test the true performance of the models here each model is trained for n-1 data points and then evaluated against the remaining 1 unseen data point indicating the true performance of the model. The cross-validation analysis indicated that the performance of the models worsens. The statistical models are calibrated using catchment descriptors from 333 catchments and are tested against the data from remaining one catchment. It shows the real performance of model with the data the model has not seen yet.

As can be seen from the Figure 6a, 6b and 7, the estimates for higher QMED values are far off the regression line, indicating that it is not suitable for high QMEDs. Meanwhile other thing that can be drawn from this is that because the high QMED estimates are so poor, it is evident that the models for smaller QMED estimations, ranging from 0-150, can be achieved with reasonable accuracy. Since the error values are almost close the ones for the original models.

Another interesting point to consider is that, in comparison to the more complex FEH models, the new non-linear model is extremely simple, yet it still produces reasonable estimates.

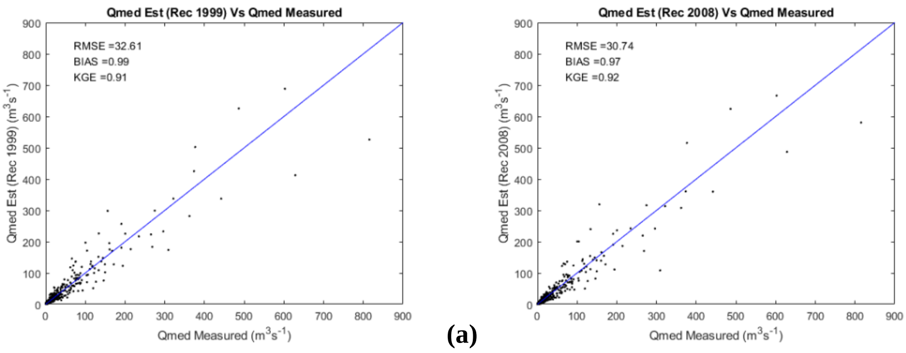


Figure 8: The plots of recalibrated estimated QMED against QMED measured using cross validation (a) displaying results of recalibrated (FEH 1999) model (b) displaying results of recalibrated (FEH 2008) model

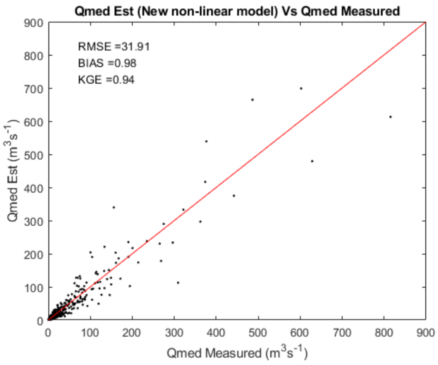


Figure 9: The results of new non-linear model obtained using multiplicative structure approach ploted estimated QMED agains measured QMED using cross validation

5.3 Artificial Neural Networks (ANNs)

In recent years, an alternate technique to flow forecasting based on the ANN has been developed (Govindoraju, et al, 2000). In this paper an attempt has been made to estimate QMED, the feed-forward networks are implemented here. Using MATLAB Bayesian Regularization training algorithm multiple configurations are executed, such as, with single hidden layer with 10 neurons, two hidden layers with 10,15 and 20 neurons, and the best two configurations of all are presented here.

Similar to the previous methodologies, cross-validation process is implemented here calibrating 333 data points while keeping one out and then validating the model with that single unseen point. The results produced by ANNs are still better compared to the non-linear regression models as shown in Figure 10a and 10b. The traditional split validation approach was not followed because of significantly small number of data points.

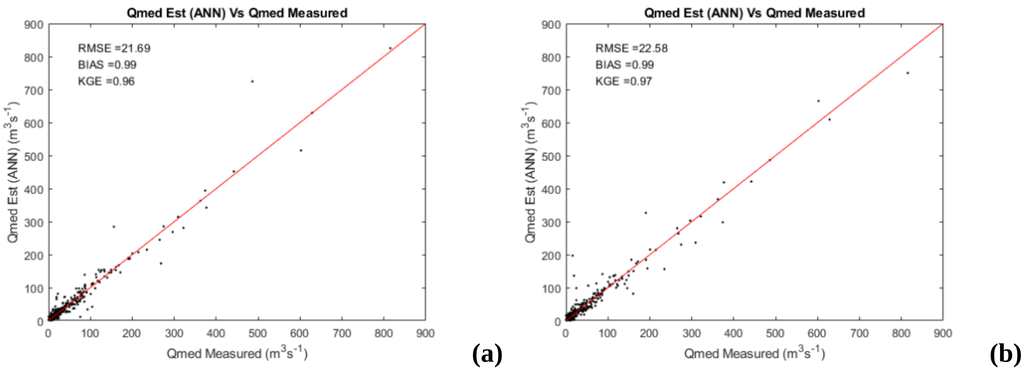


Figure 10: The plots of estimated QMED against QMED measured using ANNs (a) displaying results of a model with 2 hidden layers each with 20 neurons (b) displaying results of a model with 2 hidden layers each with 10 neurons

DISCUSSION

The traditional FEH model and the revised FEH model are tested with more flow data. The same models are re-calibrated using 100% of data and then cross-validation, as it could be expected the cross-validation process worsened the performance of models. The RMSE was indicative of poor performance but KGE and BIAS still showed performance reasonably well. Which indicates that the high QMED values are not to be estimated from these models but for the catchments with smaller catchment area (*AREA*), with small QMED values can be estimated really well.

CONCLUSIONS

The research question that we had in the start of this study is answered. The non-linear regression models showed reasonably good improvements after re-calibration. The new non-linear model based on the power law is the best among the other traditional models, for the 100% data calibration. However, ANNs outperformed every other approach that has been made for the estimation of QMED. And the computation was relatively faster than the traditional models.

LIMITATIONS OF YOUR WORK

The dataset used for the analysis is mentioned to be 'suitable for QMED' however some previous studies showed the analysis with almost double the size of data we used. Hence, they did not disregard the dataset 'suitable for pooling' and used both of them collectively making the total data points around 800 catchments.

RECOMMENDATIONS FOR FUTURE WORK

For the thorough study of relationship between catchment descriptors and QMED further work can be done such as in addition to the correlation analysis Principle Component Analysis (PCA) can be conducted.

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