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Optimal modes for spatially multiplexed free-space communication in atmospheric turbulence

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Abstract: In near-field free-space optical (FSO) communication, spatial-mode multiplexing (SMM) increases transmission capacity by transmitting independent information streams in orthogonal modes. Propagation through atmospheric turbulence causes phase and amplitude distortions that can degrade SMM performance. In this paper, we show there exist optimal modes for transmission through turbulence with minimum degradation, under a realistic assumption that a transmitter knows the turbulence statistics but not the instantaneous state of the atmosphere. These modes are determined by performing a Karhunen-Loève expansion of the optical electric field in the receiver aperture. We show that these modes are Laguerre-Gauss (LG) modes whose beam waist is chosen depending on the field coherence length in the receiver plane. These adaptive-waist LG modes, when ordered by decreasing eigenvalue, can approximate a received signal field by a finite number of modes with lowest mean-square error among all orthonormal mode sets. Hence, they represent optimal transmit and receive bases for SMM FSO. Using numerical simulation, we study SMM FSO transmission at various turbulence strengths and signal-to-noise ratios. We compare the performance using the adaptive-waist LG modes to that using fixed-waist LG modes (which assume no knowledge of turbulence statistics) and instantaneous eigenmodes (which assume knowledge of the instantaneous state of the turbulence). We also study the performance using the orbital angular momentum subsets of the adaptive-waist LG mode and fixed-waist LG mode sets.

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1. Introduction

Free-space optical (FSO) communication, which uses laser light to provide high-capacity links between distant sites, is an appealing alternative to radio-frequency communication. Spatial-mode multiplexing (SMM), a form of multi-input, multi-output (MIMO) transmission, employs spatial and polarization modes as orthogonal dimensions for increasing transmission capacity without requiring increased frequency bandwidth [1–4]. Various mode sets have been considered for SMM [5], including Laguerre-Gauss (LG) modes [6] and a subset often called orbital angular momentum (OAM) modes [7].

SMM FSO communication is applicable when, even in the presence of atmospheric turbulence, the beam size in the receiver plane is on average smaller than the receiver aperture, avoiding beam clipping and preserving orthogonality among the modes [6]. In these so-called near-field links, the maximum number of spatial modes with efficient coupling exceeds unity. Neglecting turbulence, the maximum number of efficiently coupled modes is approximated by the product of the transmitter and receiver Fresnel numbers, $N_F = (A_T A_R)/(\lambda^2 L^2)$, where A_T and A_R are transmit and receive aperture areas, λ is optical wavelength, and L is propagation range [8]. In

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links such that $N_F \leq 1$, the transmitter can couple to only one spatial mode at the receiver, and SMM cannot be employed.

SMM FSO links transmit mutually coherent, orthogonal light modes. Modes that propagate through random media, such as atmospheric turbulence, are subject to phase distortions and amplitude fluctuations, which can degrade coherence and limit the number of independent modes. The need to improve SMM FSO link robustness has motivated study of adaptive optics methods for correcting distortion of the light modes [9,10]. In this work, instead of using adaptive optics to mitigate turbulence-induced distortion, we increase SMM FSO system capacity by transmitting a set of mutually orthogonal modes that optimally exploit the spatial dimensions available after propagation through atmospheric turbulence.

In an ideal SMM FSO system, both transmitter and receiver terminals would have detailed knowledge of the instantaneous random distortions introduced by turbulence and would adapt themselves to the channel accordingly. Various effects in practical systems, including partial reciprocity and round-trip time delay, render unrealistic the assumption that the transmitter has instantaneous channel information. In practice, it is far more realistic to assume that the transmitter has access only to limited statistical information about the random distortions introduced by turbulence, as the channel statistics vary over much longer time scales than the instantaneous channel realization. Knowledge of the channel statistics can be obtained by exploiting partial link reciprocity, or by employing feedback from the receiver to the transmitter. In this work, we assume limited knowledge of the turbulent channel statistics, and determine an optimal set of modes.

Our approach to finding the optimal modes considers the second-order statistics of the propagated optical fields. It uses a Karhunen-Loève (KL) expansion [11] of the optical fields propagated through atmospheric turbulence, a statistical technique that performs an orthogonal transformation of a set of observations of correlated fields into a superposition of a set of fully spatially coherent modes with uncorrelated coefficients. These modes are the eigenfunctions of the mutual intensity and, when ordered by decreasing eigenvalues, they can approximate a received optical field by any finite number of modes with lowest mean-square error among all orthonormal mode sets. Hence, they are an optimal transmit or receive basis when the number of modes is constrained. As we show, these modes are LG modes that have their beam waist chosen appropriately relative to the transverse coherence length of the field in the receiver plane. These adaptive modes with adjustable waist can be tuned to the turbulence statistics using simple variable-magnification optics.

The remainder of this paper is organized as follows. In Section 2, we show that KL functions of the atmospheric optical fields are the optimal set of basis modes for SMM FSO. In Section 3, we study the performance of SMM in turbulence channels, comparing the optimal basis modes to other mode sets. We present conclusions in Section 4.

2. Optimal basis for random optical fields

We want to know the best set of basis modes to define independent and orthogonal communication channels for SMM FSO. In our analysis, we will define a complete orthonormal basis set of functions $\psi_1(\boldsymbol{\rho}), \psi_2(\boldsymbol{\rho}), \psi_3(\boldsymbol{\rho}), \ldots$ at any position $\boldsymbol{\rho}$ in a plane transverse to the direction of propagation.

In these near-field links through random media, we can expand the complex optical field as $U(\boldsymbol{\rho}) = U_0(\boldsymbol{\rho}) \exp[\Phi(\boldsymbol{\rho})]$, where the field without turbulence is assumed to have a Gaussian profile $U_0(\boldsymbol{\rho}) = \exp(-|\boldsymbol{\rho}|^2/2\omega_0^2)$, and the factor $\exp[\Phi(\boldsymbol{\rho})]$ represents the random distortion introduced by the turbulent medium. The Gaussian profile $U_0(\boldsymbol{\rho})$ is characterized by a *reference beam waist* ω_0, which is chosen as explained below. When the optical field propagates through turbulence, both its envelope and its phase fluctuate. The exponent of the random distortion is

$\Phi(\boldsymbol{\rho}) = \chi(\boldsymbol{\rho}) + j\phi(\boldsymbol{\rho})$, where $\chi(\boldsymbol{\rho})$ and $\phi(\boldsymbol{\rho})$ represent log-amplitude fluctuations (scintillation) and phase fluctuations (aberrations), respectively.

We begin by finding the KL expansion of the random fields across the receiver aperture. The KL expansion represents the random optical fields as a superposition of orthonormal functions $\psi_n(\boldsymbol{\rho})$ with uncorrelated coefficients α_n:

$$U(\rho) = \sum_{n=1}^{\infty} \alpha_n \psi_{\mathrm{n}}(\rho), \tag{1}$$

where

$$\int \psi_m^*(\rho)\, \psi_n(\rho)\, d\rho_1 = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \tag{2}$$

and the coefficients of the expansion are given by

$$\alpha_n = \int U(\rho)\, \psi_n^*(\rho)\, d\rho. \tag{3}$$

Among all possible orthogonal expansions, the KL expansion chooses one that has uncorrelated expansion coefficients:

$$E\{\alpha_n \alpha_m^*\} = \begin{cases} \lambda_n & m = n \\ 0 & m \neq n \end{cases}, \tag{4}$$

where $E\{\cdot\}$ denotes ensemble average. Using Eq. (3) in Eq. (4) yields

$$E\{\alpha_n \alpha_m^*\} = \int \left[\int \Gamma(\rho_1, \rho_2) \psi_n(\rho_2)\, d\rho_2 \right] \psi_m^*(\rho_1)\, d\rho_1 \tag{5}$$

and the coefficients will satisfy Eq. (4) provided that

$$\int \Gamma(\rho_1, \rho_2) \psi_n(\rho_2)\, d\rho_2 = \lambda_n \psi_n(\rho_1). \tag{6}$$

Here, $\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = U_0(\boldsymbol{\rho}_1)\; U_0^*(\boldsymbol{\rho}_2)\; E\{\exp[\Phi(\boldsymbol{\rho}_1) + \Phi^*(\boldsymbol{\rho}_2)]\}$ is the mutual intensity of the random field in the receiver aperture. $\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ characterizes the loss of coherence of an initially coherent wave propagating in the turbulent medium [13–15]. Therefore, to achieve uncorrelated coefficients, the expansion needs to select the orthogonal functions $\psi_n(\boldsymbol{\rho})$ to be eigenfunctions of the integral equation (6), and the constants λ_n are the corresponding eigenvalues [16]. We refer to the as the KL modes.

The KL expansion is optimal in the sense that no other linear decomposition can better reproduce the random fields at the receiver with the same number of modes. The eigenvalue λ_n is a measure of the power contained in the respective eigenfunction $\psi_n(\boldsymbol{\rho})$. As a consequence, when the set of KL modes $\psi_1(\boldsymbol{\rho}), \psi_2(\boldsymbol{\rho}), \psi_3(\boldsymbol{\rho}), \ldots$ is arranged in descending order of the eigenvalues $\lambda_1,\ \lambda_2, \lambda_3, \ldots,$ a linear combination of any finite subset $\{\psi_n(\boldsymbol{\rho}),\ n = 1, \ldots, N\}$ can approximate the received optical field with the *minimum mean-square error* among all orthogonal mode sets. The minimum-mean-square-error approximation of the received field is a linear combination of the $\psi_n(\boldsymbol{\rho})$ with uncorrelated coefficients, so the KL modes represent a *decorrelating basis* for the received optical field. For this reason, the KL modes represent an *optimal basis set* for transmission, reception and MIMO signal processing in SMM FSO systems when the number of modes is constrained.

Assuming the turbulence is homogeneous, isotropic and described by a classical Kolmogorov model, the ensemble average $E\{\exp[\Phi(\boldsymbol{\rho}_1) + \Phi^*(\boldsymbol{\rho}_2)]\}$ yields a spatial $5/3$-exponent law [15].

This leads to a mutual intensity $\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = U_0(\boldsymbol{\rho}_1)\, U_0^*(\boldsymbol{\rho}_2)\, \exp(-|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^{5/3}/2{\delta_0}^{5/3})$, where the field coherence length $\delta_0 = r_0/6.88^{3/5}$ is proportional to the coherence diameter r_0 describing the spatial correlation length of the random optical field $\mathrm{U}(\boldsymbol{\rho})$ in the receiver plane [15]. We would like to use this mutual intensity in the KL expansion, but, as often happens in the study of propagation effects through turbulent media, the 5/3 exponent makes analytic approaches intractable. Hence, we can only use a numerical KL expansion of the 5/3-exponent function.

To overcome the analytical difficulties caused by the 5/3-exponent function, we consider a statistical model in which the received optical field distortion is regarded as a finite set of independent cells randomly arrayed in the receiver aperture. To a good approximation, we can consider the received field distortion to comprise $(\omega_0/\delta_0)^2$ independent cells, each of radius δ_0, inside of which the field distortions are highly correlated [17]. This approach has been used in many heuristic analyses of imaging through random inhomogeneous media, and explains correctly a variety of speckle phenomena caused by atmospheric turbulence (e.g., see [18]).

Inside any of the $(\omega_0/\delta_0)^2$ independent cells, the random field has a tilted but unwarped phase front, and its correlation can be described by a Gaussian-shaped elemental function of the form $\exp(-|\boldsymbol{\rho} - \boldsymbol{\nu}|^2/2{\delta_0}^2)$, where δ_0 is the correlation length and $\boldsymbol{\nu}$ denotes the cell's random center in the receiver aperture. Considering the Gaussian profile of the transmitted field, we can assume the centers of the cells are samples of a Gaussian distribution $\boldsymbol{\nu} \sim \mathcal{N}(0,\ {\omega_0}^2\, \boldsymbol{I}\,)$ whose variance scales with the aperture radius ω_0, where $\boldsymbol{I}$ is a 2×2 identity matrix. With the received optical field expanded as a set of independent cells, we can represent the mutual intensity as a linear superposition of as many as $(\omega_0/\delta_0)^2$ Gaussian-shaped elemental functions, leading to a representation

$$\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{1}{2\pi\ {\omega_0}^2} \int \exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\nu}|^2}{2{\delta_0}^2}\right) \exp\left(-\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\nu}|^2}{2{\delta_0}^2}\right) \times \exp\left(-\frac{|\boldsymbol{\nu}|^2}{2{\omega_0}^2}\right) d\boldsymbol{\nu}. \tag{7}$$

Performing the integration in (7), we express the mutual intensity as

$$\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left(-\frac{|\boldsymbol{\rho}_1|^2 + |\boldsymbol{\rho}_2|^2}{2\sigma_g^2}\right) \exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2}{2\sigma_s^2}\right), \tag{8}$$

with σ_g^2 and σ_s^2 described in terms of ω_0^2 and ${\delta_0}^2$ as $\sigma_g^2 = (\omega_0^2 + {\delta_0}^2)$ and $\sigma_s^2 = (\omega_0^2 + {\delta_0}^2){\delta_0}^2/\omega_0^2$.

The advantage of considering the mutual intensity (8) is that its KL expansion admits an analytical solution [12]. It can be verified by direct substitution in the integral Eq. (6) that the KL modes $\psi_n(\boldsymbol{\rho})$ are a complete set of LG modes

$$\psi_n(\boldsymbol{\rho}) = \frac{1}{\omega}\sqrt{\frac{2\ p!}{\pi\ (\ p + |l|\ )!}} \left(\sqrt{2}\ \frac{\rho}{\omega}\right)^{|l|} L_p^{|l|}\left(2\ \frac{\rho^2}{\omega^2}\right)\ \exp\left(-\ \frac{\rho^2}{\omega^2}\right)\ \exp(jl\phi)\ . \tag{9}$$

We use polar coordinates to represent position $\boldsymbol{\rho} = (\rho,\ \phi)$. The orbital angular momentum (OAM) number $l = 0,\ \pm 1, \pm 2, \ldots$ indicates the rate of azimuthal twist of the phase front and $p = 0,\ 1,\ 2, \ldots$ is the radial index. $L_p^{|l|}$ is the Laguerre polynomial. ω is the *optimized beam waist* of the mode with $l = 0, p = 0$, which is the fundamental Gaussian mode.

The most important attribute of the KL modes is the optimized beam waist ω. It depends on the reference beam waist ω_0 and the field coherence length δ_0 via

$$\frac{1}{\omega^2} = \frac{1}{\omega_0^2} + \frac{1}{2{\delta_0}^2} \frac{\sqrt{1 + 2\ (\omega_o/\delta_0)^2}}{1 + (\omega_o/\delta_0)^2}\ . \tag{10}$$

The dimensionless ratio ω_0/δ_0 quantifies the loss of coherence of an initially coherent wave caused by turbulence, so we refer to it as a *turbulence strength parameter*. Figure 1(a) shows how the optimized beam waist ω varies with the turbulence strength parameter ω_0/δ_0. In the limit of weak turbulence ($\omega_0/\delta_0 \to 0$), $\omega \approx \omega_0$, while in the limit of strong turbulence ($\omega_0/\delta_0 \to \infty$), the optimized beam waist scales as $\omega \approx \left(\sqrt{2}\omega_0\delta_0\right)^{1/2}$.

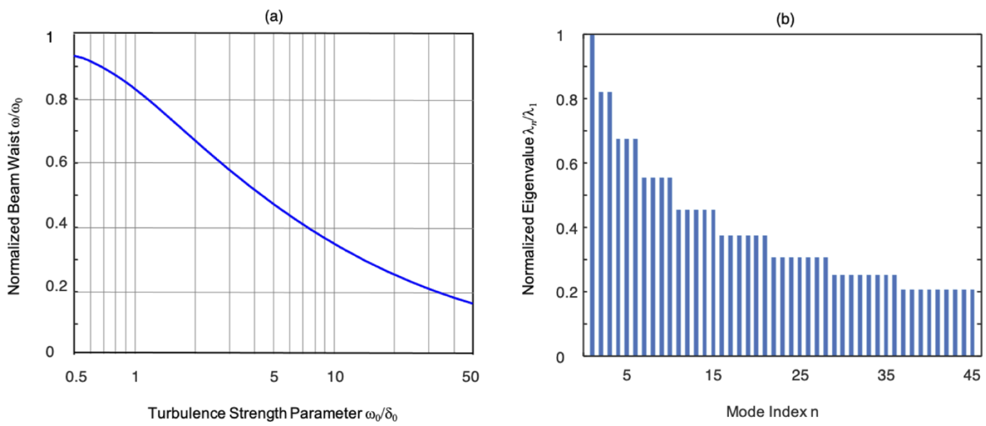

Fig. 1. (a) Normalized beam waist ω/ω_0 of KL modes as a function of turbulence strength parameter ω_0/δ. (b) Distribution of eigenvalues λ_n over the first 45 KL modes for a turbulence strength parameter $\omega_0/\delta_0 = 10$.

In the derivation of the KL modes above, the reference beam waist ω_0 imposes an upper bound on the optimized beam waist ω. In strong turbulence, if we want to minimize power coupling losses due to field mismatches, the spatial extent of the KL modes should be of the same order as the turbulence cell size at the receiver. Consequently, one might conclude that the optimized beam waist ω should depend only on the field coherence length δ_0 and not on the reference beam waist ω_0. There is, however, a nonzero probability that at a particular instant of time, the fields in two or more cells at the receiver aperture are correlated; in that case, allowing the KL modes to have a larger size to include several field cells would increase the average power coupling. As the size of the modes is ultimately limited by the reference beam waist ω_0, the optimized bream waist ω should also depend on the reference beam waist ω_0. This helps explain the dependence of the optimized beam waist in the strong-turbulence regime, $\omega \approx \left(\sqrt{2}\omega_0\delta_0\right)^{1/2}$.

In (9), n is a mode ordering index, which is a function of l and p. The KL modes, ψ_n, are ordered such that increasing mode order $n = 1, 2, \ldots$, corresponds to decreasing eigenvalue λ_n. The eigenvalues are given by

$$\lambda_n = \frac{2}{\xi + \sqrt{2\xi - 1}} \left(\frac{\xi - \sqrt{2\xi - 1}}{\xi + \sqrt{2\xi - 1}} \right)^{2p+|l|}, \tag{11}$$

where $\xi = 1 + (\omega_o/\delta_0)^2$. . For a given $2p + |l|$, modes with a lower value of l are enumerated first. The lowest-order mode ψ_1 is the fundamental Gaussian mode with $l = 0$, $p = 0$. As an example, Fig. 1(b) shows how the eigenvalues λ_n are distributed for a turbulence strength parameter $\omega_0/\delta_0 = 10$.

3. Spatial-mode multiplexing in turbulence

In this section, we evaluate the performance of SMM FSO systems on turbulence channels using various mode sets, illustrating the superior performance of the KL modes in quasi-realistic transmission scenarios. Our assumptions and analysis are similar to [6], with the major difference

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that we consider the impact of atmospheric turbulence. We assume an optical system comprising circular transmitter and receiver apertures, each containing a single thin positive lens and aligned along a common central axis. We assume similar aperture sizes at the transmitter and receiver. We consider SMM using a set of N mutually orthogonal modes, which propagate along the common central axis through an atmospheric channel. We assume that practical considerations constrain the product of transmit and receive Fresnel numbers N_F. For LG multiplexing, the number of modes $N_{LG} = (1/2)M(M+1)$ is related to the number of mode groups M by counting all permutations of p and l that satisfy $2p + |l| + 1 \leq M$. For OAM multiplexing, the same constraint onresnel number product N_F constrains M to the same value as for LG multiplexing and counting the LG modes with $p = 0$, the number of modes is estimated as $N_{OAM} = 2M - 1$.

In near-field links for SMM FSO transmission, where the beam waist does not change significantly from the transmitter to the receiver, the propagation range L lies within the Rayleigh range $z_0 = \pi\omega_0{}^2/\lambda$. For instance, at a wavelength $\lambda = 1550$ nm, two circular apertures of diameter $D = 40$ cm efficiently couple $N_F = 45$ modes out to $L \approx 12$ km, which is within the Rayleigh range of a beam with $\omega_0 = 8$ cm. A truncation parameter $\tau = 2.5$ describes the ratio of the aperture diameter D to the beam waist diameter $2\omega_0$.

We assume that $N_{LG} \leq N_F$, so that on average, even in the presence of turbulence at the levels we consider, the received beam size is smaller than the receiver aperture, avoiding beam clipping. In the presence of turbulence, the smaller optimized beam waist ω reduces the effective Rayleigh range and, consequently, the maximum propagation distance L by a factor $(\omega/\omega_0)^2$. For the previous example, under strong turbulence conditions described by $\omega_0/\delta_0 \approx 5$ and $(\omega/\omega_0)^2 \approx 0.25$ (see Fig. 1(a)), the apertures will support $N_{LG} = 45$ LG modes out to a maximum propagation range of 3 km.

We assume that modes are multiplexed at the transmitter and demultiplexed at the receiver without hardware-induced loss or crosstalk. Fundamentally lossless multi-plane converters or mode-selective photonic lanterns [19,20] can perform fixed transformations at the transmitter and receiver between separate single-mode waveguides and coaxial modes in a free-space beam. These (de)multiplexing devices were developed for SMM in optical fiber [21], and can be leveraged to help make SMM FSO systems practical. It is worth noting that most SMM FSO experiments to date (see e.g. [22–24]) have used only OAM modes, the subset of the LG modes with radial order $p = 0$.

We define a turbulence-free signal-to-noise ratio (SNR) $\gamma = P/\sigma^2$, where P is the total transmitted power in all modes and σ^2 is the receiver noise power per mode. We assume received signals are detected coherently and the dominant noise source is local oscillator shot noise, so γ equals the number of received signal photons. As we are considering near-field links, where the receiver collects virtually all the signal power leaving the transmitter, γ also equals the number of transmitted signal photons.

We quantify SMM FSO system performance using information-theoretic measures that are standard for MIMO systems over random fading channels. We consider the ergodic average *spectral efficiency* S_N, which is the ergodic average Shannon capacity per unit frequency bandwidth [25,26], and the *effective degrees of freedom* (EDOF), which represents the number of spatial modes that are effectively conveying information [27]. For a single-mode system, the SE (in bit/s/Hz) is given by $\mathrm{SE} = \log_2(1+\gamma)$ and a 2^δ-fold increase in transmit power increases the spectral efficiency by $\log_2(2^\delta) = \delta$ bit/s/Hz per polarization. In a system using EDOF independent modes in parallel, the spectral efficiency should increase by EDOF $\times\,\delta$ bit/s/Hz. Hence, the EDOF per polarization at SNR γ is given by [27]

$$\mathrm{EDOF} = (d/d\delta)\ \mathrm{SE}(2^\delta\gamma)|_{\delta=0}\ . \tag{12}$$

In our analysis, given transmit and receive bases of N_{LG} spatial modes, the ergodic average spectral efficiency per polarization is $\mathrm{SE} = \max_{\mathbf{Q}}\mathrm{SE}(\mathbf{Q})$ where $\mathrm{SE}(\mathbf{Q}) = E_{\boldsymbol{H}}\{\log_2|\boldsymbol{I} + \gamma\,\boldsymbol{H}\,\mathbf{Q}\,\boldsymbol{H}^*|\,\}$

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is the mutual information with the channel matrix $\boldsymbol{H}$ and the input matrix $\mathbf{Q}$ [28,29]. The expectation $E_{\boldsymbol{H}}\{\ \}$ is an ensemble average over realizations of the channel matrix $\boldsymbol{H}$, $\boldsymbol{I}$ is an $N_{LG} \times N_{LG}$ identity matrix, and $|\ |$ represents a matrix determinant. Here, $\boldsymbol{H}$ is an $N_{LG} \times N_{LG}$ matrix characterizing the atmospheric channel, with H_{ij} representing the complex coupling coefficient between transmit mode j and receive mode i. Also, $\mathbf{Q} = E[\boldsymbol{q}\,\boldsymbol{q}^*]$ is an $N_{LG} \times N_{LG}$ unit-trace diagonal matrix with Q_{jj} representing the fraction of the total signal power transmitted in the jth mode. The matrix $\mathbf{Q}$ is chosen to describe the SMM transmission strategy, and the spectral efficiency optimization problem involves finding the optimum $\mathbf{Q}$ to maximize SE($\mathbf{Q}$). The optimal transmission strategy $\mathbf{Q}$ depends on the amount of knowledge about the channel matrix $\boldsymbol{H}$ that is available at the transmitter [29–31]. Various transmission strategies are considered below.

First, we consider using a transmit and receive basis of N_{LG} instantaneous eigenmodes of the atmospheric channel. We assume the channel changes slowly enough that the receiver can estimate the channel matrix $\boldsymbol{H}$ using conventional MIMO signal processing techniques and feed back accurate information about $\boldsymbol{H}$ to the transmitter without significant delay. Then, using the eigende composition of the $N_{LG} \times N_{LG}$ channel matrix $\boldsymbol{H}$, the instantaneous channel realization is converted into N_{LG} parallel, non-interfering single-mode channels, or instantaneous eigenmodes [29]. In that case, the optimal transmit strategy takes the form of a water-filling over the instantaneous eigenmodes [26]. The algorithm can be understood using the analogy of pouring water (power) into a vessel of variable depth (eigenvalues). Instantaneous eigenmodes that have larger eigenvalues receive more power and the power allocated to some of the eigenmodes may be zero.

Second, we consider using a transmit and receive basis of N_{LG} KL modes (9). We assume that the beam waist of the transmit modes is adapted appropriately to the correlation statistics of the turbulence channel. This approach is of practical interest when the channel varies too rapidly for the transmitter to have perfect channel information, but the statistical properties of the atmospheric channel change sufficiently slowly for the transmitter to have an accurate knowledge of the channel correlation statistics. With this statistical knowledge of the channel, the optimum transmission strategy takes the form of a water-filling over the eigenvalues λ_n (11)**.** of the KL modes [28,30]. As a complement to the above, under identical assumptions and analysis, we also consider a transmit and receive basis of the N_{OAM} OAM modes that are the subset of the N_{LG} KL modes that have radial order $p = 0$.

Finally, we consider using a transmit and receive basis of N_{LG} conventional LG modes with fixed beam waist ω_0, even though some of these modes may not propagate efficiently through the atmosphere. This approach is the only one possible when the transmitter has no knowledge of the atmospheric channel. In that case, by analogy with results from standard MIMO systems [29,31], the transmission strategy with power distributed equally among all transmit LG modes is optimum. As with KL modes, we also consider a transmit and receive basis that only uses the N_{OAM} fixed OAM modes that are the subset of the fixed LG modes modes with radial order $p = 0$.

We use Monte Carlo simulations to assess the performance obtained using different SMM FSO transmit and receive modal bases. We approximate the field disturbance caused by turbulence using a phase screen model. Phase screens are the result of integrating atmospheric turbulence along the propagation line of sight. The phase-screen turbulence model is valid for the near-field links considered in this analysis, in which the phase aberration $\phi(\boldsymbol{\rho})$. is the most significant turbulence distortion, and power scintillation is small. Intuitively, in the presence of turbulence, which can refract and diffract light, intense scintillation occurs when a beam passing through random phase distortions interferes with a spatially shifted version of itself. In practice, this means that scintillation is most pronounced when a beam propagates over a long atmospheric path, whereas in typical near-field links, which are shorter, scintillation is weaker. We use an algorithm that simulates atmospherically distorted phase wavefronts $\phi(\boldsymbol{\rho})$ using a Zernike expansion [32]. Because the wavefront distortion in near-field links is dominated by phase fluctuations rather than amplitude fluctuations, a full-field numerical simulation of laser beam propagation including both refraction and diffraction effects would not significantly alter our findings. In our simulations, we use at least 600 Zernike polynomials for an accurate statistical generation of each phase wavefront. We introduce the influence of turbulence $\exp[\phi(\boldsymbol{\rho})]$ in the transmitting modes and consider an overlap integral to estimate the complex coupling coefficients between pairs of transmit and receive modes characterizing the random channel matrix $\boldsymbol{H}$. In the Monte Carlo experiments, we generate 10^4 instances of the channel matrix $\boldsymbol{H}$ for the different transmit and receive basis sets and use them to collect the statistics of SE and determine the EDOF.

Figure 2 summarizes the results of these studies. The figure considers five transmission strategies. The ideal transmission strategy (solid black line) uses instantaneous eigenmodes,

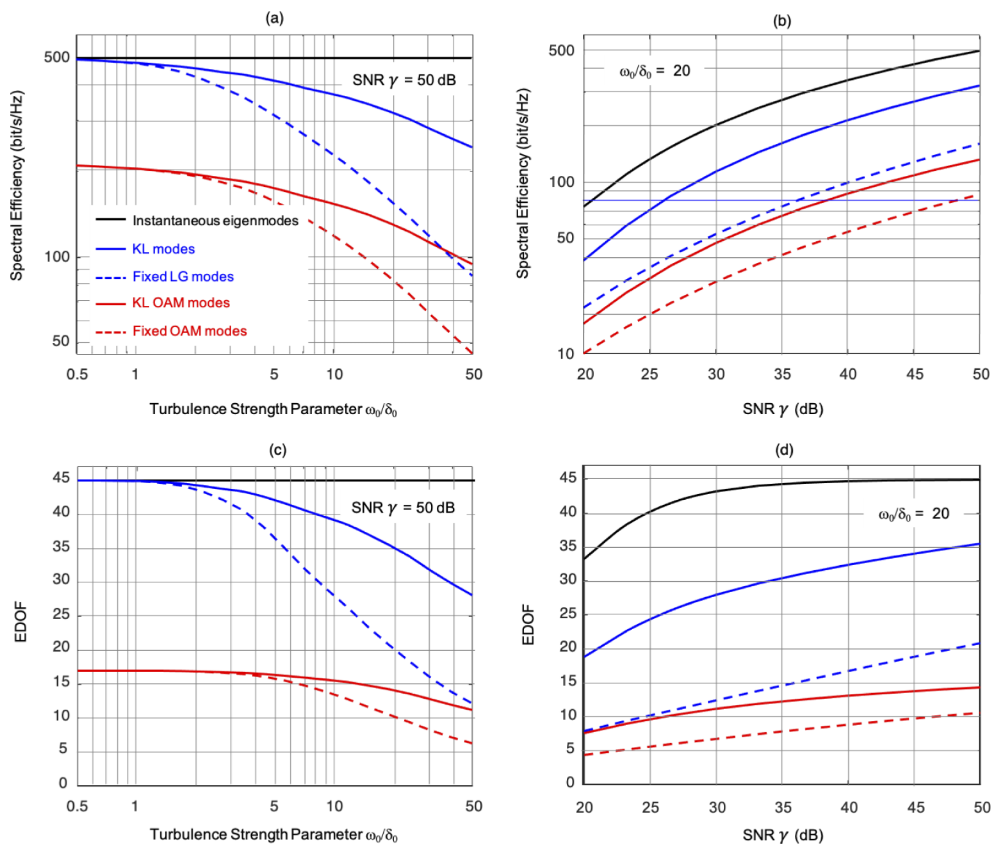

Fig. 2. Spectral efficiencies and effective degrees of freedom (EDOF) for different transmission strategies with $M = 9$ mode groups, corresponding to $N_{LG} = 45$ and $N_{OAM} = 17$ modes for SMM and OAM, respectively. (a) Spectral efficiency vs. turbulence strength parameter ω_0/δ_0 for fixed SNR $\gamma = 50$ dB. (b) Spectral efficiency vs. SNR γ for fixed turbulence strength parameter $\omega_0/\delta_0 = 20$. (c) EDOF vs. turbulence strength for fixed SNR $\gamma = 50$ dB. (d) EDOF vs. SNR γ for fixed turbulence strength parameter $\omega_0/\delta_0 = 20$. Transmission strategies considering instantaneous eigenmodes and KL modes offer much higher spectral efficiencies and EDOF than those using fixed LG modes or OAM modes under moderate-to-strong turbulence, as seen in (a) and (c). At low SNR, the efficiencies of all the transmission strategies are limited by the available transmit power, as in (b), while at high SNR, the EDOF for instantaneous eigenmodes and KL modes approaches $N_{LG} = 45$, as seen in (d).

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requiring knowledge of the instantaneous channel realization. The KL transmission strategy (solid blue line) uses KL modes, requiring only knowledge of the channel statistics to optimize the beam waist ω. The KL OAM transmission strateg(solid red line) uses the zero radial-order subset of the KL modes, and also requires knowledge of the channel statistics. The fixed LG mode transmission strategy (dashed blue line) uses fixed-waist LG modes, and requires no channel knowledge. The fixed OAM transmission strategy (dashed red line) uses fixed-waist OAM modes, as in most SMM FSO systems to date, and also requires no channel knowledge.

Figure 2 assumes $N_{LG} = 45$ modes ($M = 9$ mode groups) and assumes that the product of the transmitter and receiver Fresnel numbers is larger than the number of modes used, i.e., $N_F > N_{LG}$. For OAM multiplexing, the addressable number of LG modes with $p = 0$ is $N_{OAM} = 17$.

Figures 2(a) and 2(c) show the spectral efficiencies and EDOF for the five transmission strategies for fixed system SNR $\gamma = 50$ dB as a function of the turbulence strength parameter ω_0/δ_0. . Figures 2(b) and 2(d) show the spectral efficiencies and EDOF for the five transmission strategies for a strong turbulence $\omega_0/\delta_0 = 20$ as a function of the system SNR.

The plots in Figs. 2(a) and 2(c) show that the KL modes represent a practical transmission strategy that, under most turbulence strength scenarios, significantly outperforms fixed LG modes, yielding spectral efficiencies and EDOFs close to those expected for the ideal transmission strategy based on instantaneous eigenmodes. Under strong turbulence conditions (large ω_0/δ_0), the spectral efficiencies and EDOFs for instantaneous eigenmodes and KL modes are similarly high and are noticeably higher than those obtained for fixed LG modes. Only in the limit of very weak turbulence (small ω_0/δ_0.) are the spectral efficiencies and EDOFs for fixed LG modes and KL modes comparable. Not surprisingly, using only the OAM subset of the LG modes decreases the spectral efficiency and EDOF.

The plots in Figs. 2(b) and 2(d) show that at low SNR, all the schemes are limited by the transmit power available rather than by turbulence conditions. At very low SNR, they offer similar spectral efficiencies and EDOFs. At high SNR, the EDOFs for instantaneous eigenmodes and KL modes approach the number of independent modes N_{LG} employed. At high SNR, the EDOF for KL OAM multiplexing also coincides with the number of independent modes N_{OAM}.

4. Conclusion

We make the following conclusions based on the results presented above. First, there are optimal modes for SMM transmission through atmospheric turbulence under the assumption that the transmitter has knowledge of the channel statistics but nothe instantaneous channel realization. These modes are derived by a KL expansion of the signal field in the receiver plane. The KL modes are a set of LG modes in which the beam waist is chosen based on the transverse correlation length of the signal field in the receiver plane. Second, the KL modes are a promising basis set for SMM FSO systems, where the choice of modal basis is crucial for maximizing the number of multiplexed information streams that can be transmitted through atmospheric turbulence. The ordering of the eigenvalues of the KL modes is critical, and allows transmit power to be allocated optimally to any number of the strongest modes. Third, we have shown that the KL modes outperform other mode sets, in particular, any LG set not chosen to match the turbulence statistics. KL modes achieve spectralfficiencies and EDOFs comparable to those achieved by the ideal instantaneous eigenmodes, while avoiding the need for instantaneous knowledge of the turbulence channel. Fourth, although most prior work on SMM FSO communication has used the OAM subset of the LG modes, our analysis shows that this approach is far from optimal. The fixed OAM modes yield the lowest spectral efficiencies and EDOFs among the mode sets considered.

A distinct advantage of the proposed approach to SMM FSO transmission is the simplicity of tuning the KL modes to the regime of turbulence encountered, i.e., the shorter the field coherence length δ_0, the smaller the KL mode beam waist ω. This tuning requires measurements

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to estimate the field transverse correlation length r_0 in the receiver plane, enabling computation of δ_0, followed by adjustment of the beam waist ω of the transmitted and received modes using variable-magnification optics at the transmitter and receiver.

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